Masses, Decay Constants and Electromagnetic Form-factors with Twisted Boundary Conditions

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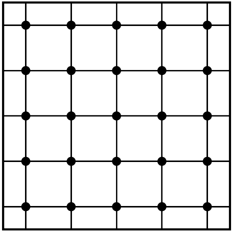
[1] J. Bijnens and J. Relefors, [arXiv:1402.1385 [hep-lat]].

Corrections in lattice QCD

Current lattice QCD techniques require corrections from theory due to

- Lattice discretization
- Finite volume
- Boundary conditions
- Quark masses

Can be analyzed using EFT techniques



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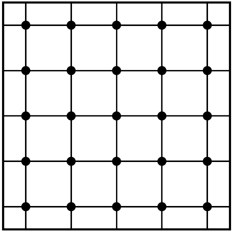

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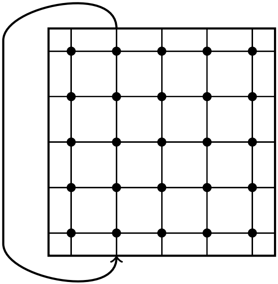
This talk: finite volume effects with twisted boundary conditions at one loop in SU(3) ChPT

Outline

- Twisted boundary conditions
- Physics motivation
- Some results

Twisted boundary conditions [1]



$$\psi(x + L) = \psi(x)$$

$$\Rightarrow p_i = \frac{2\pi}{L} n_i$$

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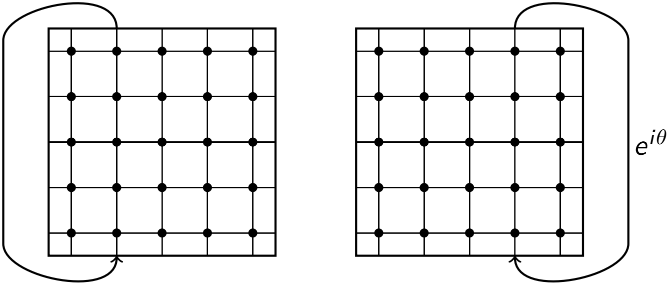

$$\psi(x+L)=\psi(x)$$
$$\Rightarrow p_i=\frac{2\pi}{L}n_i$$

$$\psi(x_i+L)=e^{i\theta_i}\psi(x_i)$$
$$\Rightarrow p_i=\frac{2\pi}{L}n_i+\theta_i/L$$

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Quarks with twist

- Quark and antiquark have opposite twists
- Different twists for each quark flavor (break flavor symmetry)
- Different twists in each direction
- Different twists for valence and sea quarks (partial twisting)

[1] C. T. Sachrajda and G. Villadoro, [hep-lat/0411033].

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- Can reinterpret as constant background field [1]

$$
\begin{aligned}
\psi(x) &= e^{i\theta}\psi(x+L) \\
\tilde{\psi}(x) &= \psi(x)e^{-i\theta x/L} \Rightarrow \tilde{\psi}(x) = \tilde{\psi}(x+L) \\
\bar{\psi}\partial_\mu\psi &= \bar{\tilde{\psi}}(\partial_\mu + iB_\mu)\tilde{\psi}, \qquad B_i = \theta_i/L
\end{aligned}
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$$\tilde{\psi}(x) = \psi(x)e^{-i\theta x/L} \Rightarrow \tilde{\psi}(x) = \tilde{\psi}(x+L)$$
$$\bar{\psi}\partial_\mu\psi = \bar{\tilde{\psi}}(\partial_\mu + iB_\mu)\tilde{\psi}, \qquad B_i = \theta_i/L$$

Momenta are $\vec{k} = \frac{2\pi}{L}\vec{n} + \frac{\vec{\theta}}{L}$ or $\vec{k} = \frac{2\pi}{L}\vec{n}$

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Twisting in meson EFT

Twist of meson $\phi_{q\bar{q}'}(x+L) = e^{i(\theta_q - \theta_{q'})}\phi_{q\bar{q}'}(x)$

- Neutral mesons have no twist
- Meson and anti-meson carry different momenta
- Isospin broken
- Discrete momenta $\int \frac{d^d k}{(2\pi)^d} \to \int \frac{d^{d-3}k}{(2\pi)^{d-3}} \frac{1}{L^3} \sum_{\vec{k}} = \int_V \frac{d^d k}{(2\pi)^d}$
- Broken reflection symmetry $\int_V \frac{d^d k}{(2\pi)^d} \frac{k^\mu}{k^2 - m^2} \neq 0$
- Loss of Lorentz and cubic symmetry gives fewer constraints on form factors

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$\sim$ normal calculation but question every step!

A note on masses

- We define $m^2 = E_0^2 - \vec{p}^2$ for fix $\vec{p} \Rightarrow m = m(p_\mu)$ at one loop
- Alternative: define $m^2 = E_0^2 - \left(\vec{p} + \vec{K}\right)^2$ where $\vec{K}$ is NLO. Momentum is renormalized at one loop [1]
- For decay constant we then get $\left\langle 0 | A_\mu^M | M(p) \right\rangle = i\sqrt{2} F_M \left(p_\mu + K'_\mu\right)$, however $\vec{K}' \neq \vec{K}$
- With our definition charge conjugation requires $p \to -p$ in order to get same NLO mass

[1] F.-J. Jiang and B. C. Tiburzi, [hep-lat/0610103].

Physics motivation

- Any momentum dependent physical quantity, e.g. form factors
- Allows variation of q^2 for fixed values of quark masses
- Charge radius of pion [1] $f_\pi(q^2) = 1 - \frac{1}{6} < r_\pi^2 > q^2 + \mathcal{O}(q^4)$
- Muon g-2 hadronic vacuum polarization peak at $p^2 \sim (m_\mu/2)^2 \Rightarrow L = 25 fm$ [2]
- K_{l3} decay at close to zero momentum relevant for V_{us} [3]

[1] B. B. Brandt, *et al.*, [arXiv:1306.2916 [hep-lat]].

[2] C. Aubin, *et al.*, [arXiv:1307.4701 [hep-lat]].

[3] P. A. Boyle *et al.*, [arXiv:1504.01692 [hep-lat]].

Our calculations

One loop results for

- Vacuum expectation values and two-point function
- Masses
- Decay constants (axial, pseudoscalar)
- Electromagnetic form-factor
- Ward identities

Pion mass

$$\Delta^V m_{\pi^\pm}^2 = \frac{\pm p^\mu}{F_0^2}[-2A_\mu^V(m_{\pi^+}^2) - A_\mu^V(m_{K^+}^2) + A_\mu^V(m_{K^0}^2)]$$
$$+ \frac{m_\pi^2}{F_0^2}\left(-\frac{1}{2}A^V(m_{\pi^0}^2) + \frac{1}{6}A^V(m_\eta^2)\right) ,$$

$$A_\mu^V(m_{\pi^+}^2) = \int_V \frac{d^d k}{(2\pi)^d}\frac{k_\mu}{k^2 - m_{\pi^+}^2}$$

$$A^V(m_{\pi^+}^2) = \int_V \frac{d^d k}{(2\pi)^d}\frac{1}{k^2 - m_{\pi^+}^2}$$

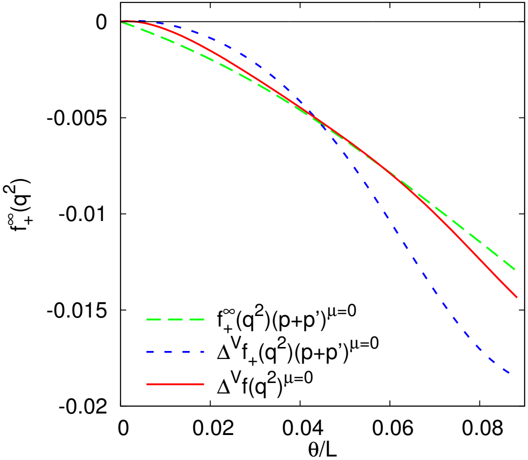
Both integrals depend on twist angles θ_i!

Electromagnetic form-factor

$$\left\langle M'(p')|j^I_\mu|M(p)\right\rangle = f_{IMM'\mu}$$
$$= f_{IMM'+}(p_\mu + p'_\mu) + f_{IMM'-}(p_\mu - p'_\mu) + h_{IMM'\mu}\,.$$

- Still satisfies Ward identity $(p_\mu - p'_\mu)f_{IMM'\mu} = 0$
- For $M = M'$ twisting gives $p_i - p'_i = \frac{2\pi}{L}(n_i - n'_i)$
- $\langle\pi^+(p')|\bar{u}\gamma_\mu u|\pi^+(p)\rangle = -\frac{1}{\sqrt{2}}\left\langle\pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\right\rangle$ from (broken) isospin

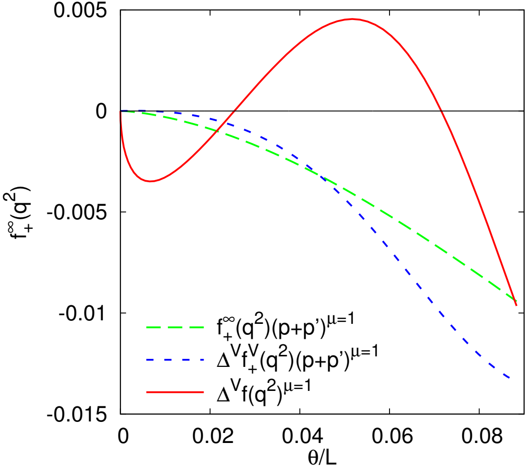
$\langle \pi^0(p') | \bar{d}\gamma_\mu u | \pi^+(p) \rangle$ numerics



- Varying q^2
- $L_9^r = 0$
- $m_\pi L = 2$
- Twist
  u-quark

$$-\frac{1}{\sqrt{2}} f_{\bar{d}\gamma u \pi^+ \pi^0 \mu} = \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$$

$\langle \pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$ numerics



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Ward identity

Let $q_\mu = (p_\mu - p'_\mu)$, then

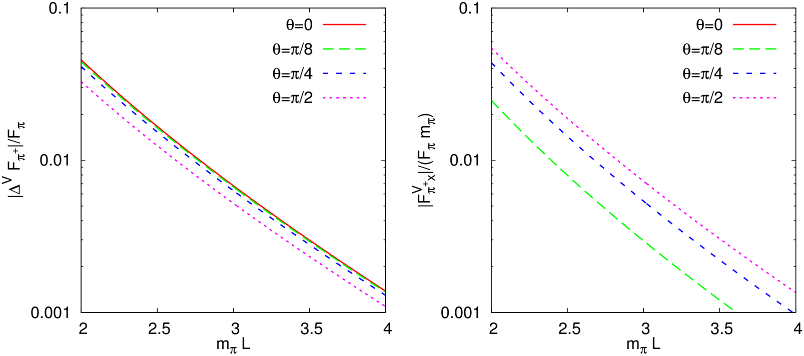
$$0 = q_\mu \left\langle M'(p')|j^I_\mu|M(p)\right\rangle$$
$$= f_{IMM'+}(p^2 - {p'}^2) + f_{IMM'-}q^2 + h_{IMM'\mu}q_\mu$$

- $p^2 - {p'}^2 \neq 0$ even for $M = M'$ since mass of in and out state differ
- Field strength for in and out states also differ for $M = M'$
- Cancellations between
  $f_{IMM'+}(p^2 - {p'}^2)$ and $f_{IMM'-}q^2$
  $f_{IMM'+}(p^2 - {p'}^2)$ and $h_{IMM'\mu}q_\mu$

Decay constants

- $\langle 0|A_\mu^M|M(p)\rangle = i\sqrt{2}F_M p_\mu + i\sqrt{2}F_{M\mu}^V$
- $\langle 0|P^M|M(p)\rangle = \frac{G_M}{\sqrt{2}}$
- For charged states $p^2 F_M + p^\mu F_{M\mu}^V = \frac{1}{2}(m_q + m_{q'})G_M$
- Isospin is broken $\Rightarrow$ decay of η through π^0 current
- $\Delta^V F_M p_\mu$ and $F_{M\mu}^V$ of similar size

Decay constants



$$\langle 0|A_\mu^M|M(p)\rangle = i\sqrt{2}F_M p_\mu + i\sqrt{2}F^V_{M\mu}$$

Kl3, $\langle \pi^- | \bar{s} \gamma_\mu u | K^0 \rangle$

- Partially quenched and partially twisted done analytically, no numerics yet
- Ongoing work: one loop finite volume corrections in partially twisted, partially quenched, staggered ChPT for K_{l3} decay [1]

[1] C. Bernard, J. Bijnens, E. Gamiz, J. Relefors, Work in progress

Summary and outlook

- Twisted boundary conditions allow for continuous variation of momenta
- Loss of reflection symmetry leads to new momentum dependence in form-factors, masses,...
- Non trivial charge conjugation and treatment of in and outgoing particles
- Size of new term in electromagnetic form factor is non-negligable
- Work in progress: partially twisted, partially quenched, staggered ChPT for K_{l3} decay [1]

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