Odd moments of nucleon charge and magnetization distribution in baryon chiral perturbation theory

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in collaboration with Vladimir Pascalutsa

The 8th International Workshop on Chiral Dynamics 2015

June 29, 2015, Pisa

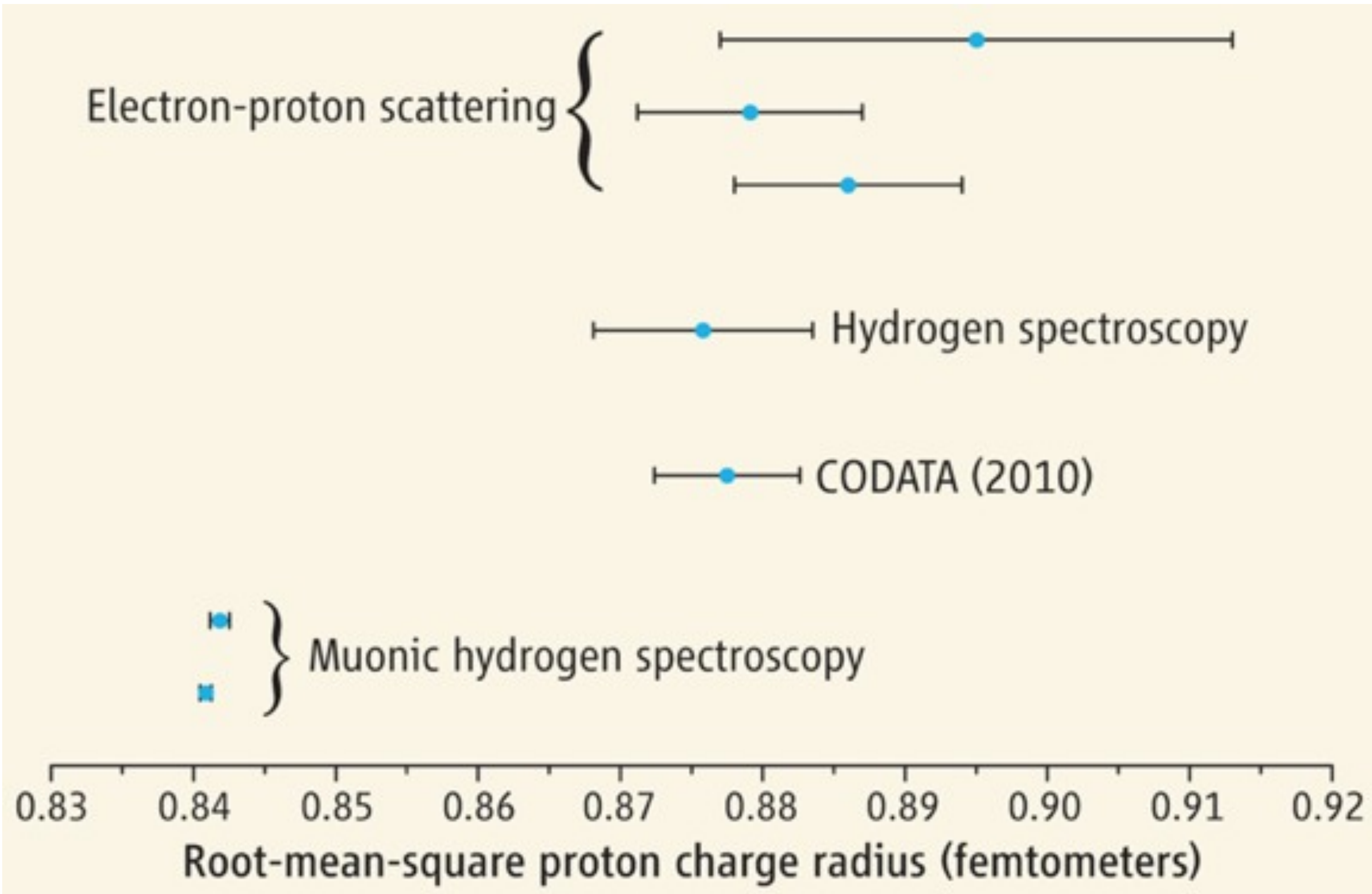
Outline

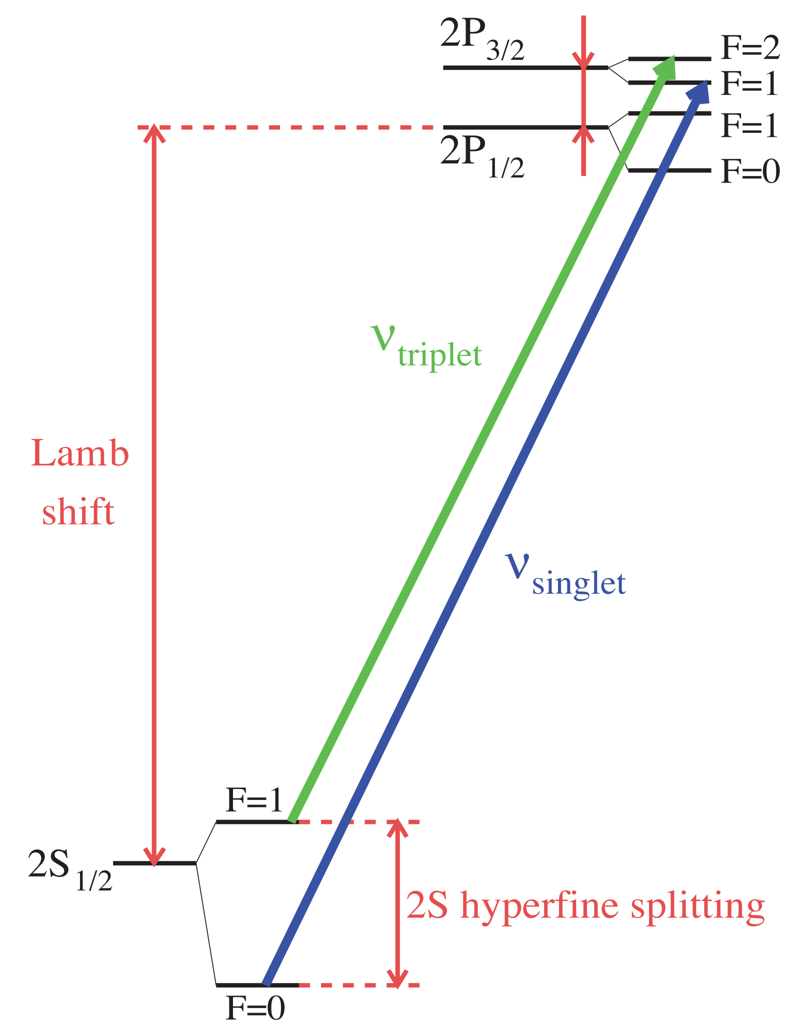
1) Motivation:
   a) Muonic hydrogen Lamb shift(LS), proton structure corrections
   b) Hyperfine splitting

2) Moments of nucleon charge distribution in BChPT

3) Summary

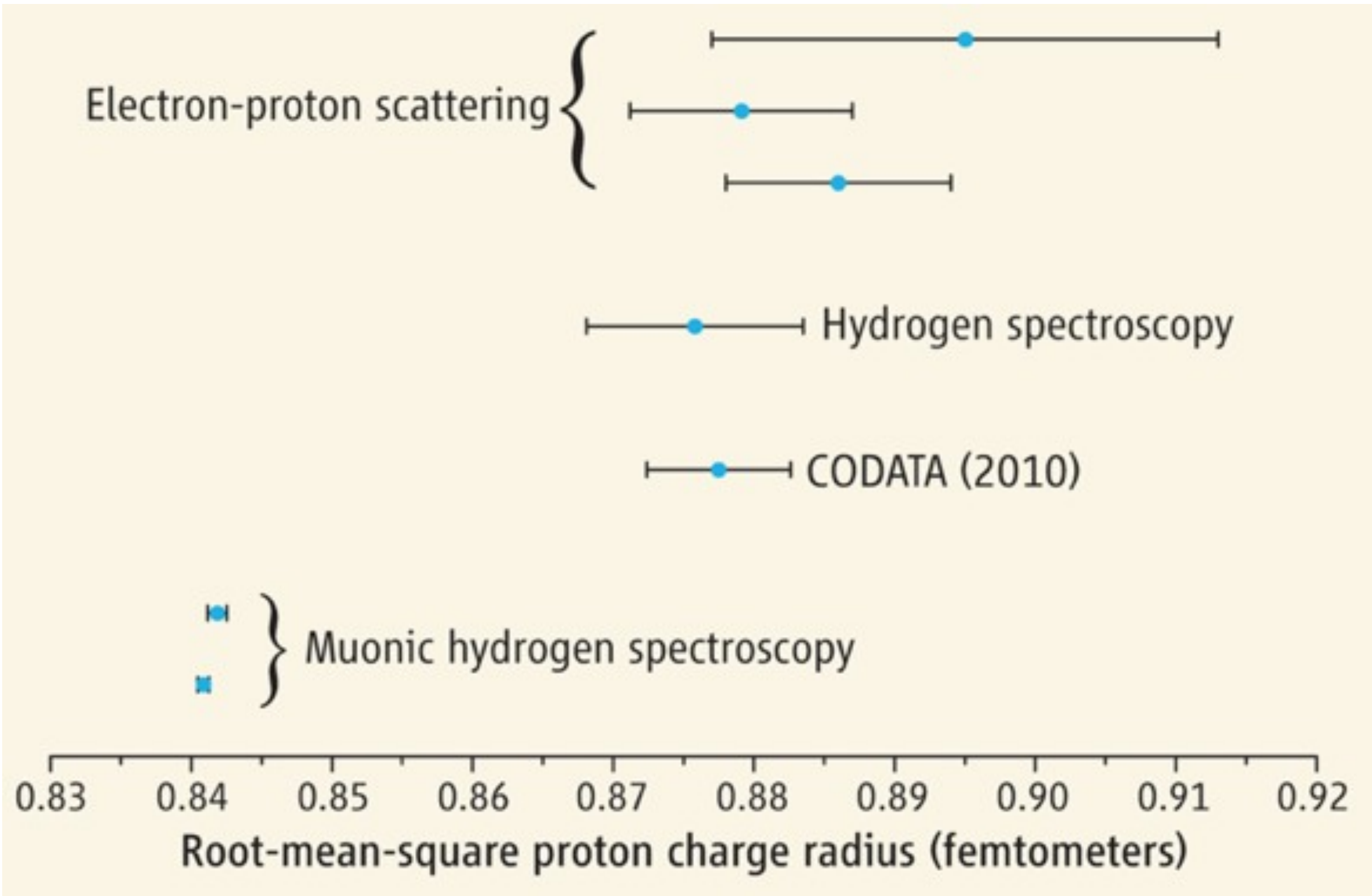
Proton size puzzle

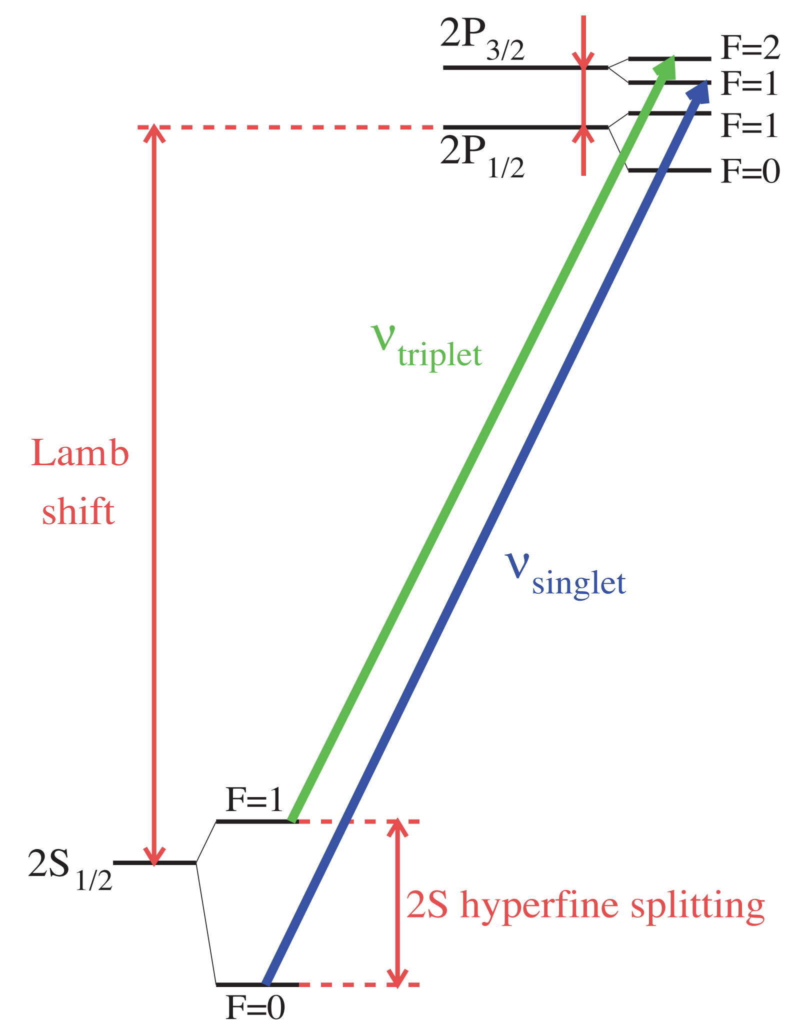




A. Antognini *et al.*, Science **339**, 417 (2013).

Proton size puzzle





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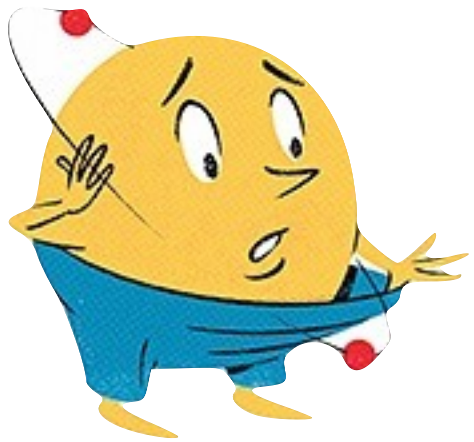
$$[R_E^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}]$$



$$[R_E^{\mathrm{CODATA\ 2010}} = 0.8775(51)\ \mathrm{fm}]$$

muH Lamb shift: Proton structure corrections

$$\Delta E_L^{\mathrm{th}}\ [\mathrm{meV}] = 206.0336(15) - 5.2275(10)\, R_E^2[\mathrm{fm}^2] + \Delta E_{\mathrm{TPE}}$$

Leading-order Finite-size correction, a^4

$$\Delta E_{nS}(\mathrm{LO}) = \frac{2(Z\alpha)^4 m_r^3}{3n^3} R_E^2$$

$$R_E^2 = -6 \lim_{Q^2 \to 0} \frac{\mathrm{d}}{\mathrm{d}Q^2} G_E(Q^2)$$

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Two-photon exchange correction, a^5

elastic:

$$\Delta E_{nS}(\mathrm{NLO}) = -\frac{(Z\alpha)^5 m_r^4}{3n^3} R_{E(2)}^3$$

3rd Zemach moment:

$$R_{E(2)}^3 = \frac{48}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^4} \left\{ G_E^2(Q^2) - 1 + \frac{1}{3} R_E^2\, Q^2 \right\}$$

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polarizability:

BChPT:

Alarcon, Lensky & Pascalutsa, EPJC (2014)

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Experimental data $\longrightarrow$ $R_{E(2)}^3$ $\xleftarrow{?}$ Chiral Perturbation Theory

Hyperfine splitting

$$\Delta E^{\mathrm{th}}_{HFS}\ [\mathrm{meV}] = 22.9763(15) - 0.1621(10)\ \langle r \rangle_Z [\mathrm{fm}] + \Delta E^{\mathrm{pol}}_{\mathrm{HFS}}$$

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Fermi energy + radiative and recoil corrections

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LO finite-size effect

Zemach radius:

$$\langle r\rangle_Z = \int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)\,G_M(Q^2)}{1+\kappa} - 1\right]$$

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Fermi energy + radiative and recoil corrections

polarizability contribution

$$\Delta E^{\mathrm{pol}}_{\mathrm{HFS}} = 0.0080(26)\ \mathrm{meV}$$

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Moments of charge distribution

Charge distribution: $$\rho_E(r) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3} G_E(\vec{q}^{\,2}) e^{-i\vec{q}\vec{r}}$$

$$G_E(Q^2) = 1 - \frac{Q^2}{\pi} \int \mathrm{dt}\, \frac{\mathrm{Im} G_E(t)}{(t+Q^2)\, t}$$

where $Q^2 = -q^2$

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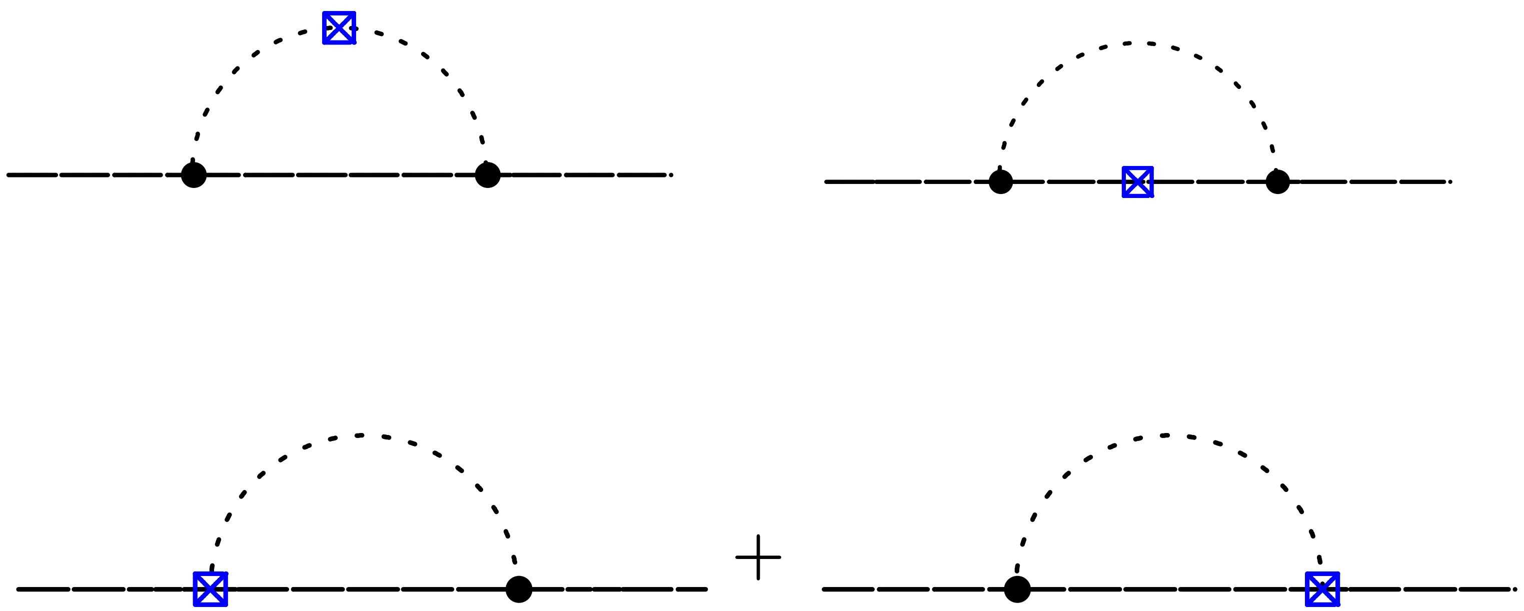
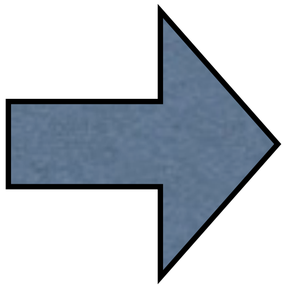
$$\boxed{\begin{aligned} \langle r^N \rangle_E &\equiv \int \mathrm{d}\vec{r}\, r^N \rho_E(r) \\ &= \frac{(N+1)!}{\pi} \int_{t_0}^{\infty} \mathrm{d}t\, \frac{\mathrm{Im}\ G_E(t)}{t^{1+N/2}} \end{aligned}}$$

Predictive orders (no new LECs)

Leading order (p^3):





LO
ImGE and ImGM

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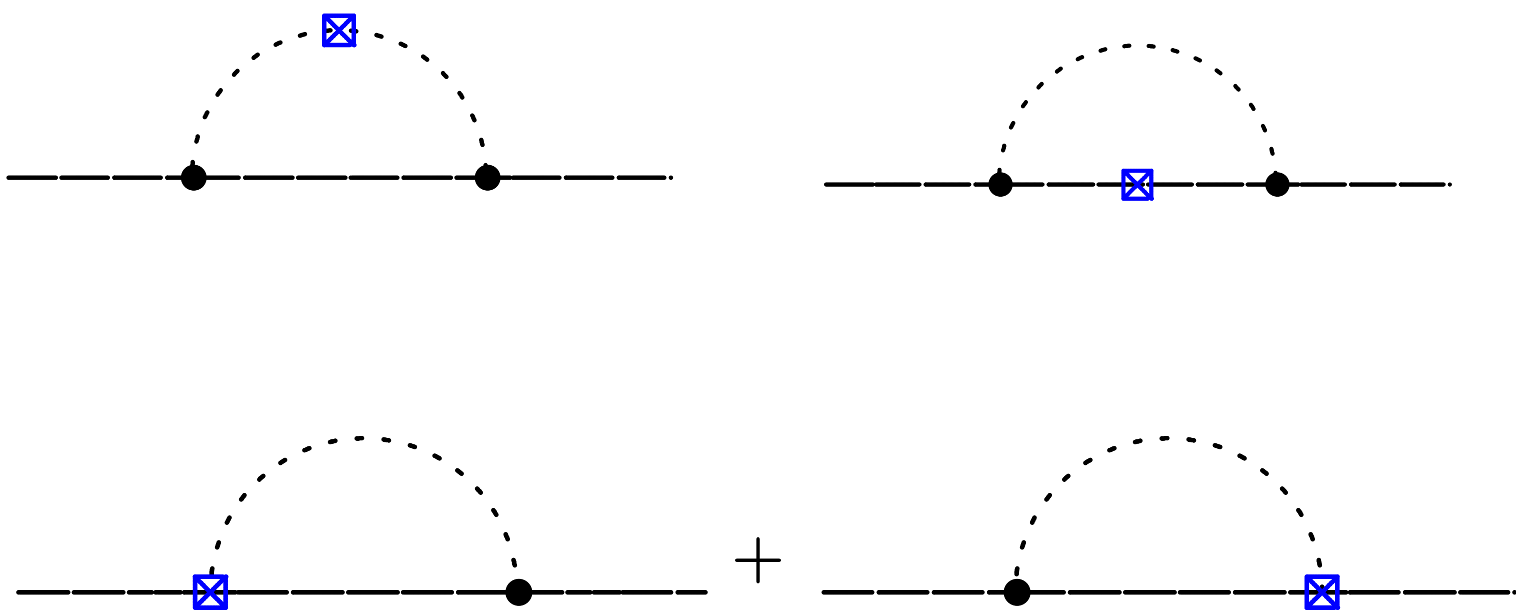

LO
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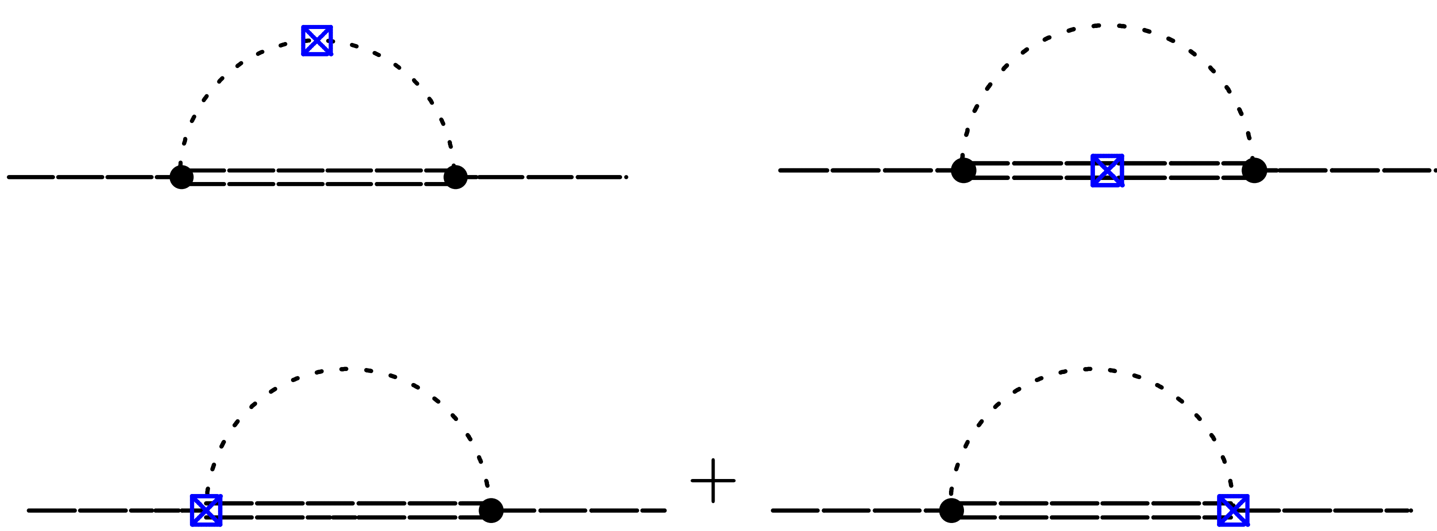
At order O(p^4/Δ):



NLO
ImGE and ImGM

Predictive orders (no new LECs)

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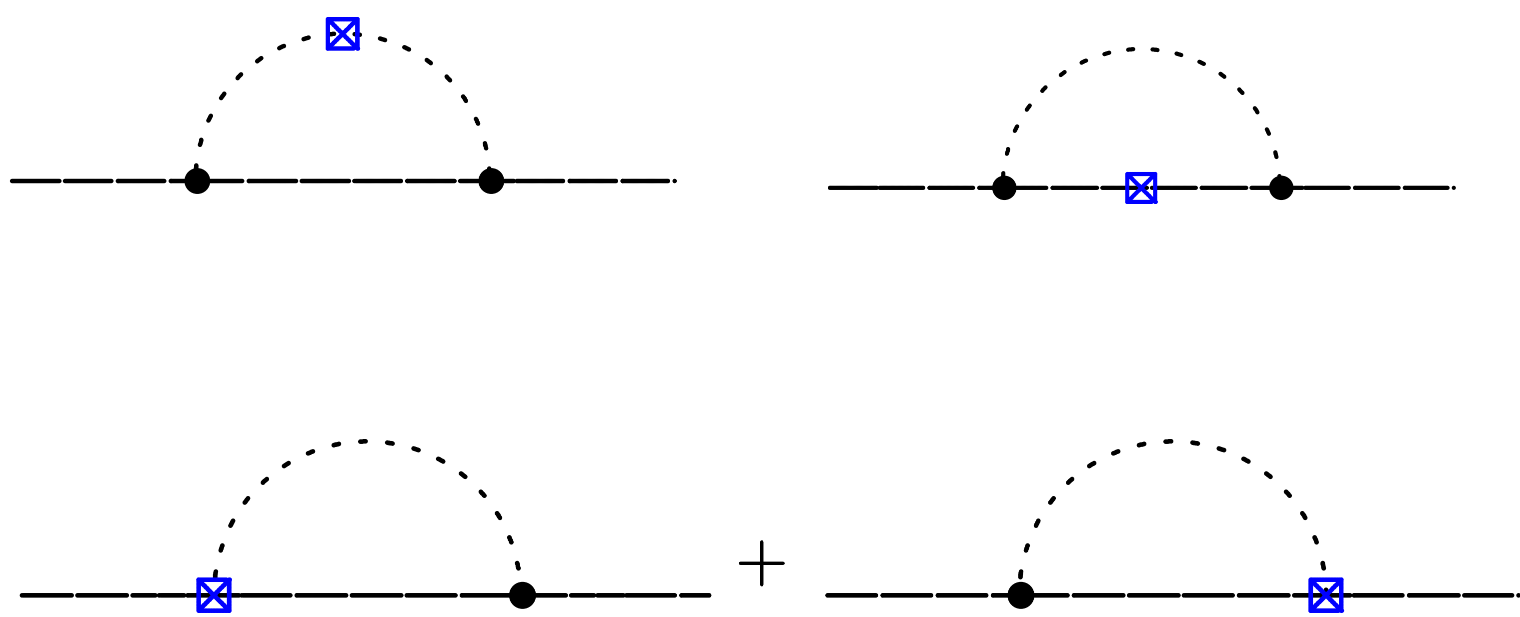

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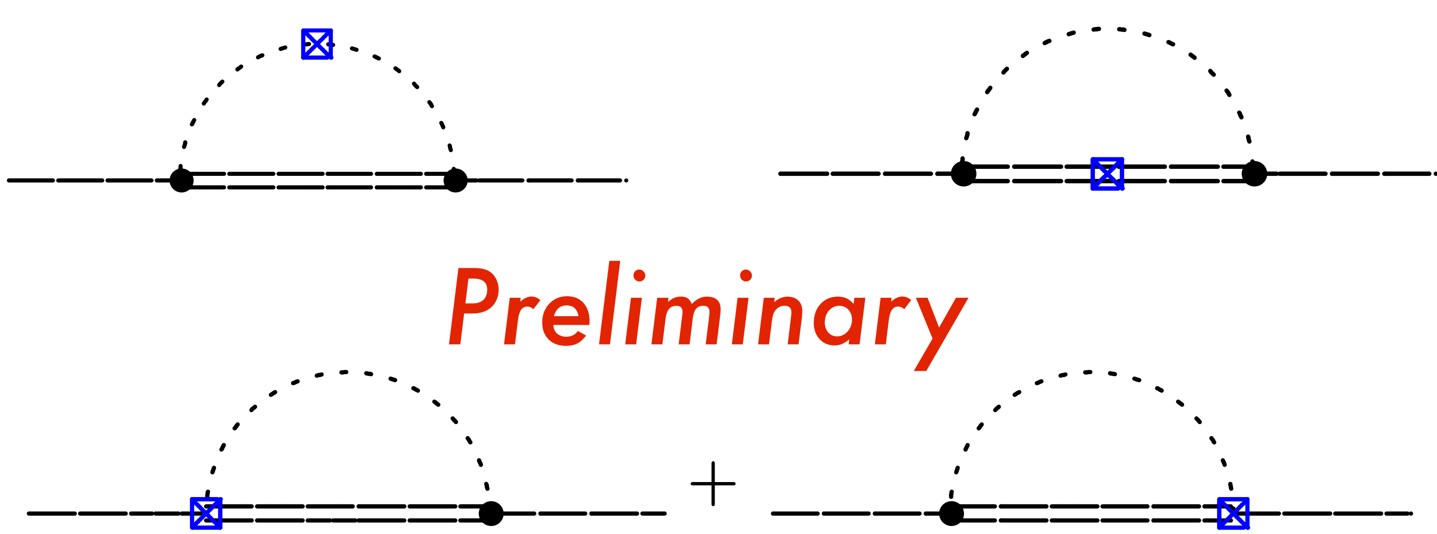

NLO
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Third moment of the charge distribution

$$\langle R^3 \rangle_E = \frac{48}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^4} \left\{ G_E(Q^2) - 1 + \frac{1}{6} \langle r^2 \rangle_E \, Q^2 \right\} = \frac{24}{\pi} \int_{t_0}^\infty \mathrm{d}t \, \frac{\mathrm{Im} G_E(t)}{t^{5/2}}$$

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LO heavy baryon ChPT:

$$\langle R^3 \rangle_E = \frac{21 g_A^2}{128 \pi f_\pi^2 m_\pi} + O\left(\frac{m_\pi}{M_p} \right)$$

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Empirical value from FF fitted to ep data:

$$\langle R^3\rangle_E = 1.16(4)\,[\mathrm{fm}^3]$$

Distler, Bernauer, Walcher, Phys. Lett. B **696**, 343 (2011).

Third Zemach moment

$$\langle R^3 \rangle_{E(2)} = 2\langle R^3 \rangle_E + \frac{24}{\pi^2} \int_{t_0}^{\infty} \mathrm{d}t \int_{t_0}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im} G_E(t) \, \mathrm{Im} G_E(t')}{(t\, t')^{3/2} (\sqrt{t} + \sqrt{t'})}$$

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Numerical result:

$$\langle R^3 \rangle_{E(2)} \simeq \underbrace{0.554}_{\text{LO}} + \underbrace{0.218}_{\text{NLO}} = 0.772 \, [\mathrm{fm}^3]$$

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Empirical value from FF fitted to ep data:

$$\langle R^3\rangle_{E(2)} = 2.85(8)\,[\mathrm{fm}^3]$$

Distler, Bernauer, Walcher, Phys. Lett. B **696**, 343 (2011).

Zemach radius

$$\langle r \rangle_Z = -\frac{2}{\pi(1+\kappa)}\Big\{\frac{1}{\pi}\int_{t_0}^{\infty} \mathrm{d}t \int_{t_0}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im}G_E(t)\,\mathrm{Im}G_M(t')}{t\,t'(\sqrt{t}+\sqrt{t'})}$$
$$-(1+\kappa)\int_{t_0}^{\infty} \mathrm{d}t \, \frac{\mathrm{Im}G_E(t)}{t^{3/2}} - \int_{t_0}^{\infty} \mathrm{d}t \, \frac{\mathrm{Im}G_M(t)}{t^{3/2}}\Big\}$$

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Numerical result:

$$\langle r \rangle_Z \simeq \underbrace{1.537}_{\text{LO}} + \underbrace{1.439}_{\text{NLO}} = 2.976\,[\mathrm{fm}]$$

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Empirical value
from FF fitted to ep data:

$$\langle r\rangle_Z = 1.045(4)\,[\mathrm{fm}]$$

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Summary of the results

Odd moment	p^3	p^4/Δ	p^3+p^4/Δ	Empirical value
$\langle R^3 \rangle_E$	0.276	0.109	0.385	1.16(4)
$\langle R^3 \rangle_{E(2)}$	0.554	0.218	0.772	2.85(8)
$\langle r \rangle_Z$	1.537	1.439	2.976	1.045(4)

Possible explanations:

i) ChTP does not work, new physics needs to be implemented

ii) Empirical values are wrong

iii) ...

Summary

(i) Calculation of the 3rd Zemach moment allows us to extract the proton radius independently on the elastic ep-scattering data.

(ii) The 3rd Zemach moment and Zemach radius can be predicted by ChPT (no new LECs).

(iii) We present the results of the leading and next-to-leading order contributions, $O(p^3)$ and $O(p^4/\Delta)$, to the third moment of the charge distribution, the 3rd Zemach moment, and Zemach radius.

(iv) More work is required to reproduce the empirical values.