η-η' mixing in Large-N_c ChPT: discussion, phenomenology, and prospects

Pere Masjuan
Johannes Gutenberg-Universität Mainz
(masjuan@kph.uni-mainz.de)

(preliminary) Work done in collaboration with
P. Bickert and S. Scherer (Mainz)

I profit as well from arXiv:1504.07742[hep-ph]
done with R. Escribano (UAB) and P. Sanchez Puertas (Mainz)

THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter

PRISMA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

CD2015, Pisa, June 30th

Outline

- Motivation and Introduction
- η-η' mixing in Large-N_c ChPT
    - strategy and preliminary results
- Conclusions and Prospects

The η-η' system

η-η' and the mixing

- Ideal for studying:
  - Symmetries
  - Symmetry breaking in QCD

$\eta, \eta' \to 3\pi$ ⟶ Quark masses

$\eta, \eta' \to 3\pi \;\; \eta' \to \pi\pi\eta$ ⟶ Chiral invariant EFT

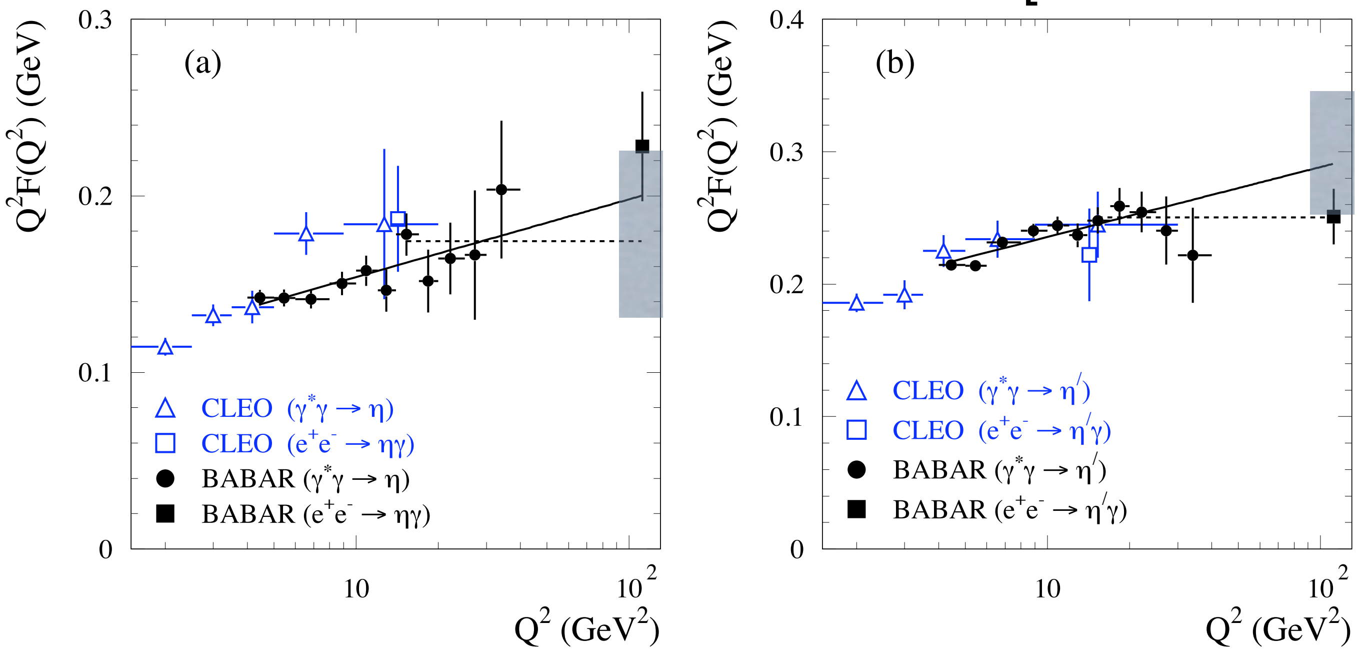
$\eta, \eta' \to 2\gamma \text{ or } \pi\pi\gamma$ ⟶ The Chiral Anomaly

$\eta, \eta' \to \gamma e^+ e^-$ ⟶ E.M. Form Factors

η-η' and the mixing

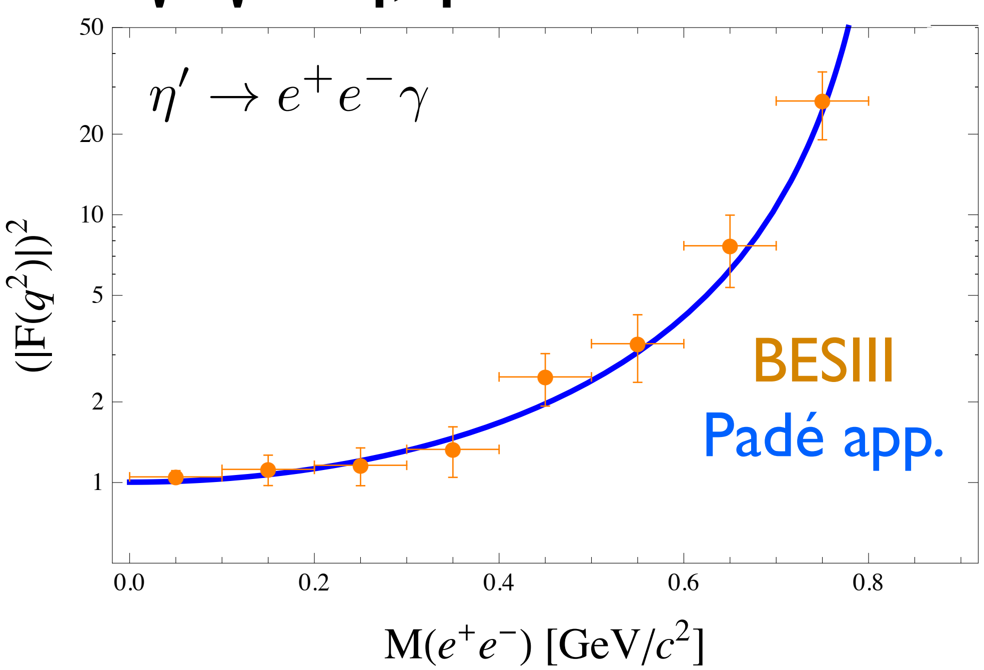
- η-η' mixing probes
  - strange quark content of light pseudoscalar
  - gluon dynamics of QCD
- Experimentally related to
  - η, η' decays
  - $\gamma^*\gamma \to \eta,\eta'$ Transition Form Factors

[P. del Amo Sanchez et. al. (BaBar Coll) '11]



η-η' and the mixing

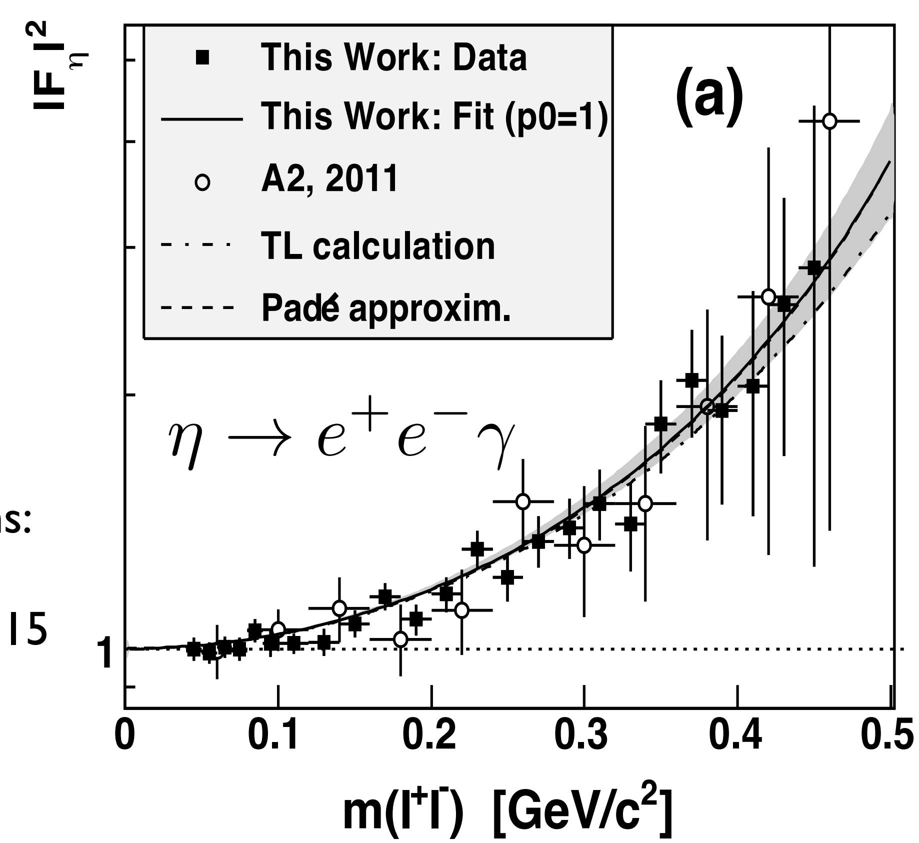
- η-η' mixing probes
  - strange quark content of light pseudoscalar
  - gluon dynamics of QCD
- Experimentally related to
  - η, η' decays
  - γ*γ→η,η' Transition Form Factors



[BESIII Coll. arXiv:1504.06016]

Padé app predictions: Escribano, PM, Sanchez-Puertas '13,'15

[A2 Coll. PRC89 2014]



η-η' and the mixing

- Intense experimental program on η' physics:
  - At BESIII (remember the talk by S. Fang):
    - $\eta' \to \eta\pi\pi \quad \eta' \to 3\pi \quad \eta' \to 4\pi \quad \eta' \to \gamma\gamma\pi^0 \quad \eta' \to \gamma e^+ e^-$
  - At MAMI
    - Goal 1.5×10^4 h^{-1} η': $\eta' \to \eta\pi\pi \quad \eta' \to \gamma\gamma\pi^0 \quad \eta' \to \gamma e^+ e^-$

- From EFTs point of view, the mixing is always there: we need a consistent approach to the mixing as well (intermediate result in a broader context of our Collaborative Research Center CRC 1044 in Mainz)

- In this talk: only small portion (interesting by itself) related to the mixing

η-η' and the mixing

- A consistent description of the mixing:
  - How?
    - Use the Effective Field Theory of QCD at low energies (Chiral Perturbation Theory) [Weinberg '79, Gasser and Leutwyler '84,'85]
    - adding consistently the η'
  - Construct the most general Lagrangian compatible with all symmetries and establish a power counting to organize the infinite number of terms
    - If $N_c \to \infty$, U(1)-anomaly is suppressed and the singlet axial current is conserved: a ninth Goldstone boson appears if $m_{uds}=0$ as well (η')
    - Explicit symmetry breaking ($m_{uds} \neq 0$): massive GB + η-η' mixing

η-η' and the mixing

- Power counting in Large-Nc ChPT: expansion parameter δ

$$p = \mathcal{O}(\sqrt{\delta}), \quad m = \mathcal{O}(\delta), \quad 1/N_c = \mathcal{O}(\delta)$$

  - Decay constant $F \sim \mathcal{O}(1/\sqrt{\delta})$
  - Flavor trace $\sim \mathcal{O}(\delta)$
  - k-meson vertex $\sim \mathcal{O}(\delta^{(k-2)/2})$
  - GB propagator $\sim \mathcal{O}(1/\delta)$
  - GB loop $\sim \dfrac{M^2}{F^2} \sim \mathcal{O}(\delta^2)$ ⟶ loops are at NNLO

Ready for constructing the Lagrangians

Relevant Lagrangians

Building blocks:

$$\phi(x) = \sum_{a=0}^{8} \lambda_a \phi_a(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \frac{F}{3}\psi & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 + \frac{F}{3}\psi & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \frac{F}{3}\psi \end{pmatrix}$$

$$U = exp\left(\frac{i\phi}{F}\right)$$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$$

$$r_\mu = v_\mu + a_\mu$$

$$l_\mu = v_\mu - a_\mu$$

$$\chi = 2BM \qquad M = \mathrm{diag}(\hat{m}, \hat{m}, m_s) \qquad \hat{m} = m_u = m_d$$

Relevant Lagrangians

Expansion in the parameter δ:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

At LO: $O(\delta^{(0)})$ [Gasser, Leutwyler '85]

$$\mathcal{L}^{(0)} = \frac{F^2}{4}\langle D_\mu U (D^\mu U)^\dagger \rangle + \frac{F^2}{4}\langle \chi U^\dagger + U\chi^\dagger \rangle - \frac{1}{2}\tau(\psi+\theta)^2$$

At NLO: $O(\delta^{(1)})$ [Kaiser, Leutwyler '00]

$$\mathcal{L}^{(1)} = L_5\langle D_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger \chi)\rangle + L_8\langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\rangle$$
$$+ \frac{F^2}{12}\Lambda_1 D_\mu\psi D^\mu\psi + i\frac{F^2}{12}\Lambda_2 \bar{\psi}\langle \chi^\dagger U - U^\dagger \chi\rangle \cdots$$

$$\bar{\psi} = \psi + \theta, \qquad D_\mu\psi = \partial_\mu\psi - 2\langle a_\mu\rangle$$

Relevant Lagrangians

Expansion in the parameter δ:

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

[Herrera-Siklody et al '96]
[Kaiser, Leutwyler '00]
[Jiang, Ge, Wang '14]
[Guo et al '15]

At NNLO: $O(\delta^{(2)})$

$$\mathcal{O}(1/N_c p^2) \quad \mathcal{O}(p^4) \quad \mathcal{O}(N_c p^6)$$

$$\mathcal{L}^{(2,N_c^{-1}p^2)} = -\frac{F^2}{4} v_2^{(2)} \bar{\psi}^2 \langle \chi U^\dagger + U \chi^\dagger \rangle$$

$$\mathcal{L}^{(2,p^4)} = L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle \chi^\dagger U + U^\dagger \chi \rangle + L_6 \langle \chi^\dagger U + U^\dagger \chi \rangle^2 + L_7 \langle \chi^\dagger U - U^\dagger \chi \rangle^2 + i L_{18} D_\mu \psi \langle D^\mu U^\dagger \chi - D^\mu U \chi^\dagger \rangle + i L_{25} \bar{\psi} \langle U^\dagger \chi U^\dagger \chi - \chi^\dagger U \chi^\dagger U \rangle + \ldots$$

Relevant Lagrangians

Expansion in the parameter δ:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

[Herrera-Siklody et al '96]
[Kaiser, Leutwyler '00]
[Jiang, Ge, Wang '14]
[Guo et al '15]

At NNLO: $O(\delta^{(2)})$

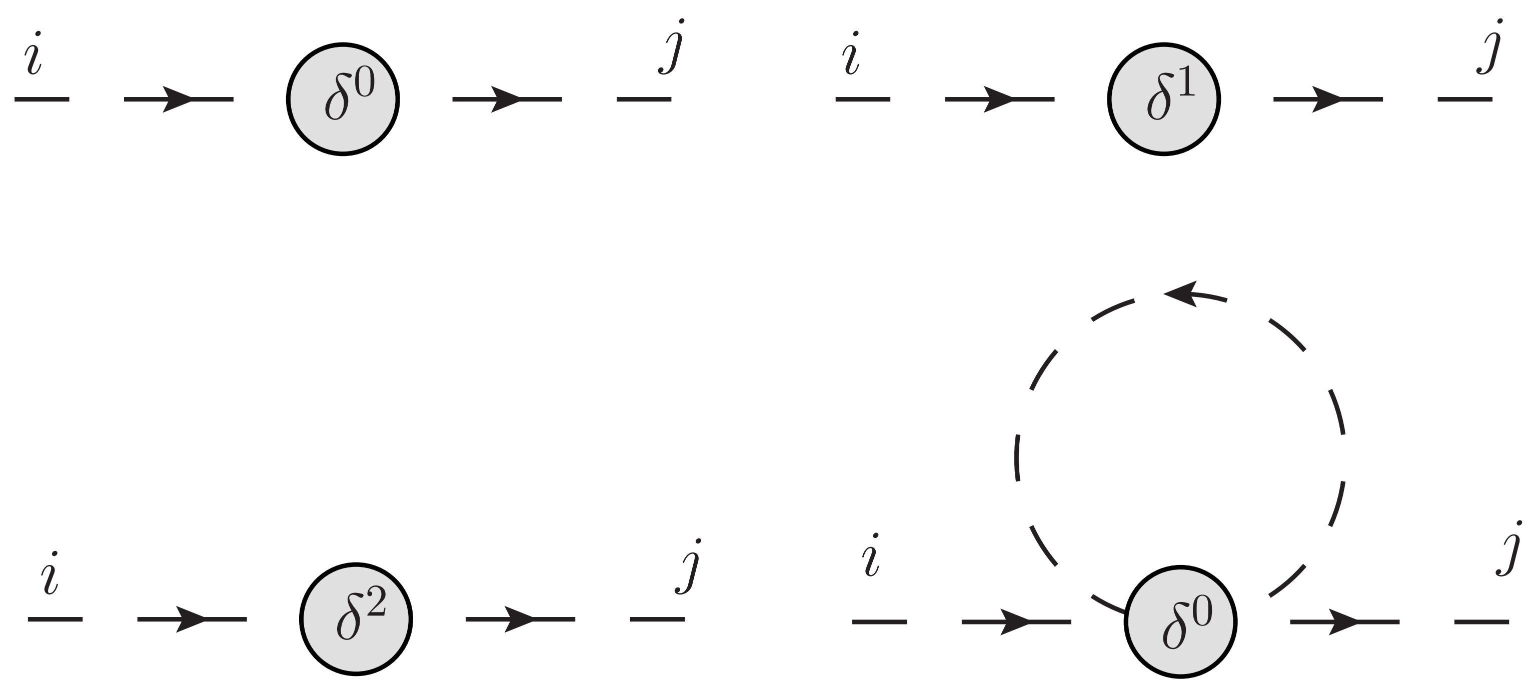
$$\mathcal{O}(1/N_c p^2) \quad \mathcal{O}(p^4) \quad \mathcal{O}(N_c p^6)$$

$$\mathcal{L}^{(2,N_c p^6)} = C_{12}\langle\chi_+ h_{\mu\nu}h^{\mu\nu}\rangle + C_{14}\langle u_\mu u^\mu \chi_+^2\rangle + C_{17}\langle\chi_+ u_\mu \chi_+ u^\mu\rangle + C_{19}\langle\chi_+^3\rangle + C_{31}\langle\chi_-^2\chi_+\rangle + \ldots$$

$$h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu,$$
$$u_\mu = i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right\},$$
$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u.$$

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X],$$
$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right\}.$$

η-η' mixing



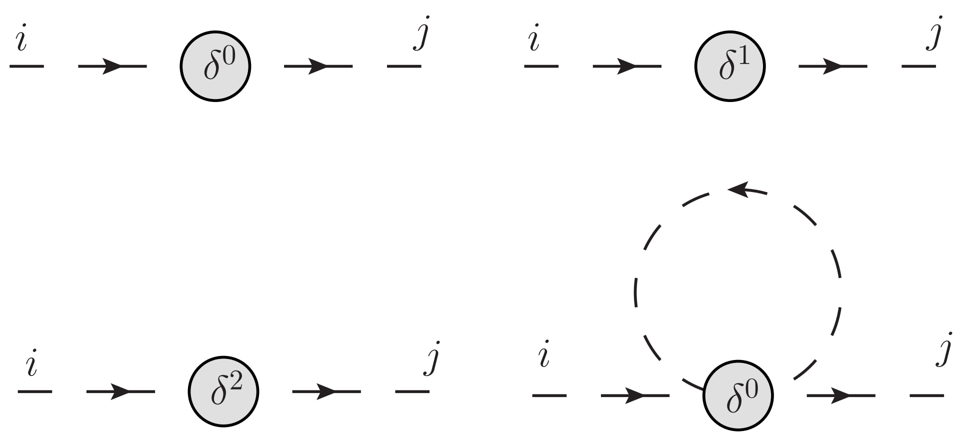
Parametrize first the mixing:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B \qquad \eta_B^T = (\eta_8, \eta_1)$$

(now, relate bare fields to physical η, η')

η-η' mixing

Mixing Lagrangian:



$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B$$

in terms of

$$\mathcal{K} = \begin{pmatrix} 1+\delta_8^{(1)}+\delta_8^{(2)} & \delta_{81}^{(1)}+\delta_{81}^{(2)} \\ \delta_{81}^{(1)}+\delta_{81}^{(2)} & 1+\delta_1^{(1)}+\delta_1^{(2)} \end{pmatrix} \quad \text{and} \quad \mathcal{M}^2 = \begin{pmatrix} M_8^2 & M_{81}^2 \\ M_{81}^2 & M_1^2 \end{pmatrix}$$

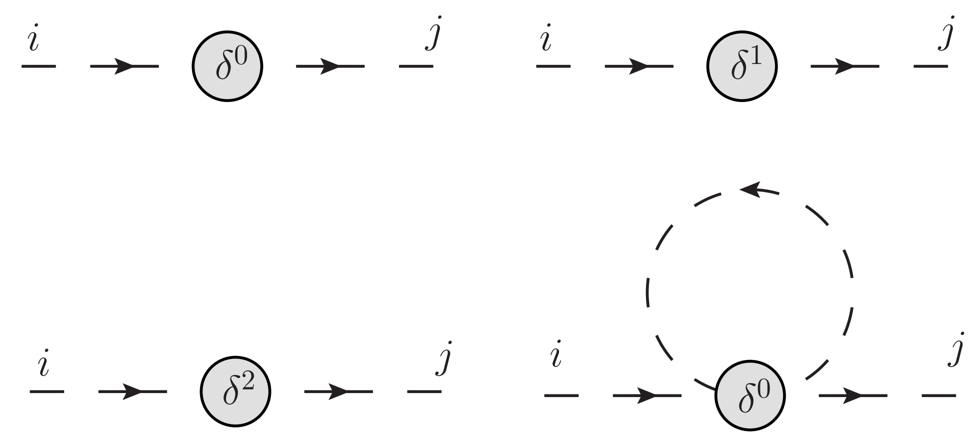
Then, diagonalize K:

$$\eta_B = {Z^{1/2}}^T \cdot \hat{\eta} \longrightarrow \hat{\mathcal{M}}^2 = Z^{1/2} \cdot \mathcal{M}^2 \cdot {Z^{1/2}}^T = \begin{pmatrix} \hat{M}_8^2 & \hat{M}_{81}^2 \\ \hat{M}_{81}^2 & \hat{M}_1^2 \end{pmatrix}$$

η-η' mixing

Mixing Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B$$



in terms of

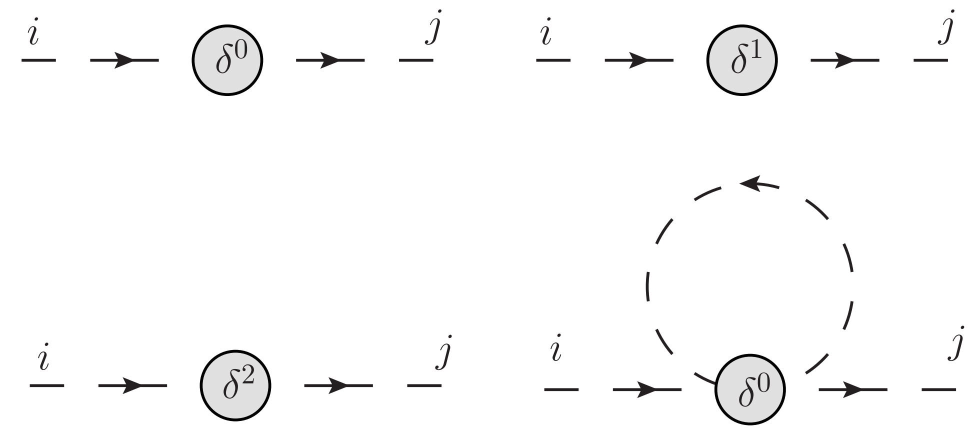
$$\mathcal{K} = \begin{pmatrix} 1+\delta_8^{(1)}+\delta_8^{(2)} & \delta_{81}^{(1)}+\delta_{81}^{(2)} \\ \delta_{81}^{(1)}+\delta_{81}^{(2)} & 1+\delta_1^{(1)}+\delta_1^{(2)} \end{pmatrix}$$ and $$\mathcal{M}^2 = \begin{pmatrix} M_8^2 & M_{81}^2 \\ M_{81}^2 & M_1^2 \end{pmatrix}$$

Then, diagonalize $\hat{\mathcal{M}}^2 \longrightarrow \hat{\mathcal{M}}^2 = R^T \cdot \mathcal{M}_D^2 \cdot R$

$$\mathcal{M}_D^2 = \begin{pmatrix} M_\eta^2 & 0 \\ 0 & M_{\eta'}^2 \end{pmatrix} \qquad R \equiv \begin{pmatrix} \cos\theta^{(2)} & -\sin\theta^{(2)} \\ \sin\theta^{(2)} & \cos\theta^{(2)} \end{pmatrix}$$

η-η’ mixing

Mixing Lagrangian:



$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B$$

From $\hat{\mathcal{M}}^2 = R^T \cdot \mathcal{M}_D^2 \cdot R$ with $\mathcal{M}_D^2 = \begin{pmatrix} M_\eta^2 & 0 \\ 0 & M_{\eta'}^2 \end{pmatrix}$

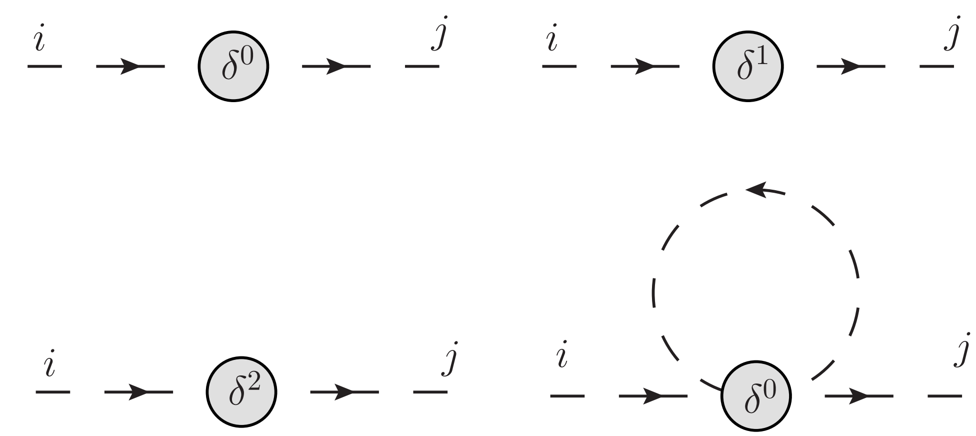
$$1)\,\hat{M}_8^2 = M_\eta^2 \cos^2\theta^{(2)} + M_{\eta'}^2 \sin^2\theta^{(2)}$$
$$2)\,\hat{M}_1^2 = M_\eta^2 \sin^2\theta^{(2)} + M_{\eta'}^2 \cos^2\theta^{(2)}$$
$$3)\,\hat{M}_{81}^2 = (M_\eta^2 - M_{\eta'}^2)\sin\theta^{(2)}\cos\theta^{(2)}$$

$$\sin 2\theta^{(2)} = \frac{2\hat{M}_{81}^2}{M_{\eta'}^2 - M_\eta^2}$$

η-η' mixing

Mixing Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta_B^T \mathcal{K} \partial^\mu \eta_B - \frac{1}{2}\eta_B^T \mathcal{M}^2 \eta_B$$



Using the Lagrangians, calculate the self-energy diagrams

$$\Sigma_{88}(p^2) = -(\delta_8^{(1)} + \delta_8^{(2)})p^2 + M_8^2,$$
$$\Sigma_{81}(p^2) = \Sigma_{18}(p^2) = -(\delta_{81}^{(1)} + \delta_{81}^{(2)})p^2 + M_{81}^2,$$
$$\Sigma_{11}(p^2) = -(\delta_1^{(1)} + \delta_1^{(2)})p^2 + M_1^2.$$

(at LO)

$$\sin 2\theta^{(0)} = -\frac{4\sqrt{2}}{3}\frac{\mathring{M}_K^2 - \mathring{M}_\pi^2}{M_{\eta'}^2 - M_\eta^2}$$

Numerical analysis

Strategy

A word on convergence

- Large-Nc ChPT (using input from Bijnens and Ecker '14):

$$F_K/F\pi \simeq \underbrace{1}_{\text{LO}} + \underbrace{0.15}_{\text{NLO}} + \underbrace{0.03}_{\text{NNLO}}\,, \qquad \text{where} \qquad \underbrace{0.03}_{\text{NNLO}} \simeq \underbrace{0.05}_{\text{loop}} - \underbrace{0.01}_{C_i} - \underbrace{0.02}_{L_i L_j}$$

$$\underbrace{0.05}_{\text{loop}} = \underbrace{0.01}_{\eta'} + \underbrace{0.04}_{\pi, K, \eta}$$

- SU(3) ChPT (from Bijnens and Ecker '14):

$$F_K/F\pi \simeq \underbrace{1}_{\text{LO}} + \underbrace{0.18}_{\text{NLO}} + \underbrace{0.02}_{\text{NNLO}}$$

Strategy

A word on convergence

- Large-Nc ChPT (using input from Bijnens and Ecker '14):

$$M_K^2/M_{K\text{phys}}^2 \simeq \overset{\text{LO}}{1.06} - \overset{\text{NLO}}{0.04} + \overset{\text{NNLO}}{0.25} \quad \text{where} \quad \overset{\text{NNLO}}{+0.25} \simeq \underbrace{0.15}_{\text{loops}} + \underbrace{0.14}_{C_i^\prime s} - \underbrace{0.03}_{L_4, L_6} - \underbrace{0.01}_{L_i L_j}$$

$$\underbrace{0.15}_{\text{loop}} = \underbrace{0.10}_{\eta'} + \underbrace{0.05}_{\pi, K, \eta}$$

- SU(3) ChPT from Bijnens and Ecker '14:

$$M_K^2/M_{K\ \text{phys}}^2 \simeq \overset{\text{LO}}{1.11} - \overset{\text{NLO}}{0.07} - \overset{\text{NNLO}}{0.04}$$

Strategy

A word on convergence

- Large-Nc ChPT (using input from Bijnens and Ecker '14):

$$M_K^2/M_{K\text{phys}}^2 \simeq \underset{\text{LO}}{1.06} - \underset{\text{NLO}}{0.04} + \underset{\text{NNLO}}{0.25} \quad \text{where} \quad \underset{\text{NNLO}}{+0.25} \simeq \underbrace{0.15}_{\text{loops}} + \underbrace{0.14}_{C_i' s} - \underbrace{0.03}_{L_4, L_6} - \underbrace{0.01}_{L_i L_j}$$

$$\underbrace{0.14}_{C_i' s} = \underbrace{+0.29}_{C_{19}} \quad \underbrace{-0.15}_{C_{12,14,17,31}}$$

- SU(3) ChPT from Bijnens and Ecker '14:

$$M_K^2/M_{K\ \text{phys}}^2 \simeq \underset{\text{LO}}{1.11} - \underset{\text{NLO}}{0.07} - \underset{\text{NNLO}}{0.04}$$

Strategy

Results: the mixing angle

General result at NNLO:

$$1)\hat{M}_8^2 = M_\eta^2 \cos^2\theta^{(2)} + M_{\eta'}^2 \sin^2\theta^{(2)}$$

$$2)\hat{M}_1^2 = M_\eta^2 \sin^2\theta^{(2)} + M_{\eta'}^2 \cos^2\theta^{(2)}$$

$$3)\hat{M}_{81}^2 = (M_\eta^2 - M_{\eta'}^2)\sin\theta^{(2)}\cos\theta^{(2)}$$

Strategy

Results: the mixing angle

General result at NNLO:

$$1)\hat{M}_8^2 = M_\eta^2 \cos^2\theta^{(2)} + M_{\eta'}^2 \sin^2\theta^{(2)}$$

$$2)\hat{M}_1^2 = M_\eta^2 \sin^2\theta^{(2)} + M_{\eta'}^2 \cos^2\theta^{(2)}$$

$$3)\hat{M}_{81}^2 = (M_\eta^2 - M_{\eta'}^2)\sin\theta^{(2)}\cos\theta^{(2)}$$

$$M_{\eta'}^2 + M_\eta^2 = \hat{M}_8^2 + \hat{M}_1^2, \quad M_{\eta'}^2 - M_\eta^2 = \sqrt{(\hat{M}_8^2 - \hat{M}_1^2)^2 + 4\hat{M}_{81}^4}$$

$$\sin 2\theta^{(2)} = \frac{2\hat{M}_{81}^2}{\mathcal{C}}$$

Strategy

Results: the mixing angle

At LO:

$$M_{\eta'}^2 + M_\eta^2 = \hat{M}_8^2 + \hat{M}_1^2, \quad M_{\eta'}^2 - M_\eta^2 = \sqrt{(\hat{M}_8^2 - \hat{M}_1^2)^2 + 4\hat{M}_{81}^4}$$

$$\sin 2\theta^{(0)} = -\frac{4\sqrt{2}}{3}\frac{\mathring{M}_K^2 - \mathring{M}_\pi^2}{M_{\eta'}^2 - M_\eta^2} \longrightarrow \theta^{(0)} = -19.6^\circ$$

Strategy

Results: the mixing angle

At LO:

$$M_{\eta'}^2 + M_\eta^2 = \hat{M}_8^2 + \hat{M}_1^2, \quad M_{\eta'}^2 - M_\eta^2 = \sqrt{(\hat{M}_8^2 - \hat{M}_1^2)^2 + 4\hat{M}_{81}^4}$$

$$\sin 2\theta^{(0)} = \frac{-4\sqrt{2}(\mathring{M}_K^2 - \mathring{M}_\pi^2)}{\sqrt{(2\,\mathring{M}_K^2 - 2\,\mathring{M}_\pi^2 - M_0^2)^2 + 32(\mathring{M}_K^2 - \mathring{M}_\pi^2)^2}} \longrightarrow \theta^{(0)} = -22.0^\circ$$

$$M_1^2 = M_0^2 + \mathring{M}_1^2 \longrightarrow$$

$$\boxed{M_0^2 = \frac{(M_{\eta'}^2 - M_\pi^2)(M_{\eta'}^2 - 2M_K^2 + M_\pi^2)}{M_{\eta'}^2 - \frac{4}{3}M_K^2 + M_\pi^2} = (0.820\ \text{GeV})^2}$$

Strategy

Results: the mixing angle

At NLO:

- We need Low-energy constants from $O(p^4)$ $L_{5,8}$ and OZI $\Lambda_{1,2}$

- Two strategies:
  - NLO I: Constrained analysis: calculate $M_\pi, M_K, M_\eta, M_0, F_K, F_\pi$
  - NLO II: Use SU(3) values from Bijnens and Ecker '14 + M_η, M_0 (after matching to U(3) and at μ=0.77 GeV)

Strategy

Results: the mixing angle

At NLO + loops:

- No new LECs ($L_{5,8}$ and $\Lambda_{1,2}$)

- Two strategies:
  - NLO I + loops: calculate $M_\pi, M_K, M_\eta, M_0, F_K, F_\pi$
  - NLO II: Use SU(3) values from Bijnens and Ecker '14 + M_η, M_0 (after matching to U(3) and at μ=0.77 GeV)

Strategy

Results: the mixing angle

At NNLO:

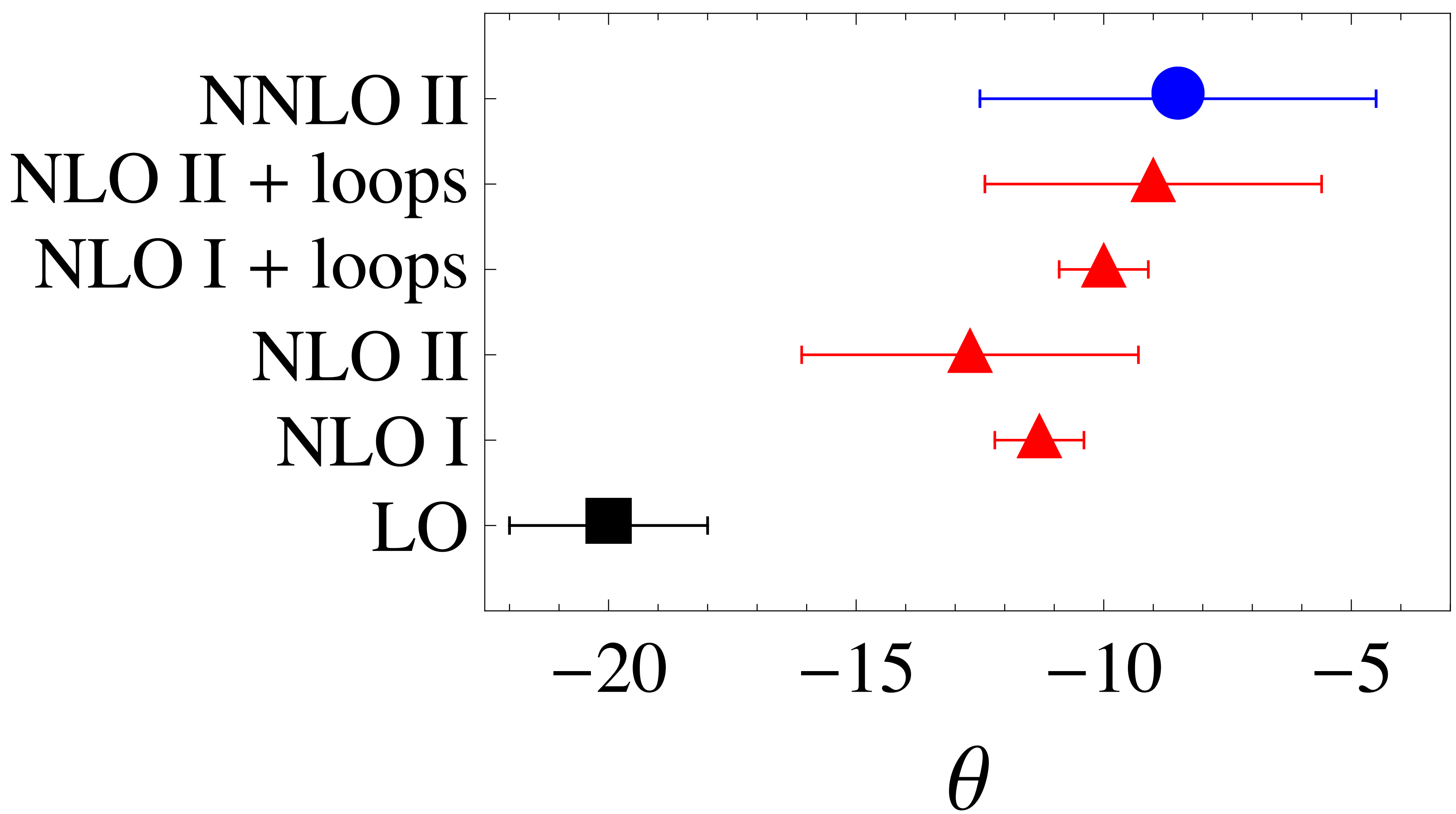
- NLO + loops + LECs $O(p^4)$ $L_{4,6,7,18,25}$ + LECs $O(p^6)$ $C_{12,14,17,19,31}$ and OZI $\Lambda_{1,2}$

- Many strategies. Here only one:
  - NNLO II: Use LECs Li and Ci at $O(p^6)$ from Bijnens and Ecker '14 + M_η, M_0
  - (lattice fits)

[Guo et al, '15]

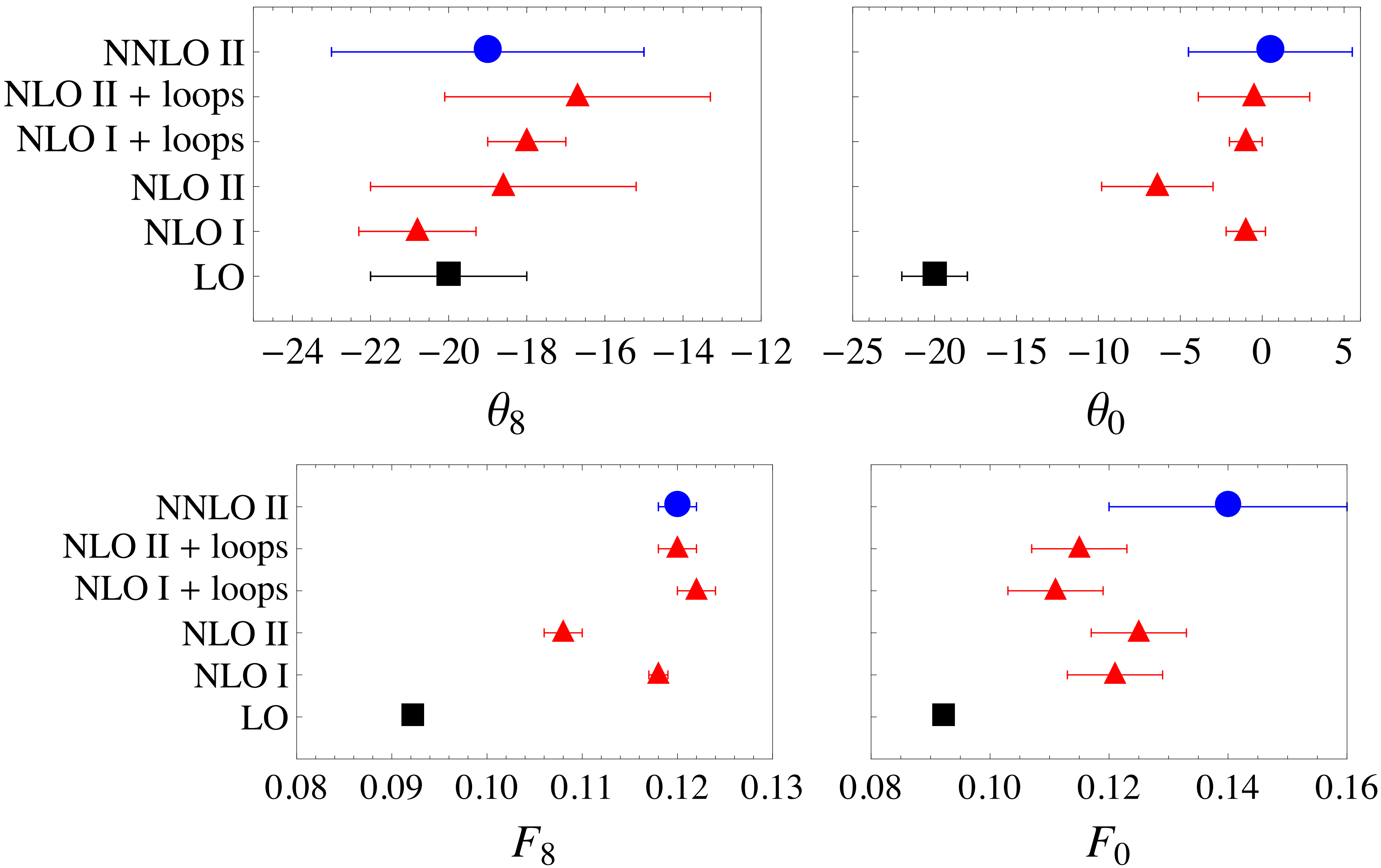
Strategy

Results: the mixing angle



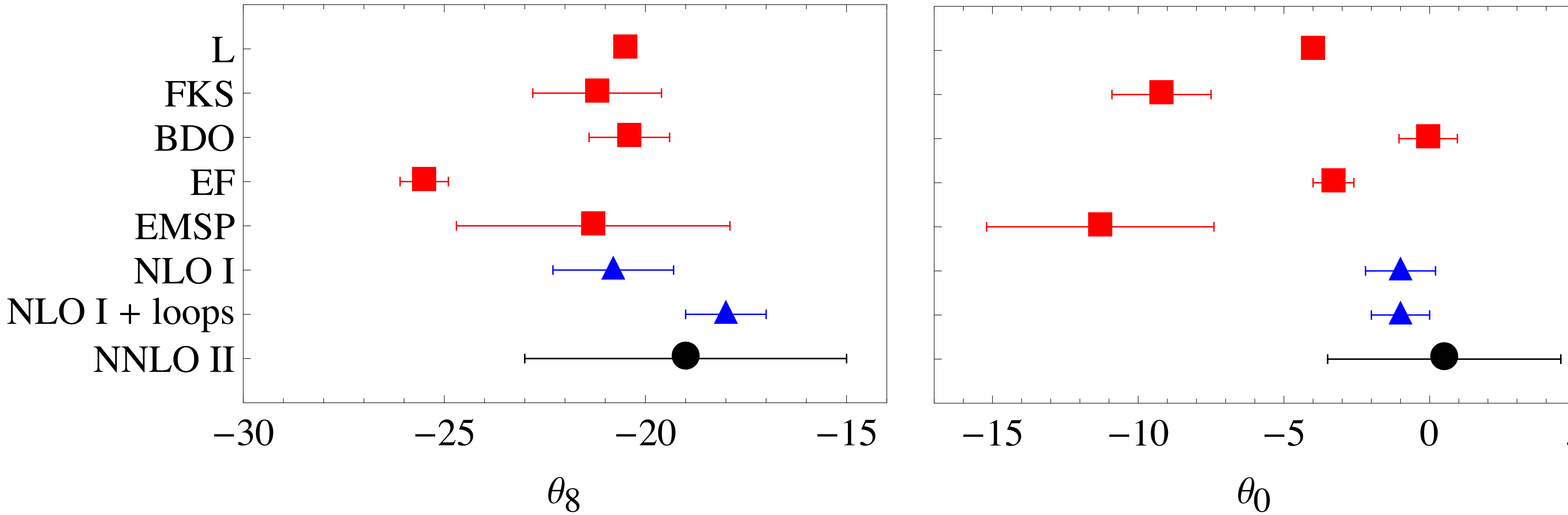
Strategy

Results: mixing in the octet-singlet basis



Strategy

comparison with phenomenological determinations

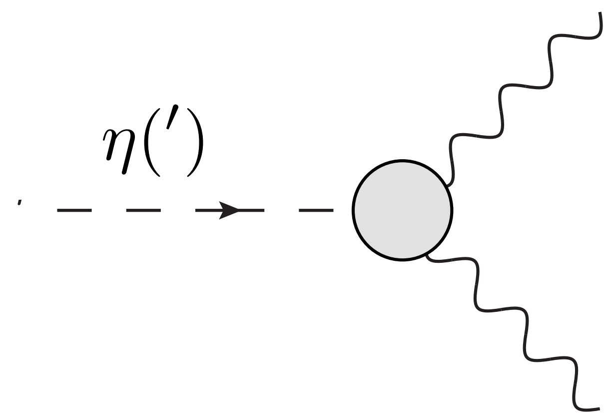


L: Leutwyler '97
FKS: Feldmann, Kroll, Stech '98
BDO: Benayoun, Del Buono, O'Connell, '00
EF: Escribano, Frere '05
EMSP: Escribano, PM, Sanchez-Puertas, '15

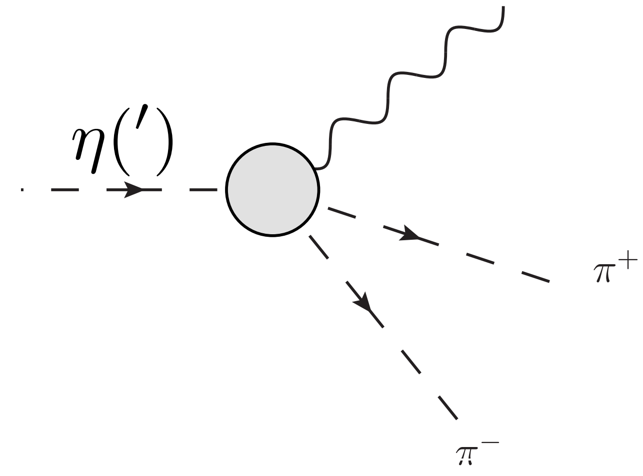
Prospects

Study of anomalous decays including mixing

$$\eta(') \to \gamma(*)\gamma(*)$$



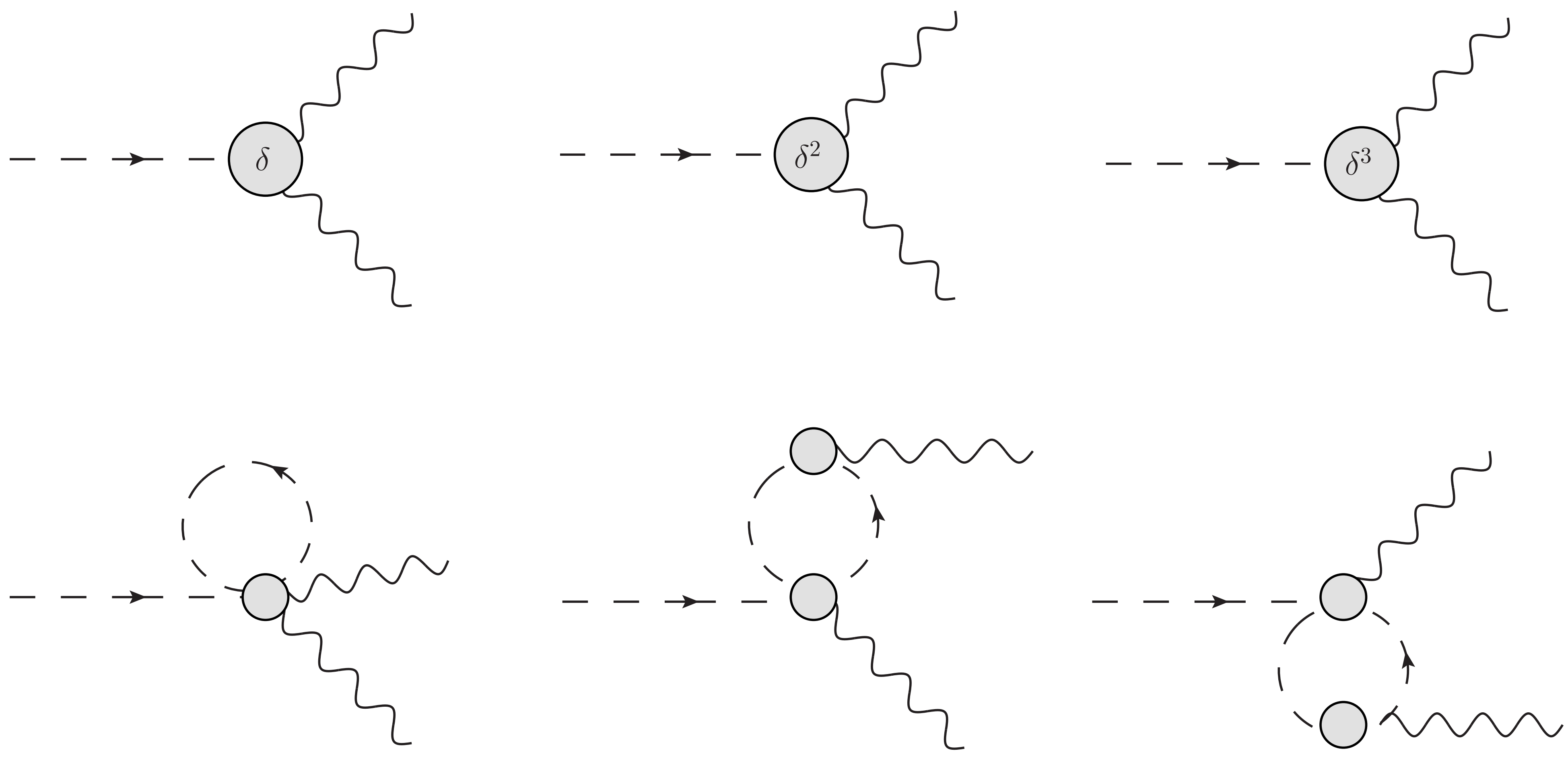
$$\eta(') \to \gamma(*)\pi\pi$$



Prospects

Study of anomalous decays including mixing

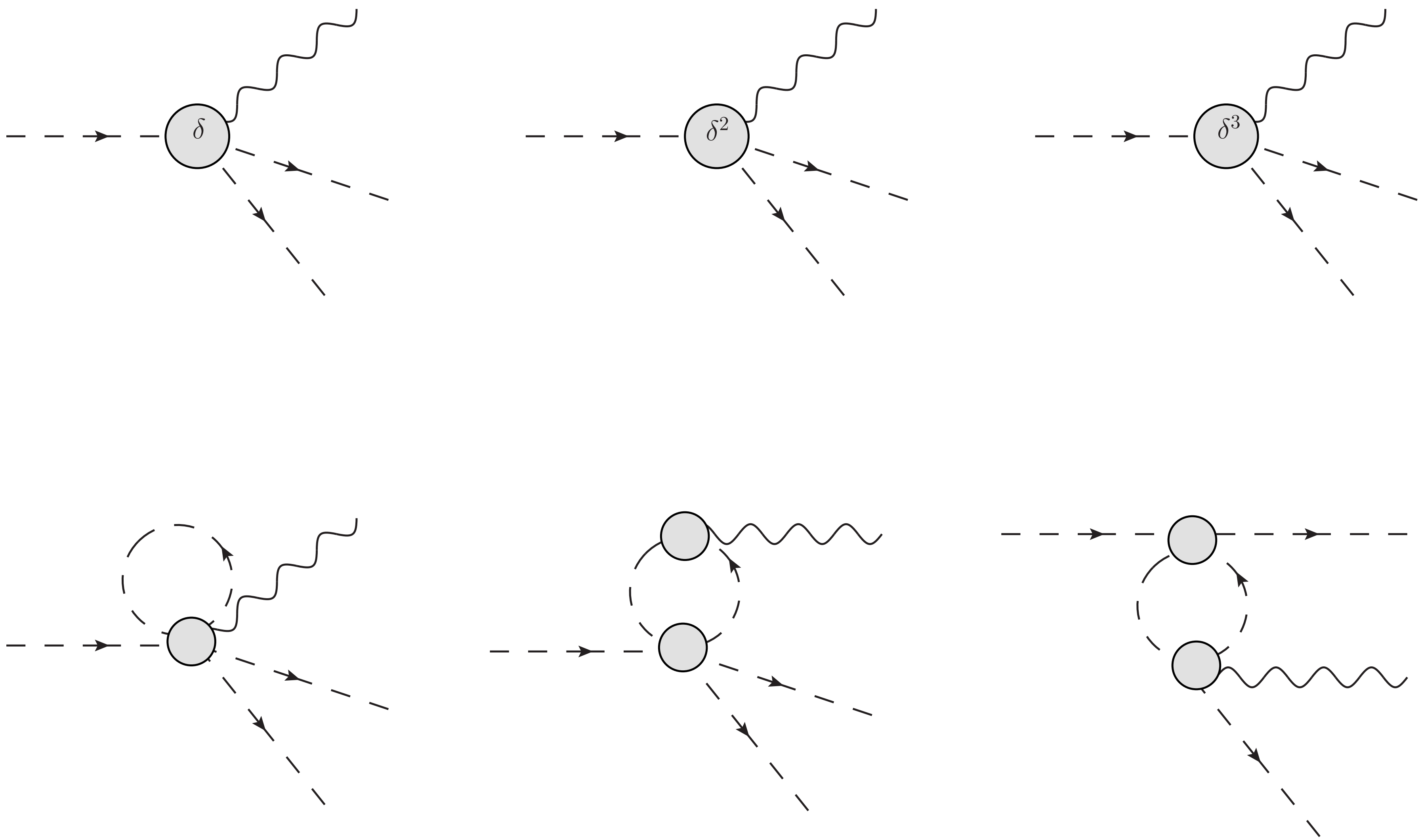
$$\eta(') \rightarrow \gamma(*)\gamma(*)$$



Prospects

Study of anomalous decays including mixing

$$\eta(') \to \gamma(*)\pi\pi$$



Conclusions

- The η-η' systems allows to study symmetries and symmetry breaking in QCD, and is a test of EFTs as well
- Today we explored the η-η' mixing at NNLO in the Large-Nc ChPT framework:
  - we discussed about the relevant Lagrangians
  - we explore the convergence, the role of loops and LECs
  - we provided with preliminary numerical results
    - while including loops is OK, the proliferation of LECs enlarges the numerical results
- With this tool at hand, we are now exploring anomalous η(') decays

Acknowledgments

- Thanks to my collaborators:
  - P. Bickert and S. Scherer for enjoyable collaboration on the calculation of the mixing at NNLO
  - R. Escribano and P. Sanchez-Puertas for enjoyable collaboration on its phenomenological study
- Special thanks to the Organizers for the encouragement and support

Thanks!

- *Notation for the mixing angle:*

mixing of mass eigenstates

octet-singlet basis	quark-flavour basis

1 mixing angle

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle$$
$$|\eta'\rangle = \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle$$

$$|\eta\rangle = \cos\phi_P|\eta_q\rangle - \sin\phi_P|\eta_s\rangle$$
$$|\eta'\rangle = \sin\phi_P|\eta_q\rangle + \cos\phi_P|\eta_s\rangle$$

with

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
$$|\eta_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

and

$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$|\eta_s\rangle = s\bar{s}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

Assumptions:
- no energy dependence
- $\Gamma_{\eta,\eta'} \ll m_{\eta,\eta'}$
- no mixing with other pseudoscalars (π^0, η_c, glueballs)

- *Notation for the mixing angles of the decay constants*

mixing of decay constants

octet-singlet basis

$$\langle 0|A^a_\mu|P(p)\rangle = i f^a_P p_\mu$$

with $A^a_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{\sqrt{2}} q$

f^a_P $(a = 8, 0; P = \eta, \eta')$

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

2 mixing angles

2 decay constants

quark-flavour basis

$$\langle 0|A^i_\mu|P(p)\rangle = i f^i_P p_\mu$$

with

$$A^q_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$

and $A^s_\mu = \bar{s}\gamma_\mu\gamma_5 s$

f^i_P $(i = q, s; P = \eta, \eta')$

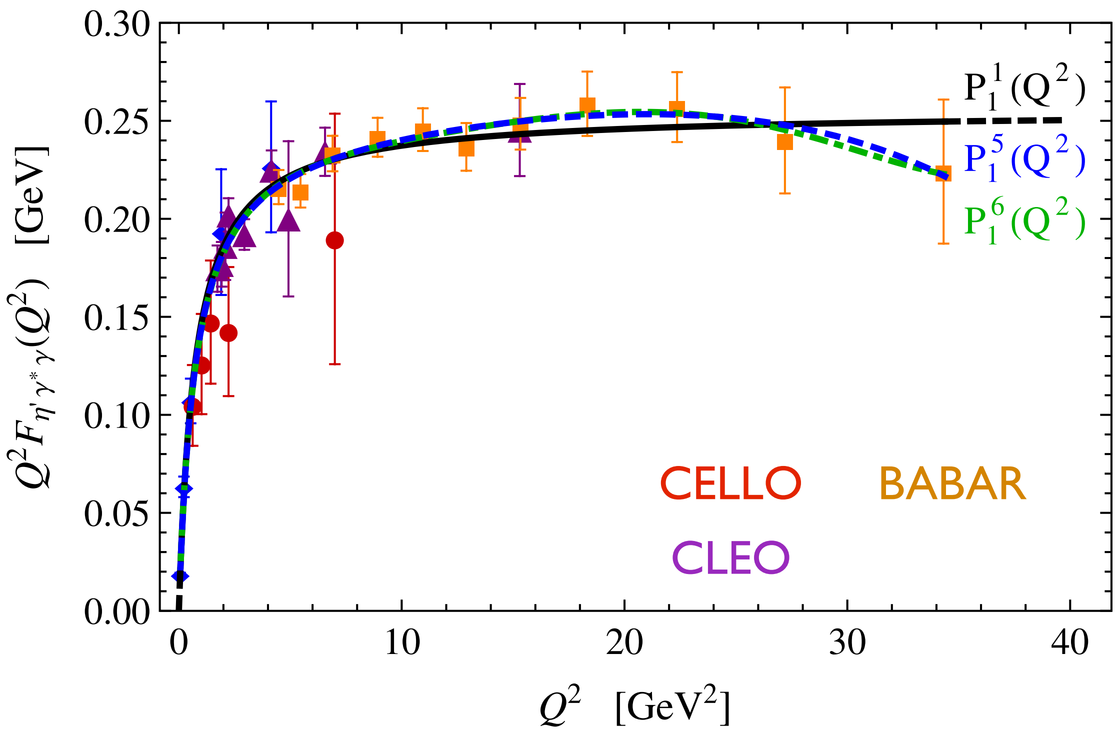
$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}$$

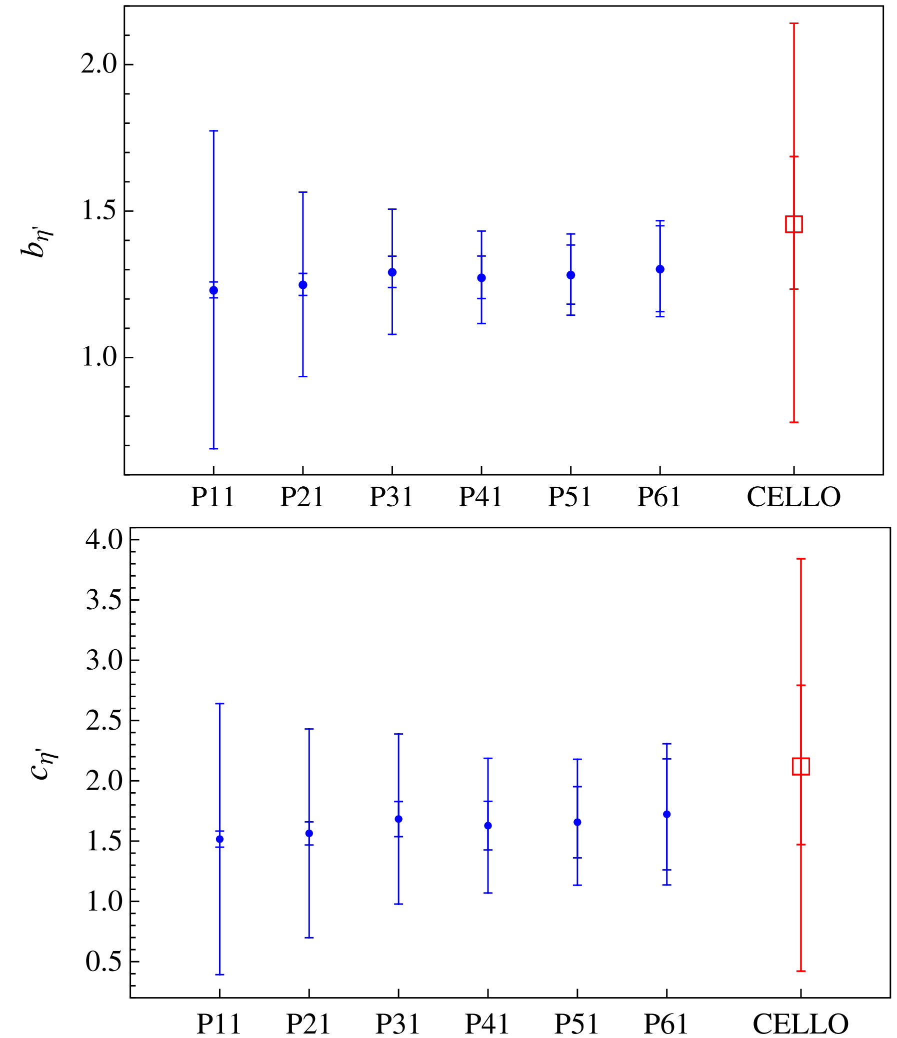
η’-TFF

Fit to Space-like data: CELLO’91, CLEO’98, L3’98, BABAR’11+ $\Gamma_{\eta' \to \gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, ’13]

$P_1^N(Q^2)$ up to N=5





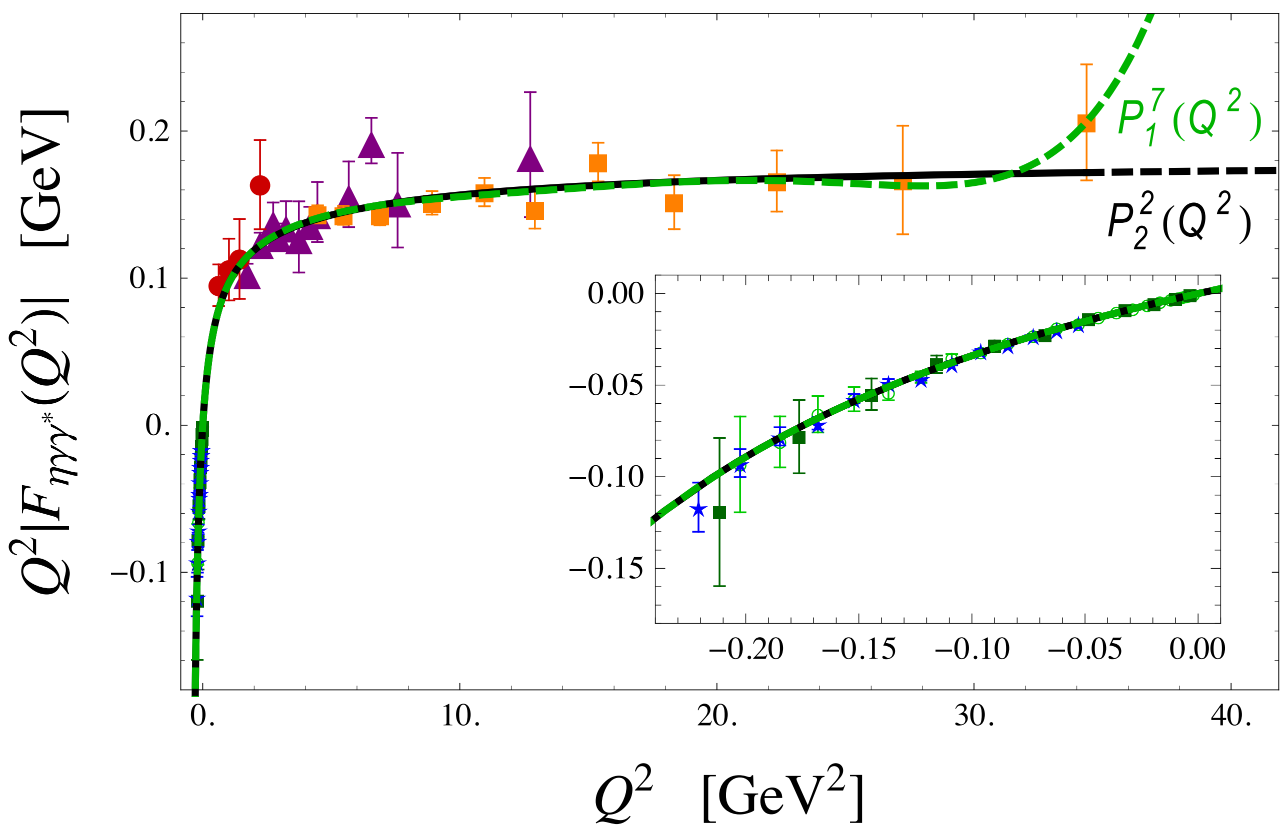
$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4) GeV$$

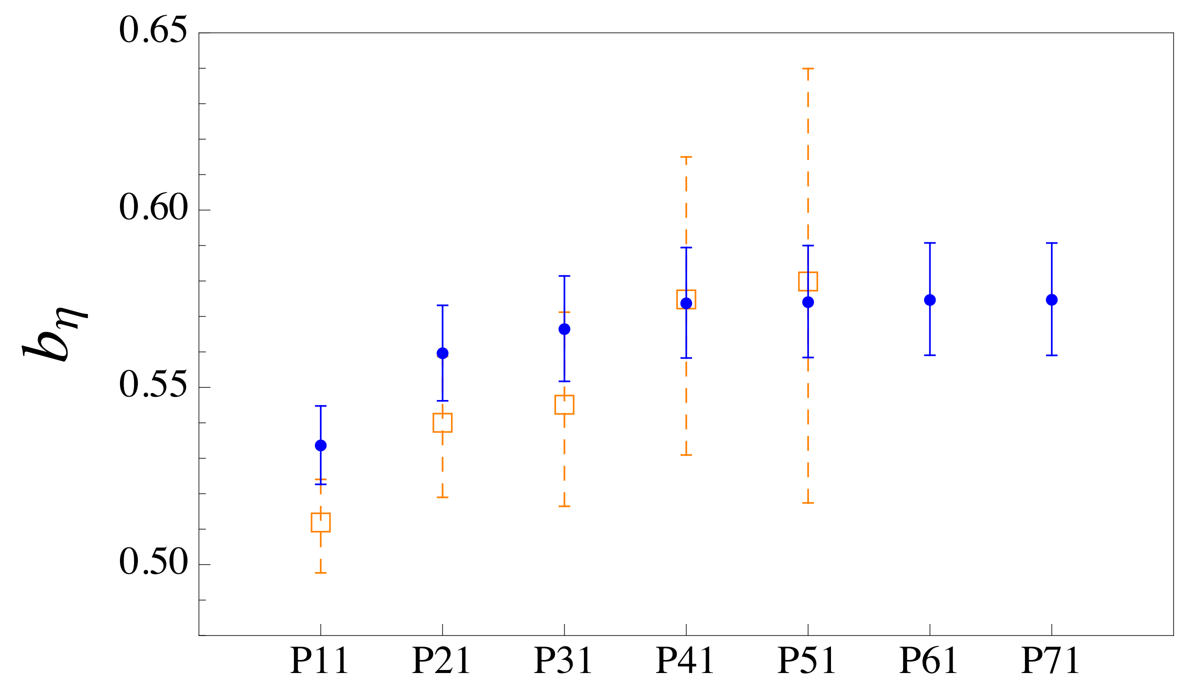
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



$P_1^N(Q^2)$ up to N=7



$P_N^N(Q^2)$ up to N=2

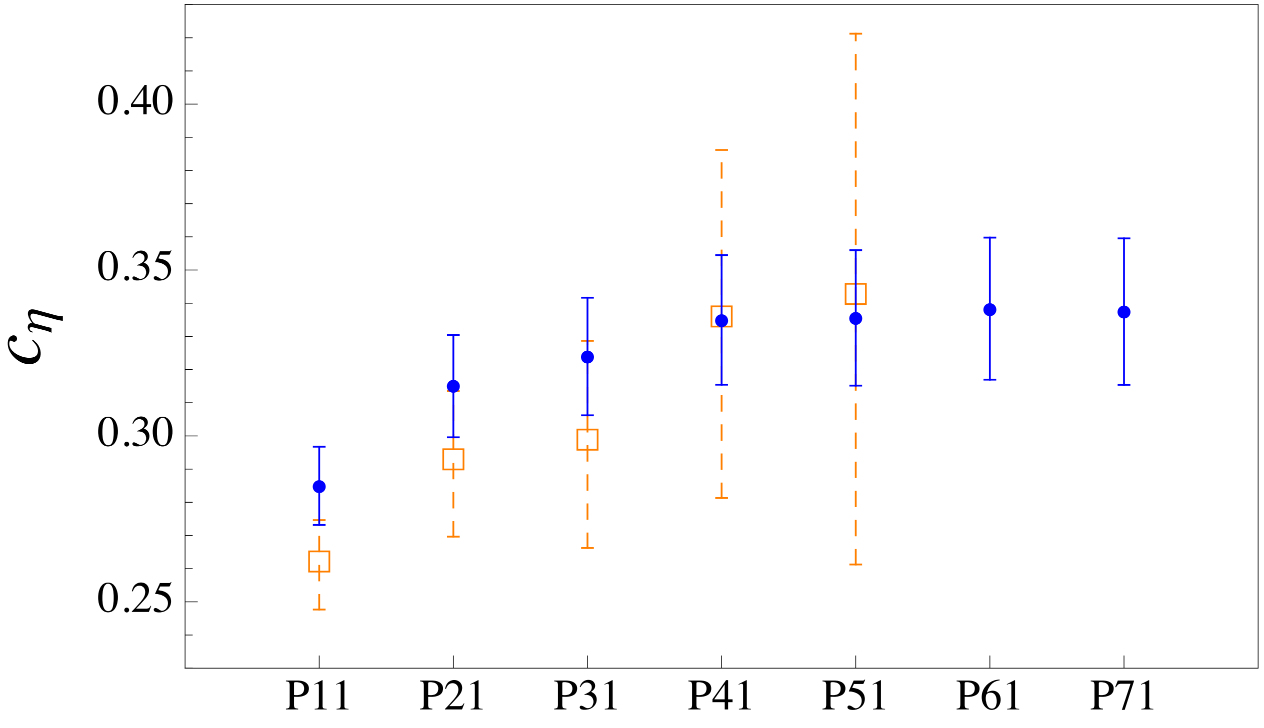
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.177(15) GeV$$

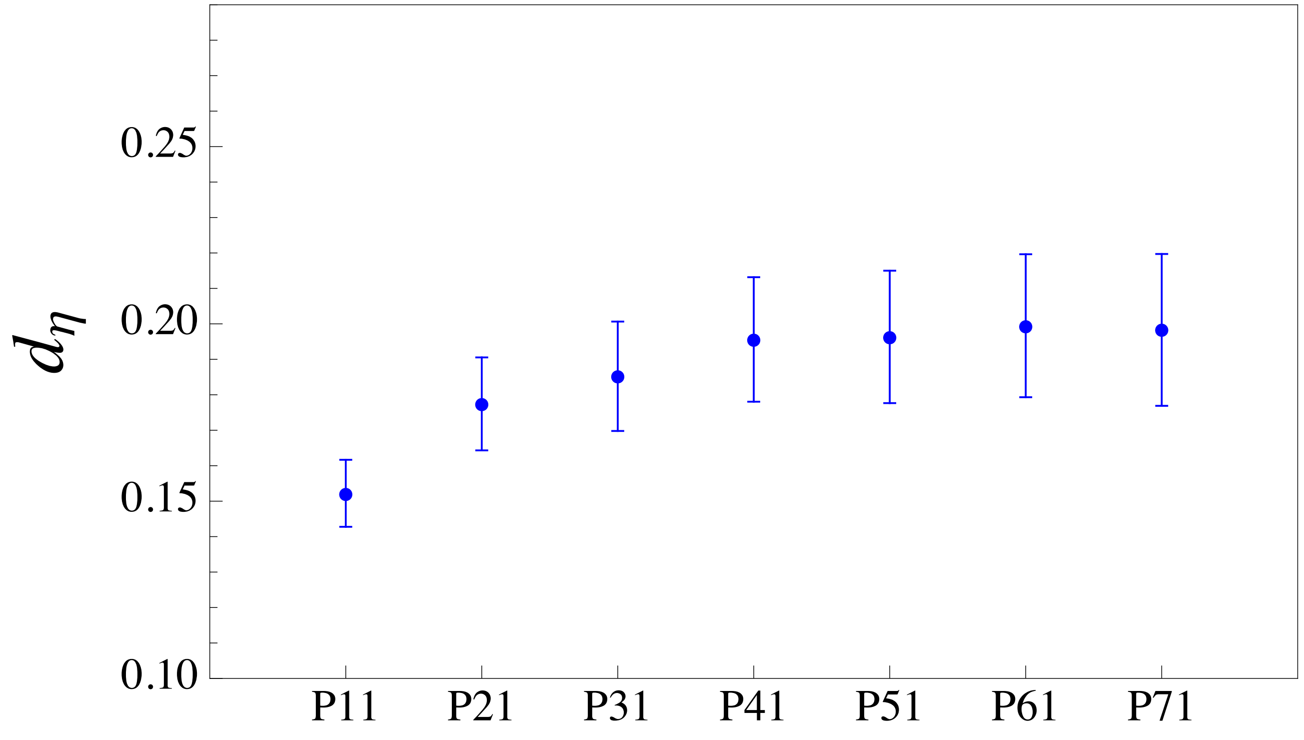
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

$P_1^N(Q^2)$ up to N=7

[R.Escribano, P.M., P. Sanchez-Puertas, '15]

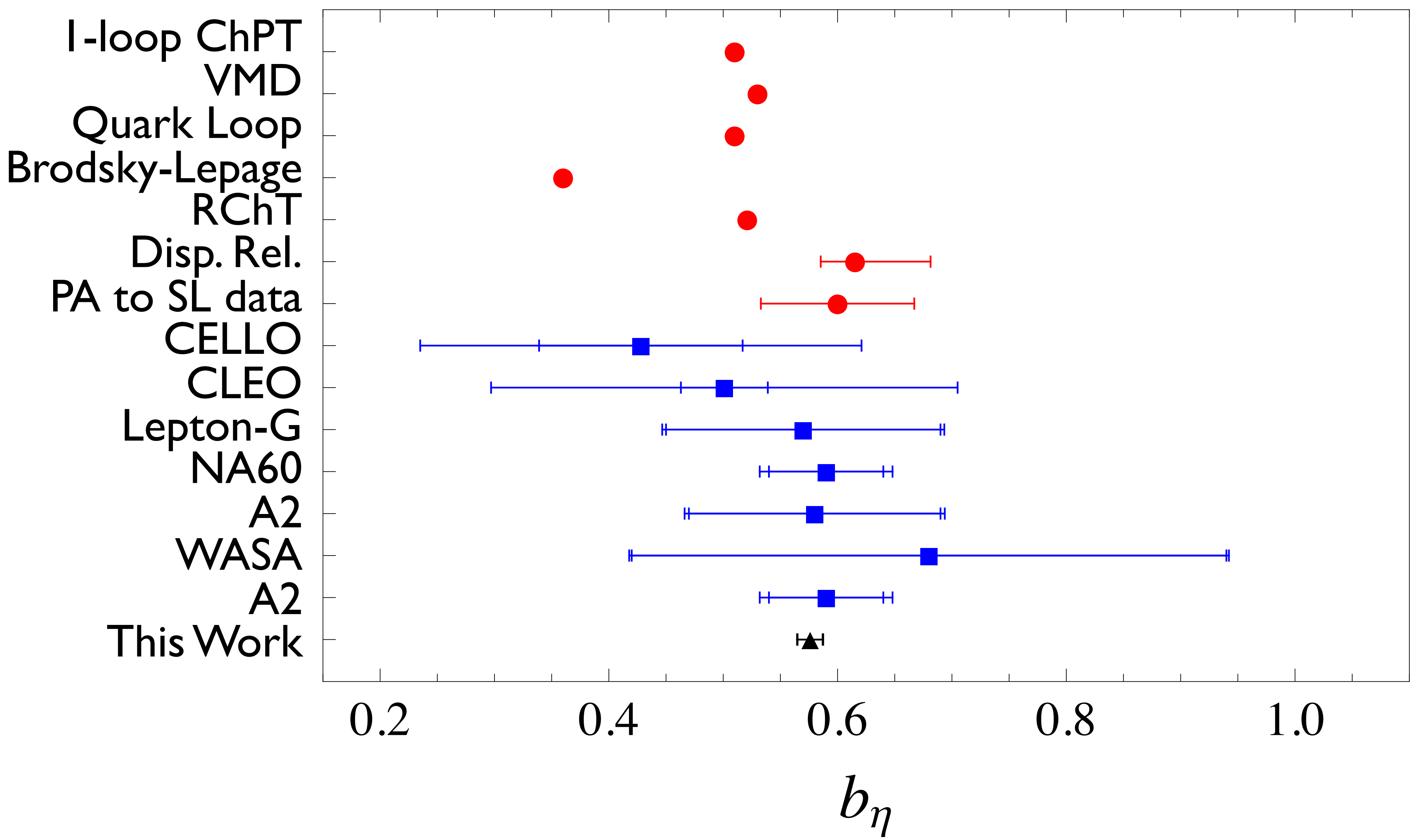




η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3} ,$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3} .$$

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ [R.Escribano, P.M., P. Sanchez-Puertas, '15]

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

[R.Escribano, P.M., P. Sanchez-Puertas, '14]

$$f_q = 1.07(1) f_\pi, \quad f_s = 1.39(14) f_\pi, \quad \phi = 39.3(1.3)^\circ$$

Update of Frere-Escribano '05 with PDG12 using 9 inputs

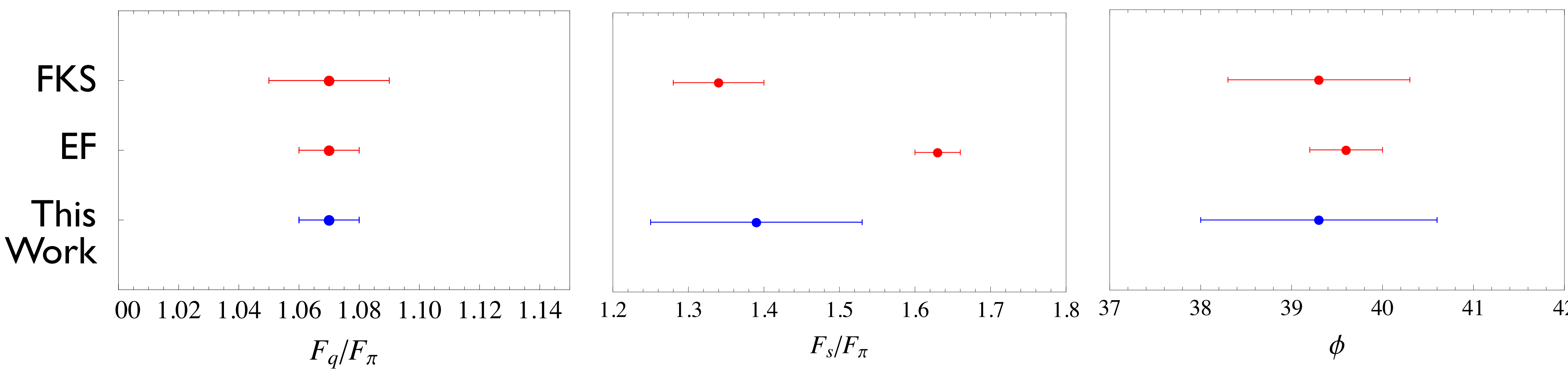
$$f_q = 1.07(1) f_\pi, \quad f_s = 1.63(2) f_\pi, \quad \phi = 40.4(0.3)^\circ$$

η-η' mixing

η-η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



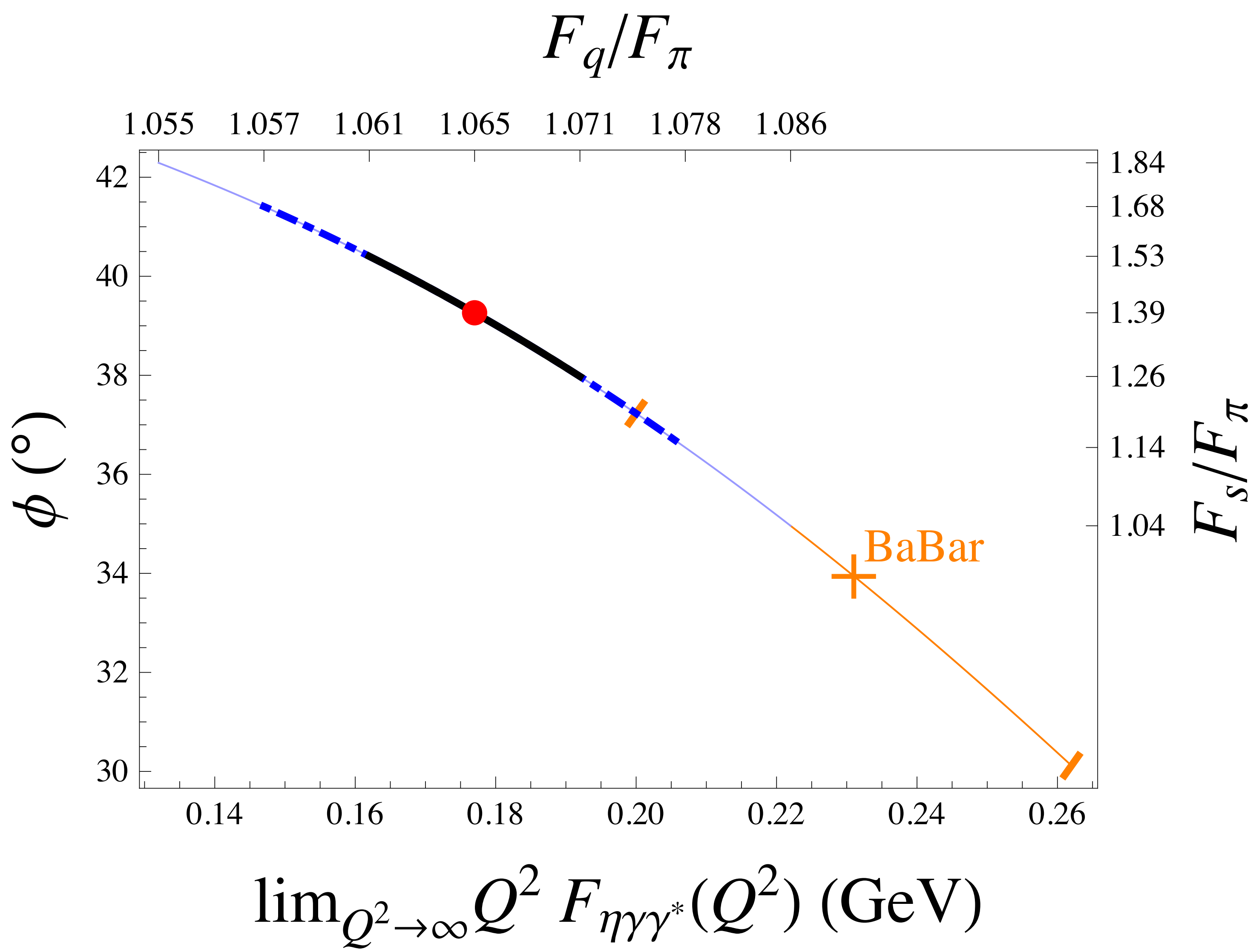
FKS: Feldmann, Kroll, Stech, PLB 449, 339, (1999)
EF: Escribano, Frere, JHEP 0506, 029 (2005) updated in Escribano, P.M, Sanchez-Puertas, 2013.

η-η' mixing

η-η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



TFF at q^2=112GeV^2

[BABAR PRD'06]

- Is 112GeV^2 not asymptotic yet?
- Could BELLE crosscheck?

η-η' mixing

From the TFFs we can determine F_q, F_s, ϕ

and the VPγ and J/Ψ decays used in FKS and EF as inputs

[R.Escribano, P.M., P. Sanchez-Puertas, '15]

(using $F_{\pi^0} = 131.5 \pm 1.4$ MeV instead of $F_{\pi^-} = 92.21 \pm 0.14$ MeV)

	Our predictions	Experimental determinations
$g_{\rho\eta\gamma}$	1.55(4)	1.58(5)
$g_{\rho\eta'\gamma}$	1.19(5)	1.32(3)
$g_{\omega\eta\gamma}$	0.56(2)	0.45(2)
$g_{\omega\eta'\gamma}$	0.54(2)	0.43(2)
$g_{\phi\eta\gamma}$	–0.83(11)	–0.69(1)
$g_{\phi\eta'\gamma}$	0.98(14)	0.72(1)
$\frac{J/\Psi\to\eta'\gamma}{J/\Psi\to\eta\gamma}$	4.74(60)	4.67(20)