Precise calculations of the deuteron quadrupole moment

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- ★ History and Overview
- ★ Covariant Spectator Theory (CST)
  - Overview
  - Off-shell couplings
- ★ Comparison between CST and χEFT calculations
- ★ Calculation of the quadrupole moment
- ★ Lessons and Conclusions

References:

★ Gross, PRC 91, 014005 (2015) — CST

★ Piarulli, et.al. PRC 87, 025503 (2013) — χEFT

★ Marcucci, et.al. JPG review (with the editors)

History and Overview

$\delta Q = (Q_{model} - Q_{exp})/Q_{exp}$

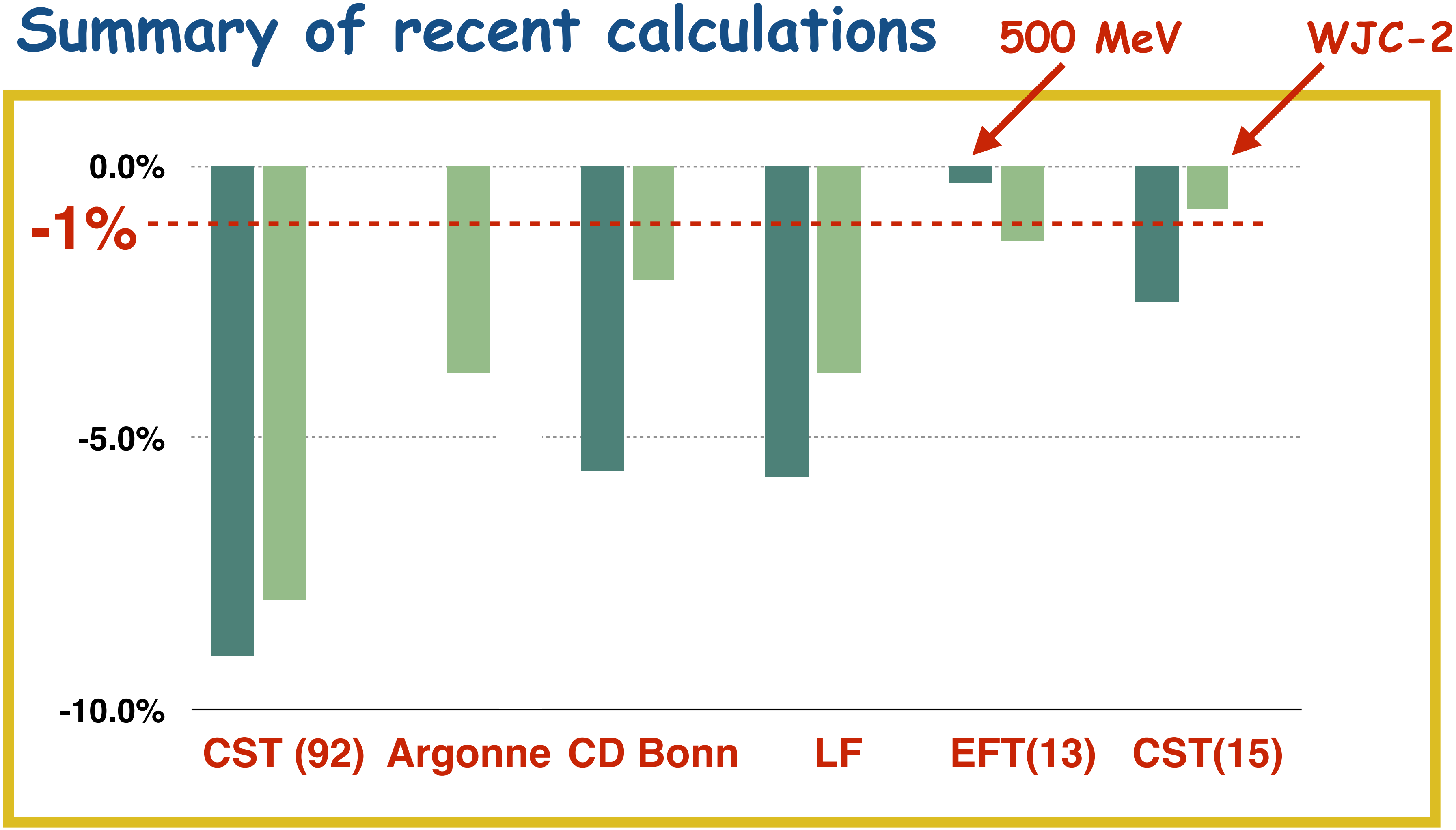
★ Summary of some recent calculations

Reference	δQ (model)	
92: Early CST (VanOrden, et.al.)	-9.0%(IIB),	-8.1%(IIB with RC)
95: Argonne (Wiringa,Stoks,Schiavilla)		-3.8% (with MEC)
01: CD Bonn (Machleidt)	-5.6% (no MEC),	-2.1% (MEC est.)
09: Light-front (Huang, Polyzou)	-5.7% (IM),	-3.8% (IM+Ex)
13: χEFT,ODU-Pisa (Piarulli, et.al.)	-0.3% (500MeV),	-1.4%(600MeV)
15: Full CST (this work)	-2.5% (WJC-1),	-0.8% (WJC-2)

★ Failure to explain the quadrupole moment (until now) an "unsolved problem"

History and Overview

★ Summary of recent calculations



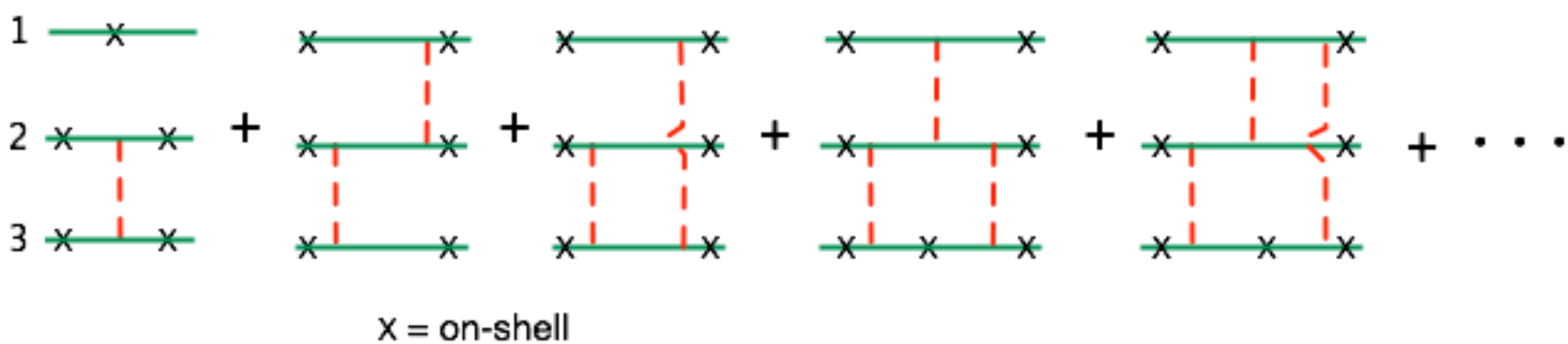
Overview of the CST — what is it?

- 1969: Covariant sum of irreducible interactions between nucleons with spectators on-shell
- 1987 (with D. O. Riska): Derived a 2-body current that is conserved, even in the presence of phenomenological form factors
- 1997 (with A. Stadler): CST OBE requires no 3 or many-body forces 2004: Using OBE models, a conserved 3-body current using the same 1-body and 2-body interaction currents is derived (with A. Stadler and M. T. Pena)
- 2008 (with A. Stadler): One boson exchange (OBE) model WJC-2 gives precise fit to np data ($\chi^2/N_{data} \sim 1.1$) requiring only 15 parameters from the exchange of only 6 bosons with $I(J^P)$:
  - pion $1(0^-)$, eta $0(0^-)$, sigma $0(0^+)$, delta $1(0^+)$, omega $0(1^-)$, rho $1(1^-)$ (and photon)
- 2014: Unique 2-body isoscalar interaction current derived

NO 3-body forces in a CST OBE model

★ Diagrammatic demonstration:

- CST ladder sums are NOT time-ordered — can always be separated into successive 2-body interactions



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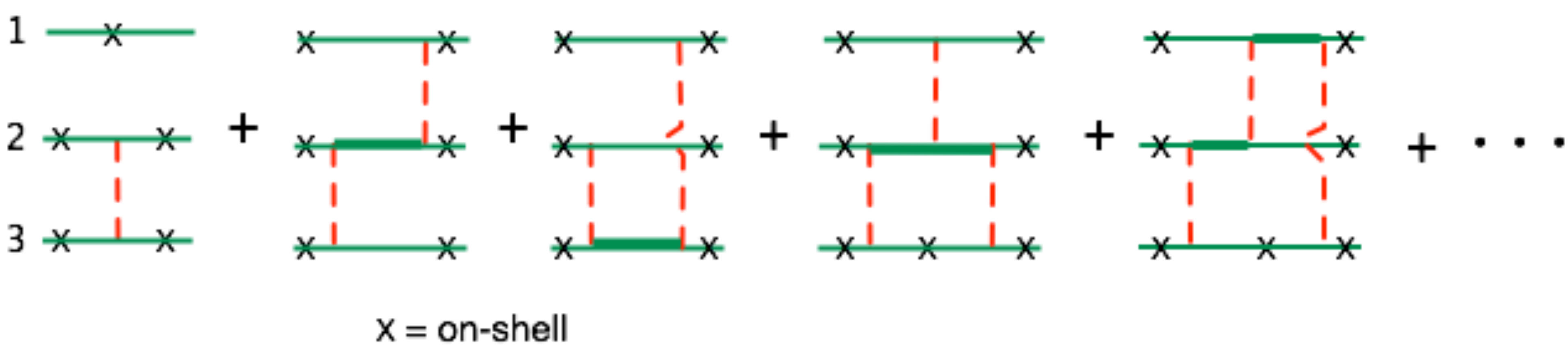

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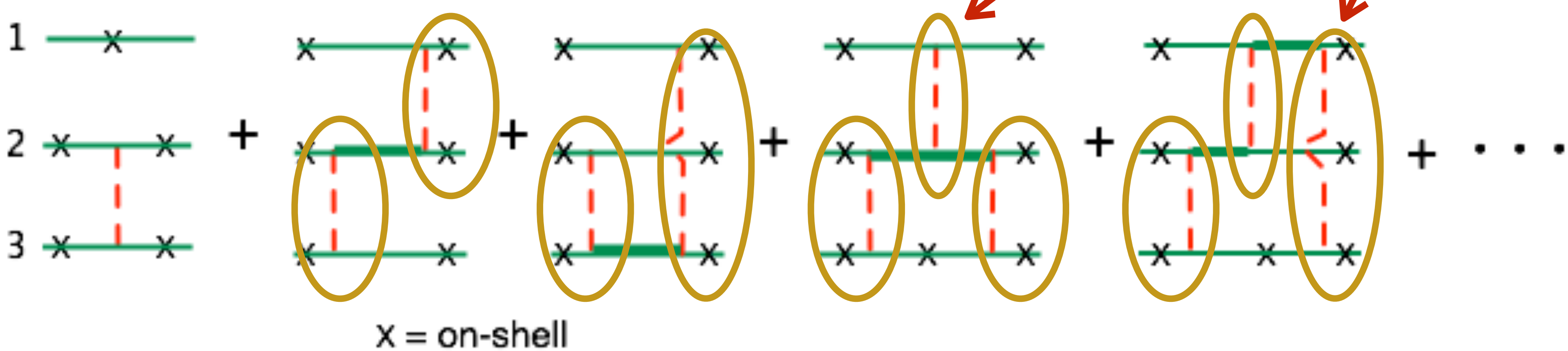
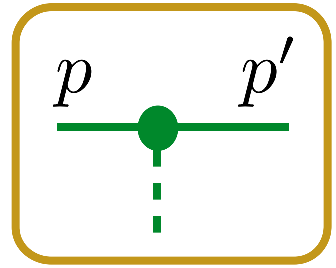
Same OBE kernel used in the 2-body problem



Off-shell couplings at meson-NN vertices



★ Examples

- Scalars

$$\Lambda^{\sigma}(p,p') = g_{\sigma}\mathbf{1} - \nu_{\sigma}[\Theta(p) + \Theta(p')], \qquad \Theta(p) = \frac{m - \not{p}}{2m}$$

- Pseudo-scalars (if pure pseudo-vector)

$$\Lambda^{\pi}(p,p') = g_{\pi}[\gamma^5 - \Theta(p)\gamma^5 - \gamma^5\Theta(p')] = \frac{g_{\pi}}{2m}\,\not{q}\gamma^5$$

- Vectors (similar terms)

★ 1997 discovery (with A. Stadler): The fit to the NN data and the 3-body binding energy is very sensitive to ν_{σ} and the value determined by NN fits gives the correct E_T !

★ Implications:

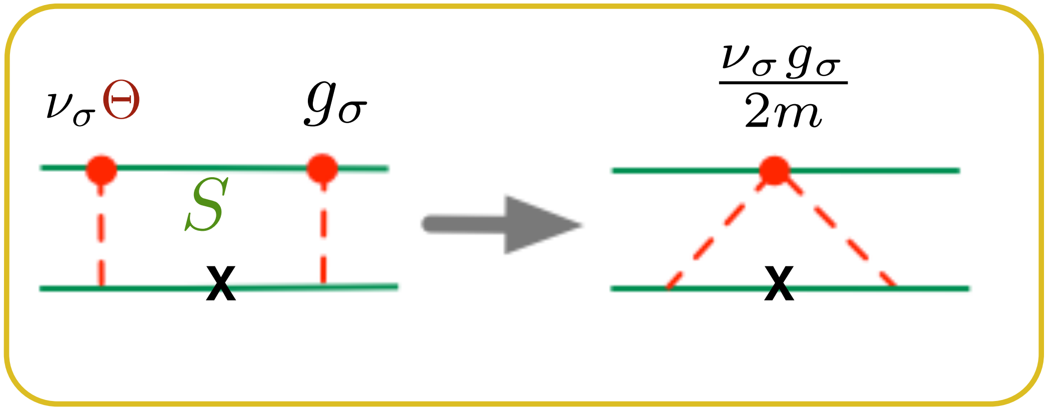
1. cancellations of the propagators —> effective contact interactions

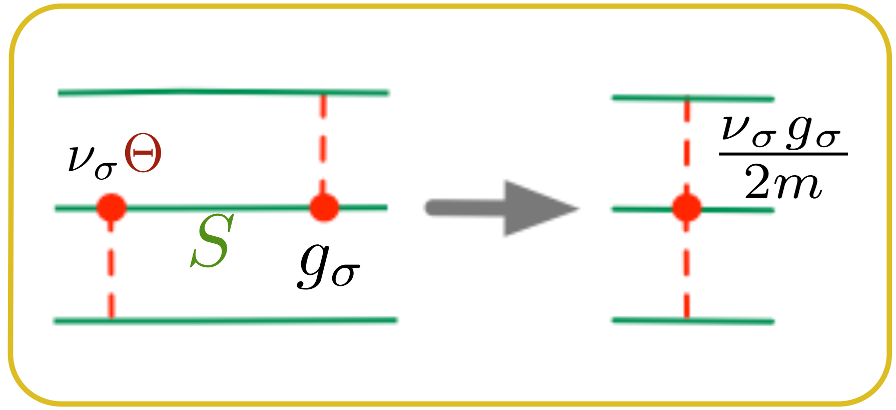
2. new effective interaction currents

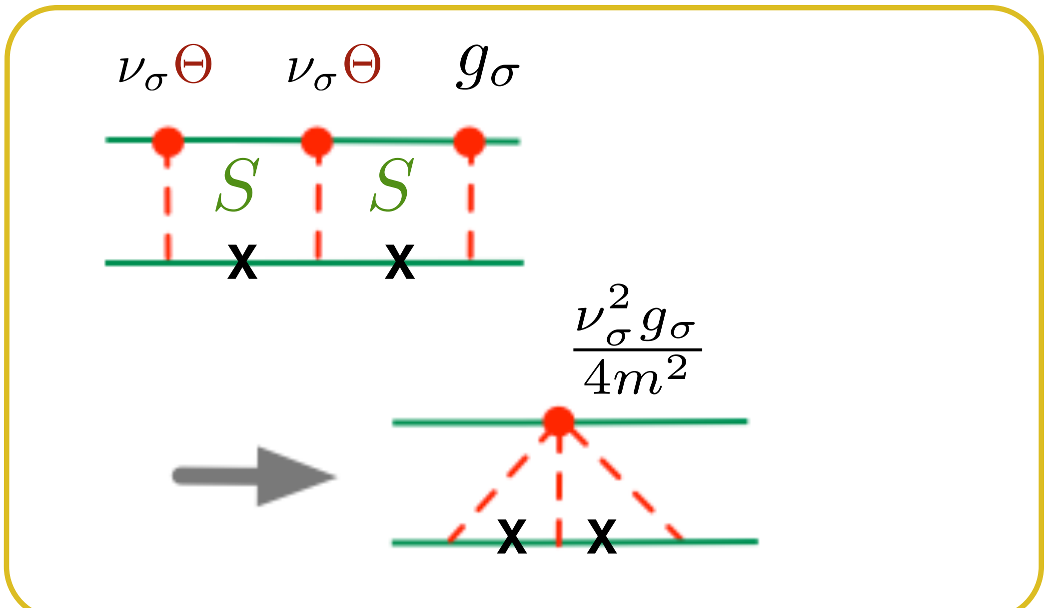
1. Cancellations of the propagators

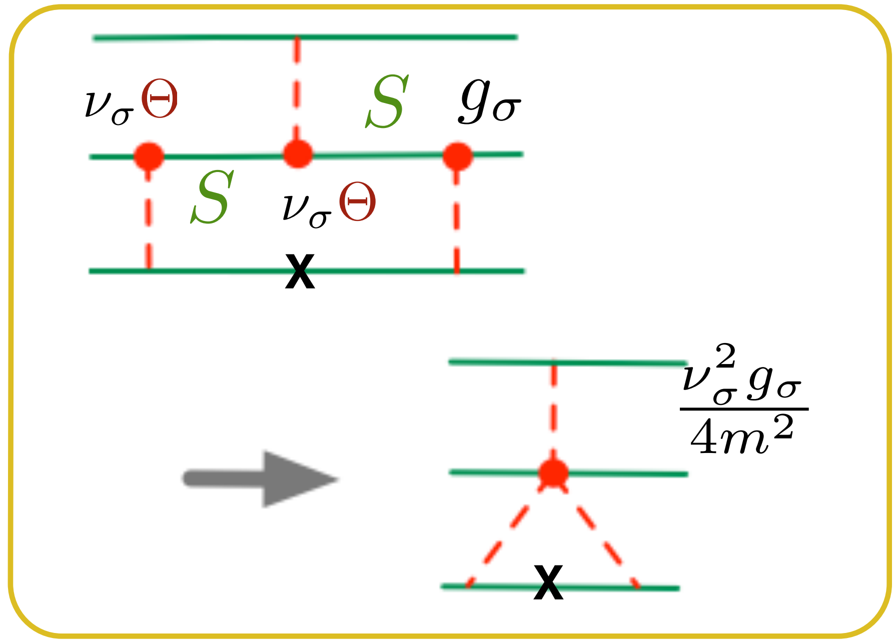
$$\Lambda^\sigma(p,p') = g_\sigma \mathbf{1} - \nu_\sigma[\Theta(p)+\Theta(p')], \qquad \Theta(p)S(p) = \frac{1}{2m}$$

Examples at 4th and 6th order









Equivalence theorem

- ★ PictureA: a CST OBE model **WITH** off-shell couplings is

  is equivalent to

- ★ Picture B: a CST model with OBE terms **WITHOUT** off-shell couplings **PLUS** an infinite sum of specific non-OBE terms, many body forces, and non-OBE interaction currents with couplings depending on combinations of only a few parameters

- ★ So.. Are there three body forces?
  - using picture A: NO (correct?)
  - using picture B: YES (incorrect?)

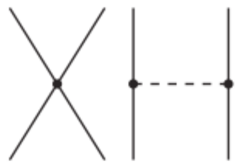
- ★ I am reminded of a statement by Peter Sauer: "Three-body forces are not made by God"

Comparison with χEFT (NN)

★ χEFT*:

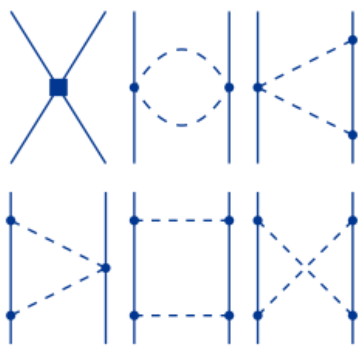
LO

$(Q/\Lambda_\chi)^0$



NLO

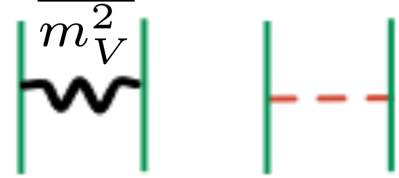
$(Q/\Lambda_\chi)^2$



Machleidt, in "NN and 3N Interactions," Blokhintsev & Strakovsky, eds, (2014)

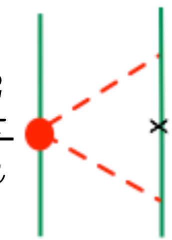
★ CST (picture B):

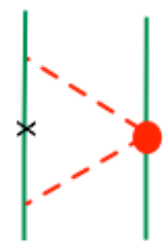
$$\frac{g_V^2}{m_V^2}$$



$$\frac{g_V^2 q^2}{m_V^4} + \frac{g_\pi^4}{m^2}(\cdots)$$

$$\frac{g_\pi^2}{m}$$





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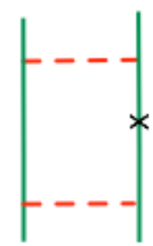

Expand the heavy meson propagators

$$\frac{1}{m_V^2 - q^2} \simeq \frac{1}{m_V^2} + \frac{q^2}{m_V^4} + \cdots$$

Comparison with χEFT (NN)

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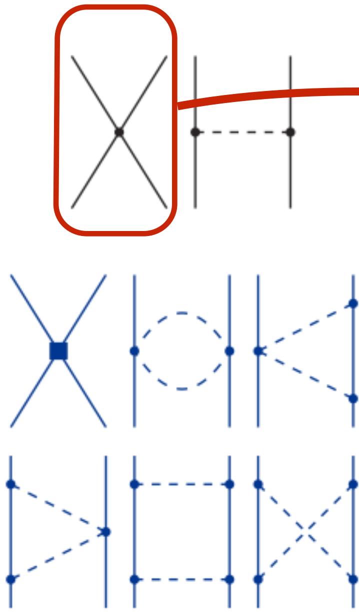

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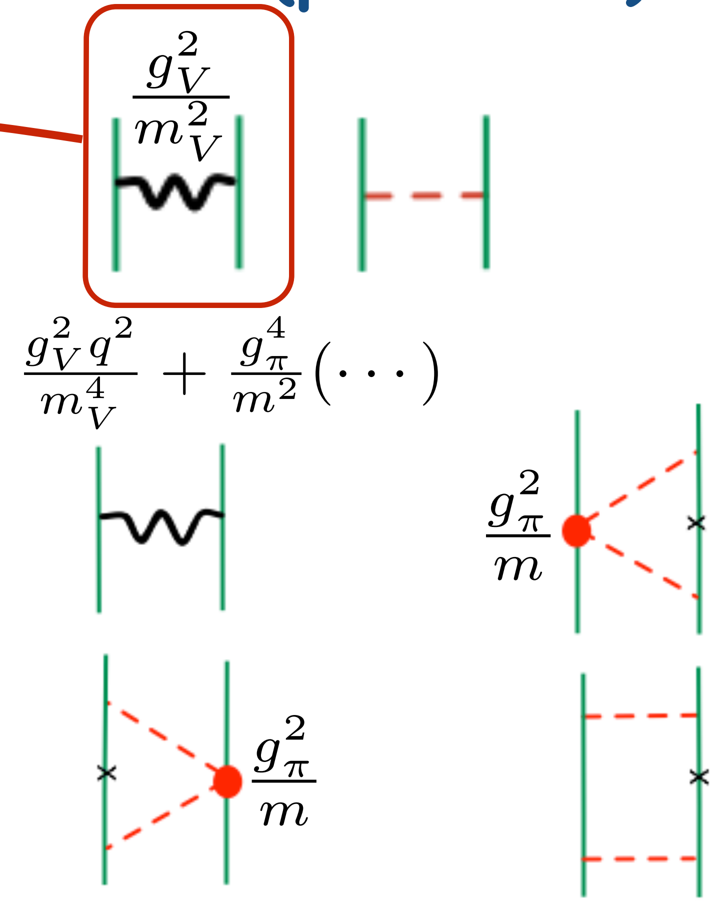

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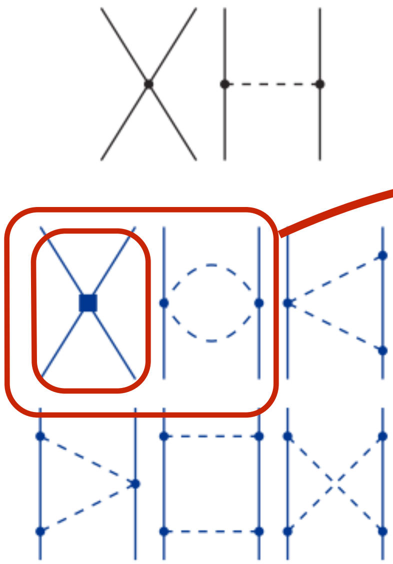

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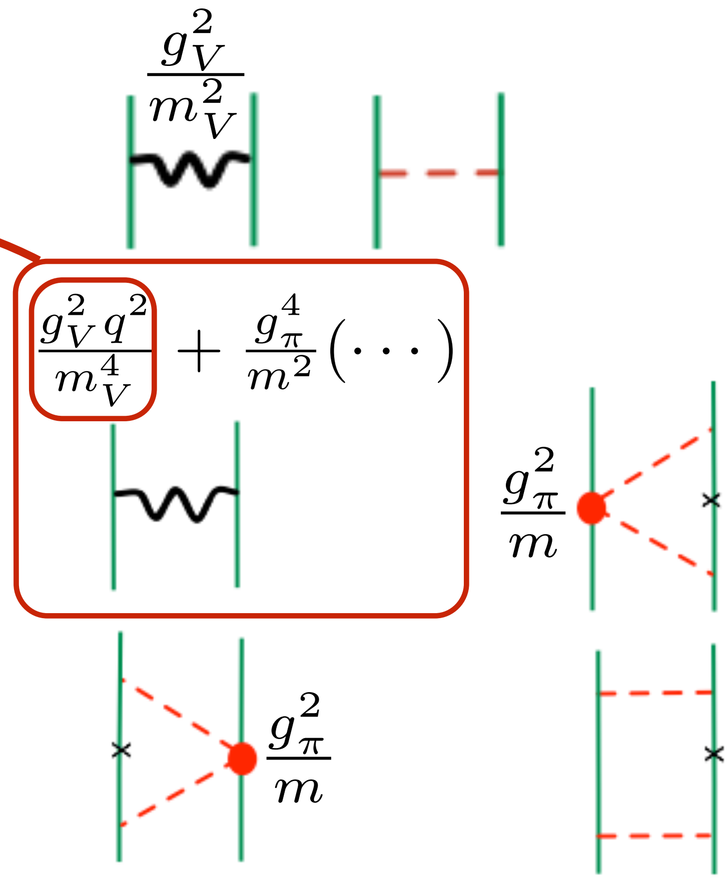

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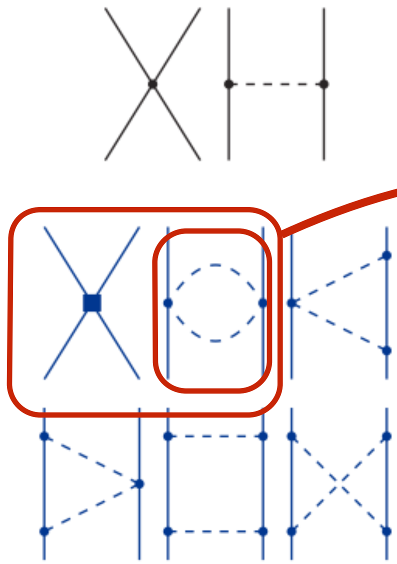

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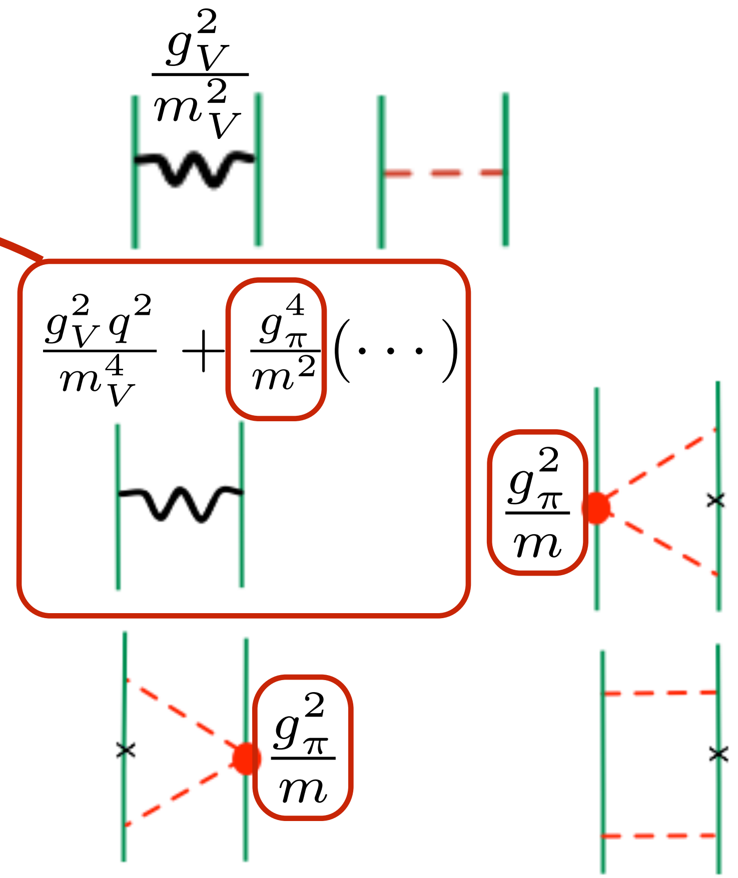

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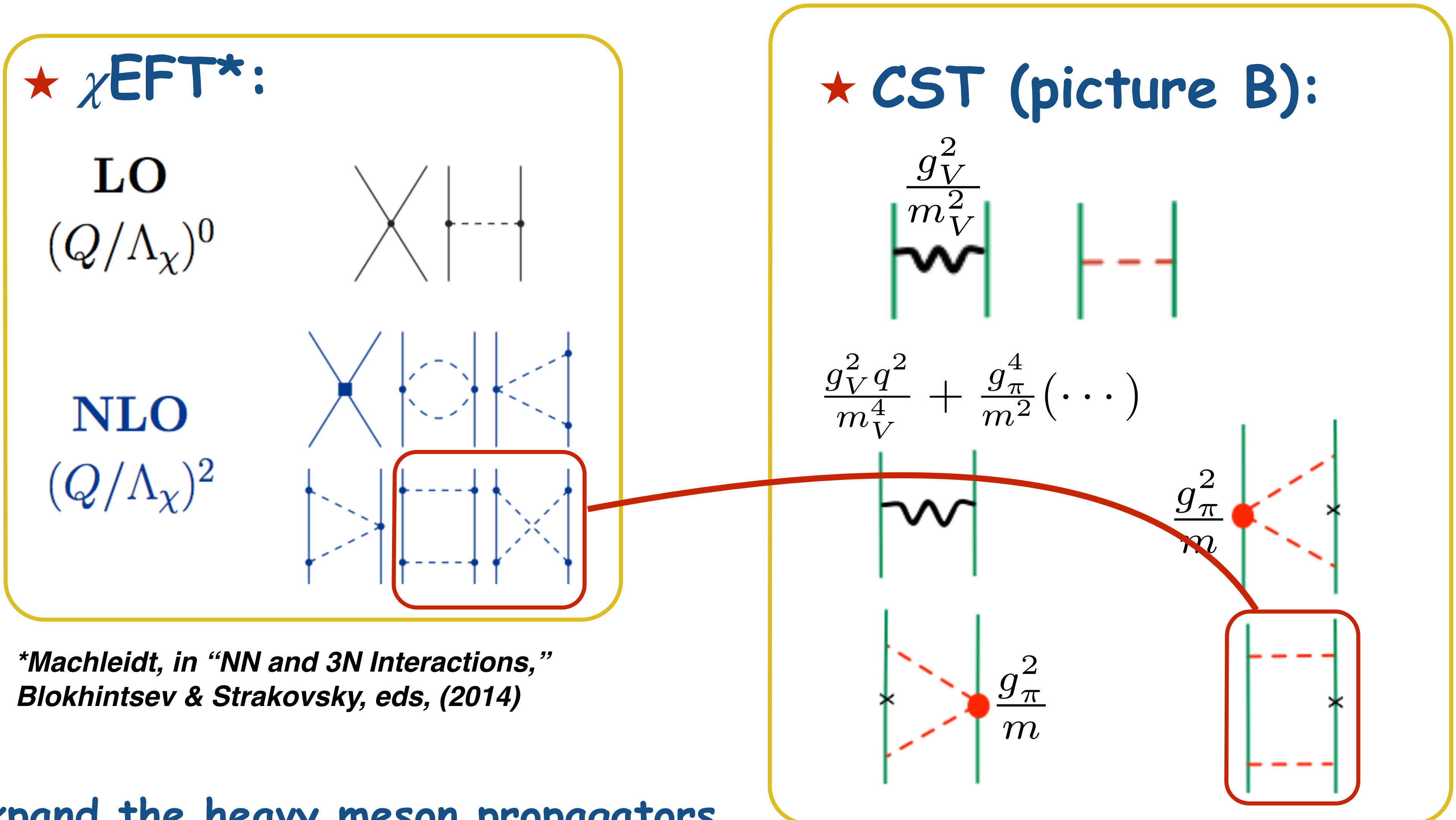

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CST: particle on shell sums the box and crossed box

Comparison with χEFT (NN)

★ χEFT*:

LO
$(Q/\Lambda_\chi)^0$

NL
$(Q/$

★ CST (picture B):

This comparison has yet to be worked out in detail !

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$$\frac{1}{m_V^2 - q^2} \simeq \frac{1}{m_V^2} + \frac{q^2}{m_V^4} + \cdots$$

Contributions to the quadrupole moment (overview)

1. Leading term looks like the non-relativistic formula

$$Q_{\rm NR} = \frac{\sqrt{2}}{10}\int_0^\infty r^2 dr \left\{uw - \frac{w^2}{\sqrt{8}}\right\}$$

2. Relativistic corrections from the normalization:

$$\int_0^\infty k^2 dk(u^2+w^2) = 1 + N_{\rm CST} = \begin{cases} 0 & \text{NR} \\ -\left\langle \frac{\partial V}{\partial m_d}\right\rangle - \int_0^\infty k^2 dk (v_t^2 + v_s^2) & \text{R} \end{cases}$$

3. Relativistic k^2/m^2 corrections to leading term

★ All other corrections from derivatives of the kernel and P-wave contributions are negligible (for WJC-2)

Contributions to the quadrupole moment (size)

1	-4.1%	Nonrelativistic formula with nonrelativistic normalization gives a result too small (in line with previous calculations)
2	+2.3%	Relativistic correction to the normalization:
		-1.8% derivative of the strong from factor
		+1.1% interaction current from off-shell particle
		+3.0% interaction current from on-shell particle
3	+1.0%	Relativistic k/m corrections to leading terms
	-0.8%	TOTAL (WJC-2)

Lessons and Conclusions

- ★ Both CST and χEFT predict the quadrupole moment within 1% (with no free parameters!)

- ★ Conjecture: CST and χEFT include the same physics

- ★ Comparing the two leads to an understanding of

  - the content of CST OBE diagrams, and the off-shell couplings in particular

  - how the many χEFT parameters might be determined from the fewer OBE parameters (will it work? — still to be done)

- ★ Currents: both approaches use measured nucleon form factors, and are therefore incomplete

- ★ ONLY CST can predict the behavior at large momentum Q (not discussed in this talk)