Recent developments in neutron-proton scattering with Lattice Effective Field Theory

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  - Provides a way to assess the theoretical errors.

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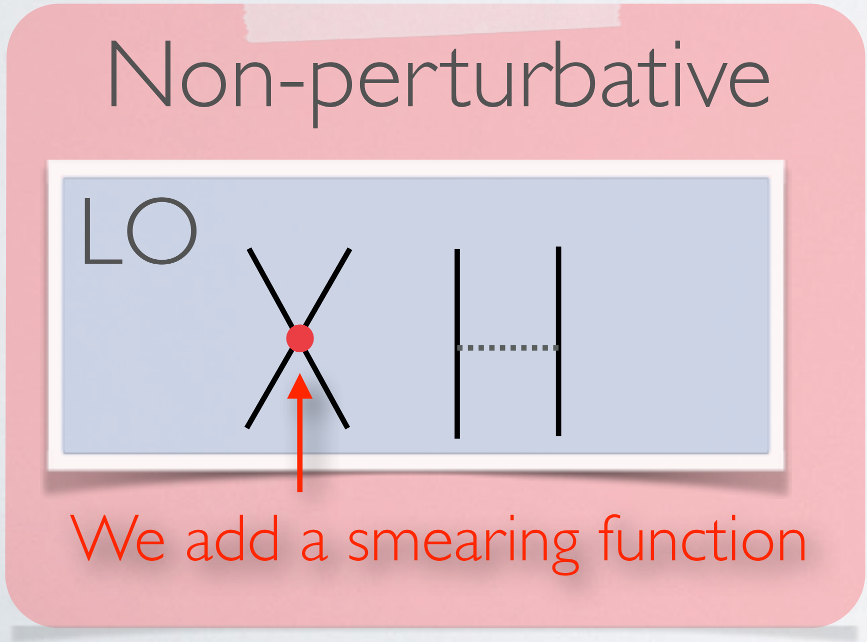

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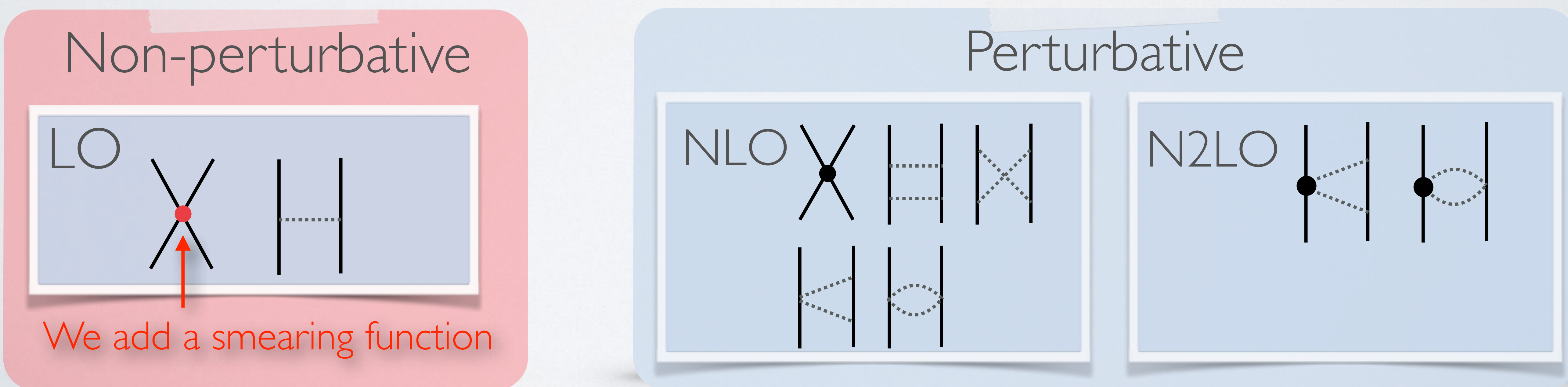

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- The χ^2 is defined as follow:

$$\chi^2 = \sum_i \left[\frac{\delta_\alpha^{latt}(p) - \delta_\alpha^{NPWA}(p)}{\Delta_\alpha(p)}\right]^2 + \left[\frac{E_B^{latt} - E_B^{exp}}{\delta E_B^{exp}}\right]^2$$

with

$$\Delta_\alpha = \max\left(\Delta_\alpha^{NPWA}, |\delta_\alpha^{NijmI} - \delta_\alpha^{NPWA}|, |\delta_\alpha^{NijmII} - \delta_\alpha^{NPWA}|, |\delta_\alpha^{Reid93} - \delta_\alpha^{NPWA}|\right)$$

[Epelbaum, Krebs and Meißner, EPJ A 51 (2015)]

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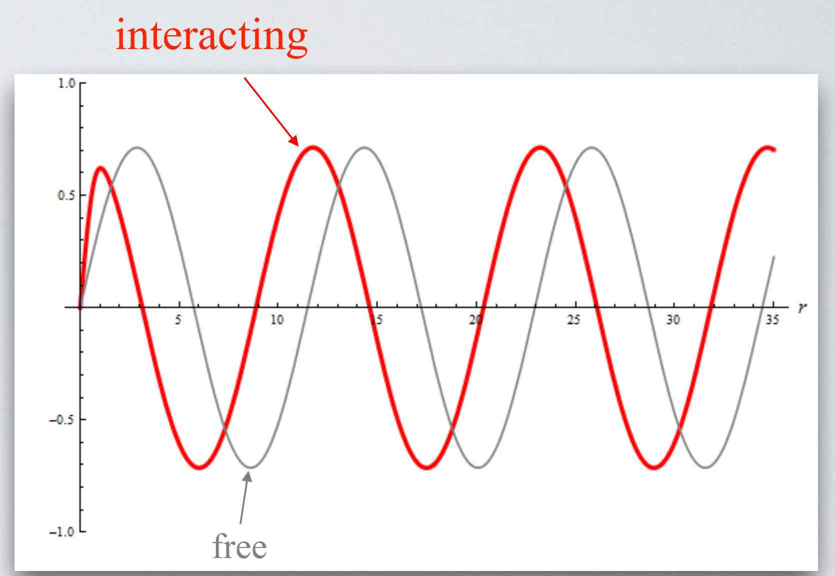
- From the definition of the phase shifts

Radial solution (non-interacting system):

$$r \cdot R(r) = r \cdot j_\ell(p\,r) \xrightarrow[r\to\infty]{} \sin(p\,r - \pi L/2)$$

Radial solution (interacting system):

$$r \cdot R(r) = r\cos\delta_\ell(p) \cdot j_\ell(p\,r) - r\sin\delta_\ell(p) \cdot y_\ell(p\,r) \xrightarrow[r\to\infty]{} \sin(p\,r - \pi L/2 + \delta_\ell(p))$$



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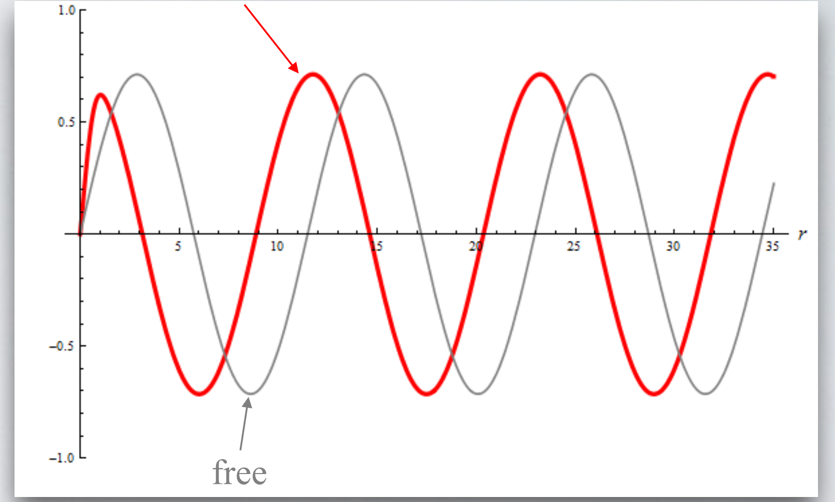
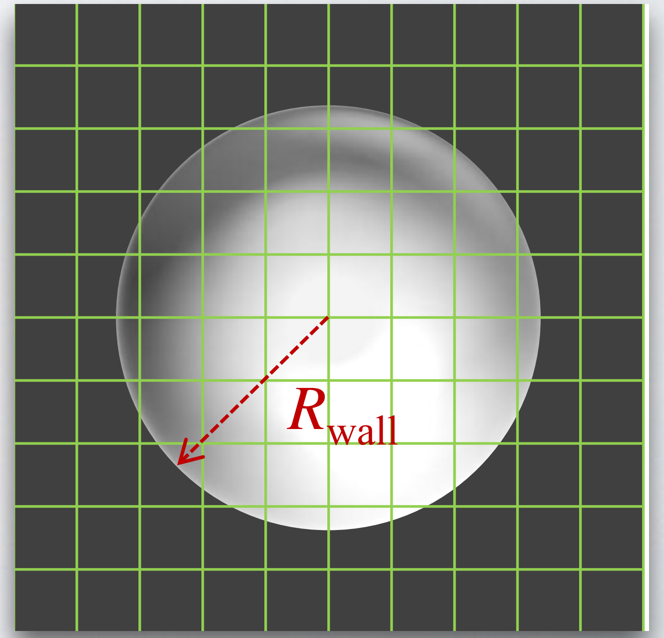


  - In order to extract $\delta(p)$, we impose a rigid wall of size $R_{wall} >> R_{interaction}$.



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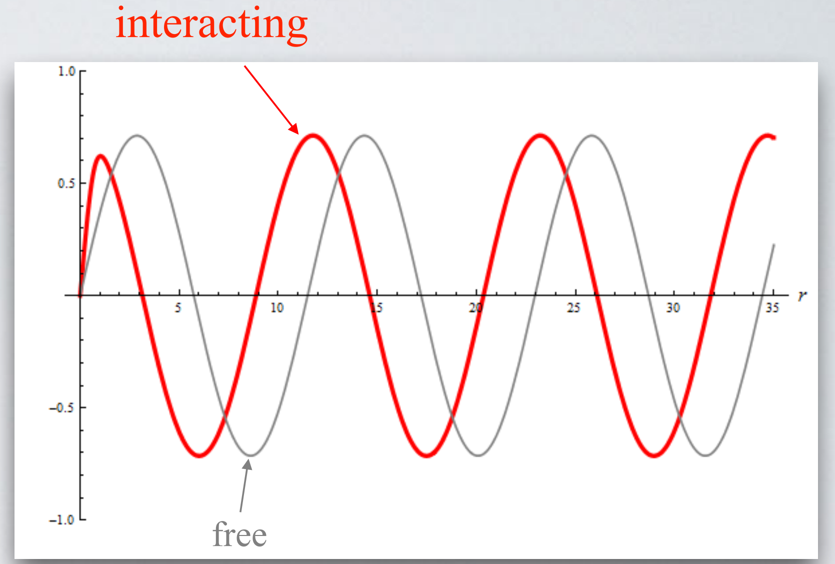

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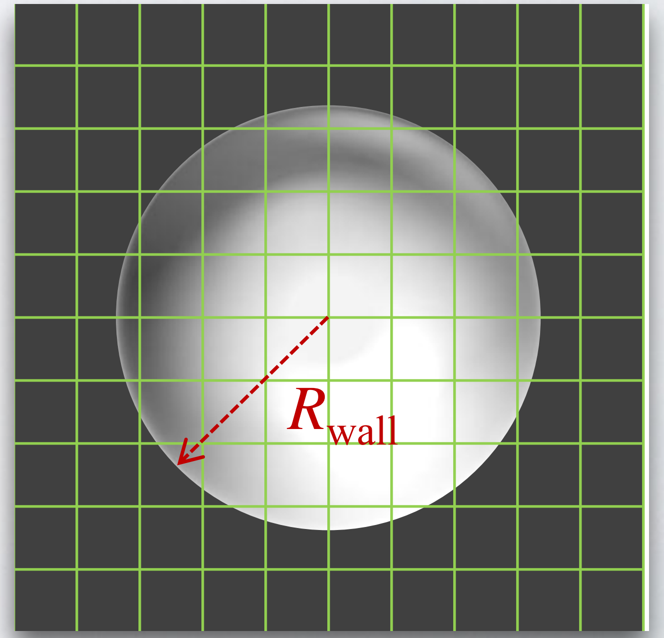
  - At the (rigid) wall, the wave function vanishes, therefore

$$\cos\delta_\ell(p) \cdot j_\ell(p\,R_{\rm wall}) = \sin\delta_\ell(p) \cdot y_\ell(p\,R_{\rm wall})$$



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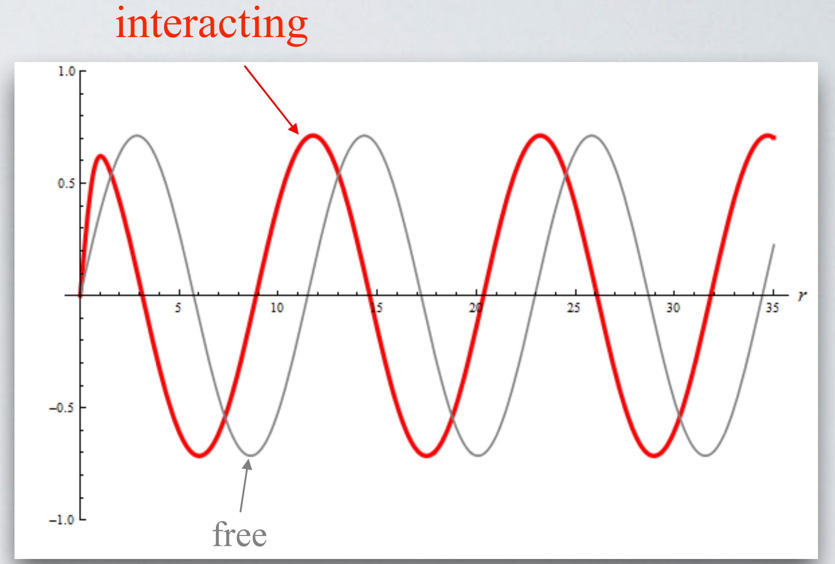

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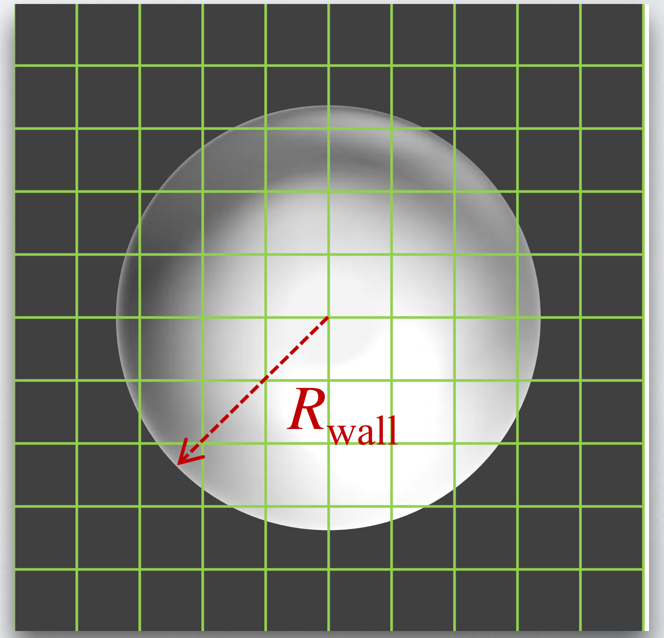

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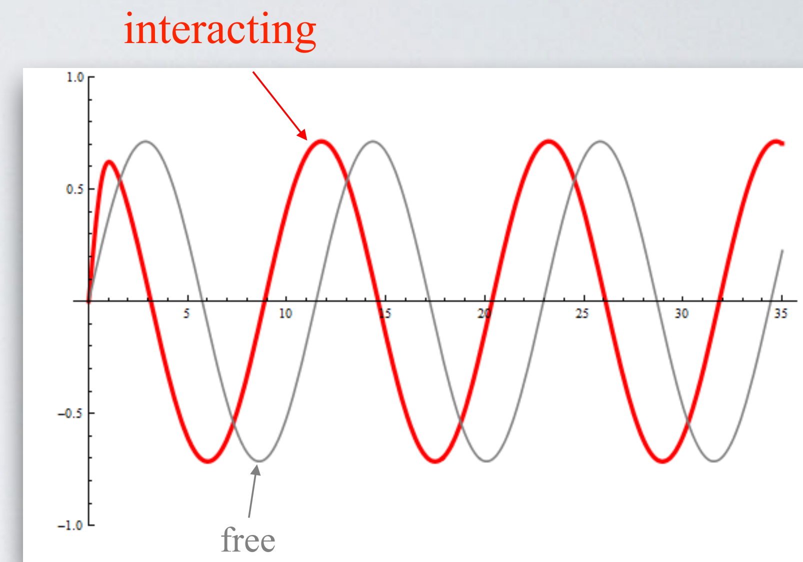

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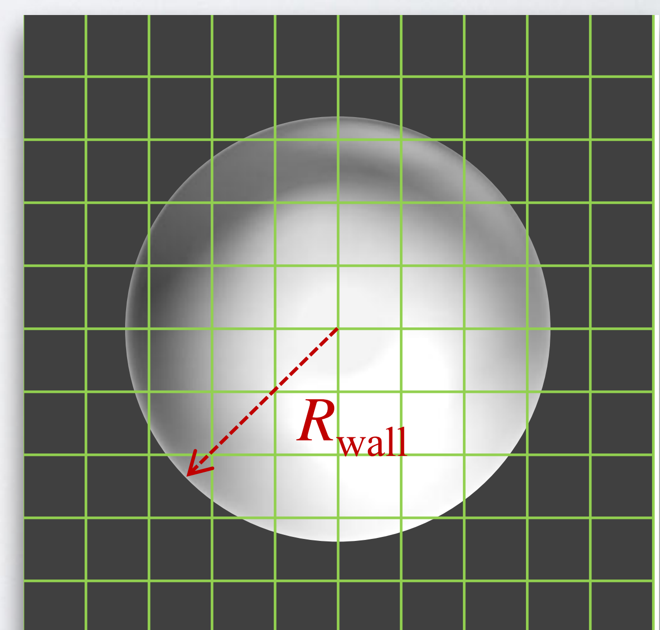
Spherical Bessel functions



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Related to the energy spectrum of the interacting system

$$E_{CM} = 2 \times \frac{p^2}{2m_N}$$

Results

Phase shifts (Preliminary)

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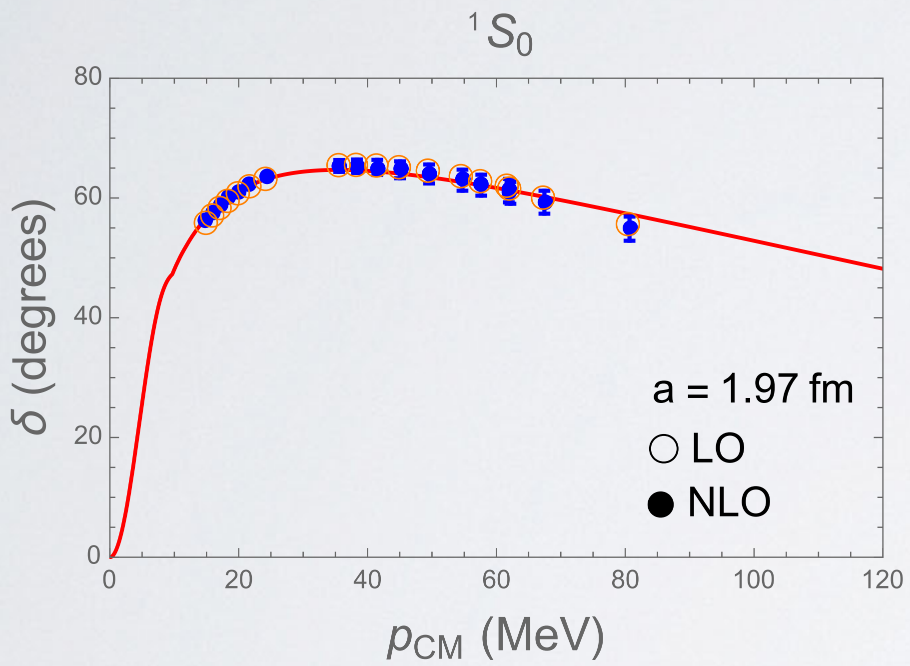

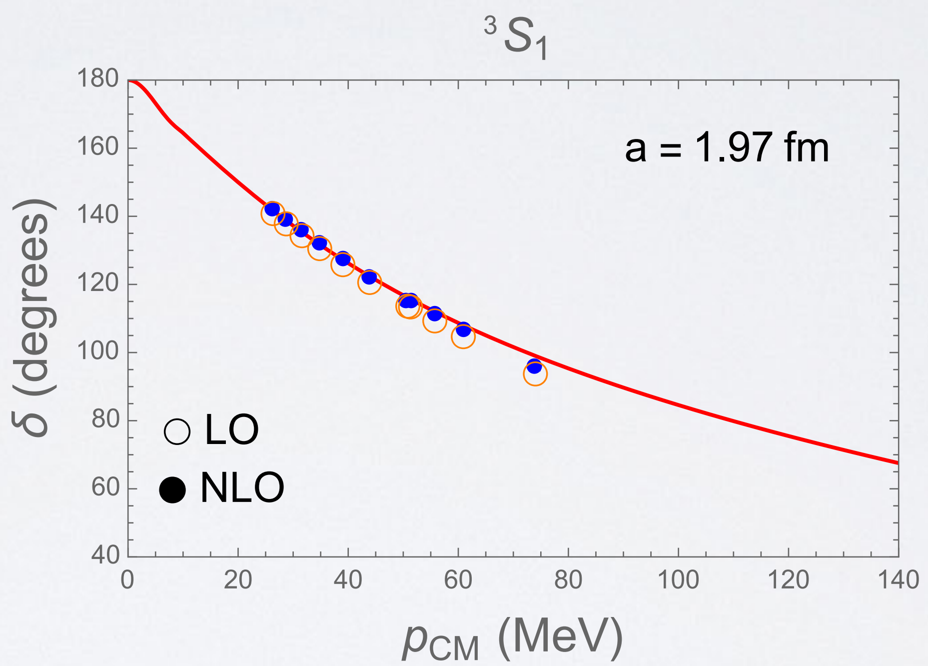

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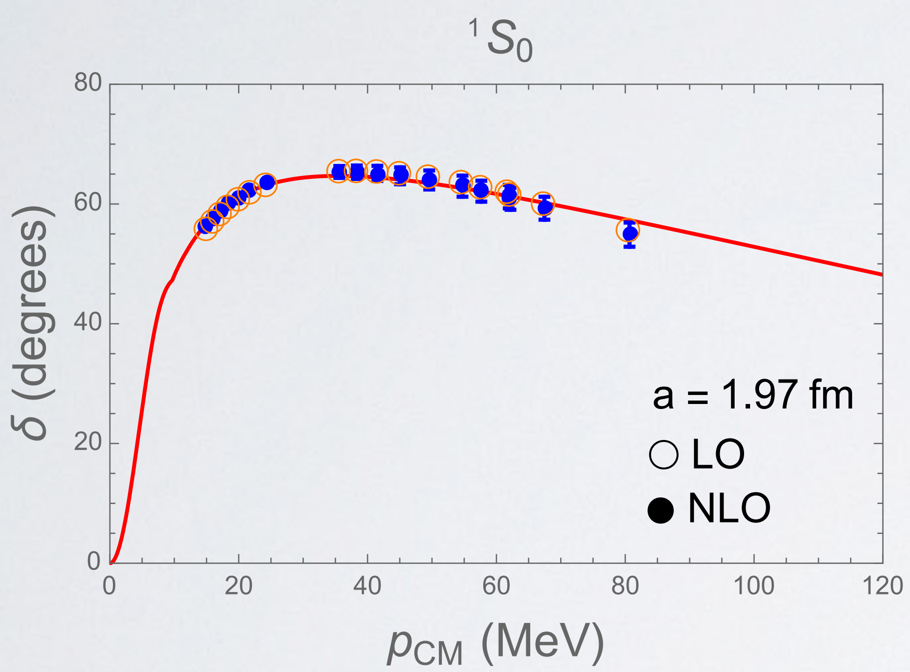

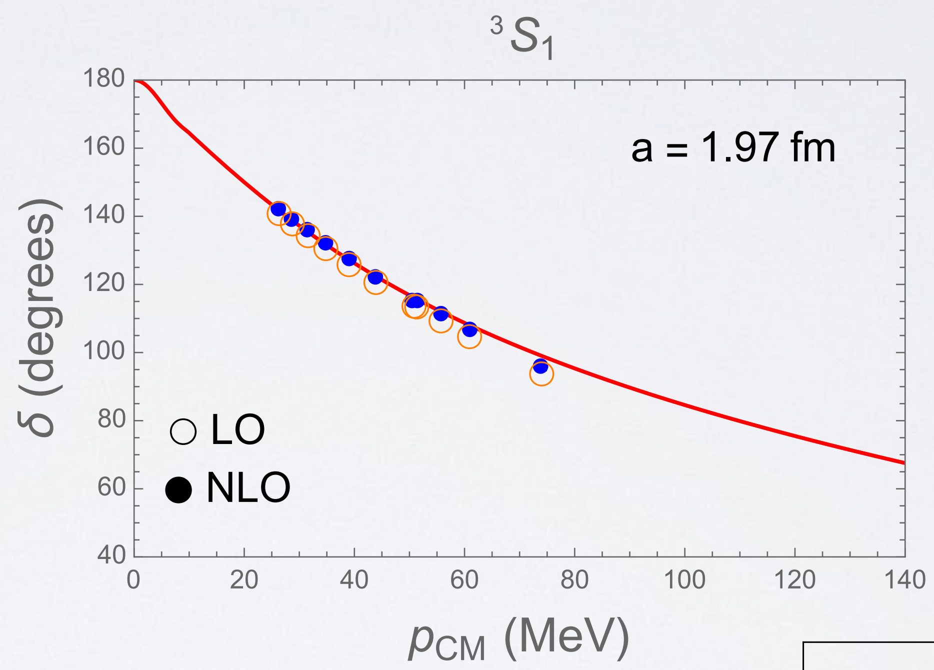

Threshold parameters			
	LO	NLO	Exp.
a_{1S0} (fm)	-23.31	-23.79	-23.76
r_{1S0} (fm)	2.38	2.57	2.75
a_{3S1} (fm)	5.26	5.23	5.42
r_{3S1} (fm)	2.05	2.04	1.76

	LO	NLO	Exp.
E_B	-2.223544	-2.224574	-2.224575(9)

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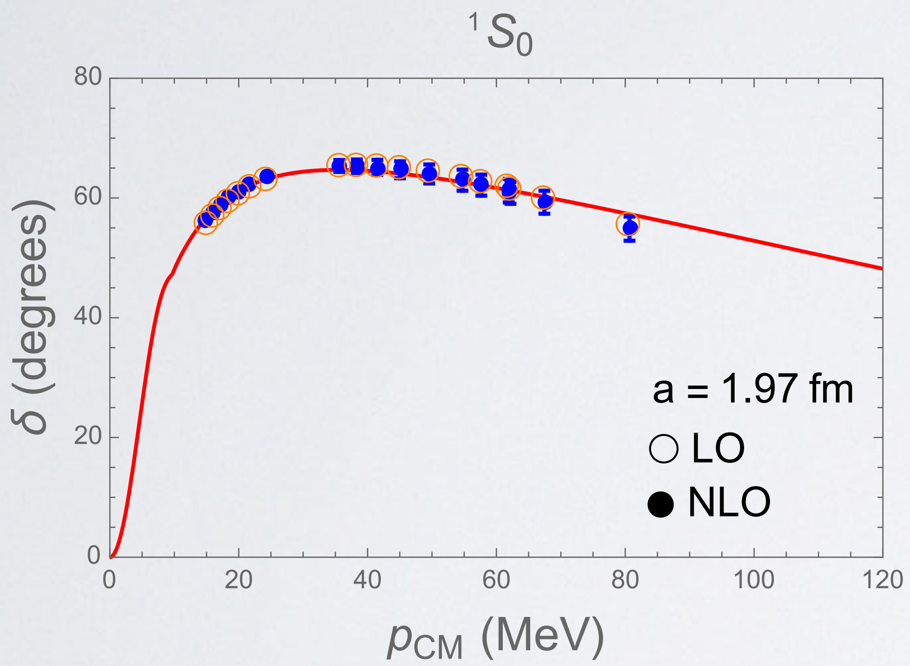

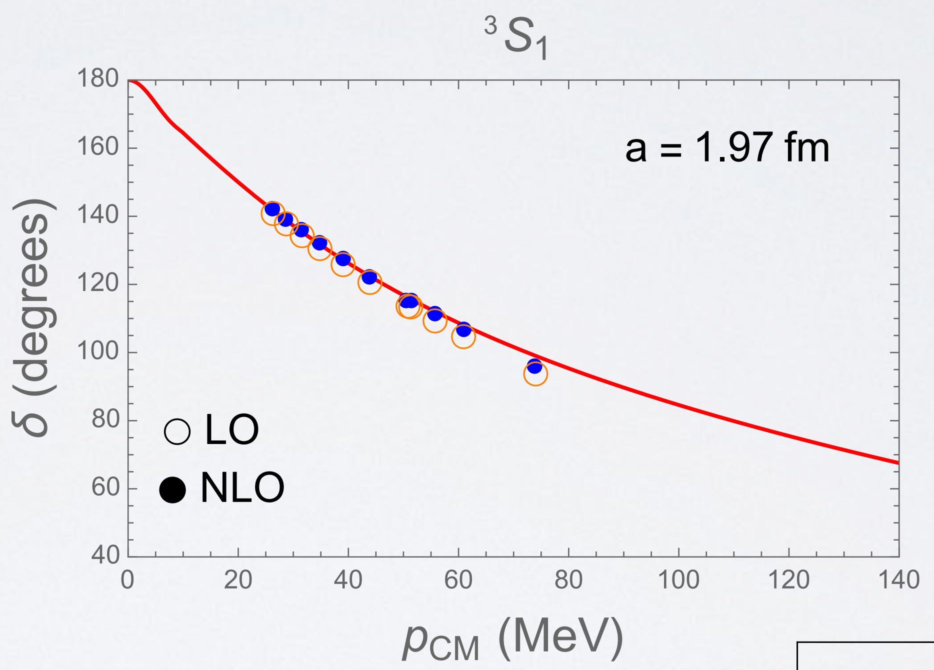

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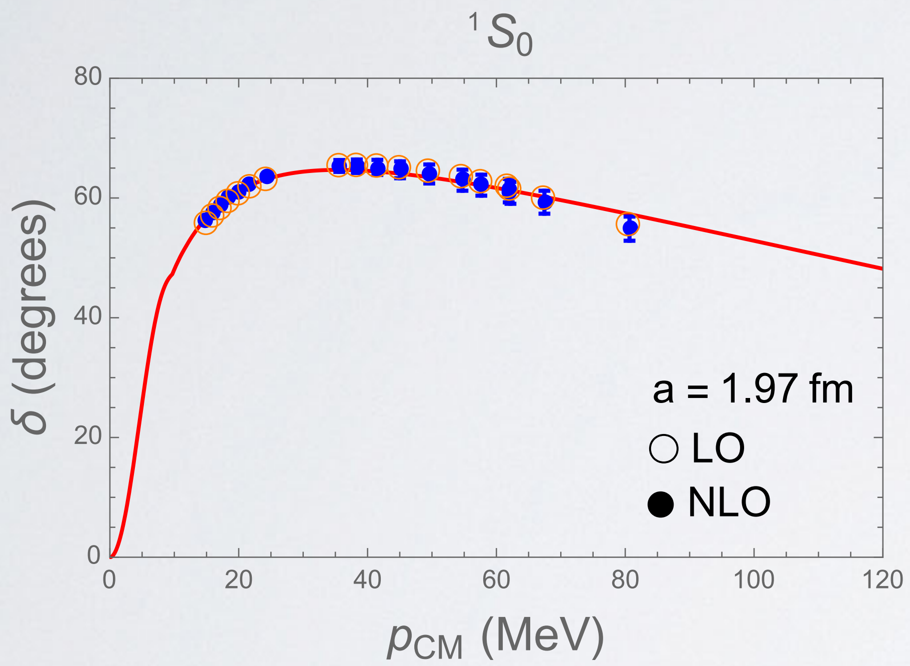

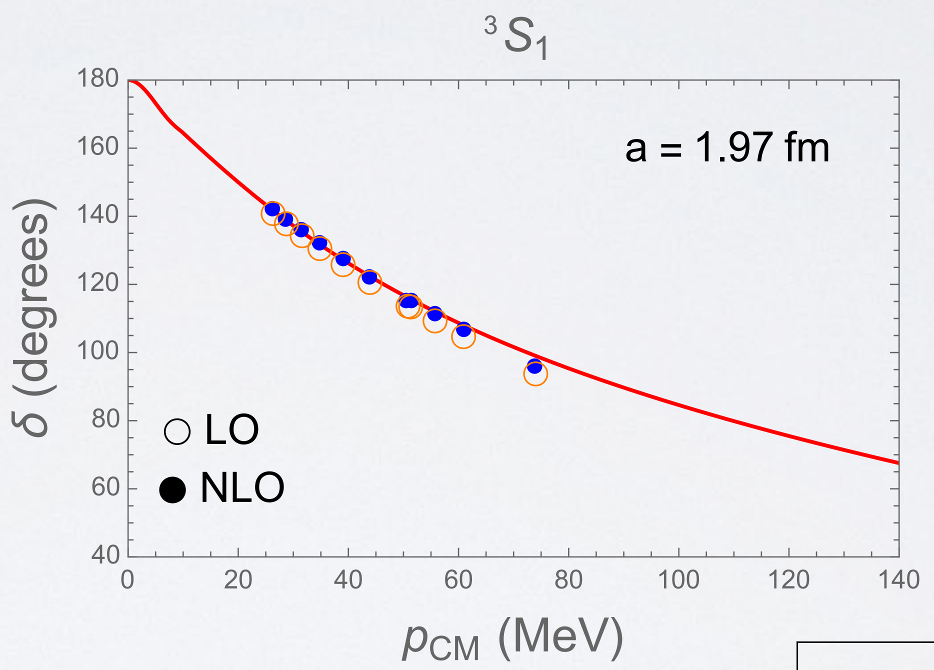

- LO provides a good description of the S-waves and threshold parameters.
- NLO corrects slightly the LO in the right direction.

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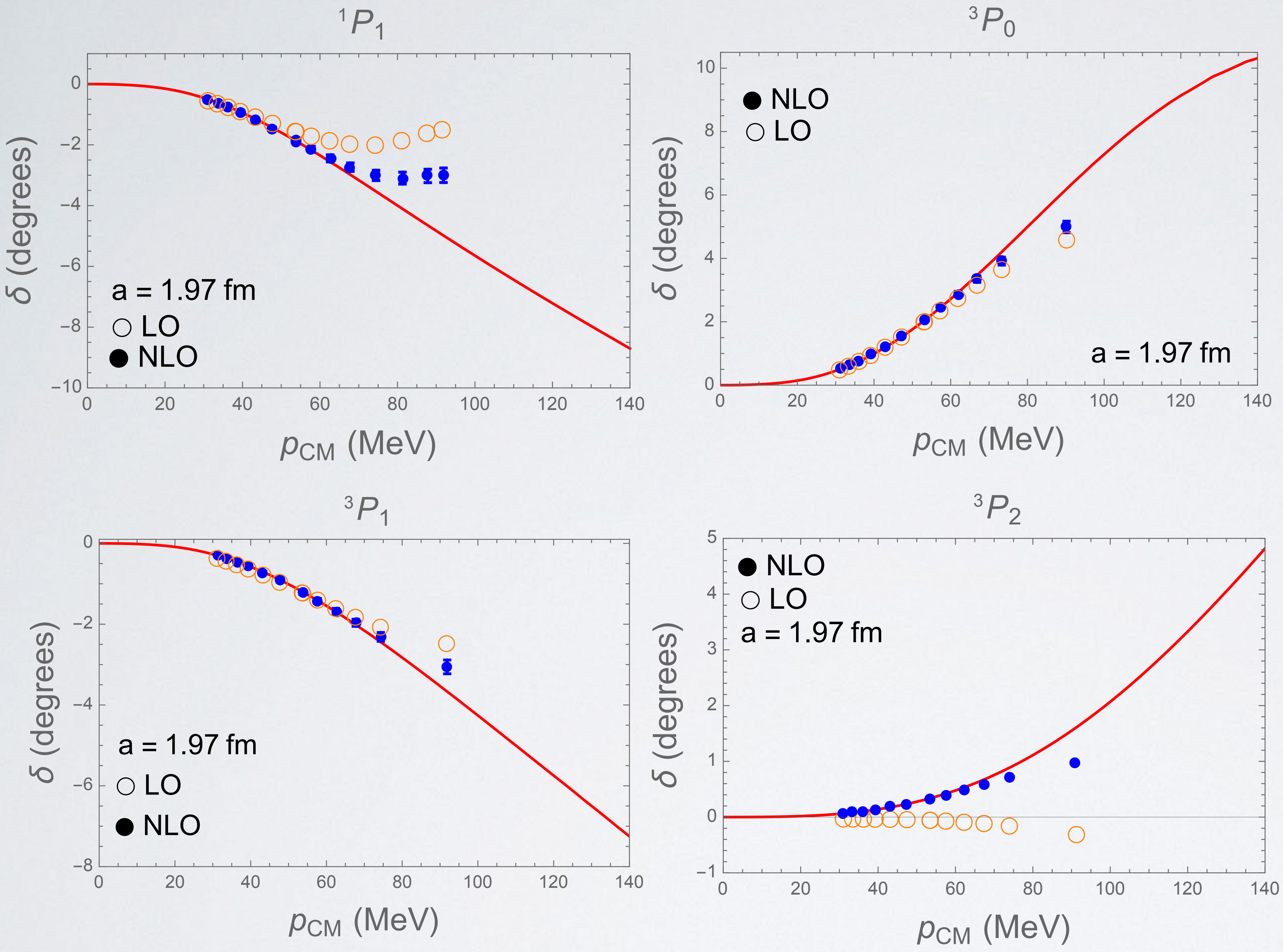

$^1P_1$
δ (degrees)
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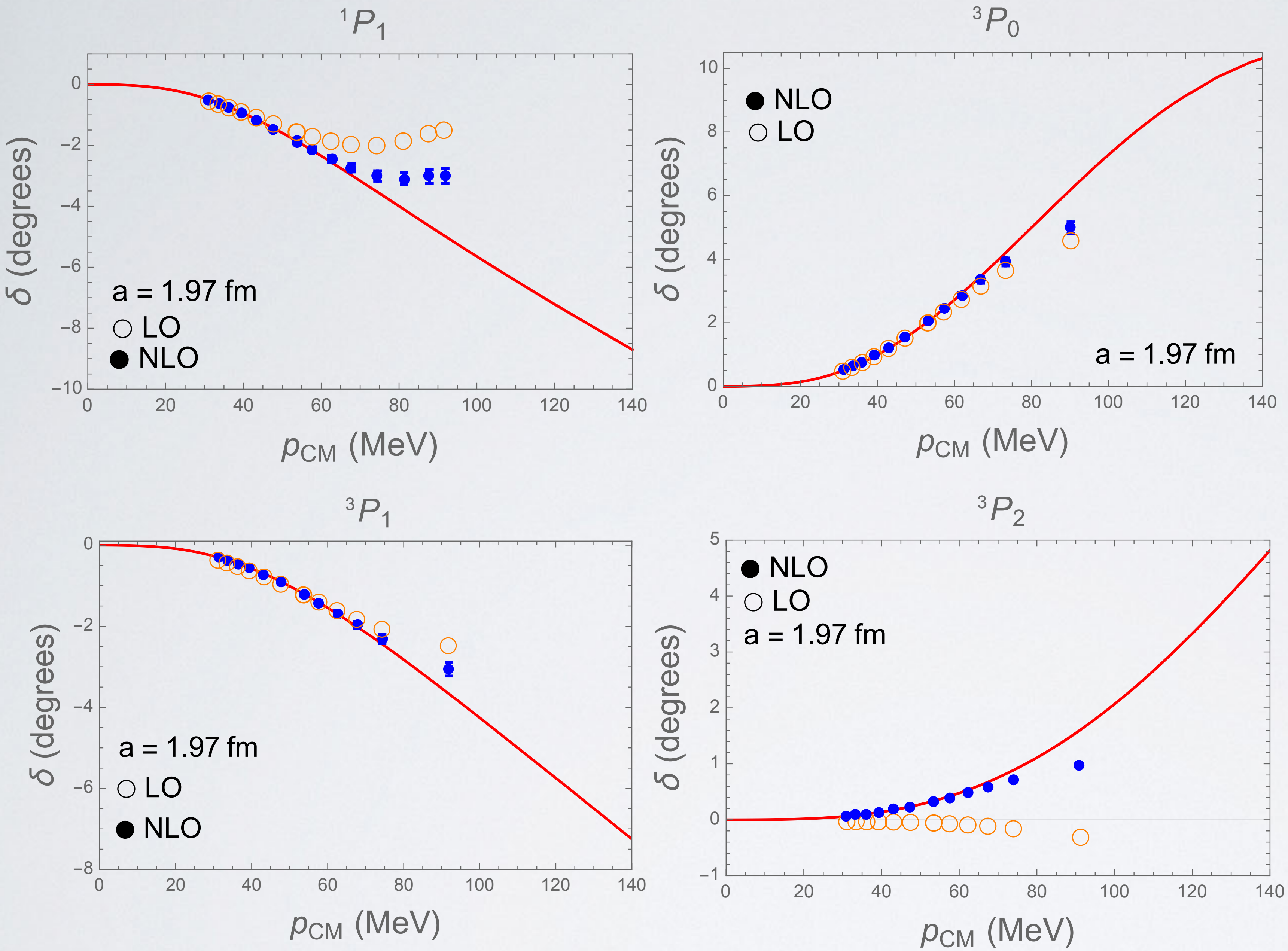

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a_{3P1} (fm^3)	1.99	1.63	1.54**
r_{3P1} (fm^{-1})	−13.57	−9.71	−8.54**
a_{3P2} (fm^3)	−0.003	−0.33	−0.29**
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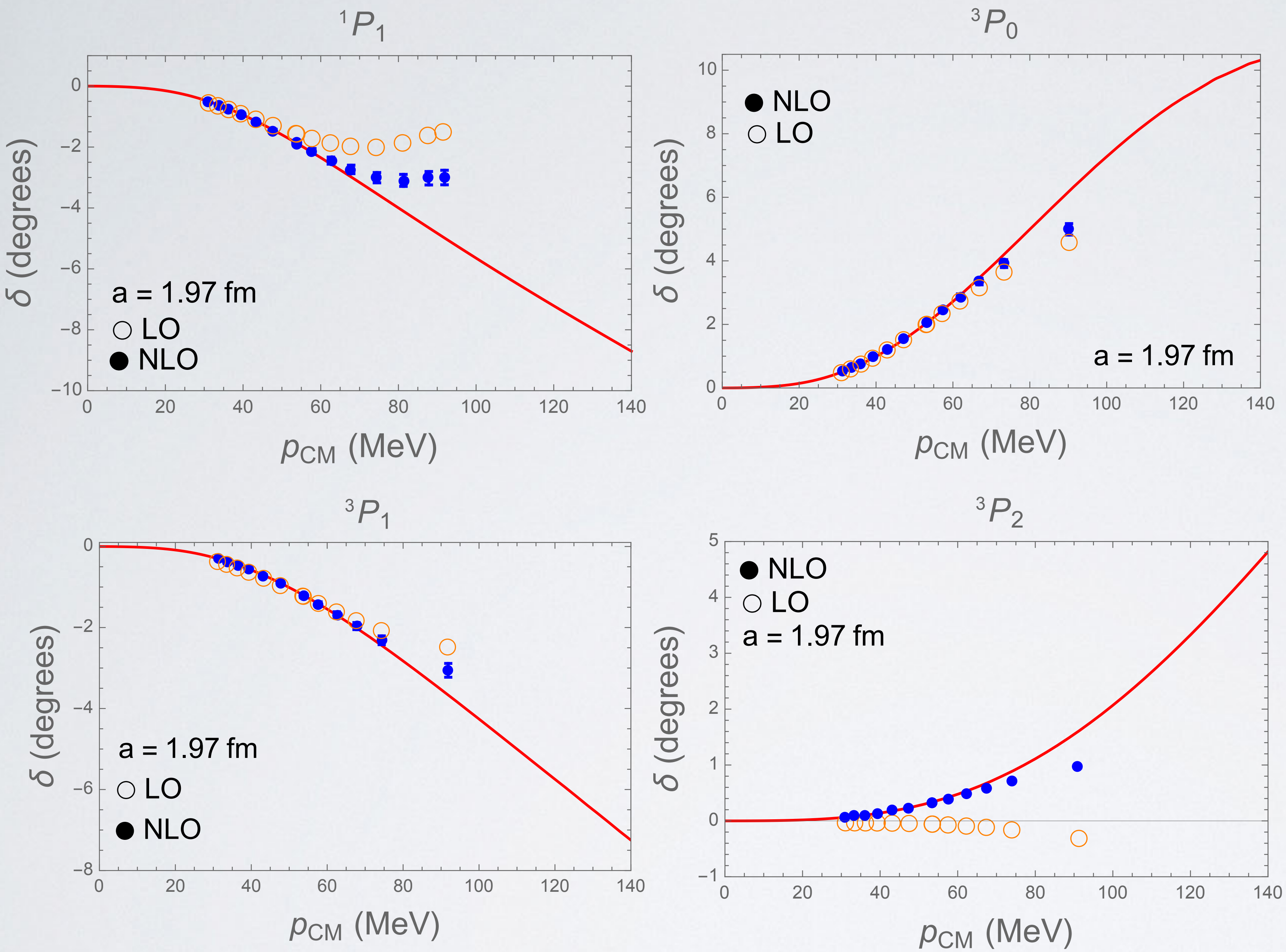

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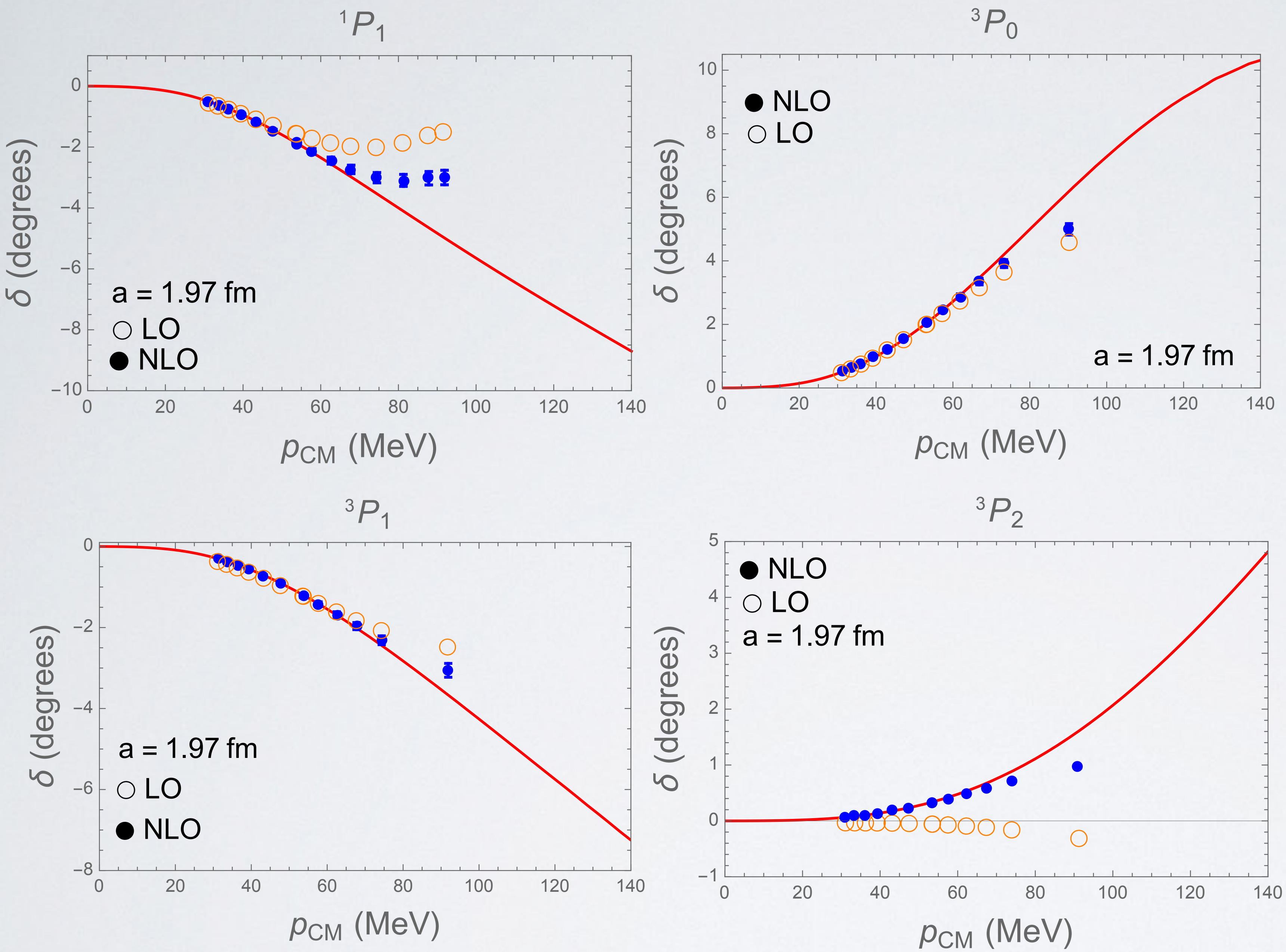

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Fulfils the expectations of an EFT approach

Lattice Spacing Dependence (Preliminary)

Lattice spacing dependence

- We study the spacing dependence of the LO LECs for $L = 32$
- Similar study in the Hamiltonian formalism [Klein, Lee, Liu, Meißner, PLB 747,(2015)]

  - a reduced, a_t constant
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a (fm)	a_t (fm)	C_{1S0} (in 10^{-5} MeV^{-2})	C_{3S1} (in 10^{-5} MeV^{-2})	b_4	E_B (MeV)
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1.80	1.09	−3.357	−4.868	0.0986	−2.224575
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1.60	0.86	−2.648	−3.776	0.1320	−2.224575
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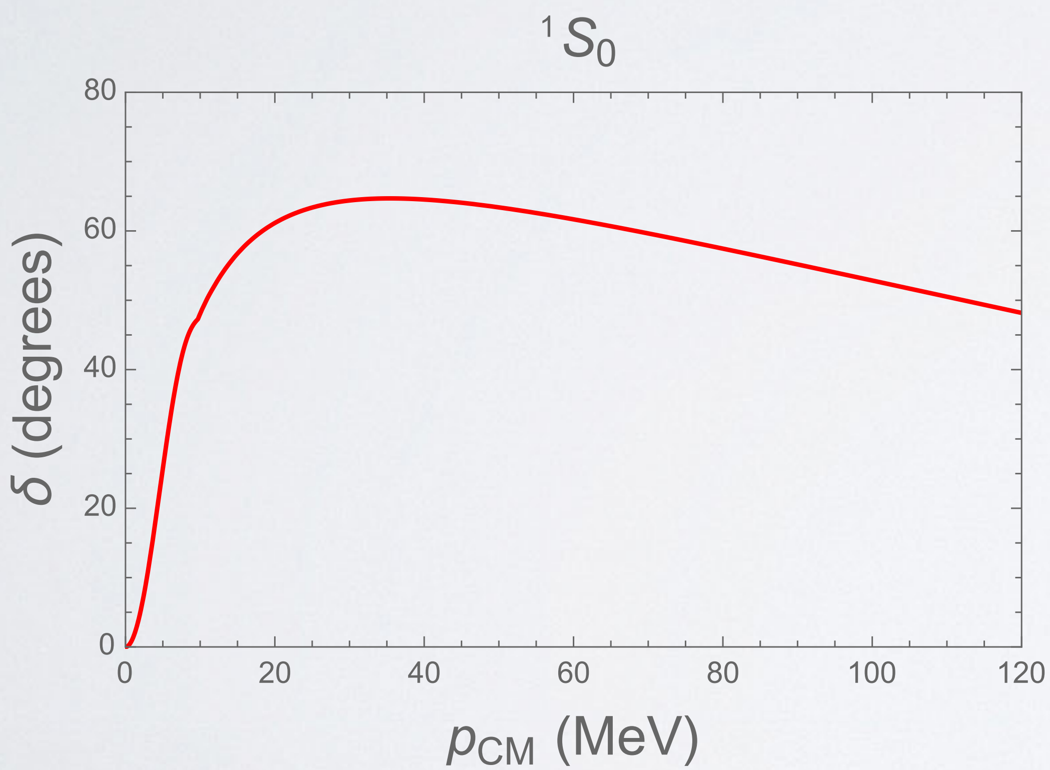
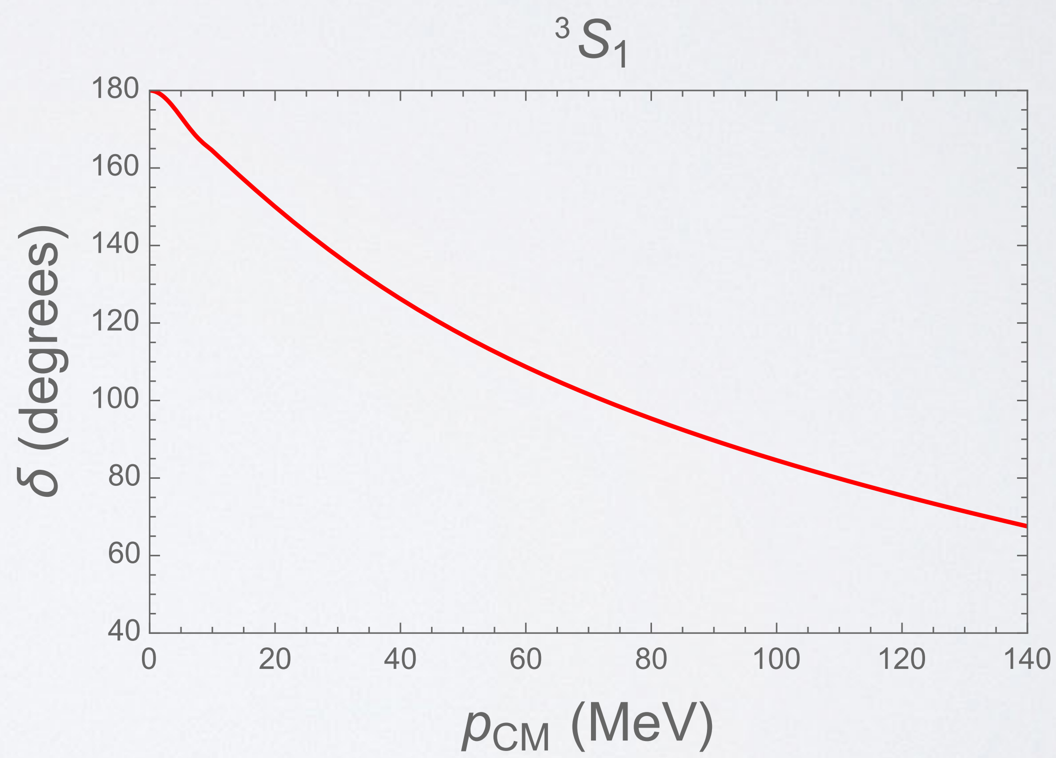
1.40	0.66	−1.975	−2.737	0.1863	−2.224575





Lattice spacing dependence

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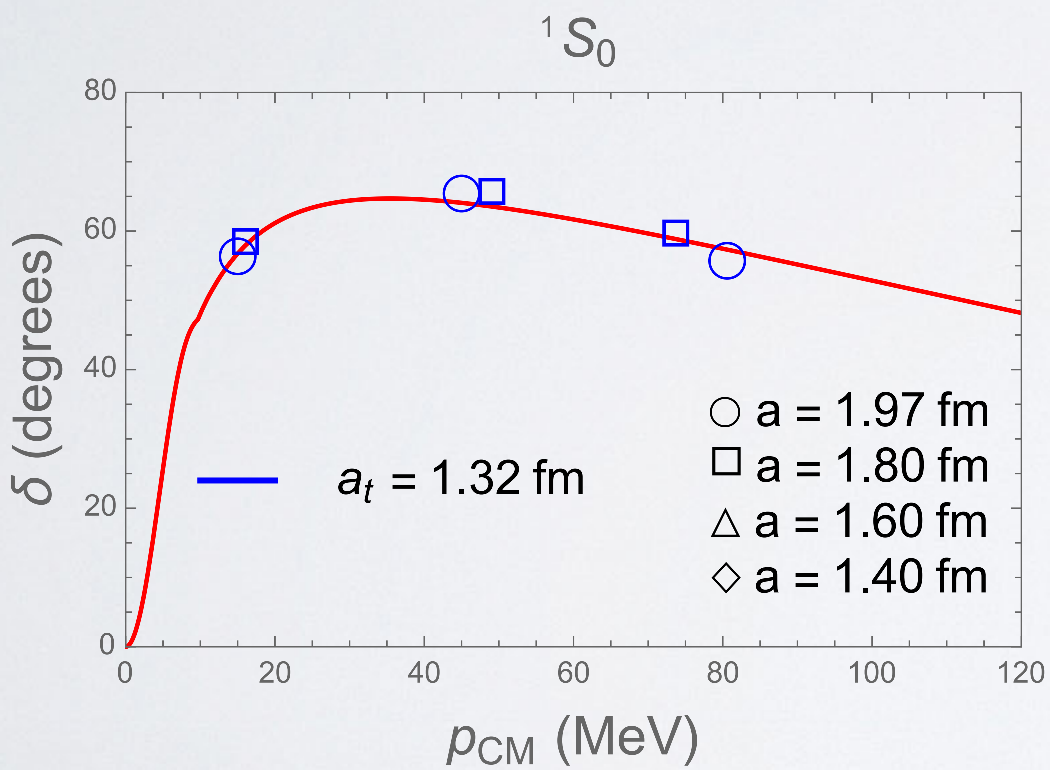

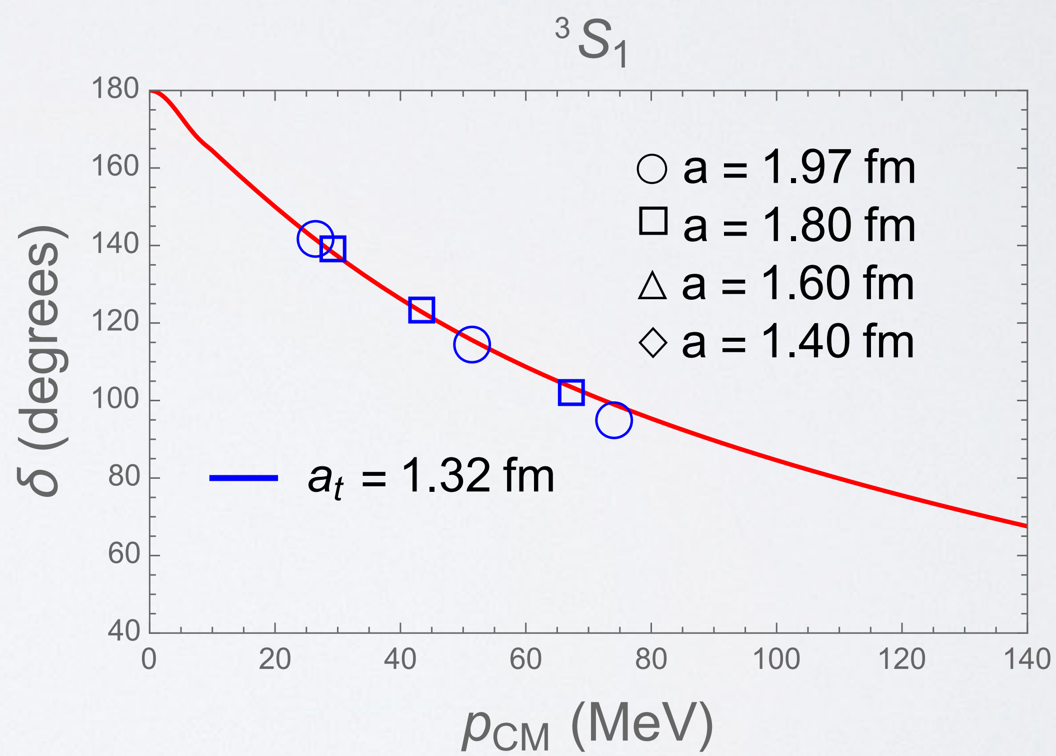

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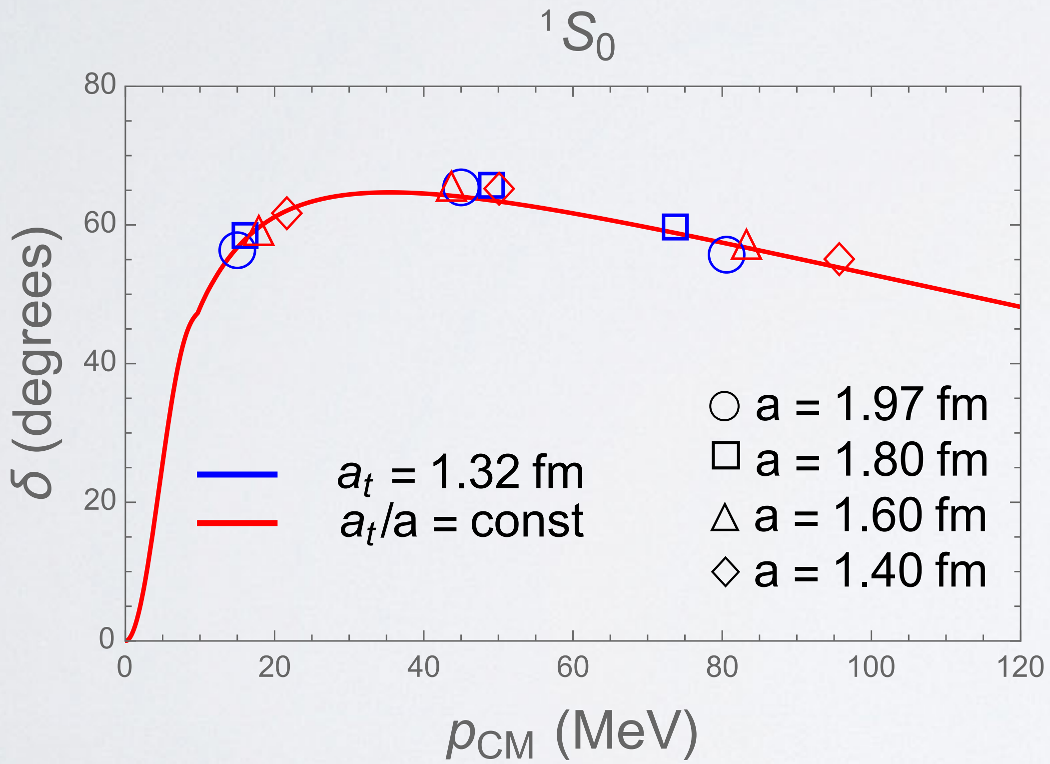

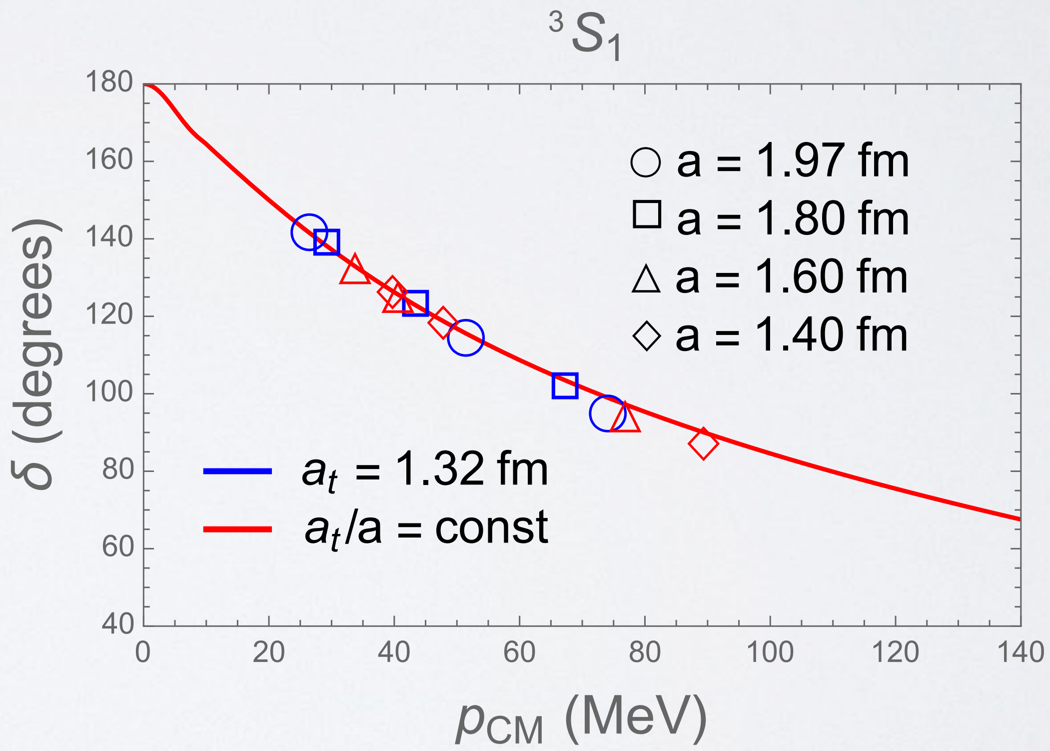

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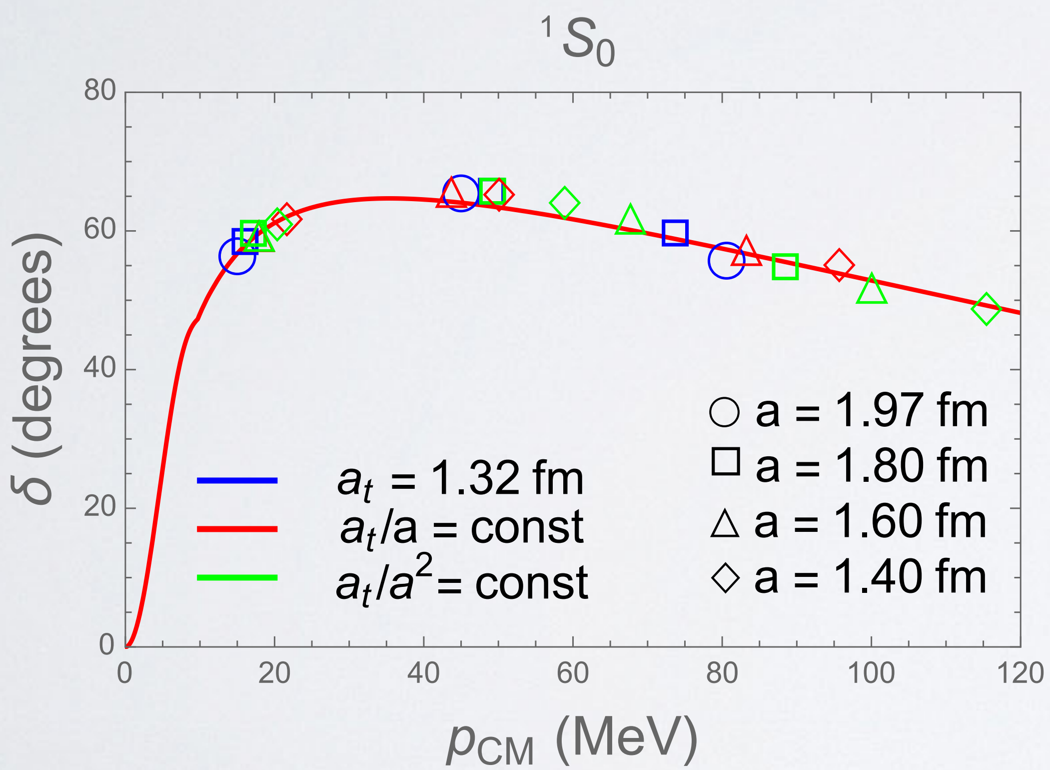

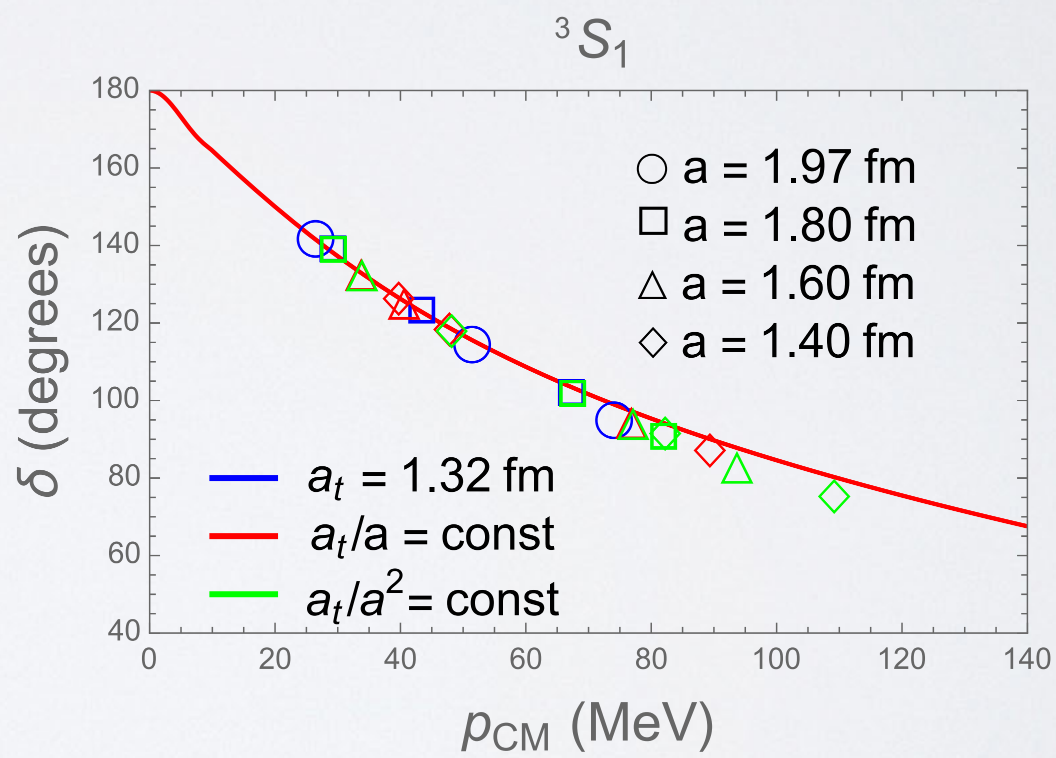

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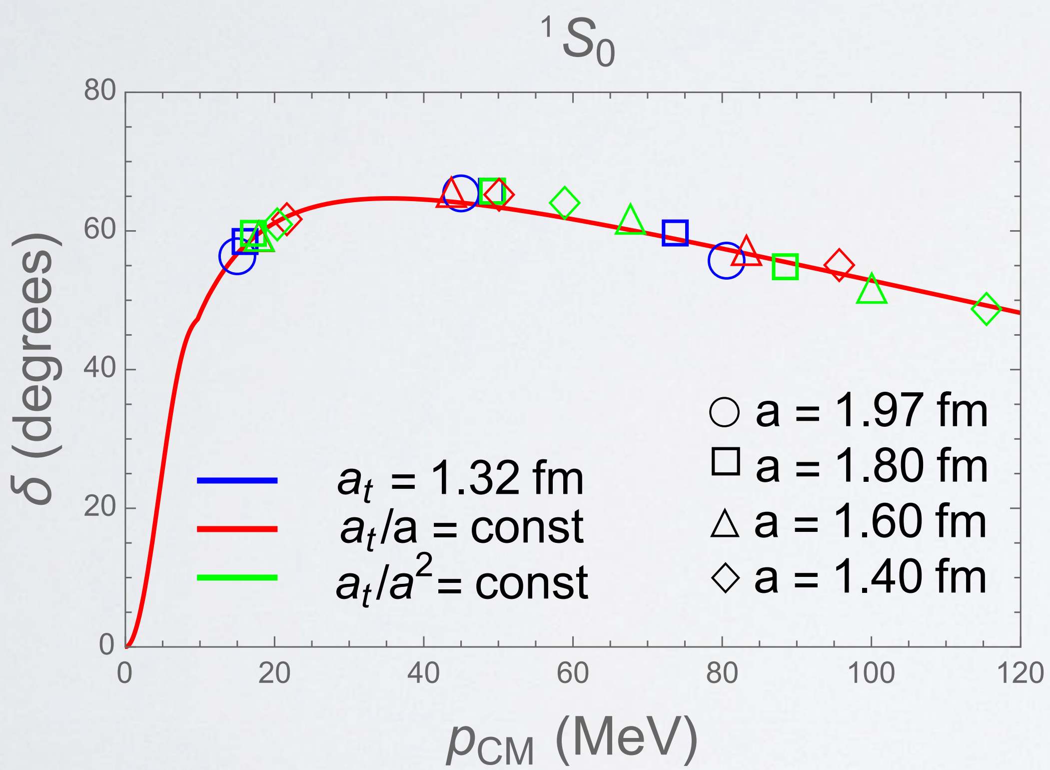

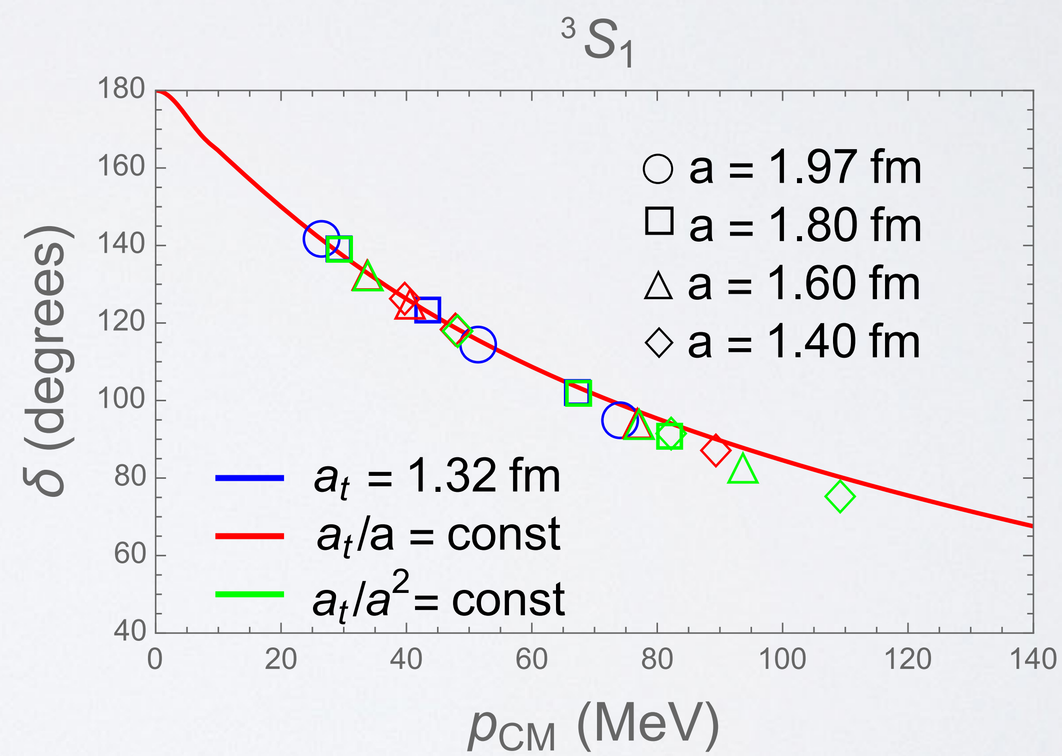

- Good description is achieved for smaller spacings.

Radial Transfer Matrix

Radial Transfer Matrix

- Radial Transfer Matrix formalism + auxiliary complex potential.

[Elhatisari et al., arXiv:1506.03513] [Lu et al., arXiv:1506.05652]

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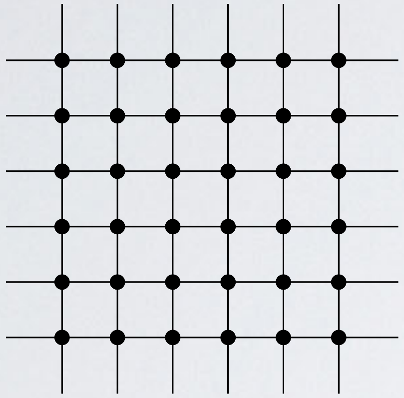

$H(x, y, z)$

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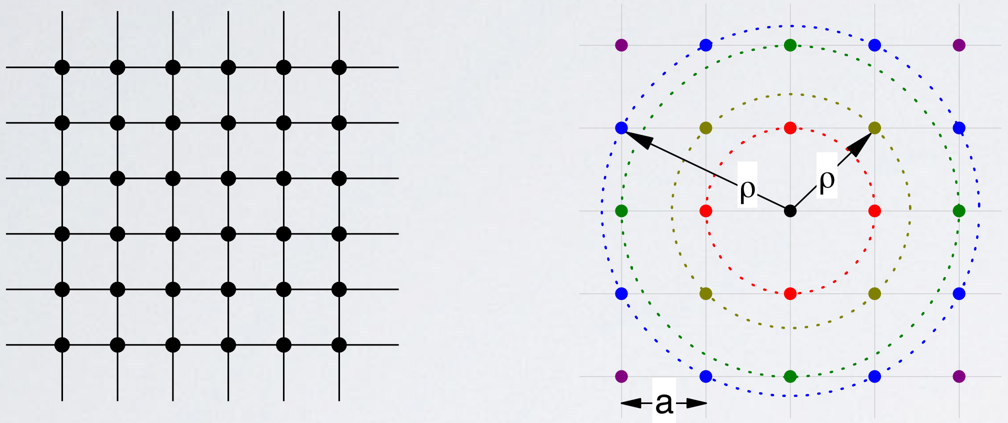

$H(x, y, z) \rightarrow H(\rho)$

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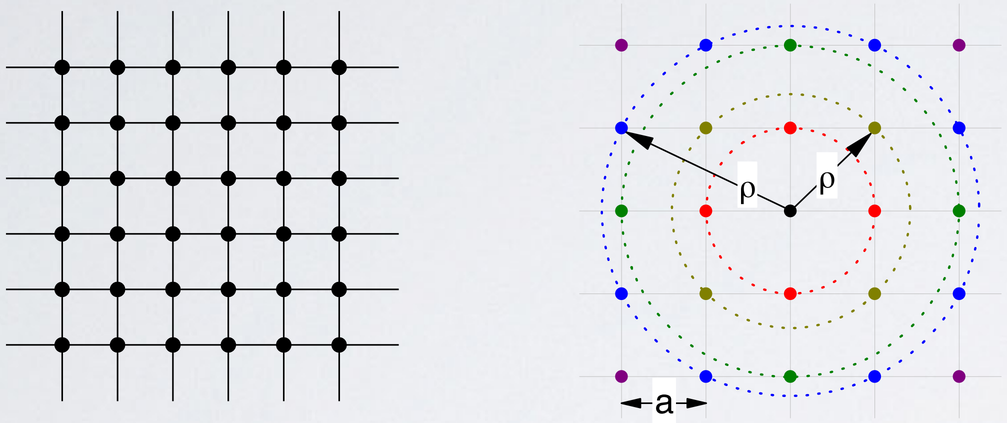

$H(x,y,z) \longrightarrow H(\rho)$

- Reduces scaling with L: $L^3 \longrightarrow L^2$

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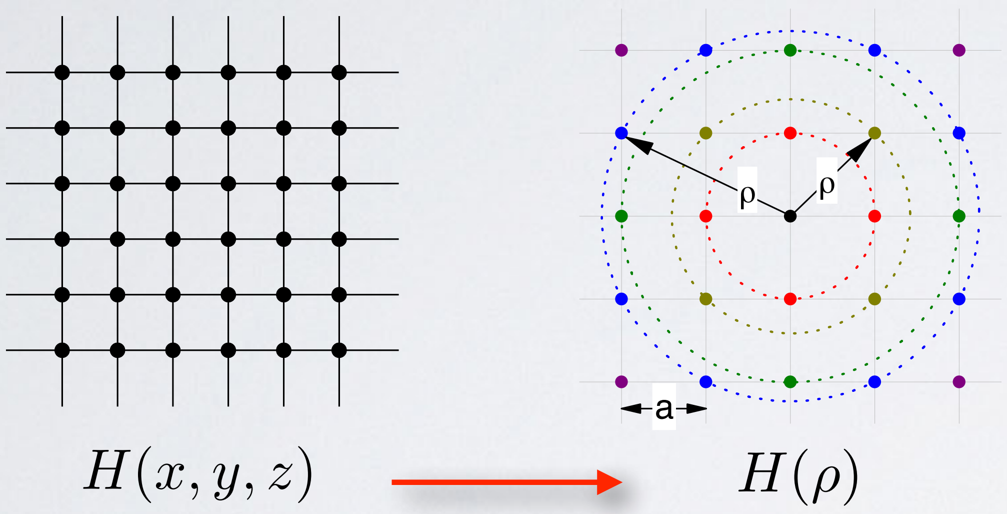

- Reduces scaling with L: $L^3 \rightarrow L^2$
- Saves CPU time.

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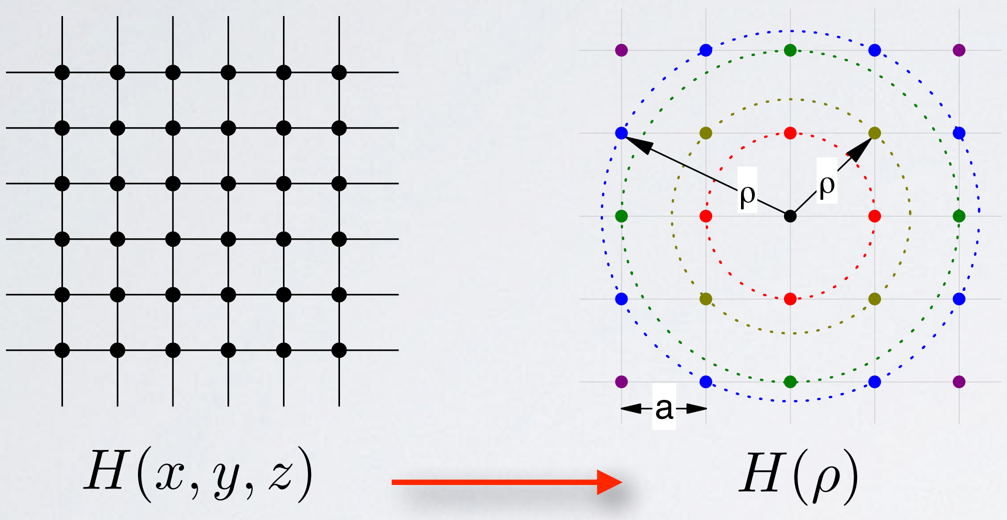

- Reduces scaling with L: $L^3 \rightarrow L^2$
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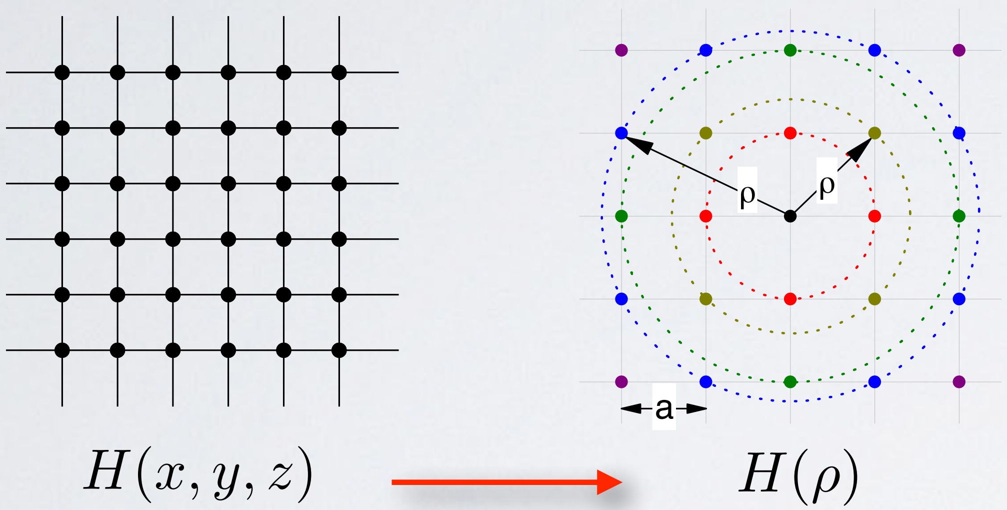

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  - Improvement in the mixing angles.

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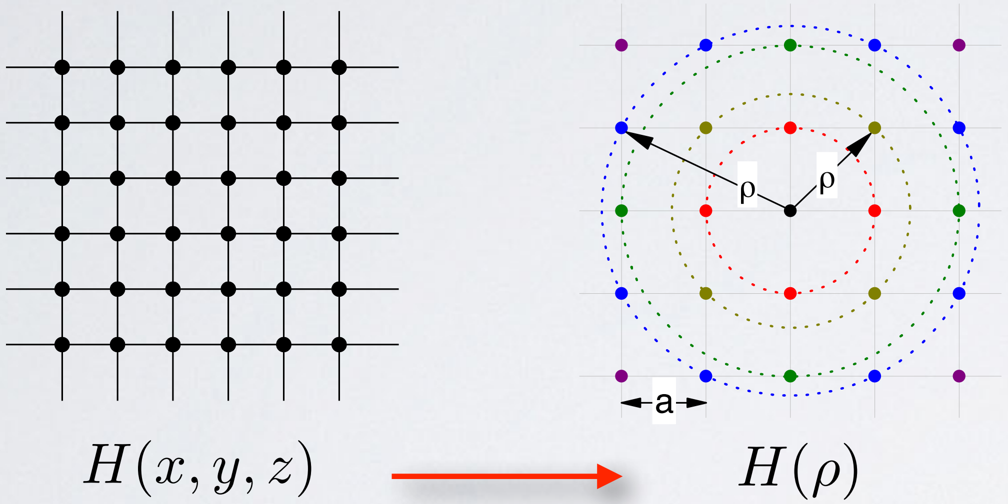

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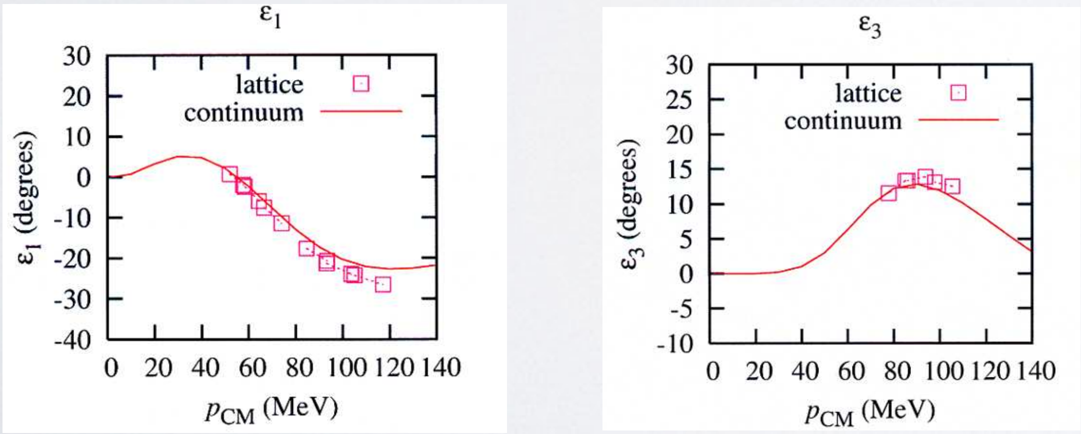
[Borasoy et al., EPJ A 34 (2007)]



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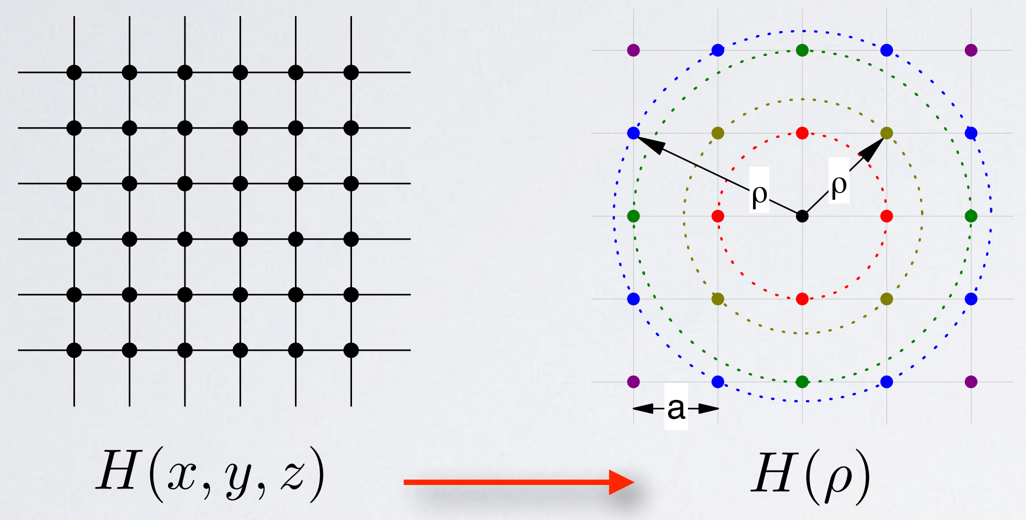

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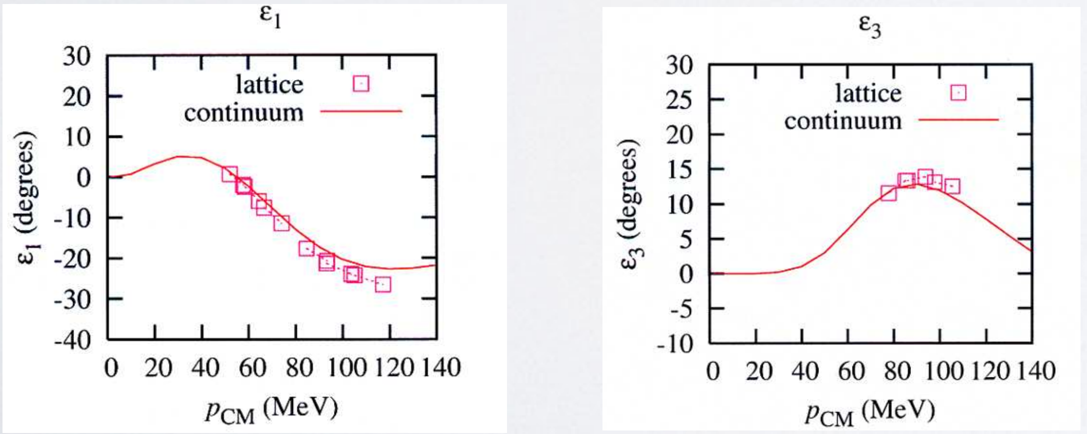

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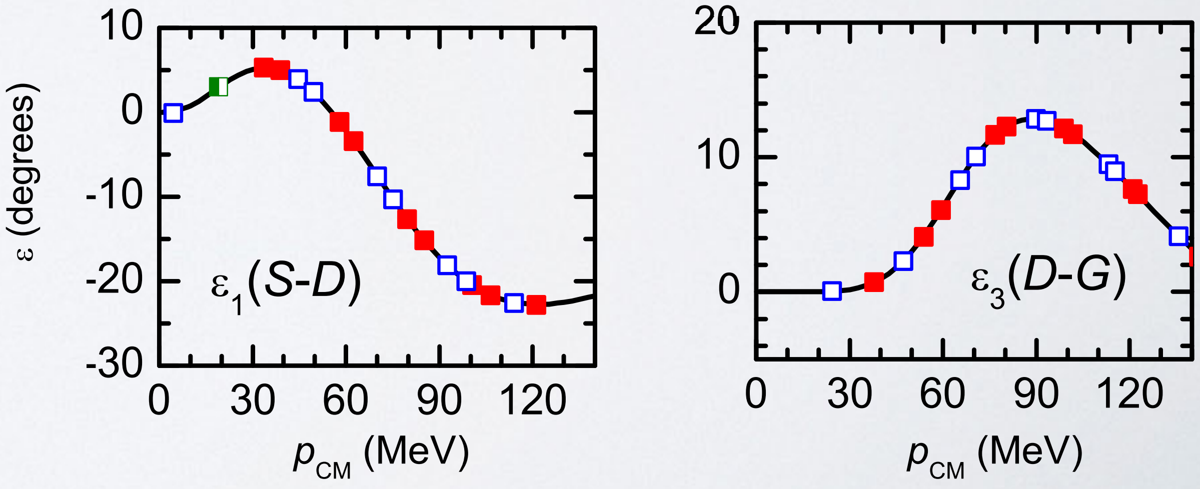


Summary and Conclusions

Summary and Conclusions

- Neutron-proton scattering on the lattice has made very important progress recently.
- Modification of the NN scattering calculation that allows:
  - Statistical analysis of the free parameters in the theory.
  - Systematic study of cutoff-dependence in the transfer matrix formalism.

  → Crucial to explore higher energies

- Radial Transfer Matrix formalism + auxiliary complex potential:
  - Improvement in the extraction of phase shifts and mixing angles.
  - Easy identification of states.

- Ready to include the TPE at NLO and N2LO.
- N3LO calculation is under way.
- Progress relevant for few-body *ab initio* calculations with NLEFT.

Stay tuned!

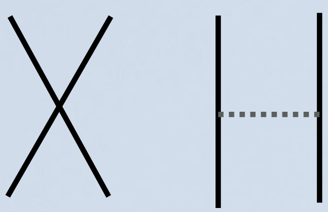
FIN

Spares

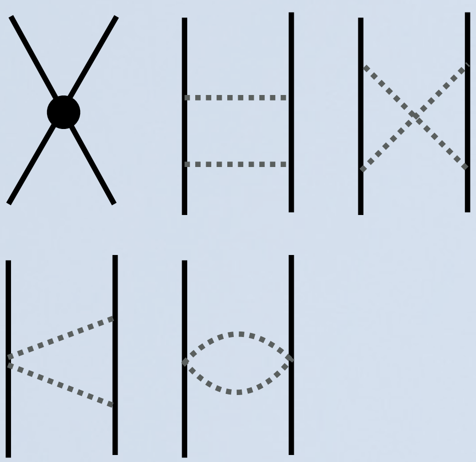
Introduction

- We calculate the energy levels in the transfer matrix formalism.
  - The same formalism used in Monte Carlo simulations.

$$\mathcal{Z} \propto \mathrm{Tr}\{M^{L_t}\} \qquad M \equiv : \exp\left[-(H_{free} + V)\alpha_t\right] : \rightarrow a_t/a$$

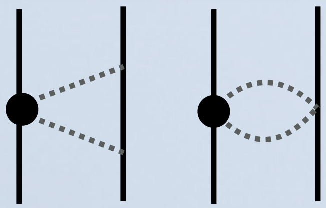


$$V^{(LO)} = \frac{1}{2} C \sum_{\vec{n}} f(\vec{n}) \left[\rho^{a^\dagger,a}(\vec{n})\right]^2 + \frac{1}{2} C_{I^2} \sum_{I} \sum_{\vec{n}} f(\vec{n}) \left[\rho_I^{a^\dagger,a}(\vec{n})\right]^2 - \frac{g_A^2 \alpha_t}{8 f_\pi^2 q_\pi} \sum_{S_1,S_2,I} \sum_{\vec{n}_1,\vec{n}_2} G_{S_1 S_2}(\vec{n}_1 - \vec{n}_2) \rho^{a^\dagger,a}_{S_1,I}(\vec{n}_1) \rho^{a^\dagger,a}_{S_2,I}(\vec{n}_2)$$



$$V^{(NLO)} = \sum_{i=1}^{7} C_i \mathcal{O}_i + \sum_{i=8}^{10} C_i \hat{\mathcal{O}}_i + \sum_{i=11}^{12} C_i \mathcal{O}_i^{(R)} + V_{TPE}^{(1)}$$

- $\sum_{i=1}^{7} C_i \mathcal{O}_i$: Continuum
- $\sum_{i=8}^{10} C_i \hat{\mathcal{O}}_i$: Cures some lattice artefacts
- $\sum_{i=11}^{12} C_i \mathcal{O}_i^{(R)}$: Rotational symmetry breaking
- $V_{TPE}^{(1)}$: Continuum



$$V^{(N^2LO)} = V_{TPE}^{(2)}$$

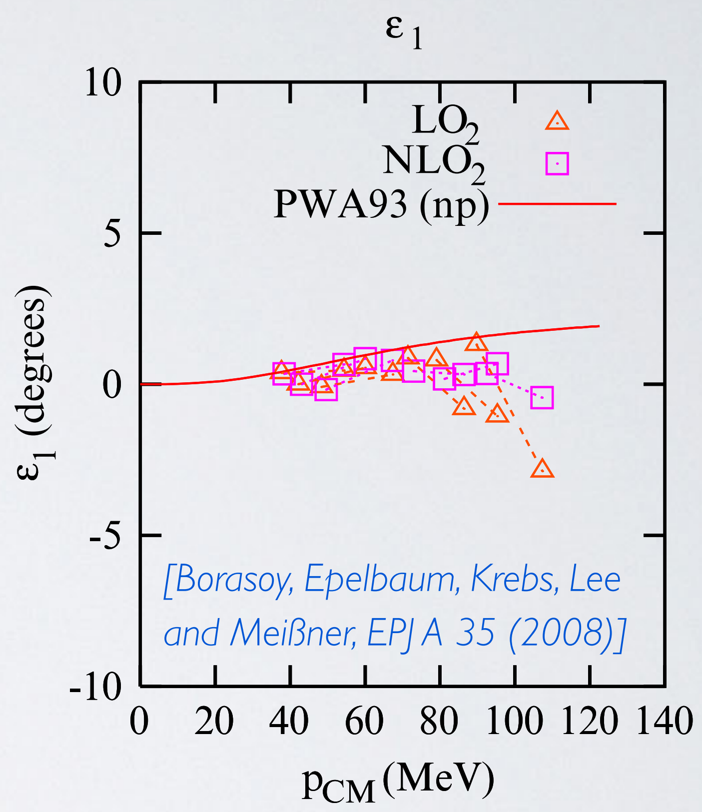
- $V_{TPE}^{(2)}$: Continuum

Mixing Angle

Mixing angle

- For spin triplet ($S=1$), channels with same total angular momentum (J) can mix.
- Lowest mixing happens in the the $\ell=0\,(S)$ and $\ell=2\,(D)$ waves.
- One can parametrize the ${}^3S_1$–${}^3D_1$ mixing in terms of one mixing angle ϵ_J, defined as:

$$S=\begin{pmatrix} e^{2i\delta_{J-1}}\cos 2\epsilon_J & ie^{2i(\delta_{J-1}+\delta_{J+1})}\sin 2\epsilon_J \\ ie^{2i(\delta_{J-1}+\delta_{J+1})}\sin 2\epsilon_J & e^{2i\delta_{J+1}}\cos 2\epsilon_J \end{pmatrix}$$



- Radial Transfer Matrix formalism + auxiliary complex potential. [Elhatisari et al., arXiv:1506.03513] [Lu et al., arXiv:1506.05652]
  - Reduces scaling with L: $L^3 \rightarrow L^2$
  - The code runs much faster.
  - Better identification of states.
    - Improvement in D-waves and mixing angle.

LECs

LECs

$$\Delta V = \frac{1}{2}\Delta C : \sum_{\vec{n}} \rho^{a^\dagger,a}(\vec{n})\rho^{a^\dagger,a}(\vec{n}) :,$$

$$\Delta V_{I^2} = \frac{1}{2}\Delta C_{I^2} : \sum_{\vec{n},I} \rho_I^{a^\dagger,a}(\vec{n})\rho_I^{a^\dagger,a}(\vec{n}) : .$$

$$V_{q^2} = -\frac{1}{2}C_{q^2} : \sum_{\vec{n},l} \rho^{a^\dagger,a}(\vec{n})\Delta_l^2\rho^{a^\dagger,a}(\vec{n}) :,$$

$$V_{I^2,q^2} = -\frac{1}{2}C_{I^2,q^2} : \sum_{\vec{n},I,l} \rho_I^{a^\dagger,a}(\vec{n})\Delta_l^2\rho_I^{a^\dagger,a}(\vec{n}) :,$$

$$V_{S^2,q^2} = -\frac{1}{2}C_{S^2,q^2} : \sum_{\vec{n},S,l} \rho_S^{a^\dagger,a}(\vec{n})\Delta_l^2\rho_S^{a^\dagger,a}(\vec{n}) :,$$

$$V_{S^2,I^2,q^2} = -\frac{1}{2}C_{S^2,I^2,q^2} : \sum_{\vec{n},S,I,l} \rho_{S,I}^{a^\dagger,a}(\vec{n})\Delta_l^2\rho_{S,I}^{a^\dagger,a}(\vec{n}) :,$$

$$V_{(q\cdot S)^2} = \frac{1}{2}C_{(q\cdot S)^2} : \sum_{\vec{n}}\sum_{S} \Delta_S\rho_S^{a^\dagger,a}(\vec{n})\sum_{S'}\Delta_{S'}\rho_{S'}^{a^\dagger,a}(\vec{n}) :,$$

$$V_{I^2,(q\cdot S)^2} = \frac{1}{2}C_{I^2,(q\cdot S)^2} : \sum_{\vec{n},I}\sum_{S} \Delta_S\rho_{S,I}^{a^\dagger,a}(\vec{n})\sum_{S'}\Delta_{S'}\rho_{S',I}^{a^\dagger,a}(\vec{n}) :,$$

$$V_{(iq\times S)\cdot k} = -\frac{i}{2}C_{(iq\times S)\cdot k} : \sum_{\vec{n},l,S,l'} \varepsilon_{l,S,l'}\left[\Pi_l^{a^\dagger,a}(\vec{n})\Delta_{l'}\rho_S^{a^\dagger,a}(\vec{n}) + \Pi_{l,S}^{a^\dagger,a}(\vec{n})\Delta_{l'}\rho^{a^\dagger,a}(\vec{n})\right] :,$$

$$V_{I^2,(iq\times S)\cdot k} = -\frac{i}{2}C_{I^2,(iq\times S)\cdot k} : \sum_{\vec{n},l,S,l'} \varepsilon_{l,S,l'}\left[\Pi_{l,I}^{a^\dagger,a}(\vec{n})\Delta_{l'}\rho_{S,I}^{a^\dagger,a}(\vec{n}) + \Pi_{l,S,I}^{a^\dagger,a}(\vec{n})\Delta_{l'}\rho_I^{a^\dagger,a}(\vec{n})\right] :,$$

$$V_{SSqq} = \frac{1}{2}C_{SSqq} : \sum_{\vec{n}}\sum_{S} \Delta_S\rho_S^{a^\dagger,a}(\vec{n})\Delta_S\rho_S^{a^\dagger,a}(\vec{n}) :,$$

$$V_{I^2,SSqq} = \frac{1}{2}C_{I^2,SSqq} : \sum_{\vec{n}}\sum_{S,I} \Delta_S\rho_{S,I}^{a^\dagger,a}(\vec{n})\Delta_S\rho_{S,I}^{a^\dagger,a}(\vec{n}) : .$$

LEC	Best values
$C_{1S0}(10^{-5}\ \mathrm{MeV}^{-2})$	$(-4.109, -3.948)$
$C_{3S1}(10^{-5}\ \mathrm{MeV}^{-2})$	$(-5.795, -5.953)$
b_4	$(0.07315, 0.08036)$
$\frac{1}{2}\Delta C$	$(-0.1001981989246, 0.069098012299509)$
$\frac{1}{2}\Delta C_{I^2}$	$(-0.1186509115258, -0.155867706639699)$
$-\frac{1}{2}C_{q^2}$	$(-0.040401953567687, 0.012600729397741)$
$-\frac{1}{2}C_{I^2,q^2}$	$(0.05827200896289, 0.087718222009940)$
$-\frac{1}{2}C_{S^2,q^2}$	$(-0.1823593021535, -0.155942762178279)$
$-\frac{1}{2}C_{S^2,I^2,q^2}$	$(0.154122107843797, 0.1564530211543)$
$\frac{1}{2}C_{(q\cdot S)^2}$	$(-0.007464222898765, -0.08246628305442)$
$\frac{1}{2}C_{I^2,(q\cdot S)^2}$	$(0.026826212664155, 0.09557831588958)$
$-\frac{i}{2}C_{i(q\times S)\cdot k}$	$(0.011724357058981, 0.01252865843888)$
$-\frac{i}{2}C_{I^2,i(q\times S)\cdot k}$	$(0.003908119019660, 0.004176219479628)$
$\frac{1}{2}C_{SSqq}$	$(0.416621988891837, 0.5407916495280)$
$\frac{1}{2}C_{I^2,SSqq}$	$(-0.416621988891837, -0.5407916495280)$

- With:

$$\Delta_l^2 f(\vec{n}) = f(\vec{n}+\hat{l}) + f(\vec{n}-\hat{l}) - 2f(\vec{n})$$

$$\Pi_l^{a^\dagger,a}(\vec{n}) = \frac{1}{4}\sum_{\nu_1,\nu_2,\nu_3=0,1}\sum_{i,j=0,1}(-1)^{\nu_l+1}a_{i,j}^\dagger(\vec{n}+\vec{\nu}(-l))a_{i,j}(\vec{n}+\vec{\nu})$$

$$\Pi_{l,S}^{a^\dagger,a}(\vec{n}) = \frac{1}{4}\sum_{\nu_1,\nu_2,\nu_3=0,1}\sum_{i,j,i'=0,1}(-1)^{\nu_l+1}a_{i,j}^\dagger(\vec{n}+\vec{\nu}(-l))[\sigma_S]_{ii'}a_{i,j}(\vec{n}+\vec{\nu})$$

$$\Pi_{l,I}^{a^\dagger,a}(\vec{n}) = \frac{1}{4}\sum_{\nu_1,\nu_2,\nu_3=0,1}\sum_{i,j,j'=0,1}(-1)^{\nu_l+1}a_{i,j}^\dagger(\vec{n}+\vec{\nu}(-l))[\tau_I]_{jj'}a_{i,j}(\vec{n}+\vec{\nu})$$

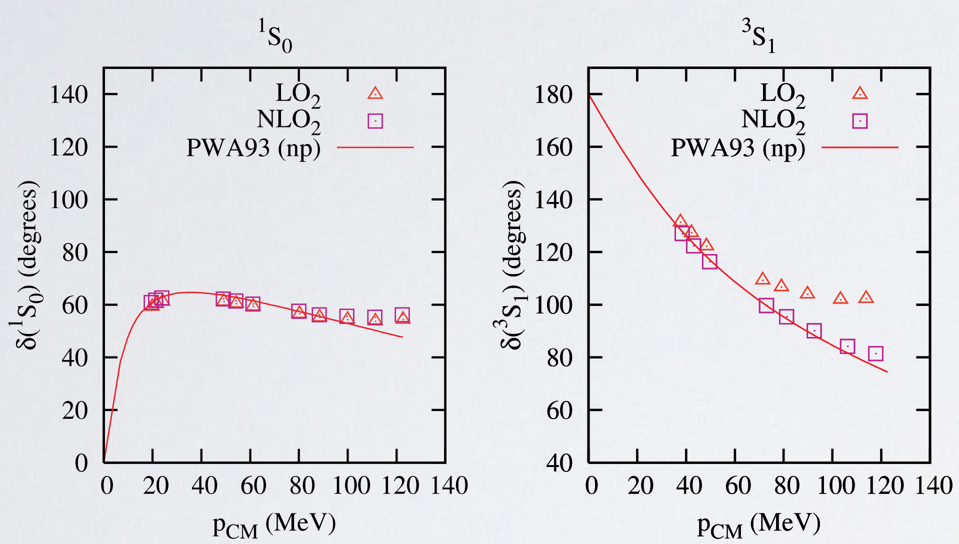
$$\Pi_{l,S,I}^{a^\dagger,a}(\vec{n}) = \frac{1}{4}\sum_{\nu_1,\nu_2,\nu_3=0,1}\sum_{i,j,i',j'=0,1}(-1)^{\nu_l+1}a_{i,j}^\dagger(\vec{n}+\vec{\nu}(-l))[\sigma_S]_{ii'}[\tau_I]_{jj'}a_{i,j}(\vec{n}+\vec{\nu})$$

Previous results

Previous Results

[Borasoy, Epelbaum, Krebs, Lee and Meißner, EPJ A 35 (2008)]

- S-waves



- P-waves