INNOWACYJNA GOSPODARKA
NARODOWA STRATEGIA SPÓJNOŚCI

Dotacje na innowacje
Inwestujemy w waszą przyszłość

UNIA EUROPEJSKA
EUROPEJSKI FUNDUSZ ROZWOJU REGIONALNEGO

FNP
Fundacja na rzecz Nauki Polskiej

Study of two and three meson decay modes of tau-lepton with Monte Carlo generator TAUOLA

O. Shekhovtsova
INP Cracow / KIPT Kharkov

Workshop on Chiral Dynamics
Pisa, 30.06.2015

τ - lepton physics

* τ mass and life time measurements

* leptonic decy modes: a test of the charged lepton universality
* search for LFV process ($\tau \to 3\mu$, $\tau \to \mu\gamma$)

m_τ = 1.78 GeV → decays in hadrons Br ($\tau \to$ hadrons) = 64.8%

Precise measurements of the hadronic τ decay modes = the low and intermediate energy study:

$$\mathcal{M}(\tau^- \to \nu_\tau h^-) = \frac{G_F}{\sqrt{2}} \mathcal{H}_h^\mu \, [\overline{\nu}_\tau \gamma_\mu (1-\gamma_5)\tau]$$

$$\mathcal{H}_h^\mu \equiv \langle h^- | \left(V_{ud}^* \, \overline{d} \, \gamma^\mu (1-\gamma_5) u + V_{us}^* \, \overline{s} \, \gamma^\mu (1-\gamma_5) u \right) |0\rangle$$

* hadronization mechanism (pQCD does not work, ChPT low tail)
  * Wess-Zumino anomaly (ex. K K π)
    * resonance parameters
      * Okuba-Zweig-Iizuka suppressed modes (ex. ϕ K)
        * second class currents (ex. $\pi\,\eta$)
* measure |*V_us*| CKM matrix (modes with K)

Experiment:

* Cleo (Dalitz plots for $\pi^- \pi^- \pi^+$), Aleph (90's)
* BaBar (preliminary data for 3 meson modes, distributions), Belle (two pion and $K\pi$ form factor)

* Belle II project (B2TiP workshop, Cracow, 04.2015)

High energy (LHC) Knowledge of the dynamics is important for Higgs polarization measurement and agreement MC/data, searched for beyond SM physics

Precise analysis of available data for 2 pion + 3 pion modes → **BaBar / Belle data**

~44% hadr Br to check

TAUOLA (Monte Carlo generator for tau decay modes)

R. Decker, S.Jadach, M.Jezabek, J.H.Kuhn, Z. Was, Comp. Phys. Comm. 76 (1993) 361; ibid 70 (1992) 69, ibid 64 (1990) 275

1. leptonic decay modes: $\tau^-(P,s) \longrightarrow \nu_\tau(N) l^-(q_1)\bar{\nu}_l(q_2), \quad l = e, \mu$

$$\bar{\mathcal{M}} = \frac{G}{\sqrt{2}}\, \bar{u}(\nu_\tau;N)\gamma^\mu(v+\gamma_5 a)u(\tau^-;P)\ \bar{u}(l^-;q_1)\gamma_\mu(1-\gamma_5)u(\nu_{l^-};q_2)$$

(general str.) → $\gamma^\mu(v+\gamma_5 a)$; (V-A) SM str → $\gamma_\mu(1-\gamma_5)$

$$d\Gamma_l = \frac{1}{2M}\left(\frac{G}{\sqrt{2}}\right)^2 32(B + H_\mu s^\mu)d\mathrm{Lips}(P;q_1,q_2,N)$$

$$B = (v+a)^2(P\cdot q_1)(N\cdot q_2) + (v-a)^2(P\cdot q_2)(N\cdot q_1) - Mm(v^2-a^2)(q_1\cdot q_2)$$

2. semi-leptonic (hadronic) decay modes $\tau(P,s) \rightarrow \nu_\tau(N)X$

$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(v+a\gamma_5)u(P)J_\mu$$

$$|\mathcal{M}|^2 = G^2\frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$

$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu)$$

$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$

$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$

$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho N_\sigma$$

$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$

CPC version

1. R. Decker, S.Jadach, M.Jezabek, J.H.Kuhn, Z. Was, Comp. Phys. Comm. 76 (1993) 361;
2. P. Golonka, B. Kersevan ,T. Pierzchala, E. Richter-Was, Z. Was, M. Worek, Comput. Phys. Commun. 174 (2006) 818;
3. A. E. Bondar, S. I. Eidelman, A. I. Milstein, T. Pierzchala, N. I. Root, Z. Was and M. Worek (4 pions), Comput. Phys. Commun. 146 (2002) 139;
4. J.H.Kuhn, Z. Was, Acta Phys. Polon. 39 (2008) 47 (5-pions), hep-ph/0602162

Hadronic modes : **π^-, K^-, $\pi^0 \pi^-$, $(\pi K)^-$, $(3\pi)^-$, $(5\pi)^-$, $(6\pi)^-$**
$(KK\pi)^-$, $\eta\pi^0 \pi^-$,$(4\pi)^-$, $\pi^0 \pi^-\gamma$ (added later)

Theoretically modelled (except for 3., a model used in e+e- data, fitted)

$$J_\mu = \langle \text{Hadrons} | (V-A)_\mu \, e^{iS_{QCD}} | 0 \rangle = \Sigma_i (\text{Lorentz Structure})^i \, F_i(Q^2, s_j)$$

lowest energy resonances (except for 3 pions : $a_1 \rightarrow (\rho ; \rho') \pi$)

* *based on VMD, i.e. 3 scalar modes BW(V1)*BW(V2) , reproduces LO ChPT limit*
* wrong normalization for 2 scalar modes, except 2π, only vector FF , no scalar FF

TAUOLA (official)

Monte Carlo generator for tau decays

***CPC* version**

***Cleo* version**

***Aleph* version**

***RChL* version**

2 pi Belle FF

**** Belle MC = Cleo version for 3 pions + 2 pion own + others modes ??***

**** BaBar MC = CPC + new modes***

$\tau^- \to h^- h^+ h^- \nu_\tau$ from BaBar and Belle

Mode	BaBar, 342 fb^{-1}	Belle, 666 fb^{-1}	PDG2006
$N_{ev}, 10^6$	1.6	8.86	–
$\mathcal{B}(\pi^-\pi^+\pi^-), 10^{-2}$	$8.83 \pm 0.01 \pm 0.13$	$8.42 \pm 0.01 \pm 0.26$	9.02 ± 0.08
$N_{ev}, 10^4$	7.0	79.4	–
$\mathcal{B}(K^-\pi^+\pi^-), 10^{-3}$	$2.73 \pm 0.02 \pm 0.09$	$3.28 \pm 0.01 \pm 0.17$	3.33 ± 0.35
$N_{ev}, 10^4$	1.8	10.8	–
$\mathcal{B}(K^-K^+\pi^-), 10^{-3}$	$1.346 \pm 0.010 \pm 0.036$	$1.53 \pm 0.01 \pm 0.05$	1.53 ± 0.10
N_{ev}	275	3160	–
$\mathcal{B}(K^-K^+K^-), 10^{-5}$	$1.58 \pm 0.13 \pm 0.12$	$2.62 \pm 0.15 \pm 0.17$	$< 3.7 \cdot 10^{-5}$

BaBar: B. Aubert et al., Phys. Rev. Lett. 100, 011801 (2008)
Belle: M.J. Lee et al., Phys. Rev. D81, 113007 (2010)

Cleo version

CPC currents + Cleo 3 pion current

$\pi^0\ \pi^0\ \pi^-\ \nu_\tau$ D. Asner et al., Phys.Rev. D61 (2000) 012002 (*)

$\pi^-\ \pi^-\ \pi^+\ \nu_\tau$ 1) (*)

2) E. I. Shibata, Nucl.Phys.Proc.Suppl.123 (2003) 40,

J.W. Hinson, PhD thesis, Purdue University (2001),

PU-99-713 (isospin transformed current)

Mechanism production, based on Dalitz plot analysis,

a1 → ...

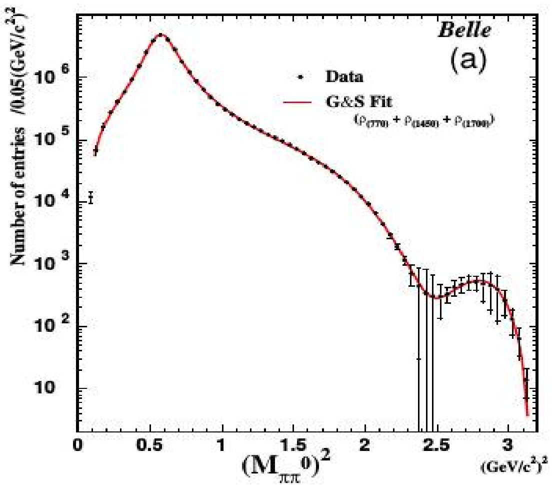
Amplitude		Branching ratio (%)
$\rho\pi$	s-wave	60.19
$\rho(1450)\pi$	s-wave	0.56 ± 0.84
$\rho\pi$	d-wave	1.30 ± 0.60
$\rho(1450)\pi$	d-wave	2.04 ± 1.20
$f_2(1270)\pi$	p-wave	1.19 ± 0.49
$\sigma\pi$	p-wave	18.76 ± 4.29
$f_0(1370)\pi$	p-wave	7.40 ± 2.71

Cleo analysis 2001, $\pi^0\ \pi^0\ \pi^-$ approved; other modes no;

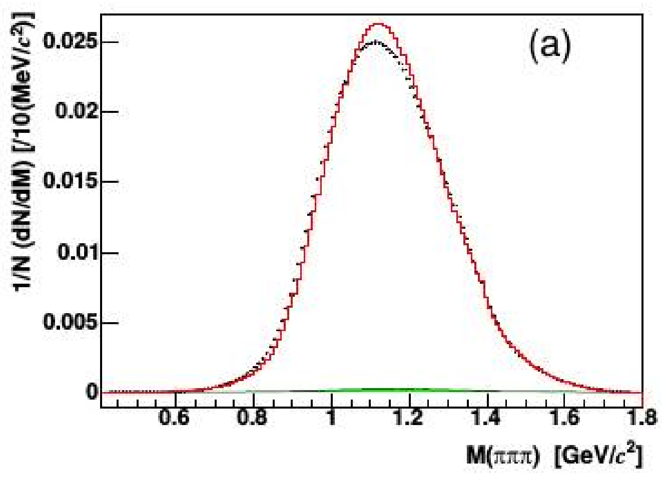
no further TAUOLA update

The Physics of the B Factories *arXiv:1406.6311*

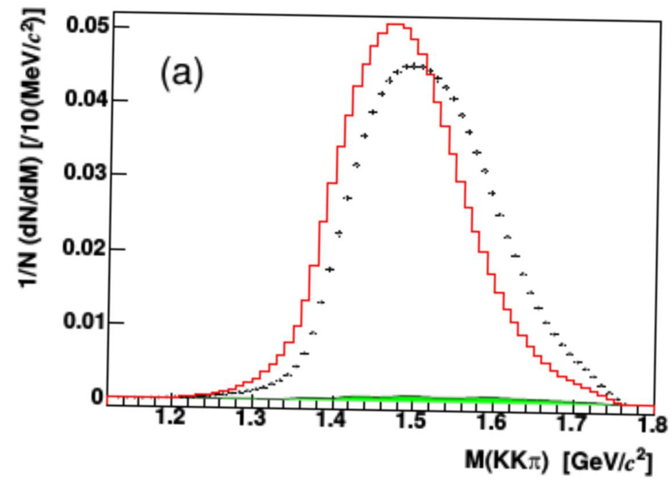
Belle MC/data



$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

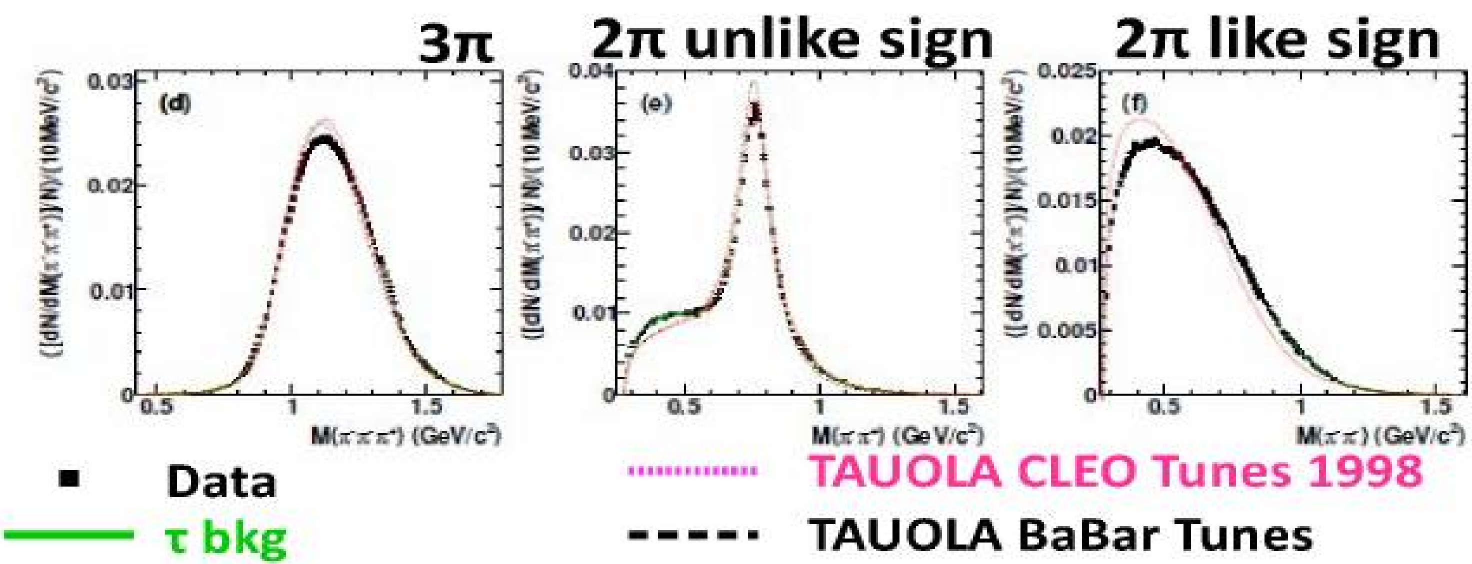


$\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$



$\tau^- \rightarrow K^-K^+\pi^-\nu_\tau$

BaBar MC/data



A. Lusiani, PhiPsi 13

RChL version

Hadronic currents for two and three meson decay modes:

$2\pi\tau$, $2K\ \tau$, $K\pi\tau$, $3\pi\tau$, $KK\pi\tau$ modes→ 88% of tau hadronic width

Hadronic form factors are:

• **Model: Resonance Chiral Lagrangian (*Chiral lagrangian with the explicit inclusion of resonances , G.Ecker et al., Nucl. Phys B321(1989)311*)**

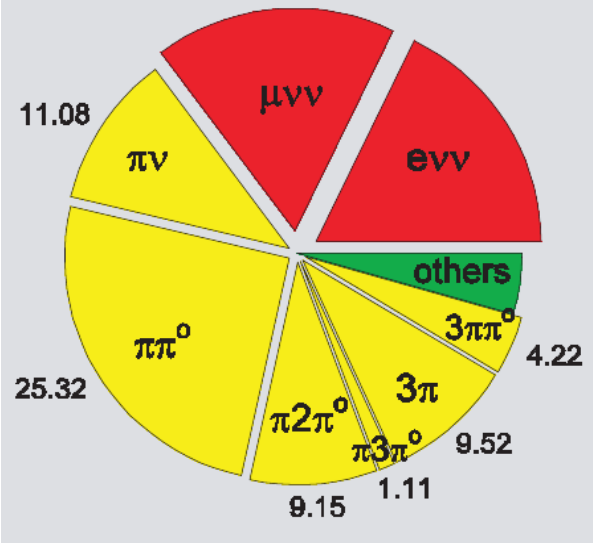
Feynman diagrams to calculate the currents

* The resonance fields ($V_{\mu\nu}$, $A_{\mu\nu}$ *antisymmetric tensor field*) is added by explicit way
* Reproduces NLO prediction of ChPT (at least)
* Correct high energy behaviour of form factors → relation between model parameters

Finite numbers of parameters *(one octet, one resonance approach: f_π , F_V, G_V, F_A)*

Analytical results for the hadronic form factors ***(Valencia IFIC group)***

Three pion decay modes $\tau^- \to (3\pi)^- \nu_\tau$



* CPC parametrization *a1/a1'* → $\rho\,\pi$

* Cleo parametrization

$\tau^- \to \pi^0\pi^0\pi^-\nu_\tau$ D. Asner et al., Phys.Rev. D61 (2000) 012002,hep-ex/9902022

$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$ E. I. Shibata, Nucl.Phys.Proc.Suppl.123 (2003)40 , hep-ex/0210039

J.W. Hinson, PhD thesis, Purdue University (2001), PU-99-713

* RChL parametrization V + A contribution (Phys.Rev. D86 (2012) 113008)

Cleo parametrization

$\tau^- \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ D. Asner et al., Phys.Rev. D61 (2000) 012002,hep-ex/9902022

Dalitz plots distributions

			Significance	Branching fraction (%)	$\lvert\beta\rvert$	phase φ/π
	$\rho\pi$	S-wave	–	68.11	1.00	0.0
	$\rho(1450)\pi$	S-wave	1.4σ	0.30 ± 0.64	0.12 ± 0.09	0.99 ± 0.25
a1(1200) →	$\rho\pi$	D-wave	5.0σ	0.36 ± 0.17	0.37 ± 0.09	-0.15 ± 0.10
	$\rho(1450)\pi$	D-wave	3.1σ	0.43 ± 0.28	0.87 ± 0.29	0.53 ± 0.16
	$f_2(1270)\pi$	P-wave	4.2σ	0.14 ± 0.06	0.71 ± 0.16	0.56 ± 0.10
	$f_0(600)\pi$	P-wave	8.2σ	16.18 ± 3.85	2.10 ± 0.27	0.23 ± 0.03
	$f_0(1370)\pi$	P-wave	5.4σ	4.29 ± 2.29	0.77 ± 0.14	-0.54 ± 0.06

$$B_Y^L(s_i) = \frac{m_{0Y}^2}{(m_{0Y}^2 - s_i) - i m_{0Y} \Gamma^{Y,L}(s_i)} \qquad \Gamma^{Y,L}(s_i) = \Gamma_0^Y \left(\frac{k_i'}{k_0'}\right)^{2L+1} \frac{m_{0Y}}{\sqrt{s_i}}$$

* fitted the beta constants + a1 mass and width
* instead o of the mass (555 MeV) and width (540 MeV) of sigma were fitted, the default version fixes sigma mass = 860 MeV and width = 880 MeV
* mass and width of the resonances were fixed to PDG'98

$\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ is not published;

Tauola (Pythia) with Cleo $\tau^- \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ parameter values

small difference in spectrum

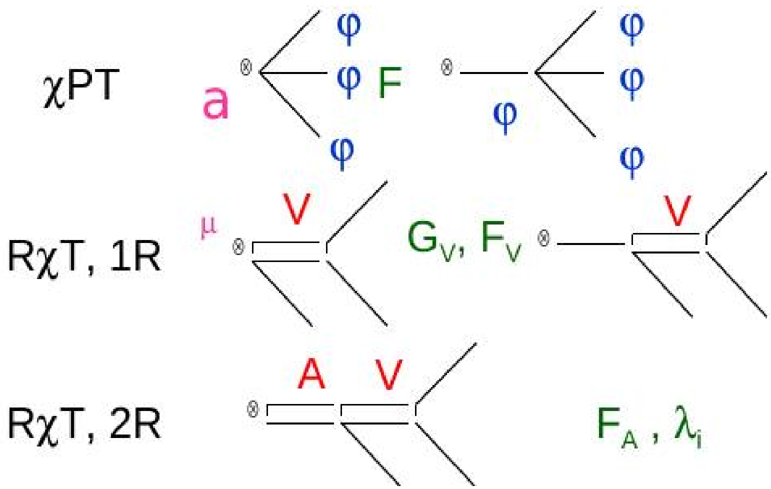
Resonance Chiral Theory results for three pion decay modes

RChL = ChPL + resonances (V, A, S, P) as new active degree of freedom

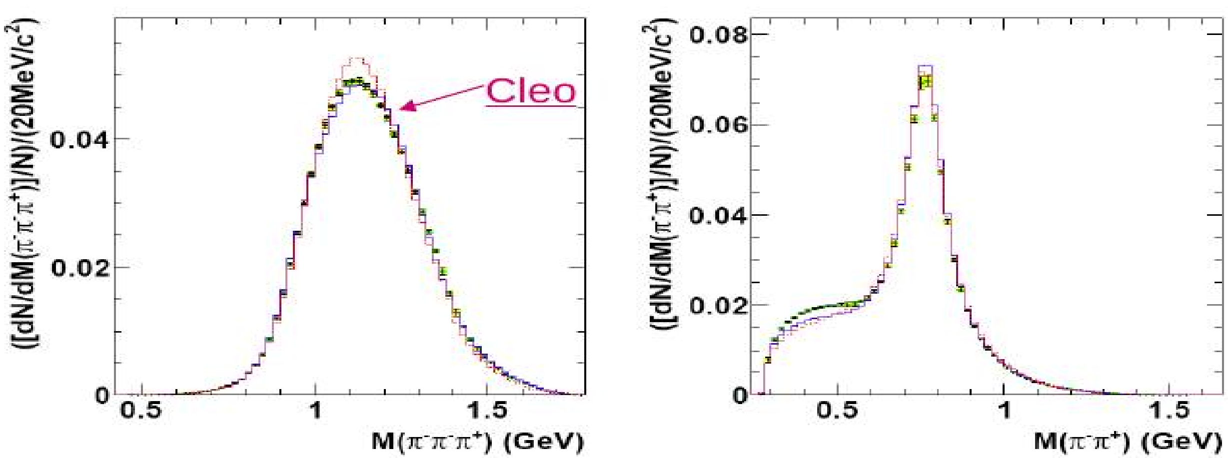
Phys.Rev. D86 (2012) 113008 → Phys. Rev. D 88, 093012 (2013)

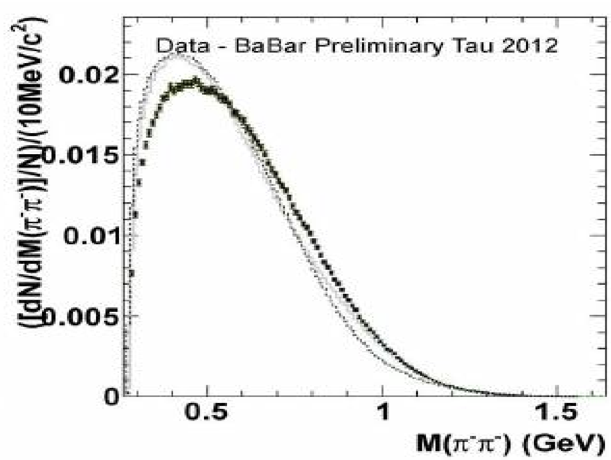
only V, A resonances

$$J^{\mu} = N \left\{ T^{\mu}_{\nu} \left[c_1 (p_2 - p_3)^{\nu} F_1 + c_2 (p_3 - p_1)^{\nu} F_2 + c_3 (p_1 - p_2)^{\nu} F_3 \right] + c_4 q^{\nu} F_4 - \frac{i}{4\pi^2 F^2} c_5 \varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5 \right\}$$



$F_2(Q^2, s_2, s_1) = - F_1(Q^2, s_1, s_2)$
the same for both 3 pion modes
$F_4 \sim m_\pi^2 / q^2$ is different





Doubts about inclusion of f0(500) = σ in RChL scheme

Cleo inspired contribution + RChL structure of FF

$\pi^- \pi^- \pi^+$

$$F_1^{\rm R} \rightarrow F_1^{\rm R} + \frac{\sqrt{2} F_V G_V}{3F^2} \left[\alpha_\sigma BW_\sigma(s_1) F_\sigma(q^2, s_1) + \beta_\sigma BW_\sigma(s_2) F_\sigma(q^2, s_2) \right]$$

$$F_1^{\rm RR} \rightarrow F_1^{\rm RR} + \frac{4 F_A G_V}{3F^2} \frac{q^2}{q^2 - M_{a_1}^2 - i M_{a_1} \Gamma_{a_1}(q^2)} \left[\gamma_\sigma BW_\sigma(s_1) F_\sigma(q^2, s_1) + \delta_\sigma BW_\sigma(s_2) F_\sigma(q^2, s_2) \right]$$

$$BW_\sigma(x) = \frac{m_\sigma^2}{m_\sigma^2 - x - i m_\sigma \Gamma_\sigma(x)} \quad \Gamma_\sigma(x) = \Gamma_\sigma \frac{\sigma_\pi(x)}{\sigma_\pi(m_\sigma^2)} \quad F_\sigma(q^2, x) = \exp\left[\frac{-\lambda(q^2, x, m_\pi^2) R_\sigma^2}{8 q^2} \right]$$

$\pi^0 \pi^0 \pi^-$

$$F_1^{\rm R} \rightarrow F_1^{\rm R} + \frac{\sqrt{2} F_V G_V}{3F^2} \alpha_\sigma^0 BW_\sigma(s_3) F_\sigma(q^2, s_3)$$

$$\alpha_\sigma = \beta_\sigma, \gamma_\sigma = \delta_\sigma$$

$$F_1^{\rm RR} \rightarrow F_1^{\rm RR} + \frac{4 F_A G_V}{3F^2} \frac{q^2}{q^2 - M_{a_1}^2 - i M_{a_1} \Gamma_{a_1}(q^2)} \gamma_\sigma^0 BW_\sigma(s_3) F_\sigma(q^2, s_3)$$

$$\alpha_\sigma^0 = \alpha_\sigma \cdot Scaling_{factor}^\gamma$$

$$\gamma_\sigma^0 = \gamma_\sigma \cdot Scaling_{factor}^\gamma$$

Our assumptions

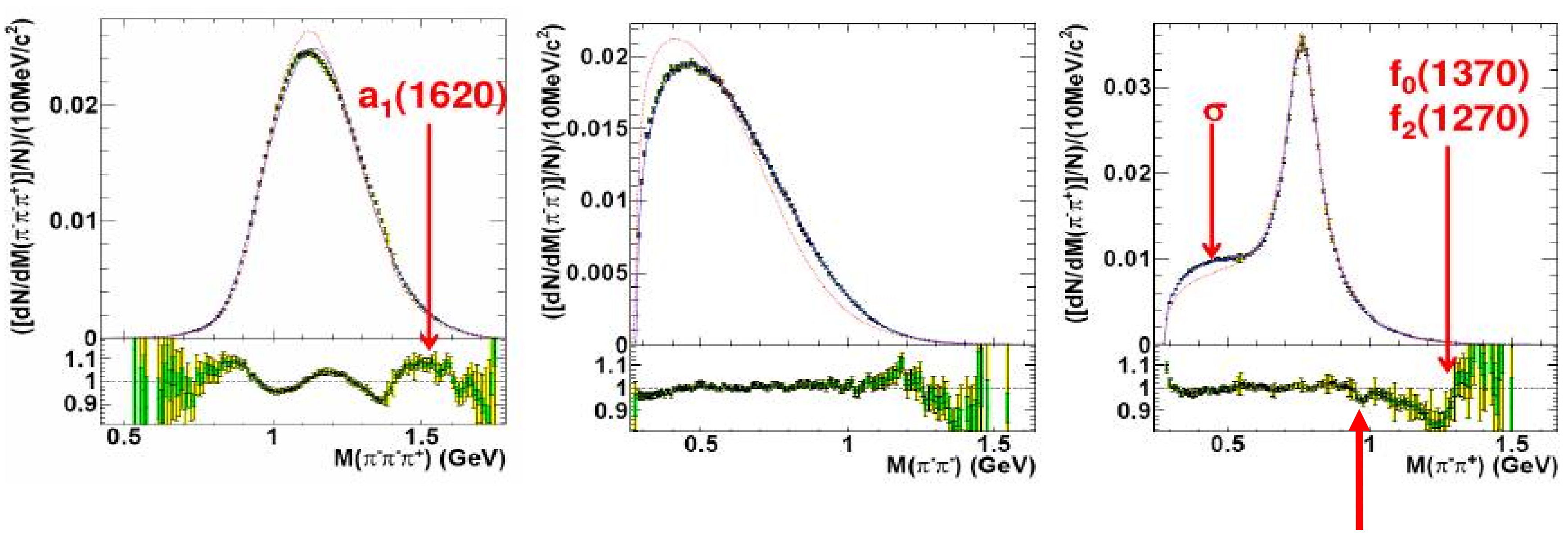
1* RChL structure of FF (but not RChL calculation)

2* simplest BW parametrization: only Im part of loop

3* two sets of parameters, different for $\pi^- \pi^- \pi^+$ and $\pi^0 \pi^0 \pi^-$

4* for $\pi^- \pi^- \pi^+$ we choose not equal parameters

Numerical results and fit to BaBar data



How include these resonances?

* a1(1260) axial-vector, the second one, analogous to rho'

* f2(1270); the lowest tensor resonance, G. Ecker, C. Zauner arXiv: 0705.0624
  + double resonance Lagrangian

* f0(980):
R. Escribano, P. Masjuan, J.J. Sanz-Cillero
ArXiv: 1011.5884
$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for I = 0

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2\phi_S}{M_{f_0}^2 - s} + \frac{\cos^2\phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

» *f0(1370); PDG m = 1200-1500 MeV; Γ = 200 – 500 MeV*

RChL parametrization to BaBar: three 1 dim mass invariant distributions;

Validation of results

* **Statistical errors and correlations between model parameters**
  - Hesse algorithm of Minuit package

* **Convergence of the fitting procedure**
  - random scan of 210 K points; select 1K with the best chi2
  - from them select 20 points with maximum distance;
    use them as a start point for the full fit and apply the full fit procedure

> 50% converge to the minimum (*others falls with number of parameters at their limits, converge to local minimum with higher chi2*)

The fitting procedure does not depend on an initial point

* **Toy MC studies to check of behaviour near the minimum**
  - 8 MC samples (different seeds) of 20 million generated with
  (I) the fit parameter values ('global minimum'), i.e. difference is “statistical error”, a set “Toy”
  (II) the set “Toy” is fitted
    (a) the starting point is the 'global' minimum
    (b) the starting point is the initial parameter values

The results of fit are consistent, i.e. the fitting procedure is stable

* **Estimation of systematic uncertainties**
  Used systematical covariance matrix from BaBar experiment to include the correlations between bins

Generalization
(under construction)

tauola_3pi.conf

```
# Configuration file for TAUOLA-FORTRAN RChL currents, mode: pi- pi- pi+
#
# See ../README for details regarding the config options
#

SET NCORES 8

#      ROOTFILE NAME    HISTO  FUNCTION
HISTO DATA.pipipi.root h12    dgams3_3pi_
HISTO DATA.pipipi.root h13_23 dgams2_3pi_
HISTO DATA.pipipi.root h123   dgamqq_3pi_

#       NAME      START_VAL     MIN         MAX
PARAM ALPSIG   -8.795938      -10.0       10.0
PARAM BETASIG   9.763701      -10.0       10.0
PARAM GAMSIG    1.264263      -10.0       10.0
PARAM DELSIG    0.656762      -5.0         5.0
PARAM RSIGMA    1.866913      -10.0       10.0
PARAM MRO       0.771849       0.767       0.780
PARAM MRHO1     1.350000       1.35        1.50
PARAM GRHO1     0.448379       0.30        0.50
PARAM MMA1      1.091865       0.99        1.25
PARAM MSIG      0.487512       0.400       0.550
PARAM GSIG      0.700000       0.400       0.700
PARAM FPI_RPT   0.091337       0.088       0.094
PARAM FV_RPT    0.168652       0.11        0.25
PARAM FA_RPT    0.131425       0.1         0.2
PARAM BETA_RHO -0.318551      -0.37       -0.17

CHANGE change_rcht_param_
INIT   rcht_3pi_init_
REINIT recalculate_a1_width_table
FINAL  check_integrals_qq_s1_s2_s3_

RUN MINUIT2 MIGRAD
RUN MINUIT2 HESSE
RUN MINUIT2 MINOS
RUN MINUIT2 PRINT
RUN MINUIT2 CHI2
RUN DRAW tauola_3pi.png tauola_3pi.eps
```

/FitFramework

- tools with Minuit2 commands
  chi2 calculation

/fitting_interface
- analytical function for fit

/tauola
- currents, models

Cross-check:

- 3 pion RChL current fit to BaBar data

New:
* multi- dim fit
* covariance matrix
* Cleo parametrization to fit BaBar data

Preliminary results + under study

1. $\tau^- \to \pi^-\pi^0\nu_\tau$ two pion form factor, fit to Belle data

TAUOLA two pion FF: $J^\mu = N\left[(p_1-p_2)^\mu F^V(s) + (p_1+p_2)^\mu F^S(s)\right]$

* KS FF from CPC

$$F_\pi^{(I=1)}(q^2) = \frac{1}{1+\beta+\gamma+\cdots}\left(BW_\rho + \beta\, BW_{\rho'} + \gamma\, BW_{\rho''} + \cdots\right) \qquad BW_\rho = \frac{M_\rho^2}{(M_\rho^2-q^2) - i\sqrt{q^2}\Gamma_\rho(q^2)}$$

* GS

$$F_\pi(s) = \frac{1}{1+\beta+\gamma}(\mathrm{BW}_\rho + \beta\cdot\mathrm{BW}_{\rho'} + \gamma\cdot\mathrm{BW}_{\rho''}) \qquad \mathrm{BW}_i^{\mathrm{GS}} = \frac{M_i^2 + d\cdot M_i\Gamma_i(s)}{(M_i^2-s)+f(s)-i\sqrt{s}\Gamma_i(s)}$$

* RChL Eur.Phys.J.C27 (2003) 587 (rho, rho')

$$F_\pi(s) = \frac{1+\sum_i \frac{F_{V_i}G_{V_i}}{f^2}\frac{q^2}{M_{V_i}^2-q^2}}{1+\left(1+\sum_i \frac{2G_{V_i}^2}{f^2}\frac{q^2}{M_{V_i}^2-q^2}\right)\frac{2q^2}{f^2}\left[B_{22}^{r,(\pi)} + \frac{1}{2}B_{22}^{r,(K)}\right]}$$

* dispersive integral + modified high energy RChL

$$F_V^\pi(s) = \exp\left[\alpha_1\, s + \frac{\alpha_2}{2}\, s^2 + \frac{s^3}{\pi}\int_{s_{\rm thr}}^\infty ds'\, \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right]$$

low energy

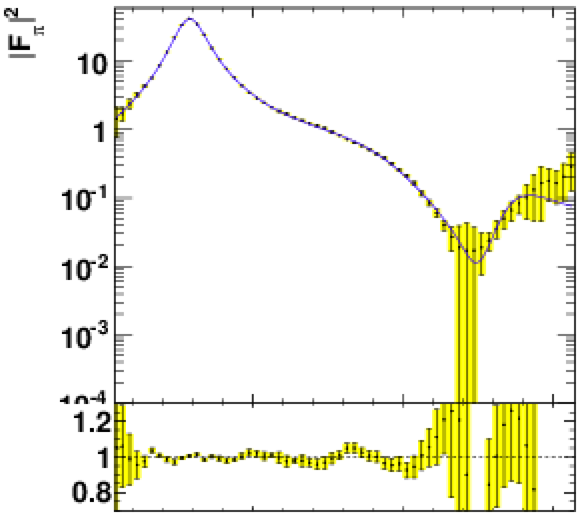
$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+\frac{1}{2}A_K(s)\right)\right]-s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\left[1+s\,C_{\rho'}A_\pi(s)\right]-s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\left[1+s\,C_{\rho''}A_\pi(s)\right]-s}$$

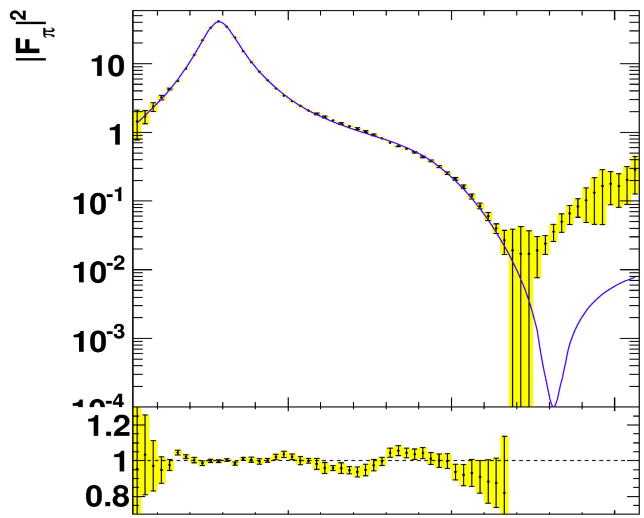
high energy

Belle parametrization

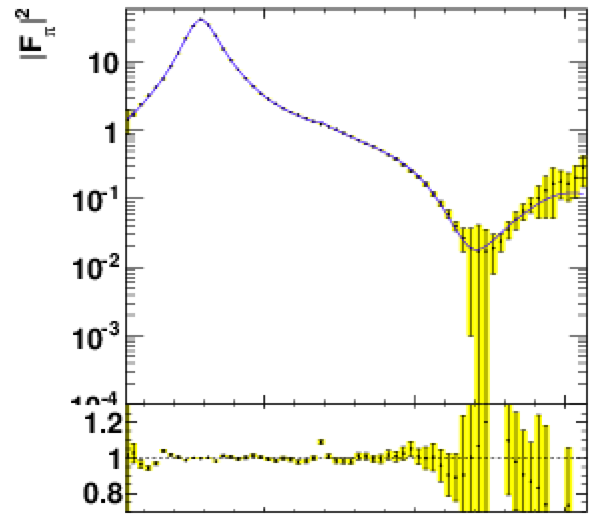
RChL parametrization

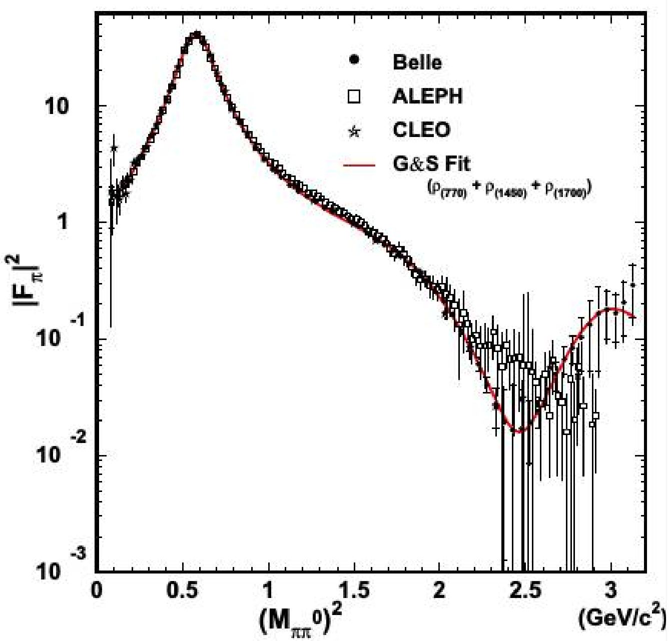
dispersive integral



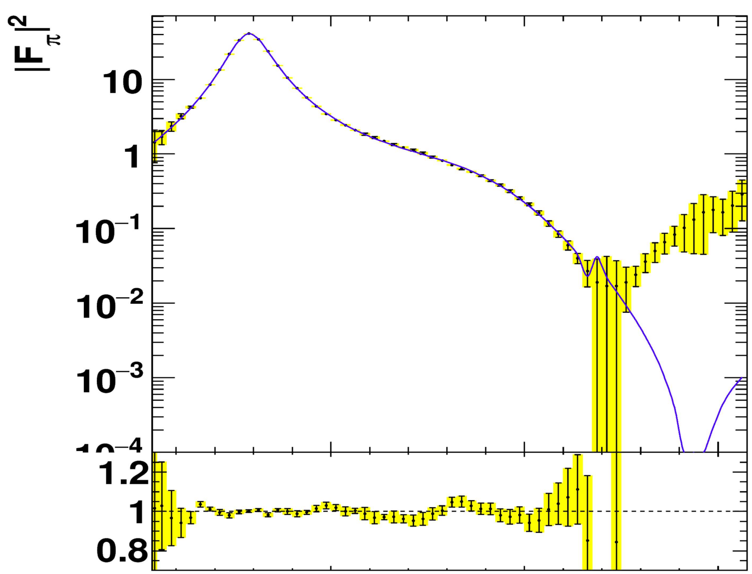


rho + rho'





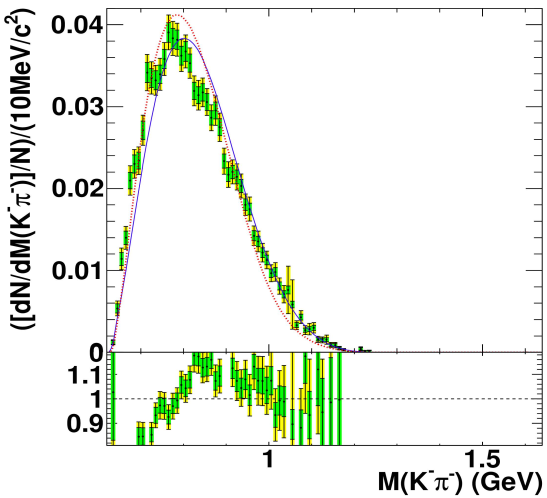
Phys Rev D 78 (2008) 072006

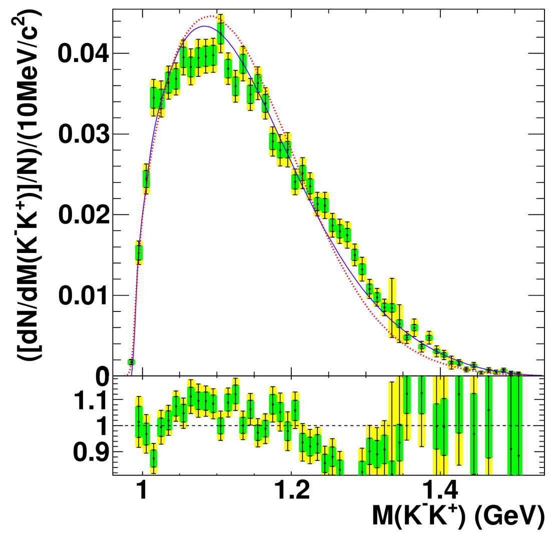


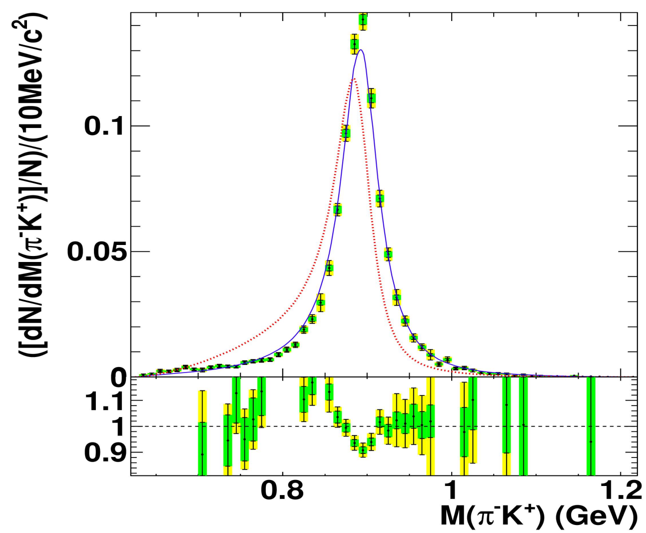
rho + rho' +rho''

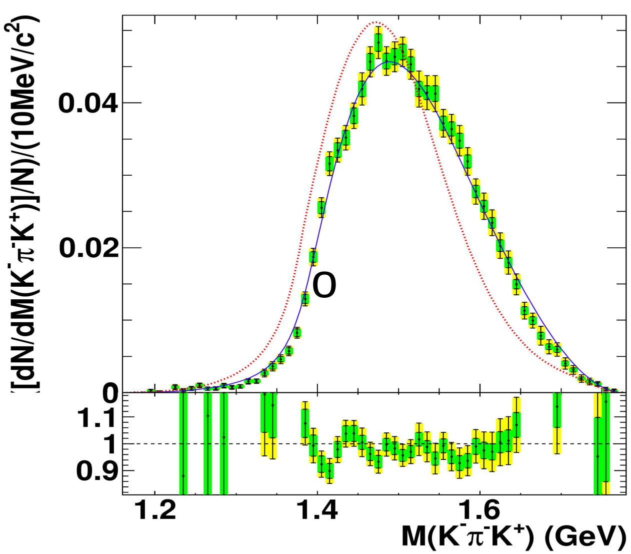
2. $\tau^- \to K^+ K^- \pi^- \nu$

Preliminary fitting results to BaBar preliminary data









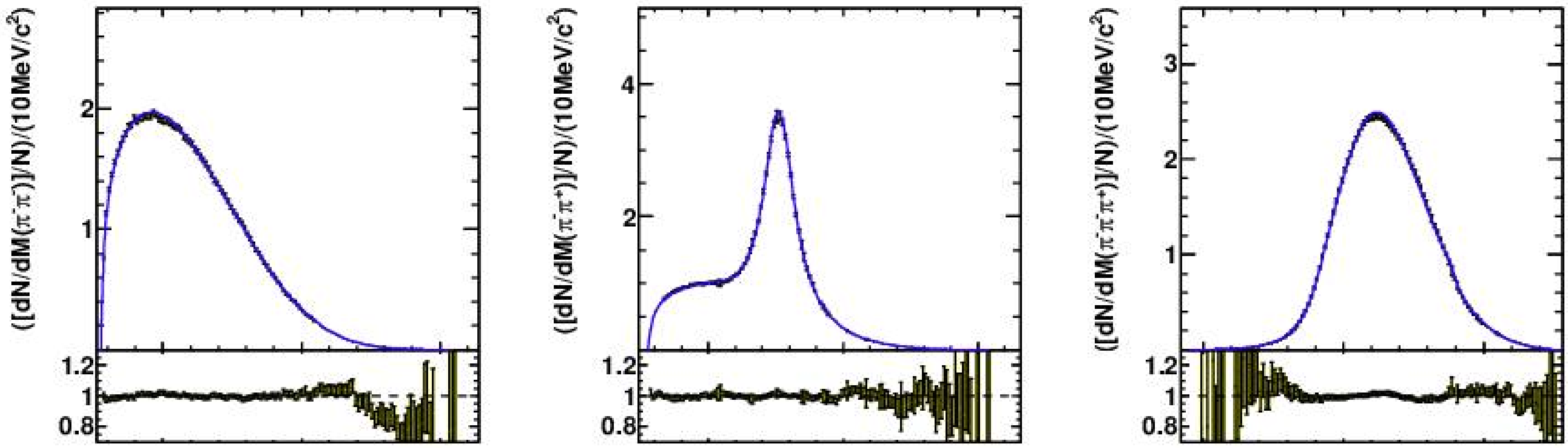
Blue – RChL   Red – Cleo

some parameters on their limits …

* generalization of 3 pion fit strategy
* in contrary to 3 pion case, no discussion of experimental systematic errors yet
* *the a1 width table corresponds to 3pion parameter values, not re-tabulated*

… common fit to $\pi^+\pi^-\pi^-$ and $K^+K^-\pi^-$

3. Cleo $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$ parametrization, fit to BaBar 1 dim histo



29.06.2015 result, no error study, no comparison with the Cleo parameter values

Scalar resonance contribution

$\pi^-\pi^-\pi^+$ $$F_1^{RR} \to F_1^{RR} + \frac{4F_A G_V}{3F^2}\frac{q^2}{q^2 - M_{a_1}^2 - iM_{a_1}\Gamma_{a_1}(q^2)}\left[\gamma_\sigma BW_\sigma(s_1)F_\sigma(q^2,s_1) + \delta_\sigma BW_\sigma(s_2)F_\sigma(q^2,s_2)\right]$$

$\pi^0\pi^0\pi^-$ $$F_1^{RR} \to F_1^{RR} + \frac{4F_A G_V}{3F^2}\frac{q^2}{q^2 - M_{a_1}^2 - iM_{a_1}\Gamma_{a_1}(q^2)}\gamma_\sigma^0 BW_\sigma(s_3)F_\sigma(q^2,s_3)$$

Resonance lagrangian:

$$\Delta\mathcal{L}_S = c_d\langle S u_\mu u^\mu \rangle + c_m\langle S\chi_+ \rangle \qquad \Delta\mathcal{L}_{AS} = \lambda_1^{AS}\langle \{\nabla_\mu S, A^{\mu\nu}\} u_\nu \rangle$$

$$F^{\pi^0\pi^0\pi^-} = -\frac{2\sqrt{2}c_d}{F^2}\frac{s_3}{3} * \left(\frac{\lambda_A F_A}{F^2}\frac{q^2}{q^2 - M_a^2 - iM_a\Gamma_a(q^2)} - \frac{\sqrt{2}c_d}{F}\right)$$

$$F^{\pi^-\pi^-\pi^+} = \frac{2\sqrt{2}c_d}{F^2}\frac{1}{3}\left(\frac{2s_2}{D_{scal}(s_2)} - \frac{s_1}{D_{scal}(s_1)}\right) * \left(\frac{\lambda_A F_A}{F^2}\frac{q^2}{q^2 - M_a^2 - iM_a\Gamma_a(q^2)} - \frac{\sqrt{2}c_d}{F}\right)$$

R. Escribano, P. Masjuan, J.J. Sanz-Cillero
ArXiv: 1011.5884

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2\phi_S}{M_{f_0}^2 - s} + \frac{\cos^2\phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for I = 0

Implementation in Tauola and fit to BaBar

tauola-bbb project

Fortran codes + C++ wrappers;
prepared to work with BaBar and Belle environment

- **Achieved:**
- TAUOLA MC with 200 decay channels, solution similar as presented on TAU04 and used by BaBar. Neutrinoless channels available.
- **Default BaBar Tauola initialization.**
- Alternatively, for 2 and 3 π's, new currents with comparison with experimental data prepared.
- Theoretically motivated currents, 4 and 5 π's decay modes, also as alternative.
- No fits to global properties such as average charged energy. For alternatives, no experimental quality stamps.
- User can re-initialize TAUOLA with own (C++ coded) currents (or matrix elements).

- **Non complete tasks:**
- Results for 3-scalar modes with K's are not incorporated, need quality fits. See e.g. Olga talk.
- Many alternative parametrizations, eg. for 2K 2π modes (BaBar) are not incorporated, even though these are missing channels, at present only flat phase space.
- Environments for fits are not well structured for model independent use.

CONCLUSION / PLANS

- study of TAUOLA models for two and three pion decay modes

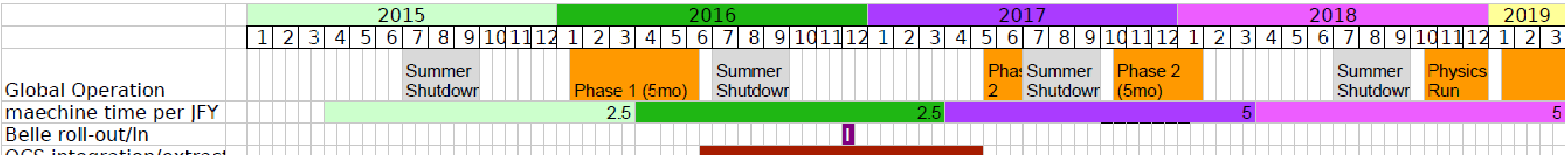
- improvements of 3 pion RChL current

- multi-dim fit; fitting strategy; fitting Cleo currents to BaBar data

- comparison Tauola with Pythia8, physics, numerical results
  (under work for 3 pion modes)

- K K π modes, models, fit

When do we start Belle II ?



Tom Browder (B2TiP meeting)

BEAST PHASE I: Starts in Jan 2016
BEAST PHASE II: Starts ~May 2017
Physics Running: Fall 2018

BACK UP

OUTLINE

* Monte Carlo generator TAUOLA and their context

* Two and three pion decay modes in TAUOLA

* Tools and fitting to BaBar $\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$ data

* tauola-bbb project

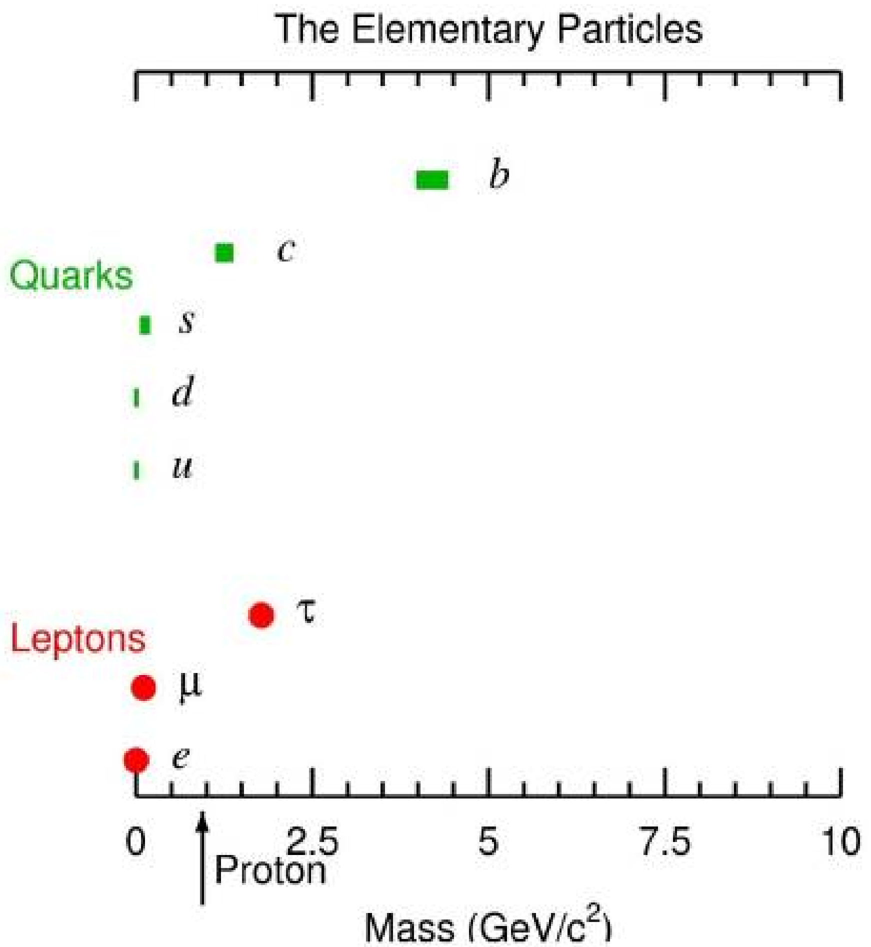
Conclusion and plans

What Physics does the τ Provide Access to?

The τ is the most massive charged lepton,

as such it provides:

- A unique environment to determine $|V_{us}|$
- Test the Charged Lepton Universality assumption in the Standard Model
- Provides a clean environment to study QCD
  - Strong coupling constant $\alpha_s(M_\tau)$
  - Search for second class currents (allowed by QCD but never observed)
  - Wess-Zumino Anomaly
  - Resonance structure
  - Okubo-Zweig-Iizuka Suppression
  - Test of Charged Vector Current (CVC)
- Search for New Physics
  - Lepton Flavour Violation (LFV)
- τ mass measurement
- τ life time measurement



> 20 modes: leptonic modes

hadronic modes **π^-, K^-, $\pi^0 \pi^-$, $K^- K^0$, $(3\pi)^-$, $(KK\pi)^-$, $\eta\pi^0 \pi^-$, $(4\pi)^-$, $(5\pi)^-$**

$(3\pi)^-$ D. Asner et al., Phys.Rev. D61 (2000) 012002, Dalitz plot analysis by Cleo

$(4\pi)^-$ fit to e+e- data, “Novosibirsk model”

others modes are from the theoretical models, include the lowest resonances

* do not reproduce the data

Features:

* based on VMD, i.e. 3 scalar modes BW(V1)*BW(V2) , reproduces LO ChPT limit

* wrong normalization for 2 scalar modes, except 2π, only vector FF , no scalar FF

** not correct low energy behaviour of the vector part for KKπ modes*

** 3 scalar mode results are not able to reproduce experimental data*

Belle (2π, $K\pi$) spectra, BaBar 3 meson invariant mass spectra

published

TAULOA

Monte Carlo generator for tau decays

***CPC* version**

***Cleo* version**

***Aleph* version**

***RChL* version**

2 pi Belle FF

*** *Belle MC = Cleo version***

TAUOLA (Monte Carlo generator for tau decay modes)

***Aleph* version** based on private communication with B. Bloch

* ***Aleph*** version in Tauola = **CPC** mechanism production with updated numerical values

* it does not include 'GS' 2 pion FF, also used by Aleph

2014 :

* M. Davier et al, Eur. Phys. J. C (2014) 74:2803
Update of the ALEPH non-strange spectral functions from hadronic τ decays

2 pion, 3 pion, 4 pion invariant mass squared distributions

http://aleph.web.lal.in2p3.fr/tau/specfun13.html

$\tau^- \rightarrow (3\pi)^- \nu_\tau$

PDG 2014 Br (ex K0)

$BR(\pi^0 \pi^0 \pi^-) = (9.3 \pm 0.11)\%$
$BR(\pi^- \pi^- \pi^+) = (9.02 \pm 0.06)\%$

Experiment data

Cleo, Aleph $\pi^0 \pi^0 \pi^-$ 1990–2000
Cleo, Aleph, BaBar, Belle $\pi^- \pi^- \pi^+$

Only BaBar measured the differential spectrum and preliminary data is available

= a1 → (intermediate resonance state = ρ, f0, π') + π

$BW(s) = m^2 /(m^2 - s - im\Gamma(s))$; vertex constant

Conclusions/plans for two pions

* fit with the Belle covariance matrix, to include bin-to-bin correlation

* kaon loop influence on the Belle parametrization
  ** ~2% at the rho peak for the RChL parametrization

* several-pion/kaon loops ($\omega\,\pi$, K* K)
  ** *Portoles, J. et al. Nucl.Phys.Proc.Suppl. 131 (2004) 170*

Resonance Chiral Curents in Tauola (RChL version)

RChL = ChPL + resonances (R =V, A, S, P) as new active degree of freedom

$$\mathcal{L}_{R\chi T} = \mathcal{L}^{(2)}_{pGB} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1,R_2} \mathcal{L}_{R_1R_2} + \sum_{R_1,R_2,R_3} \mathcal{L}_{R_1R_2R_3} + \ldots$$

One resonance part:

$$\mathscr{L}_R = \sum_i \Big\{ \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\mu\nu} \rangle + \frac{iG_{V_i}}{\sqrt{2}} \langle V_i^{\mu\nu} u_\mu u_\nu \rangle + \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\mu\nu} \rangle + c_{d_i} \langle S_i u^\mu u_\mu \rangle + c_{m_i} \langle S_i \chi_+ \rangle + i d_{m_i} \langle P_i \chi_- \rangle \Big\},$$

Antisymmetric formalizm for resonances

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu},$$

... few papers about S, P contributions

Resonance Chiral Theory results for three pion decay modes

RChT = ChPT + resonances (V, A, S, P) as new active degree of freedom

$$\mathcal{L}_{R\chi T} \;=\; \mathcal{L}^{(2)}_{pGB} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1,R_2} \mathcal{L}_{R_1 R_2} + \sum_{R_1,R_2,R_3} \mathcal{L}_{R_1 R_2 R_3} + \ldots$$

$$\mathcal{L}^{(2)}_{pGB} = \mathcal{L}^{\chi PT}_{2} = \frac{F^2}{4}\langle\, u_\mu u^\mu + \chi_+ \,\rangle$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \qquad u(\phi) = e^{\left(\frac{i}{\sqrt{2}F}\phi\right)}$$

$$\mathscr{L}_R = \sum_i \left\{ \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\mu\nu}\rangle + \frac{iG_{V_i}}{\sqrt{2}} \langle V_i^{\mu\nu} u_\mu u_\nu\rangle + \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\mu\nu}\rangle \right.$$
$$\left. + c_{d_i} \langle S_i u^\mu u_\mu\rangle + c_{m_i} \langle S_i \chi_+\rangle + i d_{m_i} \langle P_i \chi_-\rangle \right\},$$

Antisymmetric formalizm for resonances

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu},$$

Our assumptions

1 * RChT structure of FF (but not RChT calculation)
2 * simplest BW parametrization → only Im part of loop
3 * two sets of parameters, different for $\pi^-\pi^-\pi^+$ and $\pi^0\pi^0\pi^-$
4* for $\pi^-\pi^-\pi^+$ we choose not equal parameters

Preliminary answers

1* calculation within RChT will check $\mathcal{L}^S = c_d\langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle$

+ SA(u) lagrangian

2* width = Im part of loop function → + Re part of I=0 loop function

4* this point will be checked by calculation, however, in ChPT there is only one parameter → most probably we will have the same

3* calculation RchT, preliminary one (one resonance) shows equal parameters

The lowest scalar multiplet contribution: f0(980) and σ(500)

$$\mathcal{L}^S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle$$

$$S(x) = \begin{pmatrix} \frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & a^+ & \kappa^+ \\ a^- & -\frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \overline{\kappa}^0 & \frac{\sigma_0}{\sqrt{3}} - \sqrt{\frac{2}{3}}\sigma_8 \end{pmatrix}$$

ArXIv: 1011.5884; to include f0

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2\phi_S}{M_{f_0}^2 - s} + \frac{\cos^2\phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

PDG 2014: m = 990 ± 20 MeV; Γ = 40-100 MeV $\phi_S = -8^\circ$

$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for I = 0, a complex function
A real part of this function enters the nominator

Lagrangian with A S meson

Tensor resonance contribution f2(1270)

G. Ecker, C. Zauner arXiv: 0705.0624

$$\mathcal{L} = -\frac{1}{2}\langle T_{\mu\nu} D_T^{\mu\nu,\rho\sigma} T_{\rho\sigma}\rangle + \langle T_{\mu\nu} J_T^{\mu\nu}\rangle$$

$$T_{\mu\nu} = T^0_{\mu\nu}\frac{\lambda_0}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i T^{8,i}_{\mu\nu}$$

$$\frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i T^{8,i} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} \end{pmatrix}, \quad T^0 = f_2^0 .$$

No study T A (meson) ⟶ Lagrangian with A S meson

****

Fit Cleo paremetrization to BaBar data

The lowest scalar multiplet contribution: f0(980) and σ(500)

$$\mathcal{L}^S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle$$

$$S(x) = \begin{pmatrix} \frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & a^+ & \kappa^+ \\ a^- & -\frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \overline{\kappa}^0 & \frac{\sigma_0}{\sqrt{3}} - \sqrt{\frac{2}{3}}\sigma_8 \end{pmatrix}$$

ArXIv: 1011.5844; to include f0

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2\phi_S}{M_{f_0}^2 - s} + \frac{\cos^2\phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

PDG 2014: m = 990 ± 20 MeV; Γ = 40-100 MeV $\quad\quad \phi_S = -8^\circ$

$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for I = 0, a complex function
A real part of this function enters the nominator

Our assumptions

1 * RChT structure of FF (but not RChT calculation)
2 * simplest BW parametrization: only Im part of loop
3 * two sets of parameters, different for $\pi^-\pi^-\pi^+$ and $\pi^0\pi^0\pi^-$
4 * for $\pi^-\pi^-\pi^+$ we choose not equal parameters

Study

4*
$$F_1^R \to F_1^R + \frac{\sqrt{2}F_V G_V}{3F^2}\left[\alpha_\sigma BW_\sigma(s_1)F_\sigma(q^2,s_1) + \beta_\sigma BW_\sigma(s_2)F_\sigma(q^2,s_2)\right]$$

chiral prediction limits

$F(\pi^0\pi^0\pi^-) \to 1 + (16\,L1 + 8L3)/F^2\, s3 + 8L2/F^2(s2 - 2s1)$

$F(\pi^-\pi^-\pi^+) \to 1 - (16\,L1 + 8L3)/F^2\,(s2-2s1) + 8L2/F^2\, s3$

$$L_1 = \frac{G_V^2}{8M_V^2} - \frac{c_d^2}{6M_S^2} + \frac{\tilde{c}_d^2}{2M_{S_1}^2} \qquad L_2 = \frac{G_V^2}{4M_V^2} \qquad L_3 = -\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2}$$

Only V

$F(\pi^0\pi^0\pi^-) \to 1 + 16\,L1/F^2(-2s3 + s2 - 2s1)$

$-F(\pi^-\pi^-\pi^+) \to 1 + 16\,L1/F^2(-2s3 + s2 - 2s1)$

Phys Rev D 54, 4403 G. Colangelo, M. Finkemeier, R. Urech

3 pion decay of tau in general case

BR($\pi^- \pi^- \pi^0$)/BR($\pi^0 \pi^0 \pi^-$) = 1 for [210] structure, V resonance

BR($\pi^- \pi^- \pi^0$)/BR($\pi^0 \pi^0 \pi^-$) = 4 for [300] structure, S(T) resonance

F1($\pi^0 \pi^0 \pi^-$) = f(F1($\pi^- \pi^- \pi^+$))

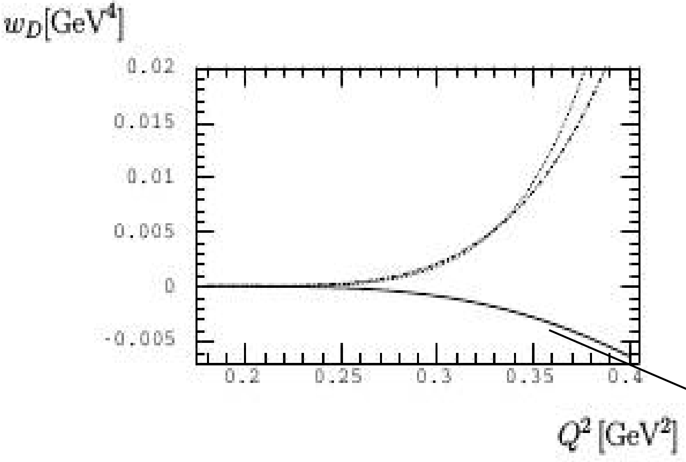
as well as calculation of F[300] and F[210] within ChPT one loop

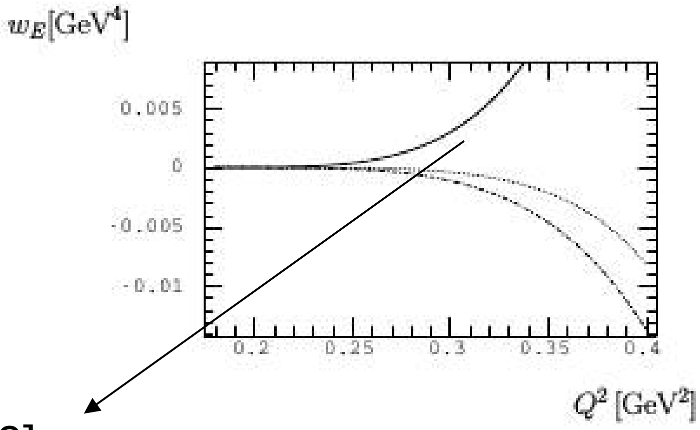
Implementation for 4 pion case → A. Pais Annals Of Physics 9 (1960) 548

Phys Rev D 54, 4403 G. Colangelo, M. Finkemeier, R. Urech

Influence of the F[300] to the integrated structure function

w_D sensitive to ReF[300], w_E to ImF[300]





No F[300]

Application for Tauola and BaBar data ???

Validation of results

* Statistical errors and correlations between model parameters
- Hesse algorithm of Minuit package

	α_σ	β_σ	γ_σ	δ_σ	R_σ	M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	Γ_σ	F_π	F_V	F_A	$\beta_{\rho'}$
α_σ	1	0.60	0.36	-0.29	-0.41	-0.69	0.46	0.68	-0.77	-0.09	0.02	0.78	0.76	0.52	-0.78
β_σ	0.60	1	0.44	-0.39	-0.42	-0.75	0.55	0.79	-0.89	-0.16	0.04	0.89	0.88	0.58	-0.88
γ_σ	0.36	0.44	1	-0.56	-0.22	-0.59	0.16	0.37	-0.47	-0.28	0.00	0.49	0.45	0.30	-0.45
δ_σ	-0.29	-0.39	-0.56	1	0.46	0.46	-0.24	-0.42	0.49	0.01	0.01	-0.49	-0.47	-0.31	0.47
R_σ	-0.41	-0.42	-0.22	0.46	1	0.42	-0.33	-0.56	0.62	0.34	0.02	-0.53	-0.56	-0.42	0.48
M_ρ	-0.69	-0.75	-0.59	0.46	0.42	1	-0.27	-0.64	0.79	0.29	-0.02	-0.83	-0.74	-0.48	0.75
$M_{\rho'}$	0.46	0.55	0.16	-0.24	-0.33	-0.27	1	0.67	-0.61	-0.13	0.03	0.61	0.66	0.37	-0.65
$\Gamma_{\rho'}$	0.68	0.79	0.37	-0.42	-0.56	-0.64	0.67	1	-0.88	-0.24	0.03	0.86	0.88	0.57	-0.88
M_{a_1}	-0.77	-0.89	-0.47	0.49	0.62	0.79	-0.61	-0.88	1	0.28	-0.03	-0.96	-0.97	-0.62	0.95
M_σ	-0.09	-0.16	-0.28	0.01	0.34	0.29	-0.13	-0.24	0.28	1	-0.02	-0.30	-0.29	-0.20	0.30
Γ_σ	0.02	0.04	0.00	0.01	0.02	-0.02	0.03	0.03	-0.03	-0.02	1	0.03	0.03	0.03	-0.04
F_π	0.78	0.89	0.49	-0.49	-0.53	-0.83	0.61	0.86	-0.96	-0.30	0.03	1	0.95	0.55	-0.97
F_V	0.76	0.88	0.45	-0.47	-0.56	-0.74	0.66	0.88	-0.97	-0.29	0.03	0.95	1	0.63	-0.96
F_A	0.52	0.58	0.30	-0.31	-0.42	-0.48	0.37	0.57	-0.62	-0.20	0.03	0.55	0.63	1	-0.56
$\beta_{\rho'}$	-0.78	-0.88	-0.45	0.47	0.48	0.75	-0.65	-0.88	0.95	0.30	-0.04	-0.97	-0.96	-0.56	1

Strong correlation > 0.95 $\quad M_{a_1}, F_\pi, F_V, \beta_{\rho'}$

Validation of results

*

* Convergence of the fitting procedure

to verify that the found minimum is a global minimum

- start with random scan of 210 K points
- select 1K with the best chi2
- from them select 20 points with maximum distance
- use them as a start point for the full fit and apply the full fit procedure

> 50% converge to the minimum

(*others falls with number of parameters at their limits, converge to local minimum with higher chi2*)

Indicates that the found minimum point is a global minimum
and the fitting procedure does not depend on an initial point

Validation of results

*

*

* Toy MC studies to check of behaviour near the minimum
  8 MC samples (different seeds) of 20 million generated with
  (I) the fit parameter values ('global minimum'), i.e. difference is “statistical error”, a set
  (II) the set “Toy” is fitted
  (a) the starting point is the 'global' minimum
  (b) the starting point is the initial parameter values

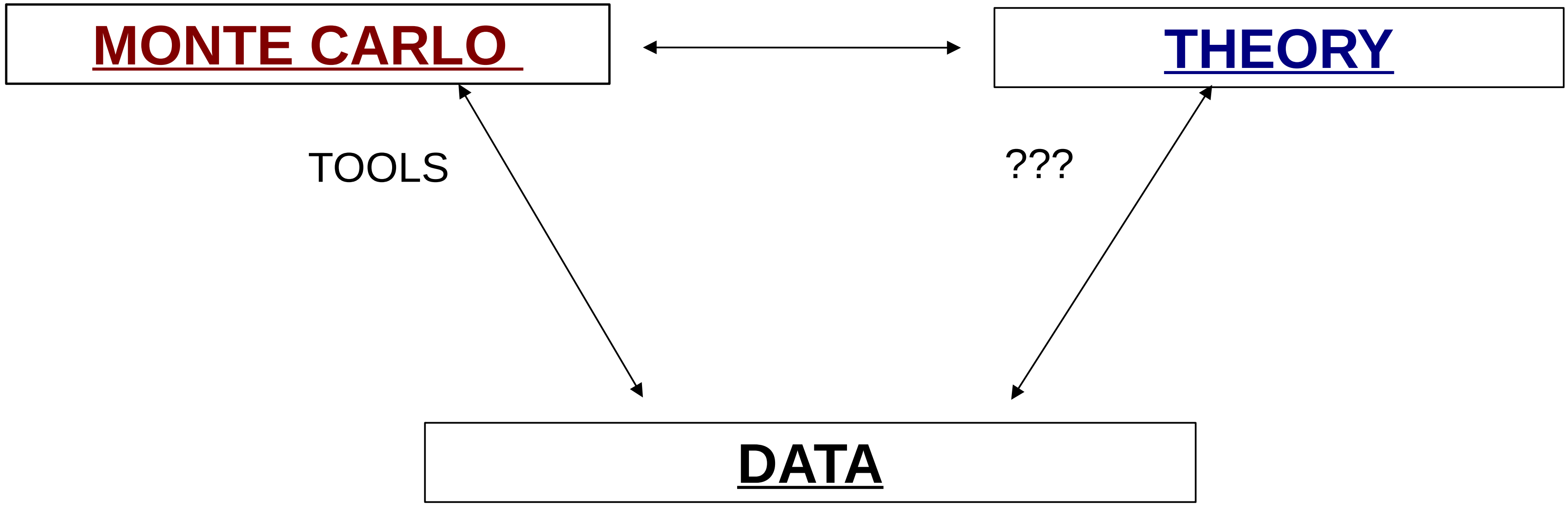
The results of fit are consistent, i.e. the fitting procedure is stable

Validation of results

*
*
*

* Estimation of systematic uncertainties

Used systematical covariance matrix from BaBar experiment to include the correlations between bins



Tools: 1dim or multi-dim