Quantum Monte Carlo calculations of electromagnetic moments and transitions in $A \leq 10$ nuclei with two-body χEFT currents

Saori Pastore @ CD15 - Pisa, Italy - July 2015

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PRC78(2008)064002 - PRC80(2009)034004 - PRC84(2011)024001 - PRC87(2013)014006 - PRC87(2013)035503 - PRL111(2013)062502 - PRC90(2014)024321

Outline

- Microscopic picture of the nucleus: the *ab initio* framework
- Many-body nuclear EM currents from χEFT
- Applications:
  - Magnetic moments and EM transitions in $A \leq 10$ systems
  - Light nuclei form factors and Zemach moments: Benchmark calculations Preliminary!
- Summary and outlook

- The nucleus is a system made of A non-relativistic interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Realistic υ_{ij} and V_{ijk} interactions are based on EXPT data fitting and fitted parameters subsume underlying QCD
- Potentials utilized in these sets of calculations to generate nuclear wave functions $|\Psi_i\rangle$ by solving $H|\Psi_i\rangle = E_i|\Psi_i\rangle$ are AV18+IL7

[Wiringa *et al.* PRC51(1995)38 + Piper *et al.* PRC64(2001)014001]

Green's function Monte Carlo

A trial w.f. Ψ_V is obtained by minimizing the H = T + AV18 + IL7 expectation value

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Ψ_V is further improved it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$

Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

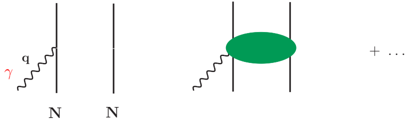
$$\langle O(\tau) \rangle = \frac{{}_f\langle \Psi(\tau) | O | \Psi(\tau) \rangle_i}{\langle \Psi(\tau) | \Psi(\tau) \rangle} \approx \langle O(\tau) \rangle^i_{\text{Mixed}} + \langle O(\tau) \rangle^f_{\text{Mixed}} - \langle O \rangle_V$$

$$\langle O(\tau) \rangle^i_{\text{Mixed}} = \frac{{}_f\langle \Psi_V | O | \Psi(\tau) \rangle_i}{{}_f\langle \Psi_V | \Psi(\tau) \rangle_i} \; ; \; \langle O(\tau) \rangle^f_{\text{Mixed}} = \frac{{}_f\langle \Psi(\tau) | O | \Psi_V \rangle_i}{{}_f\langle \Psi(\tau) | \Psi_V \rangle_i}$$

The Basic Model: Nuclear Electromagnetic Currents

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots\,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



- Longitudinal EM current operator **j** linked to the nuclear Hamiltonian via continuity eq. (**q** momentum carried by the external EM probe γ)

$$\mathbf{q}\cdot\mathbf{j} = [H, \rho] = \left[t_i + \upsilon_{ij} + V_{ijk}, \rho\right]$$

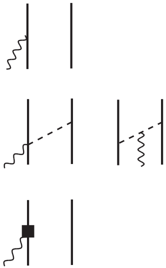
- Calculations with w.f.'s from "traditional (or conventional)" potentials and currents from χEFTs are called "hybrid calculations"

EM Current up to $n = 1$ (or up to N3LO)

LO : $\mathbf{j}^{(-2)} \sim \mathbf{eQ}^{-2}$

NLO : $\mathbf{j}^{(-1)} \sim \mathbf{eQ}^{-1}$

N^{2}LO : $\mathbf{j}^{(-0)} \sim \mathbf{eQ}^{0}$



* Two-body charge operators enter at N3LO and do not depend on LECs

- LO = IA
  N2LO = IA(relativistic-correction)
- NLO is purely isovector
- Strong contact LECs fixed from fits to *np* phases shifts—PRC**68**, 041001 (2003)
- No three-body EM currents at this order !

- 5 unknown EM LECs enter the N3LO contact and tree-level currents:
  - 2 isovector LECs entering the tree-level current are fixed by Δ-saturation
  - remaining 3 LECs fixed to reproduce $A = 2$ and 3 magnetic moments $\leftarrow$

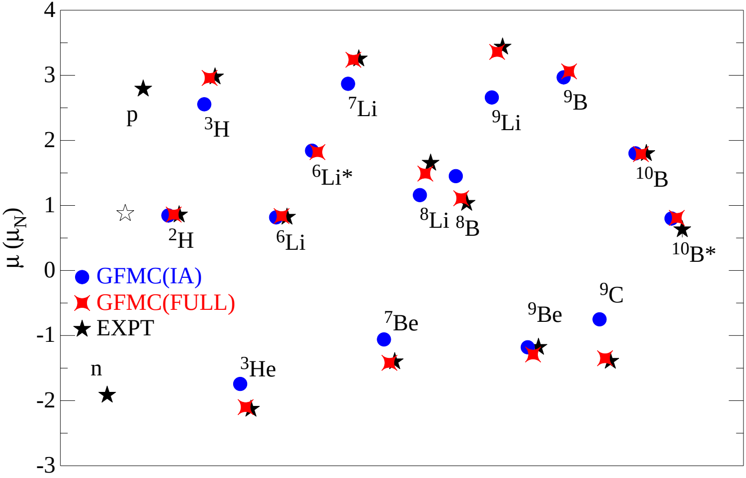
N^{3}LO: $\mathbf{j}^{(1)} \sim \mathbf{eQ}$

unknown LEC's →

PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

Magnetic Moments in $A \leq 10$ Nuclei

Predictions for $A > 3$ nuclei



- $\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$
- GFMC calculations based on H = T + AV18 + IL7

PRC87(2013)035503

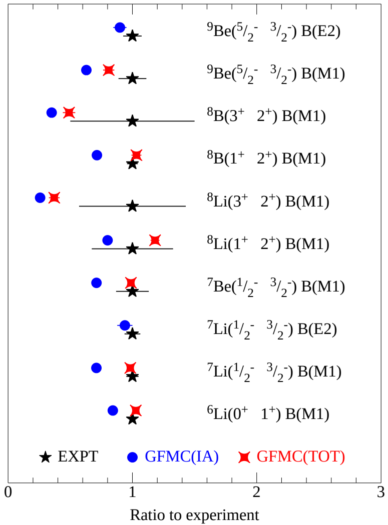
EM Transitions in $A \leq 9$ Nuclei

- Two-body EM currents bring the theory in a better agreement with the EXP
- Significant correction in $A = 9$, $T = 3/2$ systems. Up to $\sim 40\%$ correction found in ^{9}C m.m.
- Major correction ($\sim 60 - 70\%$ of total MEC) is due to the one-pion-exchange currents at NLO – purely isovector

One M1 prediction: ^{9}Li$(1/2 \rightarrow 3/2)$*

$\Gamma(\text{IA}) = 0.59(2)$ eV
$\Gamma(\text{TOT}) = 0.79(3)$ eV

+ a number of B(E2)s in IA



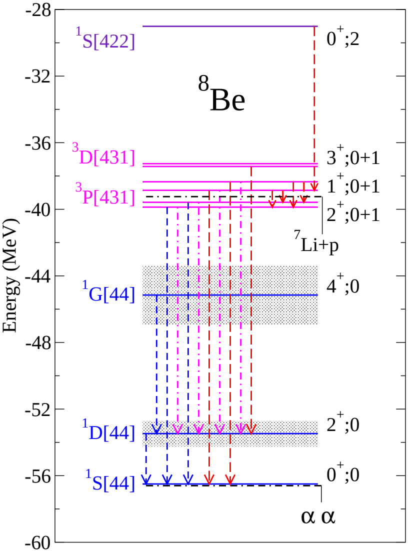
*Ricard-McCutchan *et al.* TRIUMF proposal 2014 - ongoing data analysis

PRC87(2013)035503

^{8}Be Energy Spectrum

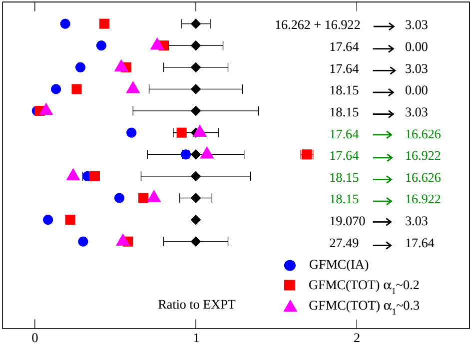
- 2^+ and 4^+ broad states at ~ 3 MeV and ~ 11 MeV
- isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- M1 transitions
- E2 transitions
- E2 + M1 transitions

$J^\pi;T$	GFMC	Iso-mixed	Experiment
0^+	-56.3(1)		-56.50
2^+	+ 3.2(2)		+ 3.03(1)
4^+	+11.2(3)		+11.35(15)
$2^+;0$	+16.8(2)	+16.746(3)	+16.626(3)
$2^+;1$	+16.8(2)	+16.802(3)	+16.922(3)
$1^+;1$	+17.5(2)	+17.67	+17.640(1)
$1^+;0$	+18.0(2)	+18.12	+18.150(4)
$3^+;1$	+19.4(2)	+19.10	+19.07(3)
$3^+;0$	+19.9(2)	+19.21	+19.235(10)



PRL111(2013)062502 & PRC90(2014)024321

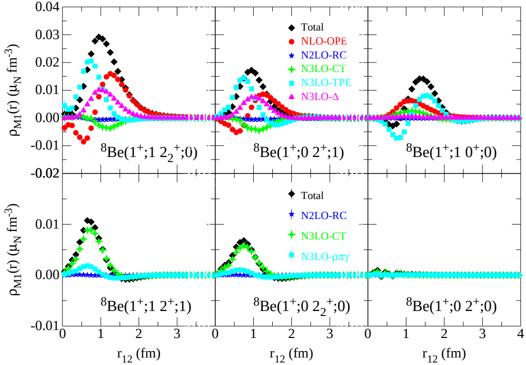
M1 Transition Widths / EXPT



- Predictions for [s.s.]-conserving transitions are in fair agreement with EXPT
- The theoretical description for this system is unsatisfactory, however, MEC provide a $\sim 20-30\%$ correction to the calculated matrix elements improving the agreement with EXPT data

PRC90(2014)024321

Two-body M1 transitions densities



$(J_i, T_i) \rightarrow (J_f, T_f)$	IA	NLO-OPE	N2LO-RC	N3LO-TPE	N3LO-CT	N3LO-Δ	MEC
$(1^+; 1) \rightarrow (2_2^+; 0)$	2.461 (13)	0.457 (3)	-0.058 (1)	0.095 (2)	-0.035 (3)	0.161 (21)	0.620 (5)

Benchmark calculations of ^{3}He Zemach Moments*

Quote: Precise moments are useful observables for the comparison with theoretical calculations, ... in particular for light nuclei where very accurate *ab initio* calculations can be performed. I. Sick - PRC90(2014)064002

$$\langle r \rangle_{(2)} \propto -\int_0^\infty \frac{dq}{q^2}\left[G_E G_M - 1\right], \qquad \langle r^3 \rangle_{(2)} \propto \int_0^\infty \frac{dq}{q^4}\left[G_E^2 - 1 + q^2 R^2/3\right]$$

	VMC(IA)	VMC(TOT)	GFMC(IA)	GFMC(TOT)	EXPT
$\langle r \rangle_{(2)}$	2.522	2.477	2.504	2.454	2.528 ± 0.016 fm
$\langle r^3 \rangle_{(2)}$	27.40	n.a.	29.30	n.a.	28.15 ± 0.70 fm^3
$\langle r_{ch}^2 \rangle^{1/2}$	1.967	n.a.	1.970	n.a.	1.973 ± 0.014 fm
$\langle r_{m}^2 \rangle^{1/2}$	2.000	1.962	2.019	1.942	1.976 ± 0.047 fm
$\langle r_{ch}^4 \rangle$	19.8	n.a.	30.0	n.a.	32.9 ± 1.60 fm^4
$\langle \mu \rangle$	-1.775	-2.134	-1.767	-2.129	-2.127 μ_N

* in collaboration with S. Bacca, C. Ji *et al.*

Preliminary!!!

Summary

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.
J.Phys.G41(2014)123002 - S.Bacca&S.P.

- Two-body EM currents from χEFT tested in $A \leq 10$ nuclei
- Two-body corrections can be sizable and improve on theory/EXPT agreement
- EM structure of $A = 2$–3 nuclei well reproduced with chiral charge and current operators for $q \lesssim 3m_\pi$, Piarulli *et al.* - PRC87(2013)014006
- $\sim 40\%$ two-body correction found in ^{9}C's m.m.
- $\sim$ 20-30% corrections found in M1 transitions in low-lying states of ^{8}Be
- Ongoing benchmark calculations of light nuclei Zemach moments

Outlook

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

J.Phys.G41(2014)123002 - S.Bacca&S.P.

* EM structure and dynamics of light nuclei
  - Charge and magnetic form factors of $A \leq 10$ systems
  - M1/E2 transitions in light nuclei
  - Radiative captures, photonuclear reactions ...
  - Fully consistent χEFT calculations with 'MEC' for $A > 4$
  - Role of Δ-resonances in 'MEC' (M. Piarulli)

* Electroweak structure and dynamics of light nuclei
  - Test axial currents (chiral and conventional) in light nuclei (A. Baroni)
  - Many-body effects in ν-d pion-production at threshold

EXTRA SLIDES

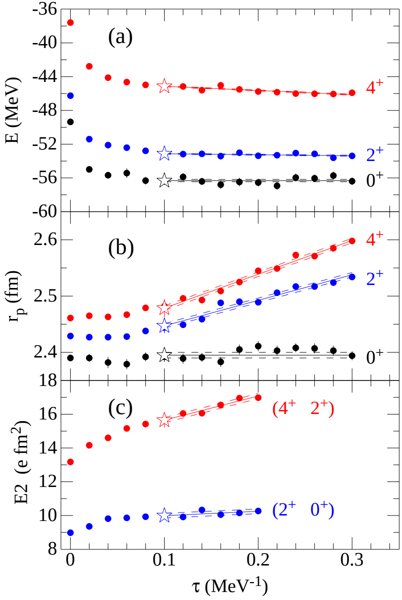
E2 transitions in ^{8}Be

- 2^+ and 4^+ broad rotational states at ~ 3 MeV and ~ 11 MeV
- $4^+ \rightarrow 2^+$ transition recently measured at BARC*, Mumbai
- Calculational challenge: 2^+ and 4^+ states tend to break up into two α as τ increases
- Results obtained by linear fitting the GFMC points and extrapolating at $\tau = 0.1$ MeV where stability is observed in the g.s. energy propagation

$J^\pi;T$	E [MeV]	B(E2) [e^2 fm^4]
0^+	-56.3(1)	
2^+	+ 3.2(2)	20.0 (8)– [$2^+ \rightarrow 0^+$]
4^+	+11.2(3)	27.2(15)– [$4^+ \rightarrow 2^+$]*

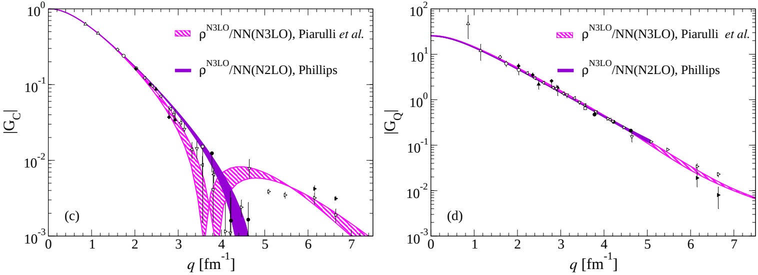
*Bhabha Atomic Research Centre

*EXPT B(E2) = 21 ± 2.3 e^2 fm^4



Applications:
EM form factors of nuclei with $A = 2$ and 3

Predictions with χEFT EM Currents for the Deuteron Charge and Quadrupole f.f.'s

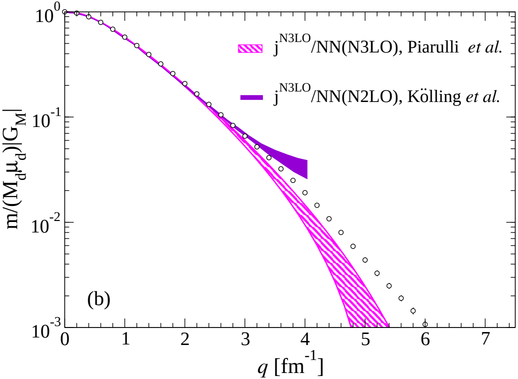


Λ MeV	$< r_d >$ (fm)	$< r_d >$ EXP	Q_d (fm^2)	Q_d (fm^2) EXP
500	1.976	1.9734(44)	0.285	0.2859(3)
600	1.968		0.282	

- Calculations include nucleonic f.f.'s taken from EXPT data
- Sensitivity to the cutoff used to regularize divergencies in the matrix elements is given by the bands' thickness

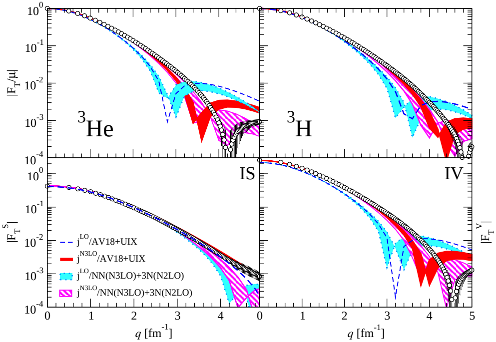
J.Phys.G34(2007)365 & PRC87(2013)014006

Predictions with χEFT EM Currents for the Deuteron Magnetic f.f.



PRC86(2012)047001 & PRC87(2013)014006

Predictions with χEFT EM Currents for ^{3}He and ^{3}H Magnetic f.f.'s



LO/N3LO with AV18+UIX – LO/N3LO with χ-potentials NN(N3LO)+3N(N2LO)

- ^{3}He/^{3}H m.m.'s used to fix EM LECs; $\sim$ 15% correction from two-body currents
- Two-body corrections crucial to improve agreement with EXPT data

	^{3}He $< r >_{EXP} = 1.976 \pm 0.047$ fm		^{3}H $< r >_{EXP} = 1.840 \pm 0.181$ fm	
Λ	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
N3LO	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

PRC87(2013)014006

Calculations with EM Currents from χEFT with π's and N's

- Park, Min, and Rho *et al.* (1996)

  applications to:
  magnetic moments and M1 properties of A=2–3 systems, and
  radiative captures in A=2–4 systems by Song, Lazauskas, Park *at al.* (2009-2011) within the hybrid approach

  ......
  * Based on EM χEFT currents from NPA596(1996)515

- Meissner and Walzl (2001);
  Kölling, Epelbaum, Krebs, and Meissner (2009–2011)

  applications to:
  d and ^{3}He photodisintegration by Rozpedzik *et al.* (2011); e-scattering (2014);
  d magnetic f.f. by Kölling, Epelbaum, Phillips (2012);
  radiative $N-d$ capture by Skibinski *et al.* (2014)

  ......
  * Based on EM χEFT currents from PRC80(2009)045502 & PRC84(2011)054008 and consistent χEFT potentials from UT method

- Phillips (2003-2007)

  applications to deuteron static properties and f.f.'s

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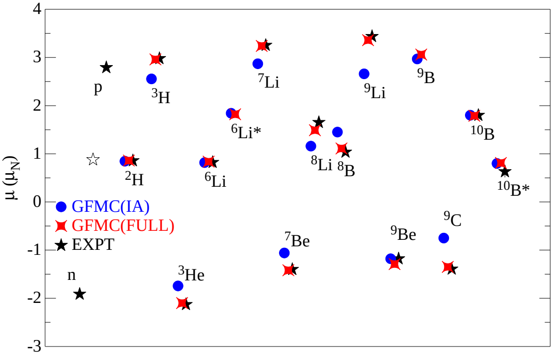
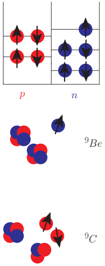
Magnetic Moments in $A \leq 10$ Nuclei - bis

Predictions for $A > 3$ nuclei

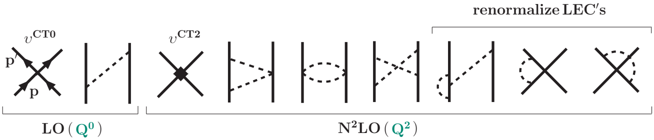




- $\mu_N(\text{IA}) = \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$
- ^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = [α,^{3}He(^{3}H),pp(nn)] $\rightarrow$ Large MEC
- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = [α, α, n(p)]

PRC87(2013)035503

NN Potential at NLO (or $Q^{n=2}$)



- Contact potential at LO (or $Q^{n=0}$) depends on 2 LECs
- Contact potential at NLO (or $Q^{n=2}$) depends on 7 additional LECs

NN potentials with π's and *N*'s

∗ van Kolck *et al.* (1994–96)

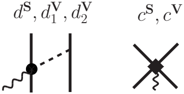
∗ Kaiser, Weise *et al.* (1997–98)

∗ Epelbaum, Glöckle, Meissner (1998–2015)

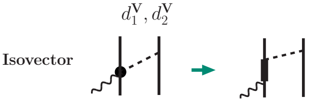
∗ Entem and Machleidt (2002–2015) ←

∗ …

χEFT EM currents at N3LO: fixing the EM LECs



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isoscalar sector:
  - * d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$
- Isovector sector:
  - * model I = c^V from EXPT $npd\gamma$ xsec.

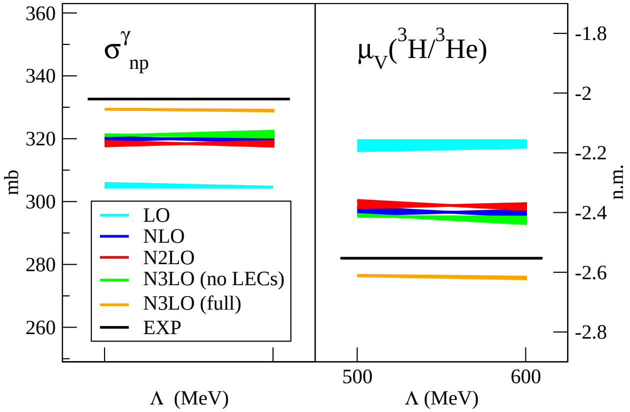
    or
  - * model II = c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. $\leftarrow$ our choice

Note that:
χEFT operators have a power law behavior $\rightarrow$ introduce a regulator to kill divergencies at large Q, *e.g.*, $C_\Lambda = e^{-(Q/\Lambda)^n}$, ...and also, pick n large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \ldots$$

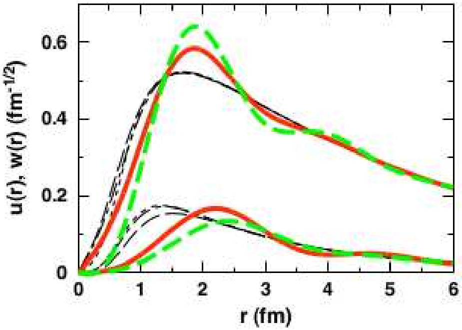
Predictions with χEFT EM currents for $A = 2$–3 systems

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)



- *npdγ* xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- Two-body currents important to reach agreement with exp data
- Negligible dependence on the cutoff entering the regulator $exp(-(k/\Lambda)^4)$

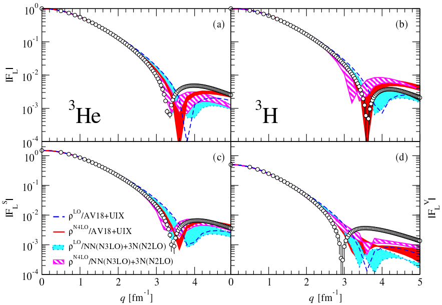
Deuteron wave functions



from Entem&Machleidt 2011 Review

- Entem&Machleidt N3LO
- Epelbaum *et al.* 2005
- black lines = conventional potentials, *i.e.* AV18, CD-Bonn, Nijm-I

^{3}He and ^{3}H charge f.f.'s



- Excellent agreement up to $q \simeq 2$ fm^{-1}
- N3LO and N4LO comparable

	^{3}He $< r >_{EXP} = 1.959 \pm 0.030$ fm		^{3}H $< r >_{EXP} = 1.755 \pm 0.086$	
Λ	500	600	500	600
LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N4LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)

Anomalous magnetic moment of ^{9}C

Mirror nuclei spin expectation value

- Charge Symmetry Conserving (CSC) picture ($p \longleftrightarrow n$) $^{\diamond}$

$$< \sigma_z >= \frac{\mu(T_z=+T)+\mu(T_z=-T)-J}{(g_s^p+g_s^n-1)/2} = \frac{2\mu(\mathrm{IS})-J}{0.3796}$$

- For $A=9$, $T=3/2$ mirror nuclei: ^{9}C and ^{9}Li
  EXP $< \sigma_z >= 1.44$ while THEORY $< \sigma_z > \sim 1$ (assuming CSC)
  possible cause: Charge Symmetry Breaking (CSB)
- Three different predictions for $< \sigma_z >$ with CSC w.f.'s (*) and CSB w.f.'s

$< \sigma_z >$	Symmetry	IA	TOT	EXP
CSB	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.05(1)	1.31(11)	1.44
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})*$	0.95 (11)	1.00 (11)	
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2})*,^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.00 (1)	1.05 (1)	

- Need both CSB in the w.f.'s and MEC!

$^{\diamond}$ Utsuno – PRC**70**, 011303(R) (2004)

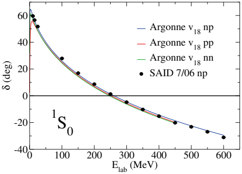
NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

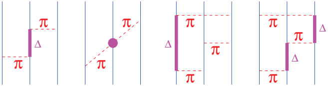
Argonne v18: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij}) O_{ij}^p$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1



Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$

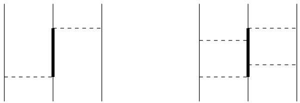


- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

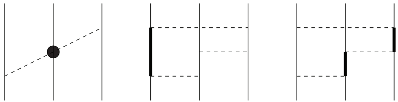
THREE-NUCLEON POTENTIALS

Urbana $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



Carlson, Pandharipande, & Wiringa, NP **A401**, 59 (1983)

Illinois $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{2\pi S} + V_{ijk}^{3\pi\Delta R} + V_{ijk}^{R}$



Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

Illinois-7 has 4 strength parameters fit to 23 energy levels in $A \leq 10$ nuclei.
In light nuclei we find (thanks to large cancellation between $\langle K \rangle$ & $\langle v_{ij} \rangle$):

$$\langle V_{ijk} \rangle \sim (0.02 \text{ to } 0.07)\ \langle v_{ij} \rangle \sim (0.15 \text{ to } 0.5)\ \langle H \rangle$$

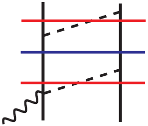
We expect $\langle \mathcal{V}_{ijkl} \rangle \sim 0.05\ \langle V_{ijk} \rangle \sim (0.01 \text{ to } 0.03)\ \langle H \rangle \sim 1$ MeV in ^{12}C .

Transition amplitude in time-ordered perturbation theory

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$
$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2}\right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\,Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



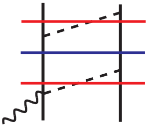
α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \ldots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

Power counting



- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\Big]|I\rangle$$

- Terms accounted into the Lippmann-Schwinger Eq. are subtracted from the reducible amplitude
- EM operators depend on the off-the-energy shell prescription adopted for the non-static OPE and TPE potentials
- Ultimately, the EM operators are unitarily equivalent: Description of physical systems is not affected by this ambiguity

Magnetic moment at N^3LO

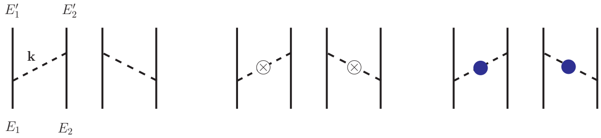
- Magnetic moment operator due to two-body current density $\mathbf{J}(\mathbf{x})$

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{r}) = \frac{1}{2}\left[\mathbf{R}\times\int \mathrm{d}\mathbf{x}\,\mathbf{J}(\mathbf{x}) + \int \mathrm{d}\mathbf{x}\,(\mathbf{x}-\mathbf{R})\times\mathbf{J}(\mathbf{x})\right]$$

- Sachs' and translationally invariant magnetic moments

$$\begin{aligned}
\boldsymbol{\mu}_{\mathrm{Sachs}}(\mathbf{R},\mathbf{r}) &= -i\,\frac{\mathbf{R}}{2}\times\int \mathrm{d}\mathbf{x}\,\mathbf{x}\,[\rho(\mathbf{x})\,,\,\upsilon_{12}] \\
\boldsymbol{\mu}_{\mathrm{T}}(\mathbf{r}) &= -\frac{i}{2}\int_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,\nabla_q\times\mathbf{j}(\mathbf{q},\mathbf{k})\Big|_{\mathbf{q}=0}
\end{aligned}$$

OPEP beyond the static limit



$v_\pi^{(0)} \sim Q^0$ $\quad$ $v_\pi^{(1)} \sim Q^1$ $\quad$ $v_\pi^{(2)} \sim Q^2$

On-the-energy-shell, non-static OPEP at N2LO (Q^2) can be equivalently written as

$$
\begin{aligned}
v_\pi^{(2)}(\nu = 0) &= v_\pi^{(0)}(\mathbf{k})\, \frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2} \\
v_\pi^{(2)}(\nu = 1) &= -v_\pi^{(0)}(\mathbf{k})\, \frac{(E_1' - E_1)\,(E_2' - E_2)}{\omega_k^2} \\
v_\pi^{(0)}(\mathbf{k}) &= -\frac{g_A^2}{F_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}
\end{aligned}
$$

$v_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1 + E_2 \neq E_1' + E_2')$

- TPE contributions are affected by the choice made for the parameter ν

From amplitudes to potentials

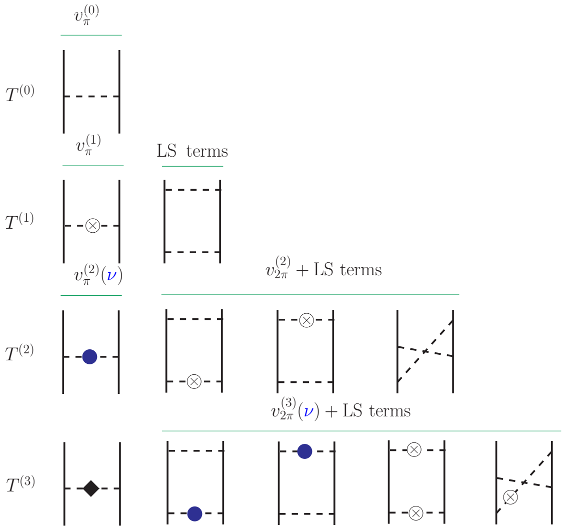
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \dots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \dots\,, \qquad\qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\,, \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\,, \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\,, \\
\upsilon^{(3)}(\nu) &= T^{(3)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] \\
&\quad \underbrace{- \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

From amplitudes to potentials: an example with OPE and TPE only



- To each $\upsilon_\pi^{(2)}(\nu)$ corresponds a $\upsilon_{2\pi}^{(3)}(\nu)$

Unitary equivalence of $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_\pi^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_\pi^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

$t^{(-1)}$ is the kinetic energy, $\upsilon_\pi^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\,\mathrm{e}^{+iU(\nu)} \,, \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

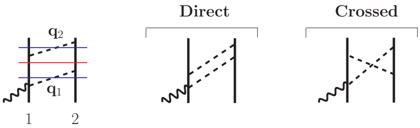
from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_\pi^{(0)}\,,\, i\,U^{(0)}(\nu)\right] + \left[t^{(-1)},\, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

Technical issue II - Recoil corrections at N^3LO

$\mathbf{j}^{\mathrm{N^3LO}} =$



- Reducible contributions

$$\mathbf{j}_{\rm red} \sim \int \upsilon^{\pi}(\mathbf{q}_2)\, \frac{1}{E_i - E_I}\, \mathbf{j}^{\rm NLO}(\mathbf{q}_1) - \int 2\, \frac{\omega_1 + \omega_2}{\omega_1\, \omega_2}\, V_{\pi NN}(2,\mathbf{q}_2)\, V_{\pi NN}(2,\mathbf{q}_1)\, V_{\pi NN}(1,\mathbf{q}_2)\, V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

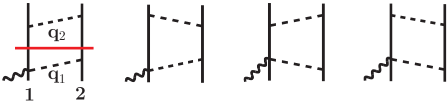
- Irreducible contributions

$$\mathbf{j}_{\rm irr} = \int 2\, \frac{\omega_1 + \omega_2}{\omega_1\, \omega_2}\, V_{\pi NN}(2,\mathbf{q}_2)\, V_{\pi NN}(2,\mathbf{q}_1)\, V_{\pi NN}(1,\mathbf{q}_2)\, V_{\gamma\pi NN}(1,\mathbf{q}_1) - \int 2\, \frac{\omega_1^2 + \omega_2^2 + \omega_1\, \omega_2}{\omega_1\, \omega_2(\omega_1 + \omega_2)} [V_{\pi NN}(2,\mathbf{q}_2), V_{\pi NN}(2,\mathbf{q}_1)]_{-}\, V_{\pi NN}(1,\mathbf{q}_2) V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

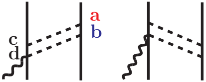
- Observed partial cancellations at N^3LO between recoil corrections to reducible diagrams and irreducible contributions

The box diagram: an example at N^{3}LO

Reducible



Irreducible direct



Irreducible crossed

$$\text{direct} = f_d(\omega_1, \omega_2)\, V_a\, V_b\, V_c\, V_d$$

$$\text{crossed} = f_c(\omega_1, \omega_2)\, V_b\, V_a\, V_c\, V_d \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

$$\begin{aligned} \text{irreducible} &= [f_d(\omega_1, \omega_2) + f_c(\omega_1, \omega_2)]\, V_a\, V_b\, V_c\, V_d \\ &- f_c(\omega_1, \omega_2)[V_a, V_b]_-\, V_c\, V_d \end{aligned}$$

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$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots$$

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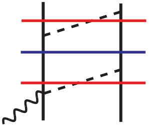

α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

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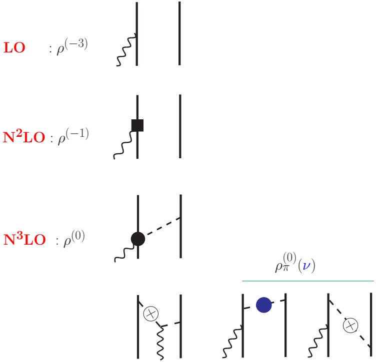
- $(N - N_K - 1)$ energy denominators expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I} |I\rangle = \frac{1}{E_i - E_N - \omega_\pi} |I\rangle \sim - \Bigg[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \ldots \Bigg] |I\rangle$$

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \ldots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

EM charge up to $n = 0$ (or up to N3LO)



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}_1')\,(1 + \tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

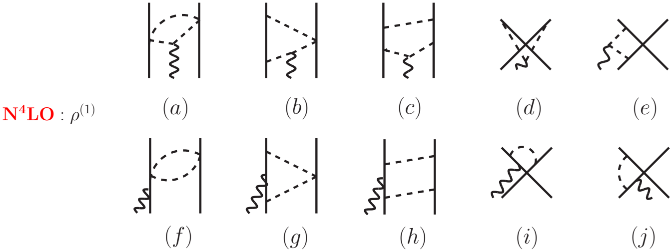
  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order (g_A, F_π)

EM charge @ $n = 1$ (or N4LO) 1.

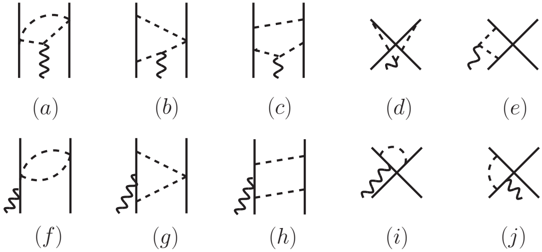


- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_{\pi}^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) \quad = \quad \rho_{\rm h}^{(1)}(\nu = 0) + \left[\rho^{(-3)}, i\,U^{(1)}(\nu)\right]$$

EM charge @ $n = 1$ (or N4LO)   2.

N^{4}LO : $\rho^{(1)}$



- Charge operators (ν-dependent included) up to $n = 1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0) = 0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0) = \int \mathrm{d}\mathbf{x}\,\rho(\mathbf{x}) = e\,\frac{(1+\tau_{1,z})}{2} + 1 \rightleftharpoons 2 = \rho^{(-3)}(\mathbf{q}=0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector