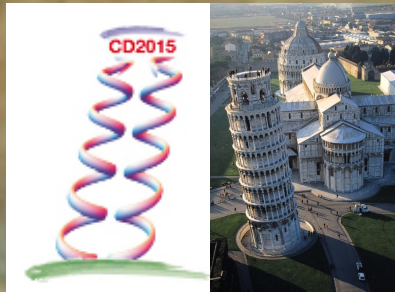


SCALING OF TETRAMER PROPERTIES CLOSE TO THE UNITARY LIMIT



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Four-bosons in 3D with zero-range interaction

Yamashita, Tomio, Delfino & Frederico

Four-boson scale near a Feshbach resonance. Europhys. Lett.75 (2006) 555

- *Tetramer ground state moves as a short-range scale collapses to zero with the trimer is fixed!*
- *coupling between a closed and open channels → many-body forces in the open channel ?*
- *Tetramer is fixed by the trimer information:*

Platter, Hammer, & Meissner,

Four-boson system with short-range interactions. Phys. Rev. A 70, 52101 (2004).

Stecher, D’Incao & Greene,

Signatures of universal four-body phenomena and their relation to the Efimov physics Nat.Phys. 5(09)417

Deltuva

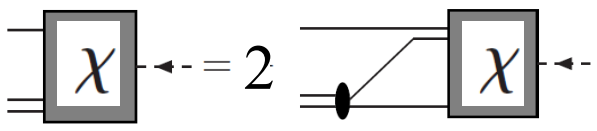
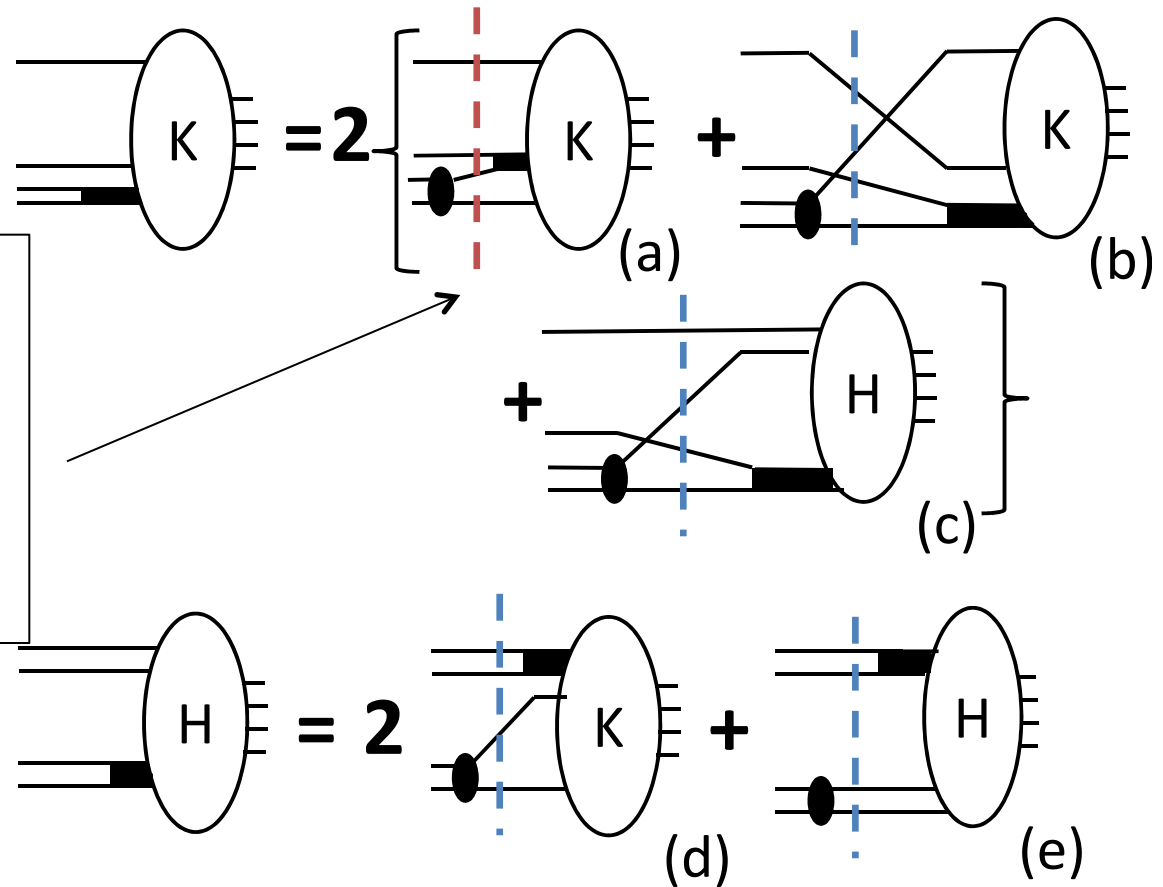
Efimov physics in bosonic atom-trimer scattering, Phys. Rev. A 82, 040701(R) (2010)

Gattobigio, Kievsky, Viviani, Birse, Hiyama...

Short range 4-body forces?

Four-bosons

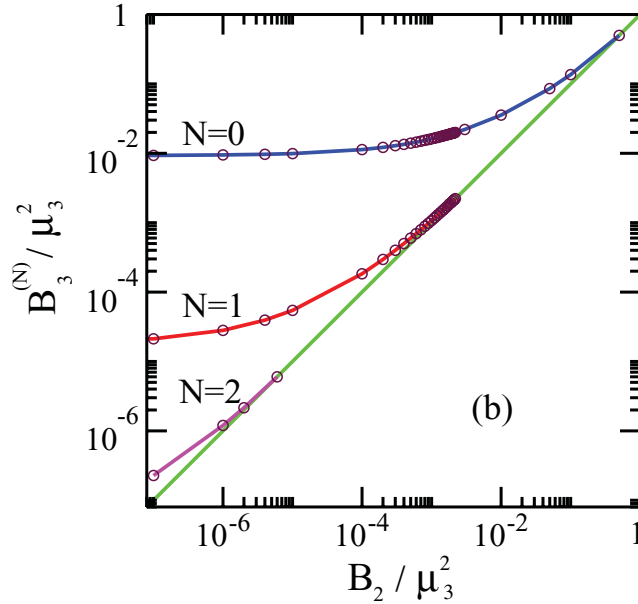
Skorniakov and Ter-Martirosian (1956)

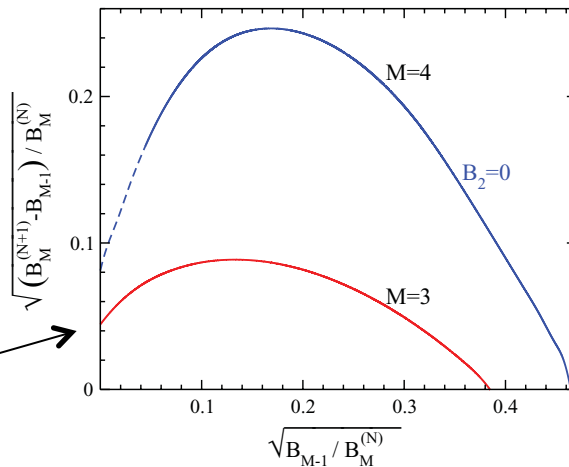
Subtracted Green's Functions: $G_0^{(N)} = \frac{1}{E-H_0} - \frac{1}{-\mu_N^2-H_0}$
 with μ_3 (RED): 3B scale & μ_4 (BLUE): 4B scale

Sliding scales: 3 and 4-body systems

Sliding 3-body scale

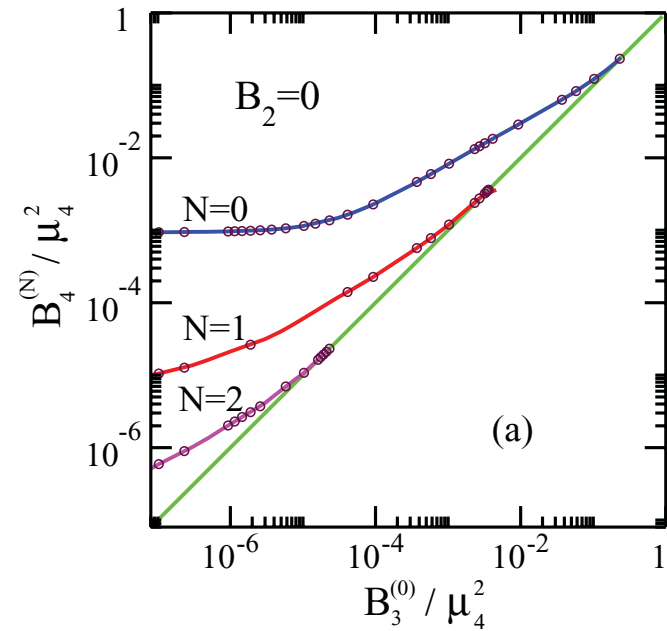


Efimov limit



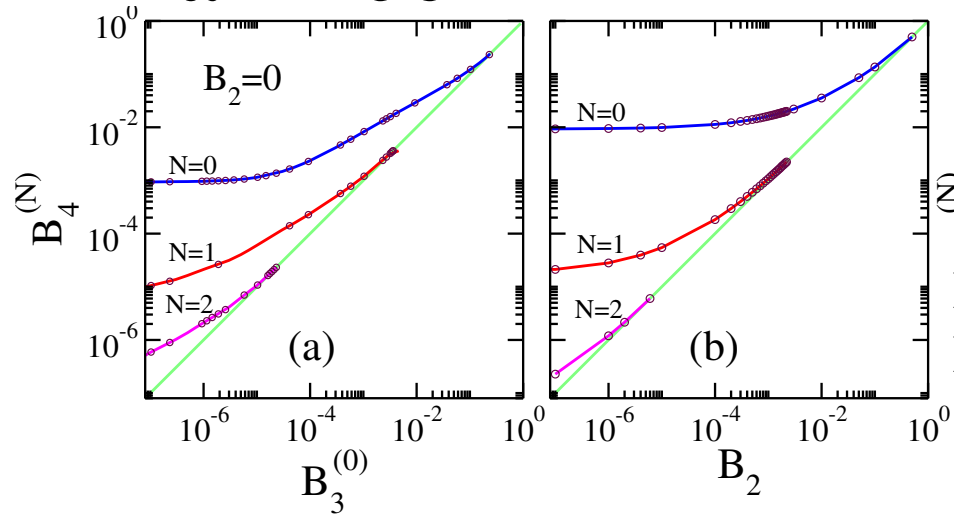
Sliding 4-body scale

Hadizadeh et al PHYSICAL REVIEW A **85**, 023610 (2012)

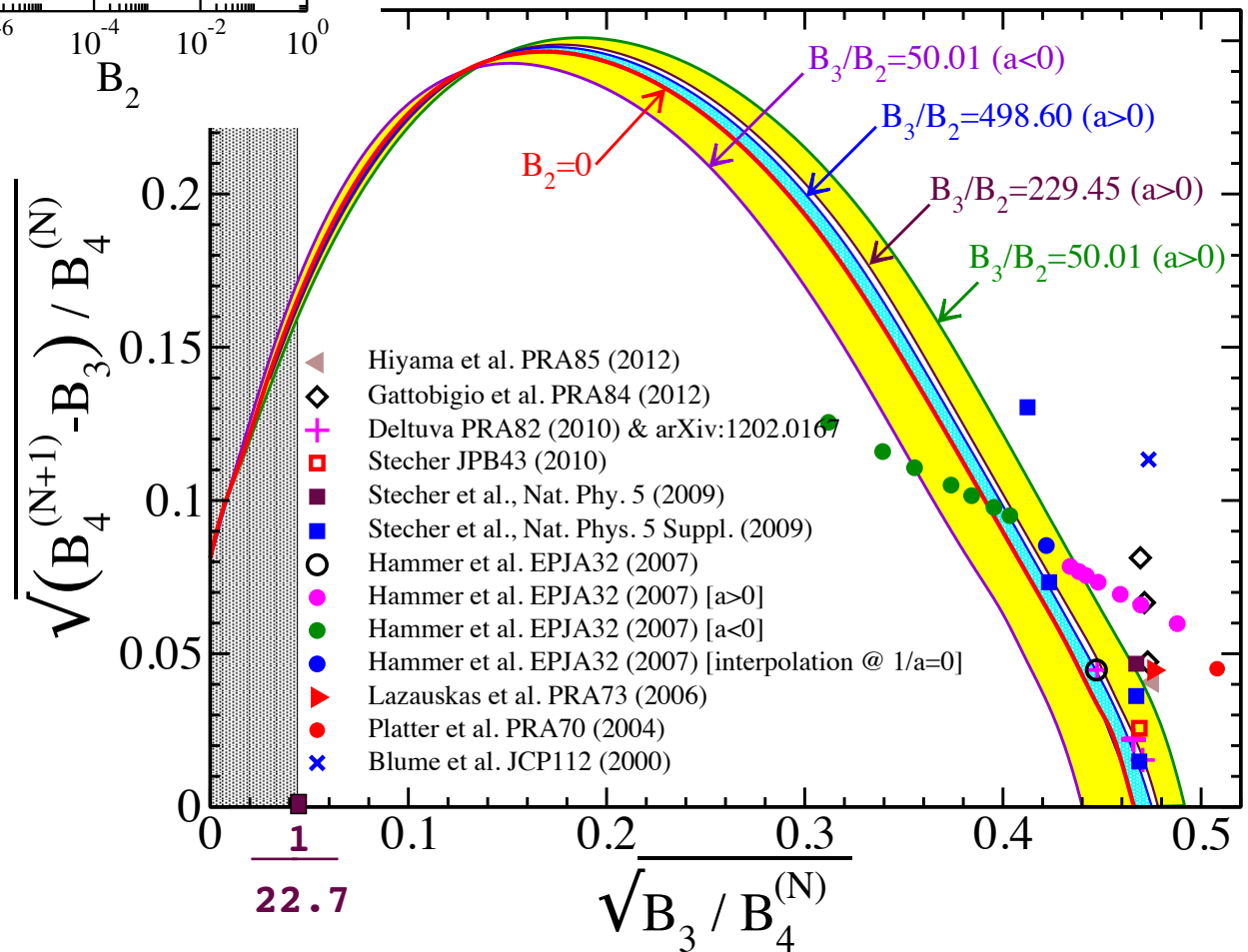


Correlation between observables:
Scaling functions

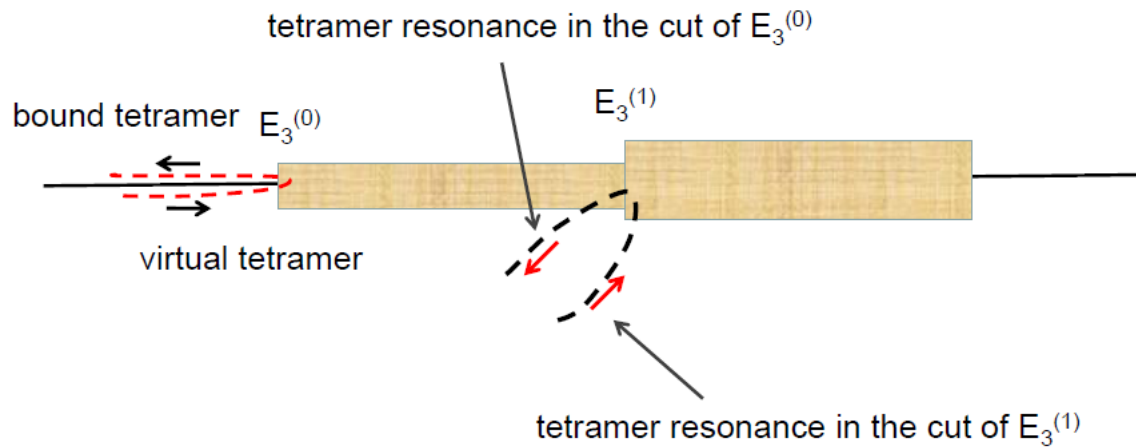
$$a = \infty$$



Hadizadeh, Yamashita, Tomio, Delfino, TF,
PRL107, 135304 (2011)



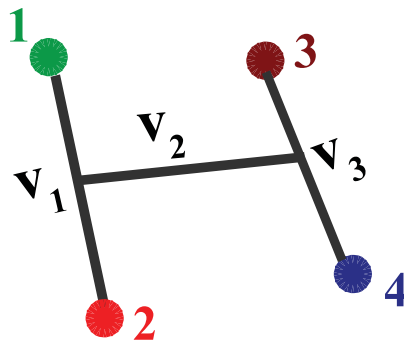
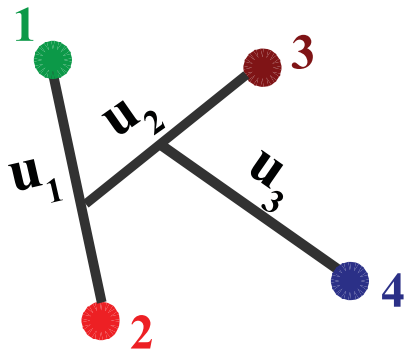
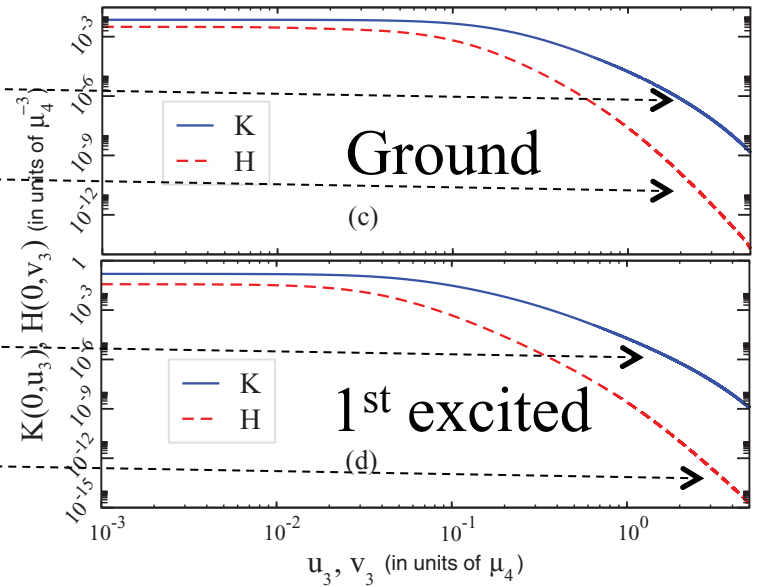
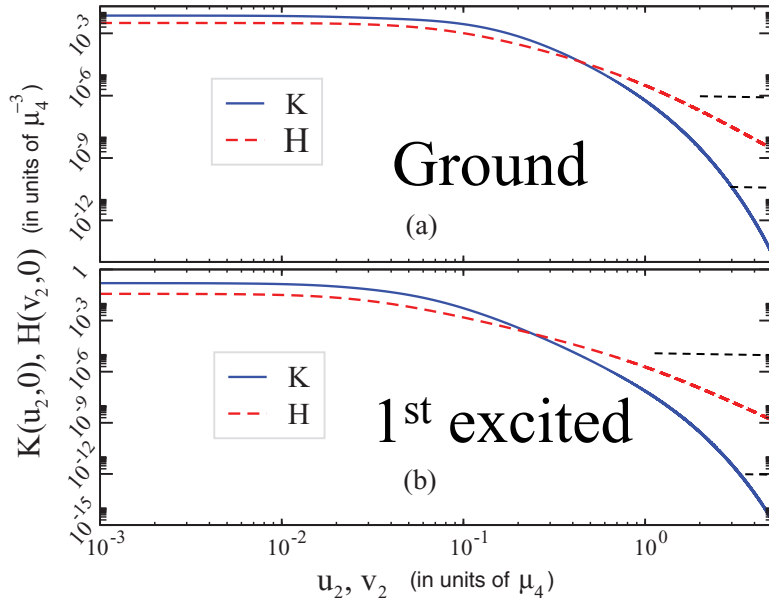
Trajectory of four-boson bound states: one scenario...



Relevance of the H component for the 4-body scale dependence

PHYSICAL REVIEW A **85**, 023610 (2012)

$$\mu_4 / \mu_3 = 5$$



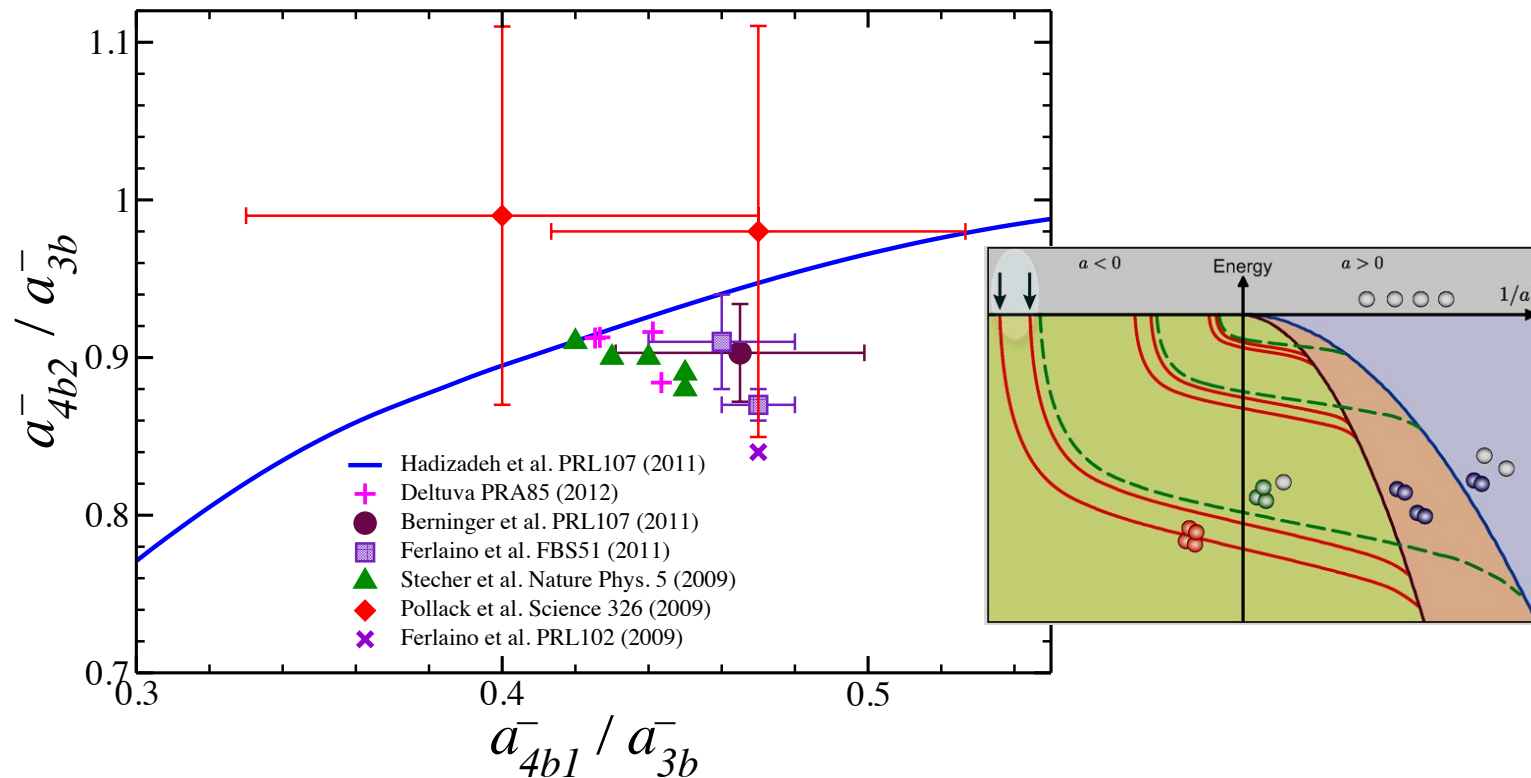
$\mu_4 / \mu_3 \nearrow H \nearrow$
(4-body force)

u_3 and v_2 associated with 4-body scale dependence

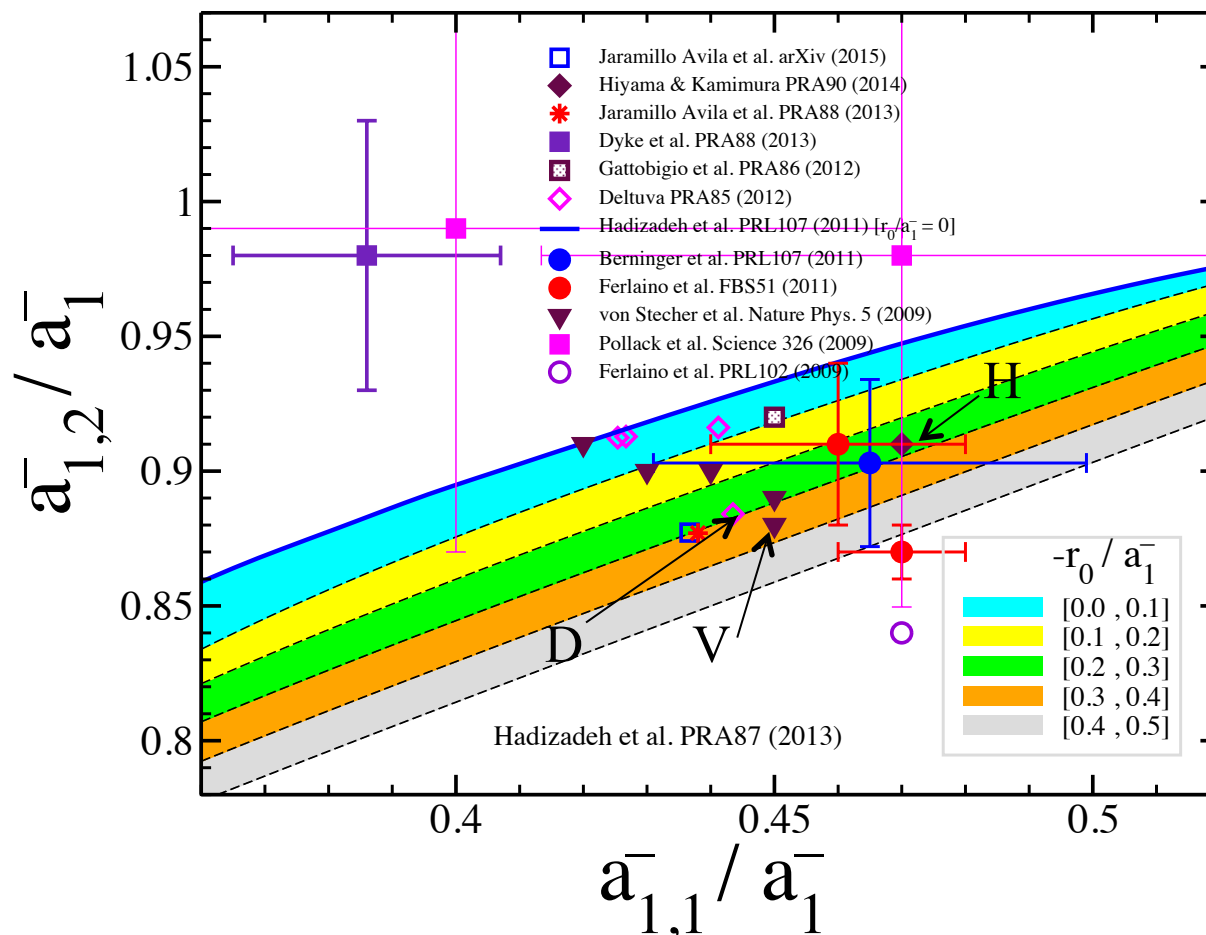
Problem: Position of four-atom resonant recombination

- ▶ Positions of four-atom recombination peaks ($a < 0$) where two successive tetramers become unbound (blue-solid line). Cesium atoms wide Feshbach resonances.
- ▶ (First point from the left corresponds to $B_4 \simeq 64 B_3$ at the unitary limit.)

Hadizadeh, et al PRL107, 135304 (2011)



RANGE CORRECTION TO THE POSITION OF 4-ATOM RESONANCE



Private communications:

Von Stecher (Gaussian local) 0.38 vs. ~ 0.37

Deltuva (separable) 0.33 vs. ~ 0.29

Hiyama (LM2M2) 0.286 vs. ~ 0.28

} Evidence 4-B scale!

r_0 from the shift of the peaks of the four-atom losses

Ref.	$a_{1,1}^T/a_1^-$	$a_{1,2}^T/a_1^-$	$a_1^- [R_{\text{vdW}}]$	$r_0 [R_{\text{vdW}}]$
Ferlaino et al PRL'09	0.47	0.84	-8.7(1)	> 5
Berniger et al PRL'11	0.465(34)	0.903(31)	-9.54(28)	2.5 ± 1.7
Ferlaino et al FBS'11	0.47(1)	0.87(1)	-8.71	4.8 ± 1.0
Ferlaino et al FBS'11	0.46(2)	0.91(3)	-9.64	2 ± 2

- ▶ $R_{\text{vdW}}^{\text{Cs}_2} = 101.0 a_0$ [Chin et al RMP82(2010)]
- ▶ $\bar{a}^{\text{Cs}_2} \simeq 0.955978 R_{\text{vdW}}^{\text{Cs}_2} = 96.5 a_0$.
- ▶ $3.5 < r_0 < 4.3 R_{\text{vdW}}$
- ▶ Weighted average for the fitted r_0 values: $3.9 \pm 0.8 R_{\text{vdW}}$

$$r_0 \simeq 2.9179 \bar{a} \left[\left(\frac{\bar{a}}{a} \right)^2 + \left(\frac{\bar{a}}{a} - 1 \right)^2 \right]$$

Gribakin and Flambaum PRA48 (1993)

Universal range correction

$$a_{N_3, N+1}^T = a_{N_3}^- \mathcal{A} \left(\frac{a_{N_3, N}^T}{a_{N_3}^-}, \frac{r_0}{a_{N_3}^-} \right) \approx a_{N_3}^- \mathcal{A} \left(\frac{a_{N_3, N}^T}{a_{N_3}^-} + \Gamma, \frac{r_0}{a_{N_3}^-} = 0 \right)$$

$$\Gamma \sim C \frac{r_0}{a_{N_3}^-}$$

Suggestion from Kievsky & Gattobigio Phys. Rev. A 87, 052719 (2013)

Summary

- ➔ Zero-range model 3B and 4B systems in 3d:
H configuration sensitive to 4-body scale
Scaling functions & limit cycles & correlation between observables
- ➔ 4B scaling function and position of the resonance
evidence of 4B scale & universal range correction

❖ Collaborators

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- Antonio Delfino – UFF/Niterói

THANK YOU!