Resonances in coupled-channel scattering from lattice QCD

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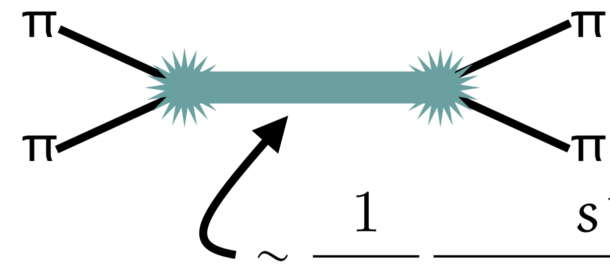
Based on work in collaboration with J.J. Dudek, R.G. Edwards and C.E. Thomas.

Chiral Dynamics
Pisa, Italy.
June 29th 2015

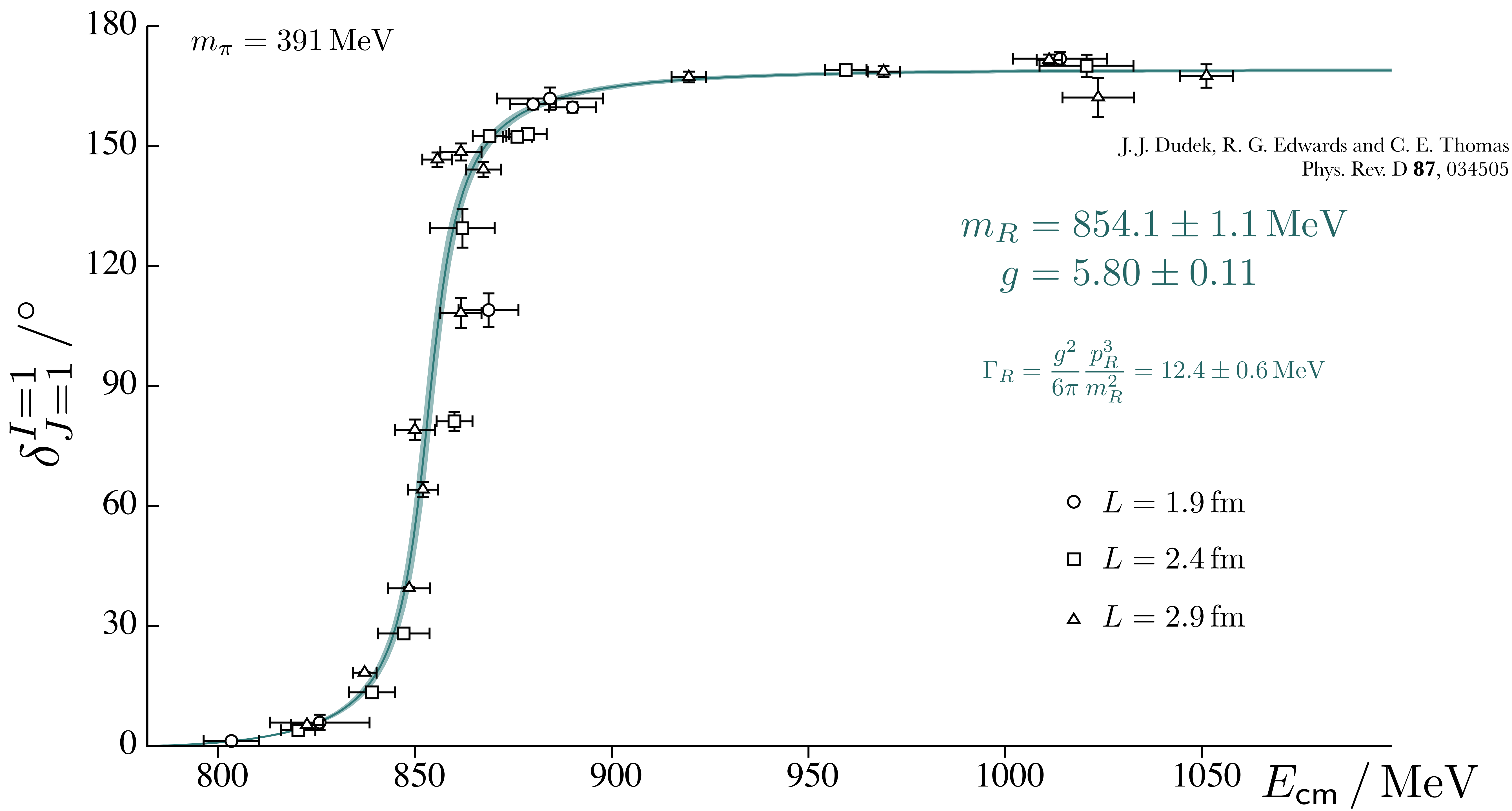
Jefferson Lab

Resonances from QCD



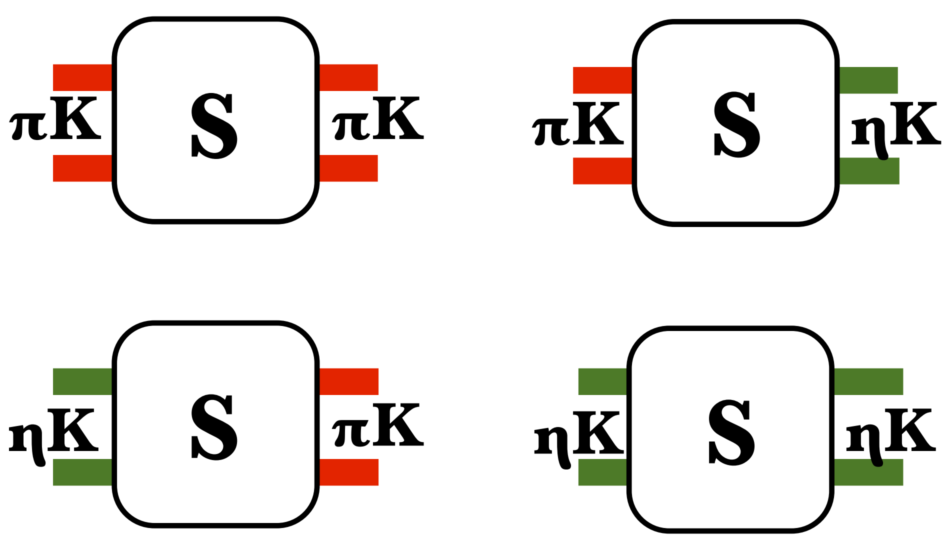
$$\sim \frac{1}{\rho(s)} \frac{s^{\frac{1}{2}}\Gamma(s)}{m_R^2 - s - i s^{\frac{1}{2}}\Gamma(s)}$$

$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k_{cm}^3}{E_{cm}^2}$$



Coupled-channel scattering

- Most physical resonances couple to multiple channels.
- To understand the physical spectrum, applying coupled-channel methods will be essential.
- We consider here **πK**, where **ηK** can also contribute in I=1/2.
- The physical amplitudes have resonances in several partial waves



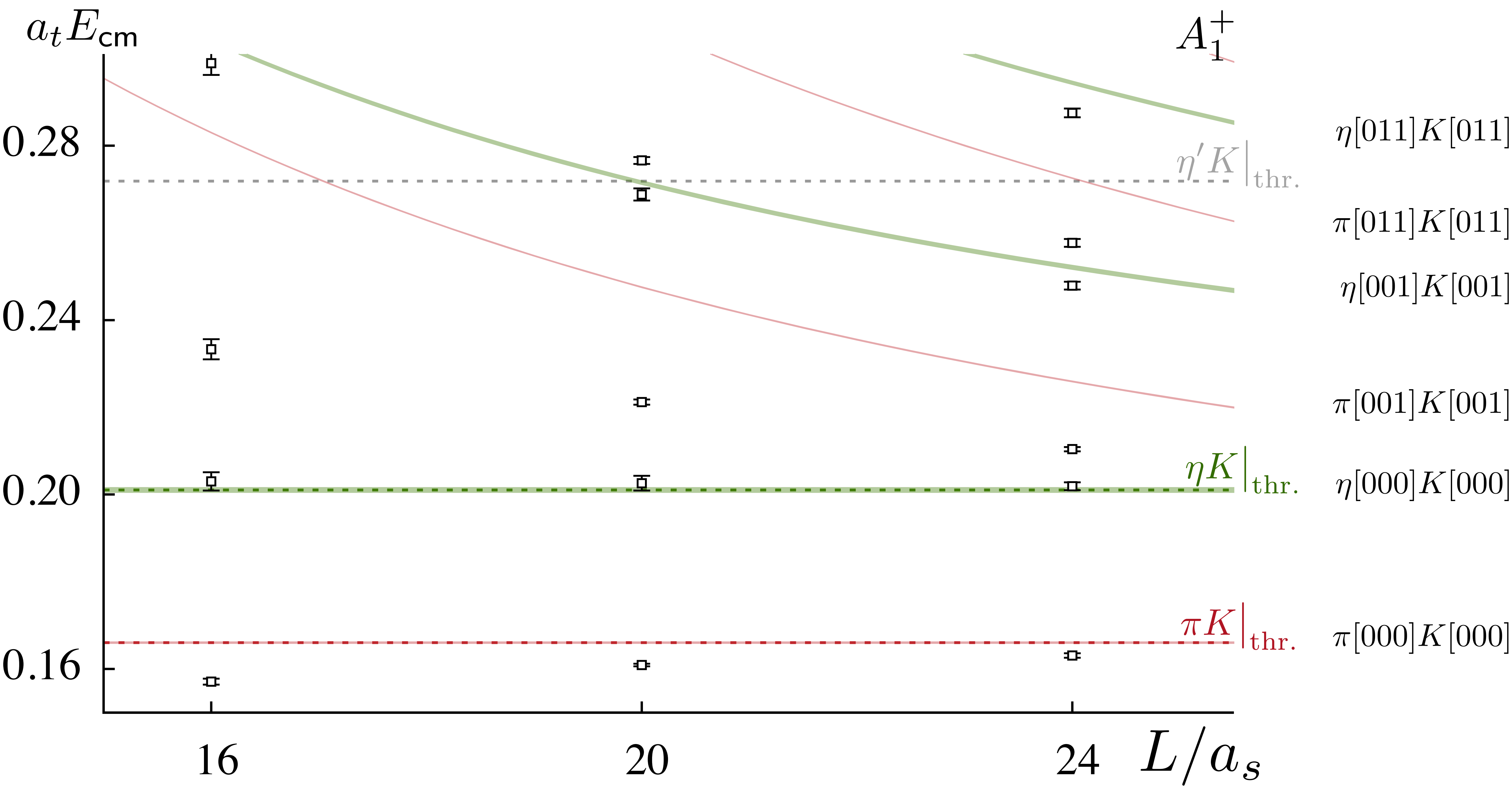
$J^P=0^+$	$\kappa(700)$, $K_0^*(1430)$, ...
$J^P=1^-$	$K^*(892)$, ...
$J^P=2^+$	$K_2^*(1430)$, ...

Aim: Obtain the scattering S-matrix from Lattice QCD

Coupled-channel scattering

Main ingredients to obtain the finite volume spectra:

- Anisotropic lattices - finer temporal spacing.
- Distillation (Peardon *et al* 2009).
- A large basis of $\bar{q}q$-like constructions.
- Pairs of optimised meson operators.
- Variational method to obtain the spectrum (Michael 1985, Lüscher & Wolff 1990).



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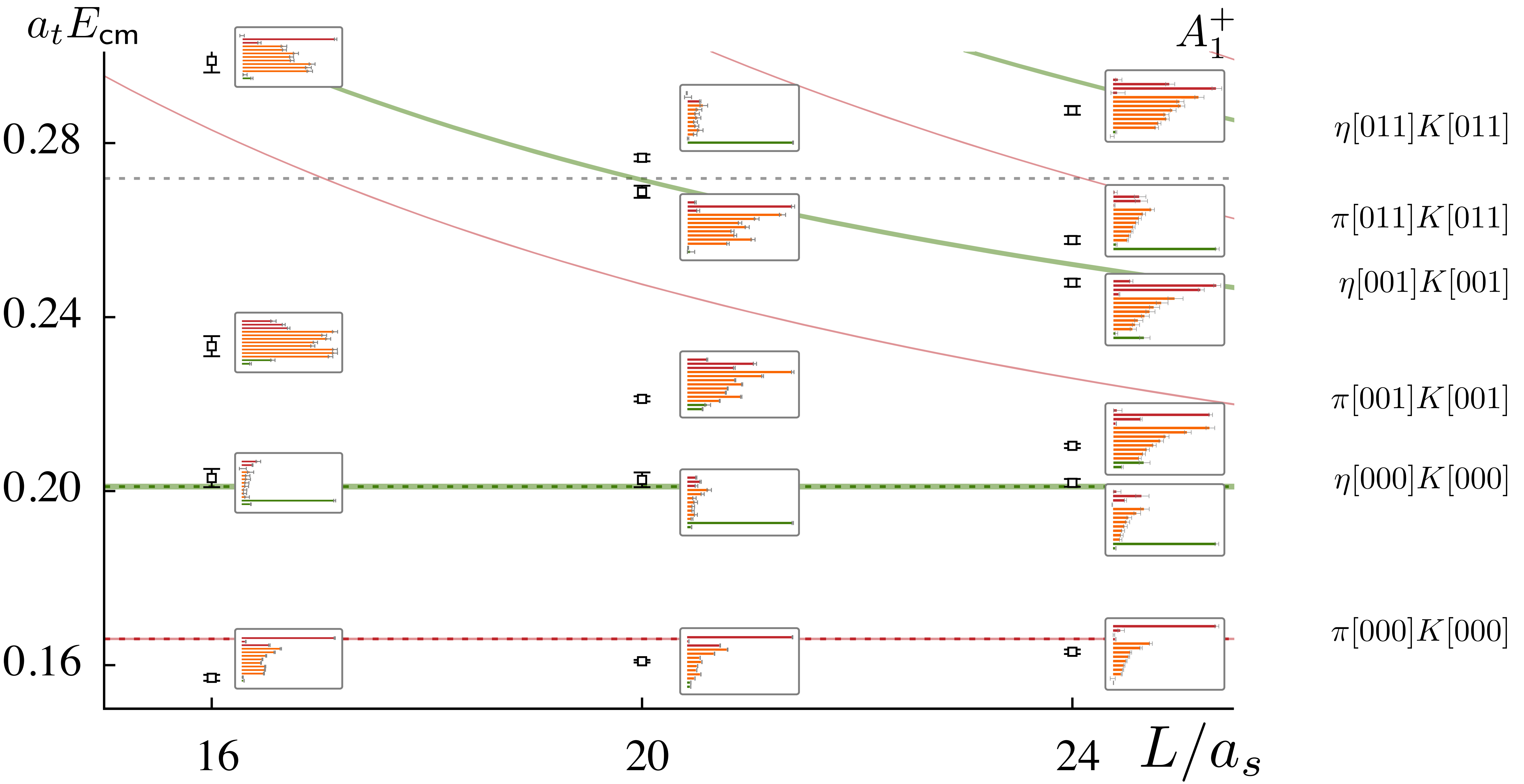

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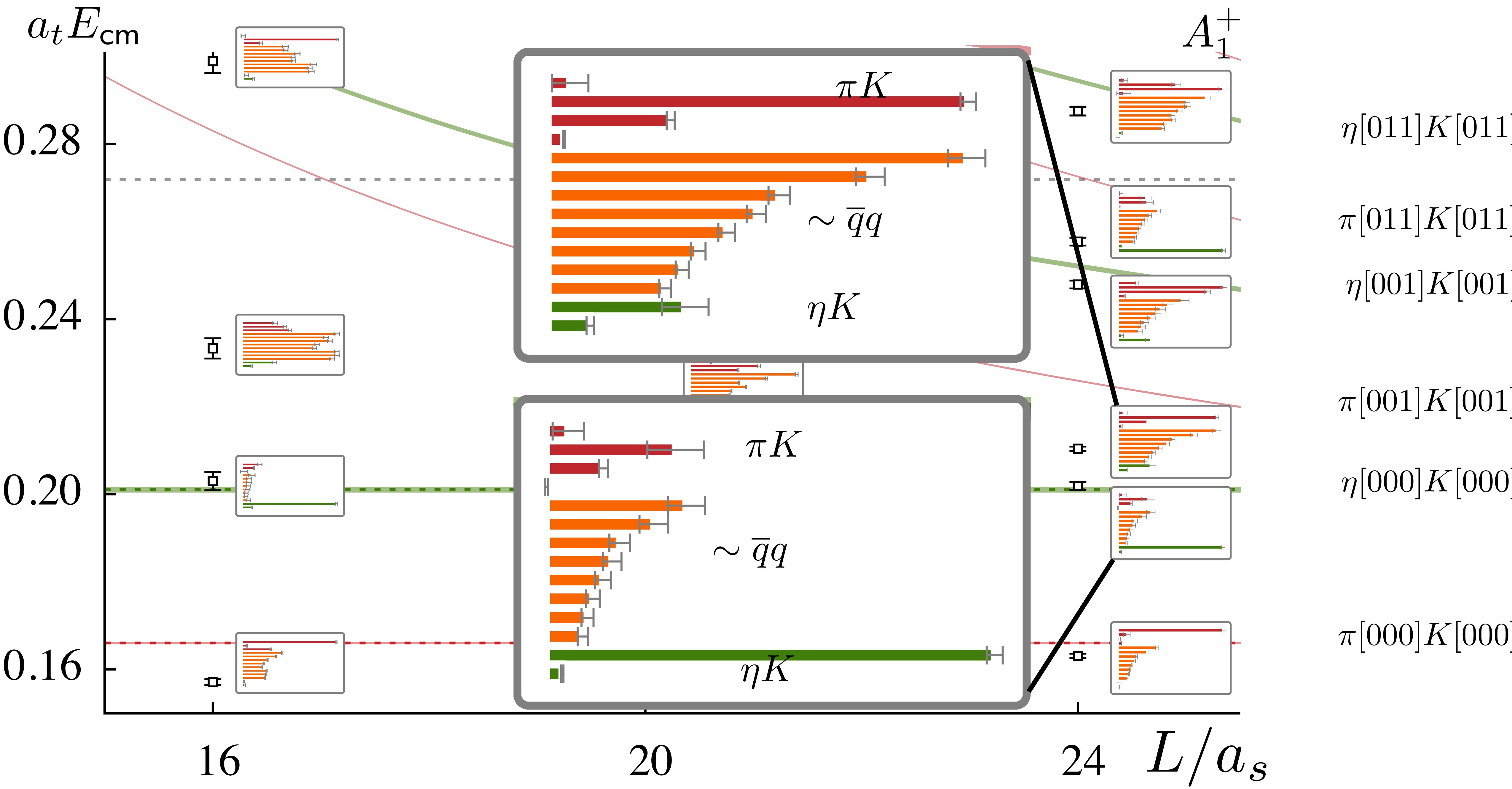

Coupled-channel extensions of Lüscher's method

$$\det\left[t^{-1}_{ij}(E) + \mathcal{M}_{ij}(E)\right] = 0$$

Many contributors:
Lüscher
Gottlieb & Rummukainen
Christ, Kim, & Yamazaki
Kim, Sachrajda & Sharpe
He, Feng & Liu
Bernard, Lage, Meissner, and Rusetsky
Leskovec & Prelovsek
Briceño & Davoudi
Hansen & Sharpe
Gockeler *et al*
Guo, Dudek, Edwards & Szczepaniak
Briceño, Davoudi, Luu
+ ...

Coupled-channel extensions of Lüscher's method

$$\det\left[t_{ij}^{-1}(E) + \mathcal{M}_{ij}(E)\right] = 0$$

infinite volume scattering
t-matrix

known finite-volume
functions

$$S = 1 + 2i\rho\, t$$

diagonal in partial waves,
mixes channels

diagonal in channels,
mixes partial waves

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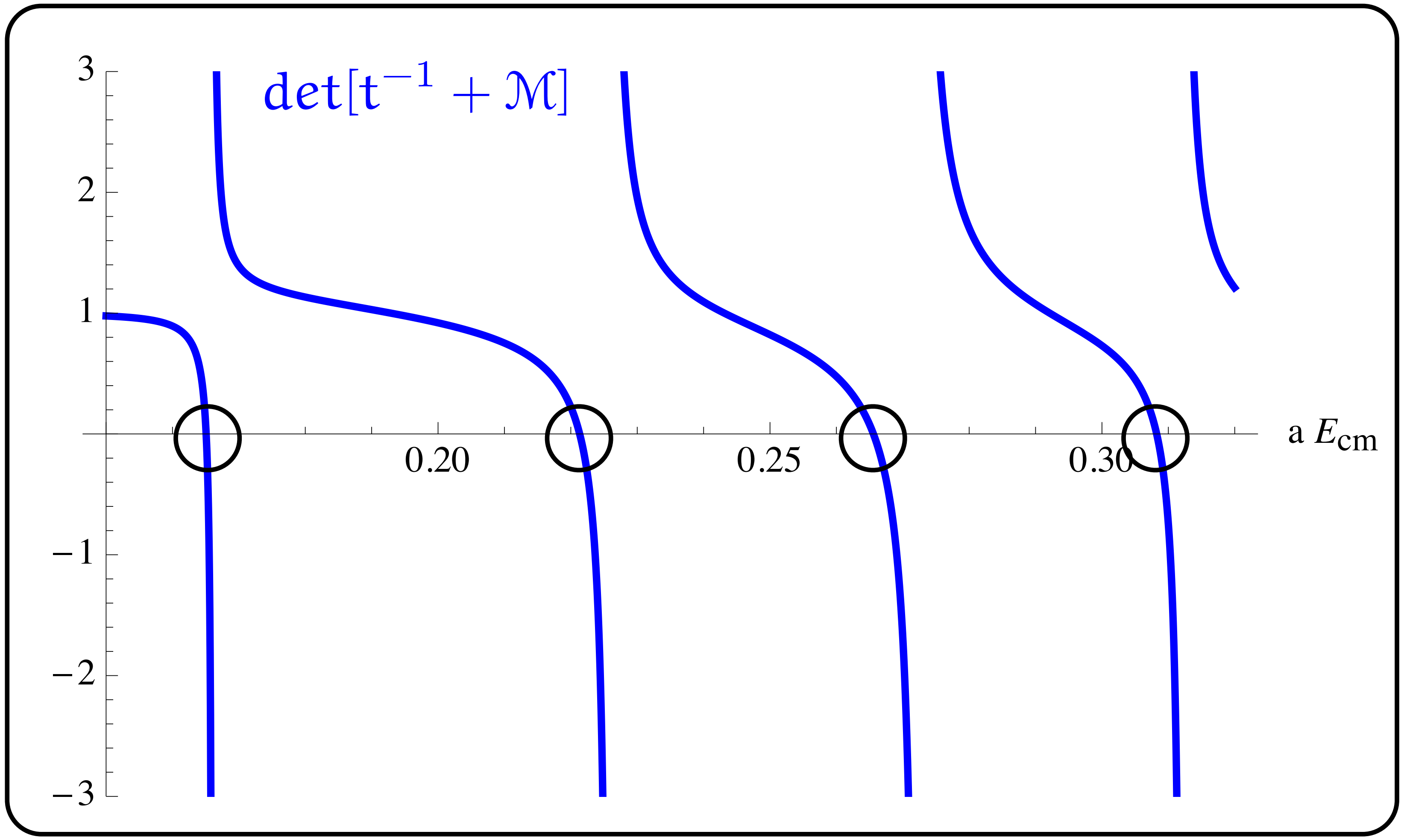


Coupled-channel scattering

Problem: Three or more unknowns for each energy level, eg:

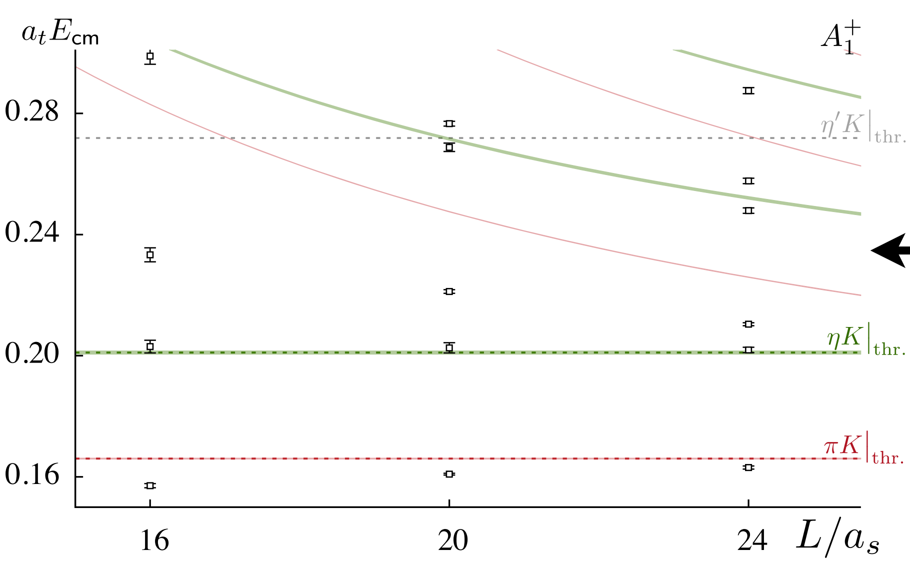
(...and even more with higher partial waves)

$$S_{11} = \eta\, e^{2i\,\delta^{\pi K}}$$

$$S_{22} = \eta\, e^{2i\,\delta^{\eta K}}$$

2x2 complex matrix (or more) but only one equation.
No one-to-one relation from energy levels to amplitudes

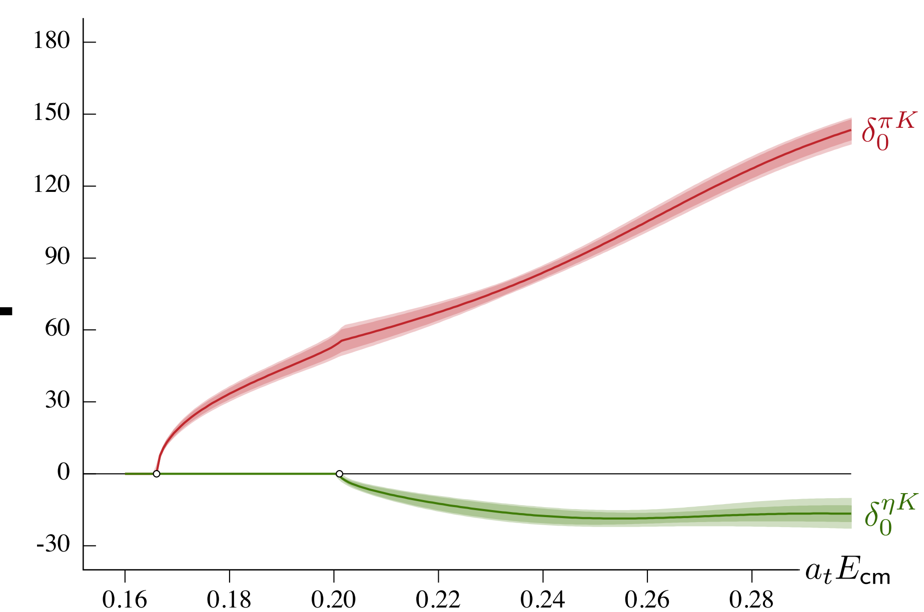
Finite volume energy levels



?

Extensions of Lüscher's method

Scattering Amplitudes



Coupled-channel scattering

Solution: Parameterise *t*-matrix, constrain parameters using many energy levels
E.g.: *K*-matrix (it's essential that we preserve unitarity)

$$S_{ij} = \delta_{ij} + 2i\,(\rho_i\rho_j)^{\frac{1}{2}}\, t_{ij}$$

$$[S^\dagger S]_{ij} = \delta_{ij}$$

$$\rightarrow \mathrm{Im}[t^{-1}]_{ij} = -\rho_i\delta_{ij}$$

- *K*-matrix contains everything that isn't constrained by unitarity

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) - i\delta_{ij}\rho_i(s)$$

- *K* must be real for real *s*. One option for two channel scattering:

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K} \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

- *m*, *g*, *γ* are real free parameters. Simple to add more - more poles, or a polynomial in *s*.
- Simple to generalise to scattering with non-zero angular momentum.
- Can be improved by adding extra physically motivated properties - eg: Chew-Mandelstam phase space.

Coupled-channel scattering

- Describe t-matrix using K-matrix in S-wave only → obtain a spectrum.
- Minimise a χ^2 to obtain the best agreement between the K-matrix and lattice energies.

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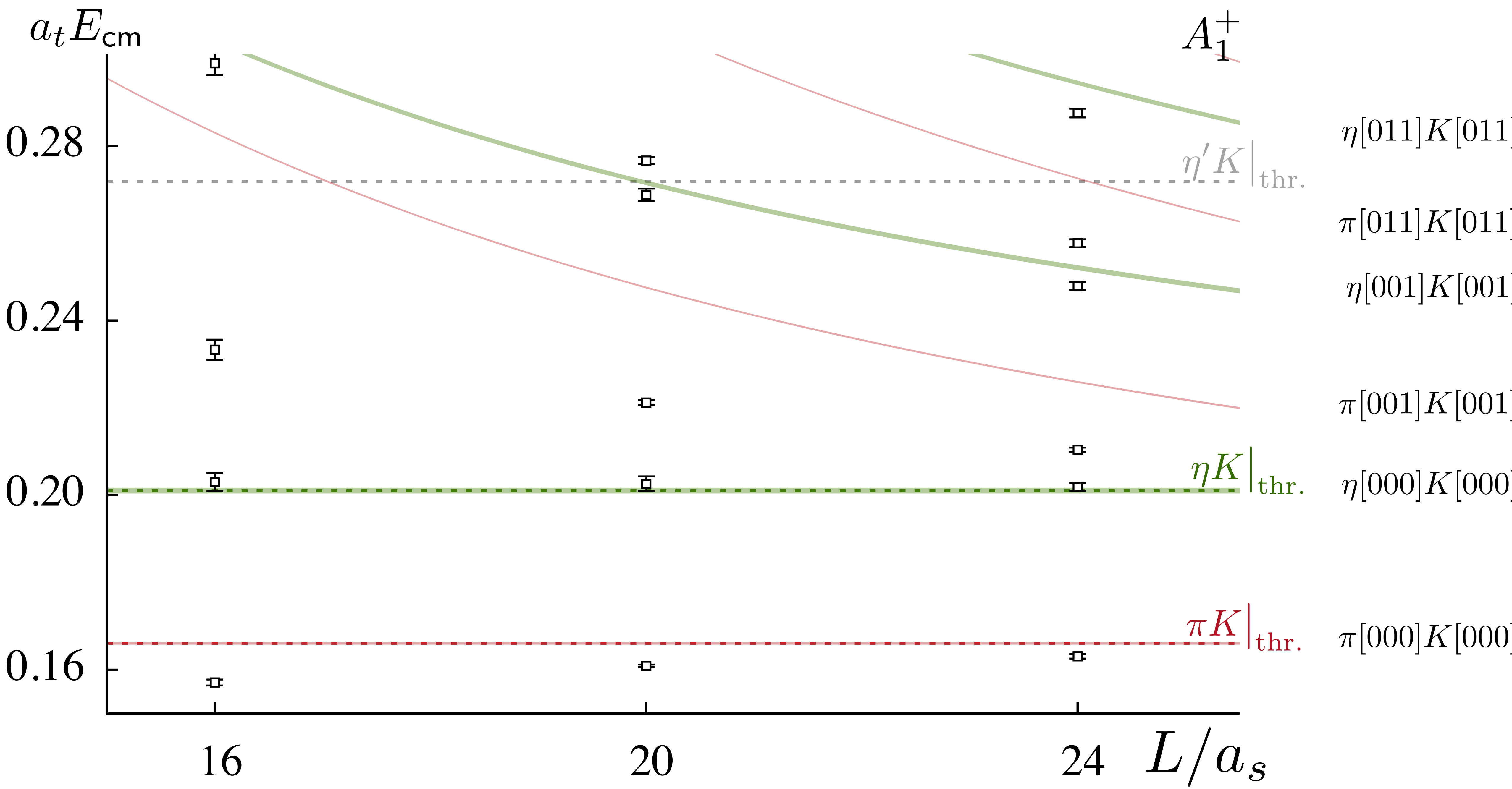

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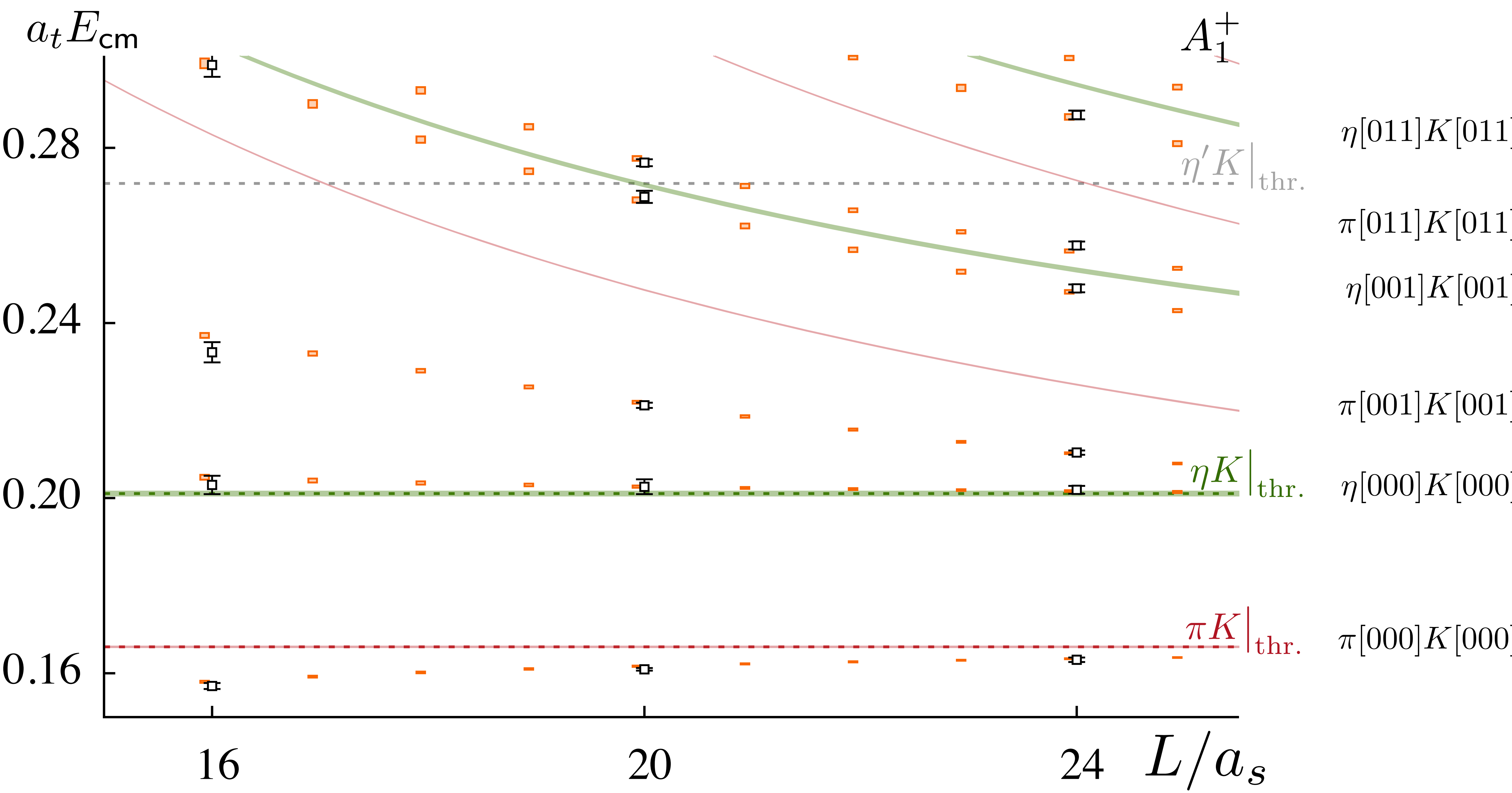
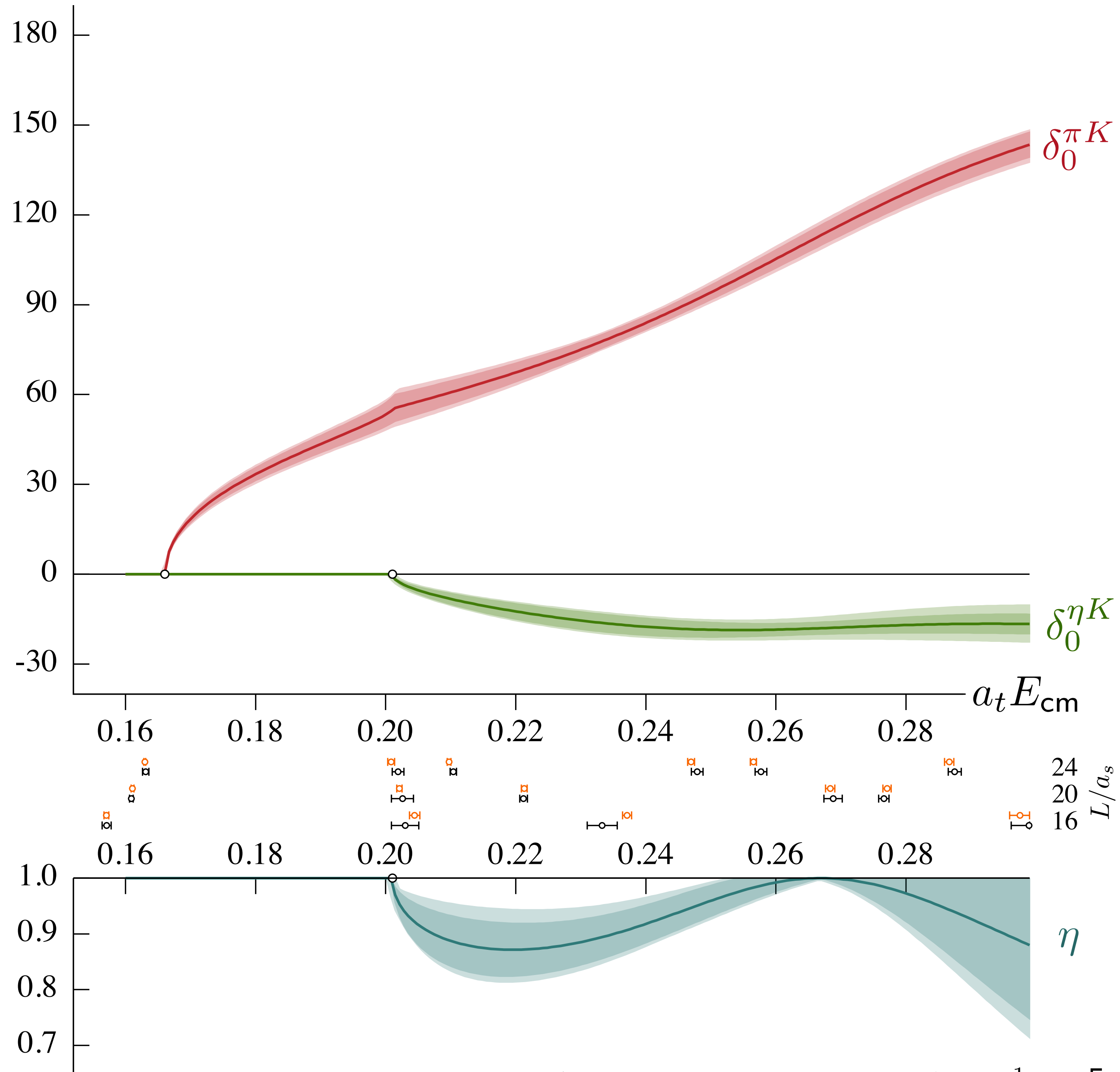
$$\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71$$



S-wave amplitudes



$$S_{11} = \eta\, e^{2i\,\delta^{\pi K}}$$

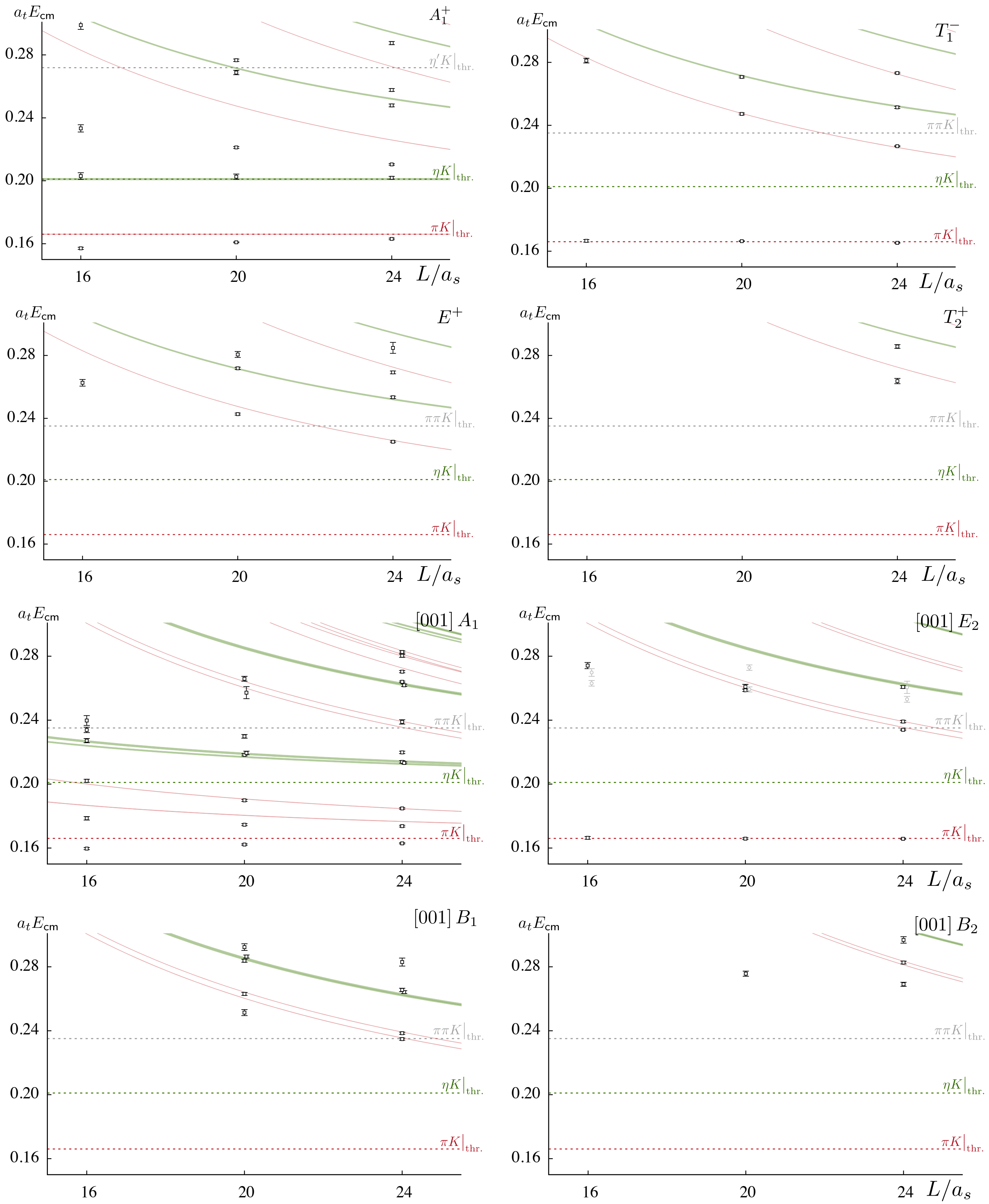
$$S_{22} = \eta\, e^{2i\,\delta^{\eta K}}$$

- Broad resonance in S-wave πK.
- ηK coupling is small.
- 3 subthreshold points, naturally included in an energy-level fit.

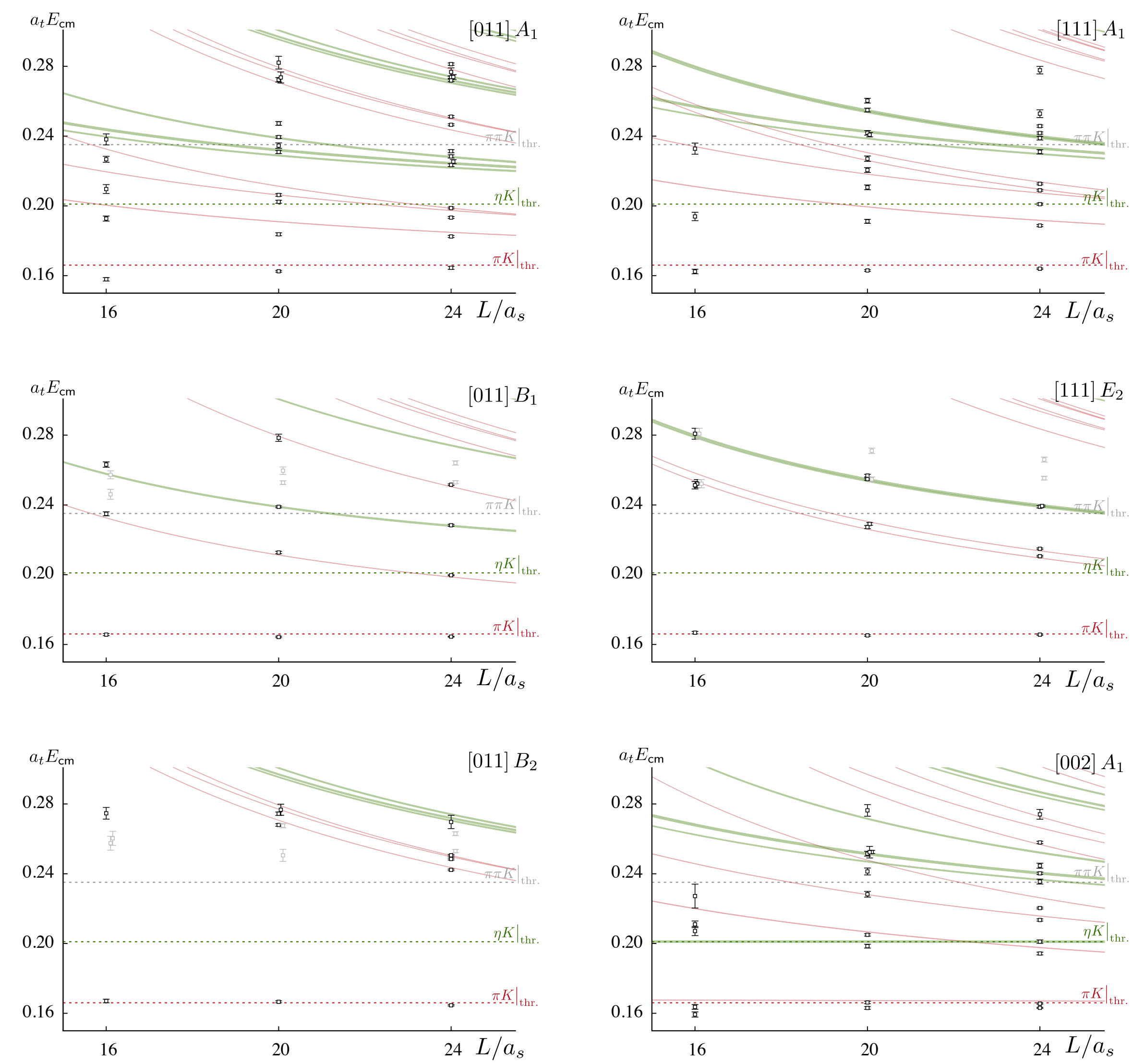
$$\begin{aligned}
\mathsf{m} &= (0.2466 \pm 0.0020 \pm 0.0009) \cdot \mathsf{a}_t^{-1} \\
\mathsf{g}_{\pi K} &= (0.165 \pm 0.006 \pm 0.002) \cdot \mathsf{a}_t^{-1} \\
\mathsf{g}_{\eta K} &= (0.033 \pm 0.010 \pm 0.003) \cdot \mathsf{a}_t^{-1} \\
\gamma_{\pi K,\pi K} &= 0.184 \pm 0.054 \pm 0.030 \\
\gamma_{\pi K,\eta K} &= -0.52 \pm 0.20 \pm 0.06 \\
\gamma_{\eta K,\eta K} &= -0.37 \pm 0.07 \pm 0.05
\end{aligned}
\qquad
\begin{bmatrix}
1 & 0.35 & -0.38 & 0.17 & 0.27 & -0.19 \\
 & 1 & -0.05 & -0.16 & 0.85 & 0.08 \\
 & & 1 & 0.26 & -0.11 & 0.64 \\
 & & & 1 & 0.10 & 0.25 \\
 & & & & 1 & 0.05 \\
 & & & & & 1
\end{bmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71\,.$$

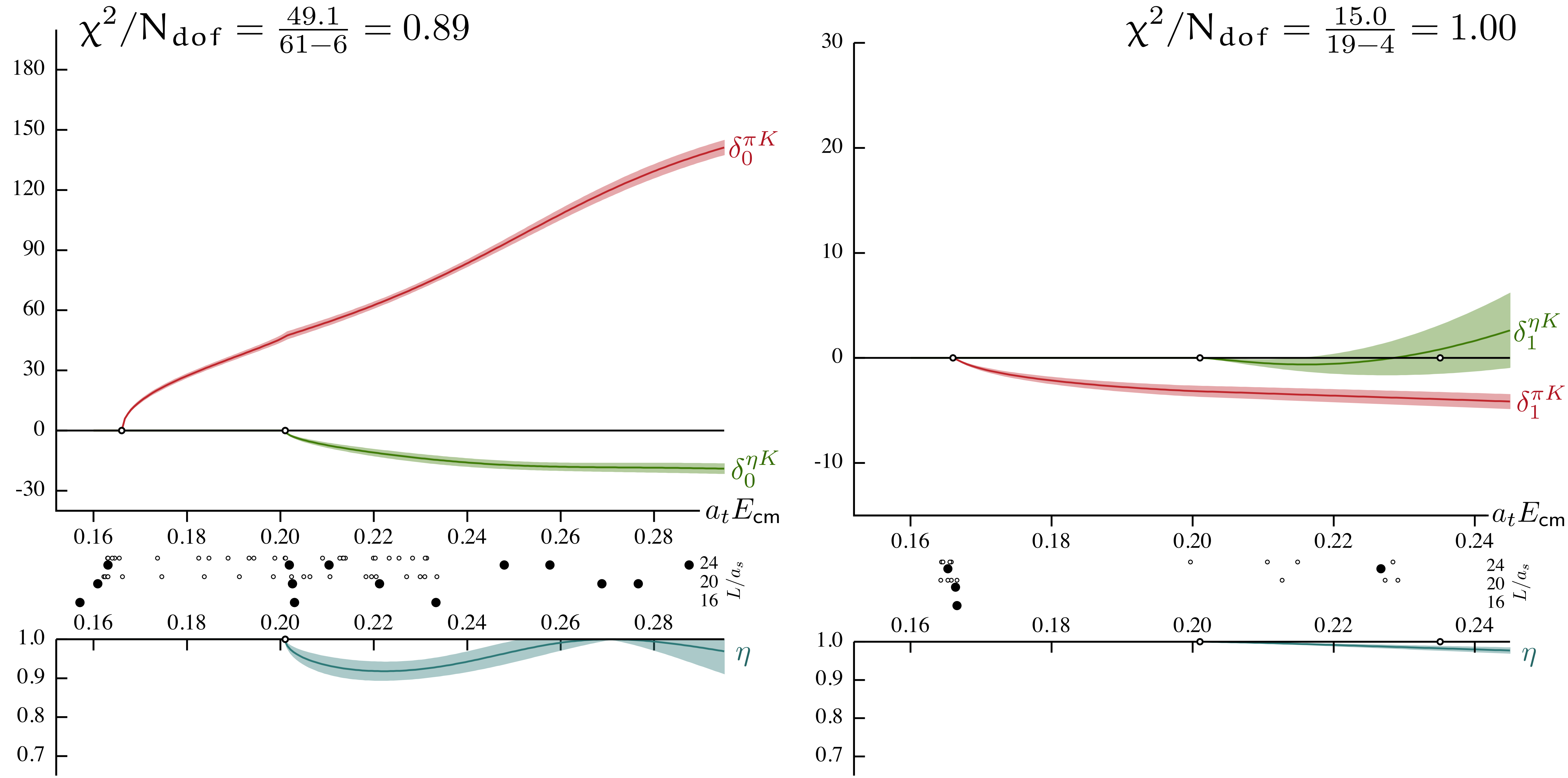
More energy levels



- Many more energy levels from irreps where the mesons are **moving with respect to the lattice**.
- More than 100 usable levels.
- Less symmetry. More **mixing of partial waves**, requiring simultaneous solutions.

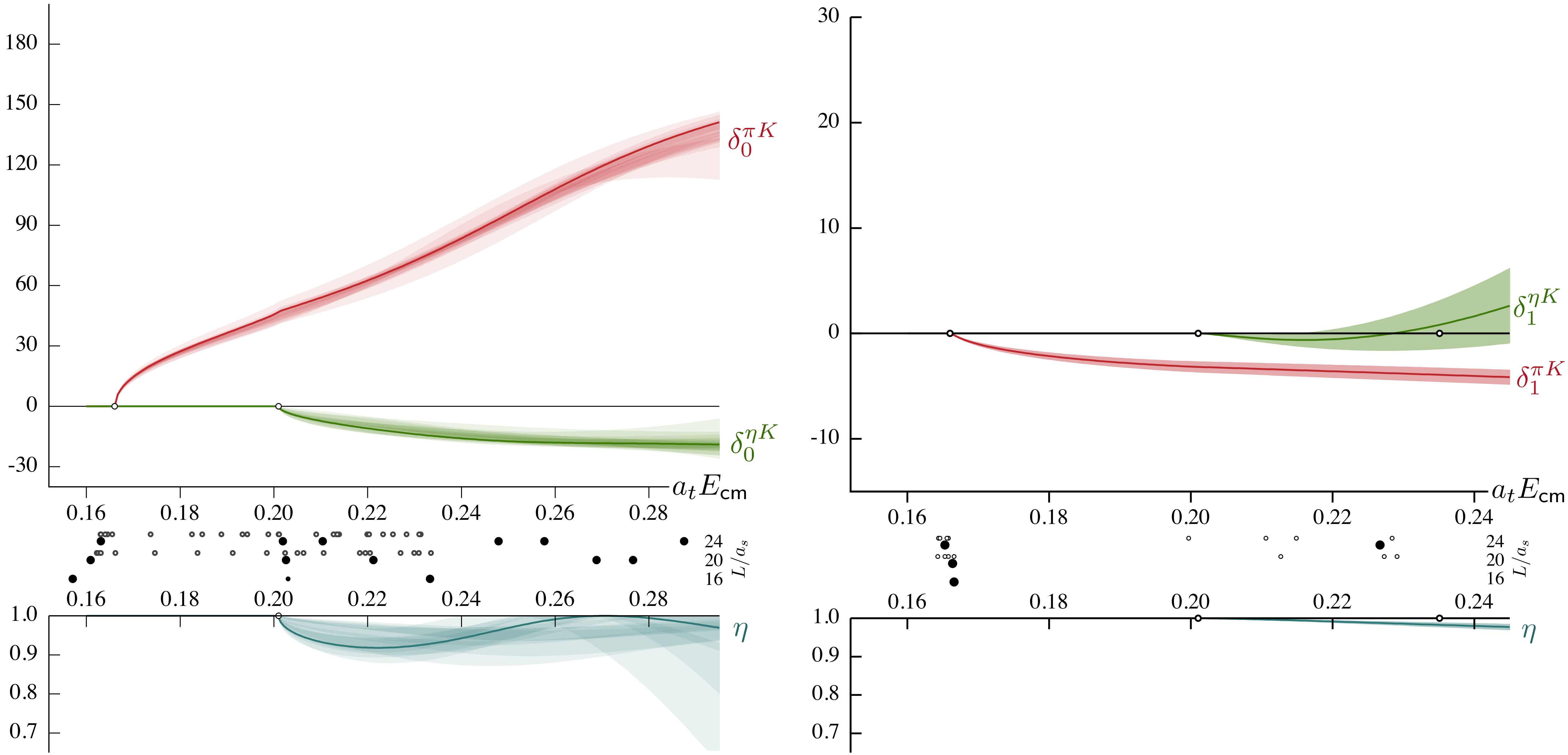


S+P-waves from 80 energy levels



- K-matrix parameterisations in S and P wave.
- Separate fits and global fits yield consistent results.
- D-wave is negligible in this region.

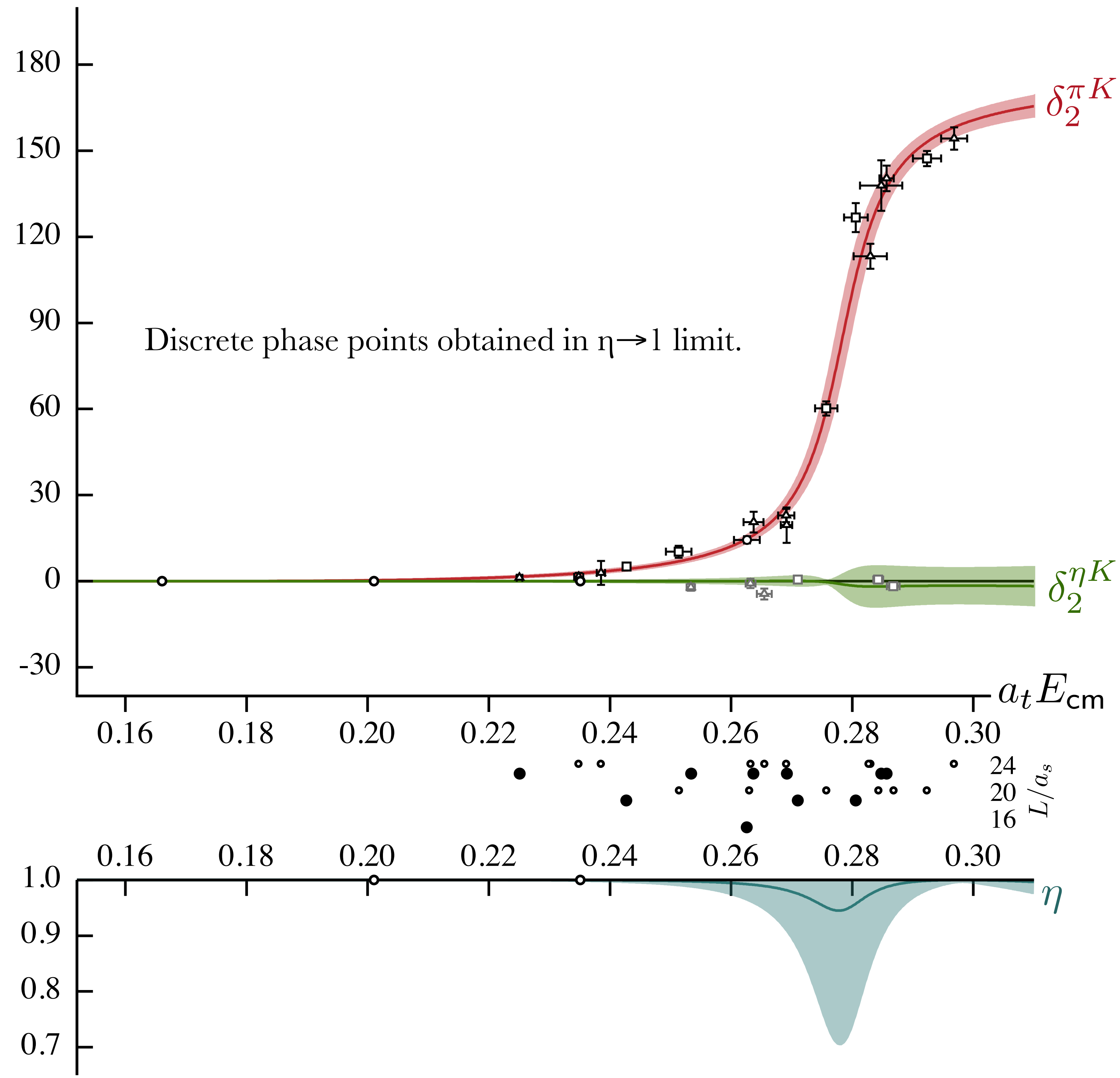
Parameterisation variation



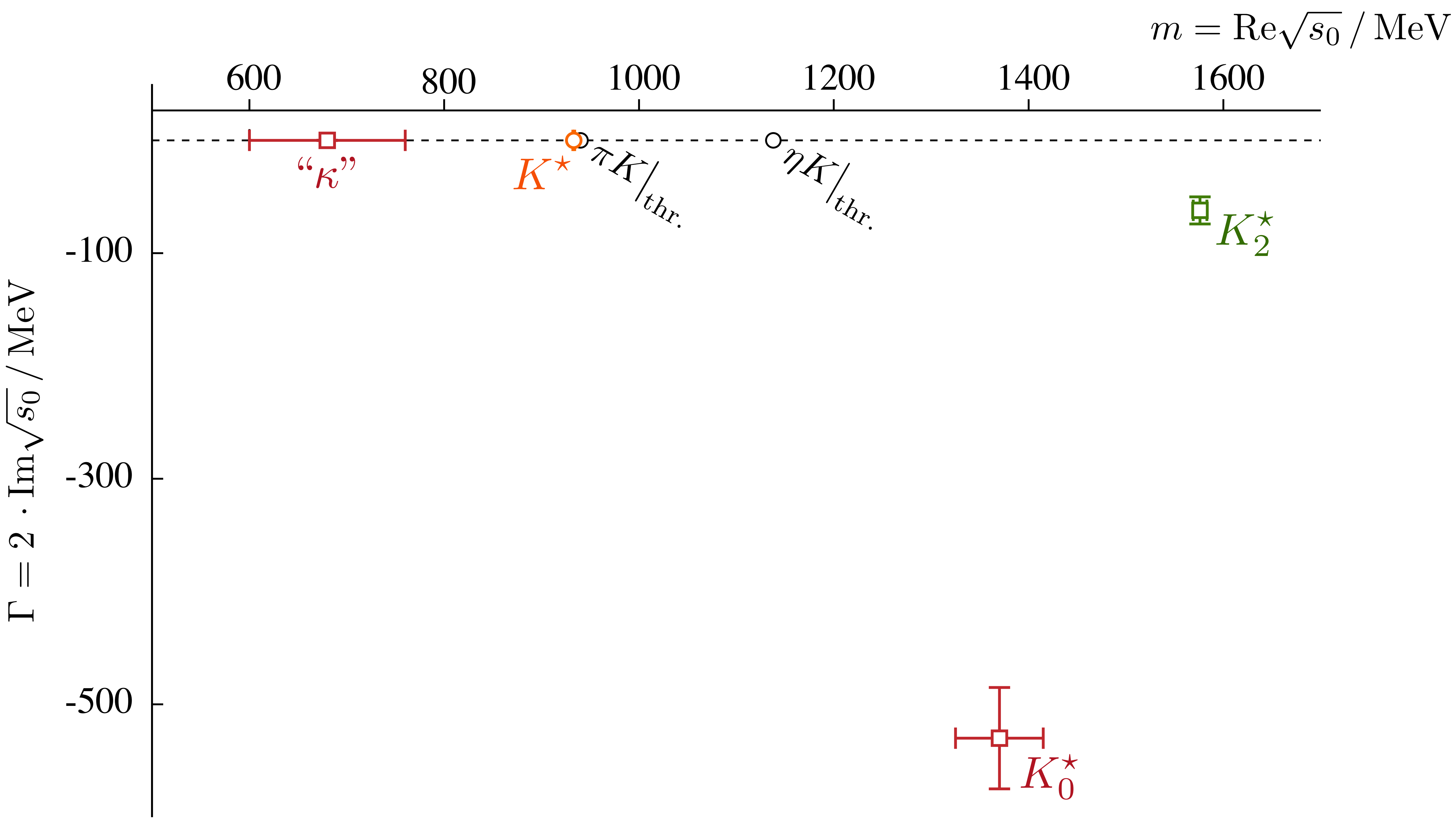
- K-matrix parameterisations in *S* and *P* wave.
- Separate fits and global fits yield consistent results.
- *D*-wave is negligible in this region.

Narrow D-wave resonance

- Many other energy levels containing scattering amplitude information.
- Using only irreps with $\mathcal{J}$=2 and higher (E^+, $T_2{}^+$, $[100]B_{1,2}$) we find a narrow resonance:
- Fit to energies.
- In $\mathcal{J}\geq1$ scattering the lowest threshold is $\pi\pi K$ at a_tE_{cm}=0.235.
- Ideally requires 3-body formalism. Although not strictly rigorous, we can apply the 2→2 formalism anyway.



S-matrix poles



Summary

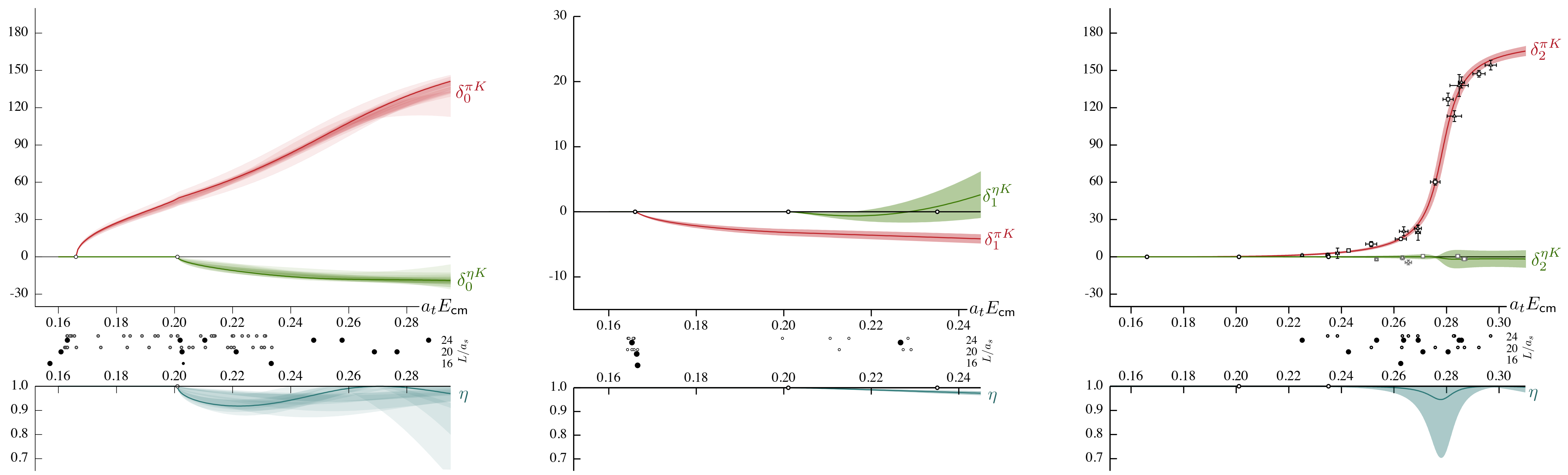
- Coupled-channel scattering amplitudes can be obtained from QCD using lattice methods.
- Using extensions of Lüscher's method, we were able to connect finite volume energy levels to infinite volume scattering amplitudes.
- There are many exciting possibilities for future calculations using similar methods:

  Strongly coupled systems like the $a_0(980)$ and $f_0(980)$ are under investigation.

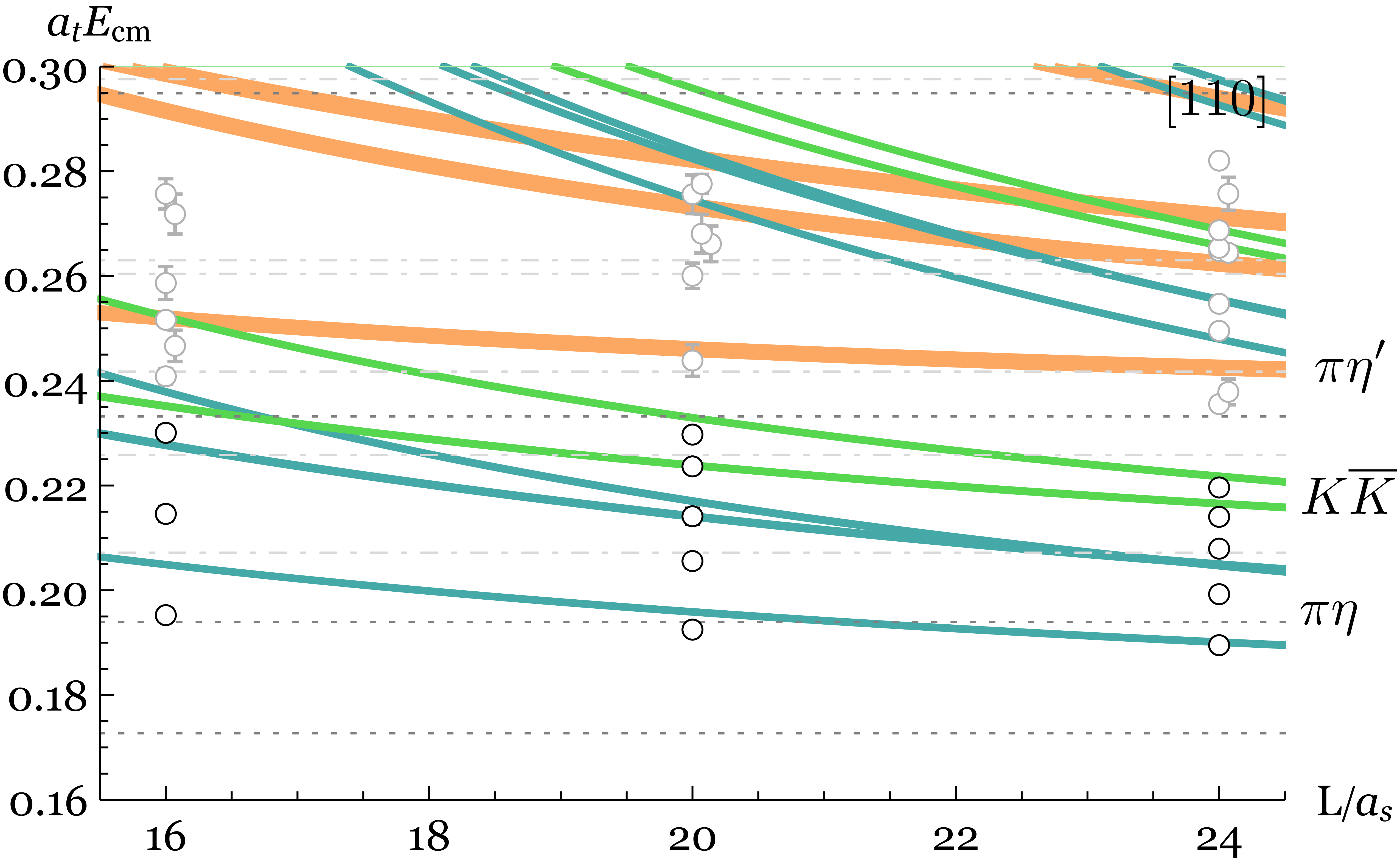
  Investigations into $\pi\gamma \rightarrow \pi\pi$ and similar processes are underway.

  Channels involving charm quarks are also under investigation by European collaborators.

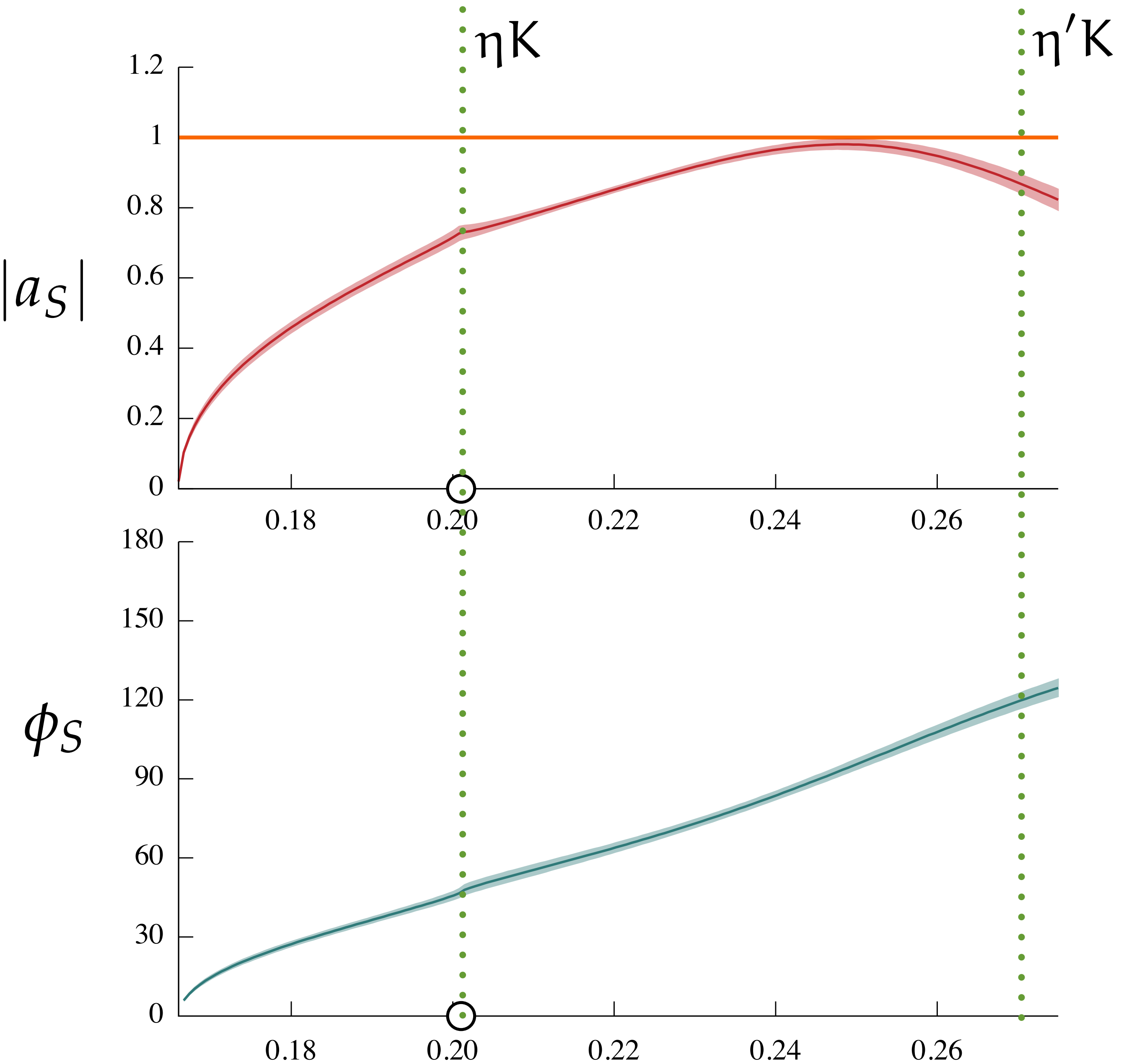
- Further in the future: $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$. Multiparticle scattering, exotics.

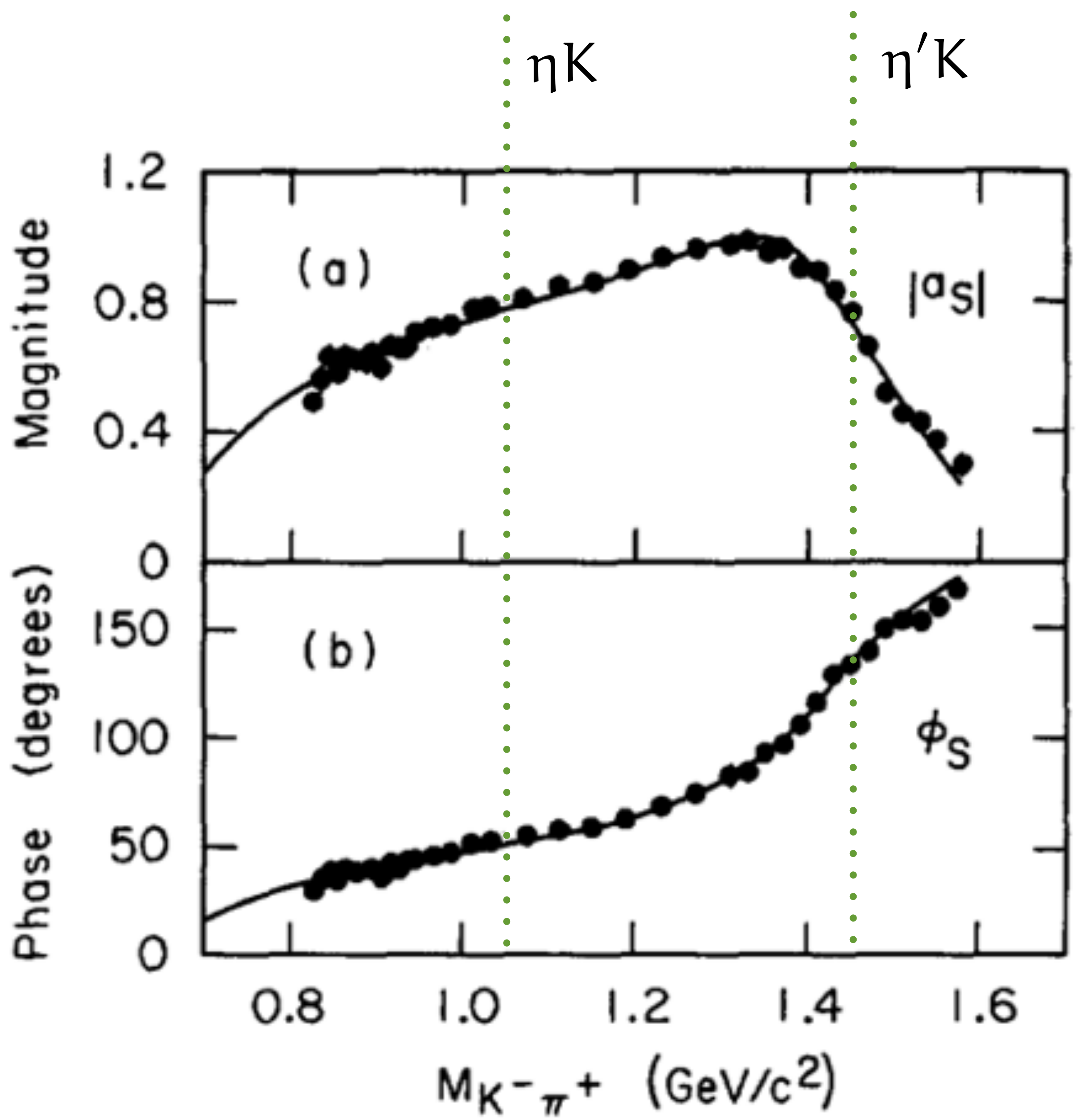


Coming soon: πη-KK̄-πη’



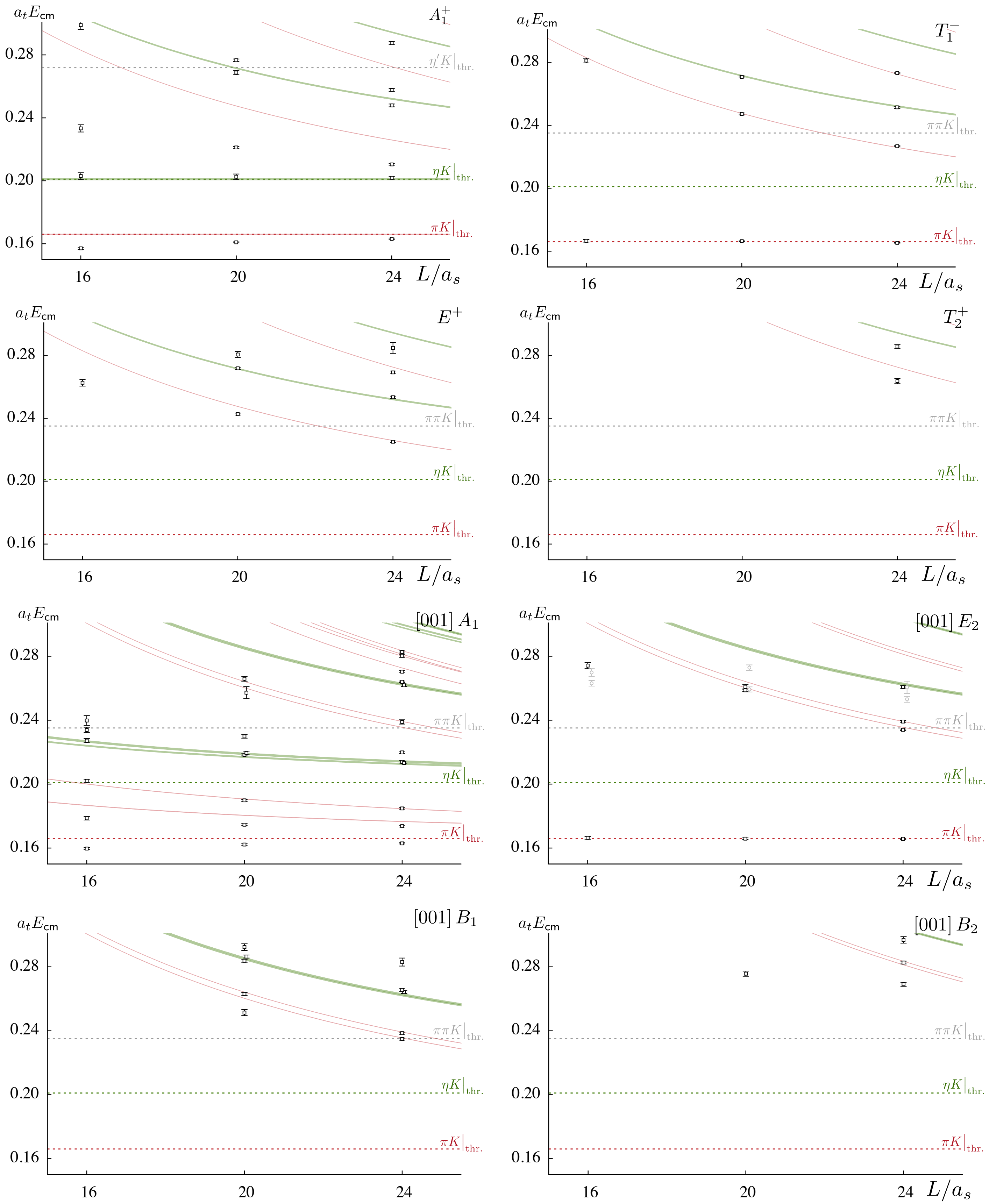
S-wave amplitudes vs experiment





Backup slides: Lattice

More energy levels



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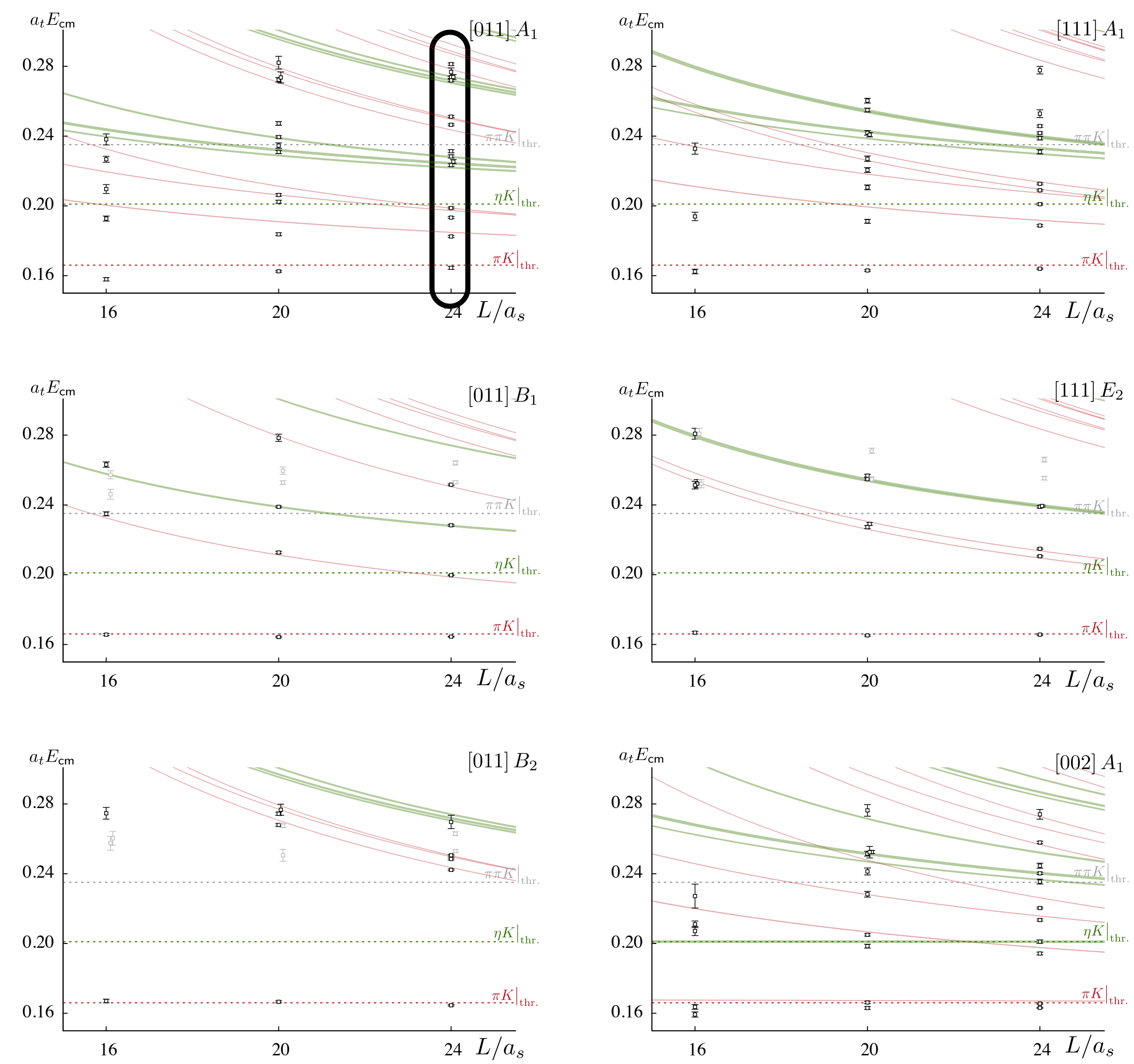
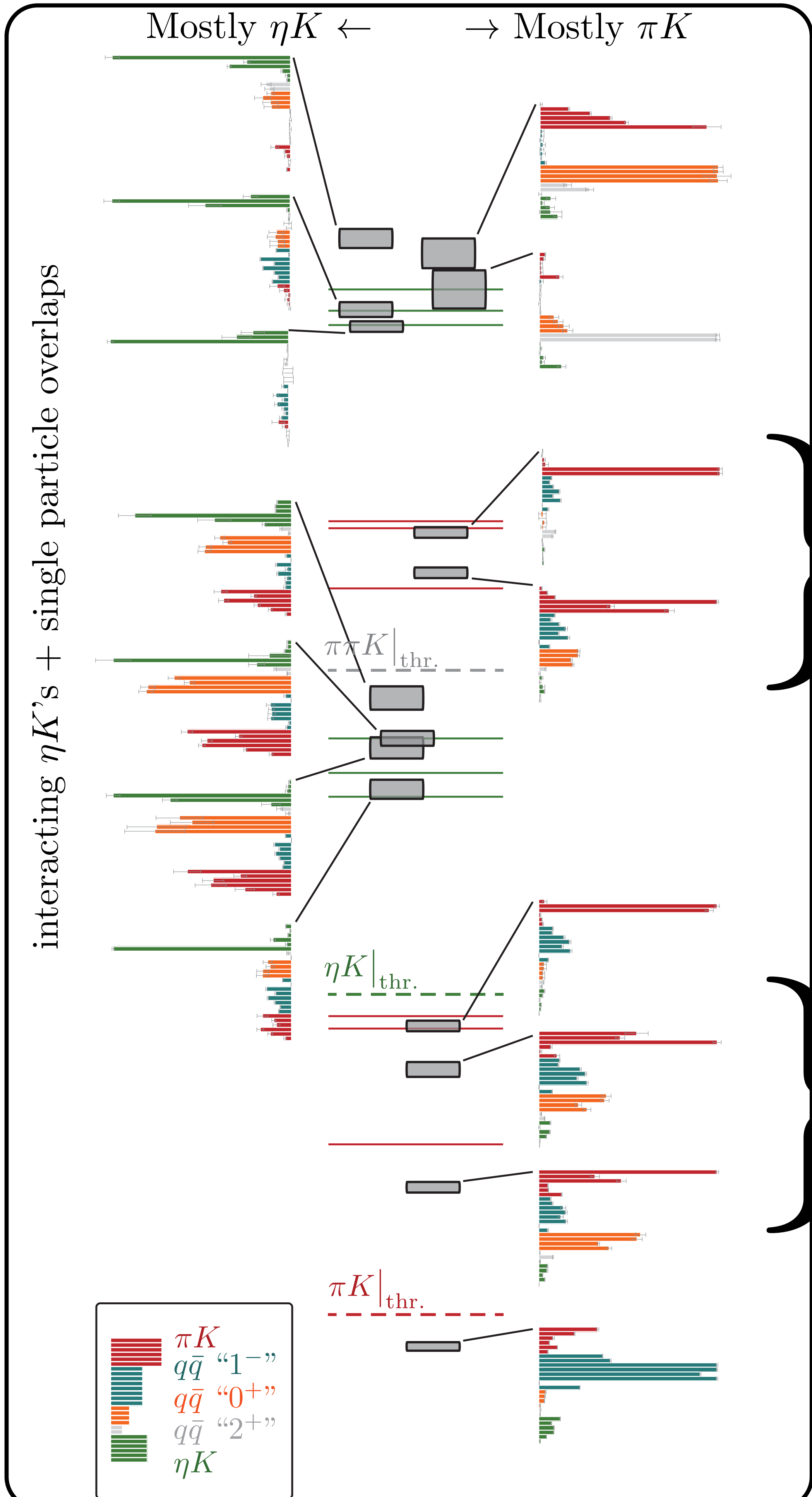
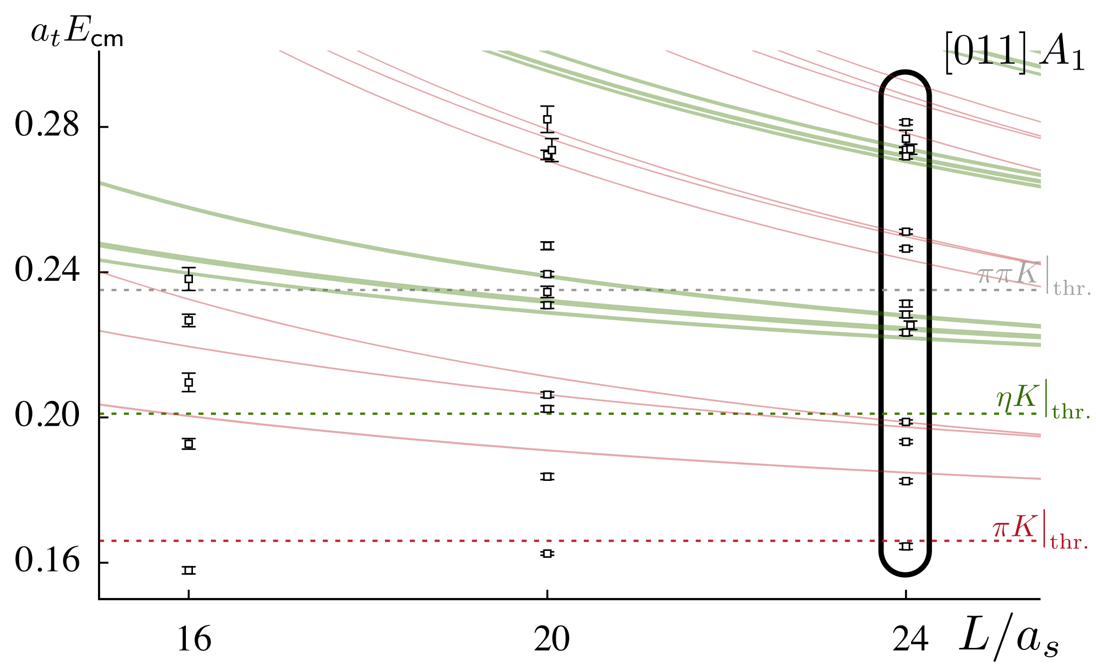


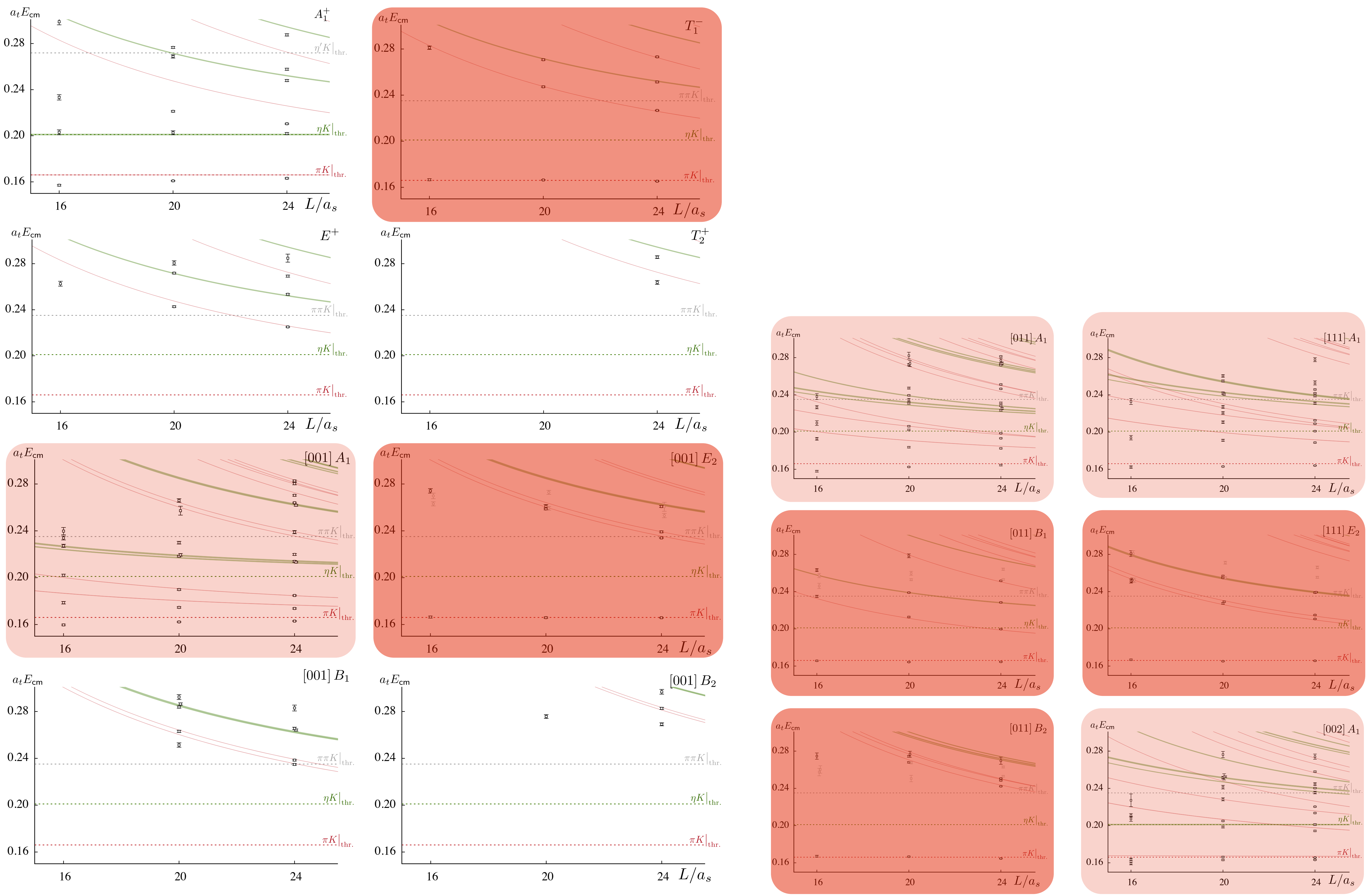
$[011]A_1$



- Overlaps ~ guide to resonant content $Z_i^{\mathfrak{n}} = \langle \mathfrak{n} | \mathcal{O}_i^\dagger | 0 \rangle$
- Shifted πK-like and ηK-like states
- $J^P=1^-$ state near to πK threshold, $J^P=2^+$ state, extra $J^P=0^+$.
- Considerable partial-wave mixing. [011] A_1 contains $J^P=0^+$, 1, 2, ...



P-wave contributions



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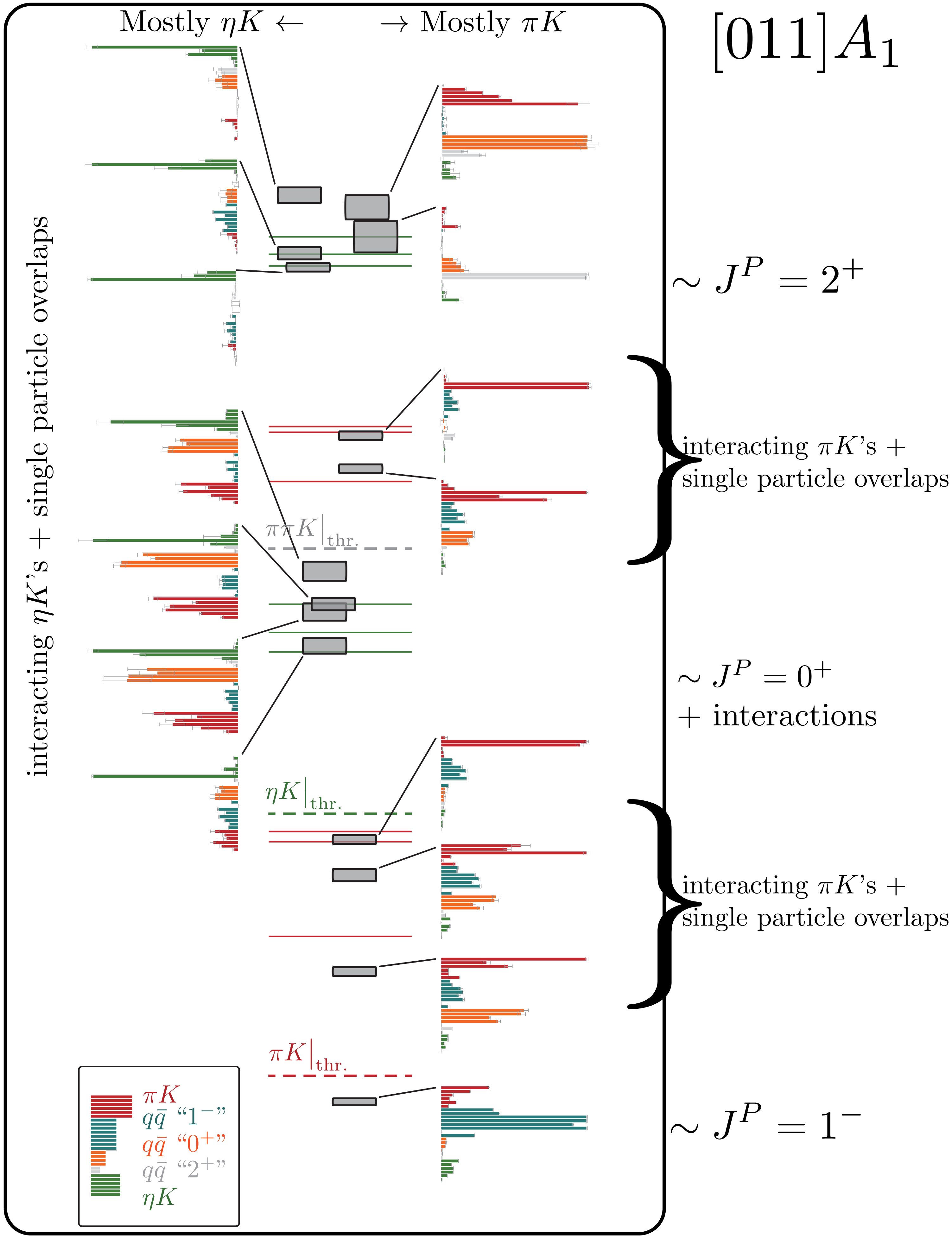

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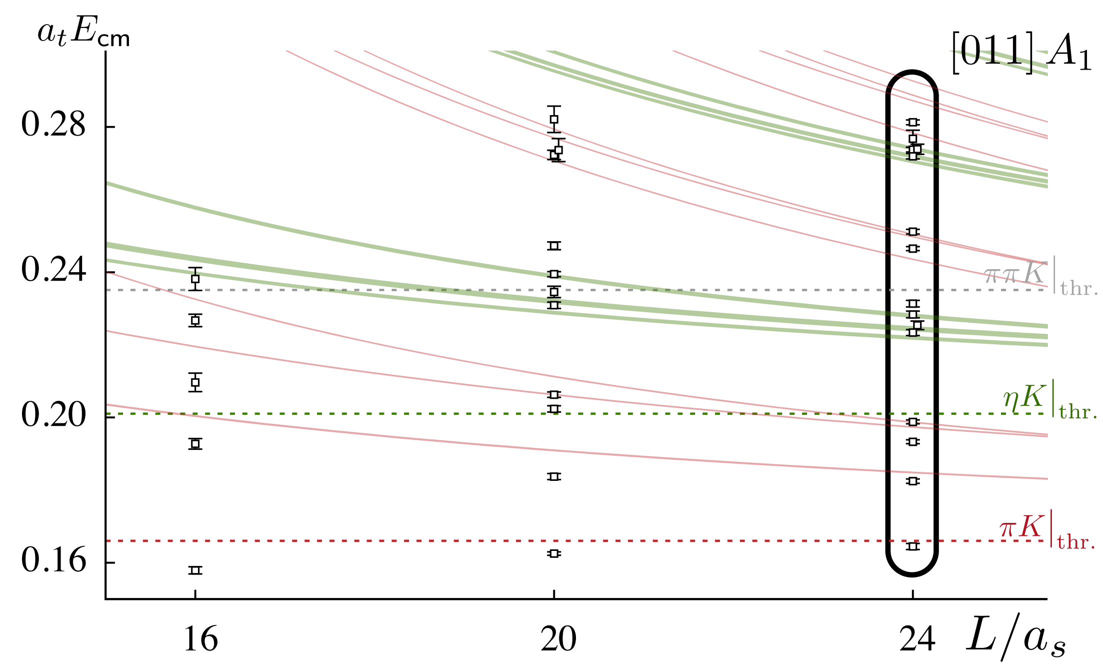


P-wave near-threshold state

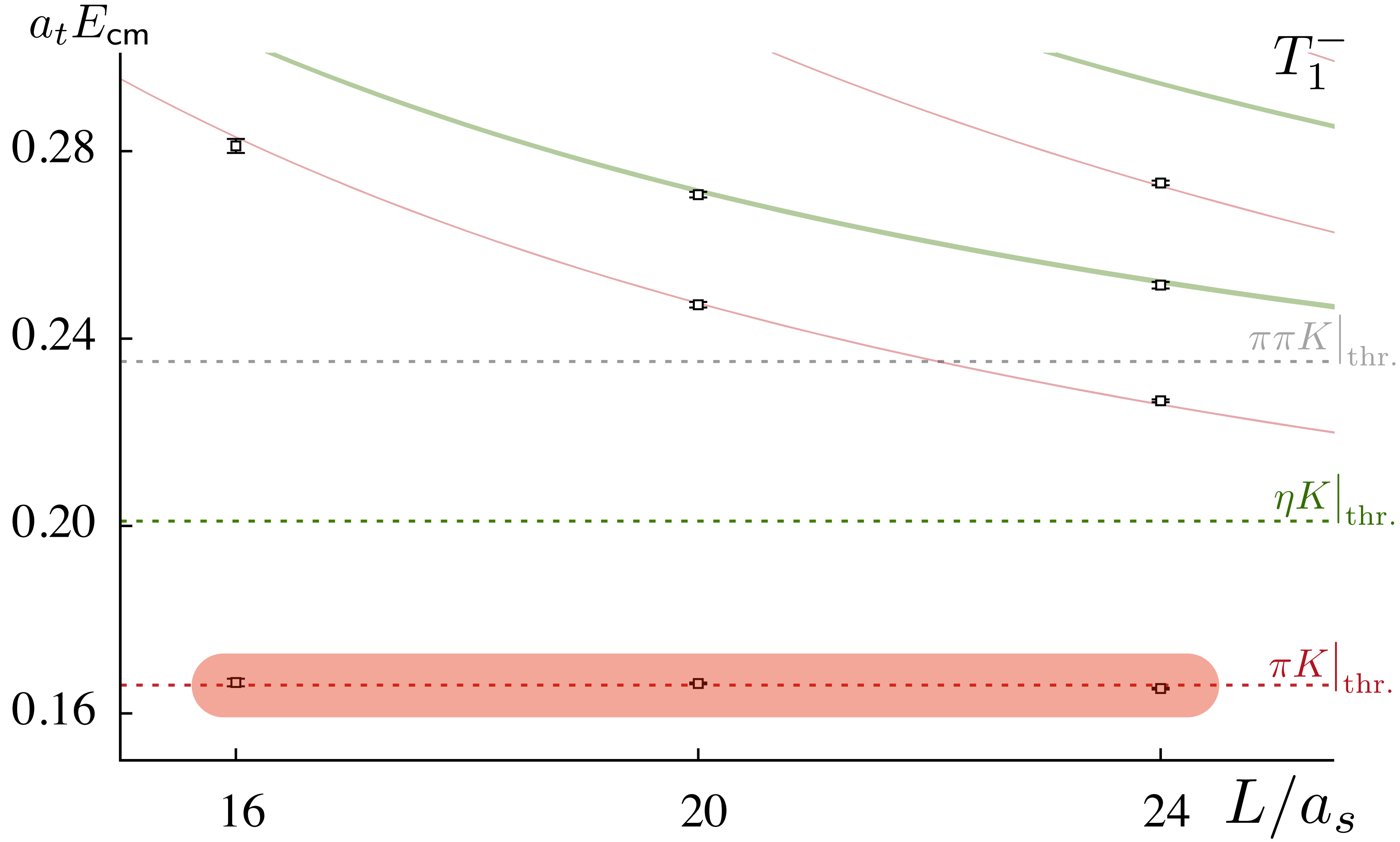
Elastic scattering just above πK threshold, no ηK to consider.

The irreps with P-wave overlap:

T_1^-, [001] A_1, [001] E_2, [011] A_1, [011] $B_{1,2}$, [111] A_1, [111] E_2, [002] A_1

all have an "extra" level near πK threshold.

Fitting the energy levels using an elastic Breit-Wigner in πK:



P-wave near-threshold state

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$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k_{cm}^3}{E_{cm}^2}$$

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$$k^3 \cot \delta_1 = (m_R^2 - s)\frac{6\pi s^{\frac{1}{2}}}{g_R^2}$$

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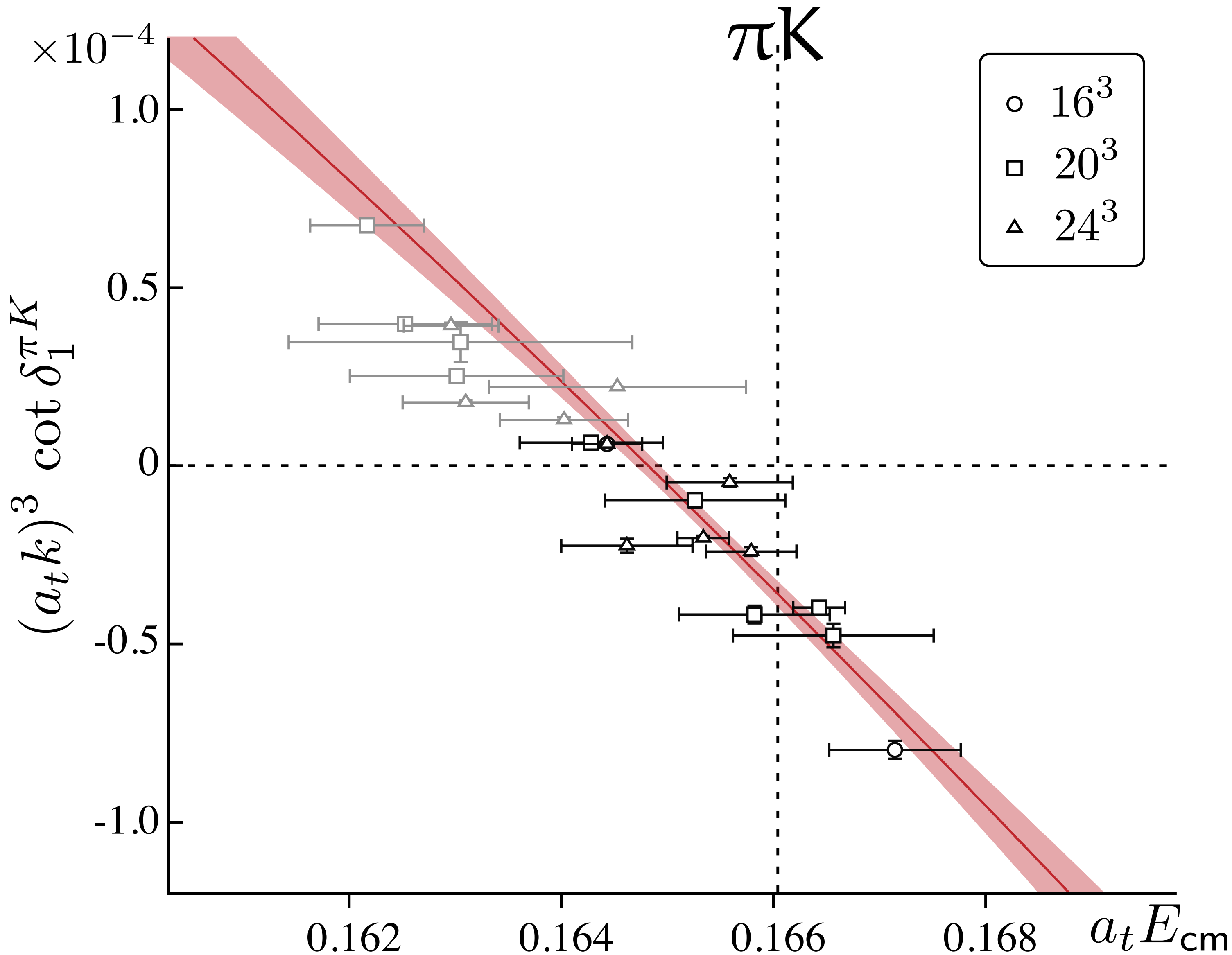
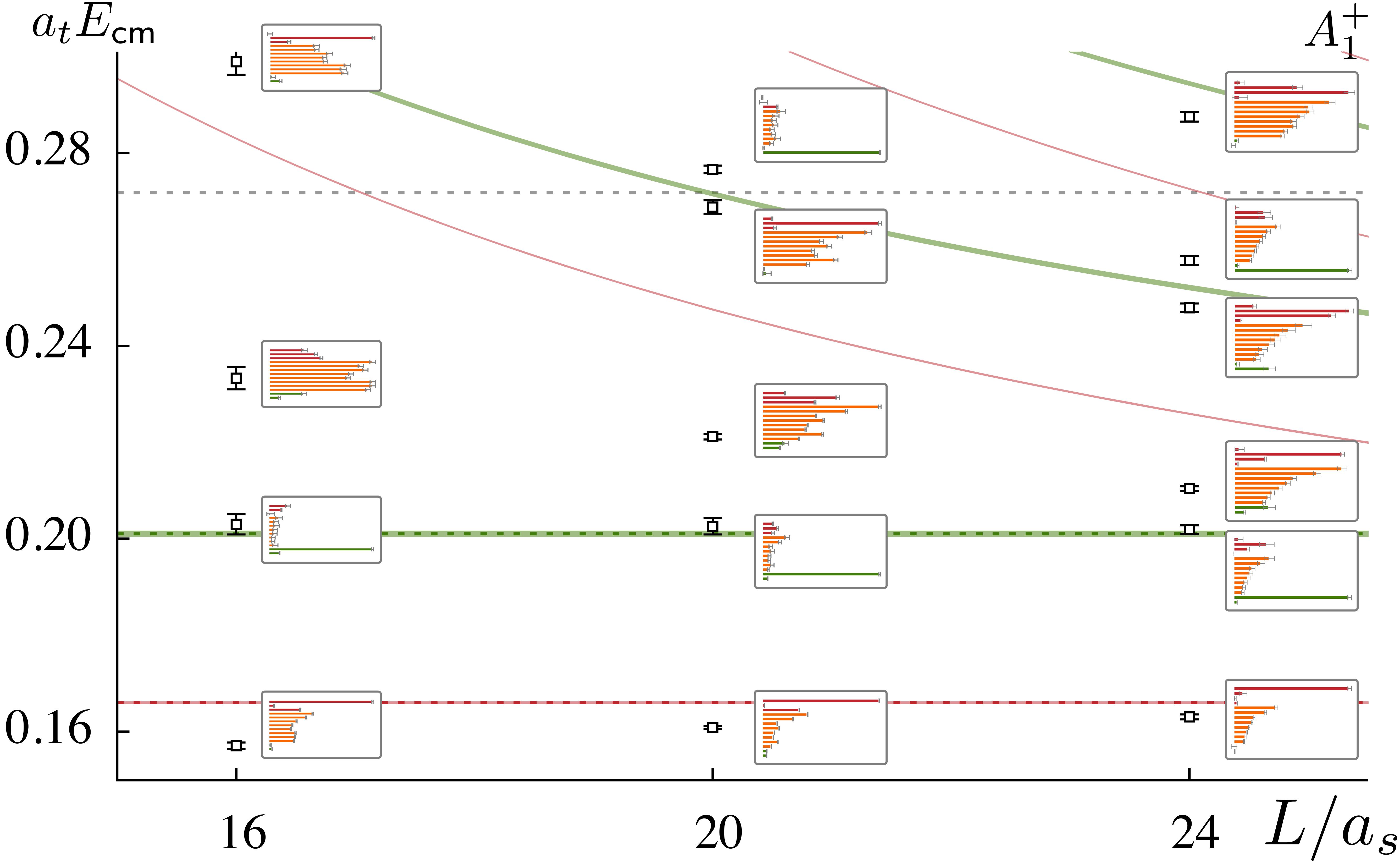


In t there is a pole on the real axis just below πK threshold:

Bound state in J^P=1⁻


$a_t E_{\mathrm{cm}}$
A_1^+
0.28
0.24
0.20
0.16
16
20
24
L/a_s

Coupled-channel calculation details

- Large basis of operators including:

"Single-meson" like operators, including bilinears and derivatives.

$$\mathcal{O}_i = \bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$$

"Meson-meson" like operators: Made from pairs of projected variationally-optimised single-meson operators at source and sink with definite momentum, e.g.:

$$\Omega_\pi(\vec{p}_1)\Omega_K(\vec{p}_2)$$

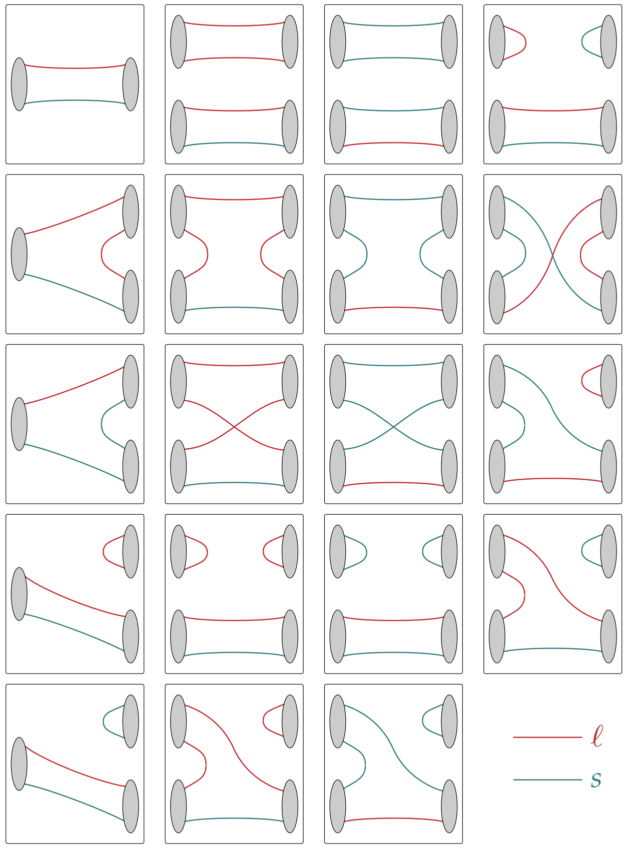
$$C(t)v^n(t) = \lambda_n(t)C(t_0)v^n(t)$$

$$\Omega_n^\dagger = \sum_i v_i^n \mathcal{O}_i^\dagger$$

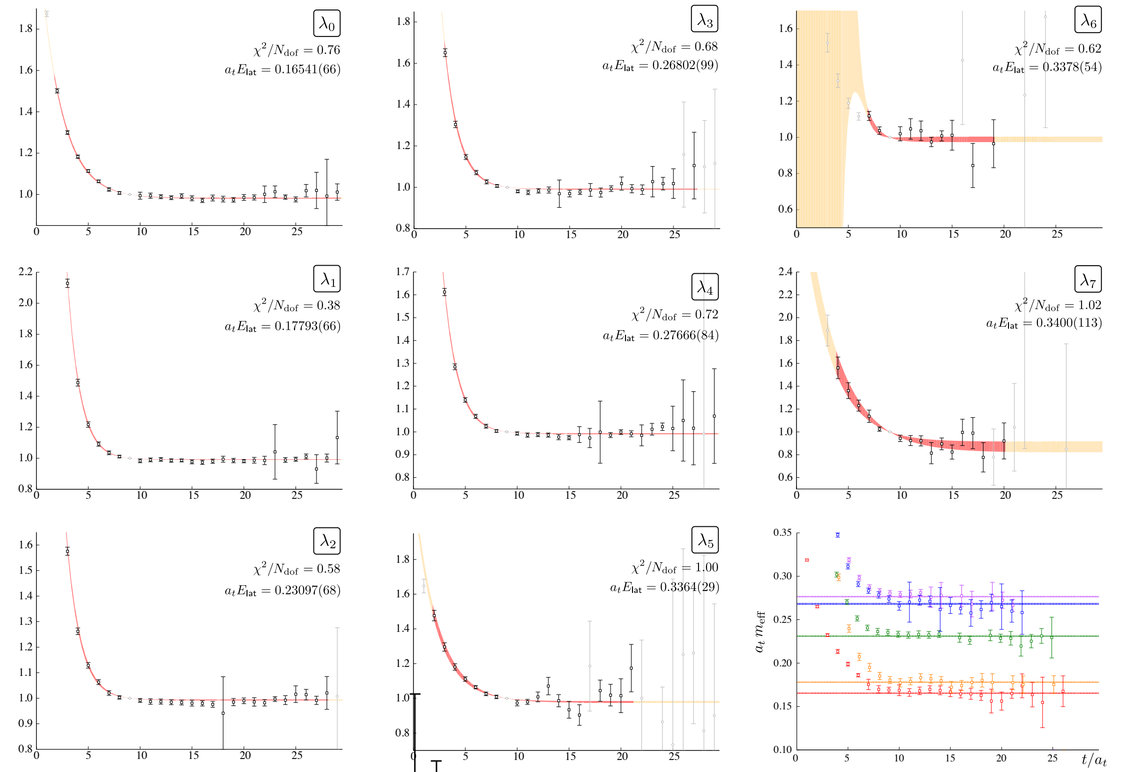
- Include all Wick contractions.

- All relevant irreps with boosts

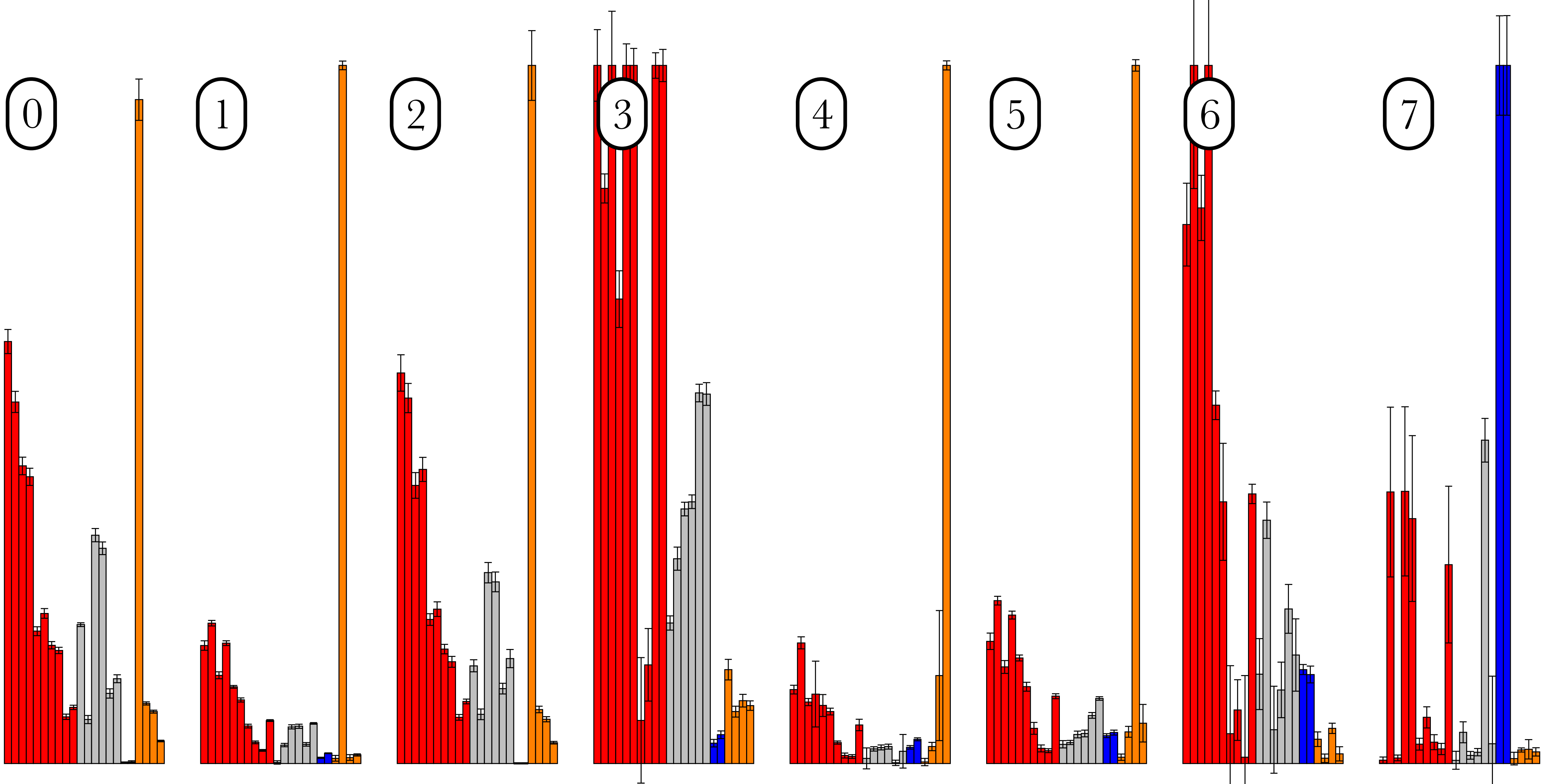
$$p^2 = |\vec{p}_1 + \vec{p}_2|^2 \leqslant 4\left(\frac{2\pi}{L}\right)^2$$



Relative operator overlaps



$$Z_i = \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$$



Operators with overall momentum

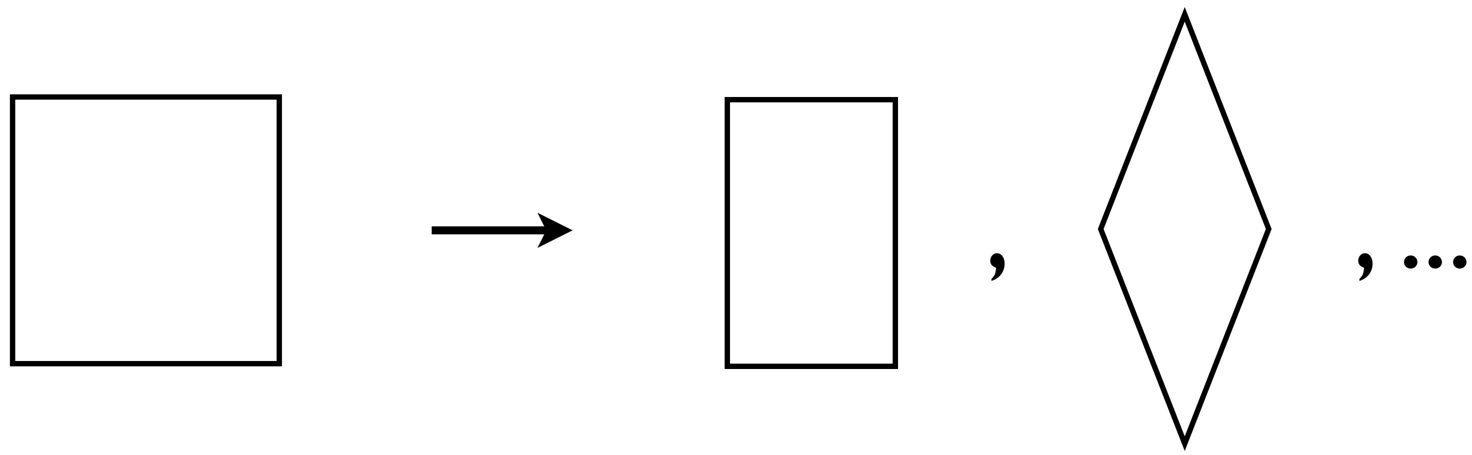
Because momentum is quantised, different energies can be accessed by considering operators with an overall momentum

Overall zero momentum: $\pi(0,0,0)\pi(0,0,0)$, $\pi(1,0,0)\pi(-1,0,0)$, ...
One unit: $\pi(1,0,0)\pi(0,0,0)$, $\pi(1,1,0)\pi(-1,0,0)$, ...

$$\vec{p} = \frac{2\pi}{\xi L}\vec{n}$$

$$E_{lat}^2 = E_{cm}^2 + \left(\frac{2\pi}{\xi L}|\vec{n}|\right)^2$$

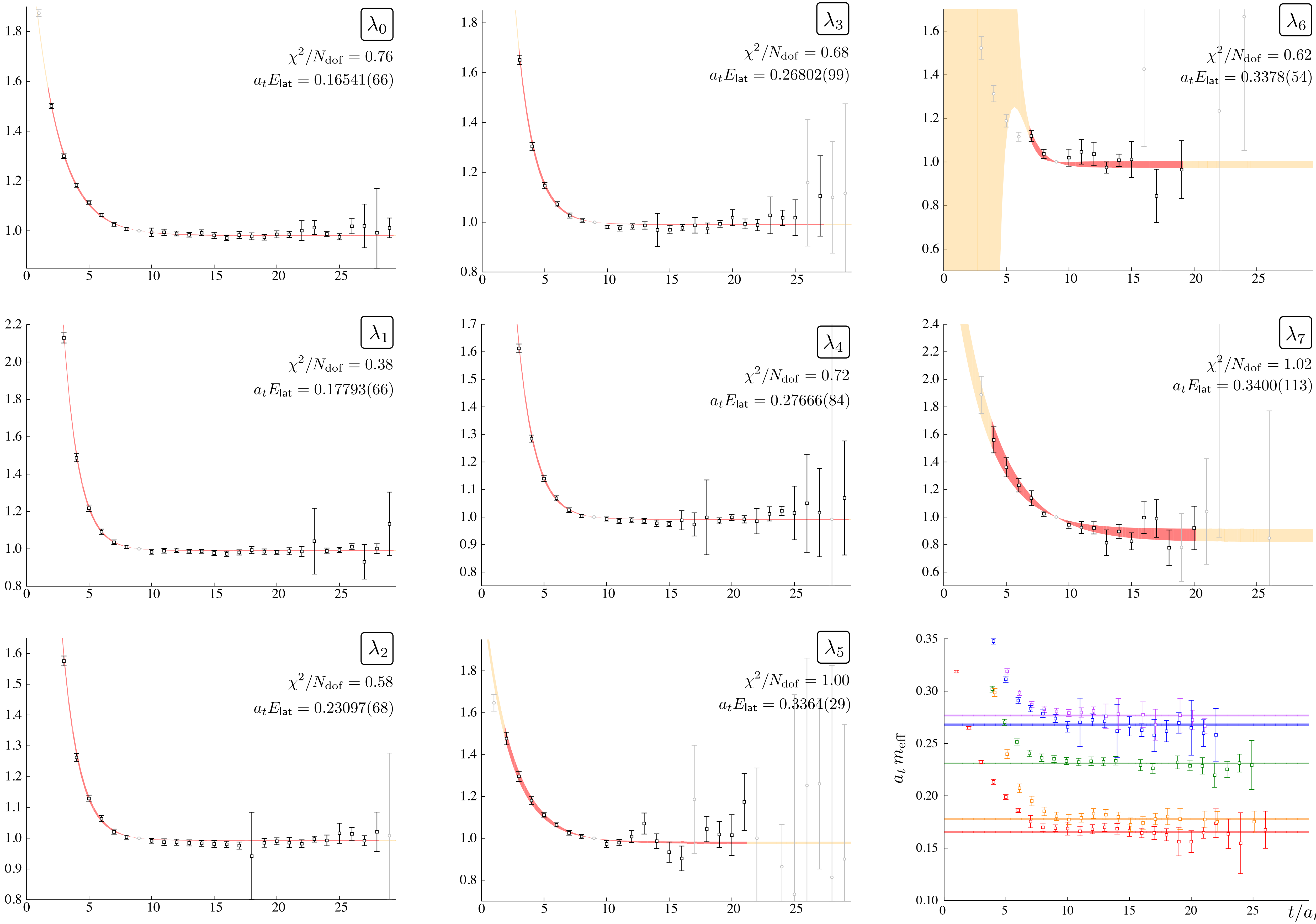
Useful to consider systems with $\vec{n} = (0,0,0),\ (1,0,0),\ (1,1,0),\ (1,1,1),\ (2,0,0)$



Less symmetry: More mixing of angular momentum!

Principal correlators

$$e^{E_{\mathfrak{n}} t} \lambda_{\mathfrak{n}}(t)$$



Extracting a spectrum

Getting the ground state is useful, but we want to extract the whole spectrum in a finite volume.

Fitting subleading exponentials doesn't get very far:
With very precise data, sometimes a second state can be found.

A solution: The variational method.

$$C_{ij}(t)v_j^n = \lambda_n(t)C_{ij}(t_0)v_j^n$$

If more than one operator overlaps onto the same state represented by some eigenvector v_i^n the generalised eigenvalue problem can be solved and then as many states as operators may be extracted.

$$\lambda_n(t) \sim e^{-E_n(t-t_0)}$$

... a large basis of operators are needed

Operators and the variational method

$$C_{ij}(t)v_j^n = \lambda_n(t)C_{ij}(t_0)v_j^n$$
$$\lambda_n(t) \sim e^{-E_n(t-t_0)}$$

Use a large basis of operators

$$\mathcal{O}_i = \bar{\psi}\Gamma\psi$$
$$\mathcal{O}_i = \bar{\psi}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$$

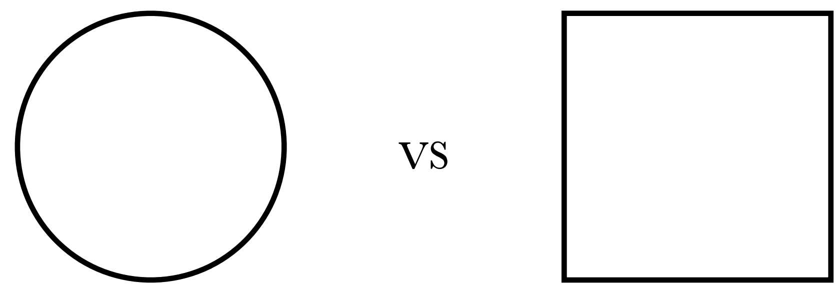
$$\Gamma_i = \{1, \gamma_0, \gamma_5, \gamma_0\gamma_5, \gamma_i, \gamma_0\gamma_i, \gamma_5\gamma_i, [\gamma_i, \gamma_j]\}$$

Use the variational method with a large correlation matrix

Symmetry on the lattice

The lattice has a cubic symmetry.
It does not have the O(3) symmetry of continuous space.

Eg: 2D QM



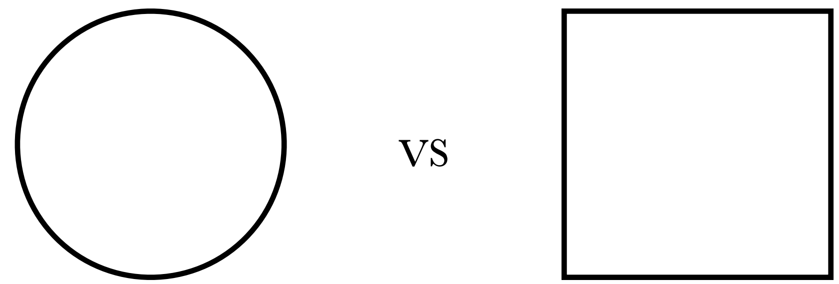
Continuous rotational spatial symmetry

$$e^{i\phi} \to e^{i\phi + i\alpha}$$

$$e^{i\phi} \to e^{i\phi + in\pi/2}$$

Only symmetric at discrete angles

Symmetry on the lattice



Continuous rotational spatial symmetry

$$e^{i\phi} \rightarrow e^{i\phi + i\alpha}$$

$$e^{i\phi} \rightarrow e^{i\phi + in\pi/2}$$

Only symmetric at discrete angles

Cubic symmetry groups mix the continuum angular momentum:

Irrep	J^P
A_1^+	$0^+, 4^+, \ldots$
T_1^-	$1^-, 3^-, \ldots$

Backup slides: Finite volume formalism

Coupled-channel extensions of Lüscher's method

$$S_{ij} = \delta_{ij} + 2i\left(\rho_i \rho_j\right)^{\frac{1}{2}} t_{ij}$$

momentum boost vector

Angular momentum

lattice irrep

scattering t-matrix

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i t_{ij}^{(\ell)}\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{d},\Lambda}_{ij,\ell n,\ell' n'}\right)\right] = 0$$

Channels:
eg πK, ηK

phase space

$$\rho_i = \frac{2k_i^{(\mathrm{cm})}}{E_{\mathrm{cm}}}$$

finite volume object - contains generalised Lüscher Zeta functions

$$\mathcal{M} \sim \frac{1}{\gamma}\sum_{\mathrm{spins}}(\mathrm{CGs})\sum_{\vec{r}}\frac{r^{\ell}Y_{\ell m}(\hat{\vec{r}})}{r^2 - q^2}$$

Symmetry of the volume mixes partial waves - *M* mixes partial waves.
t-matrix is diagonal in partial waves, but can couple scattering channels: $\pi K \rightarrow \eta K$

Coupled-channel scattering

$$\mathcal{M} \sim \frac{1}{\gamma} \sum_{\text{spins}} (\text{CGs}) \sum_{\vec{r}} \frac{r^\ell Y_{\ell m}(\hat{\vec{r}})}{r^2 - q^2}$$

scattering t-matrix, couples channels, diagonal in *l*.

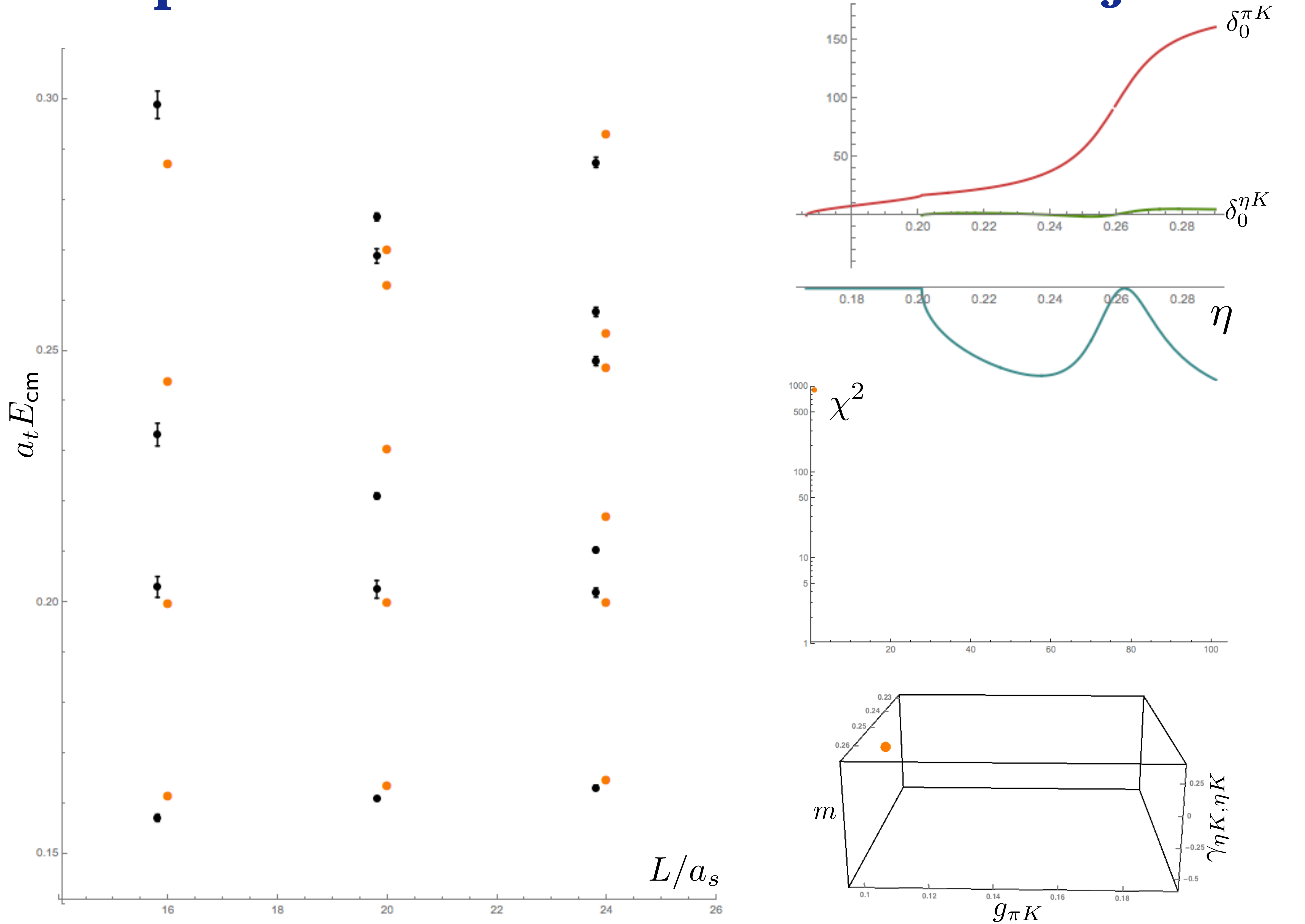
$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i t_{ij}^{(\ell)}\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}_{ij,\ell n,\ell' n'}^{\vec{d},\Lambda}\right)\right] = 0$$

finite volume object - contains
generalised Lüscher Zeta functions
mixes partial waves

- Several unknowns at each energy level: Multiple channels, multiple partial waves.
- Problem is unconstrained for a single energy level.
- Solution: Parameterise t_{ij} using a few free parameters, use many energy levels to constrain them.

Example Minimisation

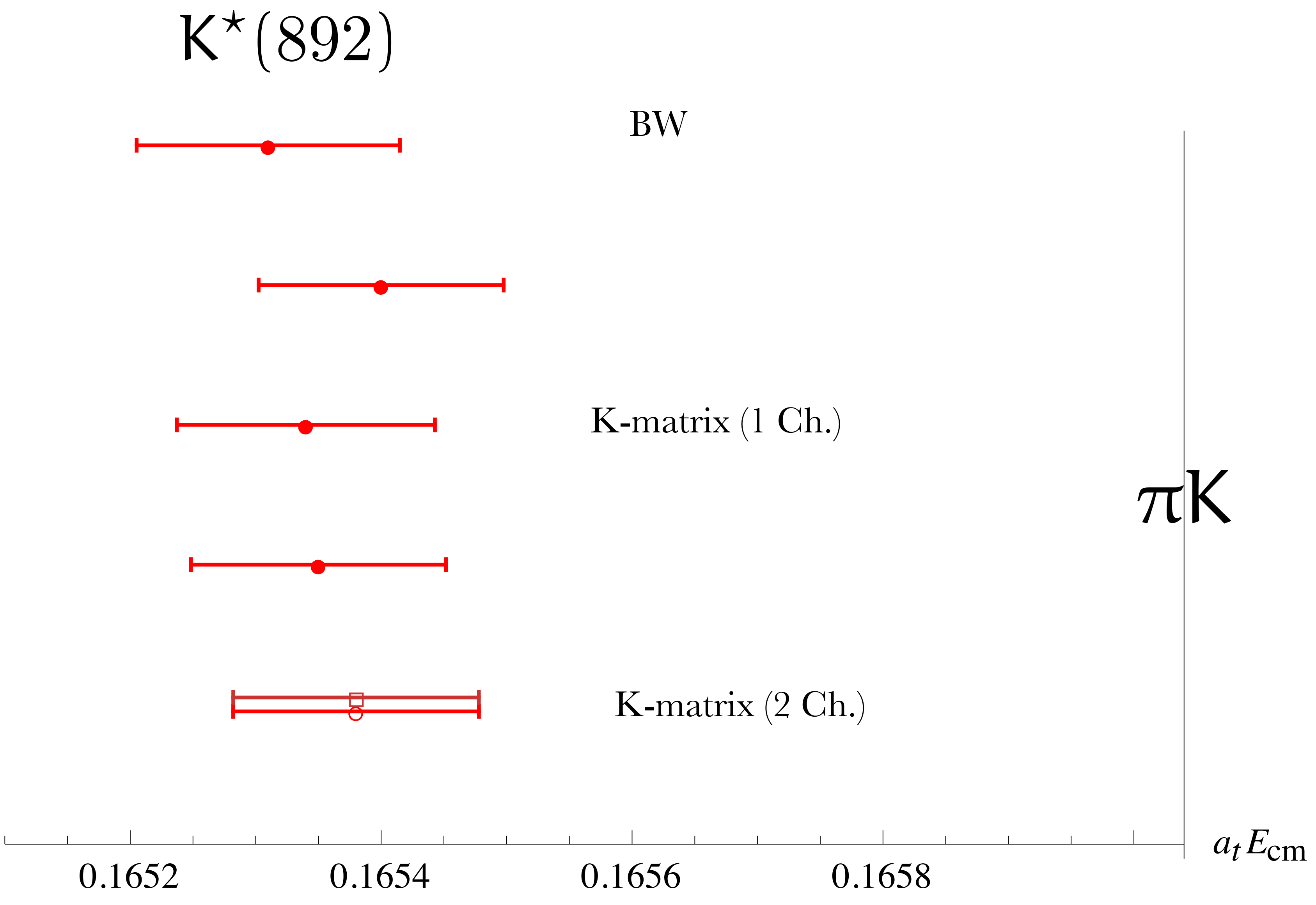
credit: Jo Dudek



Backup slides: Amplitudes

P-wave pole

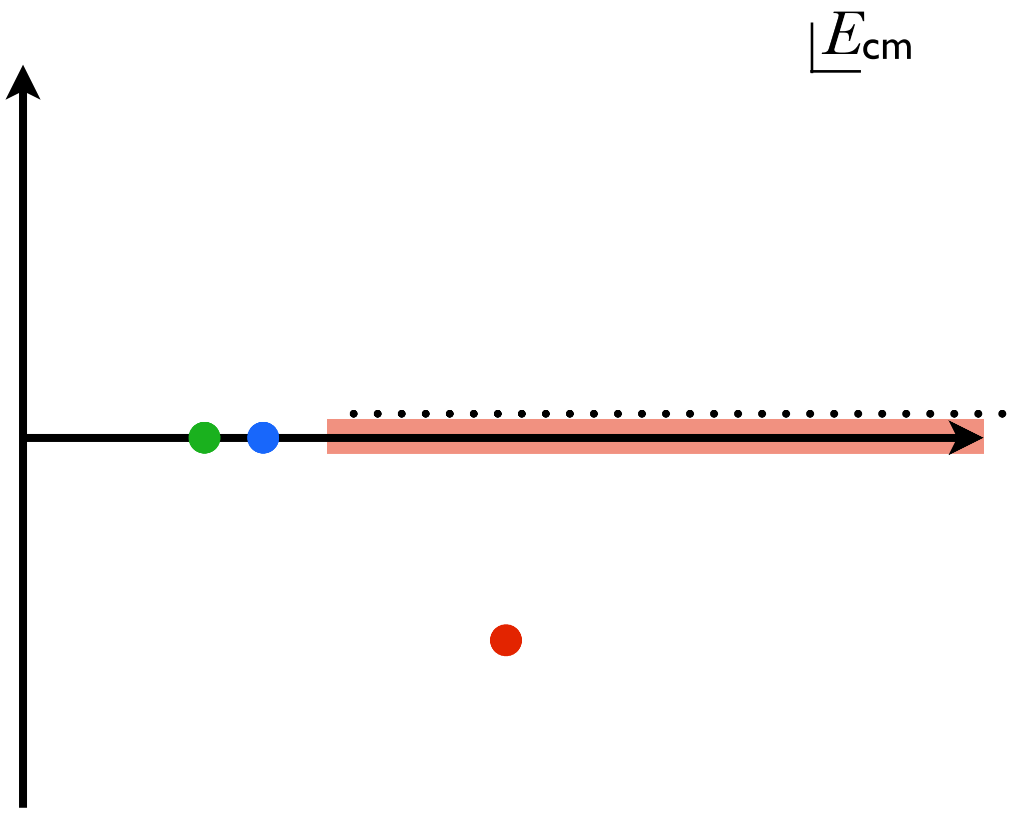
- Breit-Wigner pole from continution below threshold.
- Also used a K-matrix below threshold, found almost exactly the same result.
- Poles on physical sheet $\mathrm{Im}(k_{\mathrm{cm}}) > 0$.

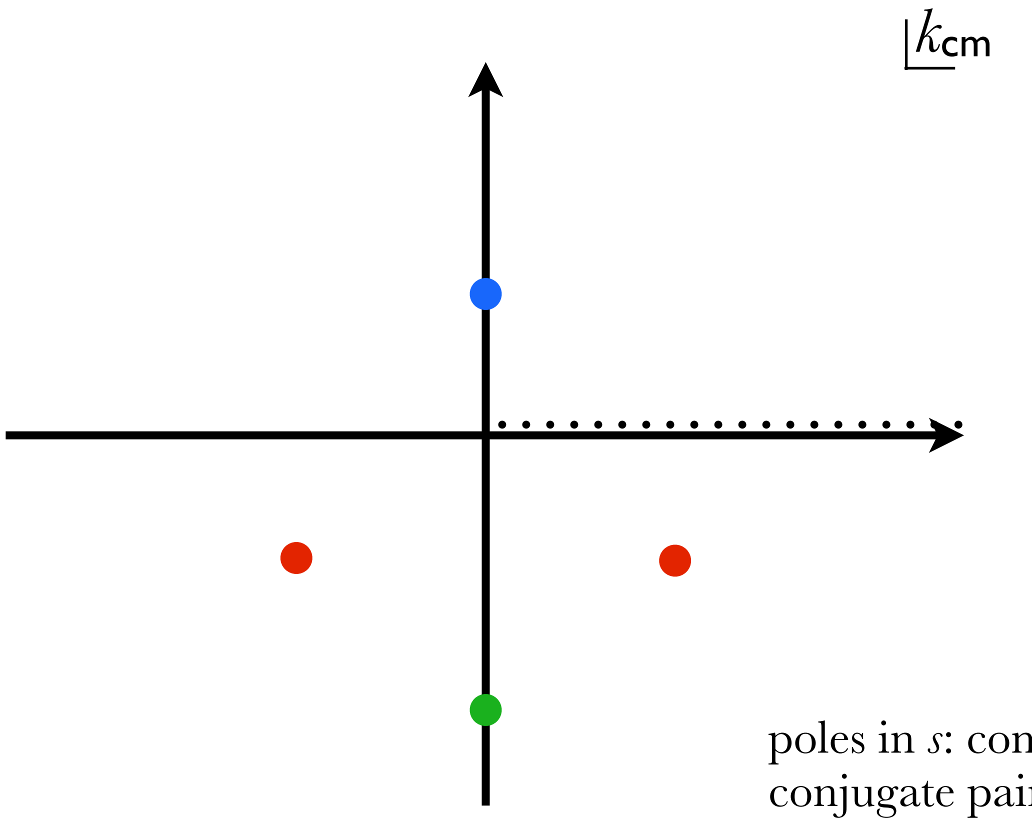


S-wave poles

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$k_{cm} = \pm\frac{1}{2}\left(E_{cm}^2 - 4m^2\right)^{\frac{1}{2}}$$





poles in s: complex conjugate pairs in k

Bound state

Resonance

Virtual Bound state

S-wave poles

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

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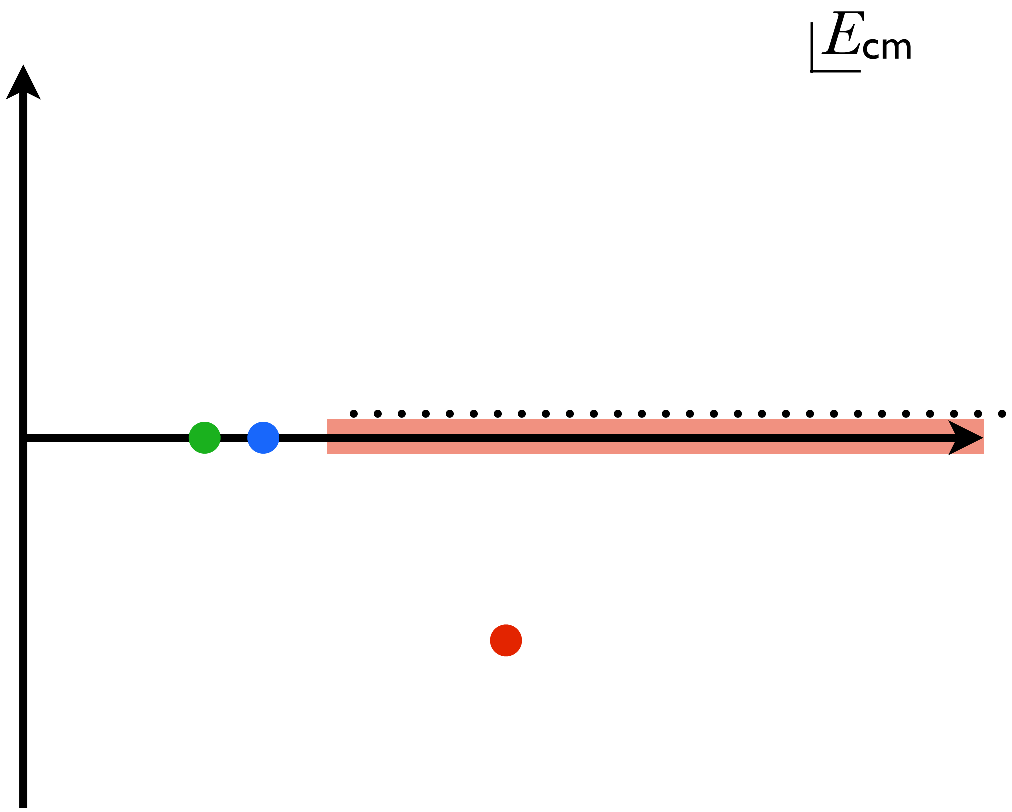

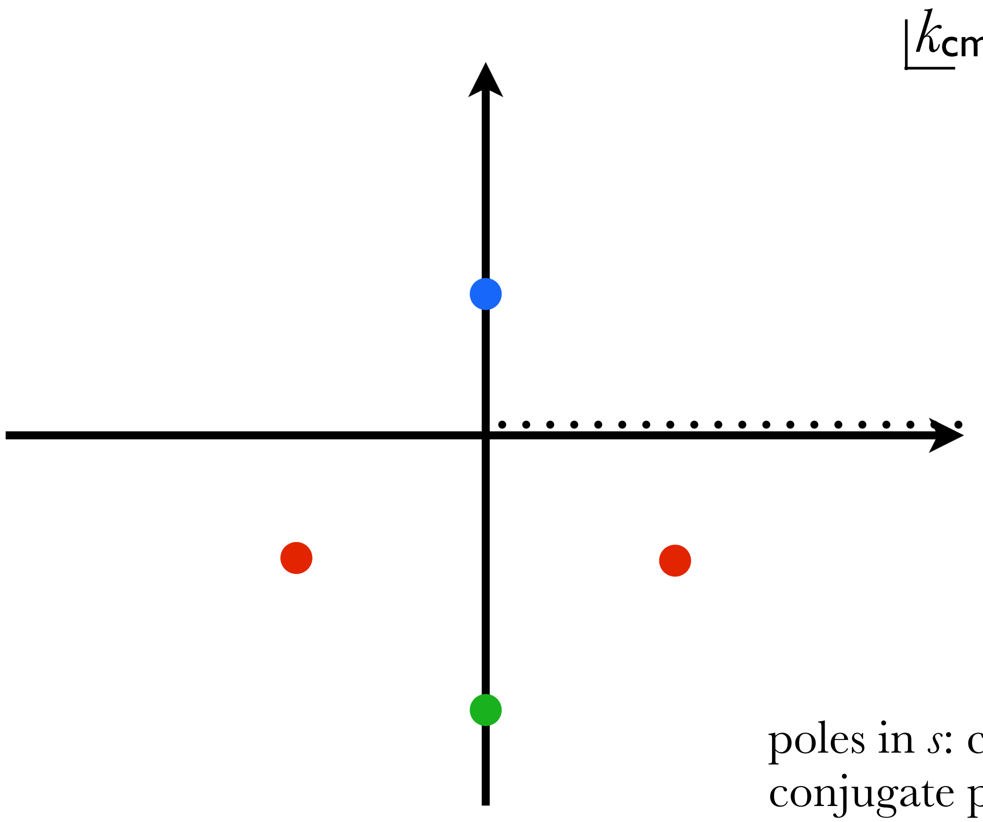

poles in s: complex conjugate pairs in k

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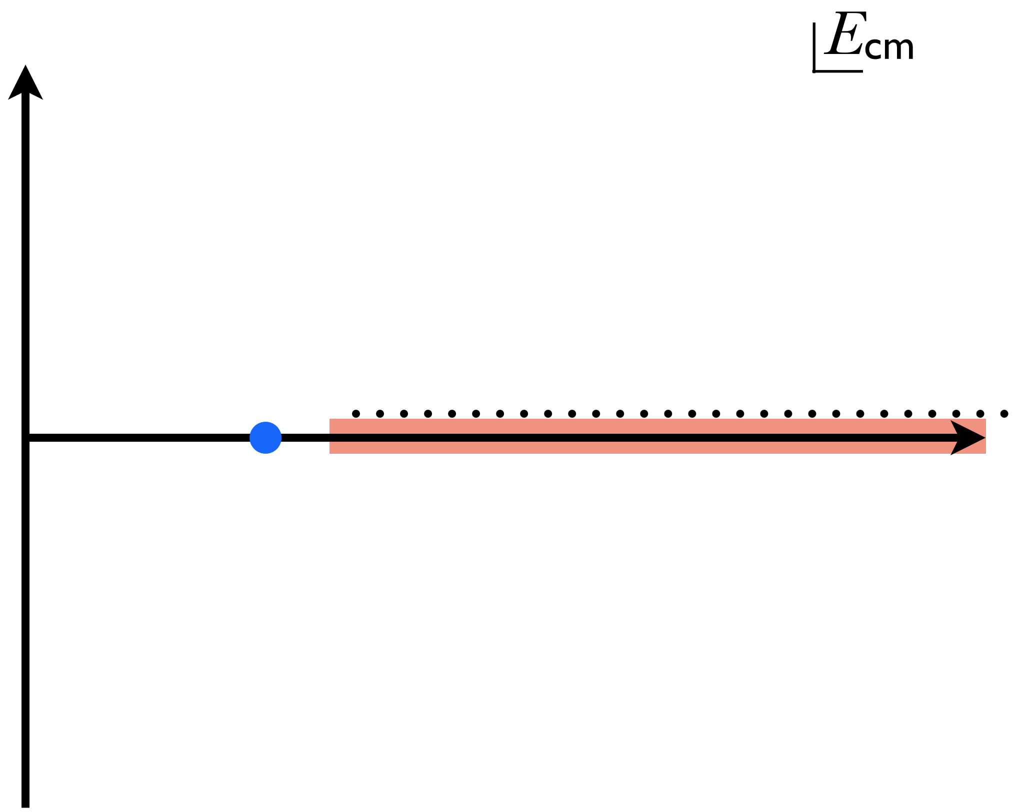
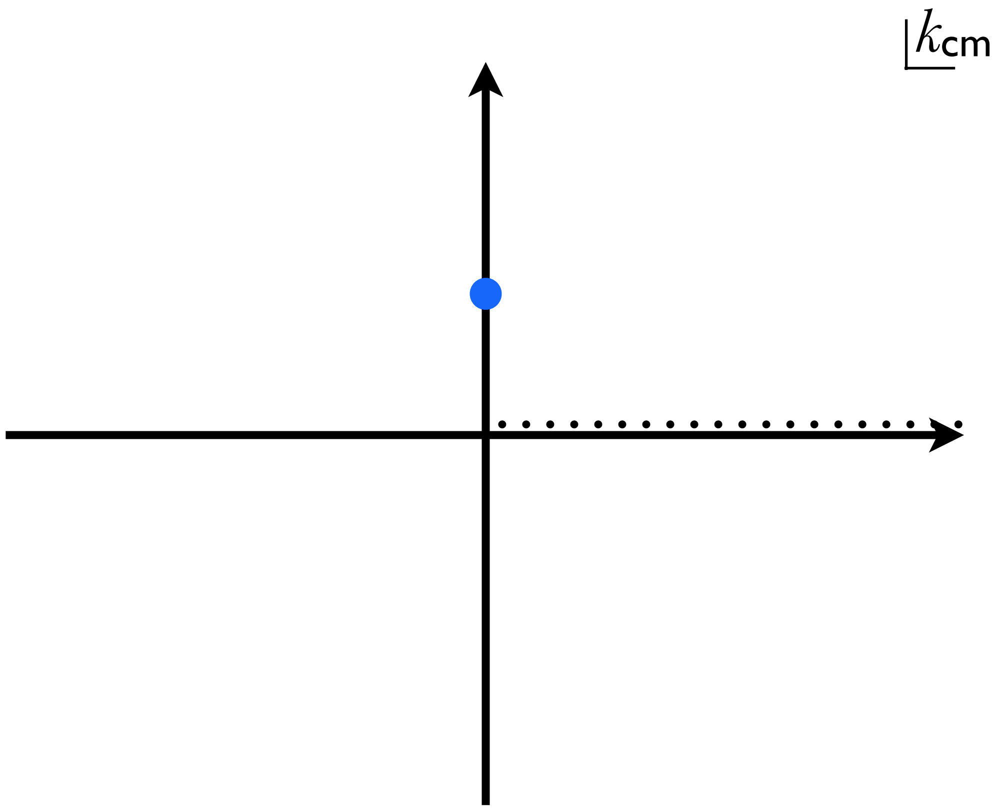


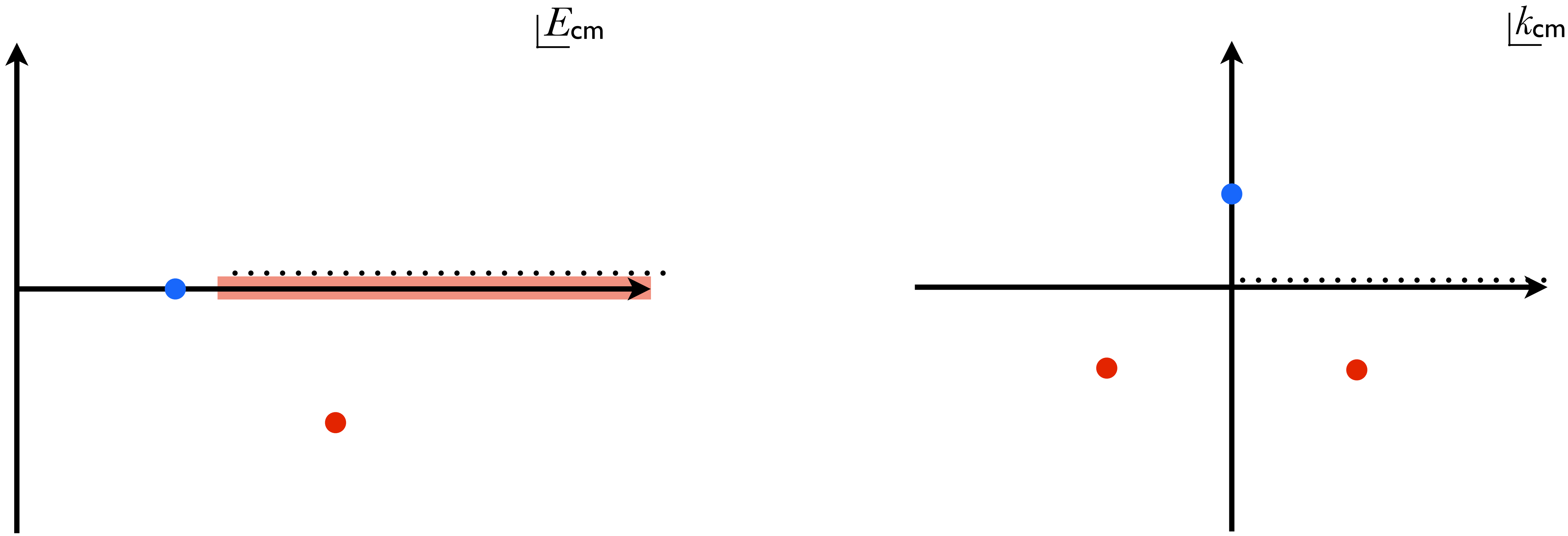


Bound state

Poles

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$k_{cm} = \pm\frac{1}{2}\left(E_{cm}^2 - 4m^2\right)^{\frac{1}{2}}$$



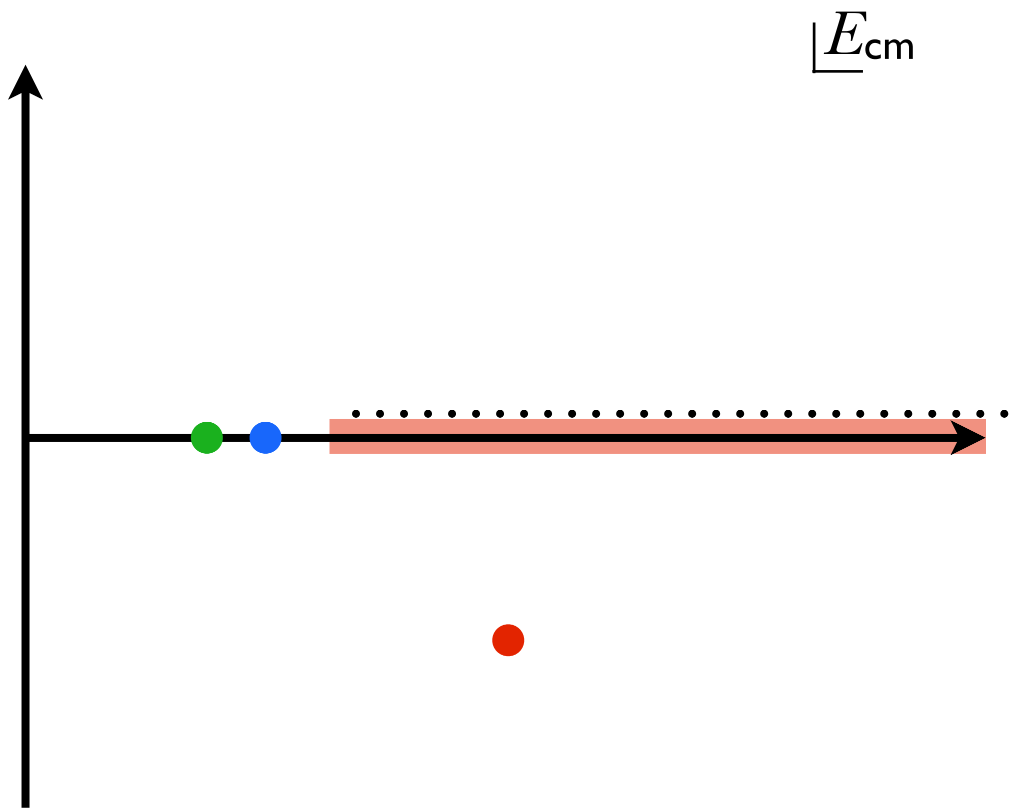
Bound state

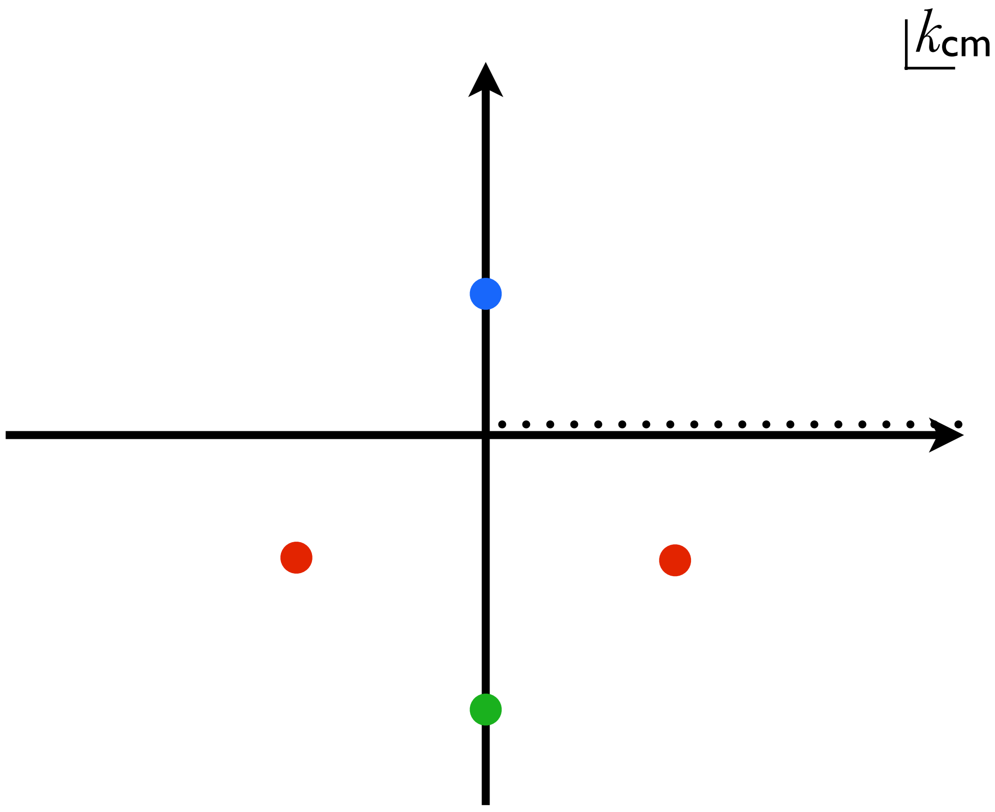
Resonance

Poles

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$k_{cm} = \pm\frac{1}{2}\left(E_{cm}^2 - 4m^2\right)^{\frac{1}{2}}$$





Bound state

Resonance

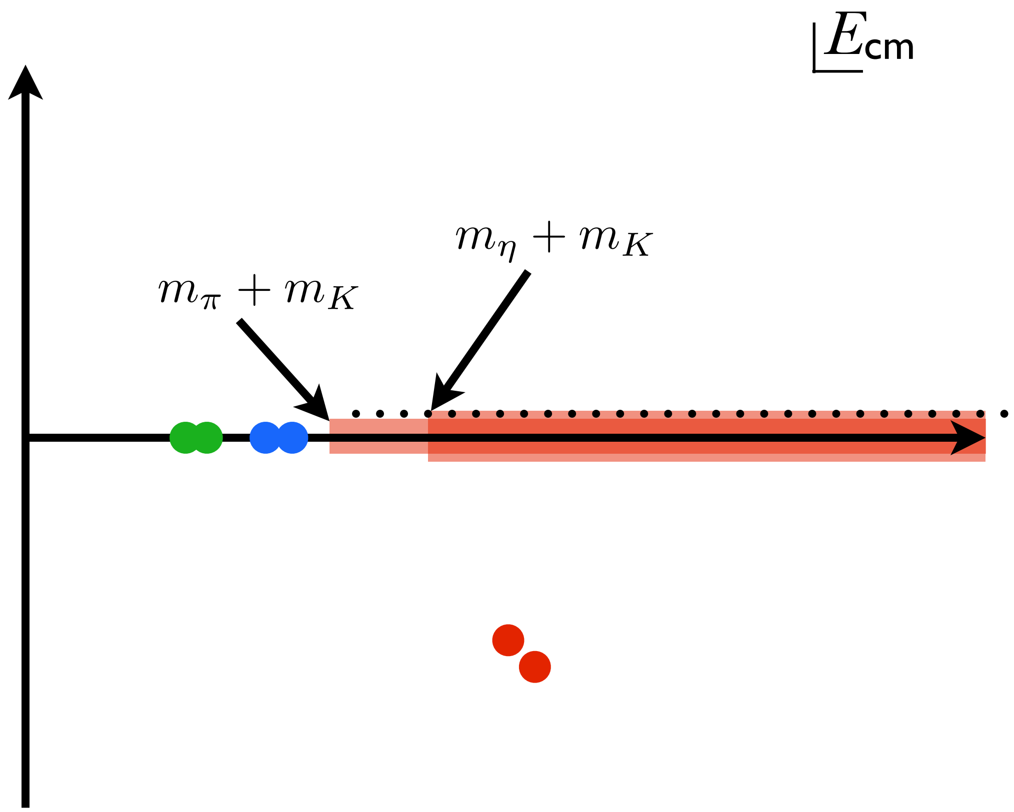
Virtual Bound state

S-wave poles

Actual situation: Unequal masses and an extra pair of sheets due to ηK scattering

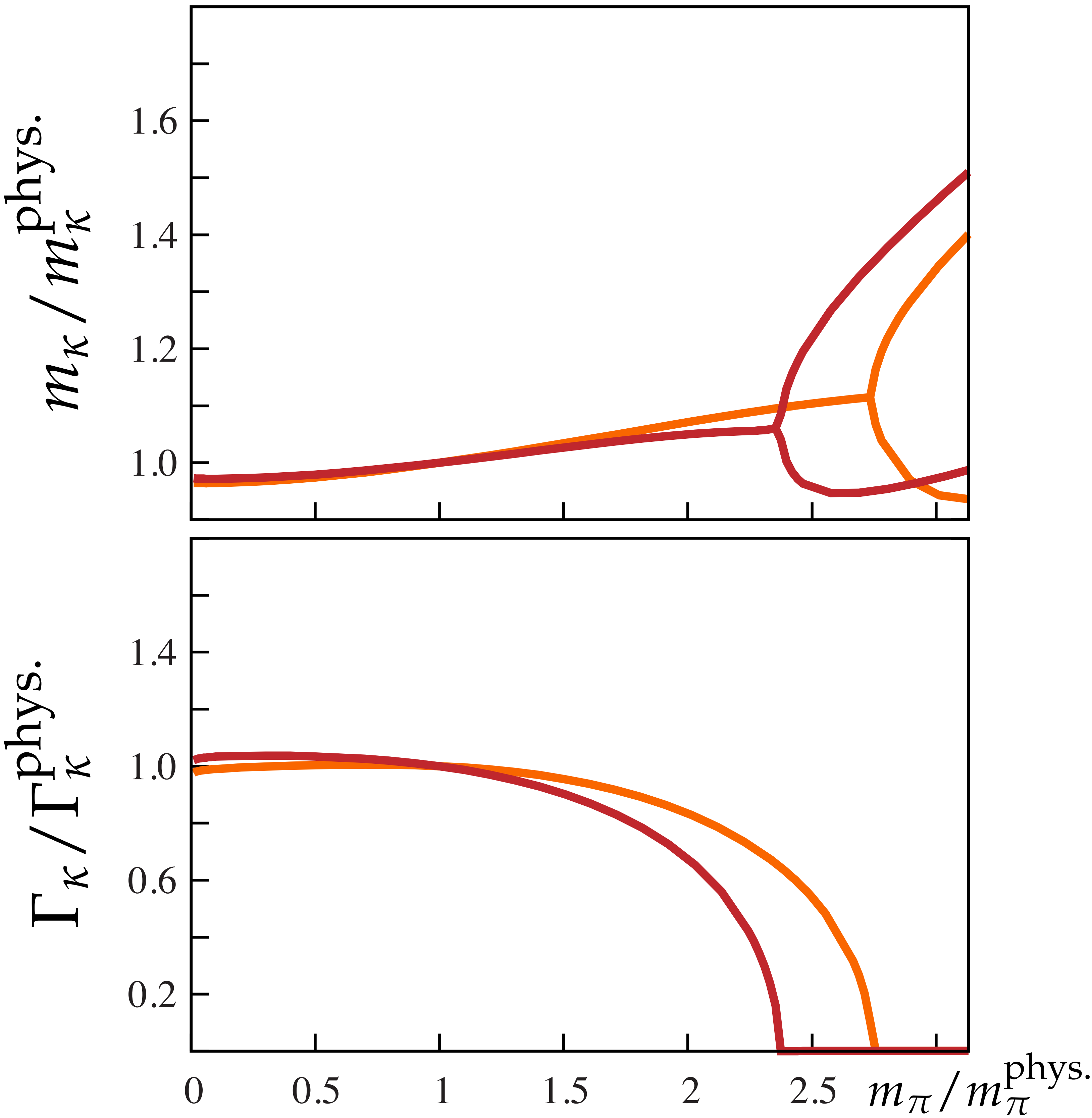
→ Poles and residues on multiple sheets.

$$k_{\mathrm{cm}} = \pm\frac{1}{2}\left(E_{\mathrm{cm}}^2 - 2\left(m_1^2 + m_2^2\right) + \frac{(m_1^2 - m_2^2)^2}{E_{\mathrm{cm}}^2}\right)^{\frac{1}{2}}$$



Virtual bound state κ

Pelaez and Nebreda using Unitiarised SU(3) Chiral Perturbation theory



More on virtual bound state

In an effective range parameterisation, strong interactions near threshold lead to a large *a*.

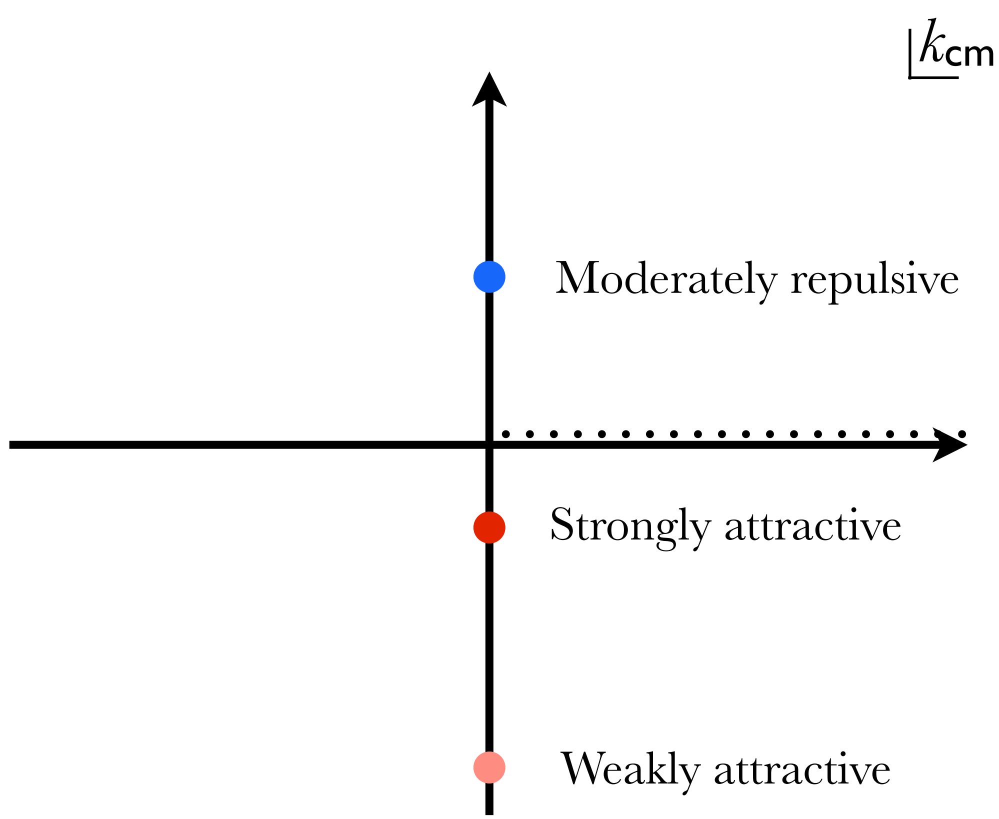
In *S*-wave large *a* automatically leads to a pole near-threshold.

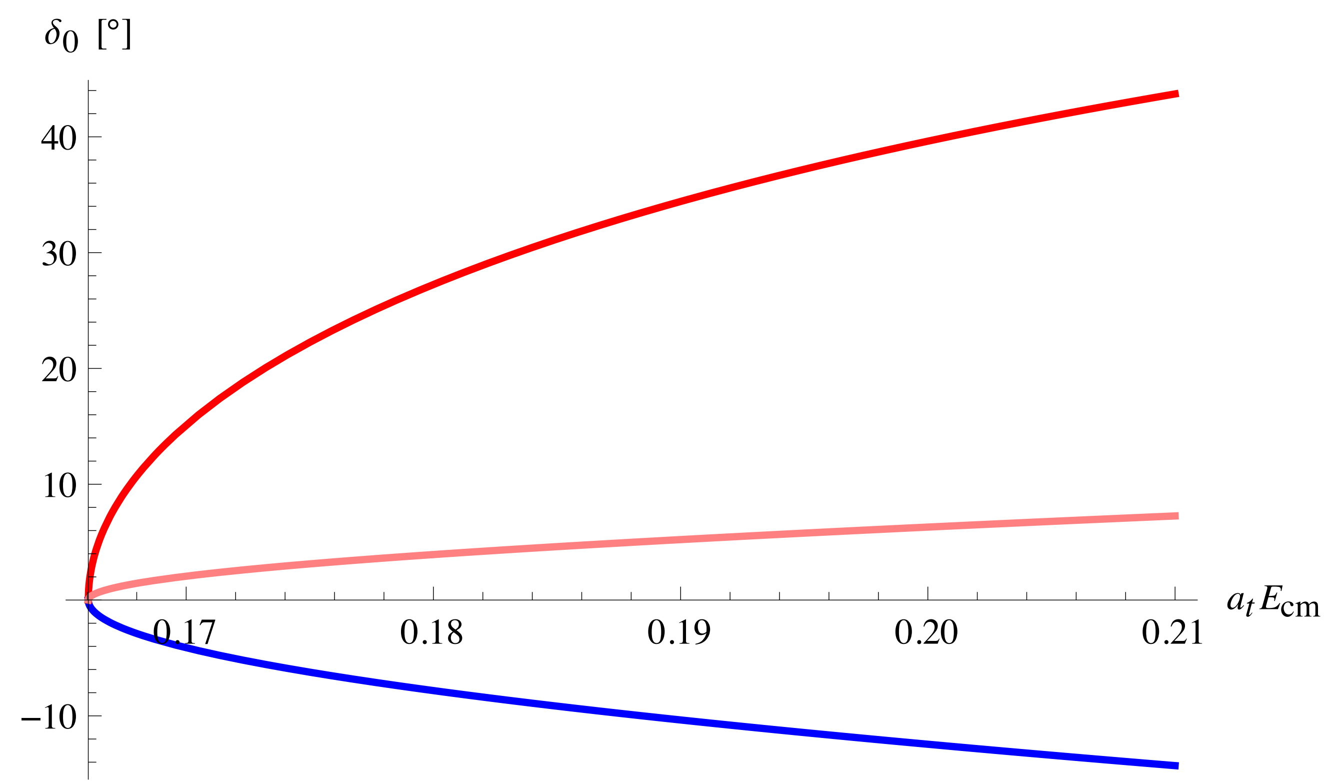
$$k_{cm} \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r k_{cm}^2$$

$$t = \frac{1}{2} \frac{E_{cm}}{k_{cm} - \frac{i}{a}} \qquad k_{cm} = \mp \frac{i}{a}$$

Arguments appear to hold for constant terms in K-matrix (slightly complicated by Chew-Mandelstam).

Appears to break down for *P*-wave and higher.





More K-matrix details

- K-matrix contains everything not constrained by unitarity

$$t_{ij}^{-1}(s) = K_{ij}^{-1}(s) - i\delta_{ij}\rho_i(s)$$

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi K}^2 & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g_{\eta K}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

- Chew-Mandelstam phase space -- include also s-channel cut along with imaginary part.

$$t_{ij}^{-1}(s) = K_{ij}^{-1}(s) + \delta_{ij}\; I_i(s)$$

$$I_i(s) = I_i(s_{\mathrm{thr}_i}) - \frac{s - s_{\mathrm{thr}_i}}{\pi}\int_{s_{\mathrm{thr}_i}}^{\infty} ds'\, \frac{\rho_i(s')}{(s'-s)(s'-s_{\mathrm{thr}_i})}$$

(Subtract at pole so that Re $I(s = m^2) = 0$)

- Threshold factors for $l>0$

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + \delta_{ij}\; I_i(s)$$

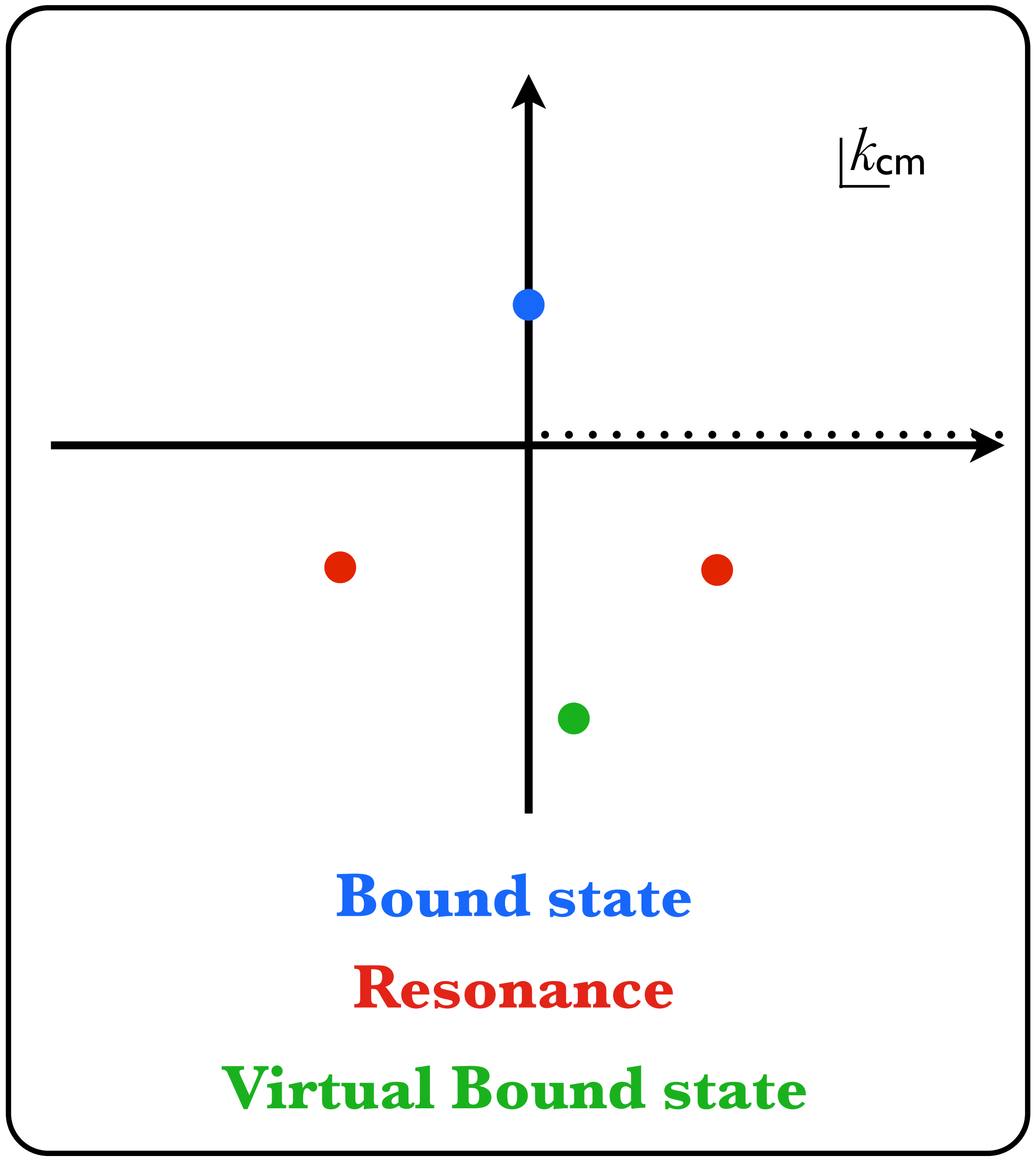
As used in Guo, Mitchell and Szczepaniak Phys.Rev. D82 (2010) 094002

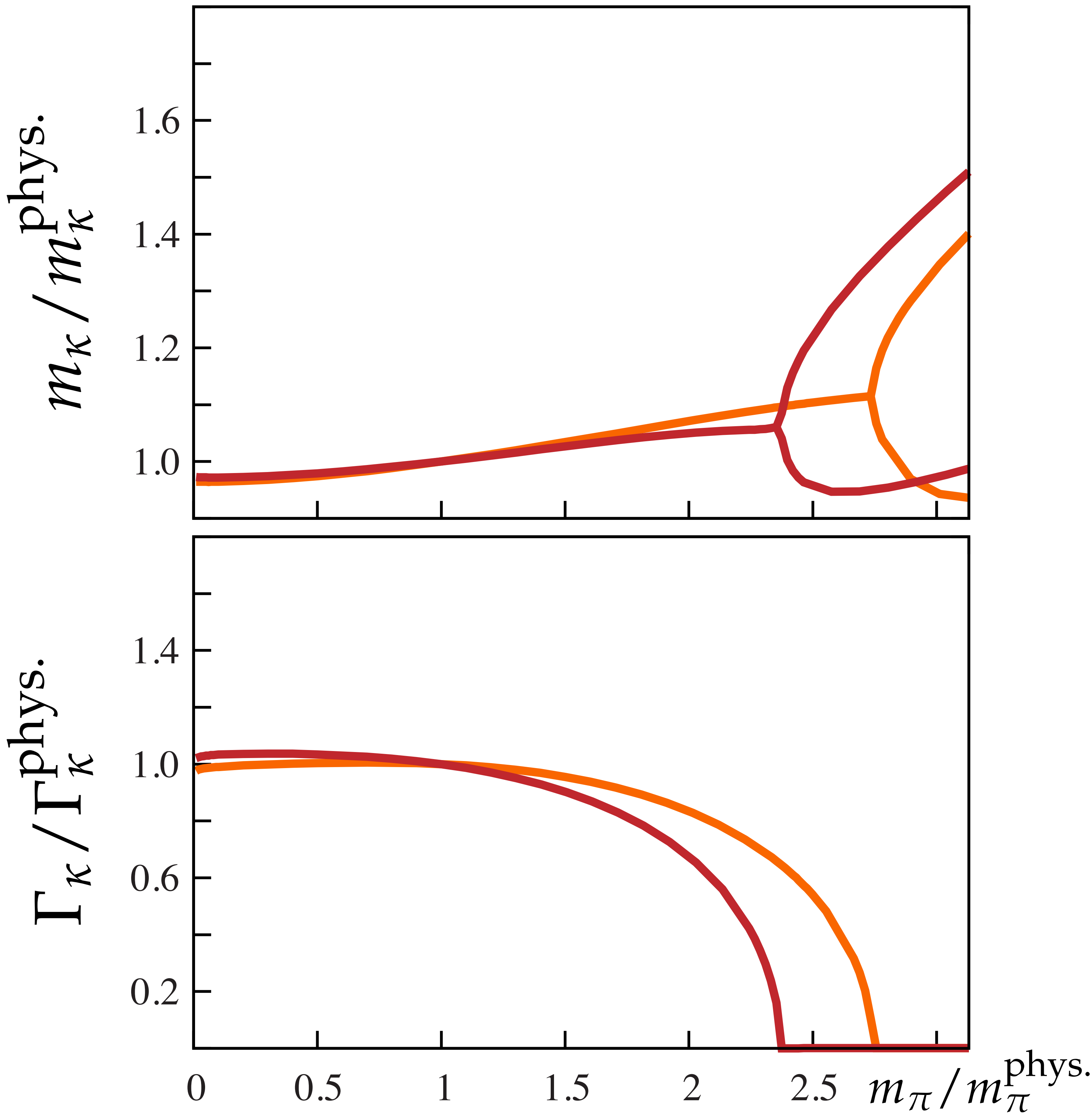
No modifications were used in I(s) for higher waves.

Also tested phase space factors instead of k_i for thresholds.

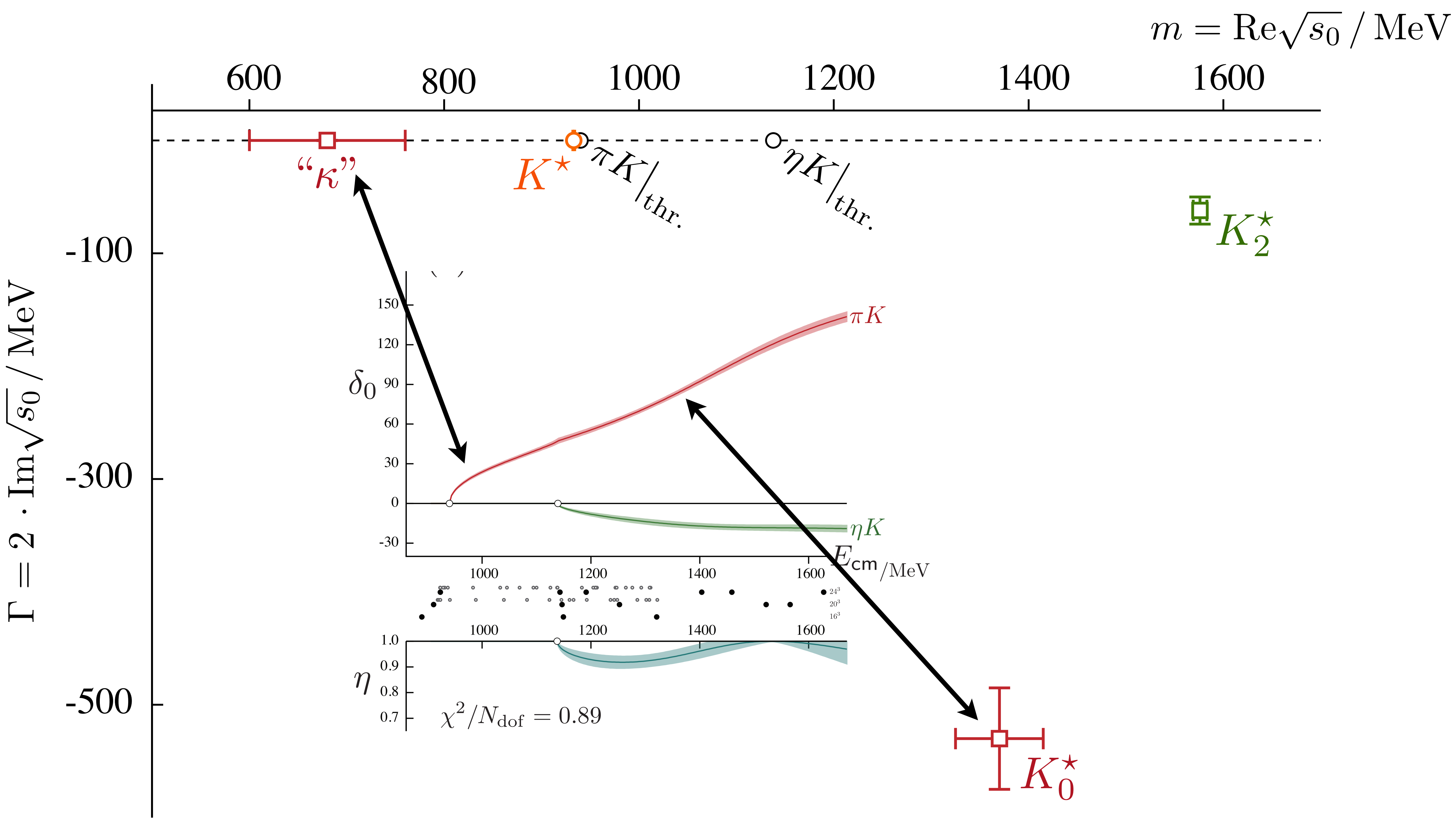
Virtual bound state κ

Pelaez and Nebreda using Unitarised SU(3) Chiral Perturbation theory

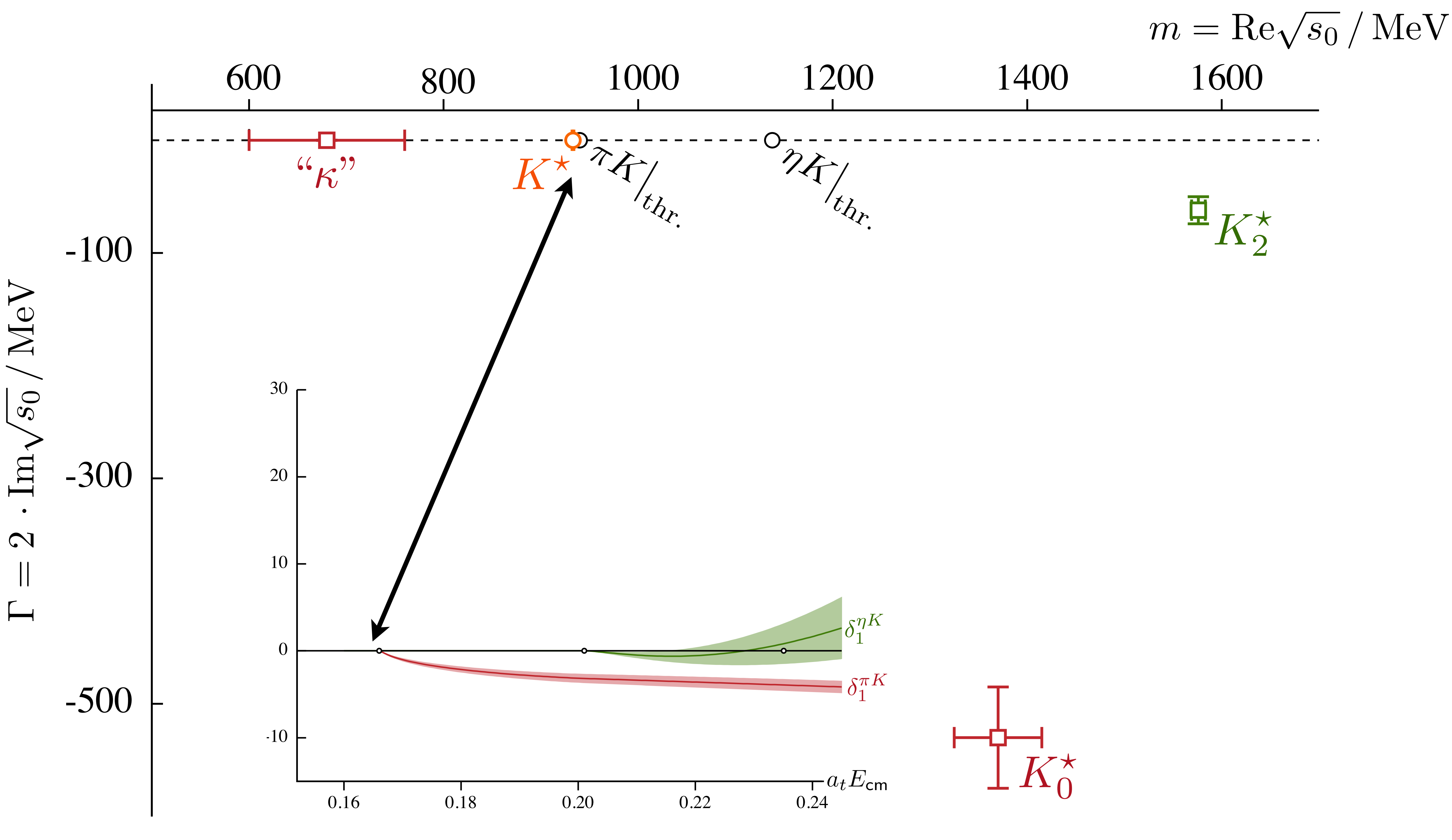




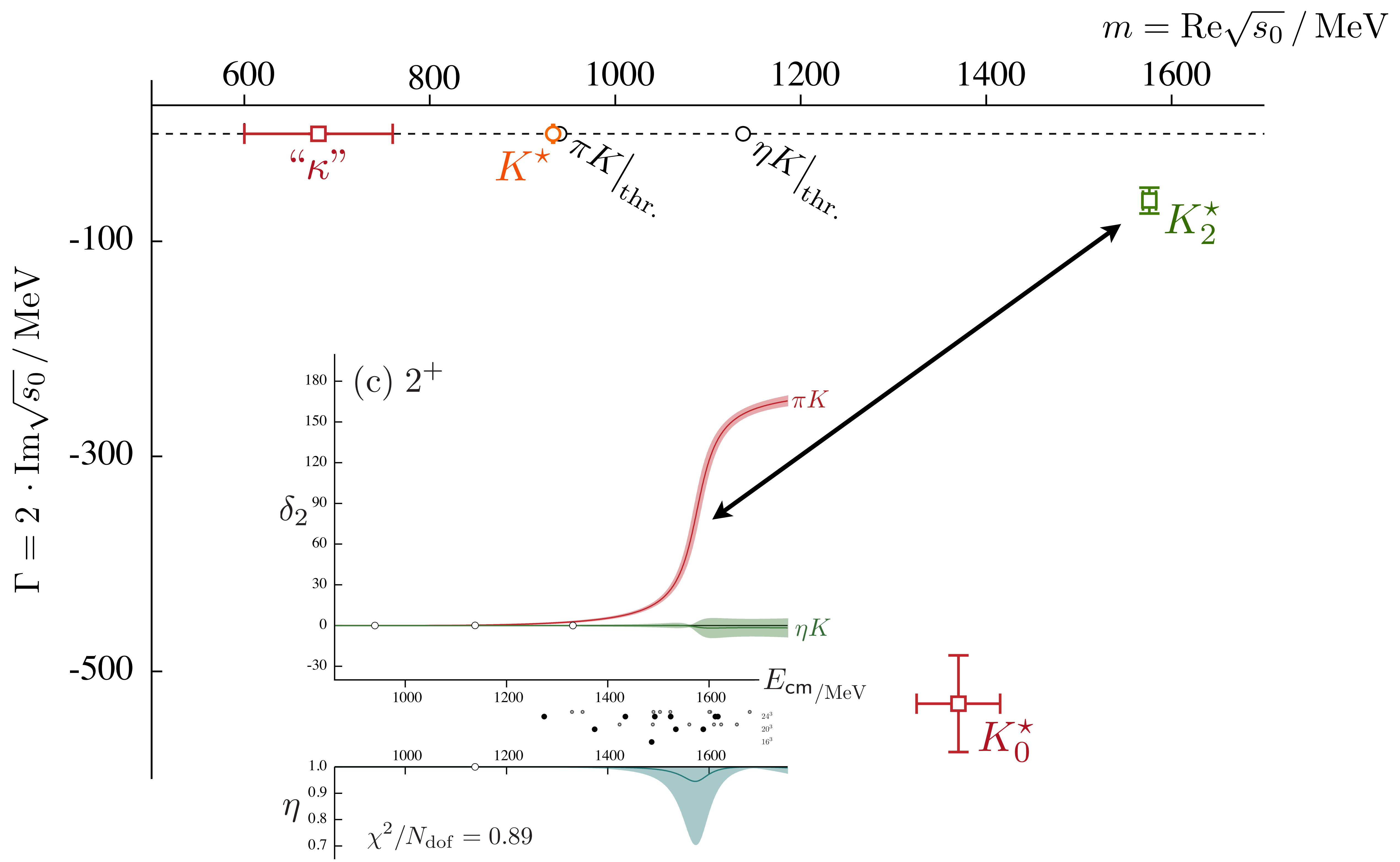
S-matrix poles



S-matrix poles

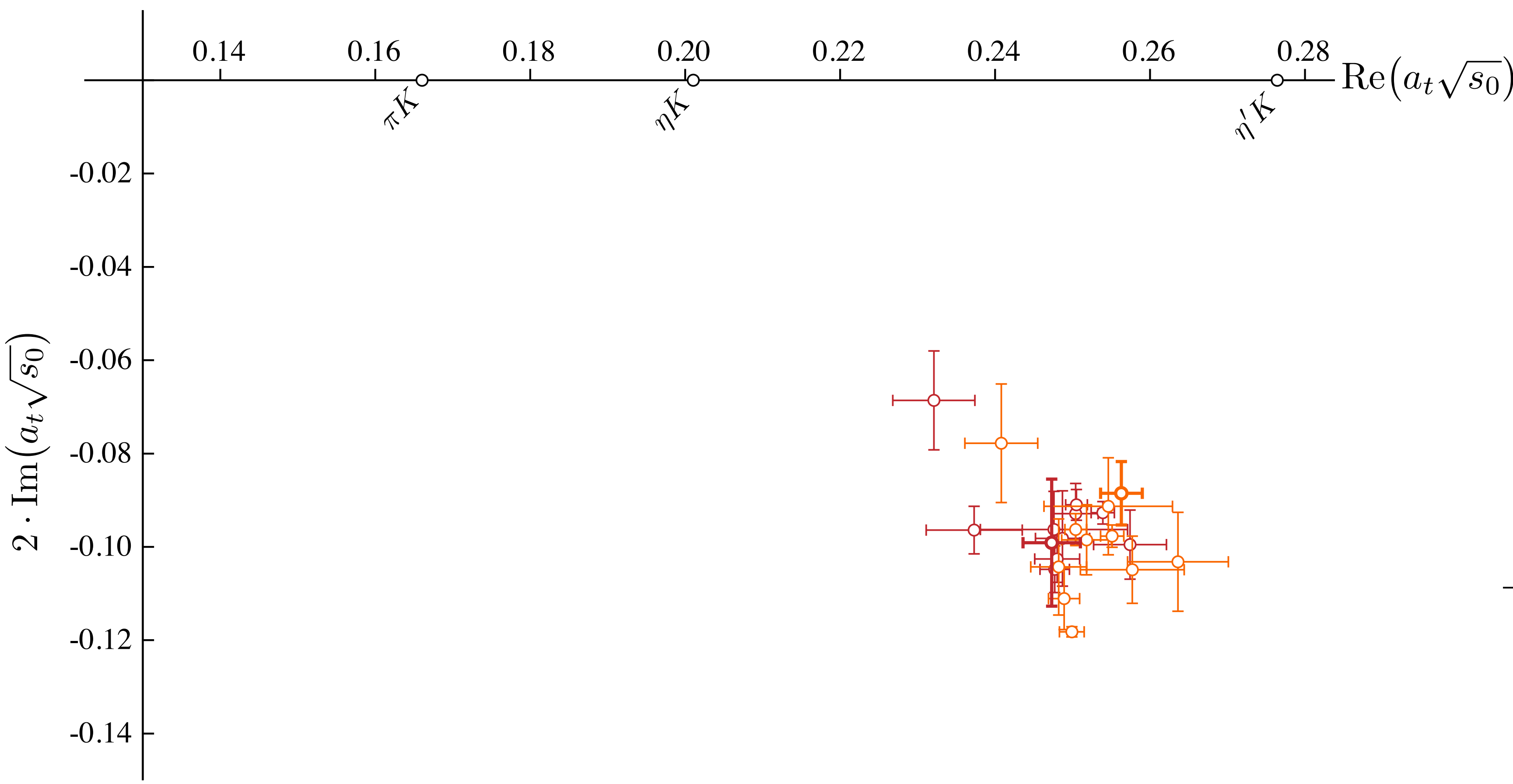


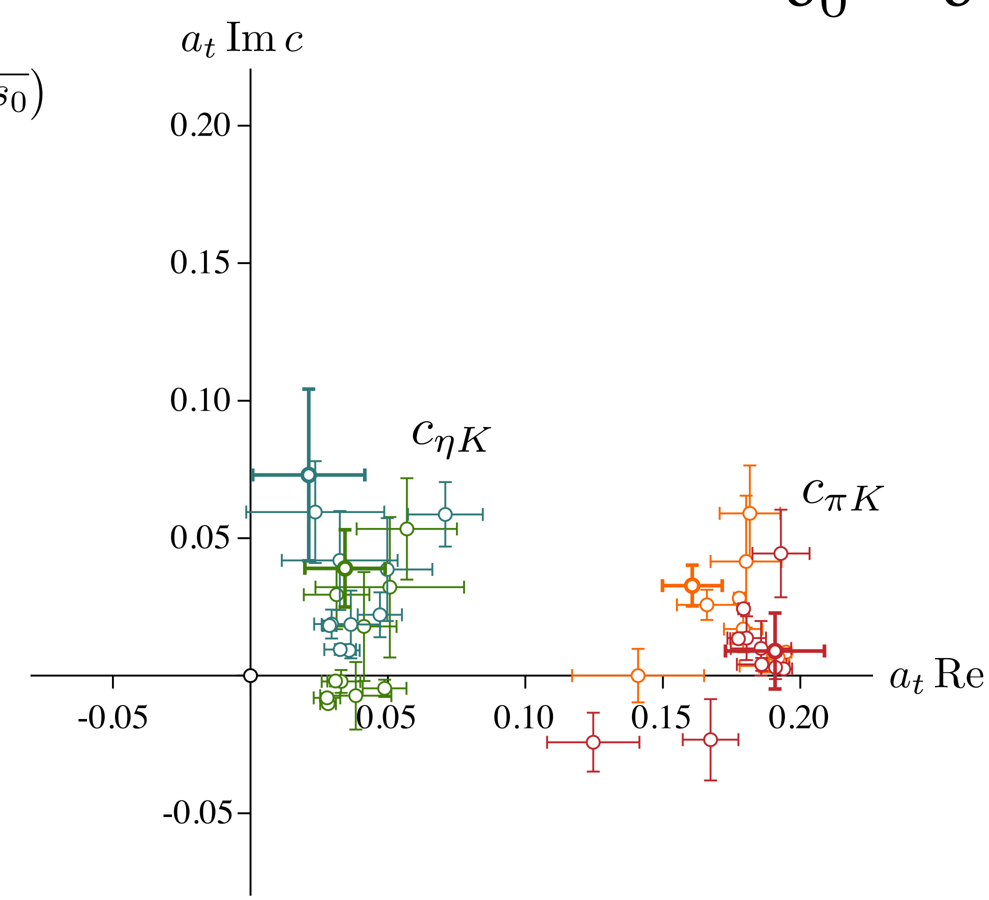
S-matrix poles

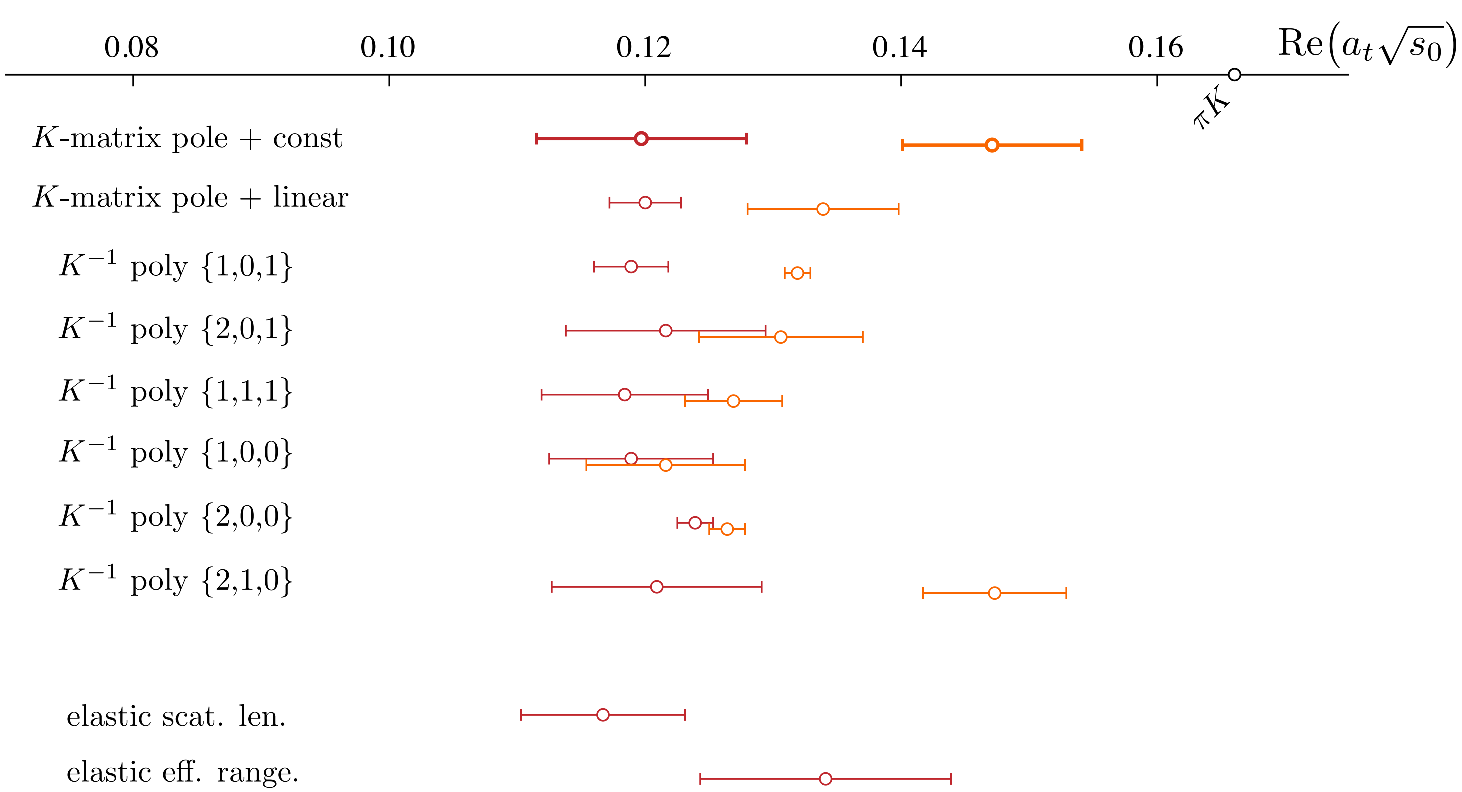


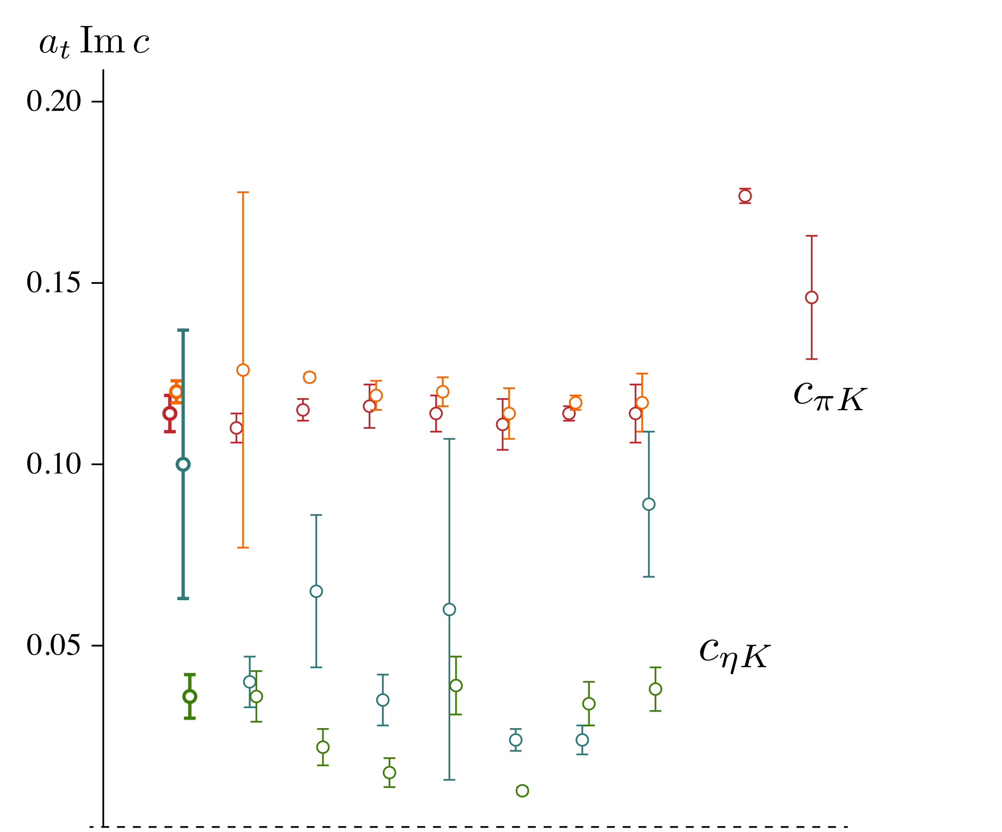
S-matrix *S*-wave poles

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$









Backup slides: ρ resonance

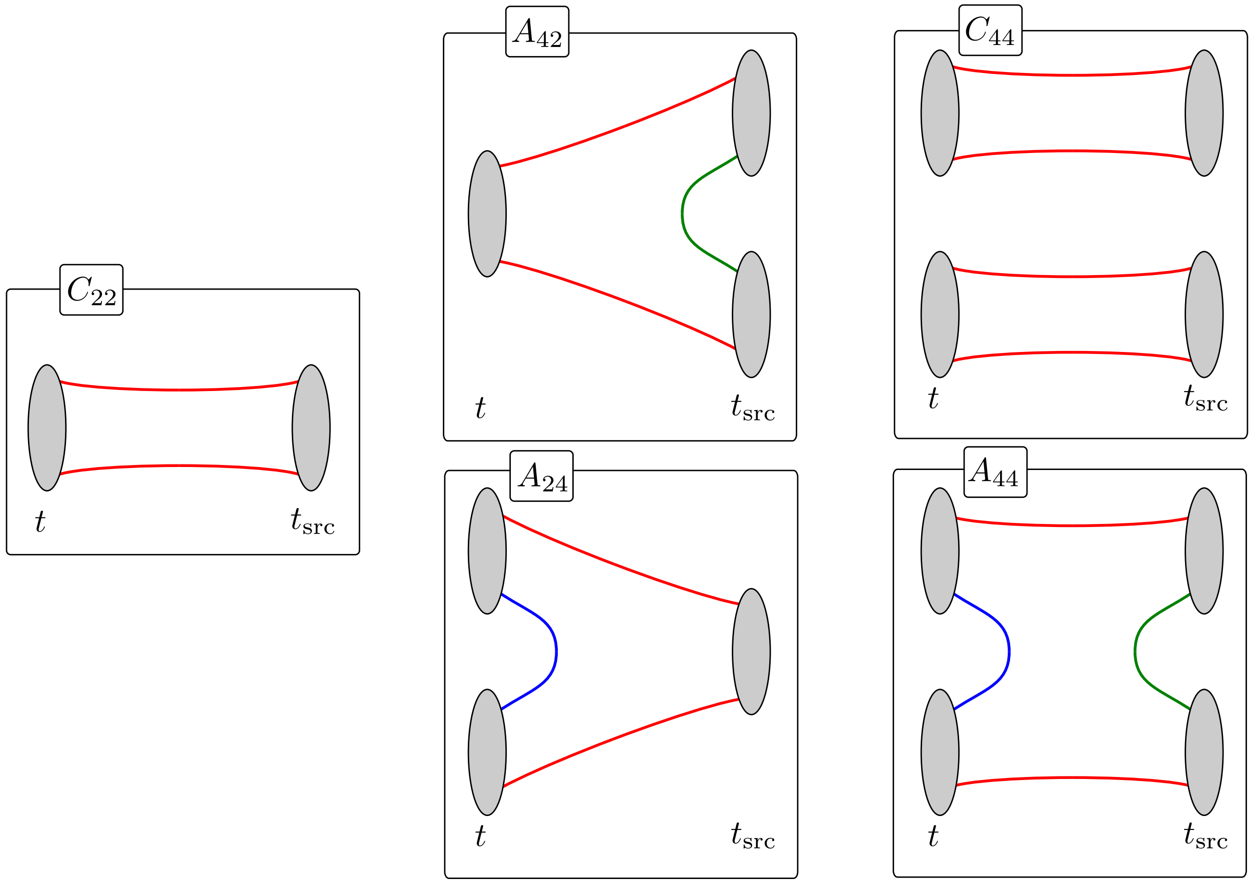
Extracting the ρ resonance

Several volumes: L=16, 20, 24.
Operators in several moving frames, upto n=(2,0,0).

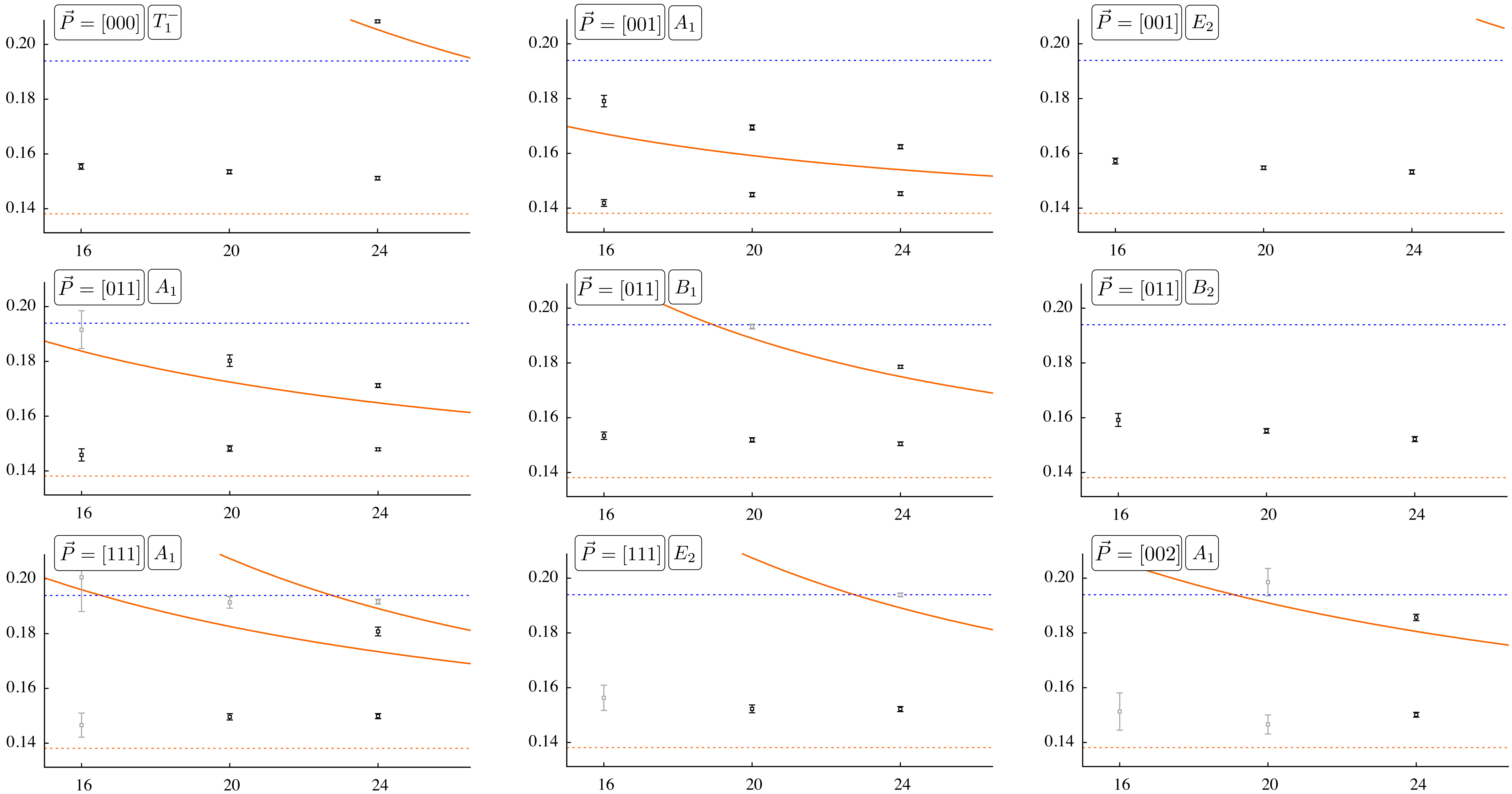
Anisotropic lattices:
temporal spacing 3.5 times finer for better energy resolution.

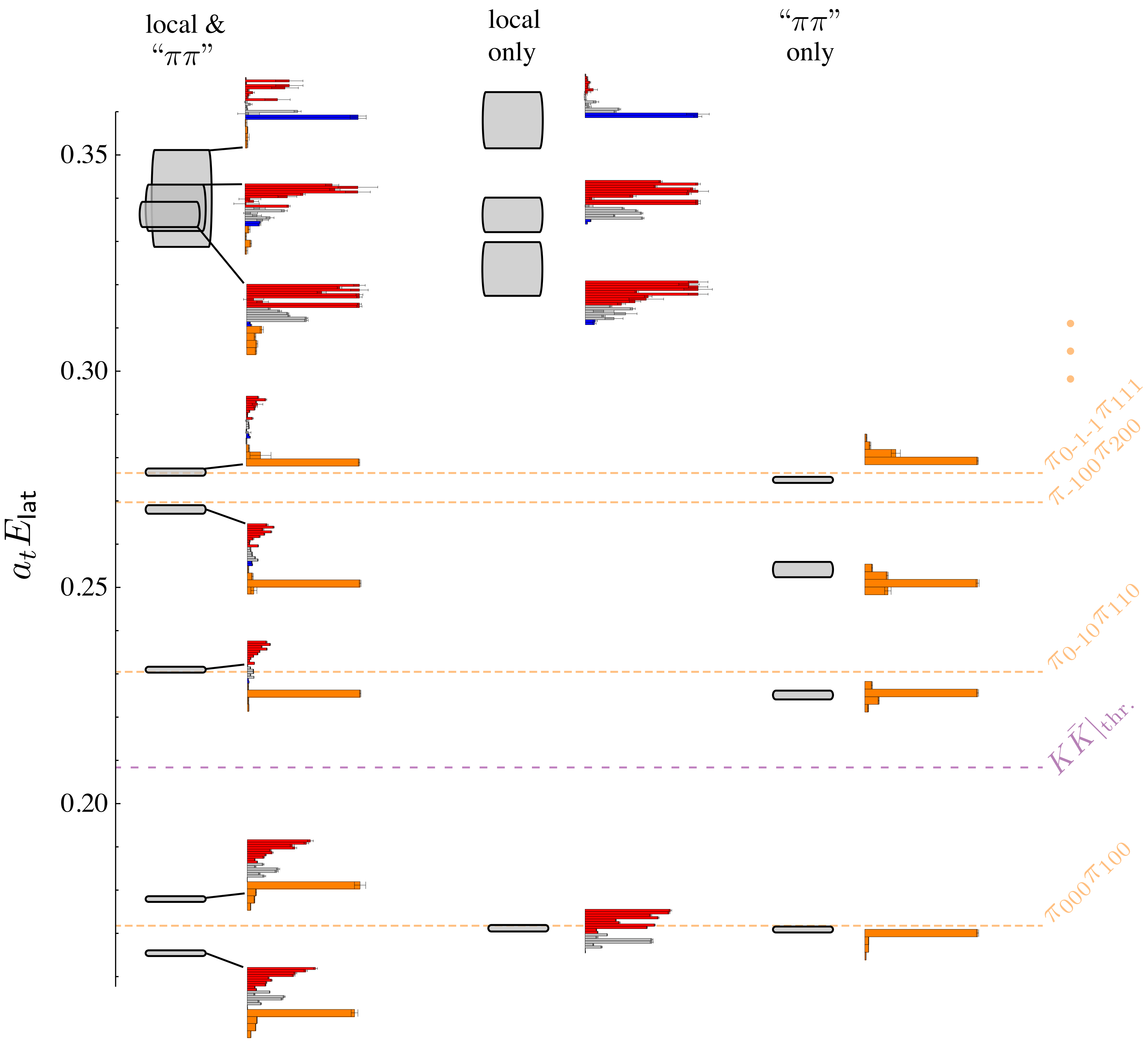
Combination of single particle and meson-meson operators.

m_π=391 MeV



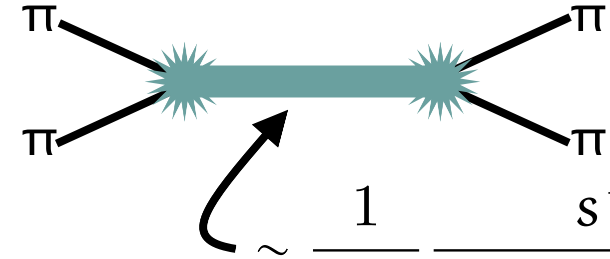
Finite volume spectra in I=1 J=1





local &
"ππ"
local
only
"ππ"
only
0.35
0.30
0.25
0.20
$a_t E_{\mathsf{lat}}$
π0-1-1π111
π-100π200
π0-10π110
$K\bar{K}|_{\mathrm{thr.}}$
π000π100

Resonances from QCD



$$\sim \frac{1}{\rho(s)} \frac{s^{\frac{1}{2}}\Gamma(s)}{m_R^2 - s - i s^{\frac{1}{2}}\Gamma(s)}$$

$$\Gamma(s) = \frac{g_R^2}{6\pi}\frac{k_{cm}^3}{E_{cm}^2}$$