From OPE to chiral perturbation theory in holographic QCD

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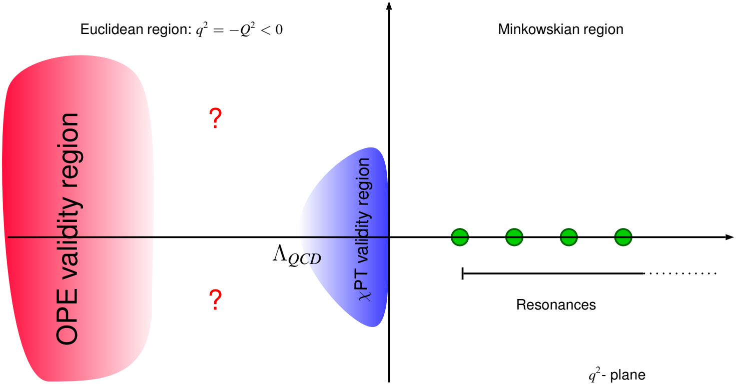
- QCD has well described different regimes: low energy (*e.g.* χPT), high energy (OPE), Minkowskian sector (resonances)
- Unfortunately the intermediate sector is unknown...
- Because of the existence of **Non-Perturbative** effects.



From now we assume Large-N$_c$ limit

Several ways to address this issue

- Treat the 2 points Green functions as Padé approximants.
  Minimal Hadronic Approximation and related models
  `S. Peris and E. de Rafael, JHEP 9805 (1998)`
  `M. Golterman and S. Peris, JHEP 0101 (2001)`
- Resummation of Large-N$_c$ resonances properties.
  `O. Cata, M. Golterman and S. Peris, JHEP 0508, 076 (2005)`
  `E. de Rafael, Indian Academy of Sciences, Vol. 78, N6 June 2012`
- Non-Analytic reconstruction method.
  `D. G. and S. Peris, Phys.Rev. D82 (2010)`
- AdS/CFT correspondence.

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)
S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998)
A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

The Maldacena conjecture

4 - dimensional		5 - dimensional
Non perturbative CFT Large N_c	$\longleftrightarrow$	Perturbative AdS Classical Theory
Green functions		Green functions

Practically...

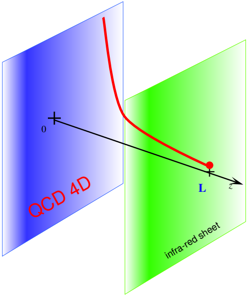
4 - dimensional		5 - dimensional
Sources coupled to currents v, a, s, p	$\underset{z \to 0}{\longleftarrow}$	Fields $\phi(x, z)$ on a gravitationnal background

Two different models: Hard - Wall and Soft-Wall

Let consider a conformally flat metric in 5D,

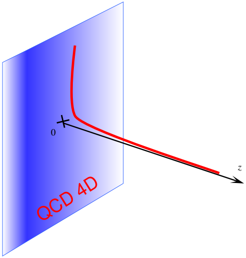
$$g_{MN}\, \mathrm{d}x^M\, \mathrm{d}x^N = w(z)^2 \left(\eta_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu - \mathrm{d}z^2\right)$$

Hard - Wall



$$0 \leqslant z \leqslant L\,, \quad w(z) = \frac{1}{z}$$

Soft - Wall



$$0 \leqslant z \leqslant \infty\,, \quad w(z) = \frac{1}{z}\mathrm{e}^{-\Phi(z)}$$

Recovered Large N_c QCD Properties

J. Hirn and V. Sanz, JHEP **0512**, 030 (2005)

J. Erlich, E. Katz, D.T. Son, M.A. Stephanov Phys. Rev. Lett. 95 (2005)

D.T. Son and M.A. Stephanov Phys. Rev. D **69** (2004)

A. Karch, E. Katz, D.T. Son, M. A. Stephanov Phys. Rev. **D** 74 (2006)

	Hard - Wall	**Soft - Wall** $\Phi(z) = \kappa^2 z^2$
Parton Log	**YES**	**YES**
OPE	**NO**	**NO**
Chiral symmetry breaking	**YES** / **NO**	**NO**
Regge-like Spectrum of resonances	**NO**	**YES**

Purpose of this work

We want to modified the dilaton field $\Phi(z)$ such that we obtain:

- The right OPE.
- A chiral symmetry breaking mechanism: axial field and a pion pole (*i.e.* $F_\pi \neq 0$)

Vectorial correlator properties

OPE

$$\Pi_V(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{24\pi^2} \ln\left(\frac{\Lambda_V^2}{Q^2}\right) + \langle\mathcal{O}_2\rangle \frac{1}{Q^2} + \langle\mathcal{O}_4\rangle \frac{1}{Q^4} + \langle\mathcal{O}_6\rangle_V \frac{1}{Q^6}$$

where in the large-N_c limit the coefficients of the OPE are given by

$$\begin{cases} \langle\mathcal{O}_2\rangle = 0 \\ \langle\mathcal{O}_4\rangle = \frac{1}{12\pi}\,\alpha_s\,\langle G^2\rangle \\ \langle\mathcal{O}_6\rangle_V = -\frac{28\pi}{9}\,\alpha_s\,\langle\bar\psi\psi\rangle^2 \end{cases}$$

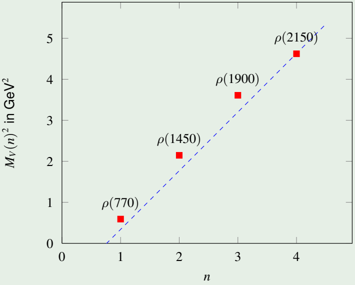
Regge Resonances spectrum

$$\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$

with $M_V(n)^2 \underset{n\to\infty}{\sim} \sigma\, n$,

σ is related to the confining string tension ($\sigma \sim 1.43(13)$ GeV^2).

P. Masjuan *et al.*, Phys. Rev. D **85**, 094006 (2012)



Lagrangian

$$S_5 = -\frac{1}{4g_5^2}\int d^4x \int_0^\infty dz \sqrt{g}\, e^{-\Phi(z)}\, g^{MN} g^{RS}\, \mathrm{Tr}\left[\mathbb{F}_{MR}\,\mathbb{F}_{NS}\right] \ ,$$

with $g = |\det g_{MN}|$, the field strength $\mathbb{F}_{MN} = \partial_M \mathbb{V}_M - \partial_N \mathbb{V}_M - i\,[\mathbb{V}_M, \mathbb{V}_N]$ and in order to reproduce the parton log

$$g_5^2 = \frac{12\pi^2}{N_c} \ .$$

The AdS/CFT correspondence prescribes that the boundary value of the 5D gauge field $\mathbb{V}_M$ has to be identified with the classical source v_μ coupled to the the 4-dimensional vectorial current $J^a_{V\ \mu} =: \bar{q}\,\gamma_\mu\, t^a q :$,

$$\lim_{z\to 0} \mathbb{V}^a_{\mu,z}(x,z) = \mathrm{v}^a_\mu(x) \ .$$

The 4D-Fourier transform f_V of $\mathbb{V}_M$ satisfies the equation of motion

$$\partial_z^2 f_V + \partial_z\left[\ln w_0(z)\right]\partial_z f_V + q^2 f_V = 0 \ ,$$

with $w_0(z) \doteq \frac{e^{-\Phi(z)}}{z} = \frac{e^{-\kappa^2 z^2}}{z}$ and the boundary condition $f_V(-q^2, 0) = 1$

First step: Dilaton profile prescription

In order to recover the right OPE of the vectorial correlator, we take in $w(z)$

$$\Phi(z) \longmapsto \Phi(z) + B(z)$$

where we **choose**:
$$B(z) = \sum_{k=1}^{K} \frac{b_{2k}}{2k} z^{2k}$$

A posteriori, since we need to recover the $1/Q^6$ we need only $K = 3$.
The purpose will be to find

$$b_{2k} \longleftrightarrow \langle \mathcal{O}_{2k} \rangle$$

Second step: Organisation of the calculation

We introduce an artificial control parameter θ such as

$$w(z) = w_0(z)\mathrm{e}^{-B(\sqrt{\theta}z)} \quad \text{with} \quad w_0(z) = \mathrm{e}^{-\Phi(z)}$$

then in $B(\sqrt{\theta}z) = \sum_{k=1}^{3} \frac{b_{2k}}{2k} z^{2k}\theta^k$ we will take $b_{2k} \propto \theta^{-k}$.

The (complicated) equation of motion

$$\partial_z^2 f_V - \left[\frac{1}{z} + 2\kappa^2 z + b_2 z + b_4 z^3 + b_6 z^6\right] \partial_z f_V + q^2 f_V = 0\ ,$$

is transformed into a hierarchical differential system in θ

$$\begin{array}{ll}
\theta^0 & \mathfrak{D} f_V^{(0)} = 0 \\
\theta^1 & \mathfrak{D} f_V^{(1)} = z\ b_2\ \partial_z f_V^{(0)} \\
\theta^2 & \mathfrak{D} f_V^{(2)} = z^3\ b_4\ \partial_z f_V^{(0)} + z\ b_2\ \partial_z f_V^{(1)} \\
\theta^3 & \mathfrak{D} f_V^{(3)} = z^5\ b_6\ \partial_z f_V^{(0)} + z^3\ b_4\ \partial_z f_V^{(1)} + z\ b_2\ \partial_z f_V^{(2)}
\end{array}$$

where $\mathfrak{D}\varphi \doteq \partial_z^2 \varphi + \partial_z \left[\ln w_0(z)\right] \partial_z \varphi - Q^2 \varphi$
that can be solved with the usual Green function method.

$$\mathfrak{D} f_V = S(Q^2, z) \quad \Leftrightarrow \quad f_V(Q^2, z) = \int_0^\infty \mathrm{d}z'\, w_0(z')\ G_V(z, z'; Q^2)\ S(Q^2, z')$$

then

$$f_V^{(n)}(Q^2, z) = \int_0^\infty \mathrm{d}x\ w_0(x)\ G_V(x, z; Q^2) \left[\sum_{k=0}^{n-1} x^{2(n-k)-1}\ b_{2(n-k)}\ \partial_x f_V^{(k)}(Q^2, x)\right]$$

Connection between the two point Green function and the 5D model

$$Q^2\Pi_V(Q^2) = \frac{1}{g_5^2} \lim_{z\to 0} w_0(z)\, f_V(Q^2,z)\, \partial_z f_V(Q^2,z)$$

Therefore we build an analytic expression of Π_V:

$$Q^2\Pi_V(Q^2) = \sum_{k,n} \mathcal{P}_{k,n}\left(\frac{Q^2}{4\kappa^2}\right)\, \psi^{(k)}\left(\frac{Q^2}{4\kappa^2}\right)$$

$\mathcal{P}_{k,n}$ a polynomial and $\psi^{(k)}$ is the k^{th} derivative of the Digamma ψ.

Just to show up...

$$\Pi_V^{(0)}(Q^2) = \frac{1}{2g_5^2}\left[\gamma_E + \psi\left(\frac{Q^2}{4\kappa^2}+1\right)\right]$$

$$\Pi_V^{(1)}(Q^2) = \frac{b_2}{4\kappa^2 g_5^2}\left(\frac{4\kappa^2}{Q^2}\right)\left[1+\left(\frac{Q^2}{4\kappa^2}\right)-\left(\frac{Q^2}{4\kappa^2}\right)^2\psi'\left(\frac{Q^2}{4\kappa^2}\right)\right]$$

$$\Pi_V^{(2)}(Q^2) = \frac{b_4}{\kappa^4 g_5^2}\left(\frac{4\kappa^2}{Q^2}\right)\left[-2-\left(\frac{Q^2}{4\kappa^2}\right)\left(5+6\frac{Q^2}{4\kappa^2}\right)+2\left(\frac{Q^2}{4\kappa^2}\right)^2\left(1+3\frac{Q^2}{4\kappa^2}\right)\psi'\left(\frac{Q^2}{4\kappa^2}\right)\right]$$
$$+\frac{b_2^2}{16\kappa^4 g_5^2}\left[-1+2\left(\frac{Q^2}{4\kappa^2}\right)\psi'\left(\frac{Q^2}{4\kappa^2}\right)+\left(\frac{Q^2}{4\kappa^2}\right)^2\psi''\left(\frac{Q^2}{4\kappa^2}\right)\right]$$

$$\Pi_V^{(3)}(Q^2) = \frac{b_6}{12\kappa^6 g_5^2}\left(\frac{4\kappa^2}{Q^2}\right)\left\{6+\left(\frac{Q^2}{4\kappa^2}\right)\left[20+30\left(\frac{Q^2}{4\kappa^2}\right)+33\left(\frac{Q^2}{4\kappa^2}\right)^2\right]\right.$$
$$\left.-6\left(\frac{Q^2}{4\kappa^2}\right)^2\left[1+3\left(\frac{Q^2}{4\kappa^2}\right)+5\left(\frac{Q^2}{4\kappa^2}\right)^2\right]\psi'\left(\frac{Q^2}{4\kappa^2}\right)\right\}$$
$$-\frac{b_2 b_4}{8\kappa^6 g_5^2}\left(\frac{4\kappa^2}{Q^2}\right)\left\{-1-\left(\frac{Q^2}{4\kappa^2}\right)\left[5+9\left(\frac{Q^2}{4\kappa^2}\right)\right]+3\left(\frac{Q^2}{4\kappa^2}\right)^2\left[1+4\left(\frac{Q^2}{4\kappa^2}\right)\right]\psi'\left(\frac{Q^2}{4\kappa^2}\right)\right.$$
$$\left.+\left(\frac{Q^2}{4\kappa^2}\right)^3\left[1+\left(\frac{Q^2}{4\kappa^2}\right)\right]\psi''\left(\frac{Q^2}{4\kappa^2}\right)\right\}$$
$$-\frac{b_2^3}{16\kappa^6 g_5^2}\left\{-2+6\left(\frac{Q^2}{4\kappa^2}\right)\psi'\left(\frac{Q^2}{4\kappa^2}\right)+6\left(\frac{Q^2}{4\kappa^2}\right)^2\psi''\left(\frac{Q^2}{4\kappa^2}\right)+\left(\frac{Q^2}{4\kappa^2}\right)^3\psi'''\left(\frac{Q^2}{4\kappa^2}\right)\right\}$$

Regge Resonances spectrum

$$Q^2\Pi_V(Q^2) = \sum_{k,n} \mathcal{P}_{k,n}\left(\frac{Q^2}{4\kappa^2}\right)\ \psi^{(k)}\left(\frac{Q^2}{4\kappa^2}\right)$$

The Digamma ψ function has its poles as $\frac{Q^2}{4\kappa^2} = -n$ then it yields to a Regge behaviour expected:

$$M(n)^2 = 4\kappa^2 n = \sigma n$$

and $\kappa \simeq 0.6$ GeV

Recovery of the OPE

$$\Pi_V(Q^2) \underset{Q^2\to\infty}{\sim} \frac{1}{2g_5^2}\ln\left(\frac{4\kappa^2 e^{-\gamma_E}}{Q^2}\right) + \underbrace{\tfrac{1}{g_5^2}\left(\kappa^2+\theta\tfrac{1}{2}b_2\right)}_{<\mathcal{O}_2>}\frac{1}{Q^2} + \underbrace{\tfrac{1}{30g_5^2}\left[-5\left(\kappa^2+\theta b_2\right)^2+20\theta^2 b_4\right]}_{<\mathcal{O}_4>}\frac{1}{Q^4}$$

$$+\underbrace{\frac{4}{5g_5^2}\left[-2\theta^2\kappa^2 b_4 - \theta^3(b_2 b_4 - 4b_6)\right]}_{<\mathcal{O}_6>}\frac{1}{Q^6}$$

one has $\Lambda_V = 2\kappa e^{-\frac{\gamma_E}{2}} \approx 1$ GeV

$$b_2 = -2\kappa^2 \qquad b_4 = \frac{3}{2}g_5^2\langle\mathcal{O}_4\rangle \qquad b_6 = \frac{5}{16}g_5^2\langle\mathcal{O}_6\rangle_V$$

(Expected) Axial correlator properties

OPE

$$\Pi_A(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{24\pi^2}\ln\left(\frac{\Lambda_A^2}{Q^2}\right) + \langle\mathcal{O}_2\rangle\frac{1}{Q^2} + \langle\mathcal{O}_4\rangle\frac{1}{Q^4} + \langle\mathcal{O}_6\rangle_A\frac{1}{Q^6}$$

where in the large-N_c limit the coefficients of the OPE are given by

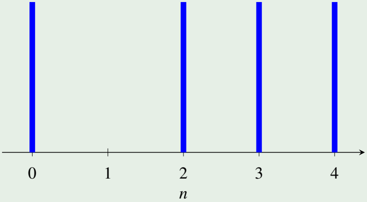
$$\begin{cases} \langle\mathcal{O}_2\rangle = 0 \\ \langle\mathcal{O}_4\rangle = \frac{1}{12\pi}\,\alpha_s\,\langle G^2\rangle \\ \langle\mathcal{O}_6\rangle_A = -\frac{11}{7}\,\langle\mathcal{O}_6\rangle_V \end{cases}$$

Axial spectrum

$$\Pi_A(Q^2) = -\frac{F_\pi^2}{Q^2} + \sum_{n=0}^{\infty}\frac{F_A(n)^2}{Q^2 + M_A(n)^2}$$

with

$$M_A(n)^2 \underset{n\to\infty}{\sim} \sigma\, n$$



H. J. Kwee and R. F. Lebed, JHEP 0801, 027 (2008)

Solution: extra-scalar field on the bulk $\mathbb{X}$

$$S_5 = \int d^4x \int_0^\infty dz \sqrt{g}\, e^{-\Phi(z)}\, \mathrm{Tr}\Big\{ g^{MN}\, (D_M\mathbb{X})^\dagger\, (D_N\mathbb{X}) - m^2\mathbb{X}^2 - \frac{1}{4g_5^2} g_{MN} g_{RS} \left(\mathbb{F}_V^{MR}\mathbb{F}_V^{NS} + \mathbb{F}_A^{MR}\mathbb{F}_A^{NS} \right) \Big\}$$

where we use

$$\begin{aligned} D^M\mathbb{X} &= \partial^M\mathbb{X} - i[\mathbb{V}^M, \mathbb{X}] - i\{\mathbb{A}^M, \mathbb{X}\} \\ \mathbb{F}_V^{MN} &= \partial^M\mathbb{V}^N - \partial^N\mathbb{V}^M - i[\mathbb{V}^M, \mathbb{V}^N] - i[\mathbb{A}^M, \mathbb{A}^N] \\ \mathbb{F}_A^{MN} &= \partial^M\mathbb{A}^N - \partial^N\mathbb{A}^M - i[\mathbb{V}^M, \mathbb{A}^N] - i[\mathbb{A}^M, \mathbb{V}^N] \end{aligned}$$

If $v(z)$ is the vacuum expectation value for the scalar field $\mathbb{X}$,

$$\mathrm{Tr}|\mathbb{X}|^2 = 2\, v(z)^2$$

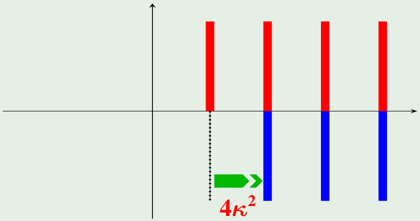
The equation of motion for the 4D F. T. space of the axial field $\mathbb{A}$, $f_A(-q^2, z)$:

$$\partial_z^2 f_A + \partial_z \left[\ln w(z)\right] \partial_z f_A - Q^2 f_A = g_5^2 \left(\frac{v(z)}{z} \right)^2 f_A$$

where $w(z) = \frac{1}{z} e^{-\Phi(z) - B(z)}$

A Chiral Symmetry Breaking Mechanism

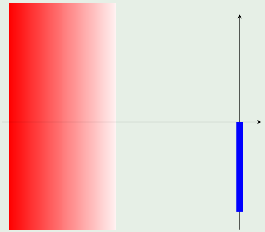
First step: recovering the axial Regge spectrum



Since $M_{a_1}^2 \simeq 2M_\rho^2 \simeq 2 \times 4\kappa^2$,

$$\left(\frac{v(z)}{z}\right)^2 = \beta_0 = \frac{4\kappa^2}{g_5^2}$$

Second step: Axial OPE and pion



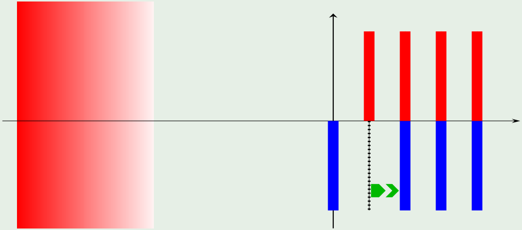
$$\Pi_A(Q^2) \underset{Q^2\to\infty}{\sim} \frac{1}{2g_5^2} \ln\left(\frac{\Lambda}{Q^2}\right) + \frac{2\kappa^2}{g_5^2}\frac{1}{Q^2}$$

Corrected with

$$\left(\frac{v(z)}{z}\right)^2 = \beta_0 + \boxed{\beta^* z\delta(z)} \longrightarrow -\frac{\beta^*}{g_5^2 Q^2}$$

This generates a pion pole.

Complete axial spectrum and axial OPE



Completing with the same method used for the vectorial sector

$$\left(\frac{v(z)}{z}\right)^2 = \beta_0 + \beta^* z\delta(z) + \beta_2 z^2 + \beta_4 z^4$$

with $\beta_2 = -\frac{6\kappa^4}{g_5^2}$ and $\beta_4 = -\frac{10\kappa^2}{3g_5^2} - 5\kappa^2 \langle \mathcal{O}_4 \rangle + \frac{45}{28} \langle \mathcal{O}_6 \rangle_V$

Conclusion

- We have the axial two point function Π_A with its correct OPE and spectrum
- What about F_π and more generally the chiral sector?

Chiral sector

$$\Pi_{LR}(Q^2) = \frac{1}{2}\left(\Pi_V(Q^2) - \Pi_A(Q^2)\right)$$

The lower Q^2 limit exists thanks to the analytic continuation of our expressions one could gives our values for the chiral constants

$$F_\pi^2 = 2\,\text{Res}\left[\Pi_{LR}\left(Q^2\right), 0\right] \qquad L_{10} = -\frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}Q^2}\left[Q^2\Pi_{LR}(Q^2)\right]\Big|_{Q^2=0}$$

F_π

$$F_\pi^2 = \frac{N_c\kappa^2\left(180\zeta(3) + 191 - 41\pi^2\right)}{72\pi^2} + \frac{5\left(\pi^2 - 10\right)}{2}\frac{\langle\mathcal{O}_4\rangle}{\kappa^2} - \frac{45\left(\pi^2 - 10\right)}{56}\frac{\langle\mathcal{O}_6\rangle_V}{\kappa^4}$$

with $N_c = 3$, $\kappa = 0.6$ GeV, $\langle\mathcal{O}_4\rangle = (-0.635 \pm 0.04)\cdot 10^{-3}$ GeV^4, $\langle\mathcal{O}_6\rangle_V = (14 \pm 3)\cdot 10^{-4}$ GeV^6

$$F_\pi \simeq \sqrt{4099.9 + 579 + 1147.8} \simeq 76\ (\pm 3)_{\text{ext.}}\ \text{MeV}$$

to be compared to the $SU(2)$ limit value : $66 < F_\pi < 84$ MeV

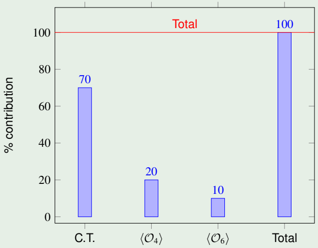
L_{10}

$$L_{10} = \frac{N_c(8010\zeta(3) + 495 - 585\pi^2 - 46\pi^4)}{8640\pi^2} + \frac{-72\zeta(3) - 12 + 11\pi^2}{64}\frac{\langle\mathcal{O}_4\rangle}{\kappa^4} + \frac{5[5216\zeta(3) + 67 - 33\pi^2]}{1792}\frac{\langle\mathcal{O}_6\rangle_V}{\kappa^6}$$

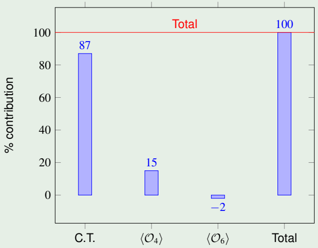
with $N_c = 3$, $\kappa = 0.6$ GeV, $\langle\mathcal{O}_4\rangle = (-0.635 \pm 0.04)\cdot 10^{-3}$ GeV^4, $\langle\mathcal{O}_6\rangle_V = (14 \pm 3)\cdot 10^{-4}$ GeV^6

$$10^3\, L_{10} = -4.6 - 0.8 + 0.1 \simeq -5.3\,(\pm 1)_{\text{ext.}}$$

to be compared to : $L_{10} = -5.3 \pm 0.13 \cdot\ 10^{-3}$



F_π



L_{10}

Conclusion

- We have shown that it is possible to recover the right OPE and the Regge behaviour of the spectrum for the vectorial field from a modified dilaton profile in the SW model
- We have shown that by the use of an extra scalar field it is possible to have axial field in SW model with the right OPE and the axial spectrum.
- We have shown a χSBM quite efficient: Taking $\langle \mathcal{O}_4 \rangle = \langle \mathcal{O}_6 \rangle = 0$, one has

$$N_C = 3.5$$

- What about the intermediate region? This method provides an unique way to do the analytic continuation from the OPE to the χPT by predicting a value for F_π and L_{10} through Π_{LR}.
- More generally we have a pion wave function (not mention here) that allows to extract prediction for the dominant L_i from the χPT Lagrangian.