Improved description of the nucleon polarizabilities with relativistic Chiral Effective Field Theory

Jose Manuel Alarcón

Helmholtz-Institut für Strahlen- und Kernphysik
University of Bonn

In collaboration with Vadim Lensky and Vladimir Pascalutsa
Introduction
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• Crucial quantities in an accurate estimation of proton-structure corrections to the Lamb shift ($\mathcal{O}(\alpha_{em}^5)$).

They have the potential to solve the “Proton radius Puzzle”.

\[
\begin{align*}
\mu & \quad \mu \\
\mu H & \quad \mu H \\
p & \quad p
\end{align*}
\]
Introduction

- VVCS in the forward region:

![Diagram](attachment:diagram.png)
**Introduction**

- VVCS in the forward region:

\[
T(\nu, Q^2) = f_L(\nu, Q^2) + (\tilde{e}'^* \cdot \tilde{e}) f_T(\nu, Q^2) + i\tilde{\sigma} \cdot (\tilde{e}'^* \times \tilde{e}) g_{TT}(\nu, Q^2) - i\tilde{\sigma} \cdot [(\tilde{e}'^* - \tilde{e}) \times \hat{q}] g_{LT}(\nu, Q^2)
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- **VWCS in the forward region:**

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\begin{align*}
  f_T(\nu, Q^2) &= f_T^{(B)}(\nu, Q^2) + 4\pi Q^2 \beta_{M1} + 4\pi (\alpha_{E1} + \beta_{M1})\nu^2 + \ldots \\
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\]

\[
H_{\text{eff}}^{(2)} = -4\pi \left[ \frac{1}{2} \alpha_{E1} \bar{E}^2 + \frac{1}{2} \beta_{M1} \bar{B}^2 \right]
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- Relativistic corrections are very important for some polarizabilities [Bernard, Kaiser and Meißner, PRL 67 (1991)], [Kao, Spitzenberg and Vanderhaeghen, PRD 67 (2003)].
We calculate the Compton scattering with relativistic Chiral EFT including the $\Delta(1232)$ up to $O(p^4/\Delta)$ in the $\delta$-counting.
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Up to this order, the EFT calculation is a prediction.
Theoretical Approach

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• We include a dipole form factor in $g_M$:

$$g_M \to \frac{g_M}{(1 + Q^2/0.71)^2}$$

[Source: Pascalutsa and Vanderhaeghen, PRD 73 (2006)]
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$$g_M \rightarrow \frac{g_M}{(1 + Q^2/0.71)^2}$$

We decompose the Compton amplitude in the relativistic form:

$$T(\nu, Q^2) = \epsilon^{\mu*}_\nu \epsilon_\nu \left\{ \left( -g^{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{m_N^2} \left( p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu} \right) \left( p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu} \right) T_2(\nu, Q^2) \right. $$

$$\left. + \frac{i}{m_N} \epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta S_1(\nu, Q^2) + \frac{i}{m_N^3} \epsilon^{\mu\nu\alpha\beta} q_\alpha (p \cdot q s_\beta - s \cdot q p_\beta) S_2(\nu, Q^2) \right\}$$

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  \[ + \frac{i}{m_N} \epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta S_1(\nu, Q^2) + \frac{i}{m_N^3} \epsilon^{\mu\nu\alpha\beta} q_\alpha (p \cdot q s_\beta - s \cdot q p_\beta) S_2(\nu, Q^2) \]
- In order to extract the polarizabilities, we relate $T_1, T_2, S_1$ and $S_2$ to $f_T, f_L, g_{TT}, g_{LT}$.
  \[ f_T(\nu, Q^2) = T_1(\nu, Q^2) \]
  \[ f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \frac{\nu^2 + Q^2}{Q^2} T_2(\nu, Q^2) \]
  \[ g_{TT}(\nu, Q^2) = \frac{\nu}{m_N} \left( S_1(\nu, Q^2) - \frac{Q^2}{m_N \nu} S_2(\nu, Q^2) \right) \]
  \[ g_{LT}(\nu, Q^2) = \frac{Q}{m_N} \left( S_1(\nu, Q^2) + \frac{\nu}{m_N} S_2(\nu, Q^2) \right) \]
Results
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For the Scalar Polarizabilities:

<table>
<thead>
<tr>
<th>Proton</th>
<th>Neutron</th>
</tr>
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<tbody>
<tr>
<td>$\alpha_{E1} + \beta_{M1}$</td>
<td>$\alpha_{L}$</td>
</tr>
<tr>
<td>This work</td>
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</tr>
<tr>
<td>$15.12(82)$</td>
<td>$18.30(99)$</td>
</tr>
<tr>
<td>$13.8(4)$</td>
<td>$14.40(66)$</td>
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References:

[4] MAID
Results

For the Spin Polarizabilities:

- For the Proton:
  - This work: \( \gamma_0 = -0.93(5) \) fm\(^4\) 
  - Empirical: \( \gamma_0 = -1.00(8)(12) \) fm\(^4\)
  - This work: \( \delta_{LT} = 1.35(7) \) fm\(^4\) 
  - Empirical: \( \delta_{LT} = 1.34 \) fm\(^4\)

- For the Neutron:
  - This work: \( \gamma_0 = 0.05(1) \) 
  - Empirical: \( \gamma_0 = -0.005 \)
  - This work: \( \delta_{LT} = 2.20(12) \) 
  - Empirical: \( \delta_{LT} = 2.03 \)

\[ \text{[1] Lensky, Alarcón and Pascalutsa, PRC 90 (2014).} \]
\[ \text{[2] Dutz, et al., PRL 91 (2003).} \]
\[ \text{[3] MAID.} \]
Results

For the Spin Polarizabilities:

- **Proton**
  - $\gamma_0$ ($10^{-4}$ fm$^4$)
    - This work: $-0.93(5)$
    - Empirical: $-1.00(8)(12)$
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    - This work: $1.35(7)$
    - Empirical: $1.34$ [3]

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  - $\gamma_0$ ($10^{-4}$ fm$^4$)
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    - Empirical: $-0.005$ [3]
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(O) Prok et al., PLB 672 (2009)
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Lamb shift
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- Intervene in the theoretical prediction \( \mathcal{O}(\alpha_{em}^5) \) of the proton radius through the Lamb shift \( \Delta E_{2P-2S} \).
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• They have the potential to solve “Proton Radius Puzzle”:

\[
\Delta E_{2P-2S}^{exp} - \Delta E_{2P-2S}^{th}(r_E^{\text{CODATA}}) = 0.31 \text{ meV} = 310 \mu\text{eV}
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- The polarizabilities contribution starts with the \( 2\gamma \) exchange.

\[
T^{\mu\nu}(P, q) = -\left( g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \right) T_1(\nu^2, Q^2) + \frac{1}{M_p^2} \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left( P^{\nu} - \frac{P \cdot q}{q^2} q^{\nu} \right) T_2(\nu^2, Q^2)
\]

\[
\Delta E_{2S}^{(pol)} \approx \frac{\alpha_{em}}{\pi} \phi_{n=2}^2 \int_0^\infty \frac{dQ}{Q^2} w(\tau_\ell) \left[ T_1^{(NB)}(0, Q^2) - T_2^{(NB)}(0, Q^2) \right] \quad T_1^{(NB)} = 4\pi Q^2 \beta_{M1}(Q^2) + \ldots
\]

\[
T_2^{(NB)} = 4\pi Q^2 [\alpha_{E1}(Q^2) + \beta_{M1}(Q^2)] + \ldots
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\[
\begin{align*}
T^{\mu\nu}(P,q) &= -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)T_1(\nu^2, Q^2) + \frac{1}{M_p^2}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)T_2(\nu^2, Q^2) \\
\Delta E_{2S}^{(\text{pol})} &\approx \frac{\alpha_{\text{em}}}{\pi} \phi_{n=2}^2 \int_0^{\infty} \frac{dQ}{Q^2} w(\tau_e) \left[T_1^{(\text{NB})}(0, Q^2) - T_2^{(\text{NB})}(0, Q^2)\right] \\
T_1^{(\text{NB})} &= 4\pi Q^2 \beta_{M1}(Q^2) + \ldots \\
T_2^{(\text{NB})} &= 4\pi Q^2 [\alpha_{E1}(Q^2) + \beta_{M1}(Q^2)] + \ldots
\end{align*}
\]

- Chiral EFT provides predictions of the leading contribution.
Lamb shift

- The main contribution to the polarizabilities comes from the low $Q^2$ region → Chiral EFT
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- Important to reduce contributions from \( Q^2 > \Lambda_{\chi SB}^2 \).

\[
\Delta E_{2S}^{(pol)} \approx \frac{\alpha_{em}}{\pi} \sum_{n=2}^\infty \frac{dQ}{Q^2} w(\tau_{\ell}) \left[ T_{1}^{(NB)}(0,Q^2) - T_{2}^{(NB)}(0,Q^2) \right]
\]

\[
w(\tau_{\ell}) = \sqrt{1 + \tau_{\ell}} - \sqrt{\tau_{\ell}}
\]

\[
\tau_{\ell} = \frac{Q^2}{4m_{\ell}^2}
\]

\[\Delta E_{2S}^{(pol)} \text{ (\( \mu\)eV)}\]

\[Q^2_{\text{max}} \text{ (GeV}^2)\]

[Alarcón, Lensky, Pascalutsa, EPJ C 74 (2014).]
**Lamb shift**

- The main contribution to the polarizabilities comes from the low $Q^2$ region $\rightarrow$ Chiral EFT
- Important to reduce contributions from $Q^2 > \Lambda_{\chi SB}^2$.

\[
\Delta E_{2S}^{(pol)}(\mu eV) \approx \frac{\alpha_{em}}{\pi} \phi_n^2 \int_0^{Q_{max}} \frac{dQ}{Q^2} w(\tau) \left[ T_1^{(NB)}(0, Q^2) - T_2^{(NB)}(0, Q^2) \right]
\]

\[
\tau = \frac{Q^2}{4m^2_{\ell}}
\]

\[
w(\tau) = \sqrt{1 + \tau - \sqrt{\tau}}
\]

\[
\Delta E_{2S}^{(pol)} (\mu eV)
\]

\[
\begin{align*}
0 & \quad 0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 & \quad 1 \\
\text{B\chiPT} & & & & \\
\text{HB\chiPT} & & & & \\
-5 & & & & \\
-10 & & & & \\
-15 & & & &
\end{align*}
\]

$\approx 10\%$ Within the uncertainty of the calculation

[Alarcón, Lensky, Pascalutsa, EPJ C 74 (2014).]
The main contribution to the polarizabilities comes from the low $Q^2$ region $\rightarrow$ Chiral EFT

Important to reduce contributions from $Q^2 > \Lambda_{\chi SB}^2$.

$$\Delta E^{(pol)}_{2S} \approx \frac{\alpha_{em}}{\pi} \phi_{n=2}^2 \int_0^{Q_{max}} \frac{dQ}{Q^2} w(\tau_\ell) \left[ T_{1}^{(NB)}(0, Q^2) - T_{2}^{(NB)}(0, Q^2) \right]$$

$w(\tau_\ell) = \sqrt{1 + \tau_\ell} - \sqrt{\tau_\ell}$

$\tau_\ell = \frac{Q^2}{4m_\ell^2}$

$\int dQ Q_{max} (\text{GeV}^2)$

$\Delta E^{(pol)}_{2S} (\mu\text{eV})$

$Q_{max}^2 (\text{GeV}^2)$

$\bullet$ Within the uncertainty of the calculation

$\bullet$ Too large contribution from $Q^2 > \Lambda_{\chi SB}^2$

$\approx 10\%$

$\approx 20\%$

[Alarcón, Lensky, Pascalutsa, EPJ C 74 (2014).]
Lamb shift

- The relativistic structure is important to agree with phenomenological determinations of $\Delta E^{(pol)}_{2S}$.

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- Chiral EFT calculations
- Phenomenological determinations (dispersion relations+data)

Lamb shift

- The relativistic structure is important to agree with phenomenological determinations of $\Delta E^{(pol)}_{2S}$. 

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Chiral EFT calculations

Phenomenological determinations (dispersion relations+data)

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Chiral EFT calculations

Phenomenological determinations (dispersion relations+data)


- Relativistic chiral EFT agrees with dispersive determinations!
Summary and Conclusions
Summary and Conclusions

• We calculate the VVCS amplitude in covariant BChPT + Δ up to \( O(p^4/\Delta) \) in the \( \delta \)-counting.
• We included a dipole structure to the magnetic coupling of the \( \Delta(1232) \) Important to reproduce electroproduction data.
• The calculation is predictive.
• Our predictions are in good agreement with experimental data and the MAID model.
• We improve the Chiral EFT results for the polarizabilities, specially the in spin-dependent case.
• The Compton amplitude can be employed to calculate the leading proton-structure corrections to the \( \mu \)H Lamb shift.
• Our prediction agrees with dispersive calculations:

<table>
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<tr>
<th>( \Delta E_{2s}^{(pol)} )</th>
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<th>Birse &amp; McGovern</th>
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FIN
Spares
Dependence on the dipole form factor
Dependence on the dipole form factor

For the Scalar Polarizabilities

\[ \alpha_{E1}^p + \beta_{M1}^p (10^{-4} \text{ fm}^3) \]

\[ \alpha_{E1}^n + \beta_{M1}^n (10^{-4} \text{ fm}^3) \]

With Dipole

Without dipole

LO HB

FIG. 6: Comparison of the full result without dipole (red solid line) with the rest of the available ChPT calculations for +. The blue solid line and its band is our total result with dipole and the blue dashed line is the leading order result of [47]. The data are the same as in Fig 4.

FIG. 7: Longitudinal polarizabilities, \( L(\mathbf{Q}^2) \), for the proton and neutron as a function of \( \mathbf{Q}^2 \). The legend is the same as in Fig. 4 except for the black dotted line that corresponds to the MAID estimate using only the \( \pi^- \) channel, as in Ref. [17].
**Dependence on the dipole form factor**

- For the Spin Polarizabilities

![Graphs showing the dependence on the dipole form factor](image)

- With Dipole
- Without dipole
- LO HB
- BChPT+$\Delta^*$

---

J. M. Alarcón (HISKP Bonn)  
Chiral Dynamics 2015
Some other interesting moments
For some interesting moments:

\[
d_2(Q^2) = \int_0^{x_0} dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)]
\]

\[
I_A(Q^2) = \frac{2M_N^2}{Q^2} \int_0^{x_0} dx g_{TT}(x, Q^2)
\]

Results

- Structure corrections to HFS
- GDH Sum Rule

**Proton**

- \(d_2(Q^2)\) for different moments and models.
- \(I_A(Q^2)\) for different moments and models.

**Neutron**

- \(d_2(Q^2)\) for different moments and models.
- \(I_A(Q^2)\) for different moments and models.

Models:

- BChPT + \(\Delta\)
- LO BChPT
- LO HB
- MAID
- BChPT + \(\Delta^*\)

[Ref. \[Kao et al., PRD 67 (2003)\]]

References:

- (1) Amarian et al. PRL 89 (2002)
- (1) Amarian et al. PRL 89 (2002)
- (d2) Amarian et al. PRL 92 (2004)