Finite Volume and Partial Quenching for Masses and Decay Constants in Meson ChPT

Thomas Rössler

Department of Theoretical Physics, Lund University

talk based on
arXiv:hep-lat/1411.6384 (JHEP 1501 (2015) 034)
arXiv:hep-lat/1507.???? work in progress
with Johan „Hans“ Bijnens

OUTLINE

Finite Volume (FV) for Masses and Decay Constants

Masses and Decay Constants in ChPT at Two-Loop
Finite Volume
Technicalities: Integral Classification and Reduction
Sunset diagram
Numerical input, numerical examples

FV for Partially Quenched (PQ) ChPT

PQ in a Nutshell: Group structure, dynamical fields, Lagrangian
Neutral propagators, Double poles, Residues
Numerical examples

CHIRON

All quantities (both unquenched and PQ) now publicly available, both analytically and to direct numerical evaluation: the CHIRON package

OUTLINE

Finite Volume (FV) for Masses and Decay Constants

Masses and Decay Constants in ChPT at Two-Loop
Finite Volume
Technicalities: Integral Classification and Reduction
Sunset diagram
Numerical input,

FV for

PQ in a Nutshe
Lagrangian
Neutral propag
Numerical exar

Motivation: Lattice

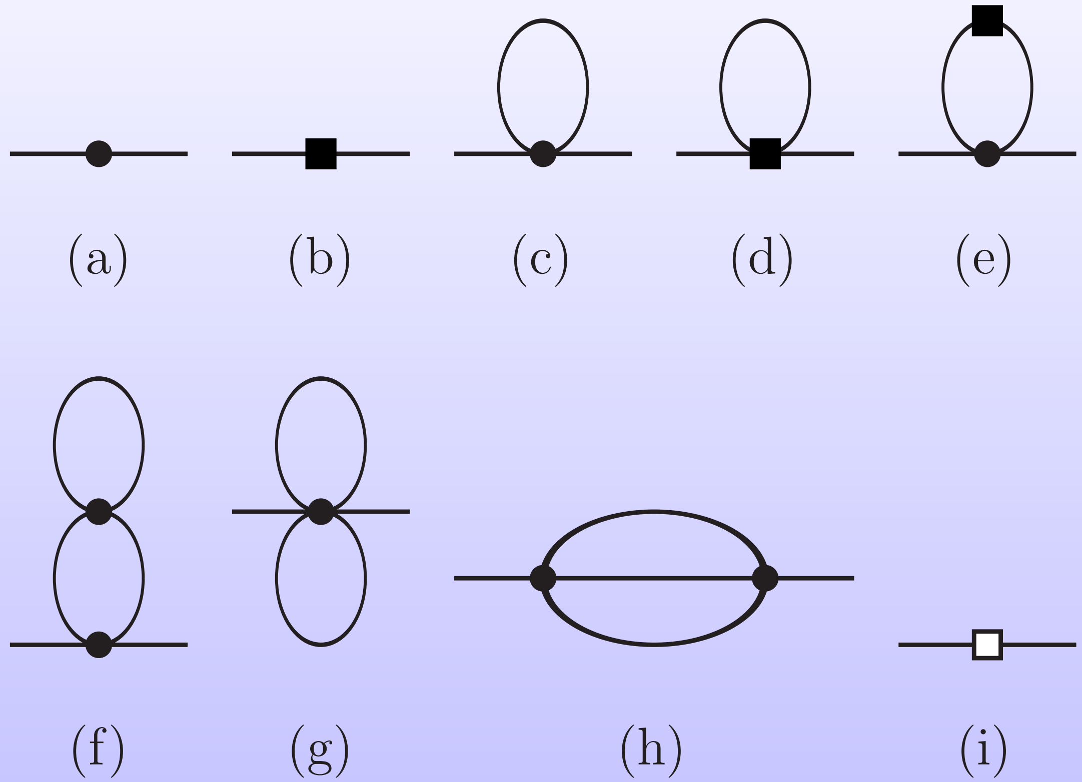
Improve Lattice extrapolations, sanity checks, determination of LECs for ChPT, …

CHIRON

All quantities (both unquenched and PQ) now publicly available, both analytically and to direct numerical evaluation: the CHIRON package

Two-loop masses and decay constants

- Ab-initio calculation using FORM
- Integral classification and reduction to a minimal set (see more later)



- Hair in the soup: Sunset!
- Framework here: Standard ChPT up to order p6, two and three flavour cases treated separately

Regularization/Renormalization: ChPT version of MSbar

loops $$A(m^2) = \frac{m^2}{16\pi^2}\left\{\lambda_0 - \ln(m^2) + \mathcal{O}(\epsilon)\right\} \qquad \lambda_0 = \frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \ln(4\pi) + 1 - \gamma_E$$

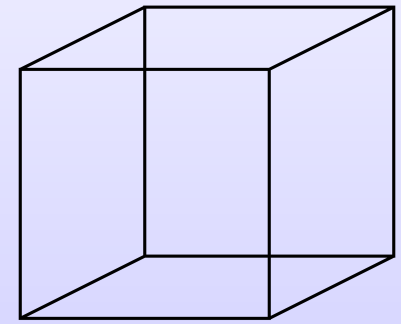
LECs $$L_i \equiv (\mu c)^{-2\epsilon}\left(\frac{-1}{32\pi^2\epsilon}\Gamma_i + L_i^r(\mu)\right) = (\mu)^{-2\epsilon}\left(\frac{-1}{32\pi^2}\Gamma_i\lambda_0 + L_i^r(\mu) + \mathcal{O}(\epsilon)\right)$$

$$C_i \equiv (\mu c)^{-4\epsilon}\left(\frac{\gamma_{2i}}{\epsilon^2} + \frac{\gamma_{1i}}{\epsilon} + C_i^r(\mu)\right) = \mu^{-4\epsilon}\left(\gamma_{2i}\lambda_2 + \gamma_{1i}\lambda_1 + C_i^r(\mu) + \mathcal{O}(\epsilon)\right)$$

Finite Volume

$$\int \frac{dp}{2\pi} F(p) \to \frac{1}{L} \sum_{n \in \mathbf{Z}} F(p_n) \equiv \int_V \frac{dp}{2\pi} F(p)$$

$$= \sum_{l_p} \int \frac{dp}{2\pi} e^{i l_p p} F(p)$$ Poisson



Improve Lattice extrapolations, ChPT fits, …

split $I = I^\infty + I^V$ appears then naturally
eliminate either integral (Bessel) or sum (Jacobi): diff. convergence behaviour
periodic BC, time kept continuous: break Lorentz

Here: Infinite volume masses serve as numerical input!

$$\Big\{ -\Sigma_4(M_0^2 \to M^2_{phys,IV}; p^2 \to M^2_{phys,IV}) \qquad \Big\}\mathcal{O}(p^4)$$

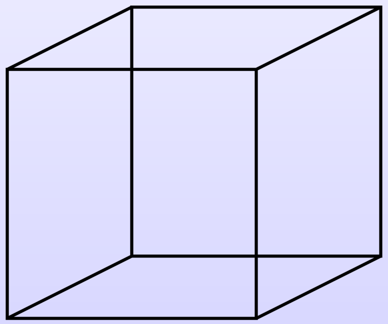
$$\left.\begin{array}{l} -\Sigma_4(M_0^2; p^2 = M_4^2 = M_0^2 + \Delta m^2_{4,full}) \\ +\Sigma_4(M_0^2 \to M^2_{phys,IV} = M_0^2 + \Delta m^2_{4,IV}; p^2 \to M^2_{phys,IV} = M_0^2 + \Delta m^2_{4,IV}) \end{array}\right\}\mathcal{O}(p^6)$$

$$\left. -\Sigma_6(M_0^2; p^2 = M_0^2) \Big\} \right|_{M_0^2 \to M^2_{phys,IV}} \qquad \Big\}\mathcal{O}(p^6)$$

Finite Volume

$$\int \frac{dp}{2\pi} F(p) \to \frac{1}{L} \sum_{n\in \mathbf{Z}} F(p_n) \equiv \int_V \frac{dp}{2\pi} F(p)$$

$$= \sum_{l_p} \int \frac{dp}{2\pi} e^{il_p p} F(p)$$ Poisson



Improve Lattice extrapolations, ChPT fits, …

split $I = I^\infty + I^V$ appears then naturally

eliminate either integral (Bessel) or sum (Jacobi): diff. convergence behaviour

periodic BC, time kept continuous: break Lorentz

Here: Infinite volume masses serve as numerical input!

$$\Big\{ -\Sigma_4(M_0^2 \to M^2_{phys,IV}; p^2 \to M^2_{phys,IV}) \qquad \Big\} \mathcal{O}(p^4)$$

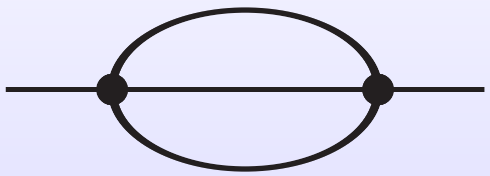
$$-\Sigma_4(M_0^2; p^2 = M_4^2 = M_0^2 + \Delta m^2_{4,full})$$

$$+\Sigma_4(M_0^2 \to M^2_{phys,IV} = M_0^2 + \Delta m^2_{4,IV}; p^2 \to M^2_{phys,IV} = M_0^2 + \Delta m^2_{4,IV}) \qquad \Big\} \mathcal{O}(p^6)$$

$$-\Sigma_6(M_0^2; p^2 = M_0^2)\Big\}\Big|_{M_0^2 \to M^2_{phys,IV}} \qquad \Big\} \mathcal{O}(p^6)$$

Integrals: The Devil is in the Sunset!

$t_\mu \equiv (1,0,0,0) \qquad t_{\mu\nu} \equiv \delta_{\mu\nu} - t_\mu t_\nu = \mathrm{diag}(0,1,1,1)$



Note: Now Euclidean (!)

$$\lfloor X \rfloor = \int_V \frac{d^d r}{(2\pi)^d} \frac{X}{(r^2+m^2)^n} \qquad \langle X \rangle = \int_V \frac{d^d r}{(2\pi)^d} \frac{X}{(r^2+m_1^2)^{n_1}((r-p)^2+m_2^2)^{n_2}} \qquad \text{(h)}$$

$$\langle\langle X \rangle\rangle \equiv \int_V \frac{d^d r}{(2\pi)^d} \frac{d^d s}{(2\pi)^d} \frac{X}{(r^2+m_1^2)^{n_1}(s^2+m_2^2)^{n_2}((r+s-p)^2+m_3^2)^{n_3}}$$

mom. indep. integrals, e.g.:

$$\lfloor 1 \rfloor^V = A^V,$$
$$\lfloor r_\mu \rfloor^V = 0,$$
$$\lfloor r_\mu r_\nu \rfloor^V = \delta_{\mu\nu} A_{22}^V + t_{\mu\nu} A_{23}^V$$
$$\lfloor r_\mu r_\nu r_\alpha \rfloor^V = 0.$$

sunsets:

$$\begin{aligned}
\langle\langle 1 \rangle\rangle^V &\equiv H^V, \\
\langle\langle r_\mu \rangle\rangle^V &\equiv H_1^V\, p_\mu + H_{3\mu}^V, \\
\langle\langle s_\mu \rangle\rangle^V &\equiv H_2^V\, p_\mu + H_{4\mu}^V, \\
\langle\langle r_\mu r_\nu \rangle\rangle^V &\equiv H_{21}^V\, p_\mu p_\nu + H_{22}^V\, \delta_{\mu\nu} + H_{27\mu\nu}^V, \\
\langle\langle r_\mu s_\nu \rangle\rangle^V &\equiv H_{23}^V\, p_\mu p_\nu + H_{24}^V\, \delta_{\mu\nu} + H_{28\mu\nu}^V, \\
\langle\langle s_\mu s_\nu \rangle\rangle^V &\equiv H_{25}^V\, p_\mu p_\nu + H_{26}^V\, \delta_{\mu\nu} + H_{29\mu\nu}^V,
\end{aligned}$$

cms →

$$\begin{aligned}
\langle\langle 1 \rangle\rangle^V &\equiv H^V, \\
\langle\langle r_\mu \rangle\rangle^V &\equiv H_1^V\, p_\mu, \\
\langle\langle s_\mu \rangle\rangle^V &\equiv H_2^V\, p_\mu, \\
\langle\langle r_\mu r_\nu \rangle\rangle^V &\equiv H_{21}^V\, p_\mu p_\nu + H_{22}^V\, \delta_{\mu\nu} + H_{27}^V\, t_{\mu\nu}, \\
\langle\langle r_\mu s_\nu \rangle\rangle^V &\equiv H_{23}^V\, p_\mu p_\nu + H_{24}^V\, \delta_{\mu\nu} + H_{28}^V\, t_{\mu\nu}, \\
\langle\langle s_\mu s_\nu \rangle\rangle^V &\equiv H_{24}^V\, p_\mu p_\nu + H_{25}^V\, \delta_{\mu\nu} + H_{29}^V\, t_{\mu\nu}.
\end{aligned}$$

each integral expansion of type $A(m^2) = \lambda_0 \frac{m^2}{16\pi^2} + \overline{A}(m^2) + A^V(m^2) + \epsilon\left(A^\epsilon(m^2) + A^{V\epsilon}(m^2)\right) + \cdots$

Simplification

integrals with diff. kind of numerator structures related
other symmetry identities (e.g. masses of the sunsets)

$$\langle\langle X \rangle\rangle \equiv \int_V \frac{d^d r}{(2\pi)^d} \frac{d^d s}{(2\pi)^d} \frac{X}{(r^2+m_1^2)^{n_1}(s^2+m_2^2)^{n_2}((r+s-p)^2+m_3^2)^{n_3}}$$

permutation symmetries, e.g.

$$H_1(m_1^2, m_2^2, m_3^2; p^2) + H_1(m_2^2, m_1^2, m_3^2; p^2) + H_1(m_3^2, m_1^2, m_2^2; p^2) = H(m_1^2, m_2^2, m_3^2; p^2)$$

Passarino-Veltman for FV:

$$p^2 H_{21} + d H_{22} + 3 H_{27} + m_1^2 H = A(m_2^2) A(m_3^2)$$

$$d A_{22}(m^2) + 3 A_{23}(m^2) + m^2 A(m^2) = 0$$

eliminate 22-integrals as in IV computation

Sunsets continued: Sanity checks

FV sunsets have residue in ε: must cancel one-loop „IV*FV" terms

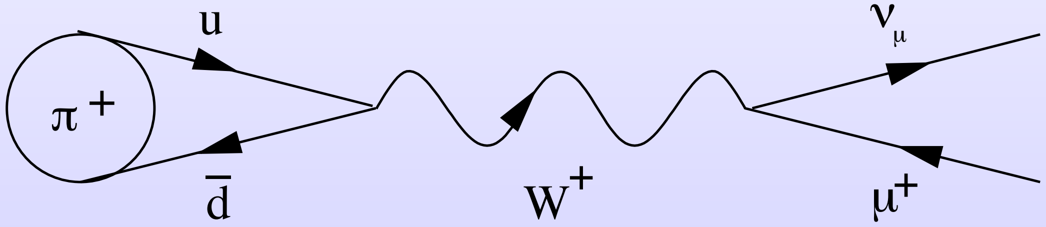
Also, finite part for FV sunsets defined differently than in the IV splits

$$\begin{aligned}
\tilde{H}^V &= \frac{\lambda_0}{16\pi^2}\left(A^V(m_1^2)+A^V(m_2^2)+A^V(m_3^2)\right)+\frac{1}{16\pi^2}\left(A^{V\epsilon}(m_1^2)+A^{V\epsilon}(m_2^2)+A^{V\epsilon}(m_3^2)\right)\\
&\quad +H^V\,,\\
\tilde{H}_1^V &= \frac{\lambda_0}{16\pi^2}\frac{1}{2}\left(A^V(m_2^2)+A^V(m_3^2)\right)+\frac{1}{16\pi^2}\frac{1}{2}\left(A^{V\epsilon}(m_2^2)+A^{V\epsilon}(m_3^2)\right)+H_1^V\,,\\
\tilde{H}_{21}^V &= \frac{\lambda_0}{16\pi^2}\frac{1}{3}\left(A^V(m_2^2)+A^V(m_3^2)\right)+\frac{1}{16\pi^2}\frac{1}{3}\left(A^{V\epsilon}(m_2^2)+A^{V\epsilon}(m_3^2)\right)+H_{21}^V\,,\\
\tilde{H}_{27}^V &= \frac{\lambda_0}{16\pi^2}\left(A_{23}^V(m_1^2)+\frac{1}{3}A_{23}(m_2^2))+\frac{1}{3}A_{23}^V(m_3^2)\right)\\
&\quad +\frac{1}{16\pi^2}\left(A_{23}^{V\epsilon}(m_1^2)+\frac{1}{3}A_{23}^{V\epsilon}(m_2^2+\frac{1}{3}A_{23}^{V\epsilon}(m_3^2))\right)+H_{27}^V\,, \qquad (9)
\end{aligned}$$

This ensures cancellation of one-loop ε-terms: Sanity check!

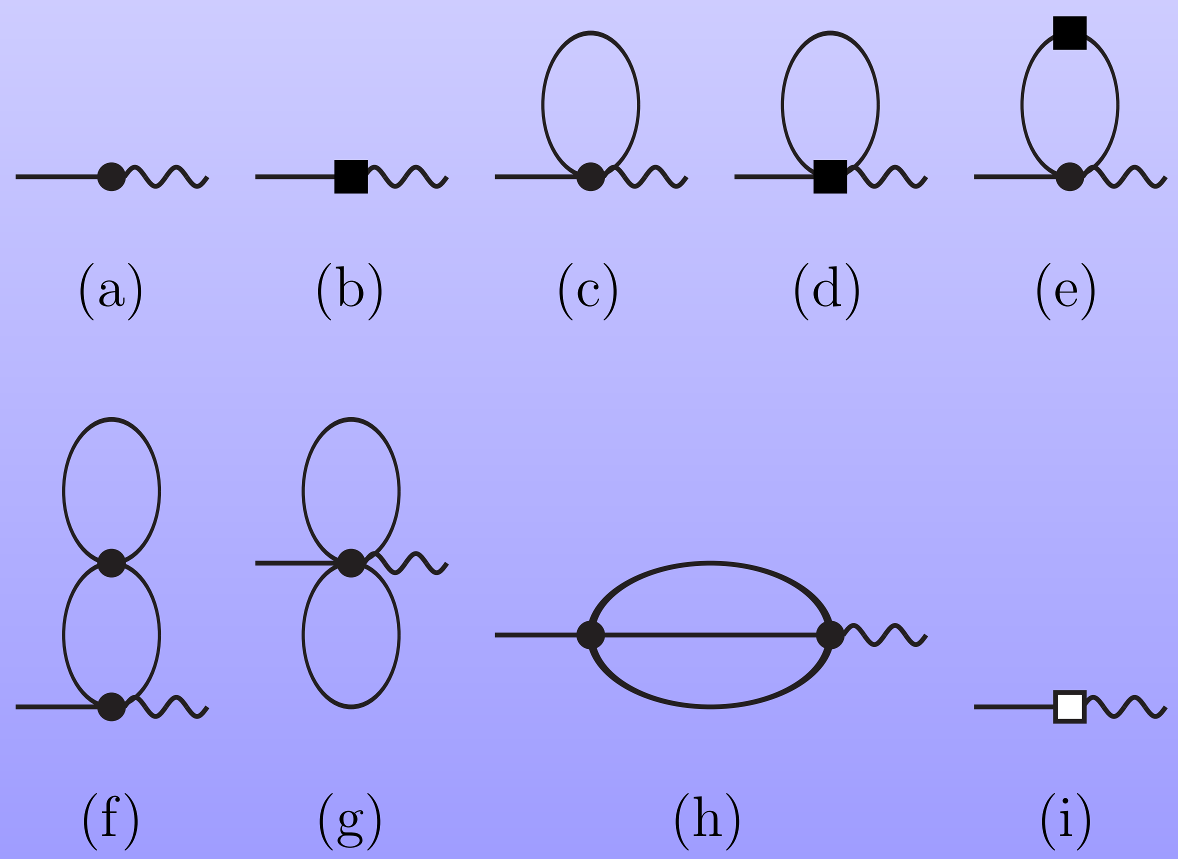
$$A(m^2)=\lambda_0\frac{m^2}{16\pi^2}+\overline{A}(m^2)+A^V(m^2)+\epsilon\left(A^\epsilon(m^2)+A^{V\epsilon}(m^2)\right)+\cdots$$

Decay Constant in a Nutshell



$$\Gamma^{(0)}(\pi \to \ell\nu) = \frac{G_F^2 |V_{ud}|^2 F_\pi^2}{4\pi} m_\pi m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

Def. $\langle 0 | A_\mu(0) | \pi^-(p) \rangle = i\sqrt{2} p_\mu F_\pi \, ;$ $\qquad A_\mu = \bar{u}\gamma_\mu\gamma_5 d$



Wavefunction renormalization (LSZ):

$${}_{out}\langle \phi_1 ... \phi_i | \phi_i ... \phi_n \rangle_{in} = \langle \phi_1 ... \phi_n \rangle = Z^{-\frac{n}{2}} G_{trunc}(\phi_1, ..., \phi_n)$$

$$\phi' = \sqrt{Z}\phi \qquad Z = 1 + \frac{d\Sigma}{dp^2}\Big|_{p^2 = M_{phys}^2}$$

$$\langle \phi\phi \rangle \simeq \frac{i}{Z(p^2 - M_{phys})^2} + \text{non-pole terms}$$

$$\langle \phi'\phi' \rangle \simeq \frac{i}{(p^2 - M_{phys})^2} + \text{non-pole terms,}$$

$$\langle \phi a^\mu \rangle \simeq \frac{i}{Z(p^2 - M_{phys})^2} i\Pi + \text{non-pole terms}$$

$$\langle \phi' a^\mu \rangle \simeq \frac{i}{\sqrt{Z}(p^2 - M_{phys}^2)} i\Pi + \text{non-pole terms}$$

Numerics: Input

SU(2) | **SU(3)**

$$F_\pi = 92.2 \text{ MeV}, \; m_\pi = m_{\pi^0} = 134.9764 \text{ MeV}$$

$$\mu = 770 \text{ MeV}$$

SU(2):

$$\bar{l}_1 = -0.4, \bar{l}_2 = 4.3, \bar{l}_3 = 3.0, \bar{l}_4 = 4.3$$

SU(3):

$$m_K = 494.53 \text{ MeV}$$

$$m_\eta = 547.30 \text{ MeV}$$

BE14

Numerics: Input

SU(2)

SU(3)

$$F_\pi = 92.2 \text{ MeV},\ m_\pi = m_{\pi^0} = 134.9764 \text{ MeV}$$

$$\mu = 770 \text{ MeV}$$

$$\bar{l}_1 = -0.4, \bar{l}_2 = 4.3, \bar{l}_3 = 3.0, \bar{l}_4 = 4.3$$

$$m_K = 494.53 \text{ MeV}$$

$$m_\eta = 547.30 \text{ MeV}$$

BE14

Self-consistent unphysical points

$$F_\pi / F = f(F_\pi, m_\pi)$$

$$\tilde{F}_\pi / F_\pi = f(\tilde{F}_\pi, \tilde{m}_\pi)/f(F_\pi, m_\pi)$$

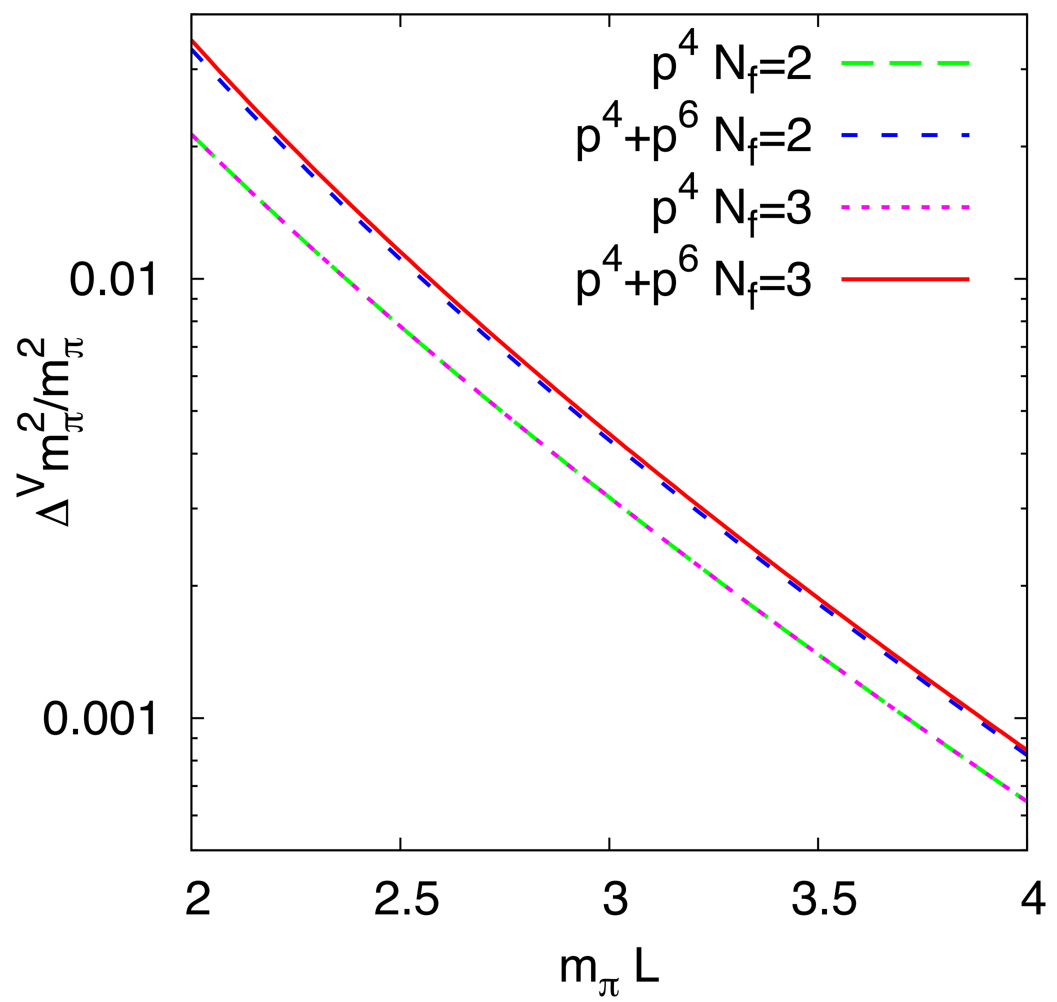
„same underlying F“ - at IV

$$c_i^r = 0$$

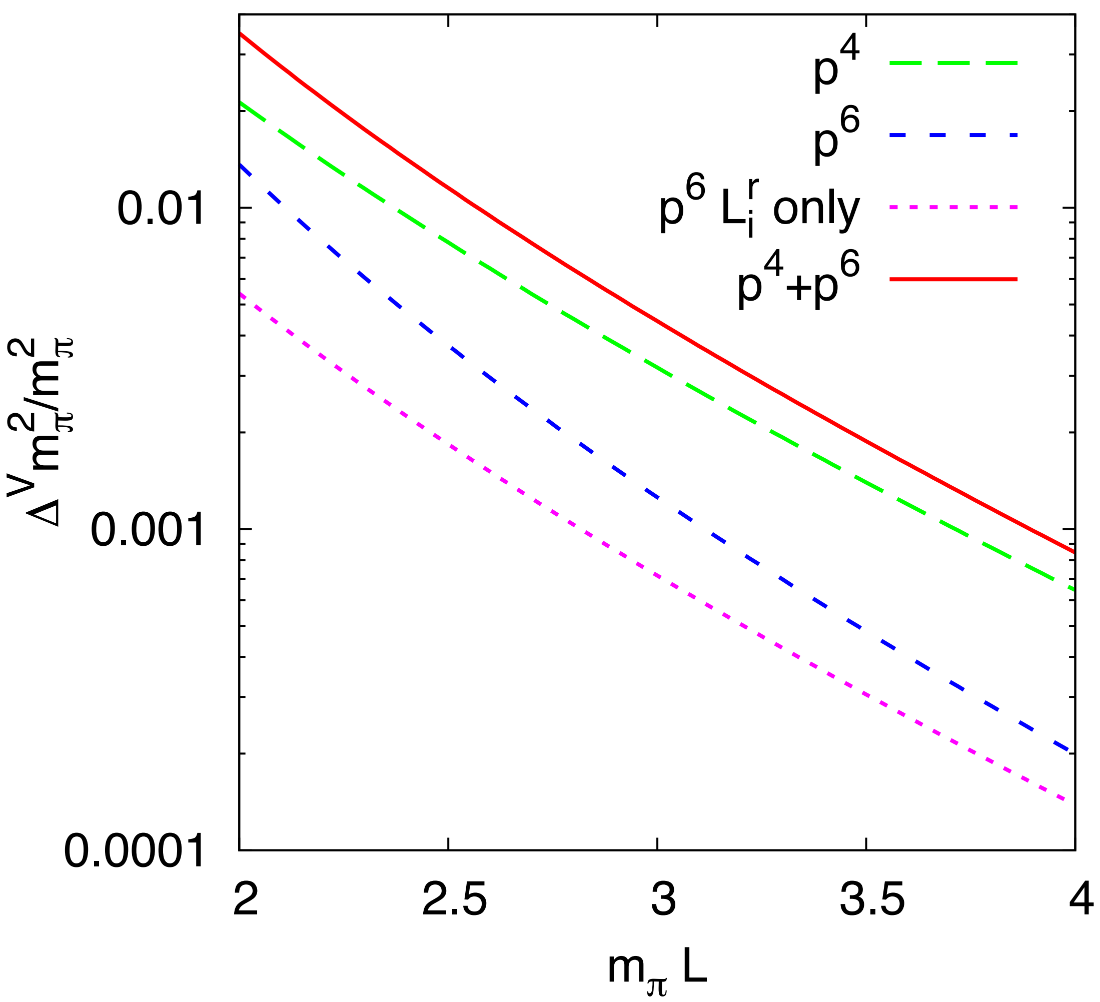
m_π	m_K	m_η	F_π	F_K/F_π	F_η/F_π	$\hat{m}/\hat{m}_{\text{phys}}$	$m_s/m_{s\text{phys}}$	$m_s/\hat{m}$
134.9764*	494.53*	545.9	92.2*	1.199	1.306	1*	1*	27.3
100	487.14	540.46	90.4	1.219	1.337	0.547	1.000	49.9
300	549.6	593.73	101.4	1.099	1.154	5.025	1.000	5.43
100	400	446.53	87.3	1.199	1.293	0.518	0.644	33.9
100	495	549.07	90.7	1.219	1.340	0.550	1.037	51.4
300	495	533.00	100.3	1.094	1.138	4.867	0.778	4.36
495	495	495.00	108.0	1	1	12.70	0.465	1

Table 1: The self consistent solution for the infinite volume values of m_η, F_π, F_K, F_η and the output quark mass ratios compared with the physical one. Units for dimensional quantities are in MeV. The input values for the physical case are starred.

Numerical examples: Pion mass

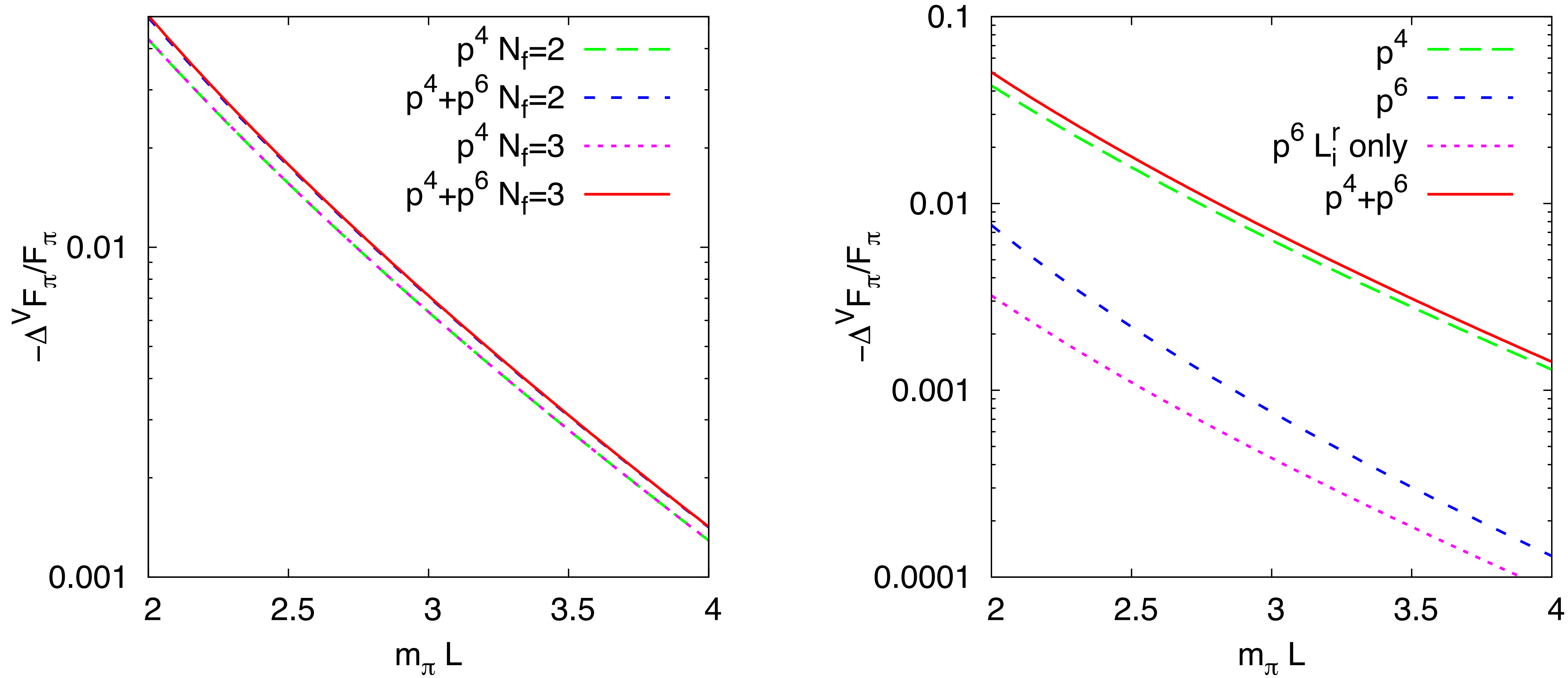


(a)



(b)

Numerical examples: Pion decay constant



Numericale examples: Three flavour Decay constants

Kaon & eta

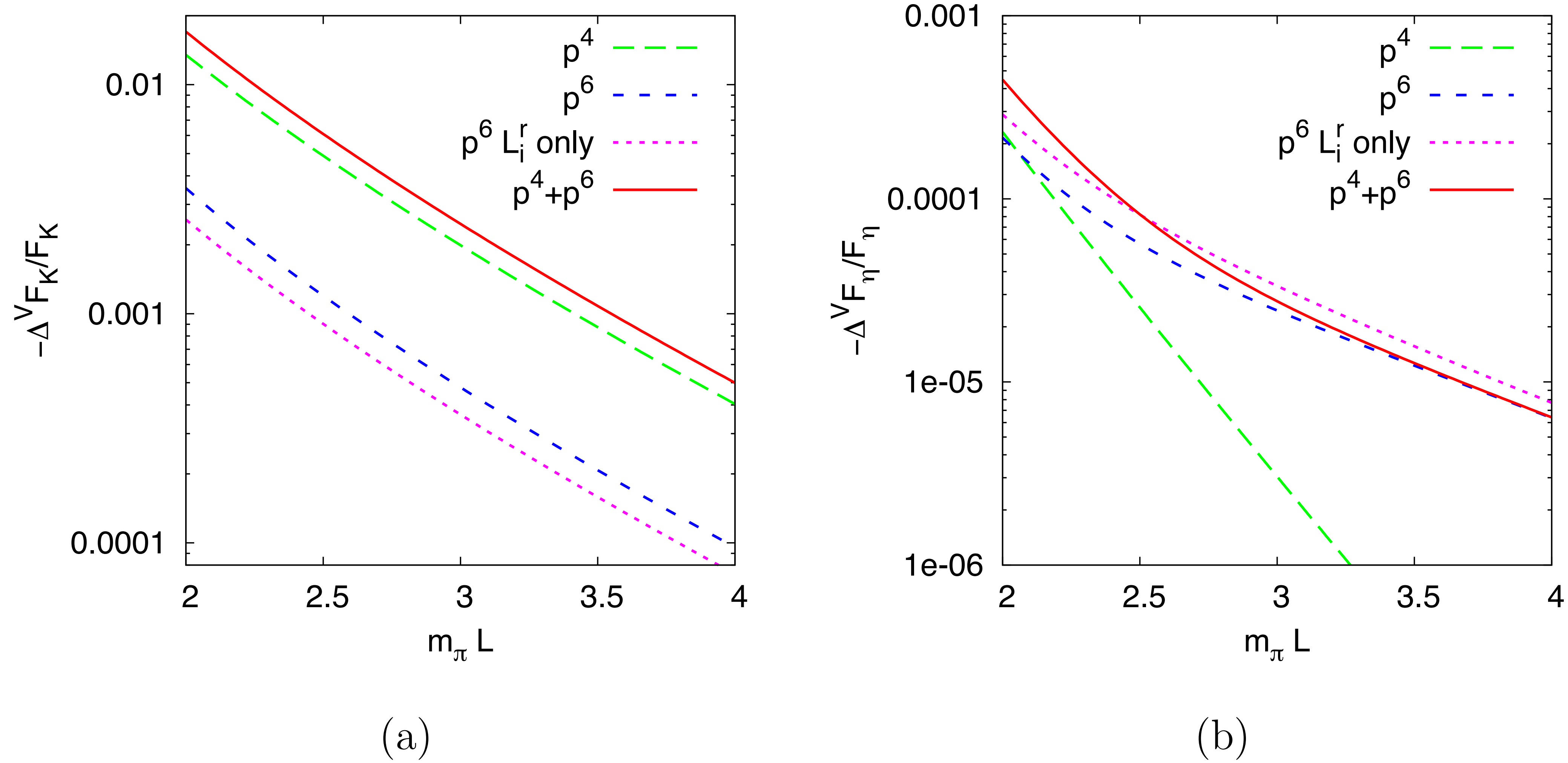
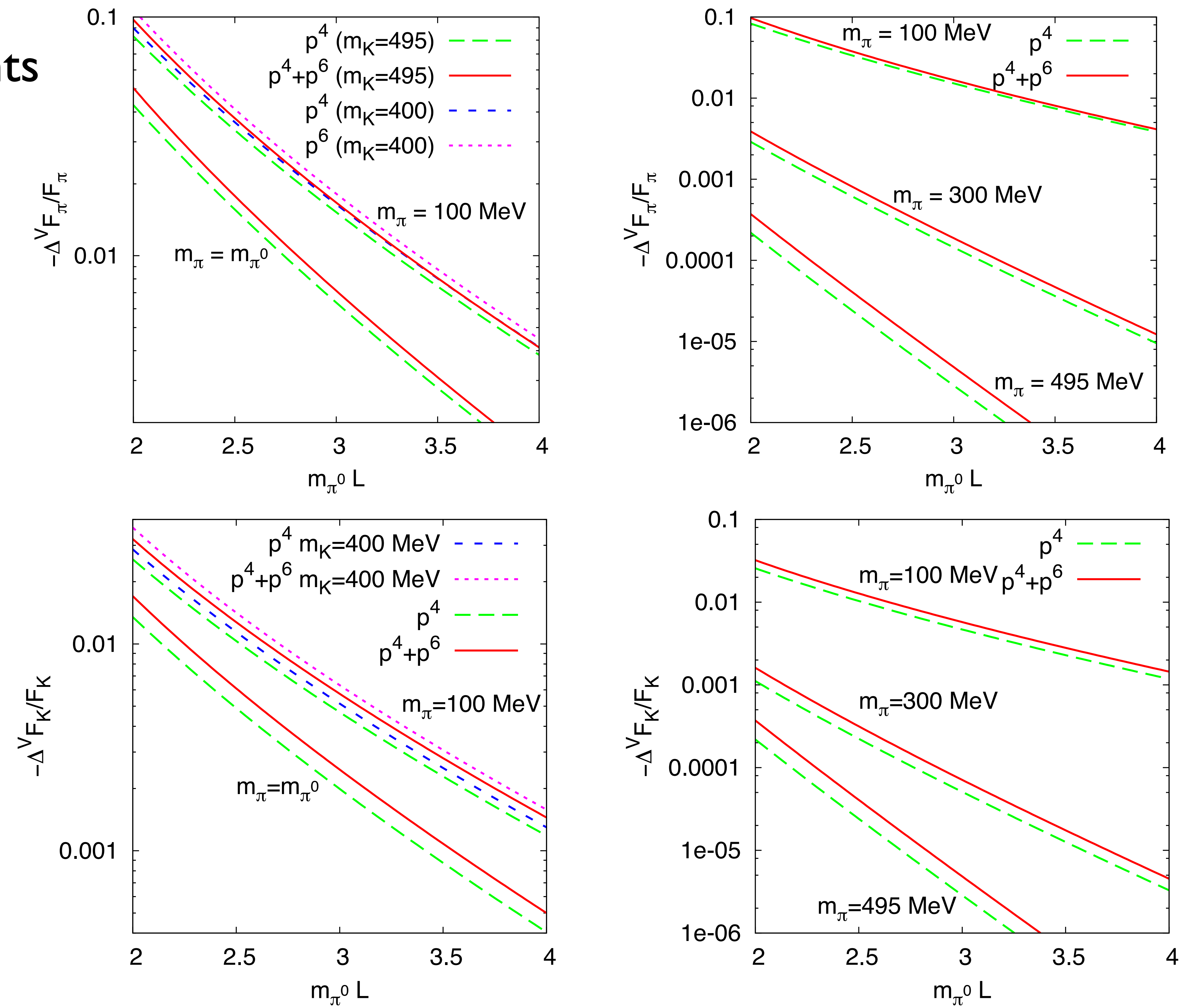


Figure 10: The corrections to the kaon and eta decay constant for the physical case. Plotted is the quantity $-(F_i^V - F_i)/F_i$ for $i = K, \eta$. Shown are the one-loop, the two-loop, the sum and the two-loop L_i^r dependent part. (a) Kaon. (b) Eta.

$$F_\pi \Delta^V F_\eta^{(4)} = A^V(m_K^2)\left(3/2\right)$$

Numericale examples: Three flavour Decay constants

Unphysical points



Now: Partially Quenched

PQ in a Nutshell

$$G = SU(n_{\text{val}} + n_{\text{sea}}|n_{\text{val}})_L \times SU(n_{\text{val}} + n_{\text{sea}}|n_{\text{val}})_R$$

- Goldstones

$$\Phi = \begin{pmatrix} [\ q_V\bar{q}_V\ ] & [\ q_V\bar{q}_S\ ] & [\ q_V\bar{q}_B\ ] \\ [\ q_S\bar{q}_V\ ] & [\ q_S\bar{q}_S\ ] & [\ q_S\bar{q}_B\ ] \\ [\ q_B\bar{q}_V\ ] & [\ q_B\bar{q}_S\ ] & [\ q_B\bar{q}_B\ ] \end{pmatrix}$$

„9x9“

„supersymmetric“

$$\text{Str}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Tr}\, A - \text{Tr}\, D$$

$\langle\Phi\rangle = \text{Str}\,(\Phi) = 0$ (Sharpe, Shoresh)

- reduction of operators:

$$\nabla^\mu u_\mu - \frac{i}{2}\hat{\chi}_- = 0$$

but no Cayley-Hamilton

n_f, n_{sea}	ChPT 2	ChPT 3	ChPT n	PQChPT 2	PQChPT 3
LO	F, B	F_0, B_0	$\hat{F}_0, \hat{B}$	F, B	F_0, B_0
NLO $n_{\text{ph}} + n_{\text{ct}}$	l_i $7+3$	L_i $10+2$	$\hat{L}_i$ $11+2$	$L_i^{(2pq)}$ $11+2$	$L_i^{(3pq)}$ $11+2$
NNLO $n_{\text{ph}} + n_{\text{ct}}$	c_i $52+4$	C_i $90+4$	K_i $112+3$	$K_i^{(2pq)}$ $112+3$	$K_i^{(3pq)}$ $112+3$

$$\begin{aligned} \mathcal{L}_4 &= \sum_{i=0}^{12} \hat{L}_i X_i + \text{contact terms} \\ &= \hat{L}_0 \langle u^\mu u^\nu u_\mu u_\nu \rangle + \hat{L}_1 \langle u^\mu u_\mu \rangle^2 + \hat{L}_2 \langle u^\mu u^\nu \rangle \langle u_\mu u_\nu \rangle \\ &+ \hat{L}_3 \langle (u^\mu u_\mu)^2 \rangle + \hat{L}_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + \hat{L}_5 \langle u^\mu u_\mu \chi_+ \rangle \\ &+ \hat{L}_6 \langle \chi_+ \rangle^2 + \hat{L}_7 \langle \chi_- \rangle^2 + \frac{\hat{L}_8}{2} \langle \chi_+^2 + \chi_-^2 \rangle \\ &- i\hat{L}_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle + \frac{\hat{L}_{10}}{4} \langle f_+^2 - f_-^2 \rangle \\ &+ i\hat{L}_{11} \left\langle \hat{\chi}_- \left(\nabla^\mu u_\mu - \frac{i}{2}\hat{\chi}_- \right) \right\rangle \\ &+ \hat{L}_{12} \left\langle \left(\nabla^\mu u_\mu - \frac{i}{2}\hat{\chi}_- \right)^2 \right\rangle \\ &+ \hat{H}_1 \langle F_L^2 + F_R^2 \rangle + \hat{H}_2 \langle \chi\chi^\dagger \rangle, \end{aligned}$$

Technicalities: Neutral propagator

$$\chi_{ij} \equiv (\chi_i + \chi_j)/2$$

$$-i\, G^c_{ij}(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\varepsilon} \quad (i \neq j)$$

$$\epsilon_j = \begin{cases} +1 & \text{for} \quad j = 1, \ldots, 6 \\ -1 & \text{for} \quad j = 7, 8, 9\,. \end{cases}$$

$$G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - G^q_{ij}(k)/n_{\text{sea}}$$

$$\chi_\pi + \chi_\eta = \frac{2}{3}(\chi_4 + \chi_5 + \chi_6)\,,$$

$$\chi_\pi \chi_\eta = \frac{1}{3}(\chi_4\chi_5 + \chi_5\chi_6 + \chi_4\chi_6)$$

$$\left.\begin{aligned} -i\, G^q_{ij}(k) &= \frac{R^i_{j\pi\eta}}{k^2 - \chi_i + i\varepsilon} + \frac{R^j_{i\pi\eta}}{k^2 - \chi_j + i\varepsilon} \\ &+ \frac{R^\pi_{\eta ij}}{k^2 - \chi_\pi + i\varepsilon} + \frac{R^\eta_{\pi ij}}{k^2 - \chi_\eta + i\varepsilon}, \end{aligned}\right\} \; i \neq j \text{ and } \chi_i \neq \chi_j$$

$$R^i_{jkl} = R^z_{i456jkl},$$

$$R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

$$\left.\begin{aligned} -i\, G^q_{ij}(k) &= \frac{R^d_i}{(k^2 - \chi_i + i\varepsilon)^2} + \frac{R^c_i}{k^2 - \chi_i + i\varepsilon} \\ &+ \frac{R^\pi_{\eta ii}}{k^2 - \chi_\pi + i\varepsilon} + \frac{R^\eta_{\pi ii}}{k^2 - \chi_\eta + i\varepsilon}, \end{aligned}\right\} \; i = j \text{ or } \chi_i = \chi_j$$

$$R^z_{ab} = \chi_a - \chi_b,$$

$$R^z_{abc} = \frac{\chi_a - \chi_b}{\chi_a - \chi_c},$$

$$R^z_{abcd} = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)}{\chi_a - \chi_d},$$

$$R^z_{abcdefg} = \frac{(\chi_a - \chi_b)(\chi_a - \chi_c)(\chi_a - \chi_d)}{(\chi_a - \chi_e)(\chi_a - \chi_f)(\chi_a - \chi_g)}$$

Taking masses degenerate requires taking the proper limit! Challenge!

Double poles

one-loop diagram topologies: up to total power 4, up to 2 different mass scales

sunset $\left\{H, H_\mu, H^s_\mu, H_{\mu\nu}, H^{rs}_{\mu\nu}, H^{ss}_{\mu\nu}\right\}(n, m_1^2, m_2^2, m_3^2, p) =$

$$\frac{1}{i^2}\int_V \frac{d^d r}{(2\pi)^d}\frac{d^d s}{(2\pi)^d}\frac{\{1, r_\mu, s_\mu, r_\mu r_\nu, r_\mu s_\nu, s_\mu s_\nu\}}{(r^2-m_1^2)^{n_1}\,(s^2-m_2^2)^{n_2}\,((r+s-p)^2-m_3^2)^{n_3}}$$

infinite and finite parts different for every n!

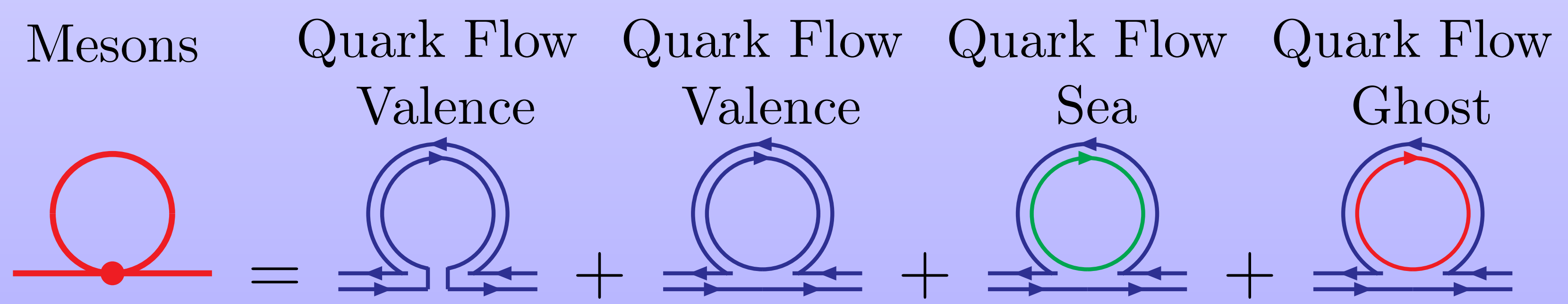
Lots of new identities between integrals of different n!

Note: related to n=1 by differentiation with respect to mass squared!
=> systematic way to obtain new identities & numerical crosscheck

	n_1	n_2	n_3
$n=1$	1	1	1
$n=2$	2	1	1
$n=3$	1	2	1
$(n=4)$	1	1	2
$n=5$	2	2	1
$(n=6)$	2	1	2
$n=7$	1	2	2
$n=8$	2	2	2

Quarkloop vs Ghost

- Two independent calculations
- Quarkloop calculation with different anatomy: „open indices“



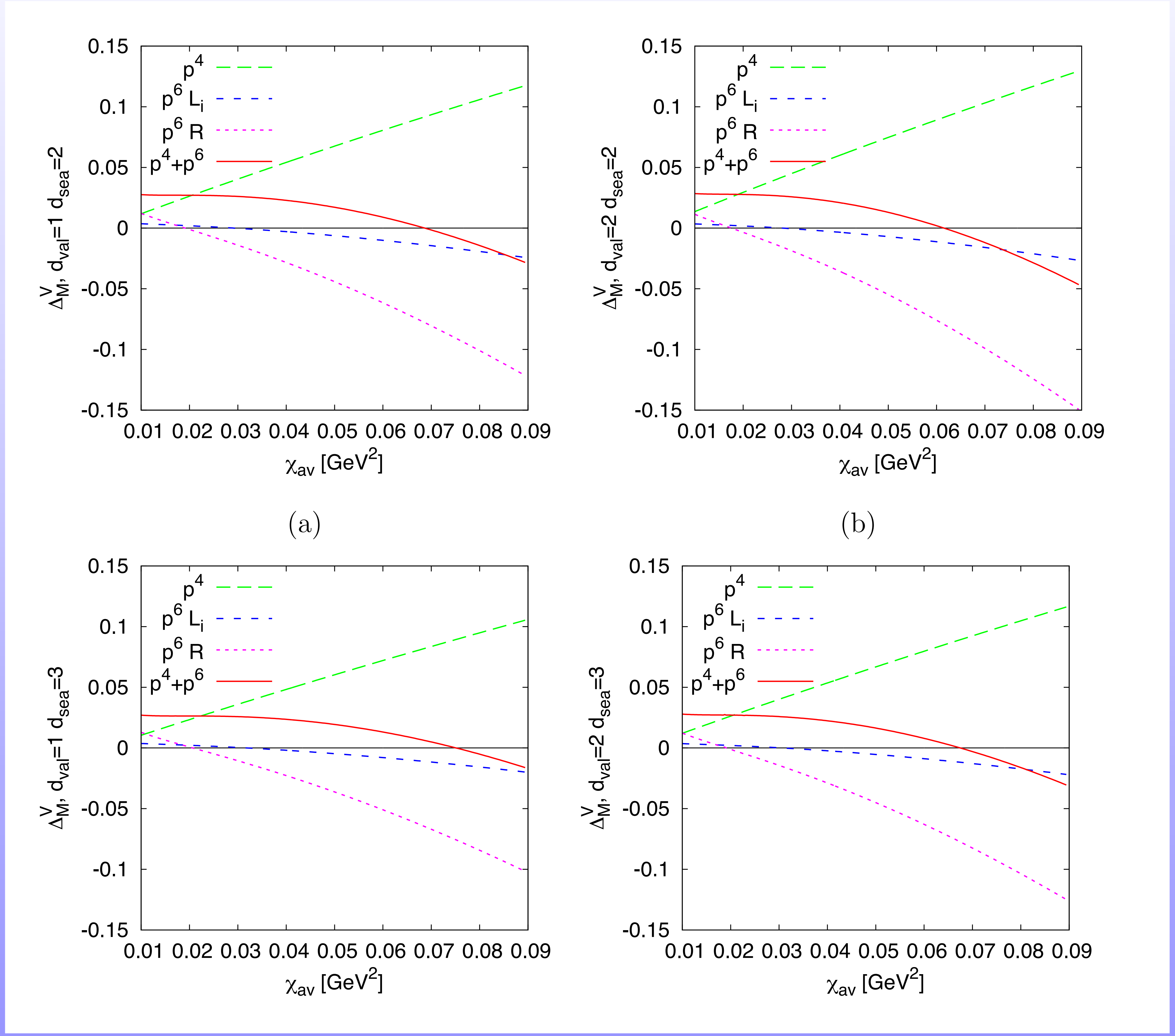
- Other checks: FV unquenched results, partially quenched IV result

Degeneracy cases/Numerical study

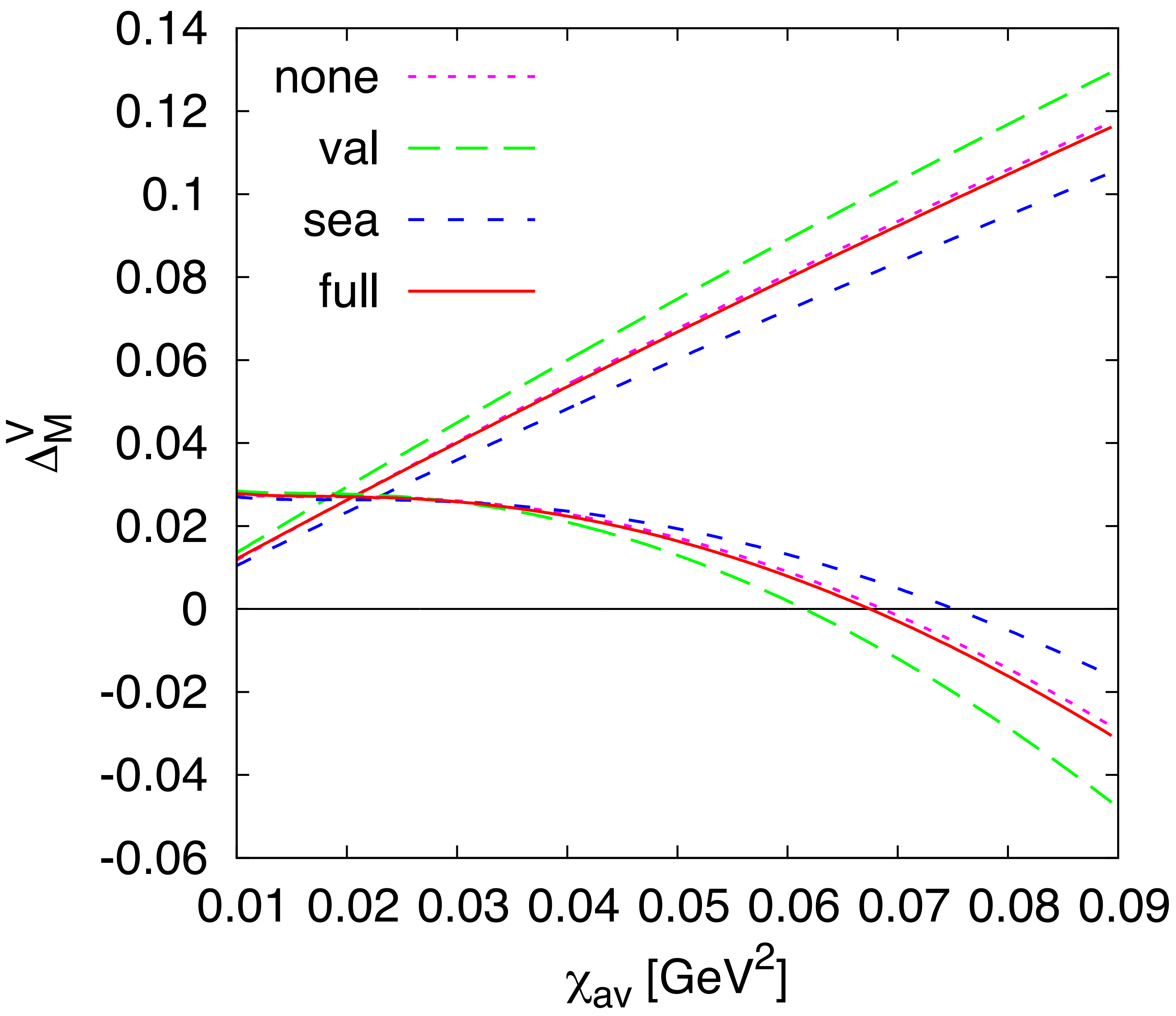
- Notation d_{val} = 1,2 d_{sea} = 1,2,3
- if two masses degenerate, use „lowest two“, i.e. for d_{sea} = 2:
  $\chi_4 = \chi_5 \neq \chi_6$
- (12), (22), (13), (23) as pions
- (22), (23) as kaons
- The up/down average mass is varied
- Input:
  BE14 (Bijnens, Ecker) with
  L0 set to zero
- ML=2 for M=0.13 GeV

$$
\begin{aligned}
\mathcal{L}_4 &= \sum_{i=0}^{12} \hat{L}_i X_i + \text{contact terms} \\
&= \hat{L}_0 \langle u^\mu u^\nu u_\mu u_\nu \rangle + \hat{L}_1 \langle u^\mu u_\mu \rangle^2 + \hat{L}_2 \langle u^\mu u^\nu \rangle \langle u_\mu u_\nu \rangle \\
&+ \hat{L}_3 \langle (u^\mu u_\mu)^2 \rangle + \hat{L}_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + \hat{L}_5 \langle u^\mu u_\mu \chi_+ \rangle \\
&+ \hat{L}_6 \langle \chi_+ \rangle^2 + \hat{L}_7 \langle \chi_- \rangle^2 + \frac{\hat{L}_8}{2} \langle \chi_+^2 + \chi_-^2 \rangle \\
&- i\hat{L}_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle + \frac{\hat{L}_{10}}{4} \langle f_+^2 - f_-^2 \rangle \\
&+ i\hat{L}_{11} \left\langle \hat{\chi}_- \left(\nabla^\mu u_\mu - \frac{i}{2}\hat{\chi}_- \right) \right\rangle \\
&+ \hat{L}_{12} \left\langle \left(\nabla^\mu u_\mu - \frac{i}{2}\hat{\chi}_- \right)^2 \right\rangle \\
&+ \hat{H}_1 \langle F_L^2 + F_R^2 \rangle + \hat{H}_2 \langle \chi \chi^\dagger \rangle,
\end{aligned}
$$

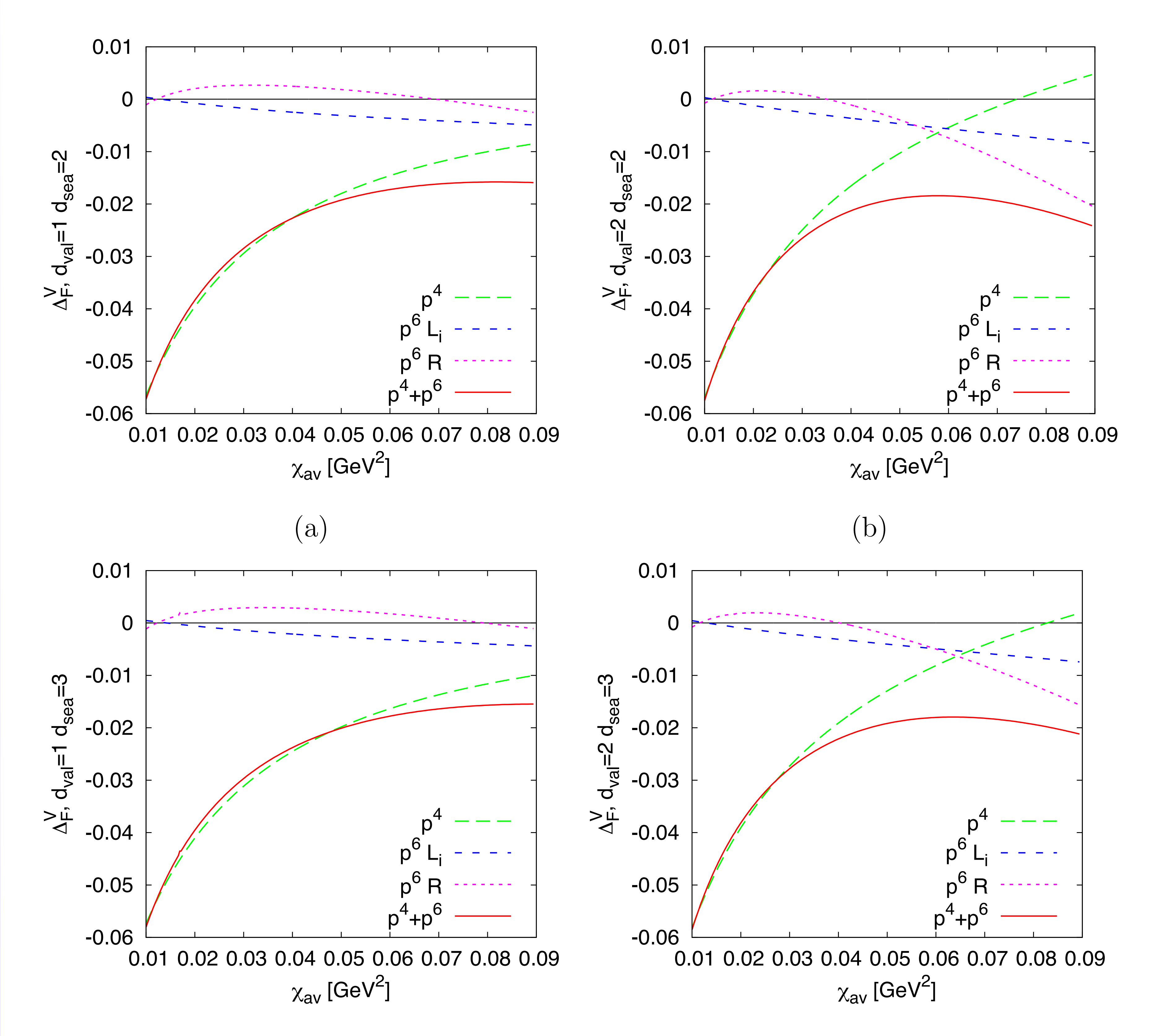
Numerical examples: „Pion“ mass



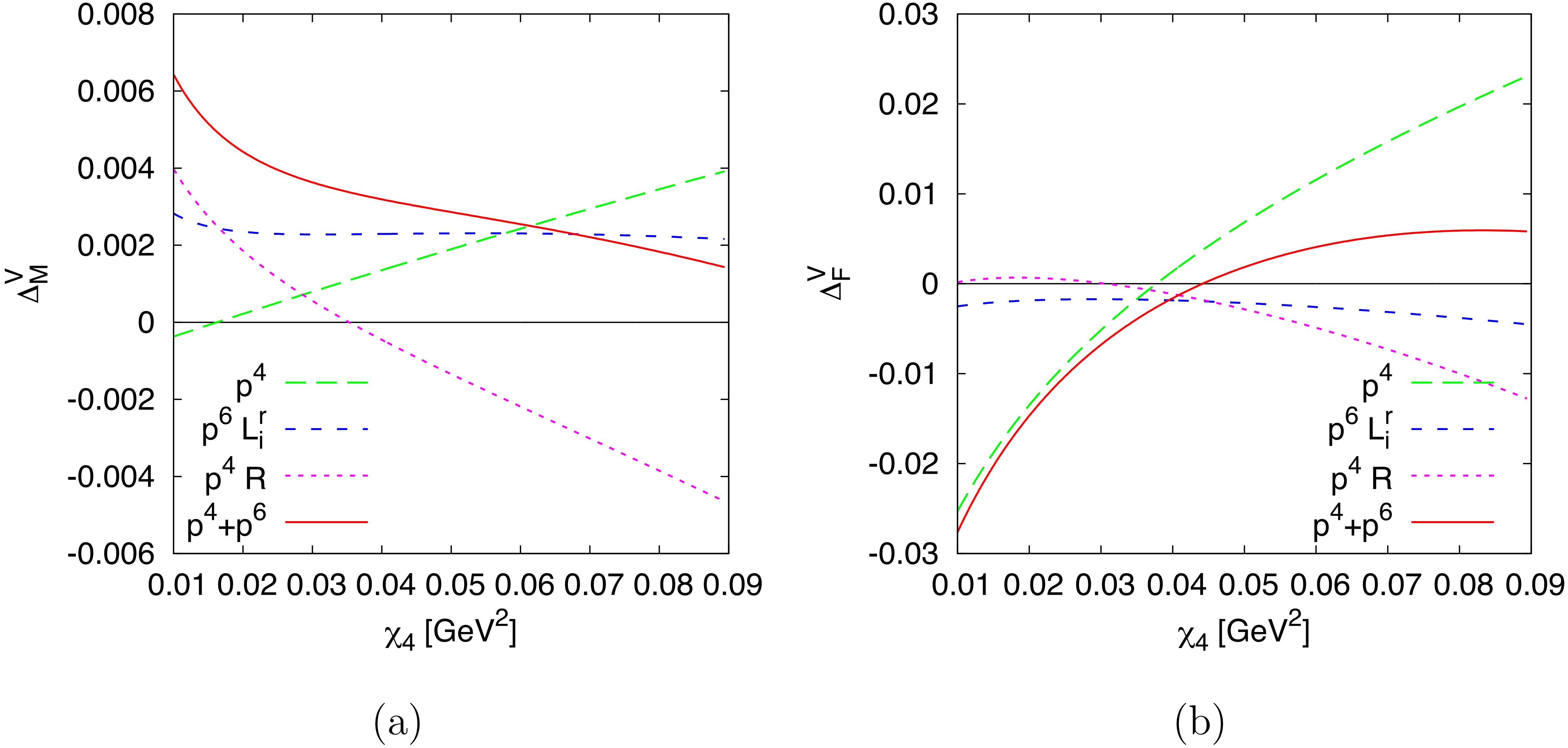
„Pion“ mass: A closer look



Numerical examples: „Pion“ decay constant



Numerical examples: „Kaon“



Summary

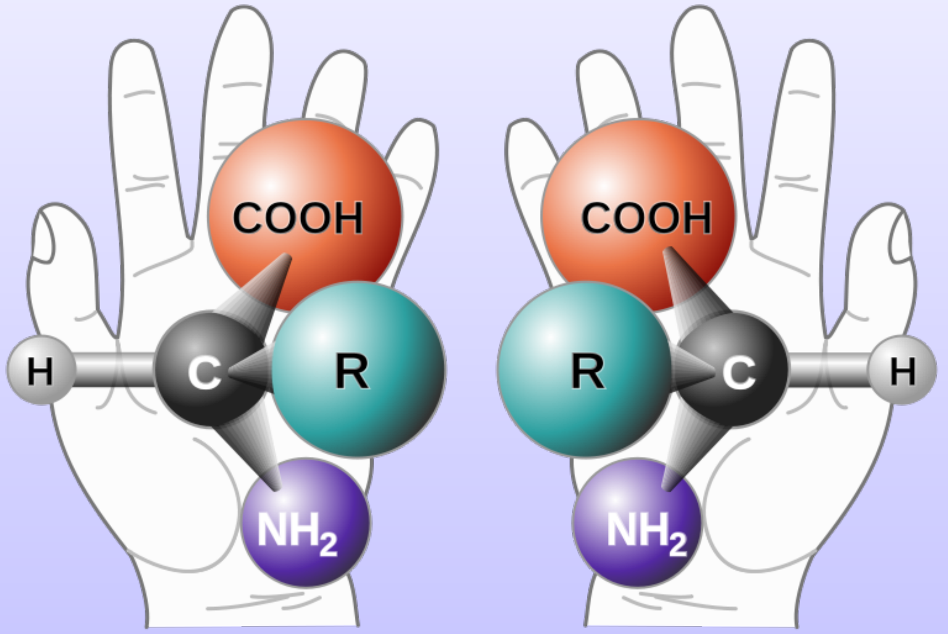
- We have calculated FV corrections up to two-loop order in two- and three-flavour ChPT. In three-flavor PQChPT, we have computed flavour-charged meson masses and decay constants with two different techniques, and also calculated the (simplified) cases of degenerate masses
- Analytical expressions, see papers and/or http://home.thep.lu.se/~bijnens/chpt/
- Examples of numerical evaluations
- CHIRON http://home.thep.lu.se/~bijnens/chiron/
  Calculate quantum corrections in ChPT with your own input parameters!
  All FV corrections from this talk (both ChPT and PQChPT) already implemented!

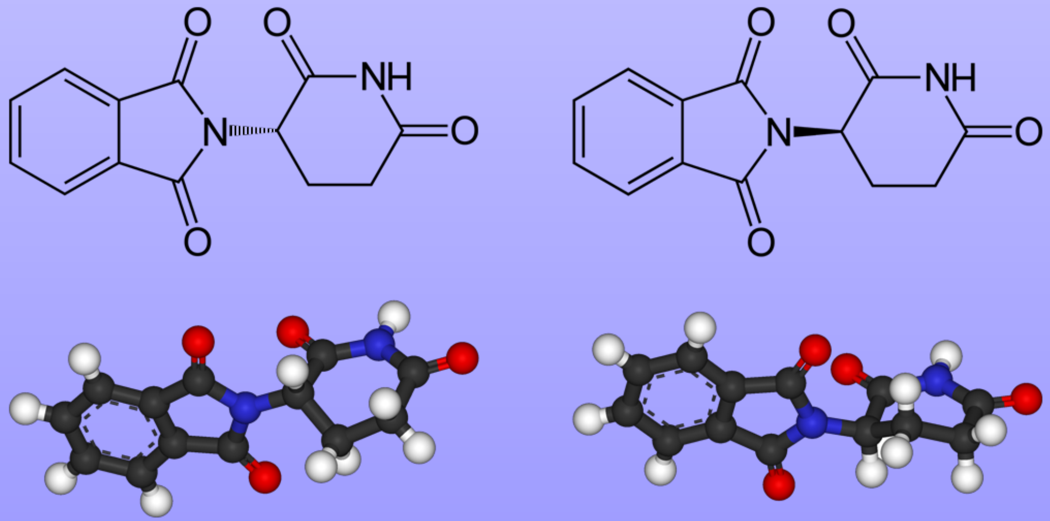
Template slide

NUMERICS

On the origin of „Chiral“

Χείρων





Chiral fields, QCD, Chiral symmetry

$$\mathcal{L}_{\rm QCD} = \sum_{\substack{i=1 \\ (u,d,s,c,b,t)}}^{6} \bar{\psi}_i (i\not{D} - m_i)\psi_i - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu} \qquad m_i = 0 \ \forall i \leq N$$

$$\underbrace{\mathrm{SU(N)}_L \times \mathrm{SU(N)}_R}\times \mathrm{U(1)}_V \times \mathrm{U(1)}_A$$

$$0 \neq \langle \bar{q} q \rangle$$

$$\mathrm{SU(N)}_V$$

Now EFT:

$$U(x) = \exp\left(i \frac{\sqrt{2}\phi(x)}{F_0} \right)$$

N=2,3; e.g. for SU(3): $$\phi(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta \end{pmatrix}$$

Lowest order

- Chiral symmetry, parity, time-reversal, Lorentz: most general Lagrangian

$$U \mapsto RUL^\dagger$$

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)$$

- Explicit breaking … generates masses: „Pseudo Goldstones“

$$\chi = 2B_0 \begin{pmatrix} \hat{m} & 0 & 0 \\ 0 & \hat{m} & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$\begin{aligned} M_{\pi,2}^2 &= 2B_0\hat{m}, \\ M_{K,2}^2 &= B_0(\hat{m}+m_s), \\ M_{\eta,2}^2 &= \frac{2}{3}B_0\left(\hat{m}+2m_s\right) \end{aligned}$$

$$4M_K^2 = 3M_\eta^2 + M_\pi^2$$ GMO

no unique elimination of *m*’s in terms of *M*’s: difference of higher order

Higher orders

general structure:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

e. g. SU(3)

$$\begin{aligned}\mathcal{L}_4 = & L_1 \left\{\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger]\right\}^2 + L_2 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger\right] \mathrm{Tr}\left[D^\mu U (D^\nu U)^\dagger\right] \\ & + L_3 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger\right] + L_4 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\ & + L_5 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger)\right] + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\ & + L_7 \left[\mathrm{Tr}\left(\chi U^\dagger - U \chi^\dagger\right)\right]^2 + L_8 \mathrm{Tr}\left(U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger\right) \\ & - i L_9 \mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + f^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U\right] + L_{10} \mathrm{Tr}\left(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}\right) \\ & + H_1 \mathrm{Tr}\left(f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu}\right) + H_2 \mathrm{Tr}\left(\chi \chi^\dagger\right)\end{aligned}$$

Gasser, Leutwyler 1984/85

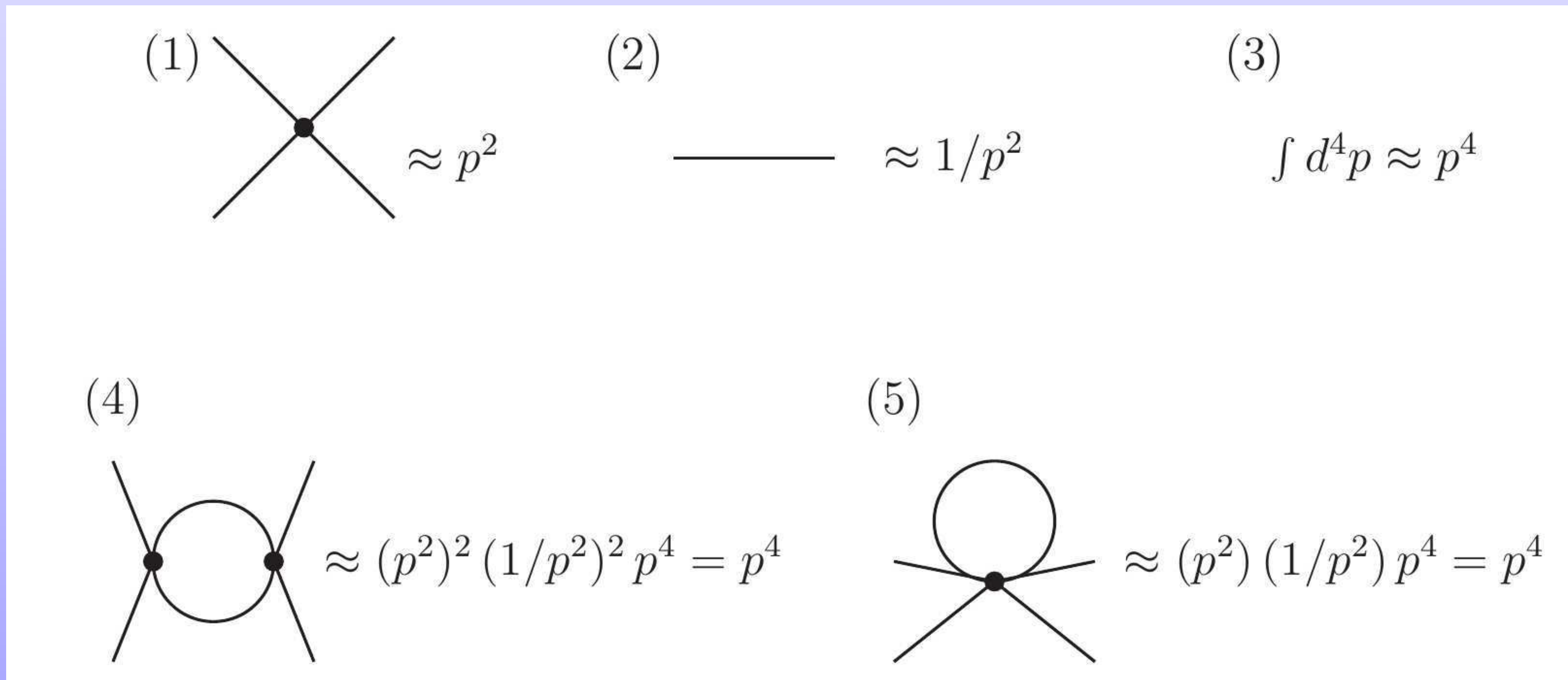
Most general Lagrangian consistent with symmetry,
reduction operators to minimal set via EOM, Cayley-Hamilton

Low Energy Constants (LECs): to be determined via experiment/lattice

Powercounting: Weinberg

Chiral dimension of a diagram

$$D = 4N_L - 2N_I + \sum_{n=1}^{\infty} 2nN_{2n}$$



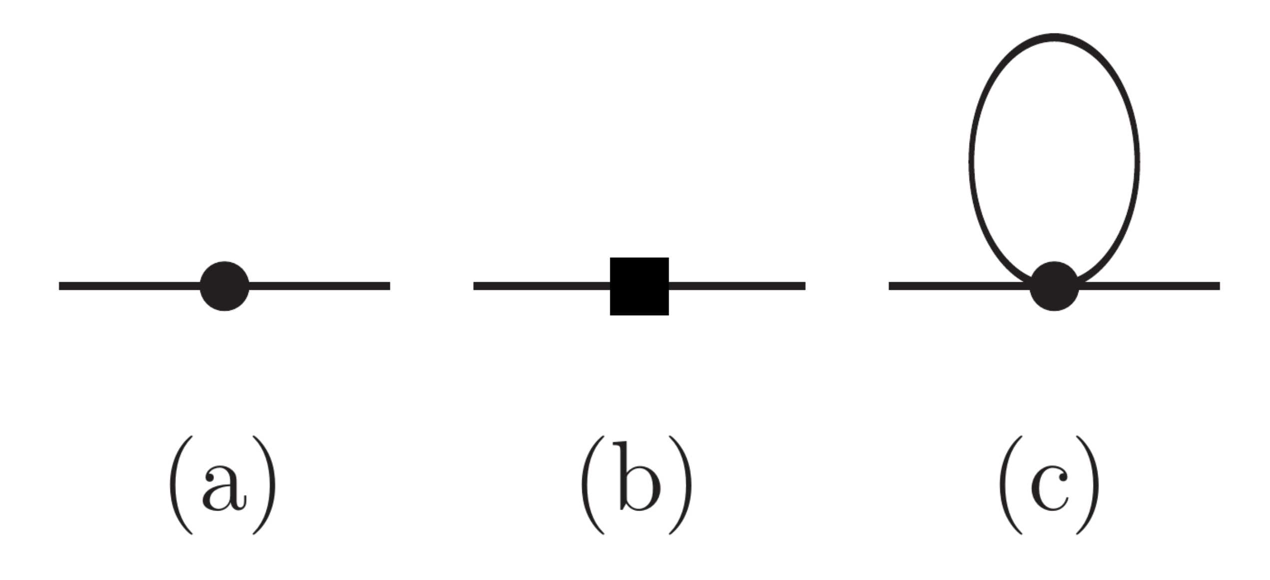
loops clearly bounded $D = 2 + \sum_{n=1}^{\infty} 2(n-1)N_{2n} + 2N_L$

Simple(st) example: Mass @ NLO

Chiral dimension of a diagram

$$D = 4N_L - 2N_I + \sum_{n=1}^{\infty} 2n N_{2n}$$

Self energy:



$$M^2 - M_0^2 - \Sigma(M^2) = 0$$

$$M^2 = M_0^2 + \Sigma(M^2) = M_0^2 + \Sigma(M_0^2) + \mathcal{O}(p^6)$$

$$\Sigma_4^{\phi}(p^2) = A_{\phi} + B_{\phi} p^2$$

$$M^2 = M_0^2 + A_{\phi} + B_{\phi} M_0^2 + \mathcal{O}(p^6)$$

Simple(st) example: Mass @ NLO

Expanding......

$$\mathcal{L}_2^{4\phi} = \frac{1}{24F_0^2}\left\{\mathrm{Tr}([\phi,\partial_\mu\phi]\phi\partial^\mu\phi) + B_0\mathrm{Tr}(\mathcal{M}\phi^4)\right\}.$$

$$\begin{aligned}\mathcal{L}_4^{2\phi} = \frac{1}{F_0^2}\Big\{ & L_4 \quad 8 \quad B_0\mathrm{Tr}(\partial_\mu\phi\partial^\mu\phi)\mathrm{Tr}(\mathcal{M}) \\ & +L_5 \quad 8 \quad B_0\mathrm{Tr}(\partial_\mu\phi\partial^\mu\phi\mathcal{M}) \\ & +L_6 \quad (-32) \quad B_0^2\mathrm{Tr}(\mathcal{M})\mathrm{Tr}(\mathcal{M}\phi^2) \\ & +L_7 \quad (-32) \quad B_0^2[\mathrm{Tr}(\mathcal{M}\phi)]^2 \\ & +L_8 \quad (-16) \quad B_0^2(\mathrm{Tr}(\phi\mathcal{M}\phi\mathcal{M}) + \mathrm{Tr}(\phi^2\mathcal{M}^2))\Big\}\end{aligned}$$

Regularization/Renormalization: ChPT version of MSbar

loops $A(m^2) = \frac{m^2}{16\pi^2}\left\{\lambda_0 - \ln(m^2) + \mathcal{O}(\epsilon)\right\}$ $\quad \lambda_0 = \frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \ln(4\pi) + 1 - \gamma_E$

LECs $L_i \equiv (\mu c)^{-2\epsilon}\left(\frac{-1}{32\pi^2\epsilon}\Gamma_i + L_i^r(\mu)\right) = (\mu)^{-2\epsilon}\left(\frac{-1}{32\pi^2}\Gamma_i\lambda_0 + L_i^r(\mu) + \mathcal{O}(\epsilon)\right)$

Simple(st) example: Mass @ NLO

Combinatorics, symmetry factors…
Result: e.g. pion

	ϕ^4 term	derivative term	sum of both
Pion loop	$\frac{B_0\hat{m}}{6F_0^2}10 = \frac{1}{6F_0^2}(5m_\pi^2)$	$\frac{1}{6F_0^2}(-4m_\pi^2 - 4p^2)$	$\frac{1}{6F_0^2}(m_\pi^2 - 4p^2)$
Kaon loop	$\frac{B_0(3\hat{m}+m_s)}{6F_0^2}2 = \frac{1}{6F_0^2}(2m_K^2 + 2m_\pi^2)$	$\frac{1}{6F_0^2}(-2m_K^2 - 2p^2)$	$\frac{1}{6F_0^2}(2m_\pi^2 - 2p^2)$
Eta loop	$\frac{B_0\hat{m}}{6F_0^2}2 = \frac{1}{6F_0^2}(m_\pi^2)$	-	$\frac{1}{6F_0^2}(m_\pi^2)$

Table 1: Coefficients of the one-loop diagram contribution to the self-energy $\Sigma_4(p^2)$ for the pion, split up according to which operator of $\mathcal{L}_2^{4\phi}$ contributes and which virtual particle occupies the loop, given in units of the divergent integral $A(m^2)$. Note that derivatives can come with the loop particles, thus introducing their masses into the result, as well as with the external particles, introducing their own squared momenta. Lowest order mass relations were applied to the symmetry-breaking terms. Observe also the cancellation of the kaon mass dependence: In an unbroken SU(2), the pion has to remain massless.

$$M_{\pi,4}^2 = M_{\pi,2}^2\left\{1 + \frac{M_{\pi,2}^2}{32\pi^2F_0^2}\ln\left(\frac{M_{\pi,2}^2}{\mu^2}\right) - \frac{M_{\eta,2}^2}{96\pi^2F_0^2}\ln\left(\frac{M_{\eta,2}^2}{\mu^2}\right)\right.$$
$$\left. + \frac{16}{F_0^2}\left[(2m+m_s)B_0(2L_6^r - L_4^r) + mB_0(2L_8^r - L_5^r)\right]\right\}$$

Note already GMO!

Example: Kaon mass correction :-)

$$
\begin{aligned}
F_\pi^4 \Delta m_K^{2(6)} = {} & +64\, m_K^6\, C_{32}^r + 32\, m_K^6\, C_{31}^r + 192\, m_K^6\, C_{21}^r + 128\, m_K^6\, C_{20}^r + 96\, m_K^6\, C_{19}^r - 64\, m_K^6\, C_{16}^r \\
& -32\, m_K^6\, C_{15}^r - 32\, m_K^6\, C_{14}^r - 64\, m_K^6\, C_{13}^r - 32\, m_K^6\, C_{12}^r + 32\, m_\pi^2\, m_K^4\, C_{32}^r + 192\, m_\pi^2\, m_K^4\, C_{21}^r \\
& -32\, m_\pi^2\, m_K^4\, C_{20}^r - 96\, m_\pi^2\, m_K^4\, C_{19}^r - 32\, m_\pi^2\, m_K^4\, C_{17}^r + 64\, m_\pi^2\, m_K^4\, C_{16}^r - 16\, m_\pi^2\, m_K^4\, C_{15}^r \\
& +32\, m_\pi^2\, m_K^4\, C_{14}^r - 32\, m_\pi^2\, m_K^4\, C_{13}^r + 48\, m_\pi^4\, m_K^2\, C_{21}^r + 48\, m_\pi^4\, m_K^2\, C_{20}^r + 48\, m_\pi^4\, m_K^2\, C_{19}^r \\
& +16\, m_\pi^4\, m_K^2\, C_{17}^r - 48\, m_\pi^4\, m_K^2\, C_{16}^r - 16\, m_\pi^4\, m_K^2\, C_{14}^r \\
& -512\, (L_8^r)^2\, m_K^6 - 2048\, L_6^r L_8^r m_K^6 - 768\, L_6^r L_8^r m_\pi^2\, m_K^4 - 256\, L_6^r L_8^r m_\pi^4\, m_K^2 - 2048\, (L_6^r)^2\, m_K^6 \\
& -2048\, (L_6^r)^2\, m_\pi^2\, m_K^4 - 512\, (L_6^r)^2\, m_\pi^4\, m_K^2 + 384\, L_5^r L_8^r m_K^6 + 128\, L_5^r L_8^r m_\pi^2\, m_K^4 + 768\, L_5^r L_6^r m_K^6 \\
& +512\, L_5^r L_6^r m_\pi^2\, m_K^4 + 256\, L_5^r L_6^r m_\pi^4\, m_K^2 - 64\, (L_5^r)^2\, m_K^6 - 64\, (L_5^r)^2\, m_\pi^2\, m_K^4 + 1024\, L_4^r L_8^r m_K^6 \\
& +384\, L_4^r L_8^r m_\pi^2\, m_K^4 + 128\, L_4^r L_8^r m_\pi^4\, m_K^2 + 2048\, L_4^r L_6^r m_K^6 + 2048\, L_4^r L_6^r m_\pi^2\, m_K^4 + 512\, L_4^r L_6^r m_\pi^4\, m_K^2 \\
& -384\, L_4^r L_5^r m_K^6 - 256\, L_4^r L_5^r m_\pi^2\, m_K^4 - 128\, L_4^r L_5^r m_\pi^4\, m_K^2 - 512\, (L_4^r)^2\, m_K^6 - 512\, (L_4^r)^2\, m_\pi^2\, m_K^4 \\
& -128\, (L_4^r)^2\, m_\pi^4\, m_K^2 + 89/27\, \frac{1}{16\pi^2}\, L_3^r m_K^6 - 4/27\, \frac{1}{16\pi^2}\, L_3^r m_\pi^2\, m_K^4 + 41/27\, \frac{1}{16\pi^2}\, L_3^r m_\pi^4\, m_K^2 \\
& +122/9\, \frac{1}{16\pi^2}\, L_2^r m_K^6 - 16/9\, \frac{1}{16\pi^2}\, L_2^r m_\pi^2\, m_K^4 + 56/9\, \frac{1}{16\pi^2}\, L_2^r m_\pi^4\, m_K^2 + 4\, \frac{1}{16\pi^2}\, L_1^r m_K^6 \\
& \left(\frac{1}{16\pi^2}\right)^2 \Big(- 4709/1728\, m_K^6 - 19/108\, m_\pi^2\, m_K^4 - 13/24\, m_\pi^4\, m_K^2 \\
& -763/1296\, \pi^2\, m_K^6 - 73/648\, \pi^2\, m_\pi^2\, m_K^4 - 1/8\, \pi^2\, m_\pi^4\, m_K^2 \Big) \\
& +\overline{A}(m_\pi^2)^2 \Big(- 1/2\, m_\pi^{-2}\, m_K^4 - 27/32\, m_K^2 \Big) + \overline{A}(m_\pi^2)\, \overline{A}(m_K^2) \Big(- 3/4\, m_K^2 \Big) \\
& +\overline{A}(m_\pi^2)\, \overline{A}(m_\eta^2) \Big(- 41/48\, m_K^2 + 1/12\, m_\pi^2 \Big) \\
& +\overline{A}(m_\pi^2) \Big(+ 32\, L_8^r m_K^4 + 24\, L_8^r m_\pi^2\, m_K^2 + 64\, L_6^r m_K^4 + 88\, L_6^r m_\pi^2\, m_K^2 - 16\, L_5^r m_K^4 - 12\, L_5^r m_\pi^2\, m_K^2 \\
& -32\, L_4^r m_K^4 - 68\, L_4^r m_\pi^2\, m_K^2 + 15\, L_3^r m_\pi^2\, m_K^2 + 12\, L_2^r m_\pi^2\, m_K^2 + 48\, L_1^r m_\pi^2\, m_K^2 + 3/4\, \frac{1}{16\pi^2}\, m_K^4 \Big) \\
& +\overline{A}(m_K^2)^2 \Big(- 251/72\, m_K^2 - 3/8\, m_\pi^2 \Big) + \overline{A}(m_K^2)\, \overline{A}(m_\eta^2) \Big(- 2/3\, m_K^2 \Big) \\
& +\overline{A}(m_K^2) \Big(+ 64\, L_8^r m_K^4 + 128\, L_6^r m_K^4 + 16\, L_6^r m_\pi^2\, m_K^2 - 32\, L_5^r m_K^4 - 80\, L_4^r m_K^4 - 8\, L_4^r m_\pi^2\, m_K^2 \\
& +30\, L_3^r m_K^4 + 36\, L_2^r m_K^4 + 72\, L_1^r m_K^4 + 3/4\, \frac{1}{16\pi^2}\, m_K^4 + 3/4\, \frac{1}{16\pi^2}\, m_\pi^2\, m_K^2 \Big) \\
& +\overline{A}(m_\eta^2)^2 \Big(- 5/36\, m_K^2 - 25/128\, m_\pi^2 - 43/1152\, m_\pi^4\, m_\eta^{-2} \Big) \\
& +\overline{A}(m_\eta^2) \Big(+ 32\, L_8^r m_K^4 - 56/3\, L_8^r m_\pi^2\, m_K^2 + 16/3\, L_8^r m_\pi^4 + 64/3\, L_7^r m_K^4 - 32\, L_7^r m_\pi^2\, m_K^2 + 32/3\, L_7^r m_\pi^4 \\
& +128/3\, L_6^r m_K^4 - 8/3\, L_6^r m_\pi^2\, m_K^2 - 112/9\, L_5^r m_K^4 + 4/3\, L_5^r m_\pi^2\, m_K^2 - 8/9\, L_5^r m_\pi^4 - 32\, L_4^r m_K^4 \\
& +4\, L_4^r m_\pi^2\, m_K^2 + 28/3\, L_3^r m_K^4 - 7/3\, L_3^r m_\pi^2\, m_K^2 + 16/3\, L_2^r m_K^4 - 4/3\, L_2^r m_\pi^2\, m_K^2 + 64/3\, L_1^r m_K^4 \\
& -16/3\, L_1^r m_\pi^2\, m_K^2 + 1/2\, \frac{1}{16\pi^2}\, m_K^4 + 1/4\, \frac{1}{16\pi^2}\, m_\pi^2\, m_K^2 \Big) \\
& -15/32\, H(m_\pi^2, m_\pi^2, m_K^2, m_K^2)\, m_K^4 + 3/4\, H(m_\pi^2, m_\pi^2, m_K^2, m_K^2)\, m_\pi^2\, m_K^2 \\
& +29/16\, H(m_\pi^2, m_K^2, m_\eta^2, m_K^2)\, m_K^4 + 3/4\, H(m_K^2, m_K^2, m_K^2, m_K^2)\, m_K^4 \\
& +181/288\, H(m_K^2, m_\eta^2, m_\eta^2, m_K^2)\, m_K^4 - H_1(m_\pi^2, m_K^2, m_\eta^2, m_K^2)\, m_K^4 \\
& +3/4\, H_1(m_K^2, m_\pi^2, m_\pi^2, m_K^2)\, m_K^4 - 5/2\, H_1(m_K^2, m_\pi^2, m_\eta^2, m_K^2)\, m_K^4 \\
& -5/4\, H_1(m_K^2, m_\eta^2, m_\eta^2, m_K^2)\, m_K^4 - H_1(m_\eta^2, m_\pi^2, m_K^2, m_K^2)\, m_K^4 \\
& +9/4\, H_{21}(m_\pi^2, m_\pi^2, m_K^2, m_K^2)\, m_K^4 - 9/32\, H_{21}(m_K^2, m_\pi^2, m_\pi^2, m_K^2)\, m_K^4 \\
& +27/16\, H_{21}(m_K^2, m_\pi^2, m_\eta^2, m_K^2)\, m_K^4 + 9/4\, H_{21}(m_K^2, m_K^2, m_K^2, m_K^2)\, m_K^4 \\
& +27/32\, H_{21}(m_K^2, m_\eta^2, m_\eta^2, m_K^2)\, m_K^4
\end{aligned}
$$

with implicit one-loop choice

$$
\begin{aligned}
F_\pi^4 \Delta m_K^{2(4)} & = +\overline{A}(m_\eta^2) \Big(- 1/4\, m_\eta^2 - 1/12\, m_\pi^2 \Big) \\
& \quad +8 \Big(2L_6^r - L_4^r \Big) \Big(2m_K^2 + m_\pi^2 \Big) - 8m_K^4 \Big(L_5^r - 2L_8^r \Big)
\end{aligned}
$$

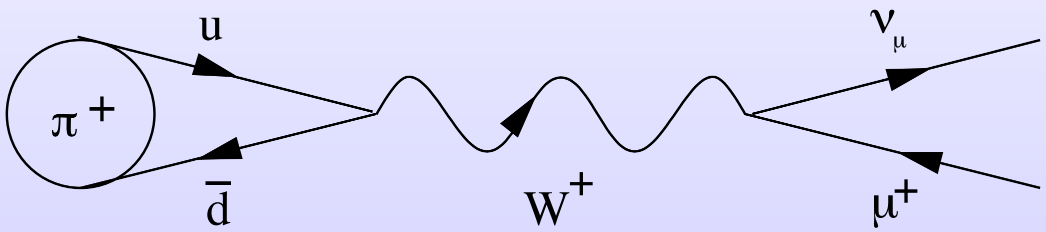
ChPT at NNLO: Two loop mass

- Pole eqn $$M^2 - M_0^2 \underbrace{-\Sigma_4(M_0^2)}_{\mathcal{O}(p^4)} \underbrace{-\Sigma_4(M_4^2) + \Sigma_4(M_0^2)}_{\mathcal{O}(p^6)} \underbrace{-\Sigma_6(M_0^2)}_{\mathcal{O}(p^6)} = \mathcal{O}(p^8)$$

- „renormalization part“ is infinite, so is the new diagrammatic part
- dependence on choice at NLO
- can safely put physical masses (since „we subtract what we add“)

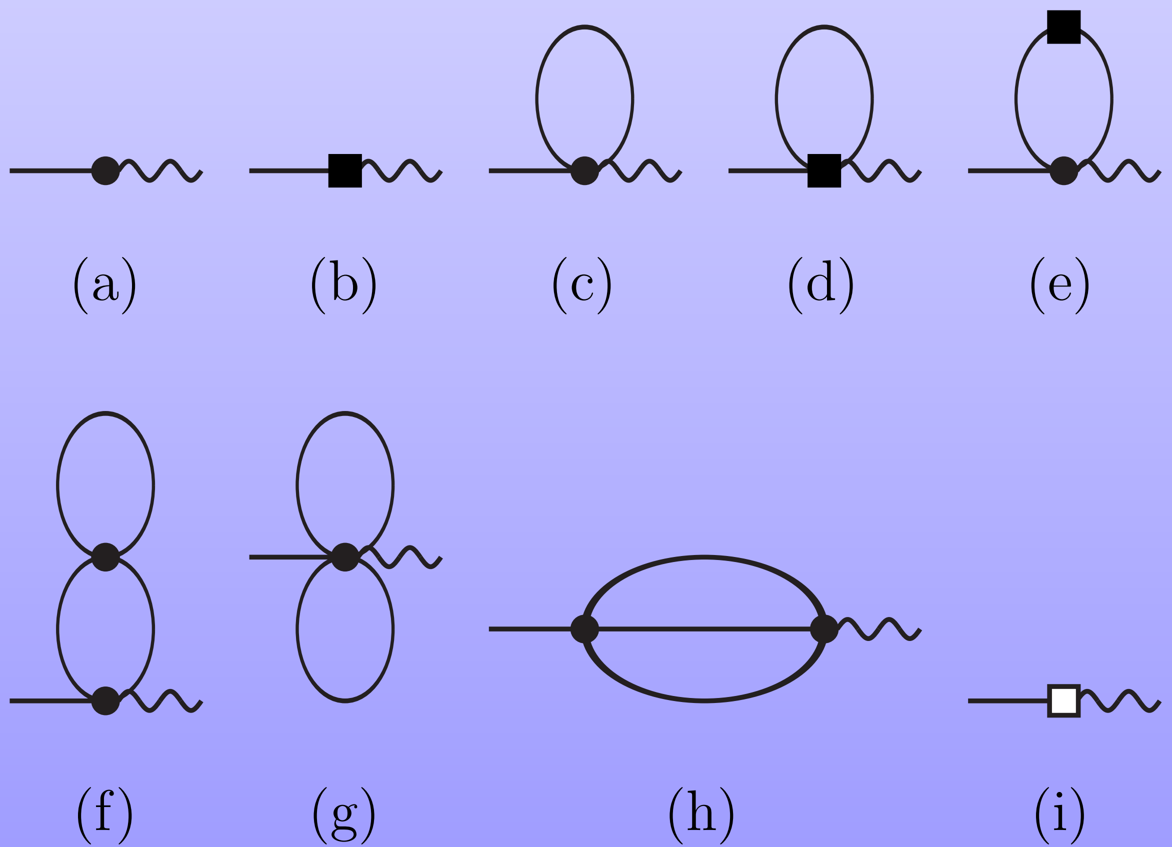
$$C_i \equiv (\mu c)^{-4\epsilon}\left(\frac{\gamma_{2i}}{\epsilon^2} + \frac{\gamma_{1i}}{\epsilon} + C_i^r(\mu)\right) = \mu^{-4\epsilon}\left(\gamma_{2i}\lambda_2 + \gamma_{1i}\lambda_1 + C_i^r(\mu) + \mathcal{O}(\epsilon)\right)$$

Decay constant



$$\Gamma^{(0)}(\pi \to \ell\nu) = \frac{G_F^2 |V_{ud}|^2 F_\pi^2}{4\pi} m_\pi m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

Def. $\langle 0|A_\mu(0)|\pi^-(p)\rangle = i\sqrt{2} p_\mu F_\pi$; $\quad A_\mu = \bar{u}\gamma_\mu\gamma_5 d$



Wavefunction renormalization (LSZ):

$$_{out}\langle\phi_1 ... \phi_i|\phi_i ... \phi_n\rangle_{in} = \langle\phi_1 ... \phi_n\rangle = Z^{-\frac{n}{2}} G_{trunc}(\phi_1, ..., \phi_n)$$

$$\phi' = \sqrt{Z}\phi \qquad Z = 1 + \frac{d\Sigma}{dp^2}\Big|_{p^2 = M_{phys}^2}$$

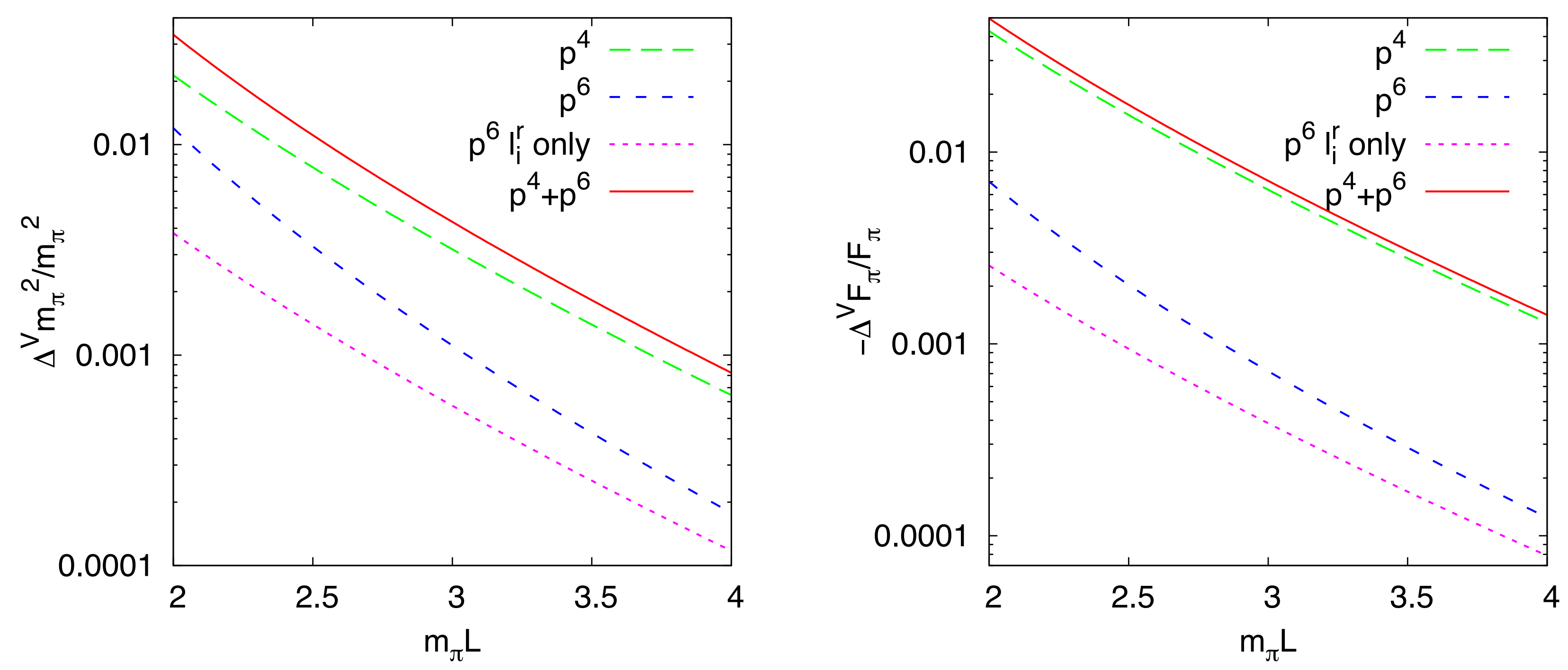
$$\langle\phi\phi\rangle \simeq \frac{i}{Z(p^2 - M_{phys})^2} + \text{non-pole terms}$$

$$\langle\phi'\phi'\rangle \simeq \frac{i}{(p^2 - M_{phys})^2} + \text{non-pole terms,}$$

$$\langle\phi a^\mu\rangle \simeq \frac{i}{Z(p^2 - M_{phys})^2} i\Pi + \text{non-pole terms}$$

$$\langle\phi' a^\mu\rangle \simeq \frac{i}{\sqrt{Z}(p^2 - M_{phys}^2)} i\Pi + \text{non-pole terms}$$

Numerics: Two flavour



(a) (b)

Figure 2: The relative finite volume corrections for the mass squared and decay constant of the pion in two-flavour ChPT at a fixed infinite volume pion mass $m_\pi = m_{\pi^0}$. Shown are the one-loop or p^4 corrections, the full p^6 result and the part only dependent on the l_i^r, $p^6 l_i^r$ and the sum of the p^4 and p^6 result. $m_\pi L = 2, 4$ correspond to $L \approx 2.9, 5.8$ fm. (a) The pion mass, plotted is $(m_\pi^{V2} - m_\pi^2)/m_\pi^2$. (b) The pion decay constant. Plotted is $-(F_\pi^V - F_\pi)/F_\pi$.

Numerics: Two flavour unphysical

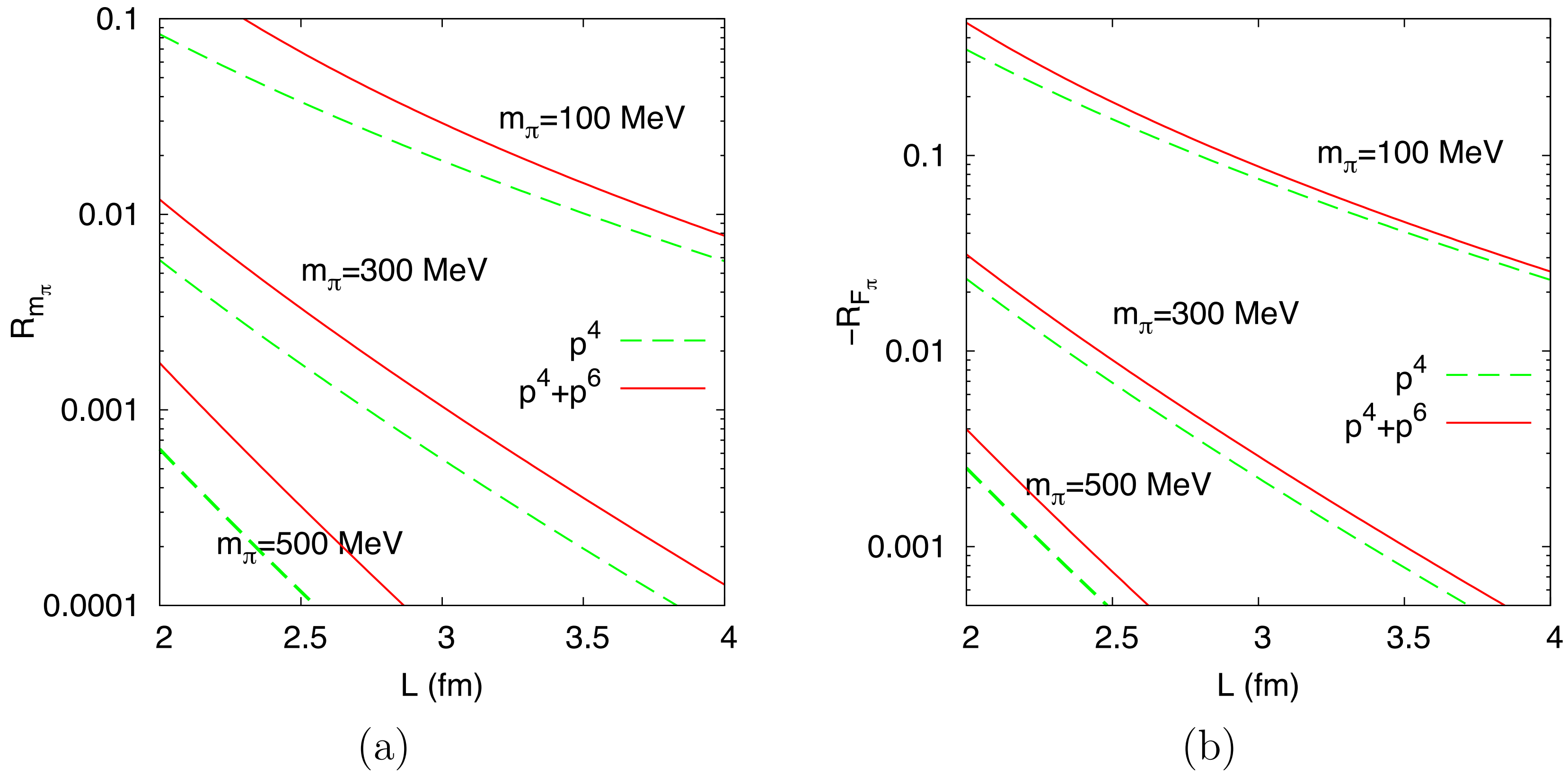
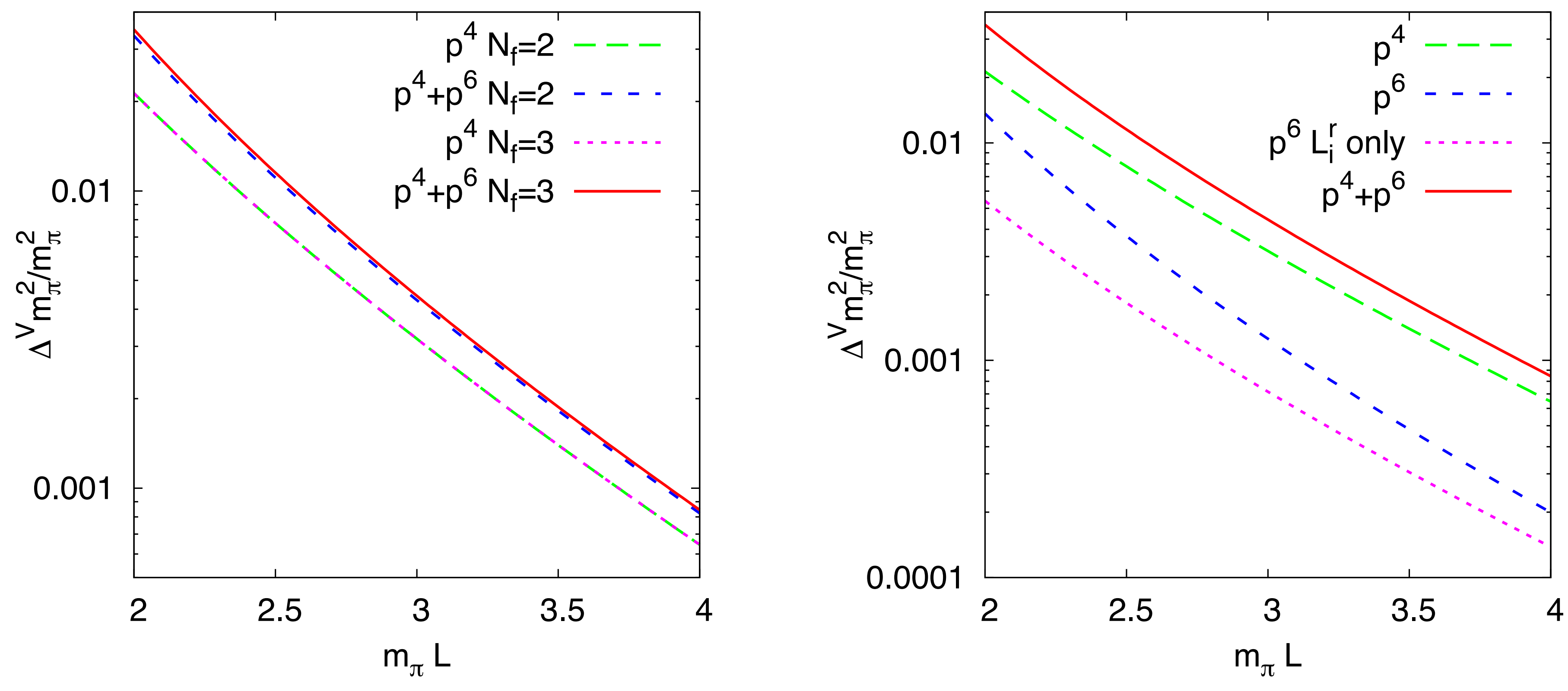


Figure 3: The relative finite volume corrections for the mass and decay constant of the pion in two-flavour ChPT at three values of the infinite volume pion mass. (a) $R_{m_\pi} = m_\pi^V/m_\pi - 1$. (b) $R_{F_\pi} = F_\pi^V/F_\pi - 1$, plotted is $-R_{F_\pi}$.

Numerics: Three flavour



(a)

(b)

Numerics: Kaon and eta mass

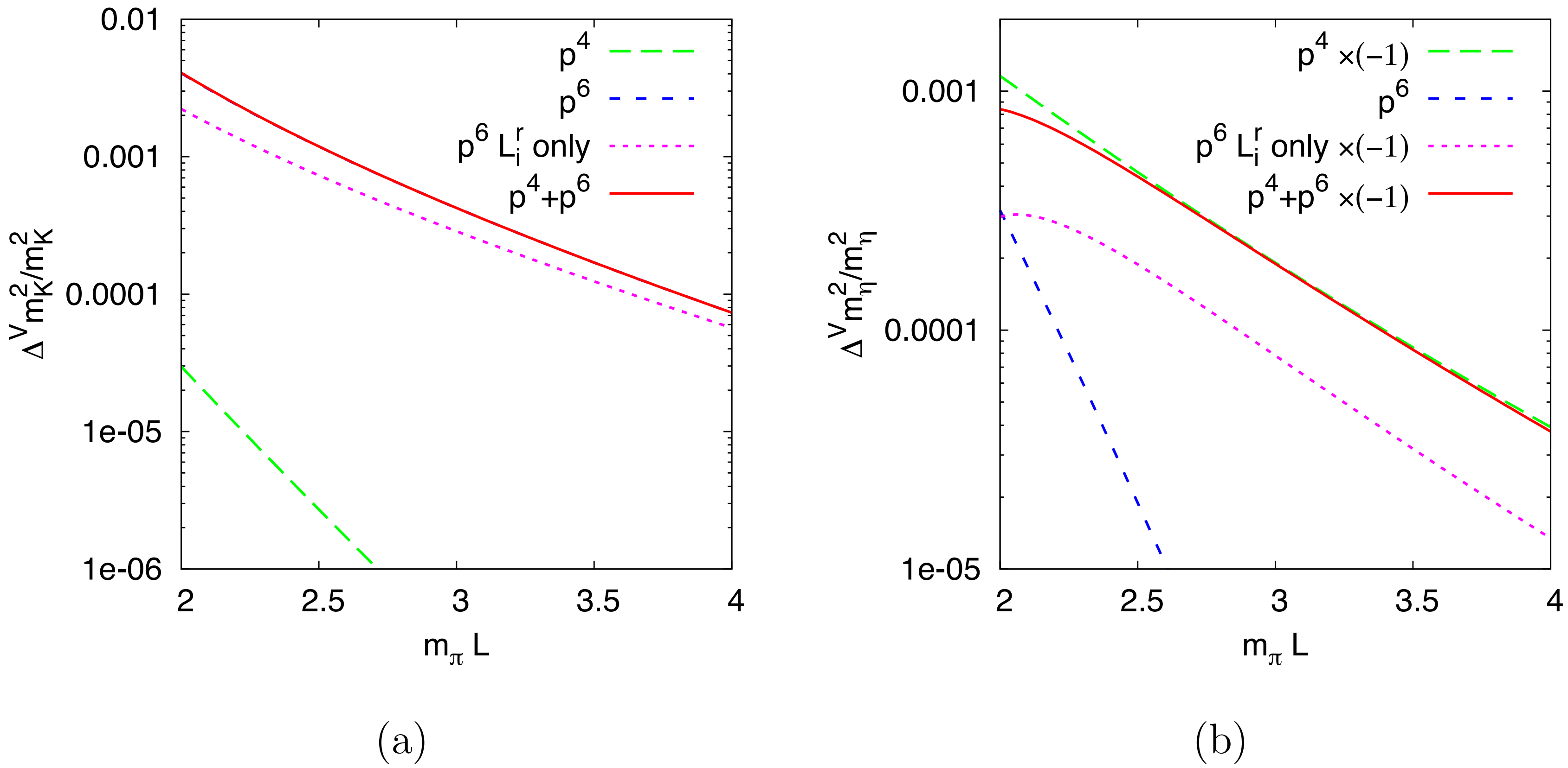


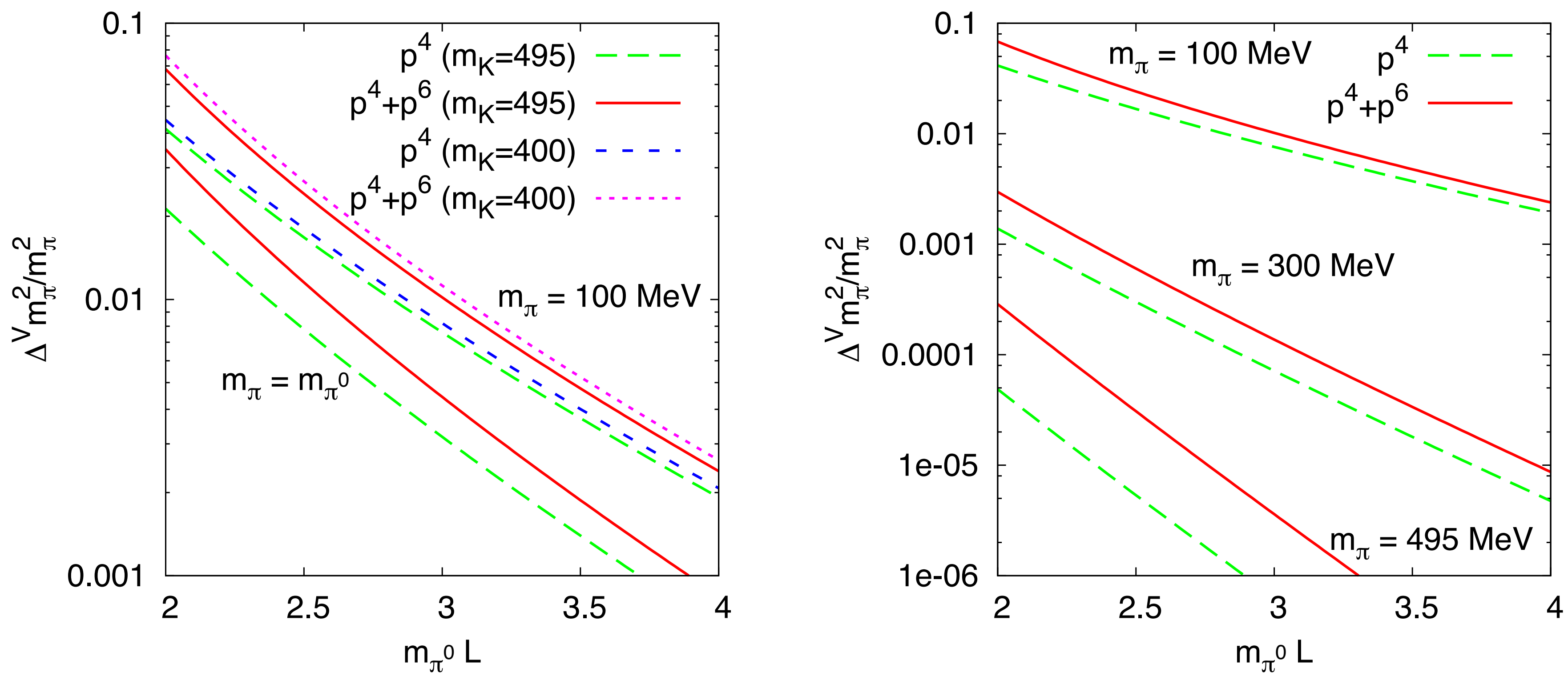
Figure 5: The corrections to the kaon and eta mass squared for the physical case. Plotted is the quantity $(m_i^{V2} - m_i^2)/m_i^2$ for $i = K, \eta$. Shown are the one-loop, the two-loop, the sum and the two-loop L_i^r dependent part. (a) Kaon, the p^4 is so small that p^6 and $p^4 + p^6$ are indistinguishable. (b) Eta, note the signs, some parts are negative.

Kaon: $F_\pi^2 \Delta^V m_K^{2(4)} \;=\; A^V(m_\eta^2)\left(-1/4\, m_\eta^2 - 1/12\, m_\pi^2\right)$

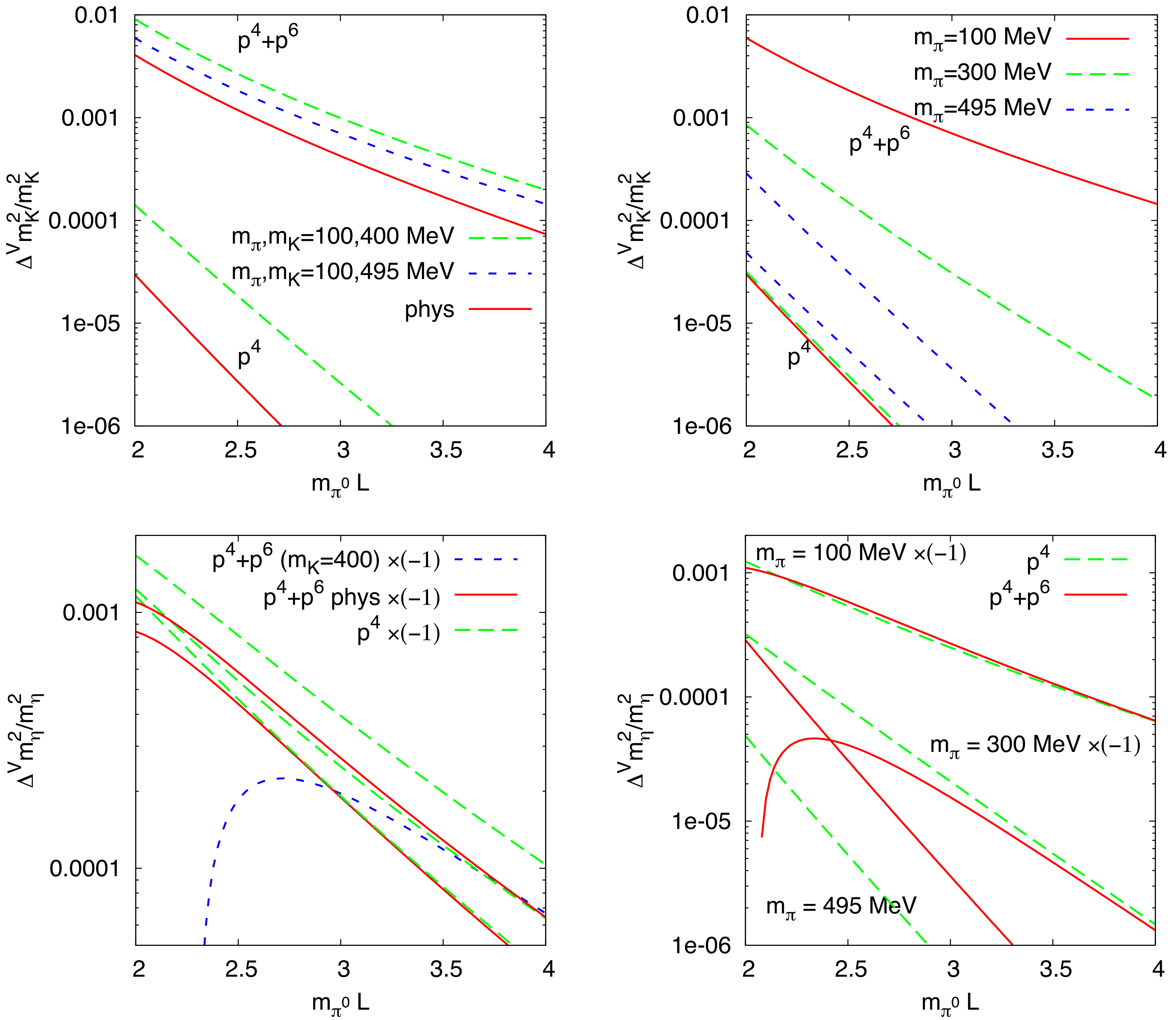
Eta: note cancellation at two-loops; note negative

Numerics: Three flavour unphysical

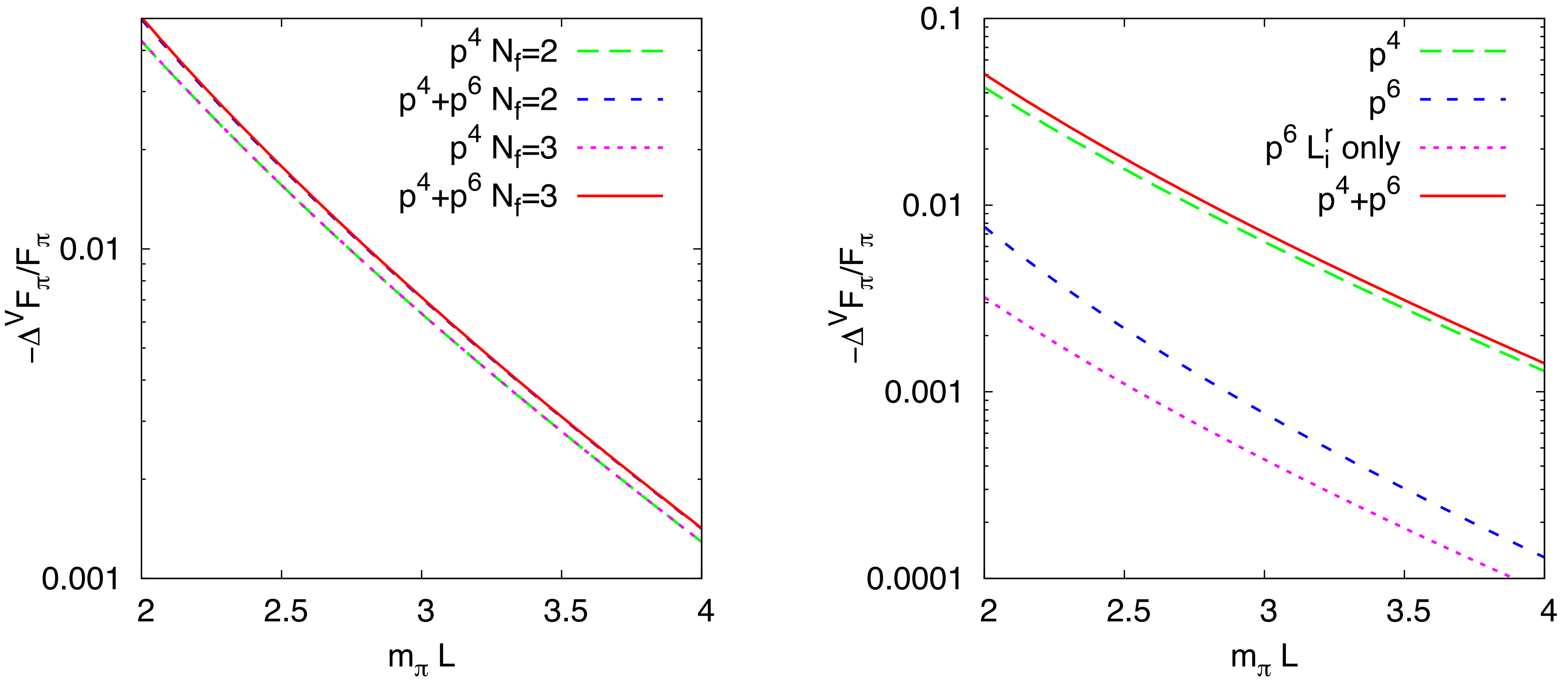
Pion mass dominance



Numerics: Kaon and eta mass unphysical

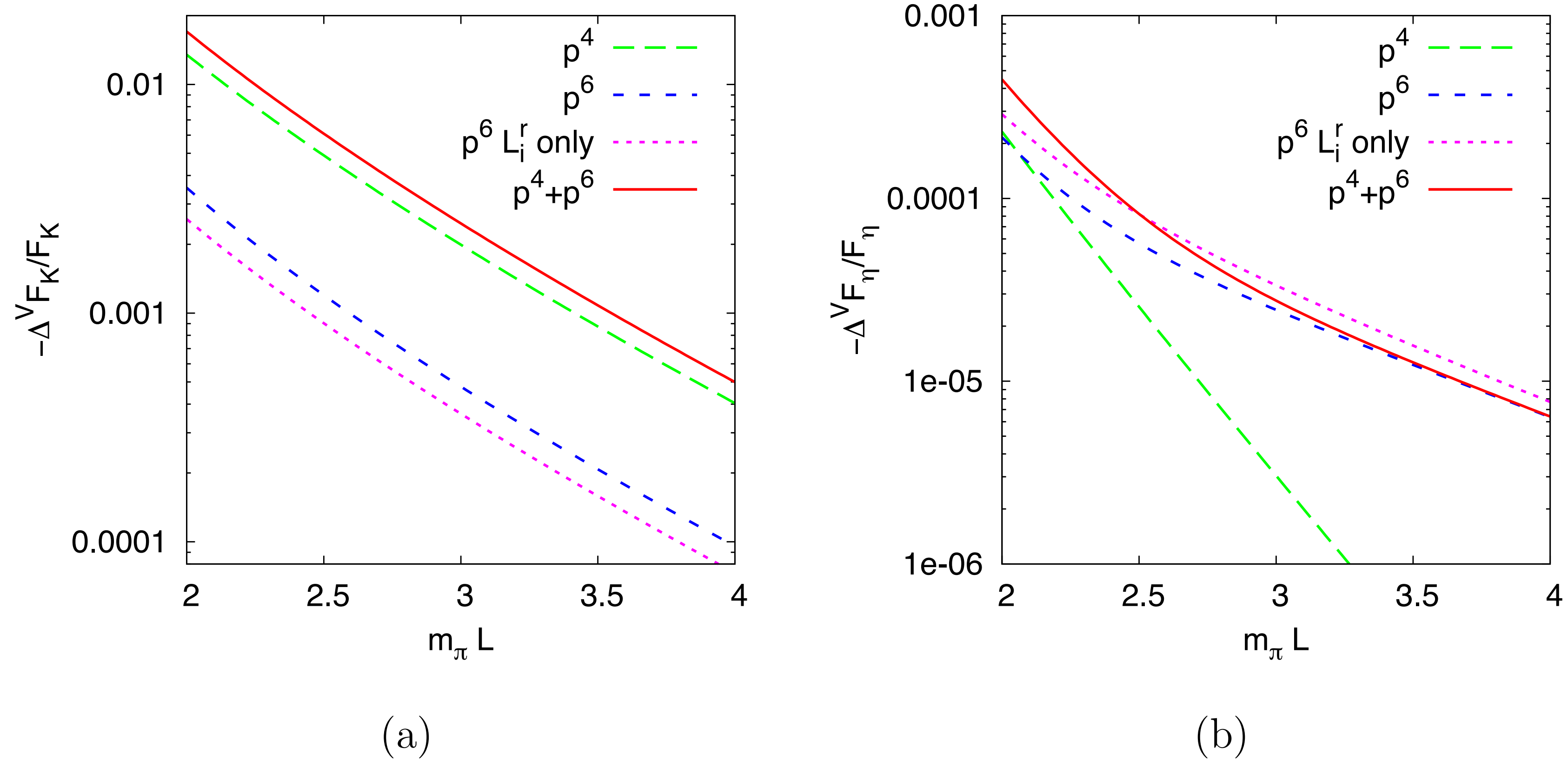


Numerics: Three flavour Decay constant



Numerics: Three flavour Decay constant

Kaon & eta



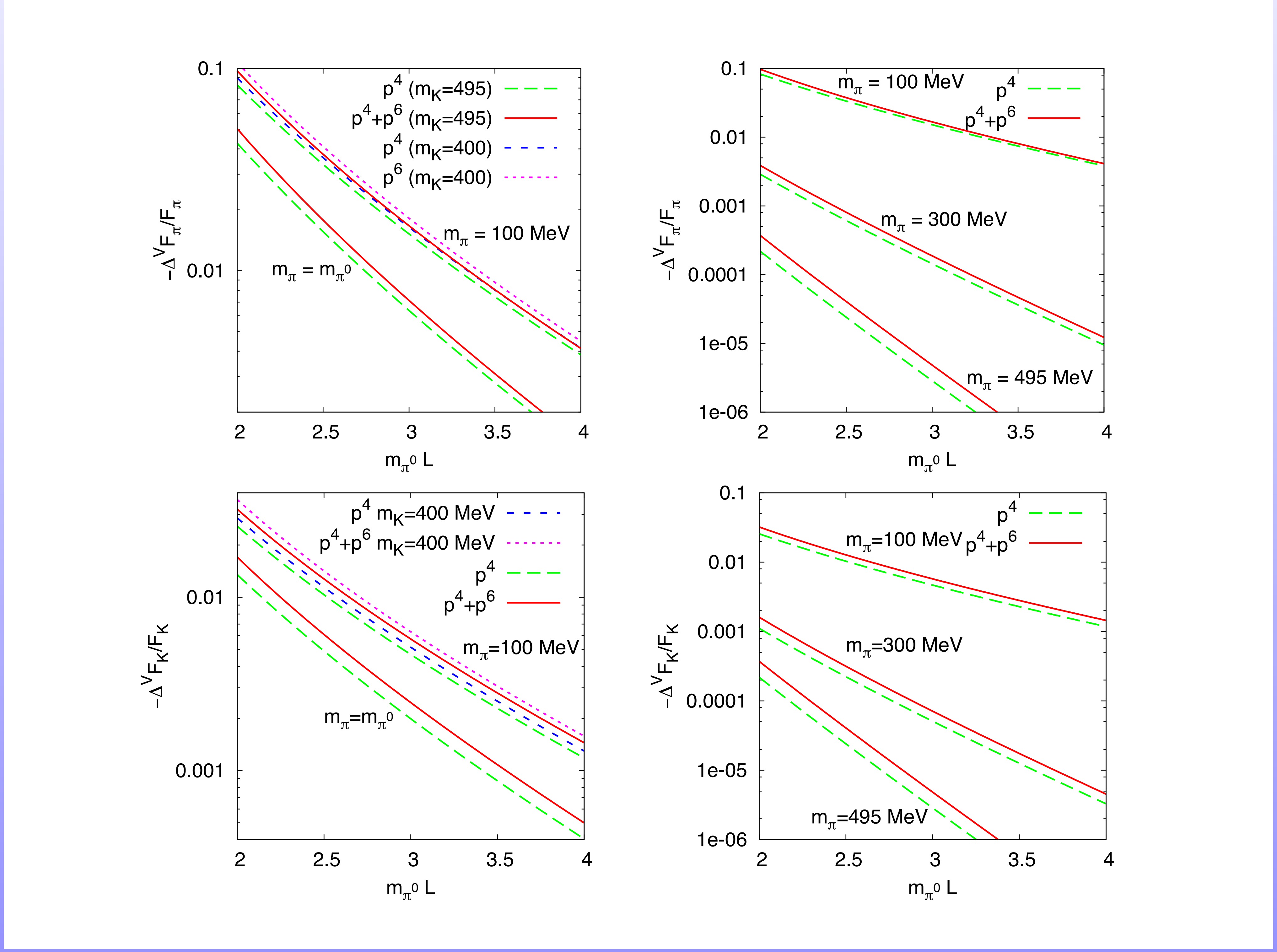
(a) (b)

Figure 10: The corrections to the kaon and eta decay constant for the physical case. Plotted is the quantity $-(F_i^V - F_i)/F_i$ for $i = K, \eta$. Shown are the one-loop, the two-loop, the sum and the two-loop L_i^r dependent part. (a) Kaon. (b) Eta.

$$F_\pi \Delta^V F_\eta^{(4)} = A^V(m_K^2)\left(3/2\right)$$

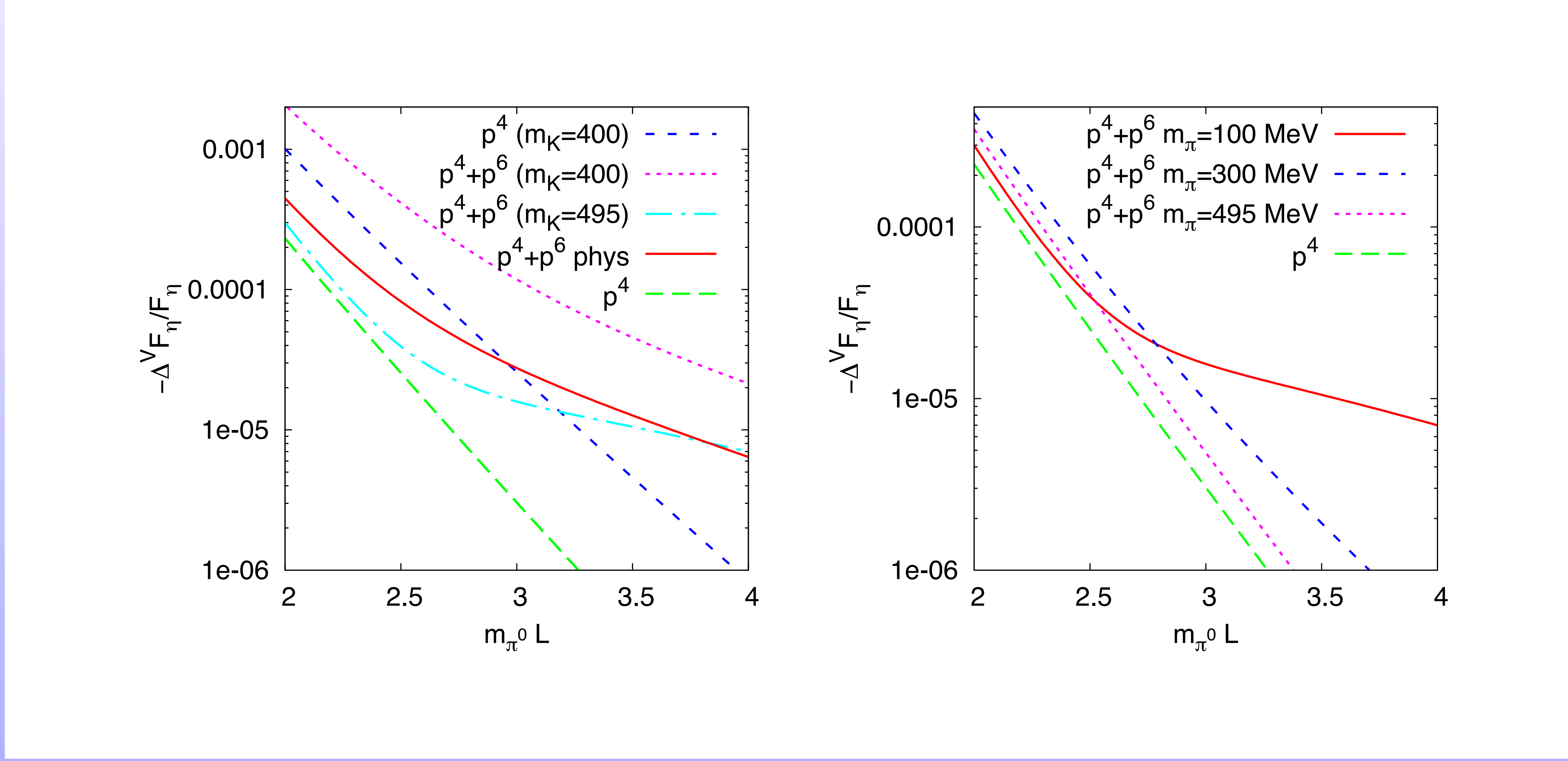
Numerics: Three flavour Decay constant

Unphysical input



Numerics: Three flavour Decay constant

Appreciating the small things in life…



$$F_\pi \Delta^V F_\eta^{(4)} \;=\; A^V(m_K^2)\left(3/2\right)$$

Conclusions and outlook

- We have calculated FV corrections up to two-loop order in two- and three-flavour ChPT. Analytical expressions, see paper.
- At one-loop, full agreement with literature.
- Comparisons of two-loop terms, wherever possible, with existing work, in particular papers by G. Colangelo et al.: one analytical deviation found, single pre-factor (details see text)
- FV corrections evaluated numerically. Found to be necessary for pion mass and decay constant and kaon decay constant (less relevant for kaon mass, negligible for the eta quantities)

- Next: PQChPT. We really see the need to publish longer expressions.