The pion quasiparticle in the low-temperature phase of QCD

Chiral dynamics in the low temperature phase of QCD

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Introduction...

Initial thoughts...

- The success of the HRG in describing equilibrium properties (e.g. quark number susceptibilities) has led to assuming that the vacuum spectrum does not change "too much" until temperatures near T_C.

$$\log Z_i^M(T, V, \mu_{X^a}, M_i) = -\frac{Vd_i}{2\pi^2}\int_0^\infty dkk^2 \log(1 - z_i e^{-E_i/T})$$

- Individual excitations of the system may have modified properties compared to $T = 0$.
- $\rightarrow$ In particular, it is worth asking what becomes of the relevant degrees of freedom that dominate the low-temperature regime ($T < T_C$) $\rightarrow \pi$ states

Goal

- Extensive study of the dispersion relation of the pion quasiparticle at finite temperature below the phase transition.

Pion dispersion relation...

- The ordinary pion dispersion relation dictated by Lorentz symmetry is $E = \sqrt{\mathbf{k}^2 + m_\pi^2}$.
- We want to a study a modified dispersion relation that takes the following form:

$$\omega_{\mathbf{p}} = u(T)\sqrt{\mathbf{p}^2 + m_\pi^2}$$

D.T. Son & M. Stephanov '02, [hep-ph/0204226]

- m_π is the screening mass (inverse correlation length).
- A chiral expansion was derived around the point $(T, m_q = 0)$ with $T < T_C$ as opposed to the usual $(T = 0, m_q = 0)$ ChPT expansion of Gasser and Leutwyler.
- We rederived it starting with Chiral Ward Identities arising from the PCAC relation and studied the $\mathbf{p} = 0$ case.

B. Brandt, A. Francis, H. Meyer & D.R., Phys. Rev. **D**90, 054509 (2014)

The double role of $u(T)$:

$$\omega_{\mathbf{p}} = u(T)\sqrt{\mathbf{p}^2 + m_\pi^2}$$

- At zero momentum it determines the ratio of the pion quasiparticle mass with respect to the screening mass.

$$\omega_{\mathbf{0}} = u(T) m_\pi \tag{1}$$

- In the chiral limit it truly corresponds to the group velocity of a massless pion excitation.

$$\mathbf{v}_g = \frac{d\omega_{\mathbf{p}}}{d\mathbf{p}} = u(T) < c. \tag{2}$$

Lattice approach

A priori it looks like a very simple problem...

- We should measure nothing but the pion ground states at finite temperature and finite momenta.

But the problem is...

- Because of how finite temperature is implemented on the Lattice, there is no hope on doing spectroscopy along the very short time direction ($N_\tau \sim 12, 16, 20, 24, ...$).
- The kernel appearing in euclidean time dependent correlators $K(\omega, x_0) = \frac{\cosh(\omega(\beta/2 - x_0))}{\sinh(\omega\beta/2)}$ falls off very slowly with x_0. $\beta \equiv 1/T$

Solutions...

- $\rightarrow$ Use *screening correlators* together with the Chiral expansion!
- Spectral function reconstruction: Maximum Entropy Method (MEM) (model dependent).
- Backus-Gilbert method (model independent).

Effective chiral expansion around $(T, m_q = 0)$

- In '02 Son & Stephanov demonstrated that the dispersion relation of the pion is fully determined by *static quantities*, which in principle can be measured accurately enough on finite T lattices.

- We exploit thermal Ward Identities arising from the PCAC relation to determine the residues of the relevant correlators in the chiral limit.
- Then, use those at small but finite quark mass in order complete the expansion.

For example: $\rho_A(\omega, \mathbf{p}) = \underbrace{f_\pi^2(m_\pi^2 + \mathbf{p}^2)}_{\mathrm{Res}(\omega_\mathbf{p})} \delta(\omega^2 - \omega_\mathbf{p}^2)$

[arXiv:1506.05732]

Limitations...

- Quark condensate is assumed to be different from zero.
- The quark mass has to be small.
- The width Γ is neglected (parametrically small if sufficiently closed to the chiral limit)
- Correlation functions containing pion states have to be dominated by the pion itself (at least at large distances).

Lattice estimators for $u(T)$ at $\mathbf{p} = 0$

$$\rho_A(\omega, 0) = \mathrm{sgn}(\omega) f_\pi^2 m_\pi^2 \delta(\omega^2 - \omega_{\mathbf{0}}^2) + \ldots$$

and finally by using:

$$\omega_{\mathbf{0}}^2 = \left.\frac{\partial_0^2 G_A(x_0, \mathbf{0})}{G_A(x_0, \mathbf{0})}\right|_{x_0=\beta/2} = -4m^2 \left.\frac{G_P(x_0, \mathbf{0})}{G_A(x_0, \mathbf{0})}\right|_{x_0=\beta/2}$$

$$u_f = \frac{f_\pi^2 m_\pi}{2 G_A(\beta/2, \mathbf{0}) \sinh(u_f m_\pi \beta/2)}$$

$$u_m = -\frac{4m^2}{m_\pi^2} \left.\frac{G_P(x_0, \mathbf{0})}{G_A(x_0, \mathbf{0})}\right|_{x_0=\beta/2}$$

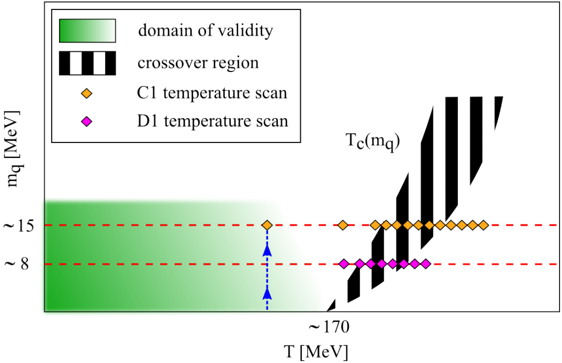
Relevant quantities...

- f_π, m_π
- $m \leftrightarrow \overline{m}^{\overline{\mathrm{MS}}}(\mu = 2\mathrm{GeV})$
- $G_A(\beta/2, \mathbf{0}),\ G_P(\beta/2, \mathbf{0})$

$\longrightarrow$ $u(T)$ is a RGI quantity!!

Lattice setup ...

- Two temperature scans (**C_1**, **D_1**) at constant renormalized awi-mass with $N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions.
- Lattice sizes are 16×32^3 covering a temperature range from 150 MeV to 235 MeV.



$\longrightarrow$ Additional CLS zero temperature ensemble (**A_5**) 64×32^3 equivalent to **C_1** at $m_\pi = 290$MeV: *test ensemble*.

Pion velocity results in the $\mathbf{C_1}$ scan. ...

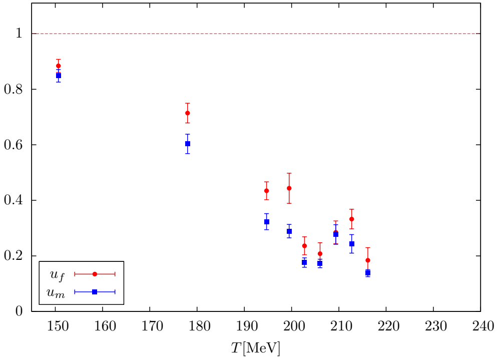


Figure: Pion velocitiy $u(T)$ Lattice estimators.

ChPT perturbative calculations of $u(T)$ in Real Time Formalism

- *Pion Propagation at finite temperature*, A. Schenk (1993)
- *Pion Dynamics at finite femperature*, D. Toublan (1997)

$\Longrightarrow$ Both find a significant reduction of the pion quasiparticle mass.

Going for a "better" finite T lattice ...

B. Brandt, A. Francis, H. Meyer & D.R. [arXiv:1506.05732]
submitted to Phys.Rev. D

Single finite T lattice ($N_f = 2$), $T = 169$MeV

- Finer lattice spacing ~ 0.05 fm than the ensembles of the scan ~ 0.08 fm.
- Bigger Volume 24×64^3: less finite volume and cutoff effects.
- Smaller pion mass ~ 270MeV.
- High statistics.
- $T = 0$ CLS ensemble 128×64^3 (O7) to compare with.

	$T = 169$MeV
u_f	0.76(1)
u_m	0.74(1)
u_f/u_m	1.02(1)

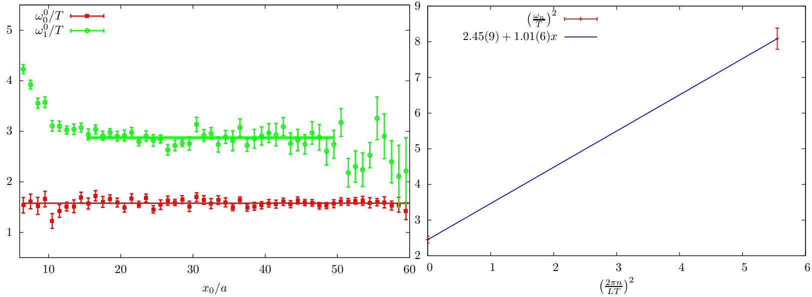
$$\frac{\text{Pion mass at } T = 169\text{MeV}}{\text{Pion mass at } T = 0} = 0.836(14)$$

$$\frac{\text{Pion decay constant at } T = 169\text{MeV}}{\text{Pion decay constant at } T = 0} = 1.03(2)$$

$\Longrightarrow$ Pion quasiparticle pole is shifted by $\sim 16\%$, while time-like pion decay constant remains constant!

Going for $\mathbf{p} \neq 0$, first the $T = 0$ case...

- Effective mass plots of axial charge euclidean correlators: $\int d^3x\ e^{i\mathbf{px}} \langle A_0(x) A_0(0) \rangle$



$\Longrightarrow$ No violation of boost invariance. Expected behavior of the dispersion relation $\rightarrow u(T \rightarrow 0) \sim 1$

Analyzing $G_A(x_0, T = 169\text{MeV}, \mathbf{p})$ at $\mathbf{p} \neq 0$

- Ground state extraction "à la $T = 0$" becomes unreliable:

1. Too few points available on the temporal extension of a finite T lattice.
2. Excited state contamination due to axial vector mesons contributing at $\mathbf{p} \neq 0$: $m_{a_1} \approx 1.2\text{GeV}$

$\rightarrow$ Analysis based on direct fits to $G_A(x_0, T, \mathbf{p})$. Include non-pion contributions:

$$\rho_A(\omega \rightarrow \infty, T, \mathbf{p}) = \theta(\omega^2 - 4m^2 - \mathbf{p}^2)\frac{N_c}{24\pi^2}(\mathbf{p}^2 + 6m^2).$$

$$\rho_A(\omega, T, \mathbf{p}) = A_1(\mathbf{p})\sinh(\omega\beta/2)\delta(\omega - \omega_{\mathbf{p}}) + A_2(\mathbf{p})\frac{N_c}{24\pi^2}\left(1 - e^{-\omega\beta}\right)\theta(\omega - c)$$

$$\Downarrow$$

$$\boxed{G_A(x_0, T, \mathbf{p}) = A_1(\mathbf{p})\cosh(\omega_{\mathbf{p}}(\beta/2 - x_0)) + A_2(\mathbf{p})\frac{N_c}{24\pi^2}\left(\frac{e^{-cx_0}}{x_0} + \frac{e^{-c(\beta - x_0)}}{\beta - x_0}\right)}$$

Fit results for $\mathbf{p} = (0, 0, 2\pi n/L)$

- Fit interval is $x_0/a \in [5, 12]$ to avoid cutoff effects present at small distances.
- 8 points for 4 parameters leads to poor constrained fits $\rightarrow$ $\omega_n = u_m\sqrt{m_\pi^2 + \mathbf{p}^2}$ is NOT a fit parameter. $u_m \sim 0.74(1)$ from the screeening analysis.

n	ω_n/T	$\tilde{A}_2 = A_2(\mathbf{p})/\mathbf{p}^2$	c/T	$\mathrm{Res}(\omega_\mathbf{p})$	$b(\mathbf{p})$	$\chi^2/d.o.f$
1	2.19(3)	1.78(8)	6.7(3)	1.72(6)	−0.08(3)	0.06
2	3.73(6)	1.26(2)	6.1(1)	3.3(2)	−0.39(4)	0.15
3	5.40(9)	1.19(1)	7.7(1)	3.9(5)	−0.65(4)	0.35
4	7.1(1)	1.15(1)	9.67(9)	4.21(7)	−0.78(3)	0.49

$$\begin{aligned}\rho(\omega, \mathbf{p}) &= \mathrm{Res}(\omega_\mathbf{p})\delta(\omega^2 - \omega_\mathbf{p}^2) \\ \mathrm{Res}(\omega_\mathbf{p}) &= 2A_1(\mathbf{p})\omega_\mathbf{p}\sinh(\omega_\mathbf{p}\beta/2) \\ &= f_\pi^2(m_\pi^2 + \mathbf{p}^2)(1 + b(\mathbf{p}))\end{aligned}$$

- Value of $\tilde{A}_2 = A_2(\mathbf{p})/\mathbf{p}^2 \rightarrow 1$
- $b(\mathbf{p})$ is small for $n = 1$. Chiral prediction fulfilled up to $|\mathbf{p}| \approx 400\text{MeV}$.

Backus-Gilbert method for $\rho_A(\omega, \mathbf{p})$

- Method for inverting integral equations. NOT a new method!

  The B.G. method revisited: background, implementation and examples,
  Hararioa & Somersalo (1987)

- Model independent approach for spectral function reconstruction

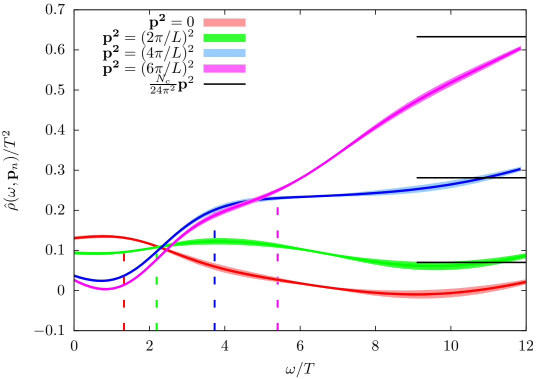
$$G_A(x_0, T, \mathbf{p}) = \int_0^\infty d\omega \left(\frac{\rho_A(\omega, \mathbf{p})}{\tanh(\omega/2)} \right) \underbrace{\left(\frac{\cosh(\omega(\beta/2 - x_0))}{\cosh(\omega\beta/2)} \right)}_{K(x_0,\omega)}.$$

- Consists in defining an estimator for the true spectral function $\rho_A(\omega, \mathbf{p})$

$$\hat{\rho}(\bar{\omega}, \mathbf{p}_n) = \int_0^\infty d\omega \; \hat{\delta}(\bar{\omega}, \omega) \left(\frac{\rho_A(\omega, \mathbf{p}_n)}{\tanh(\omega\beta/2)} \right) = \sum_{i=1}^{n} G(x_0^i, T, \mathbf{p}_n) q_i(\bar{\omega}).$$

- The resolution function $\hat{\delta}(\bar{\omega}, \omega)$ is a smooth function peaked around $\bar{\omega}$ which "smears" the true spectral function.
- The coefficients $q_i(\bar{\omega})$ are determined by minimizing the width of the resolution function subject to the condition that the area under the curve is normalized to 1.
- This is carried out by inverting a "close to singular" matrix. Regulating with the covariance matrix of the data is necessary!

BG results for $\hat{\rho}(\bar{\omega}, \mathbf{p}_n)$



What do we learn?

- The asymptotic value of the spectral function is reproduced very well. We obtain consistent results with the fits: $\tilde{A}_2 \to 1$.

$\rightarrow$ The method is exact if the spectral function is a constant.

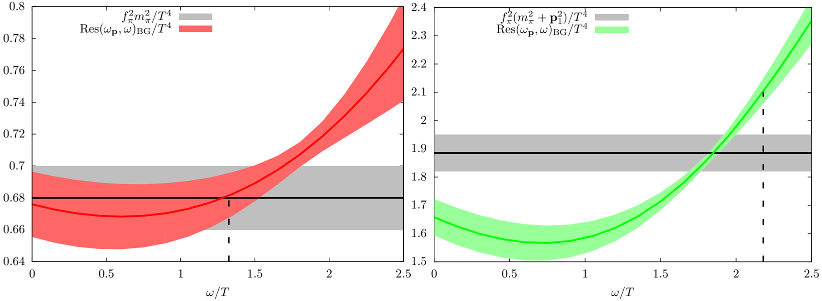
$\rightarrow$ With this estimator we cannot really resolve the low-frequency region.

Testing the chiral prediction for $\mathrm{Res}(\omega_{\mathbf{p}})$ with BG method

If we now plug in the expected value of $\omega_{\mathbf{p}}$ at a given $\bar{\omega}$, we can define an estimator

$$\mathrm{Res}(\omega_{\mathbf{p}}, \omega)_{\mathrm{BG}} = \frac{2\omega_{\mathbf{p}} \tanh(\omega_{\mathbf{p}}\beta/2)\hat{\rho}(\omega, \mathbf{p}_n)}{\hat{\delta}(\omega, \omega_{\mathbf{p}})}.$$

$|\mathbf{p}| = 0$ $\qquad\qquad$ $|\mathbf{p}| = 400\mathrm{MeV}$



$\Longrightarrow$ If evaluated at $\omega = \omega_{\mathbf{p}}$, where $\omega_{\mathbf{p}} = u_m\sqrt{m_\pi^2 + \mathbf{p}^2}$, the value of the residue is consistent with the chiral prediction at the 10% level and with the results of the fits.

Conclusions & Outlook

- Strong evidence that the pion dispersion relation at finite T is governed at small momenta by a single parameter $u(T) \sim 0.75$ up to $|\mathbf{p}| \sim 400\text{MeV}$ (boost invariance violated?).
- Splitting in the masses while the time-like pion decay constant remains unaffected (Implications for HRG?)

$$T = 0 : \qquad \text{pion mass} = 267(2)\,\text{MeV}$$

$$\swarrow \qquad\qquad \searrow$$

$$T = 169\text{MeV} : \qquad \begin{array}{l}\text{quasiparticle}\\ \text{mass} = 223(4)\text{MeV}\end{array} \qquad \begin{array}{l}\text{screening}\\ \text{mass} = 303(4)\text{MeV}\end{array}$$

- Various two-loop ChPT calculation at finite T support our findings: A. Schenk, '93, D. Toublan, '97
- The BG method has been found to be a useful tool for spectral function reconstruction within QCD.

- Extend the calculation to even lighter quark masses.
- Detailed study of finite size effects and cutoff dependence.

Backup

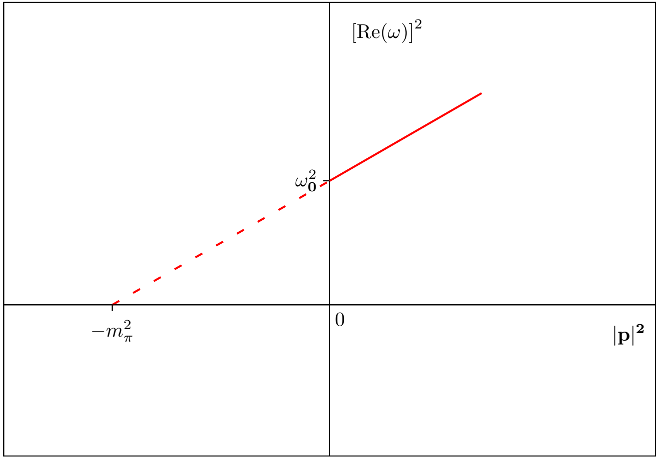


Figure: Trajectory of the pole in the pseudoscalar retarded correlator $G_R(\omega, \mathbf{p})$.

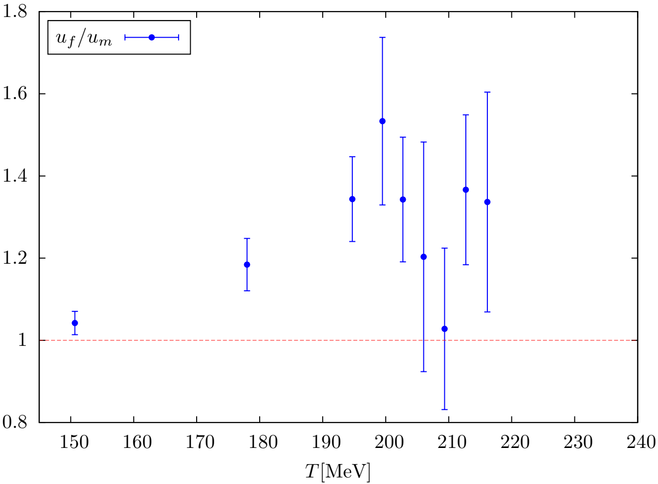
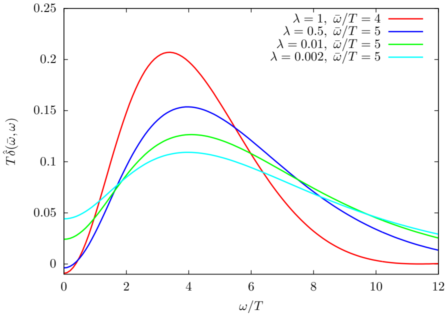


Figure: Ratio of estimators u_f/u_m

$\Longrightarrow$ Well in the deconfined phase $u_f/u_m \sim \mathcal{O}(T/m)$.

Some equations on the BG method

$$
\begin{aligned}
G(x_i) &= \int_0^\infty d\omega f(\omega) K(x_i,\omega) \\
\hat{f}(\bar{\omega}) &= \int_0^\infty d\omega \hat{\delta}(\bar{\omega},\omega) f(\omega) \\
\hat{\delta}(\bar{\omega},\omega) &= \sum_i q_i(\bar{\omega}) K_i(\omega) \\
q_i(\bar{\omega}) &= \frac{\sum_j W_{ij}^{-1}(\bar{\omega}) R(x_j)}{\sum_{k,l} R(t_k) W_{kl}^{-1}(\bar{\omega}) R(x_l)} \\
W_{ij}(\bar{\omega}) &= \int_0^\infty d\omega K(x_i,\omega)(\omega-\bar{\omega})^2 K(x_j,\omega) \\
R(x_i) &= \int_0^\infty d\omega K(x_i,\omega) \\
W_{ij} &\to \lambda W_{ij} + (1-\lambda) S_{ij}
\end{aligned}
$$



A more detailed derivation ...

In the massless theory ($m = 0$) we notice that $\langle P\mathbf{A}\rangle$ is fully determined by WI's:

$$G_{AP}(x_0, \mathbf{0}) = \int d^3x \, \langle P(x) A_0(0) \rangle = \frac{\langle \bar{\psi}\psi \rangle}{2\beta}(x_0 - \beta/2)$$

$$G_{AP}(x_0, \mathbf{k}) = \int d^3x e^{-i\mathbf{kx}} \langle P(x) A_0(0) \rangle = \int_0^\infty d\omega \rho_{AP}(\omega, k) \frac{\sinh(\omega(\beta/2 - x_0))}{\sinh(\omega\beta/2)}$$

One conclude easily that at zero momentum:

$$\rho_{AP}(\omega, 0) = -\frac{\langle \bar{\psi}\psi \rangle}{2}\delta(\omega)$$

a massless excitation persists at finite temperature for any temperature below T_C.

We define the screening mass m_π at *small but finite quark mass*, by making use of the results for the $\langle P\mathbf{A}\rangle$ correlator and the GOR relation:

$$f_\pi^2 m_\pi^2 = -m\left\langle\bar{\psi}\psi\right\rangle$$

Chiral Ward Identities imply for the static $\langle PP\rangle$ correlator:

$$\int dx_0 \,\langle P(0)P(x)\rangle = -\frac{\left\langle\bar{\psi}\psi\right\rangle^2}{4f_\pi^2}\frac{\exp(-m_\pi r)}{4\pi r} \qquad r\to\infty$$

Now, we use the following Ansatz

$$\rho_{\mathrm{P}}(\omega,k) = \mathrm{sgn}(\omega)C(k^2)\delta(\omega^2-\omega_{\mathbf{k}}^2)+...$$

$$\begin{aligned}\int dx_0\,\langle P(0)P(x)\rangle &= 2\lim_{\epsilon\to 0}\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kx}}\int_0^\infty\frac{d\omega}{\omega}e^{-\epsilon\omega}\rho_{\mathrm{P}}(\omega,k)\\ &= \int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kx}}\frac{C(k^2)}{\omega_{\mathbf{k}}^2}+...\end{aligned}$$

One last observation ...

By comparing the last two equation one concludes easily that

$$\omega_{\mathbf{k}}^2 \propto (\mathbf{k}^2 + m_\pi^2)$$

with

$$C(k^2) = -\frac{\left\langle \bar{\psi}\psi \right\rangle^2 u^2}{4 f_\pi^2}$$

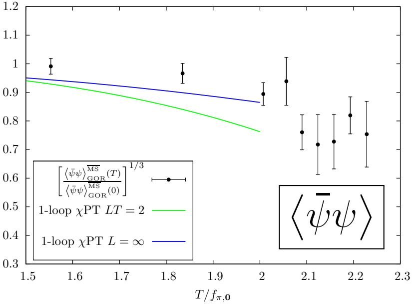
and f_π is defined by

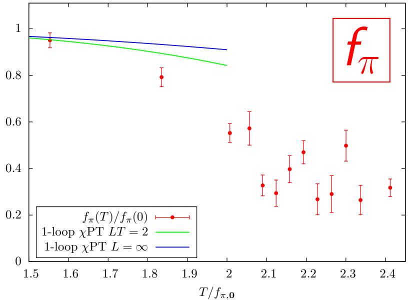
$$\int dx_0 d^2 x_\perp \left\langle A_3(x) A_3(0) \right\rangle = \frac{1}{2} f_\pi^2 m_\pi e^{-m_\pi |x_3|} \qquad |x_3| \to \infty$$

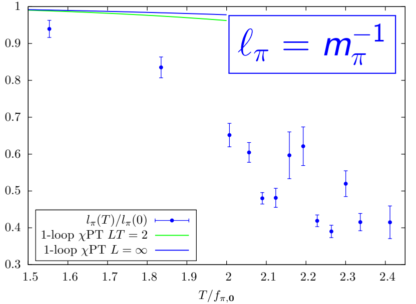
Conclusion:
We have proven our formula for the modified dispersion relation and showed that it is compatible with chiral WI's in the limit of small quark mass.

Test of chiral predictions ($\mathbf{A_5}$ comparison) ...

m_π [MeV]	305(5)
f_π [MeV]	93(2)
$\left\|\langle\bar\psi\psi\rangle^{\overline{\rm MS}}_{\rm GOR}\right\|^{1/3}$ $(\mu = 2{\rm GeV})$ [MeV]	364(7)
$\omega_{\mathbf{0}}$ [MeV]	294(4)
$f_{\pi,\mathbf{0}}$ [MeV]	97(3)
$\left\|\langle\bar\psi\psi\rangle^{\overline{\rm MS}}_{{\rm GOR},\mathbf{0}}\right\|^{1/3}$ $(\mu = 2{\rm GeV})$ [MeV]	368(9)
u_f	0.96(2)
u_m	0.92(6)
u_f / u_m	1.04(4)
$\omega_{\mathbf{0}} / m_\pi$	0.96(2)







Cross check. Maximum Entropy Method (MEM) ...

Goal: To reproduce the spectral function from the Euclidean correlator via different models:

Recalling the form of the spectral function for G_{A}:

$$\rho_{\mathsf{A}}(\omega,\mathbf{0}) = \frac{f_\pi^2 m_\pi}{2u}\delta(\omega-\omega_{\mathbf{0}}) \ + \ \dots \implies \mathcal{A}(\Lambda) \equiv 2\int_0^\Lambda \frac{d\omega}{\omega}\rho_{\mathsf{A}}(\omega,\mathbf{0}) = \frac{f_\pi^2}{u^2}$$

One introduces a strong systematic with MEM. One has to check the model independency of the results very carefully!

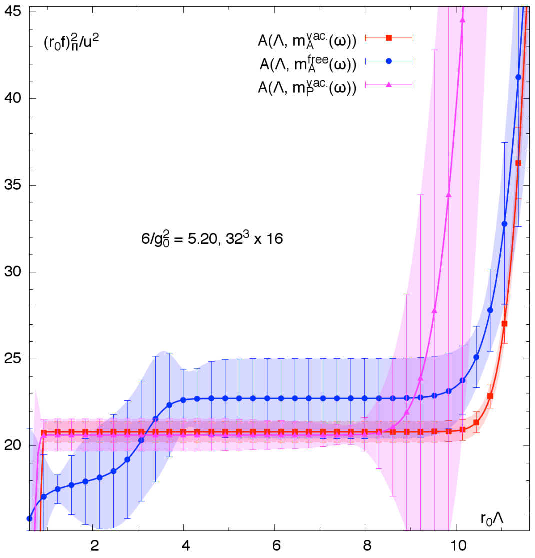


Figure: Cutoff Λ-dependence of $\mathcal{A}(\Lambda, m(\omega))$

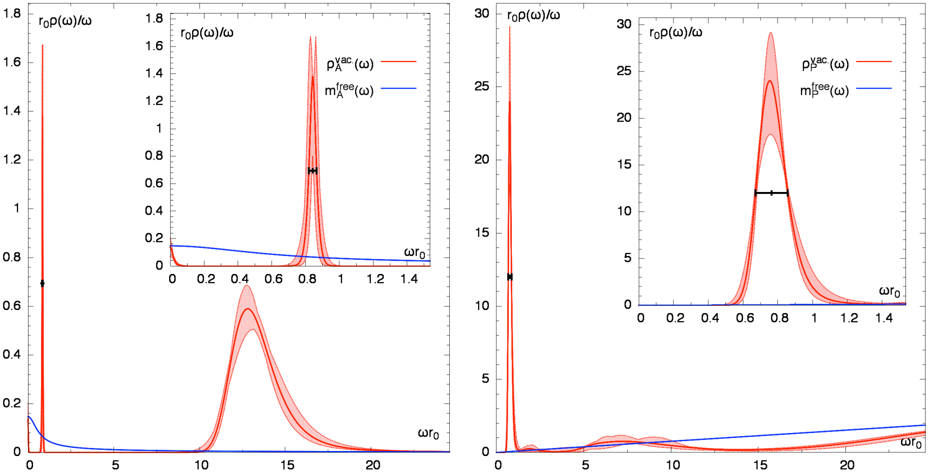


Figure: Left: $\langle PP \rangle (x_0)$ channel. Right: $\langle A_0 A_0 \rangle (x_0)$ channel.

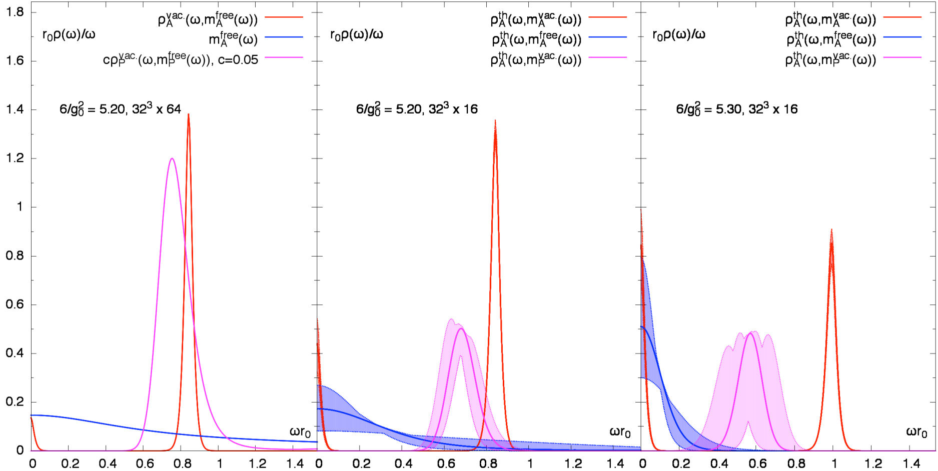


Figure: $\langle A_0 A_0 \rangle$ reconstruction for the 3 different default models.

Summary of MEM results ...