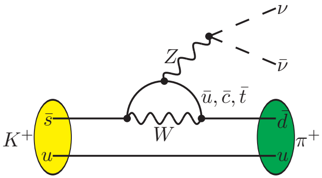
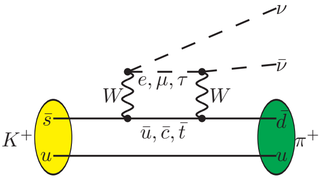
Exploratory lattice QCD study of $K^+ \to \pi^+\nu\bar{\nu}$





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CD15@Pisa, 07/02/2015

Collaborators

- on behalf of RBC-UKQCD collaboration

- people involved in this project

UKQCD	**RBC**
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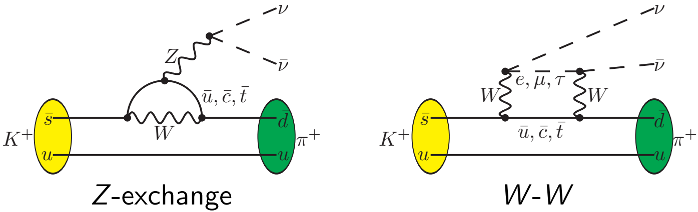
$K \to \pi\nu\bar{\nu}$: FCNC process

$K \to \pi\nu\bar{\nu}$ decays are flavor changing neutral current processes

in SM such decay can happen only through W-Z or W-W exchange

therefore, it is a second-order weak interaction



SM effects highly suppressed in the second order → ideal probes for NP

Past experimental searches

$K^+ \to \pi^+ \nu\bar{\nu}$, from 1969 to 2008

1969	first upper limit on $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$ set	PRL 23('69) 326
1997	1^{st} event observed by E787 at BNL	PRL 79('97) 2204
2000	no new events, but more constraint on Br.	PRL 84('00) 3768
2002	2^{nd} event by E787	PRL 88('02) 041803
2002	3^{rd} event by E787	PLB 537('02) 211
2004	4^{th} event by E949 at BNL	PRL 93('04) 031801
2008	another 3 events by E949	PRL 101('08) 191802

a series of PRL publications and 7 events of $K^+ \to \pi^+ \nu\bar{\nu}$ observed

experimental measurement of branching ratio

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

search for 1 candidate among every 6 billion events of K^+ decays, difficult!

New experiments (plenary talk by A. Ceccucci)

new generation of experiment: NA62 at CERN aims at

- observation of $O(100)$ events in two years
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$

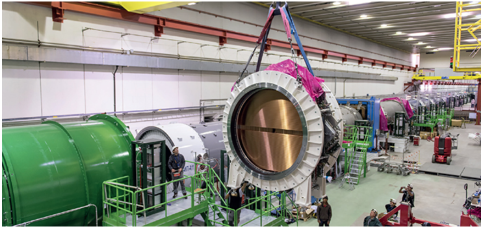


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

$K_L \to \pi^0 \nu\bar{\nu}$

- even more challenging since π^0 decays quickly to two photons
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe K_L decays

Recent SM prediction

$K \to \pi\nu\bar{\nu}$ known to be top-quark dominated $\Rightarrow$ theoretically very clean

- dominated SM uncertainty from CKM matrix V_{td}, $V_{ts} \Leftarrow V_{ub}$, V_{cb}, γ
- two ways to determine V_{ub}, V_{cb}, γ
  [Buras, Buttazzo, Girrbach-Noe, Knegjens, arXiv:1503.02693]
  - using tree-level measurement of $b \to c$, $b \to u$ transitions

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = (8.4 \pm 1.0) \times 10^{-11}$$

  - using loop-level observables, ε_K, $\Delta M_{d,s}$, $S_{\psi K_S}$

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = (9.11 \pm 0.72) \times 10^{-11}$$

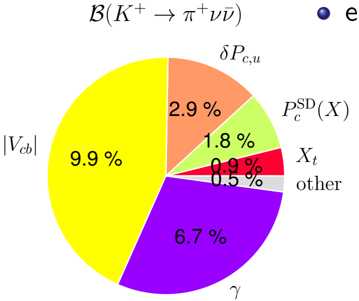
recall past experimental measurement

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10} \qquad > 60\% \text{ err}$$

$\mathrm{Br}_{\mathrm{exp}}$ is 2 times larger than $\mathrm{Br}_{\mathrm{SM}}$, but still consistent with $> 60\%$ error

SM error budget for $K^+ \to \pi^+\nu\bar{\nu}$

new experiment confronts SM soon ⇒ can we do better?



- error budget from Buras et. al.
  - $|V_{cb}|$, γ: CKM inputs for $|V_{td}|$, $|V_{ts}|$
  - $\delta P_{c,u}$: LD contribution
  - P_c: c-quark contribution (SD part)
  - X_t: t-quark contribution
  - other: remaining SM parameters

- most important thing is to reduce the error from CKM inputs
- long-distance contribution yields a sub-dominat uncertainty
- phenomenological ansatz involving χPT and OPE [hep-ph/0503107] yields $\delta P_{c,u} = 0.04 \pm 0.02$ ⇒ branching ratio enhanced by 6%
  - 50% err in $\delta P_{c,u}$ is a guess rather than a controlled error
  - $\delta P_{c,u}$ may be much larger or even smaller

can lattice QCD provide a result with controlled uncertainty?

Lattice methodology

Three-step LQCD development strategy for $K^+ \to \pi^+ \nu\bar{\nu}$

1) this talk, using domain wall fermion configs. generated by RBC-UKQCD

- $16^3 \times 32$, $m_\pi = 420$ MeV, $m_c = 860$ MeV, $a^{-1} = 1.73$ GeV
- set up the calculation at unphysical kinematics

2) USQCD proposal this year (awarded with 27 million BG/Q core hours)

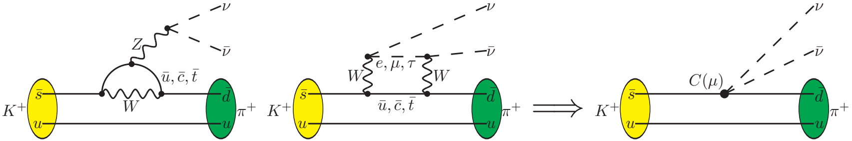
- $32^3 \times 64$, $m_\pi = 170$ MeV, $m_c = 750$ MeV, $a^{-1} = 1.37$ GeV
- further control the unphysical effects from pion mass
- $a^{-1} = 1.37$ GeV $\Rightarrow$ expect large lattice artifacts from m_c

3) for the future

- $80^2 \times 96 \times 192$, $m_\pi = 140$ MeV, $m_c = 1.3$ GeV, $a^{-1} = 3$ GeV
- control all systematic effect, produce $\delta P_{c,u}$ with 20% err $\Rightarrow$ 1% in Br

Perturbation theory vs lattice QCD

- perturbation theory (high energy scale above charm quark mass)
  - effective Hamiltonian described by a dim-6 operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$



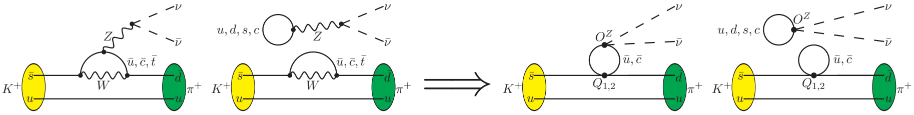
$$\mathcal{H}_{eff}^{(6)} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} (\lambda_c X_c^\ell + \lambda_t X_t)(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$$

  - X_t and X_c^ℓ are Inami-Lim loop function for top and charm contribution
- lattice QCD (low energy scale at or below charm quark mass)
  - non-local effects given by two local operators O_1 and O_2

$$\mathcal{H}_{eff}^{BL} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \lambda_c \left(\frac{\pi^2}{M_W^2} \int d^4x\, O_1(x)\, O_2(0) \right)_{u-c}$$

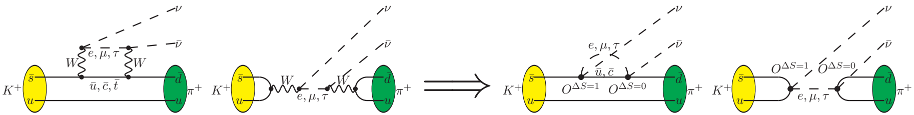
Bilocal structure [Isidori, Mescia, Smith, hep-ph/0503107]

Z-exchange diagrams:



$$\int d^4x \, \langle \pi^+ \nu \bar{\nu} | T\{Q_{1,2}(x) O^Z(0)\} | K^+ \rangle$$

W-W diagrams:



$$\int d^4x \, \langle \pi^+ \nu \bar{\nu} | T\{O^{\Delta S=1}(x) O^{\Delta S=0}(0)\} | K^+ \rangle$$

Bilocal LD contribution vs local SD contribution

- compare LD and SD contribution

$$\langle\pi\nu\bar{\nu}|\left(\frac{\pi^2}{M_W^2}\int d^4x\, O_1(x)\, O_2(0)\right)_{u-c}|K\rangle \qquad X_c^\ell\langle\pi\nu\bar{\nu}|(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}|K\rangle$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

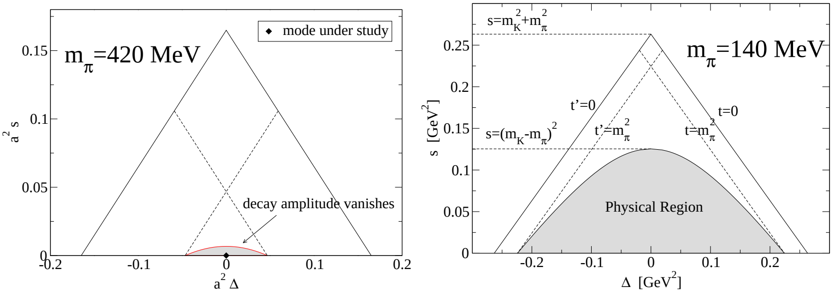
$$X_{BL} \qquad\qquad\qquad\qquad X_c^\ell$$

- we define X_{BL} as a ratio so that it can be compared to X_c^ℓ directly

$$X_{BL} = \frac{\langle\pi\nu\bar{\nu}|\left(\frac{\pi^2}{M_W^2}\int d^4x\, O_1(x)\, O_2(0)\right)_{u-c}|K\rangle}{\langle\pi\nu\bar{\nu}|(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}|K\rangle} = \frac{\frac{\pi^2}{M_W^2}F_{BL}(p_K,p_\pi,p_\nu)}{F_{SD}(p_K,p_\pi)}$$

  - Z-exchange diag: ν $\bar{\nu}$ are localized $\Rightarrow$ X_{BL} depends on p_K, p_π only
  - W-W diag: non-local neutrino structure, X_{BL} also rely on p_ν, $p_{\bar{\nu}}$

Dalitz plot

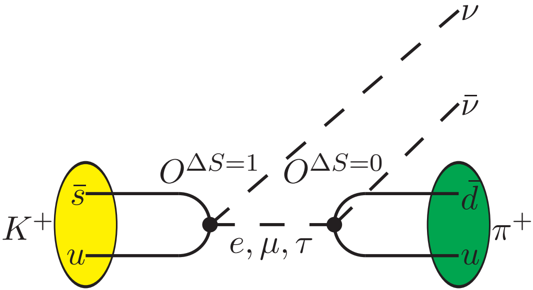


- $S = (p_\nu + p_{\bar\nu})^2, \quad \Delta = (p_K - p_{\bar\nu})^2 - (p_K - p_\nu)^2$
- allowed momentum region highly suppressed at $m_\pi = 420$ MeV
- on-shell massless neutrinos $\rightarrow$ modulus of decay amplitude vanishes at the edge of the Dalitz plot
- away from edge $(\Delta, s) = (0, 0) \quad \Rightarrow \quad \vec{p}_\nu = \vec{p}_{\bar\nu},\ \vec{p}_\pi = -\vec{p}_\nu - \vec{p}_{\bar\nu}$

Preliminary lattice results

(use W-W diagram as example)

Type 1 diagram, gluon exchanges not drawn



Exponential growing contamination

given a non-local matrix element in Minkowski space

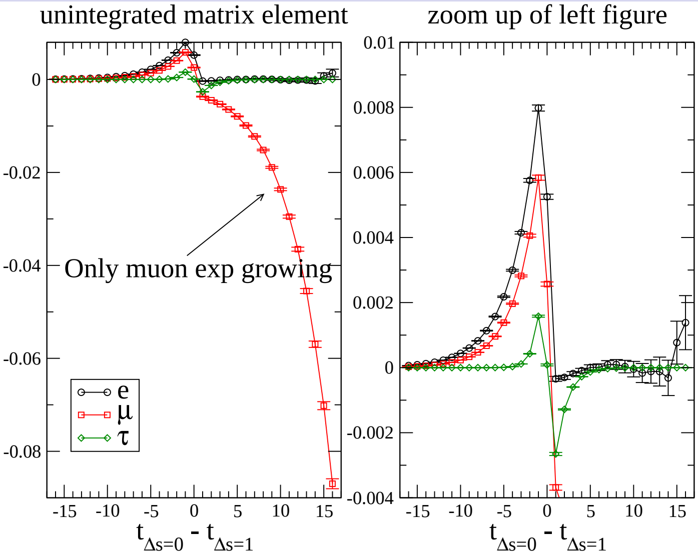
$$\mathcal{T}^M = i\int dt\,\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f+i\epsilon} - \sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n+i\epsilon}$$

in Euclidean space

$$\mathcal{T}^E = \sum_{t=-T_a}^{T_b}\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(1-e^{(E_f-E_{n_s})T_b}\right)$$
$$-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(1-e^{(E_K-E_n)T_a}\right)$$

if $E_n < E_K$, remove exp growing contamination, $\mathcal{T}^E \Rightarrow \mathcal{T}^M$

Unintegrated matrix element



- e mode: no exp growing contamination due to helicity suppression
- τ mode: no exp growing contamination since τ is heavy

F_{BL}^{ℓ} for type 1 diagram

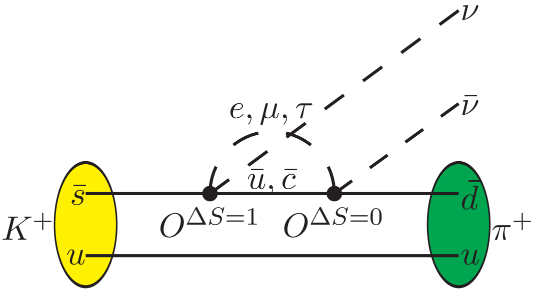
F_{BL}^{ℓ}	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

- vacuum saturation approximation assumes only single-lepton contribution in the intermediate state

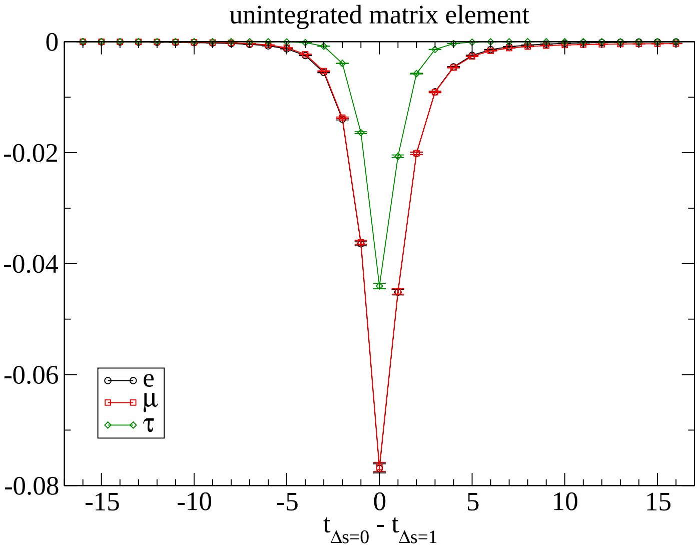
$$f_K f_\pi \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) \frac{\not{q}}{q^2 - m_\ell^2} \not{p}_\pi (1-\gamma_5) v(p_{\bar{\nu}})$$
$$= f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2} \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) v(p_{\bar{\nu}})$$

  with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$
- in the above table, model results are given by $Z_A^{-2} f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2}$
- non-local form factor F_{BL}^{ℓ} can be converted to X_{BL}^{ℓ}

Type 2 diagram, gluon exchanges not drawn



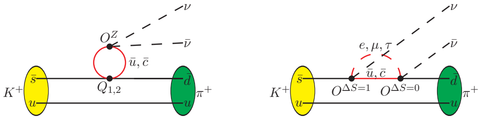
Type 2 diagram



intermediate state is given by $\ell + \pi^0$, since pion is heavy, we don't observe significant exponential growing effects

Short distance divergence

- by dimensional counting the loop integrals are quadratically divergent



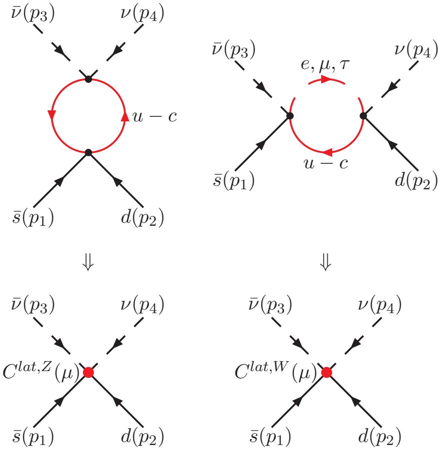
- GIM mechanism reduces the divergence to logarithmic
- in the physical world, the SD divergence is cut off by physical M_W
- in the lattice calculation it is cut off by an energy scale $\Lambda_{lat} \sim \frac{1}{a}$
- correction can be made through $A - A_{SD}^{lat} + A_{SD}^{cont} =$

$$\int d^4x \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu)O_{SD}|K\rangle + \langle f|C^{cont}(\mu)O_{SD}|K\rangle$$

- $C^{lat}(\mu)$ is determined non-perturbatively using RI/SMOM approach
- $C^{cont}(\mu)$ can be calculated perturbatively, currently in LO

Rome-Southampton method (RI/SMOM approach)

- evaluate off-shell Green's function with $p_i^2 \gg \Lambda_{QCD}^2$
- energy scale of internal momentum, μ^2, is forced to be larger than p_i^2
- at high energy scale μ, mainly SD contribution to off-shell Green's function
- correctly represented by a SD operator multiplying with Wilson coefficient $C^{lat}(\mu)$



Pauli-Villars method

- Rome-Southampton

$$X_{BL}^{\ell} - Z_{V,A}\frac{\pi^2}{M_W^2}C^{lat}(\mu^2) + X_c^{\ell}(\mu^2)$$

  - μ^2 is the scale of loop momenta
  - $C^{lat}(\mu^2)$ contains the lattice cutoff effects, but replaced by $X_c^{\ell}(\mu^2)$

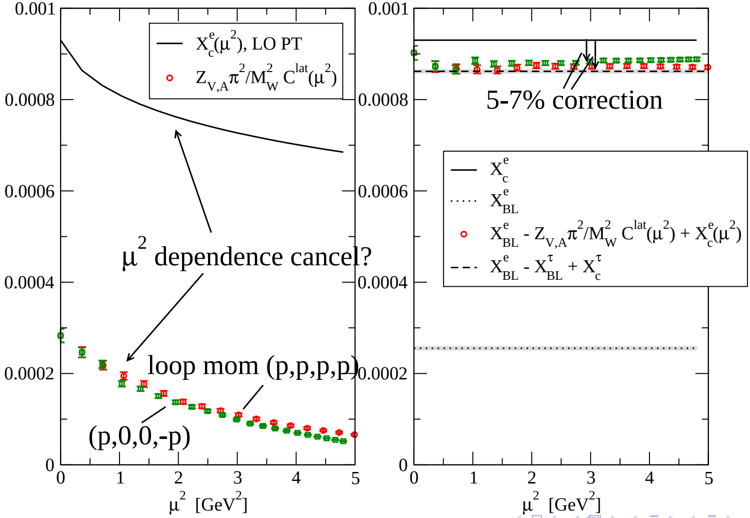
- Pauli-Villars

$$X_{BL}^{\ell} - X_{BL}^{M} + X_c^{M}$$

  - we use a heavy lepton M as a regulator
  - X_{BL}^{M} contains the lattice cutoff effects, but replaced by correct SD X_c^{M}
  - we calculated X_{BL}^{ℓ} with $\ell = e, \mu, \tau$, thus we use $M = \tau$ as a regulator

Short-distance matching

$X_c^e(\mu^2)$: LO perturbative result of loop function with W prop insertion
$C^{lat}(\mu^2)$: non-perturbative lattice matching coeff at scale μ^2



Outlook

- we have established method and shown that the lattice QCD calculation of the LD contribution to rare kaon decay is feasible.
- the new allocation of USQCD computer time starts from July, so we will look at $m_\pi = 170$ MeV very soon
- we also want to include the physical charm quark mass as soon as $a^{-1} = 3$ GeV ensembles available
- considering the huge experimental efforts to search for $K^+ \to \pi^+\nu\bar{\nu}$, it is important to determine the LD contribution to $K^+ \to \pi^+\nu\bar{\nu}$ with a controlled uncertainty

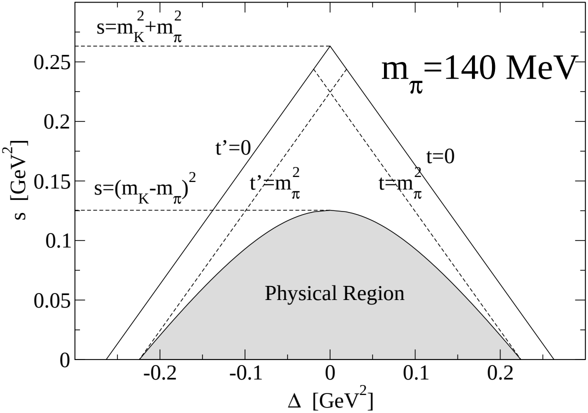
Backup slides

Dalitz plot

three Lorentz invariants s, t, t'

$$s = (p_K - p_\pi)^2 = (p_\nu + p_{\bar\nu})^2, \quad t = (p_K - p_\nu)^2 = (p_{\bar\nu} + p_\pi)^2$$
$$t' = (p_K - p_{\bar\nu})^2 = (p_\nu + p_\pi)^2, \quad s + t + t' = m_K^2 + m_\pi^2$$

two independent variables: s and $\Delta = t' - t$



Infinite volume vs finite volume

- above two-pion threshold, $\sum_n$ and $\sum_{n_s}$ shall be replaced by $\displaystyle\sum\!\!\!\!\!\!\int_n$ and $\displaystyle\sum\!\!\!\!\!\!\int_{n_s}$
- for infinite volume, integral is well defined using principal value

$$\mathcal{I}^\infty = \mathcal{P}\sum\!\!\!\!\!\!\!\int_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{\infty\infty}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

- for finite volume, energy states are always discrete, we still have

$$\mathcal{I}^L = \sum_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{LL}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

- finite-volume correction $\mathcal{I}^\infty = \mathcal{I}^L - \delta\mathcal{I}$

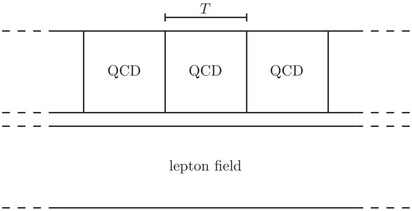
$$\delta\mathcal{I} = \cot(\phi(E)+\delta(E))(\phi'(E)+\delta'(E))\langle f|O^{\Delta S=0}|\pi\pi, E\rangle^{LL}\langle\pi\pi, E|O^{\Delta S=1}|K\rangle\Big|_{E=m_K}$$

  - $\phi(E)$: a known function depending on L; $\delta(E)$: $\pi\pi$ scattering phase
  - $\cot(\phi(E)+\delta(E))$ is singular at $E = E_n$; it cancels the singularity of $\mathcal{I}^L$
- for a complete derivation of $\delta\mathcal{I}$, see [N. Christ, XF, G. Martinelli, C. Sachrajda, arXiv:1504.01170]

Evaluation of non-local matrix element

$$\int dt \, \langle \pi^+ \nu \bar{\nu} | T\{ O^{\Delta S=1}(t) O^{\Delta S=0}(0) \} | K^+ \rangle$$

- construct 4-point correlator $\langle \phi_\pi(t_\pi) O^{\Delta S=1}(t_1) O^{\Delta S=0}(t_0) \phi_K^\dagger(t_K) \rangle$
- perform time translation average → statistical error reduced by $\sqrt{T}$
  - propagators generated on all time slices, quite a lot of cost
  - use low-mode deflation w. 100 low-lying eigenvectors to accelerate CG
  - time required to generate light quark propagators is reduced to 10%
- use overlap fermion for lepton propagator
  - time extent for lepton is infinite



Extract the form factor

$$\mathcal{T} = \int dt \, \langle \pi^+ \nu \bar{\nu} | T\{ O^{\Delta S=1}(t) O^{\Delta S=0}(0) \} | K^+ \rangle$$

when operators $O^{\Delta S=1}$, $O^{\Delta S=0}$ act on $\langle \nu \bar{\nu} |$, it produces $\bar{u}(p_\nu)$, $v(p_{\bar{\nu}})$

$$\mathcal{T} = \bar{u}(p_\nu) \left(\int dt \, \langle \pi^+ | T\{ O^{\Delta S=1}(t) O^{\Delta S=0}(0) \} | K^+ \rangle \right) v(p_{\bar{\nu}})$$

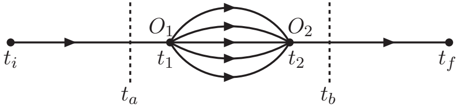
due to the chiral property of $\bar{u}(p_\nu)$ and $v(p_{\bar{\nu}})$, $\mathcal{T}$ can be written as

$$\mathcal{T} = F^\ell(p_K, p_\nu, p_{\bar{\nu}}) \bar{u}(p_\nu) \not{p}_K (1 - \gamma_5) v(p_{\bar{\nu}})$$

we therefore extract the form factor $F^\ell(p_K, p_\nu, p_{\bar{\nu}})$, $\ell = e, \mu, \tau$

final result does not have explicit dependence on spinors

Double integration



perform the double integration to gain a better precision

$$\sum_{t_1=t_a}^{t_b}\sum_{t_2=t_a}^{t_b}\langle f|T[O^{\Delta S=1}(t_2)O^{\Delta S=0}(t_1)]|K\rangle\, e^{m_K t_1} e^{-m_f t_1}$$

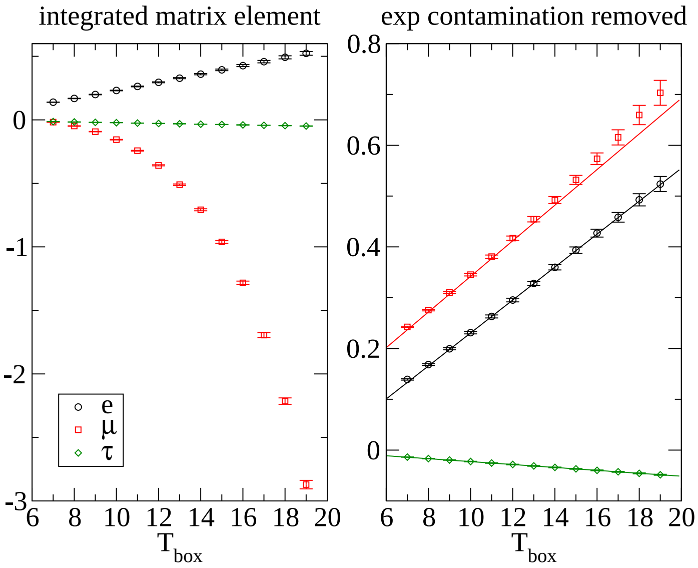
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(T_{\text{box}} - \frac{1-e^{(E_f-E_{n_s})T_{\text{box}}}}{E_{n_s}-E_f}\right)$$

$$-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(T_{\text{box}} + \frac{1-e^{(E_K-E_n)T_{\text{box}}}}{E_K-E_n}\right)$$

here $T_{\text{box}} = t_b - t_a + 1$ is defined as size of the integral window

remove the exponential growing contamination, and fit with $a + bT_{box}$, the slope b is what we want

Integrated matrix element



right figure: the slope of the curve gives $F^{\ell}(p_K, p_\nu, p_{\bar{\nu}})$

Preliminary results for Z-exchange diagrams

Evaluation of non-local matrix element

$$T_\mu^Z = \int dt\, \langle \pi^+ | T\{Q_{1,2}(t) J_\mu^Z(0)\} | K^+ \rangle$$

- Z-exchange diagrams do not require on-shell neutrinos
  - we use $\vec{p}_K = \vec{p}_\pi = 0, \quad J_\mu^Z,\ \mu = t$
- hadronic current J_μ^Z has vector and axial vector component
  - for the vector current, according to Ward identity (WI), we have

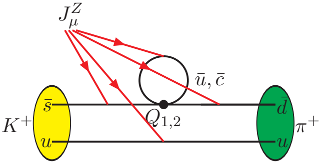
$$T_\mu^{Z,V} = F^{Z,V}(q^2)\left(q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2)q_\mu\right), \quad q = p_K - p_\pi$$

  - with $\vec{p}_K = \vec{p}_\pi = 0 \quad \Rightarrow \quad q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2)q_\mu = 0$
  - WI suggests $T_\mu^{Z,V} = 0$, this is confirmed by our numerical calculation
- in the following, I will present the results for axial vector current

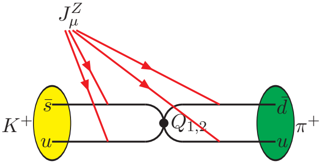
Summary of Z-exchange diagrams

four classes: Type 1 (Q_1), Type 1 (Q_2), Type 2 (Q_1), Type 2 (Q_2)

- connected diagrams, J_μ^Z can be inserted into all the possible quark line

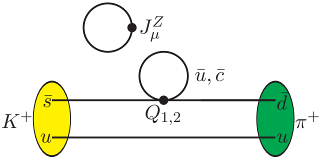


Type 1 diagram

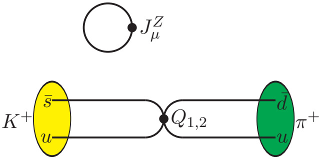


Type 2 diagram

- disconnected diagrams (usually excluded in lattice calculation since they are noisy and difficult to calculate)

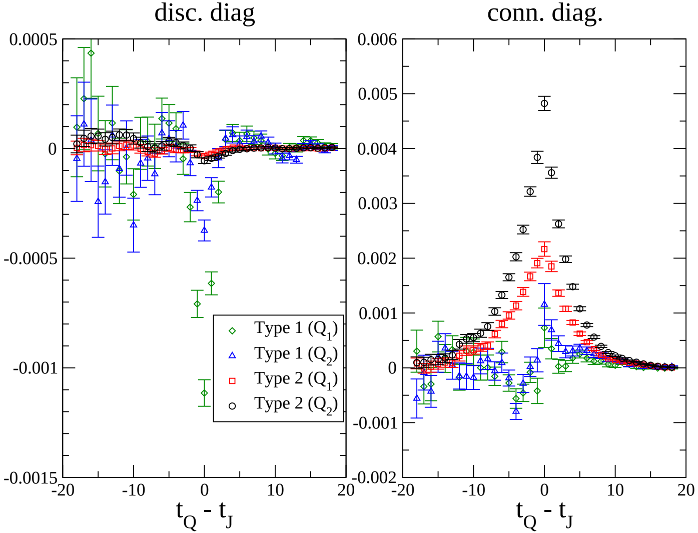


Type 1 diagram



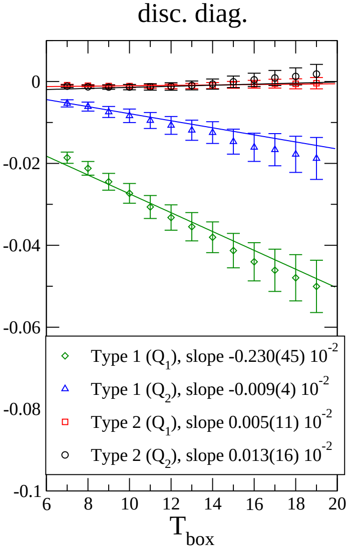
Type 2 diagram

Disc. diag. vs Conn. diag.



- although noisy, clear signal from disc. diagram
- scale of y-axis in disc. plot is 4 times smaller than that in conn. plot

Integrated matrix element



disc. diag. is relatively noisy, but its contribution is small. Adding the disc. part does not affect the conn. part significantly