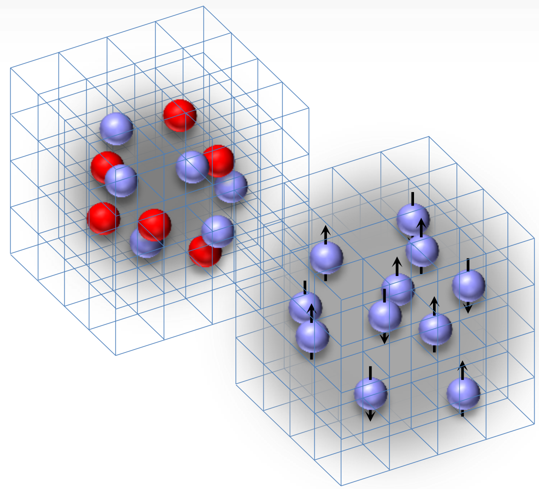
Ab initio alpha-alpha scattering



Serdar Elhatisari

The 8th Chrial Dynamics Workshop

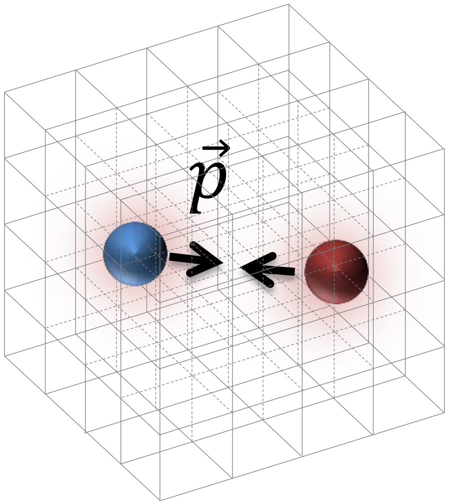
29 Jun - 3 July 2015,

Pisa, Italy

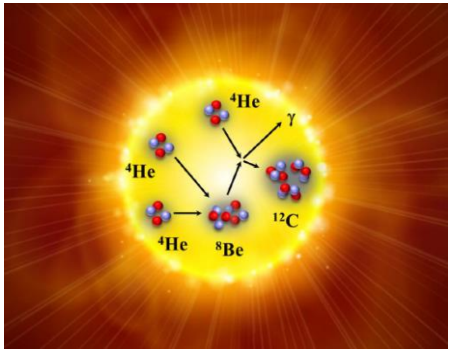
Nuclear LEFT Collaborators [Scattering and reactions]

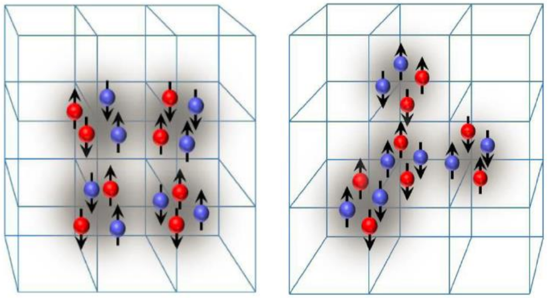
Evgeny Epelbaum
Hermann Krebs
Timo Lähde
Dean Lee
Thomas Luu
Ulf-G. Meißner
Michelle Pine (NCSU)
Alexander Rokash (Bochum)
Gautam Rupak

Two-body scattering on the lattice



- Two spin-1/2 particles scattering.
  Borasoy, Epelbaum, Krebs, Lee, Meißner, *Eur.Phys.J.A34:185-196,2007*
- The LECs by fitting the experimental *NN* scattering data
  Borasoy, Epelbaum, Krebs, Lee, Meißner, *Eur.Phys.J.A35:343-355,2008*
- Regularization methods for NLEFT
  Klein,Lee, Liu, Meißner, *PLB 747, 2015.*
- New developments in *NN* scattering.
  Alarcón, Du, Ni, Klein, Lähde, Lee, Meißner, *work in progress*

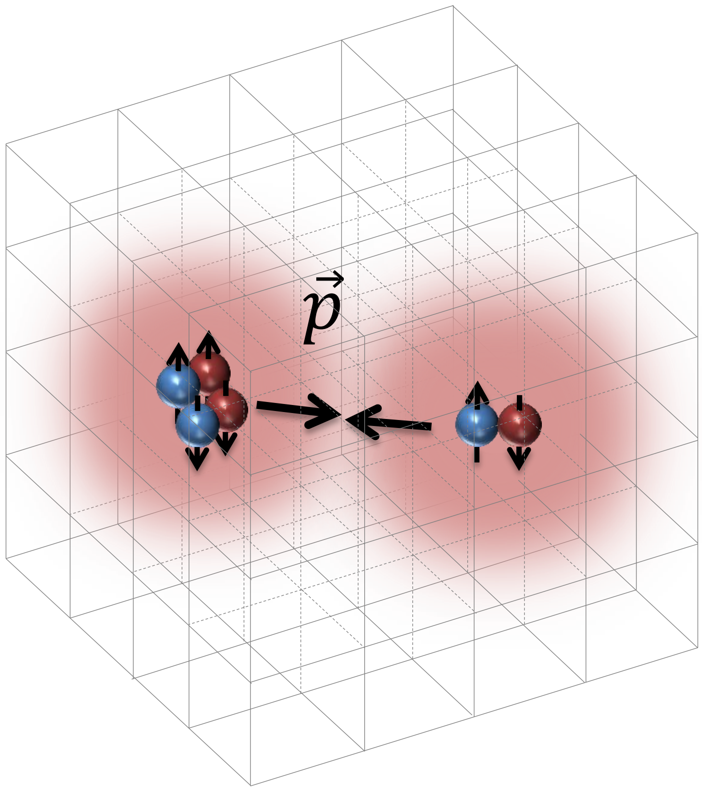




Some highlighted work…

Nuclear lattice EFT collaboration
PRL 106 (2011) 192501;
PRL 109 (2012) 252501;
PRL 110 (2013) 112502;
PRL 112 (2014) 102501.

Two-body scattering on the lattice



Processes involving alpha particles and alpha-like nuclei comprise a major part of stellar nucleosynthesis, and control production of some elements in stellar evolution.

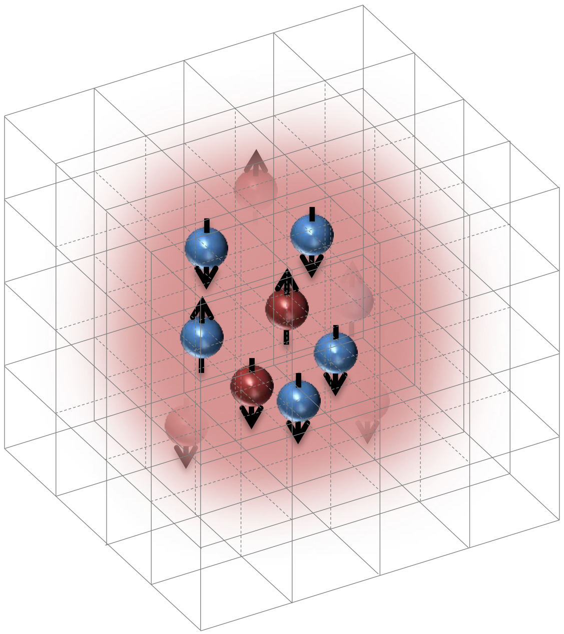
Ab initio calculations of scattering and reactions suffer from the computational scaling with the number of nucleons in clusters.

Lattice EFT computational scaling $\Rightarrow$ $(A_1 + A_2)^2$

Rupak, Lee. *Phys. Rev. Lett. 111, 032502 (2013)*
Pine, Lee, Rupak. *Eur. Phys. J. A (2013) 49: 151*
SE, Lee. *Phys. Rev. C 90:064001 (2014)*
Rokash, Pine, SE, Lee, Epelbaum, Krebs. *arXiv:1505.02967*
S.E., Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, *arXiv:1506.03513*

Outline

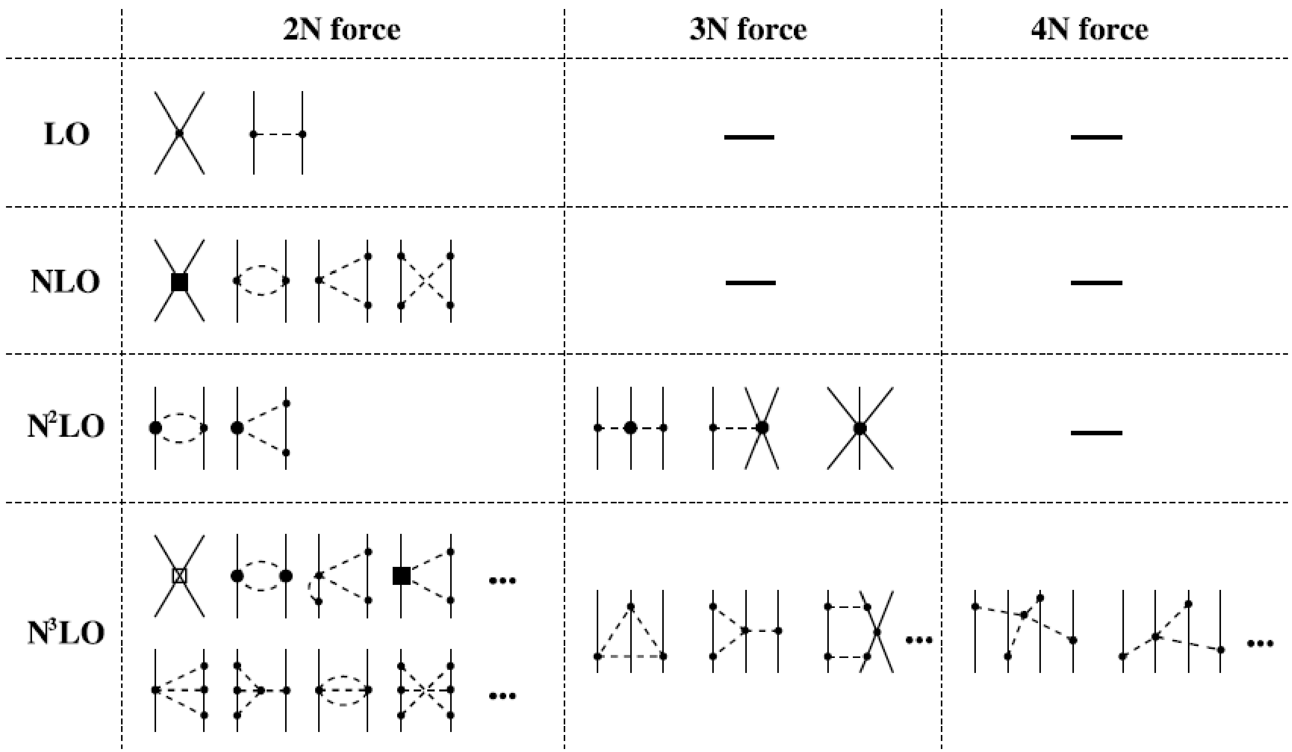
- Motivation & Introduction
- Lattice effective field theory
- Adiabatic projection method
- Alpha-alpha scattering
- Summary



Lattice effective field theory

Lattice effective field theory is a powerful numerical method formulated in the framework of effective field theory.

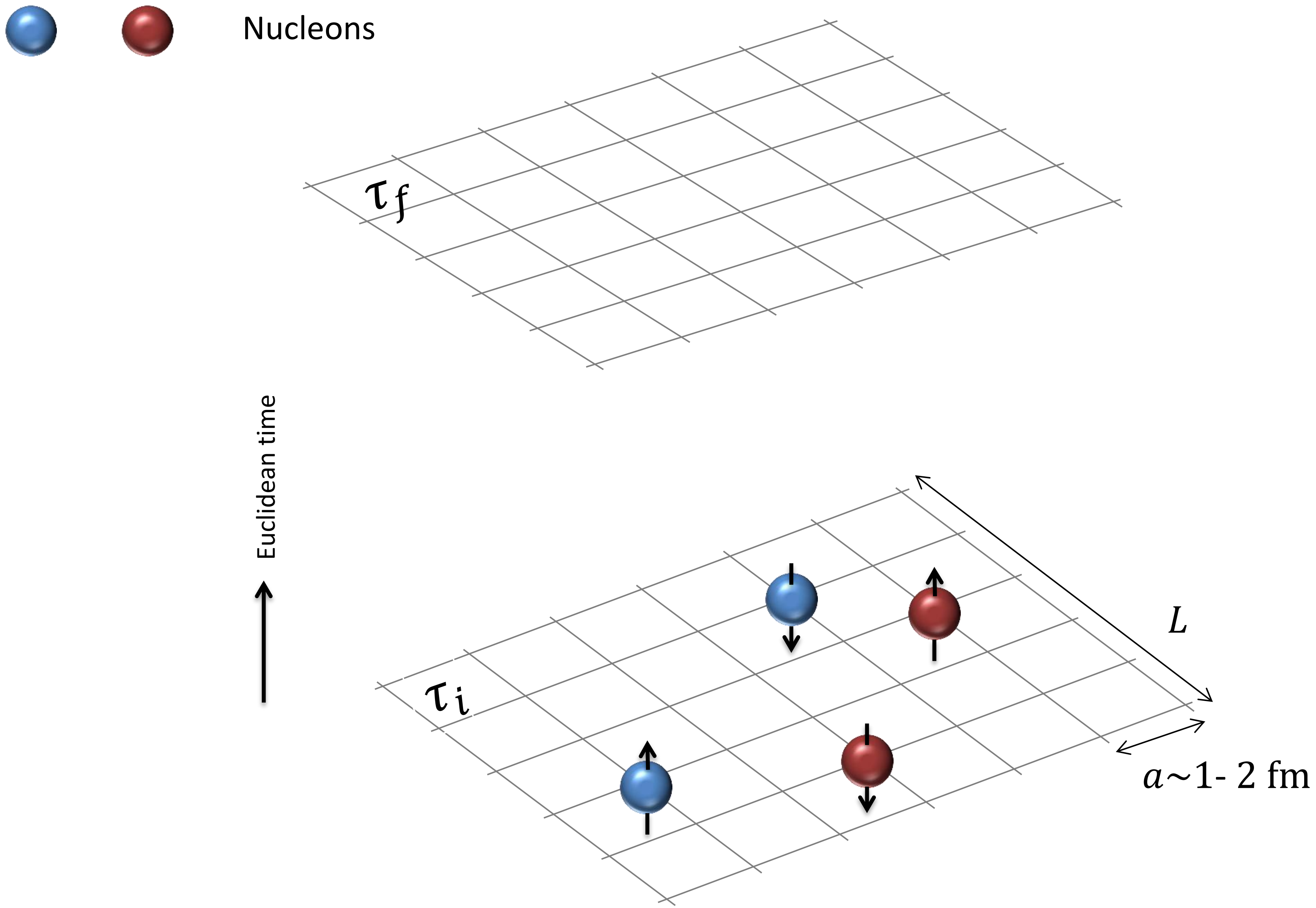
Effective field theory organizes the nuclear interactions as an expansion in powers of momenta and other low energy scales such as the pion mass.



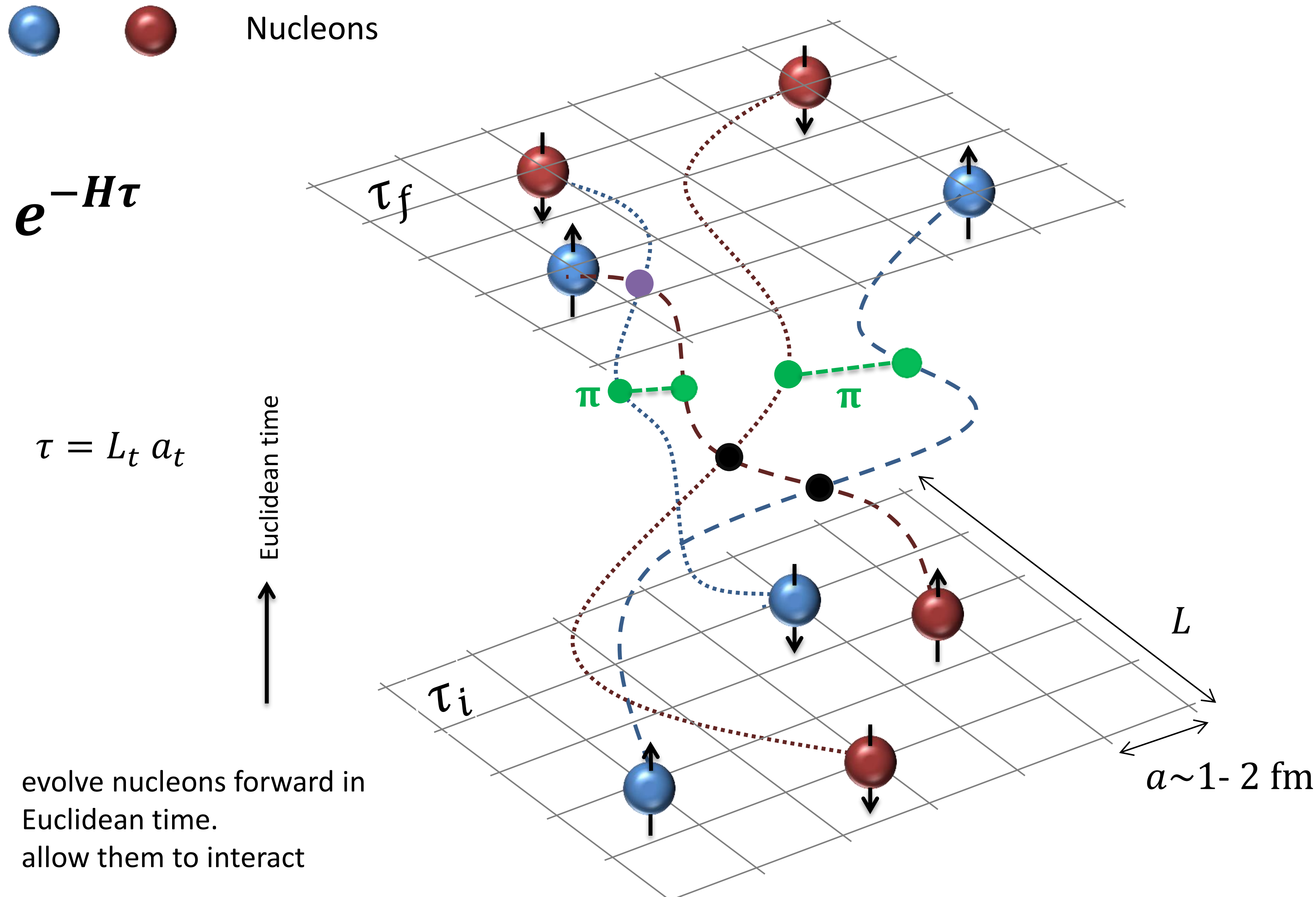
Ordonez et al. ’94;
Friar & Coon ’94;
Kaiser et al. ’97;
Epelbaum et al. ’98,’03,’05;
Kaiser ’99-’01;
Higa et al. ’03; …

Fig. courtesy E.Epelbaum

Lattice effective field theory – Euclidean time projection



Lattice effective field theory – Euclidean time projection



- evolve nucleons forward in Euclidean time.
- allow them to interact

Lattice Monte Carlo calculations

 $= M_{\rm LO}$

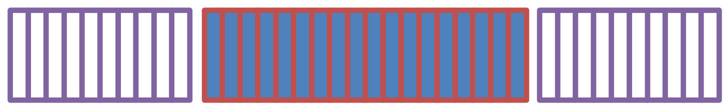
 $= O_{\rm observable}$

 $= M_{\rm approx}$

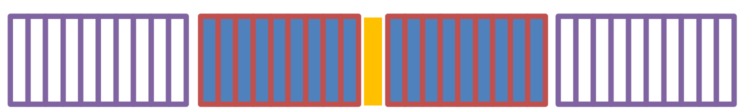
A pionless SU(4)-symmetric transfer matrix is an approximation to LO transfer matrix ($M_{\rm LO}$). Significant supression of sign oscillation.

Chen, Lee, Schäfer, *PRL 93 (2004) 242302*

Hybrid Monte Carlo sampling

$Z_{\rm LO}^{(L_t)} = \langle \psi_{\vec{p}} |$  $| \psi_{\vec{p}} \rangle$

$$e^{-E_{0,\rm LO}\, a_t} = \lim_{L_t \to \infty} Z_{\rm LO}(L_t + 1)/Z_{\rm LO}(L_t)$$

$Z_{\langle O \rangle,\rm LO}^{(L_t)} = \langle \psi_{\vec{p}} |$  $| \psi_{\vec{p}} \rangle$

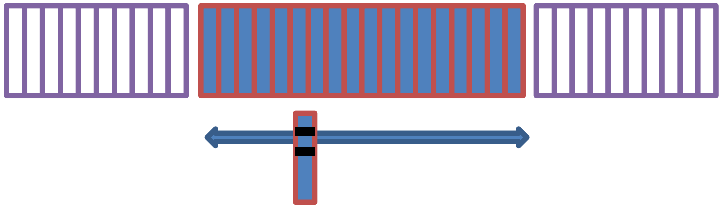
$$\langle O \rangle_{0,\rm LO} = \lim_{L_t \to \infty} Z_{\rm LO}^{\langle O \rangle}(L_t)/Z_{\rm LO}(L_t)$$

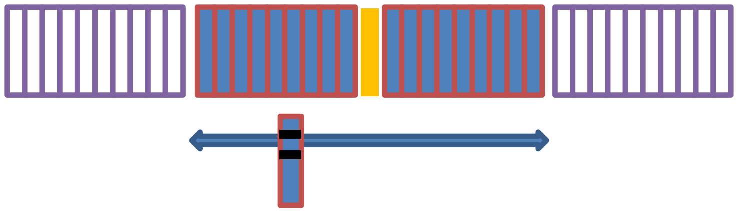
Lattice Monte Carlo calculations

 $= M_{\mathrm{LO}}$  $= O_{\mathrm{observable}}$  $= M_{\mathrm{approx}}$

 $= M_{\mathrm{NLO}}$  $= M_{\mathrm{NNLO}}$

Hybrid Monte Carlo sampling

$Z_{\mathrm{NLO}}^{(L_t)} = \langle \psi_{\vec{p}} |$  $| \psi_{\vec{p}} \rangle$

$Z_{\langle O \rangle,\mathrm{NLO}}^{(L_t)} = \langle \psi_{\vec{p}} |$  $| \psi_{\vec{p}} \rangle$

$$\langle O \rangle_{0,\mathrm{NLO}} = \lim_{L_t \to \infty} Z_{\langle O \rangle,\mathrm{NLO}}^{(L_t)} / Z_{\mathrm{NLO}}^{(L_t)}$$

Adiabatic projection method

Split the problem into two parts.

The first part

use Euclidean time projection to construct an *ab initio* low-energy cluster Hamiltonian, called the adiabatic Hamiltonian.

The second part

compute the two-cluster scattering phase shifts or reaction amplitudes using the adiabatic Hamiltonian.

Rupak, Lee., *PRL 111 (2013) 032502.*

Pine, Lee, Rupak, *EPJA 49 (2013) 151.*

S.E., Lee, *Phys. Rev.* **C90**, 064001 (2014).

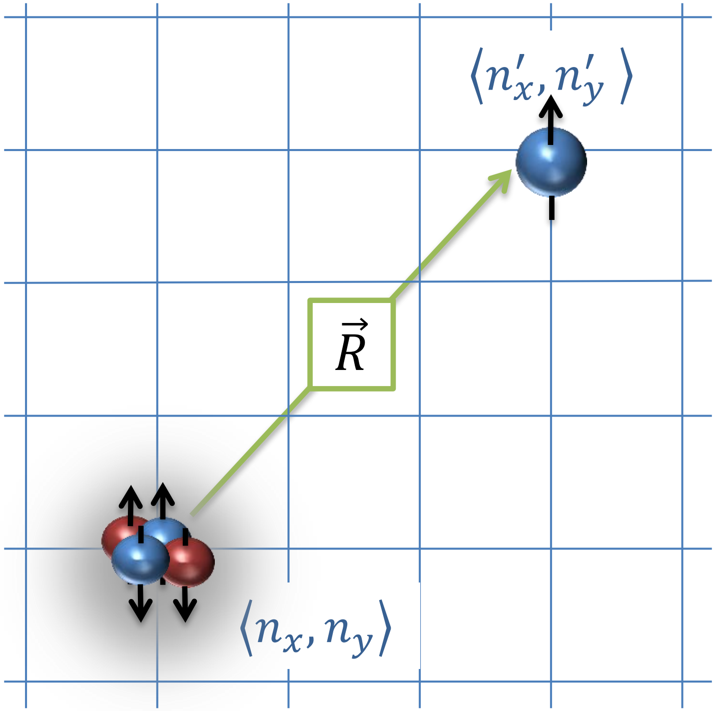
Rokash, Pine, S.E., Lee, Epelbaum, Krebs, arXiv:1505.02967

Adiabatic projection method

Constructs a low-energy effective theory for clusters.

Use initial states parameterized by the relative spatial separation between clusters, and project them in Euclidean time.

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes |\vec{r}\rangle$$



$$|\vec{R}\rangle_\tau = e^{-H\tau}|\vec{R}\rangle$$

Dressed cluster states

The adiabatic projection in Euclidean time gives a systematically improvable description of the low-lying scattering cluster states.

In the limit of large Euclidean projection time the description becomes exact.

Pine, Lee, Rupak, *EPJA 49 (2013) 151.*

Rokash, Pine, S.E., Lee, Epelbaum, Krebs, arXiv:1505.02967

Adiabatic projection method

$$|\vec{R}\rangle_\tau = e^{-H\tau}|\vec{R}\rangle$$

Normal matrix

$$[N_\tau]_{\vec{R}\,\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau \qquad [H_\tau]_{\vec{R}\,\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

$$[H_\tau^a]_{\vec{R}\,\vec{R}'} = \sum_{\vec{R}_n\vec{R}_m}\left[N_\tau^{-1/2}\right]_{\vec{R}\vec{R}_n}[H_\tau]_{\vec{R}_n\vec{R}_m}\left[N_\tau^{-1/2}\right]_{\vec{R}_m\vec{R}'}$$

The structure of the adiabatic Hamiltonian, $[H_\tau^a]_{\vec{R}\,\vec{R}'}$, is similar to the Hamiltonian matrix used in recent calculations of *ab initio* NCSM/RGM.

Navratil, Quaglioni, Phys. Rev. C 83, 044609 (2011).
Navratil, Roth, Quaglioni, Phys. Lett. B 704, 379 (2011).
Navratil, Quaglioni, Phys. Rev. Lett. 108, 042503 (2012).

Adiabatic projection method

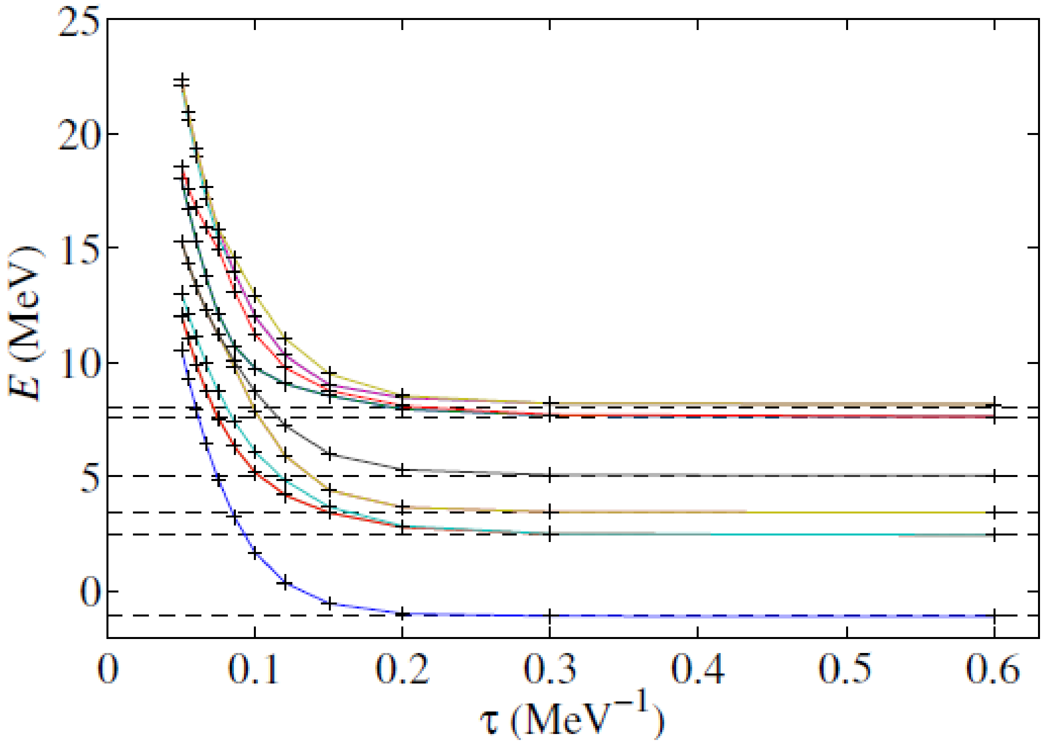
Microscopic Hamiltonian
$L^{3(A-1)} \times L^{3(A-1)}$

Two-cluster (adiabatic) Hamiltonian
$L^3 \times L^3$

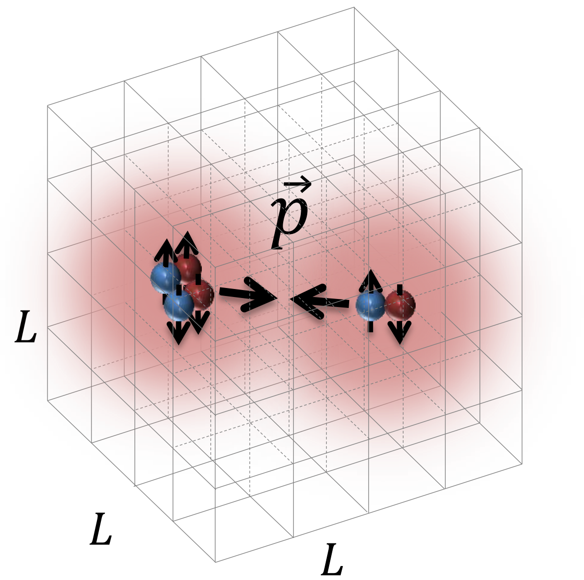
fermion-dimer scattering

Pine, Lee, Rupak, *EPJA 49 (2013) 151*

lattice size is 7^3 , $a = 1.93$ fm



Scattering phase shifts from lattice EFT



Lüscher's method

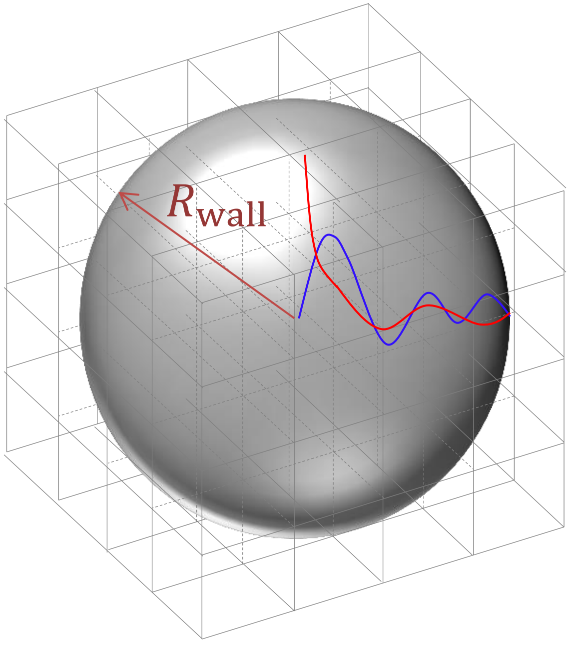
Two-body energy levels below the inelastic threshold in a periodic lattice are related to *the scattering phase shifts in continuum*.

Lüscher, *Comm. Math. Phys. 105 (1986) 153; NPB 354 (1991) 531*

Spherical wall method

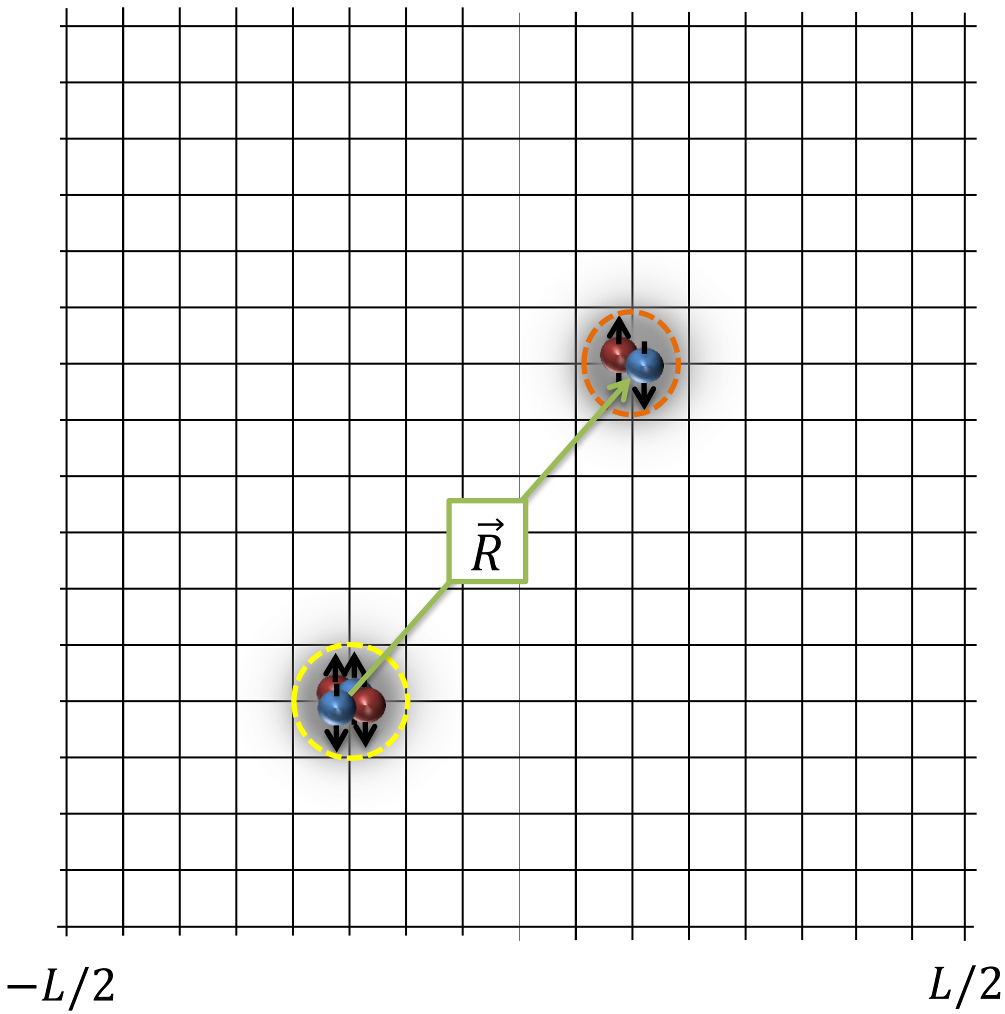
Use the spectrum of the adiabatic Hamiltonian and the fact that the cluster wave function vanishes for $r = R_{\text{wall}}$ to compute the scattering phase shifts directly from

$$\psi_\ell(r) = N\,[\cos\delta_\ell(p)\,F_\ell(p\,r) + \sin\delta_\ell(p)G_\ell(p\,r)]$$



Borasoy, Epelbaum, Krebs, Lee, Meißner, *EPJA 34 (2007) 185*

Scattering cluster wave functions



During the Euclidean time interval τ_ϵ, each cluster undergoes spatial diffusion

$$d_{\epsilon,i} = \sqrt{\tau_\epsilon / M_i}$$

$$|\vec{R}| \gg d_{\epsilon,i} \quad \Rightarrow \quad |\vec{R}\rangle_{\tau_\epsilon}$$

only non-overlapping clusters.

Define the asymptotic region where the amount of overlap between cluster wave packages is less than ϵ

$$|\vec{R}| > R_\epsilon$$

Rokash, Pine, S.E., Lee, Epelbaum, Krebs, *arXiv:1505.02967*

Scattering cluster wave functions

For $|\vec{R}| > R_\epsilon$, the dressed cluster states are widely separated, and we can describe the system in terms of an effective cluster Hamiltonian, H^{eff}, which is a free lattice Hamiltonian for two clusters plus infinite-range interactions.

$$[N_\tau]_{\vec{R}\,\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau = c\left[e^{-2H^{\mathrm{eff}}\tau}\right]_{\vec{R}\vec{R}'}$$

$$[H_\tau]_{\vec{R}\,\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau = c\left[e^{-H^{\mathrm{eff}}\tau}\,H^{\mathrm{eff}}e^{-H^{\mathrm{eff}}\tau}\right]_{\vec{R}\vec{R}'}$$

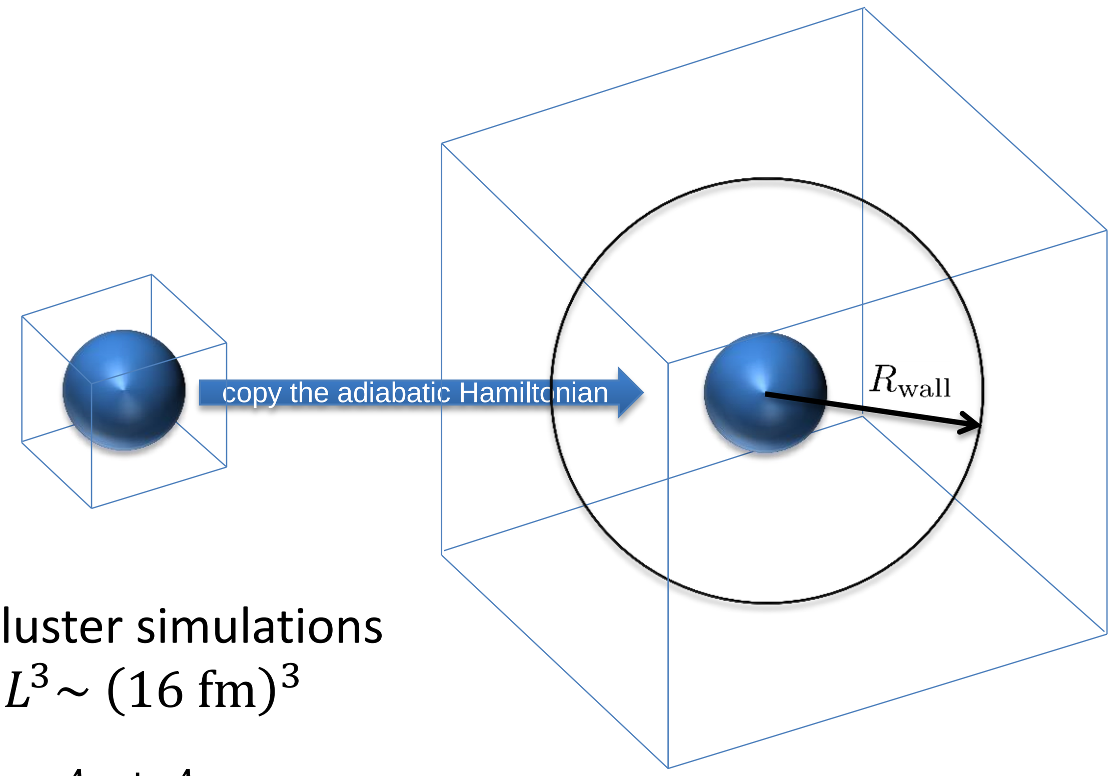
$$[H_\tau^a]_{\vec{R}\,\vec{R}'} = \left[H^{\mathrm{eff}}\right]_{\vec{R}\vec{R}'}$$

Rokash, Pine, S.E., Lee, Epelbaum, Krebs, *arXiv:1505.02967*

Adiabatic Hamiltonian

single cluster simulations
$L^3 \sim (120 \text{ fm})^3$

cluster A_1 and cluster A_2



two-cluster simulations
$L^3 \sim (16 \text{ fm})^3$

$A_1 + A_2$

includes only infinite-range interactions such as Coulomb between the clusters.

Radial adiabatic Hamiltonian

microscopic Hamiltonian
$L^{3(A-1)} \times L^{3(A-1)}$

two-cluster (adiabatic) Hamiltonian
$L^3 \times L^3$

Define "radial adiabatic Hamiltonian" by coherently adding 3D position states $|n_x, n_y, n_z\rangle$ weighted by the spherical harmonics

$$|R\rangle^{\ell,\ell_z} = \sum_{\vec{R}'} Y_{\ell,\ell_z}\left(\hat{R}'\right)\ \delta_{R,|\vec{R}'|}\ |\vec{R}'\rangle$$

Moinard, S.E., Lu, Lähde, Lee, Meißner, *work in progress*
S.E., Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, arXiv:1506.03513

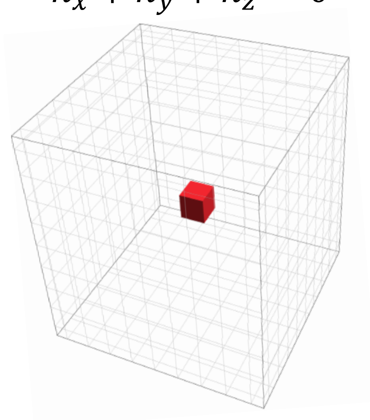
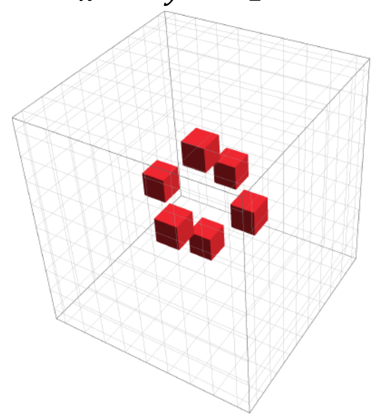
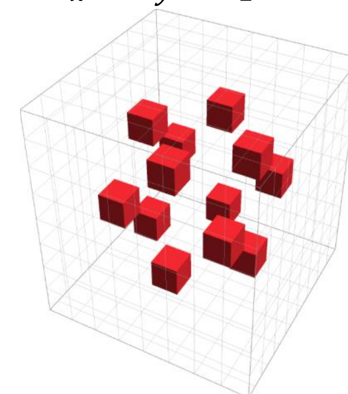
Precise determination of lattice phase shifts and mixing angles

Lu, Lähde, Lee, Meißner, *arXiv:1506.05652*

Radial adiabatic Hamiltonian

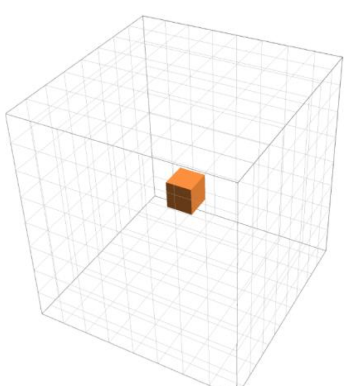
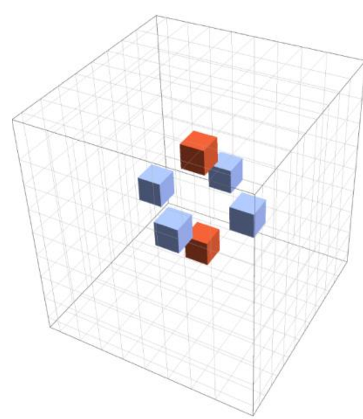
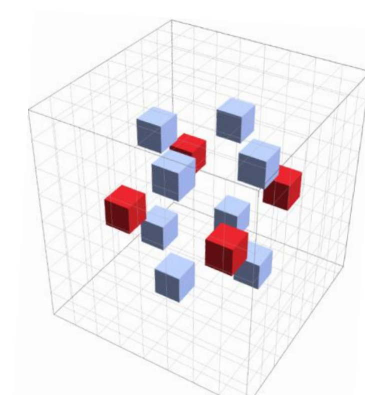
$n_x^2 + n_y^2 + n_z^2 = 0$ | $n_x^2 + n_y^2 + n_z^2 = 1$ | $n_x^2 + n_y^2 + n_z^2 = 2$

$Y_{0,0}\ (\hat{R}')$

$$|R\rangle^{\ell,\ell_z} = \sum_{\vec{R}'} Y_{\ell,\ell_z}\,(\hat{R}')\ \delta_{R,|\vec{R}'|}\ |\vec{R}'\rangle$$

$Y_{2,0}\ (\hat{R}')$

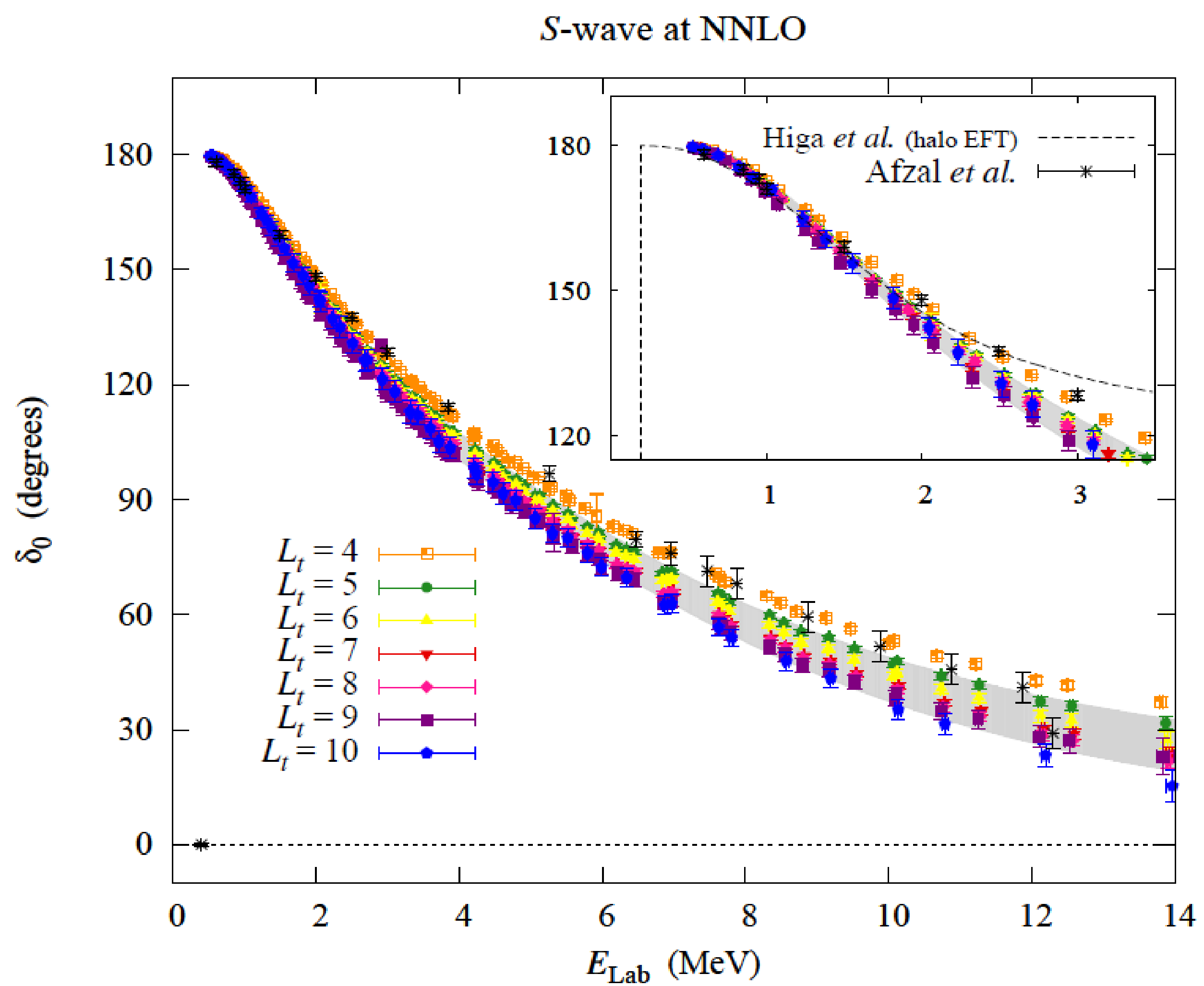




Alpha-alpha scattering

- the same lattice action as in the Hoyle state of $^{12}\mathrm{C}$ and the structure of $^{16}\mathrm{O}$,
- a new algorithm for Monte Carlo updates and alpha clusters,
- the adiabatic projection method to construct a two-alpha (adiabatic) Hamiltonian ,
- the spherical wall method to extract the scattering phase shifts.

S.E., Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, *arXiv:1506.03513*

Alpha-alpha scattering



$a = 1.97$ fm

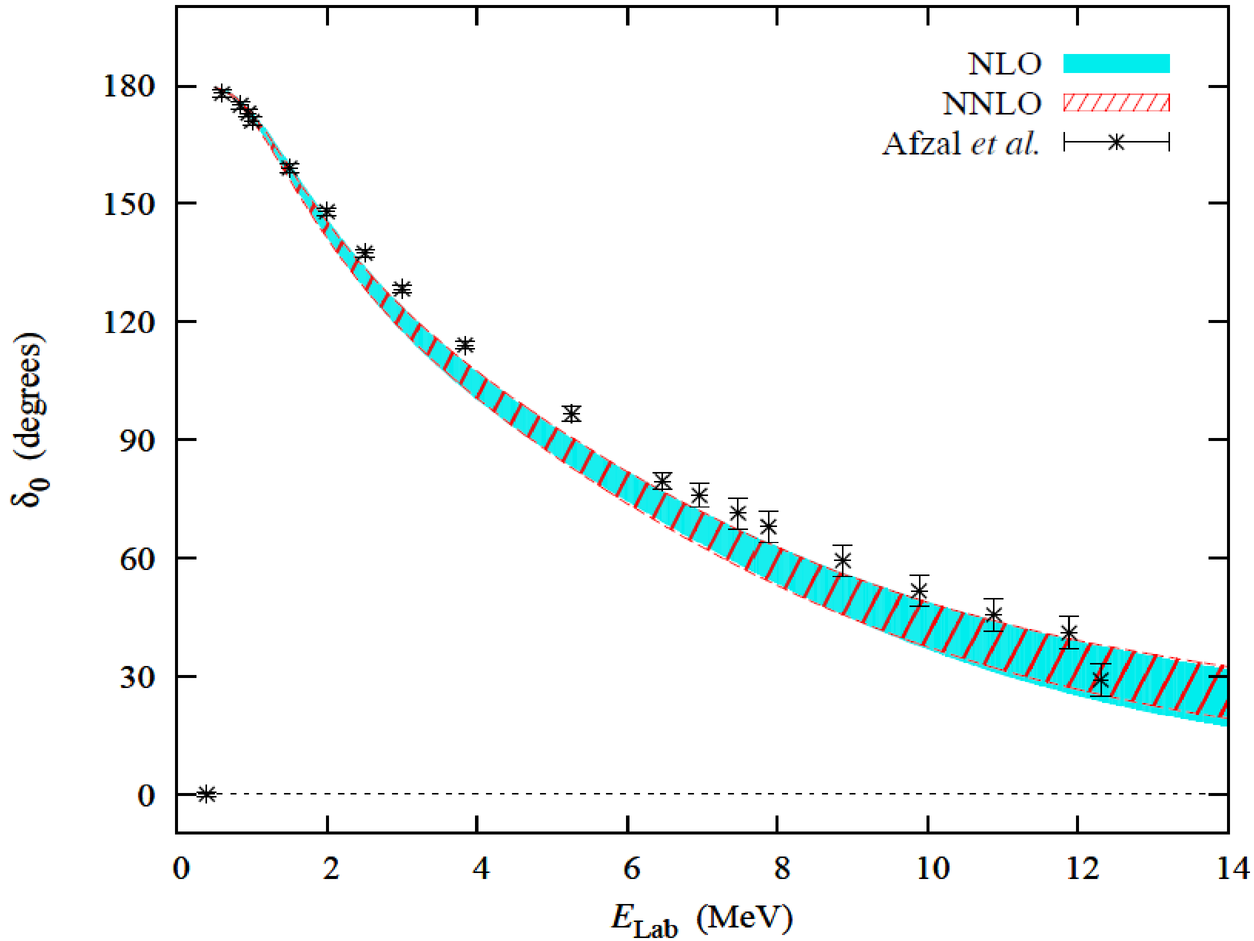
$a_t = 1.32$ fm

Afzal, Ahmad, Ali, *Rev. Mod. Phys. 41, 247 (1969)*

Higa, Hammer, van Kolck, *Nucl.Phys. A809, 171 (2008)*

S.E., Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, *arXiv:1506.03513*

Alpha-alpha scattering



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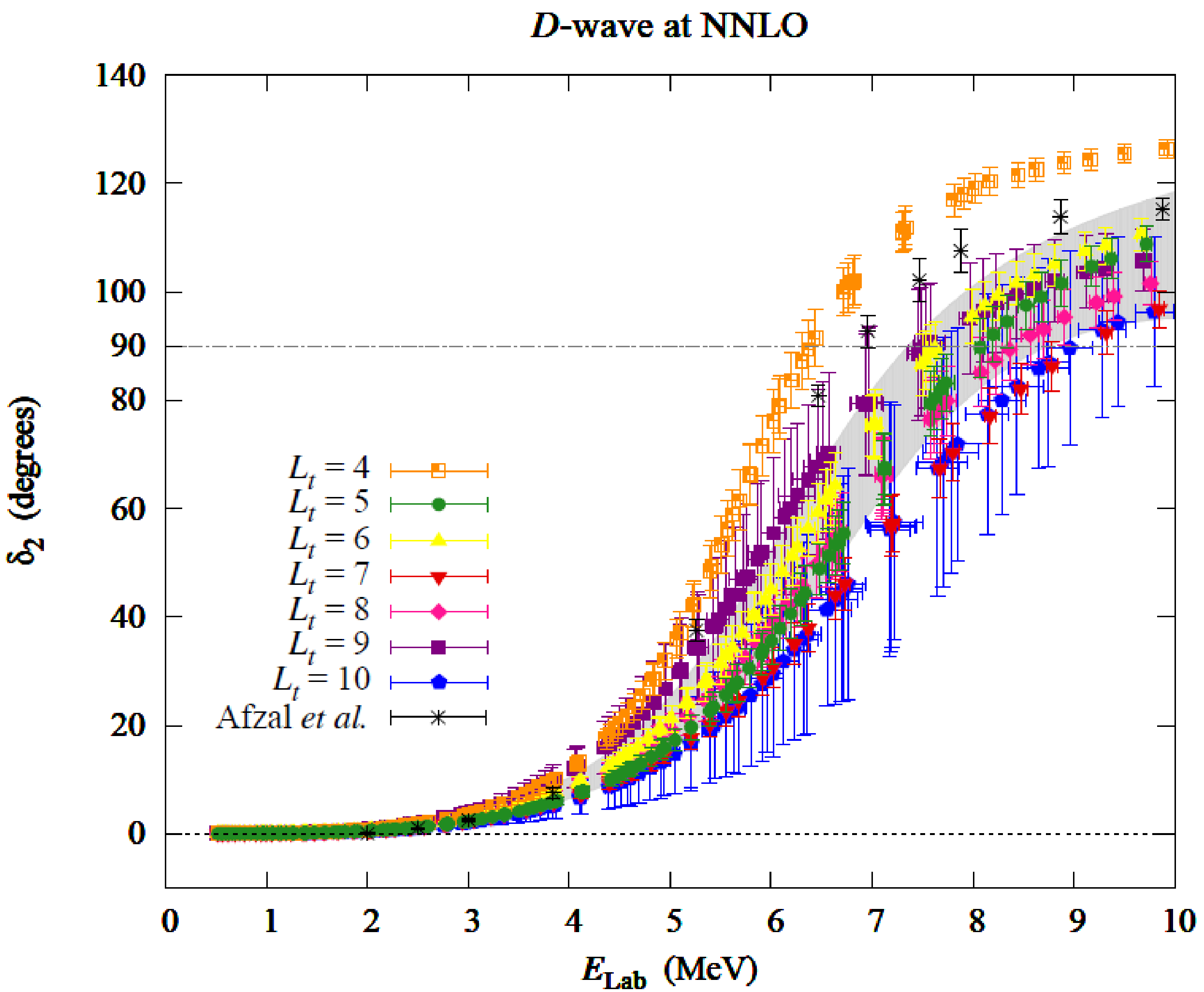
Alpha-alpha scattering



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Afzal, Ahmad, Ali, *Rev. Mod. Phys. 41, 247 (1969)*

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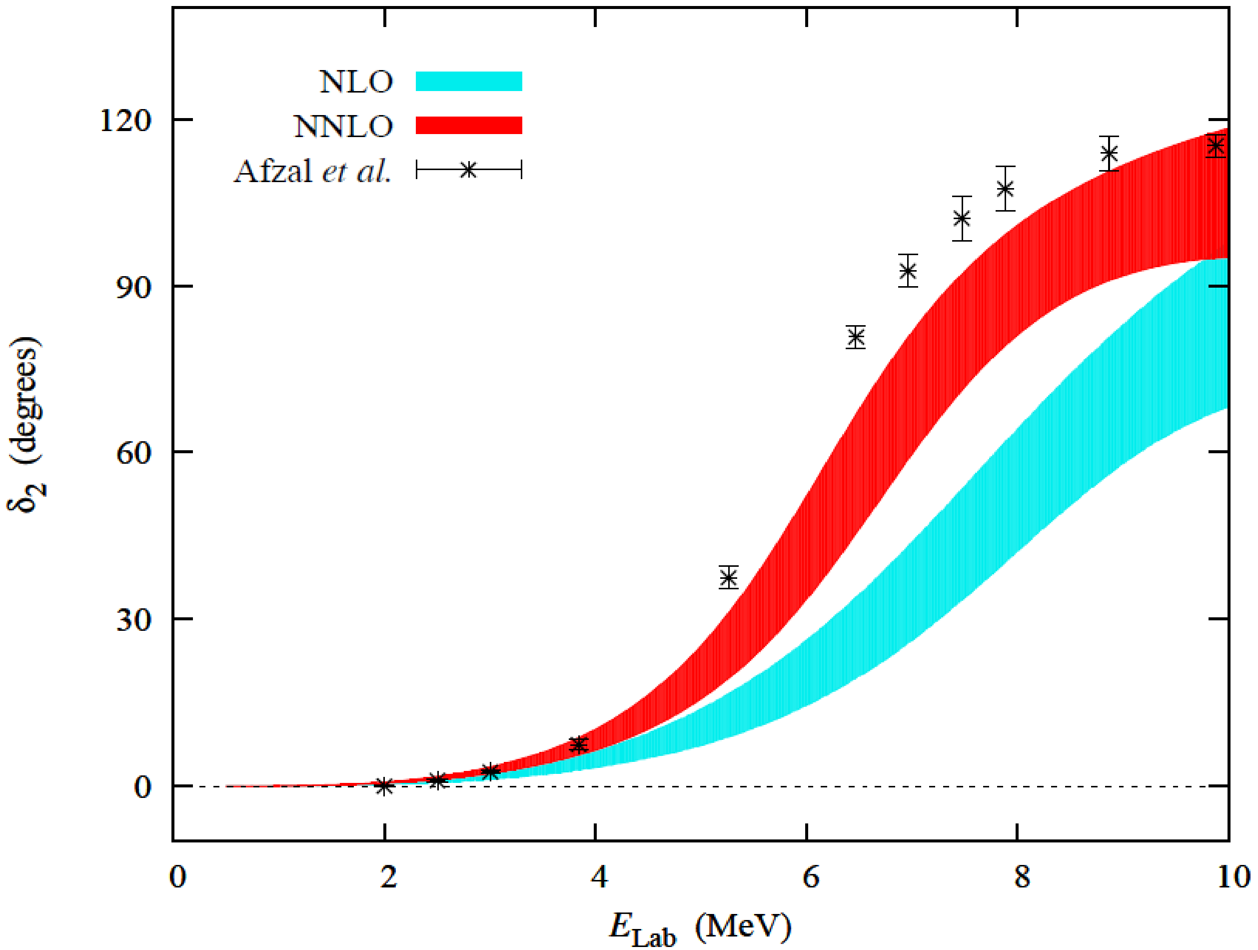
Alpha-alpha scattering



$a = 1.97$ fm

$a_t = 1.32$ fm

Afzal, Ahmad, Ali, *Rev. Mod. Phys. 41, 247 (1969)*

S.E., Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, *arXiv:1506.03513*

Summary

- The adiabatic projection method is a general framework for scattering and reactions on the lattice.

- Scattering and reaction processes involving alpha particle are in reach of *ab initio* methods.

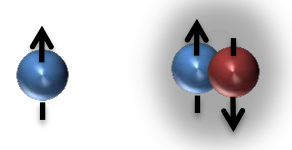
- The computational scaling for A_1-body and A_2-body clusters is
$$(A_1 + A_2)^2$$

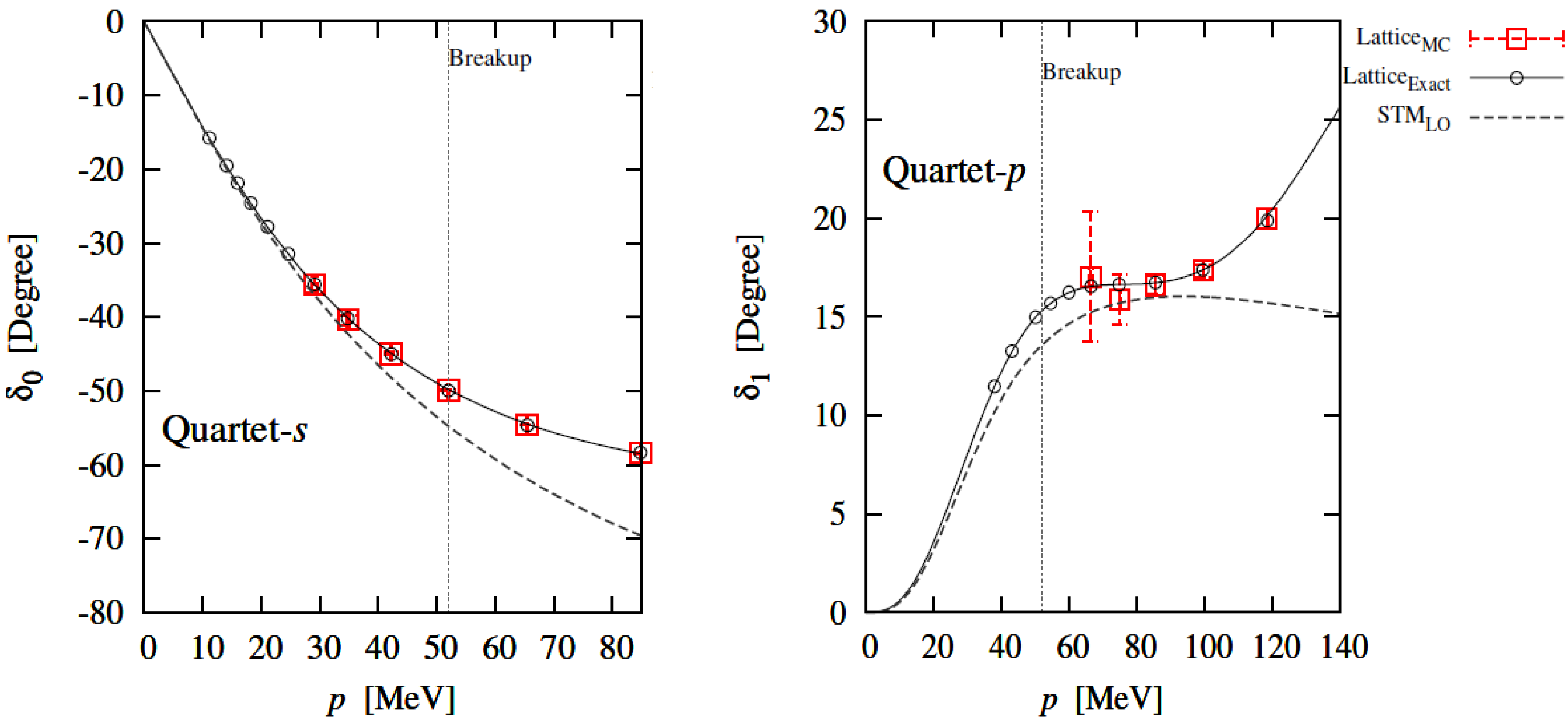
- The problem of sign oscillations is greatly suppressed for alpha-like nuclei, and this approach appears to be a viable method to study important alpha processes involving heavier nuclei.

Thanks for your attention!

Extras

neutron-deuteron scattering (pionless EFT)

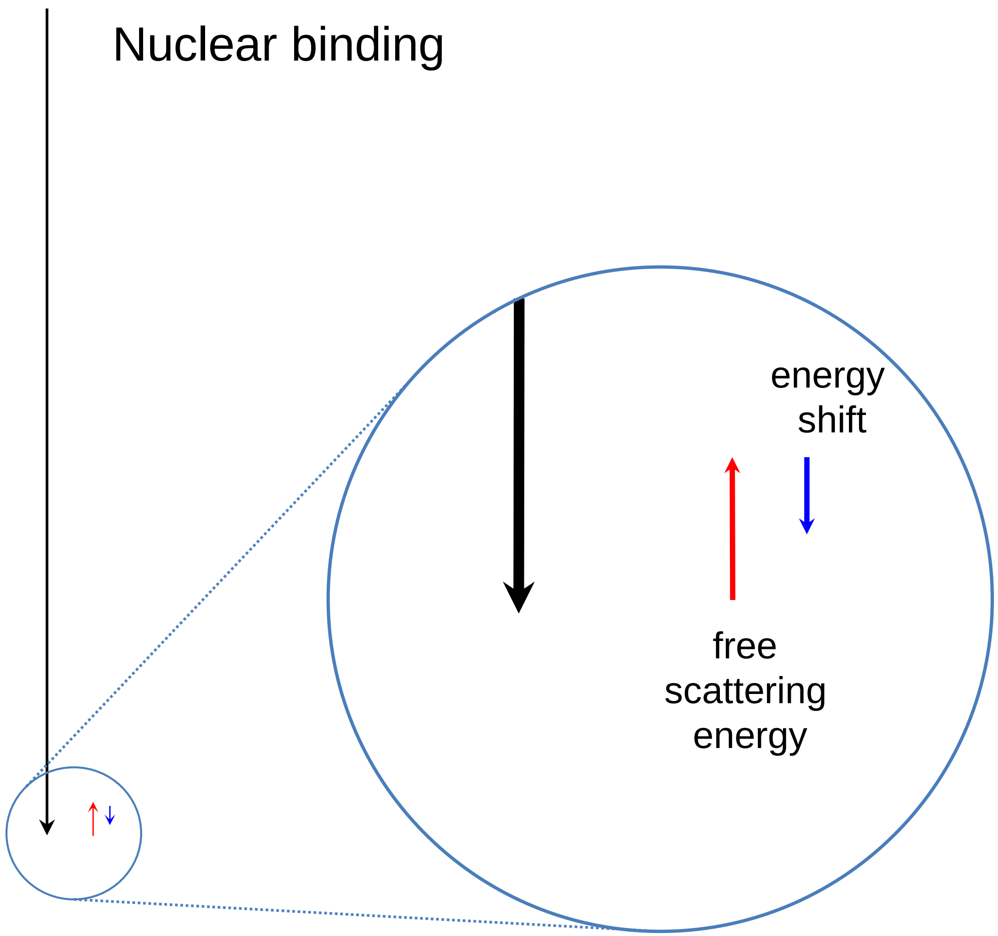




S.E., Lee, *Phys. Rev.* **C90**, 064001 (2014).

Gabbiani, Bedaque, Grießhammer, *NPA 675 (2000) 601*

Two clusters spectrum



Signal-to-noise problems for finite-volume energy extraction

- Low-energy phase shifts are computed using very large box sizes where the level spacing is small.
- This magnifies any small error in the energy values.
- For very large boxes very long Euclidean time projection is required.