# Excited-State Contamination in Nucleon Correlation Functions from Chiral Perturbation Theory

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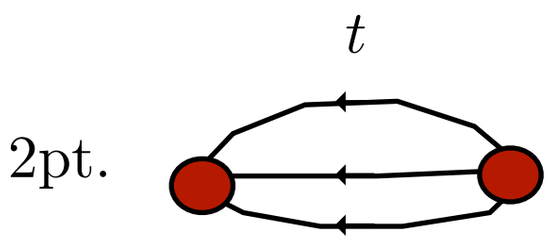
30 June 2015

# Lattice QCD Motivation

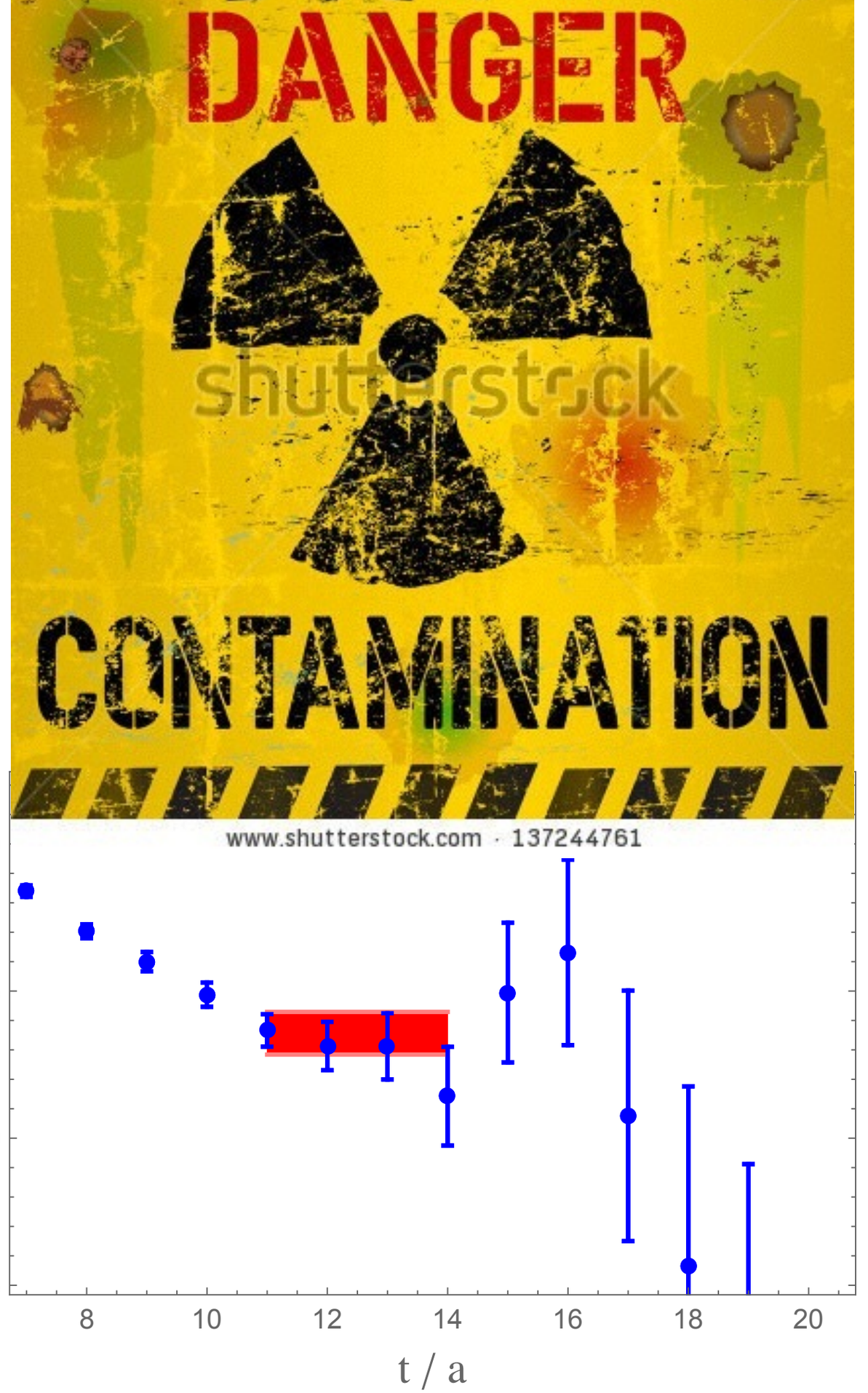
- Large (Euclidean) time used as filter for ground-state hadrons




$$M_{\text{eff}}(t) = M_N + Ze^{-(M'-M_N)t} + \cdots$$

- Baryon correlators suffer signal-to-noise problem at large times

- Statistically precise behavior at early times contaminated by excited states



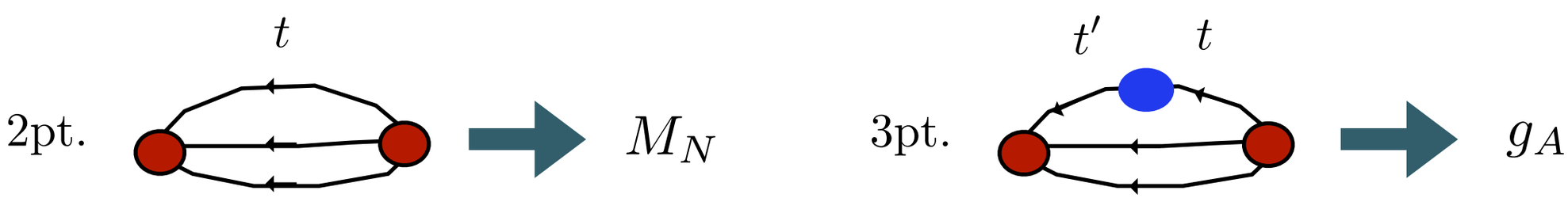
# Outline

- Address **πN** (πΔ) contributions to nucleon correlators … using *ChPT*

  Continuum of states [BCT, Phys. Rev. D**80**:014002 (2009)]

  Discrete states [BCT, Phys. Rev. D**91**:094510 (2015)]

- **Two**-point and axial-vector **Three**-point correlators




- Assess impact on lattice calculations: **t**, **L** dependence

- Aid in construction of better lattice interpolating operators

# Nucleon in ChPT

Local operator $N(x)$

Iso-doublet $N \xrightarrow{SU(2)_V} VN$

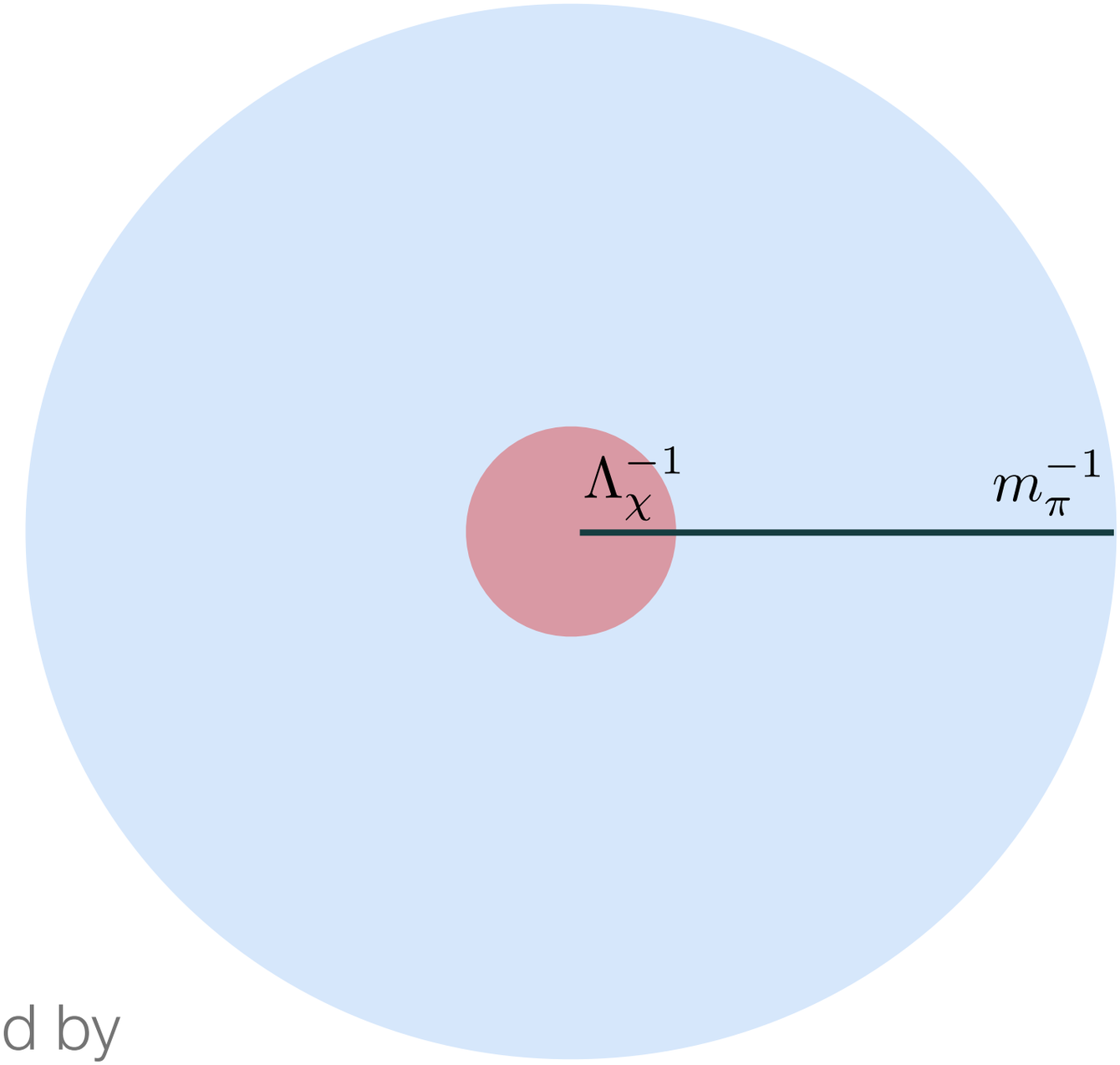
Chiral multiplet

$$N \xrightarrow{SU(2)_L \times SU(2)_R} ?$$

… not an ingredient

$$N \to U\Big(L, R, \xi(x)\Big) N$$

convenient choice




**πN** (πΔ) interactions constrained by pattern of chiral symmetry breaking

# Nucleon in Lattice QCD

Interpolating operator $\mathcal{O}_N(x)$

e.g. $\mathcal{O}_N \sim q\left(q^T C\gamma_5\tau^2 q\right)$

Chiral multiplet

$$(\mathbf{2}_L, \mathbf{1}_R) \oplus (\mathbf{1}_L, \mathbf{2}_R)$$

Maximize overlap with nucleon

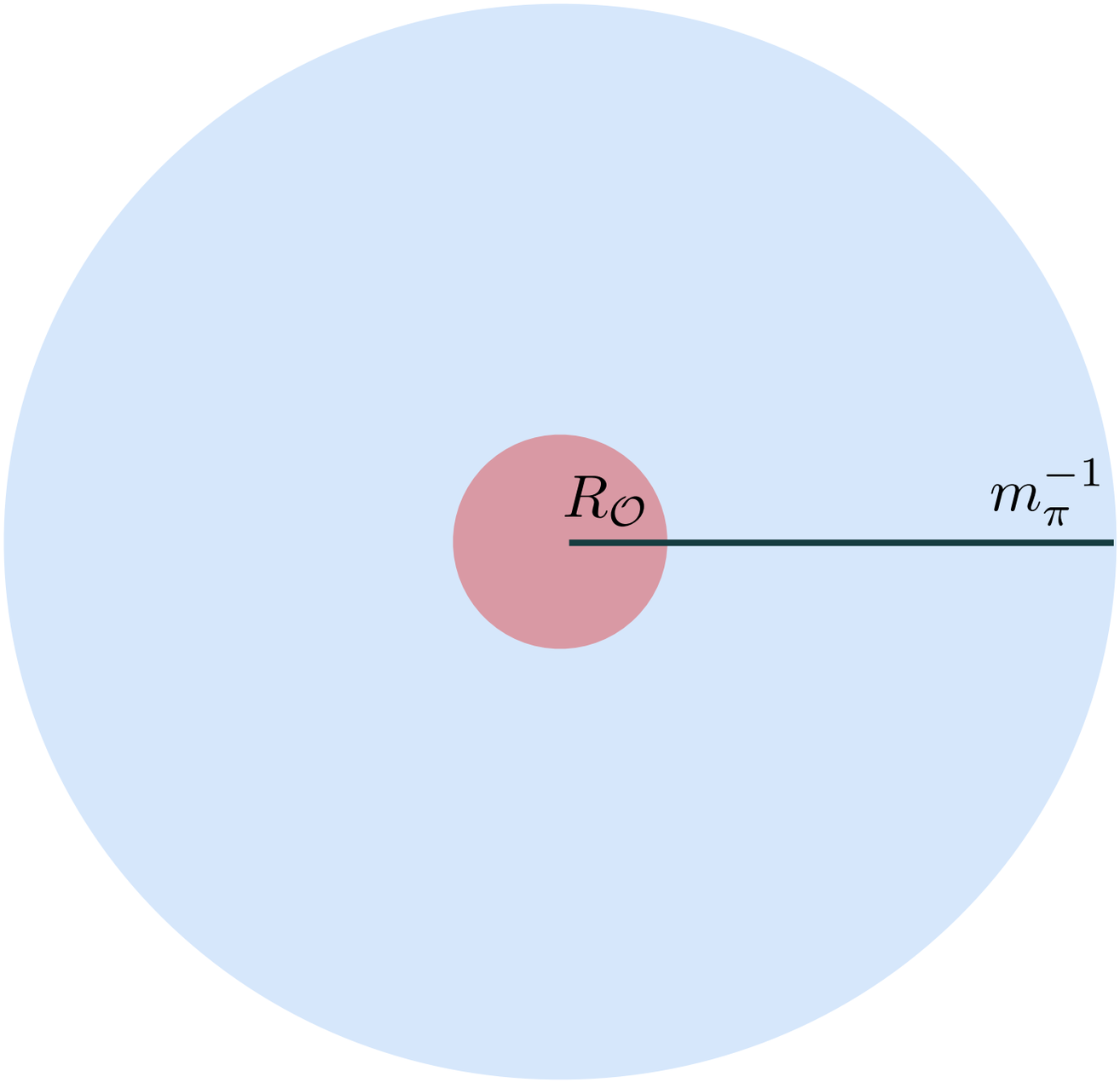
$$Z_{\mathcal{O}} = \langle 0|\mathcal{O}_N|N\rangle$$

πN excited-state couplings

$$Z_{\pi N} = \langle 0|\mathcal{O}_N|(\pi N)_p\rangle$$

… constrained by pattern of chiral symmetry breaking

[Bär, arXiv:1503.03649]



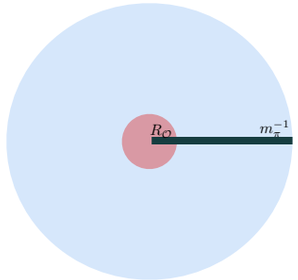

*Related ideas*: ChPT for gradient flow

[Bär & Golterman, Phys.Rev.D**89**:034505(2014)]

# Procedure to Determine Contamination


Lattice interpolating operator $\mathcal{O}_N(x)$

$$R_{\mathcal{O}} \sim M_N^{-1}, \Lambda_\chi^{-1} \ll m_\pi^{-1}$$




ChPT operator for lattice interpolator

Chiral transformation of interpolators

[Nagata *et al.*, Eur.Phys.J.C**57**:557 (2008)]

$$N_{\mathcal{O}} = N_{\mathcal{O}}(N, \xi)$$

Needed for moments of nucleon DA in ChPT

[Wein *et al.*, Eur.Phys.J.A**47**:149 (2011)]

E.g.

$$N_{(\mathbf{2}_L,\mathbf{1}_R)} = \frac{1}{2}\xi N + \mathcal{O}(M_N^{-1})$$

$$N_{(\mathbf{1}_L,\mathbf{2}_R)} = \frac{1}{2}\xi^\dagger N + \mathcal{O}(M_N^{-1})$$

$$N_{(\mathbf{2}_L,\mathbf{1}_R)\oplus(\mathbf{1}_L,\mathbf{2}_R)} = \frac{1}{2}Z_{\mathcal{O}}(\xi + \xi^\dagger)N + \ldots = Z_{\mathcal{O}}\left(1 - \frac{\pi^2}{2f^2}\right)N + \ldots$$

[Bär, arXiv:1503.03649]

[BCT, Phys.Rev.D**91**:094510 (2015)]

+ ordinary pion loop contributions

# Chiral Contamination in Nucleon 2-pt. Function

$$G(\tau) = \sum_{\vec{x}} \langle 0 | N_{\mathcal{O}}(\vec{x},\tau) N^{\dagger}_{\mathcal{O}}(\vec{0},0) | 0 \rangle = |Z_{\mathcal{O}}|^2 e^{-M_N \tau} \left[ 1 + \sum_n |Z_n|^2 e^{-\delta E_n \tau} \right]$$

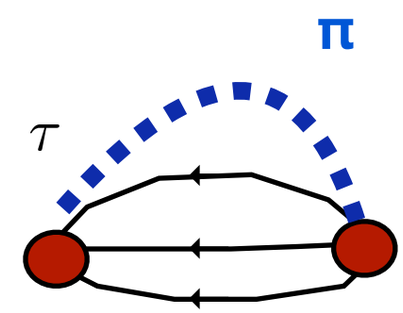
ChPT gives relative contribution from low-lying excitations

Continuum of **πN** states —> spectral density

$$\rho_{\pi N} = \frac{6 g_A^2}{(4\pi f)^2} \frac{(\vec{k}_\pi^2)^{\frac{1}{2}+1}}{E_\pi^2}$$

non-analytic,
2 body phase space,
πN relative p-wave

[BCT, Phys. Rev. D**80**:014002 (2009)]

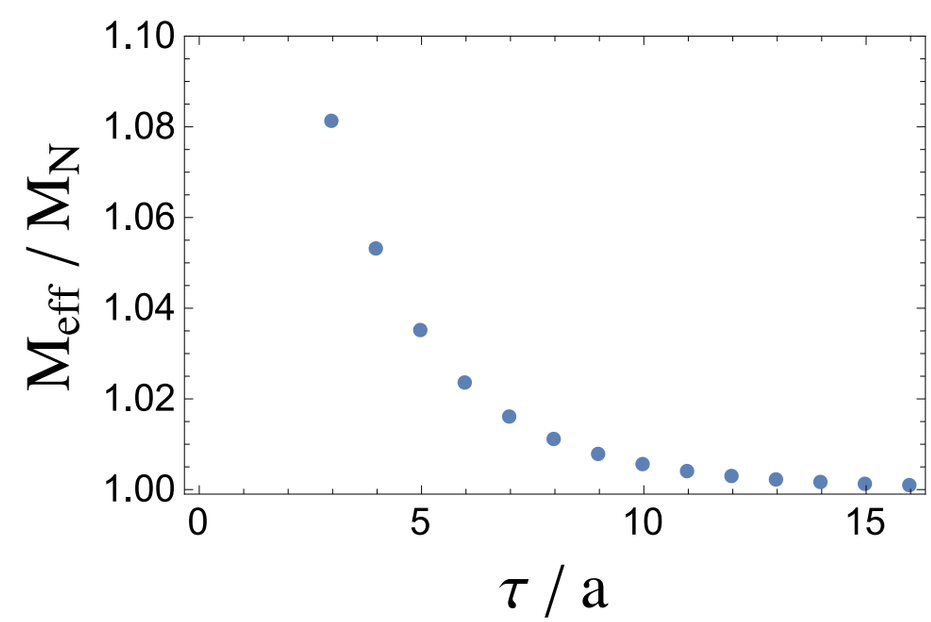



Discrete **πN** states on spatial torus

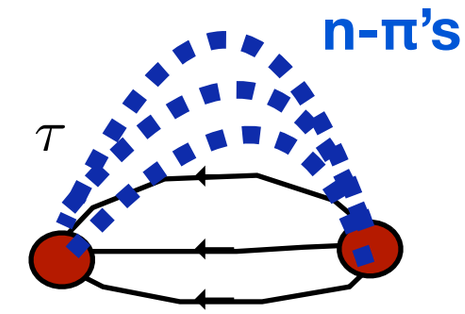
$$|Z_{\pi N}|^2 = \frac{3 g_A^2}{4 f^2 L^3} \frac{\vec{k}_\pi^2}{E_\pi^3}$$

analytic,
≈ no phase space,
πN relative p-wave

[BCT, Phys. Rev. D**91**:094510 (2015)]



# Suppression of Multi-particle States




- (model-independent)-independent

Spectral density near threshold: phase-space + orbital angular momentum

$$\rho_{(\pi)^n N}(E) \propto \sqrt{E - E_{\rm Th}}^{\,3n+\ell-2}$$

Analogous result for spatial torus

$$|Z_{(\pi)^n N}|^2 \propto \sqrt{E - E_{\rm Th}}^{\,n+\ell-1}$$

- model-independent

ChPT gives proportionality constant

$$(g_A/f)^{2n}$$

[FV: no suppression (πN) s-wave near threshold]

**Expansion near threshold valid for**

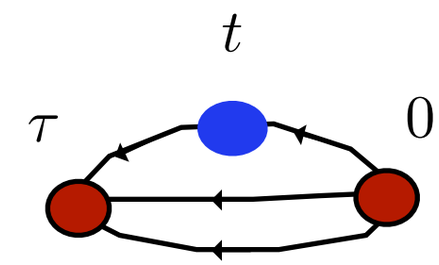
$$E - E_{\rm Th} \approx \frac{\vec{k}^2}{2m_\pi} \ll 1 \quad \Rightarrow \quad m_\pi L \gg 1$$

**Competition with exponential suppression**

Lower energy gap $e^{-E\tau}$ $\uparrow$

1. $(m_\pi \text{ fixed}, \uparrow L)$
2. $(\downarrow m_\pi, L \text{ fixed})$

# Axial Current in the Nucleon




Ratio of 3-pt. to 2-pt. correlators leads to lattice extraction of axial charge

$$G_A(\tau, t) = g_A + \sum_{n.m} \mathfrak{g}_{n,m} e^{-\delta E_n(\tau - t)} e^{-\delta E_m t}$$

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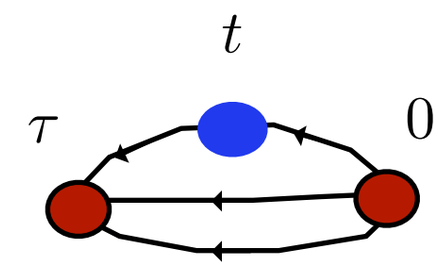


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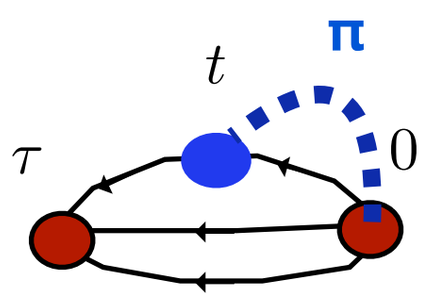
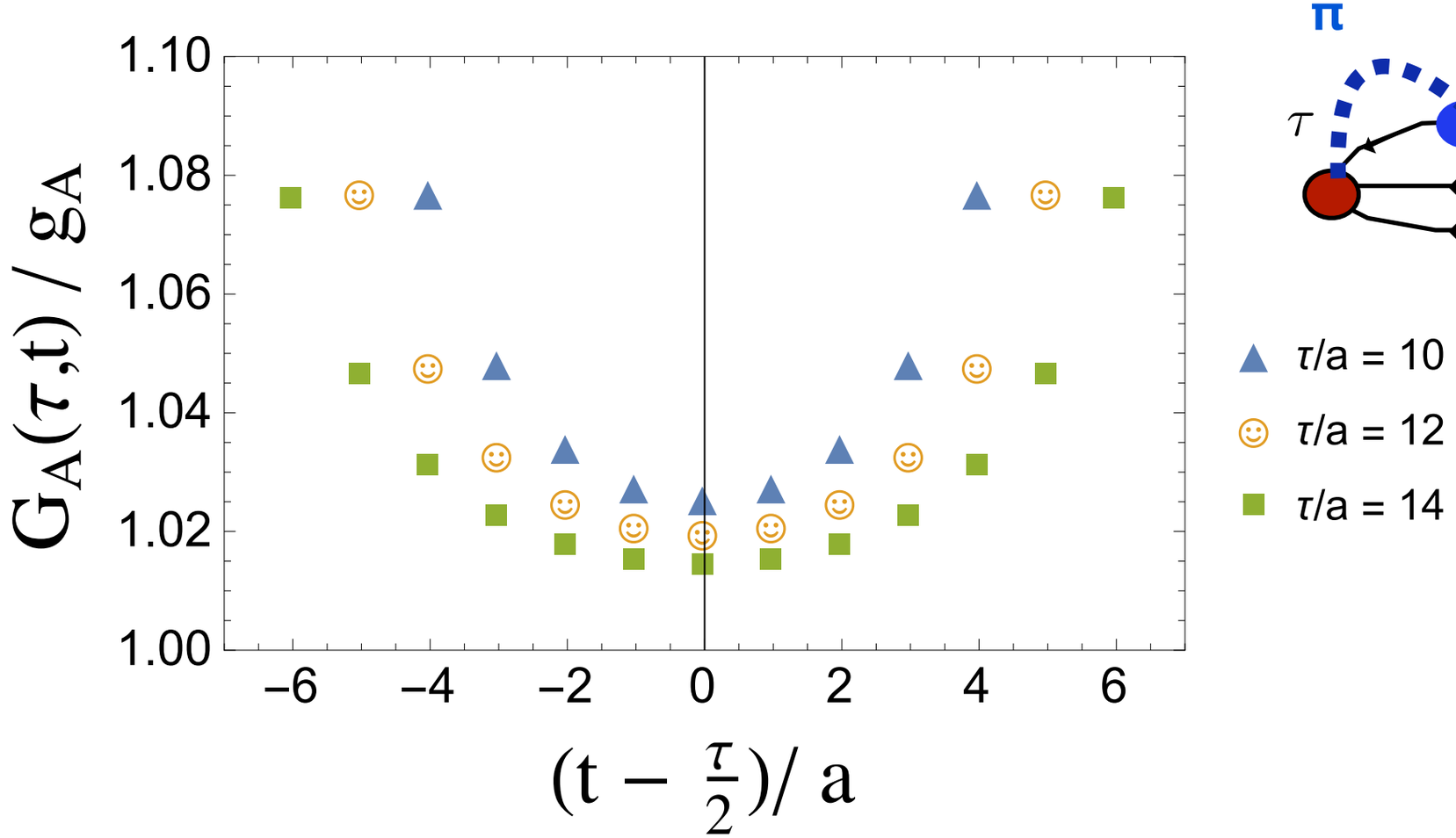
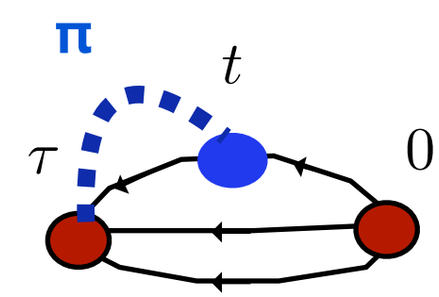
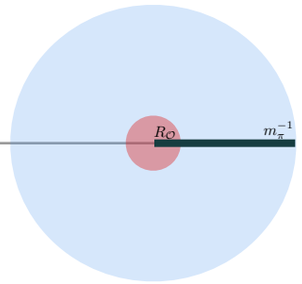
Chiral contamination



# Summary




- ChPT addresses **πN** (πΔ) contributions to nucleon correlators provided operator smearing: $R_{\mathcal{O}} \ll m_\pi^{-1}$

- Two-point functions: phase-space arguments disfavor **πN** contributions. In ChPT, additional suppression $\left(\frac{g_A}{fL}\right)^2$

  Excited states ≈ Excited hadrons (bound)

- Axial-vector current: **N-πN** contribution drives three-point function upwards for insufficient time separation

  Underestimation of axial charge ≠ πN contamination

- **πN** in quark momentum fraction?

- Resonances: need **πN**-type operators [no surprise to Lattice community]