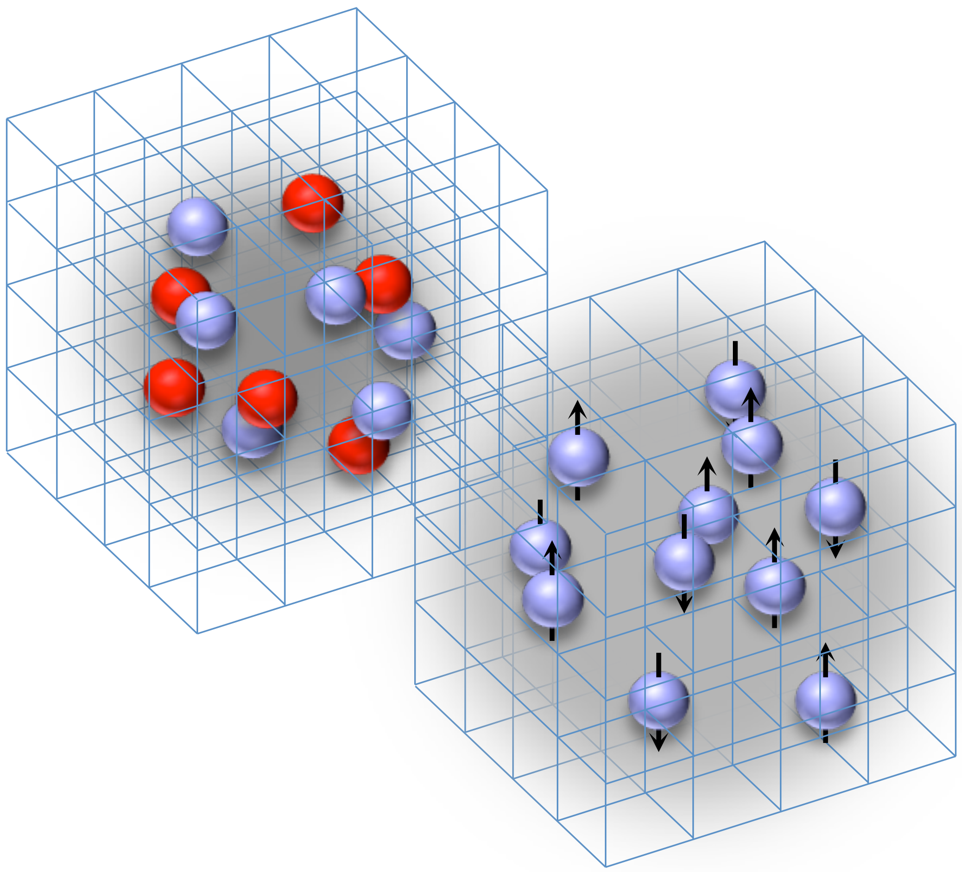
Alexander Rokash

The 8th International Workshop on
Chiral Dynamics, 29 June-03 July 2015

Scattering cluster wave functions on the lattice using the adiabatic projection method



Serdar Elhatisari - Bonn
Michelle Pine - NC State
Dean Lee - NC State
Evgeny Epelbaum - RUB
Hermann Krebs - RUB

Outline

- Introduction
- Adiabatic projection method
- Asymptotic cluster wave functions
- One and three dimensional examples
- Summary

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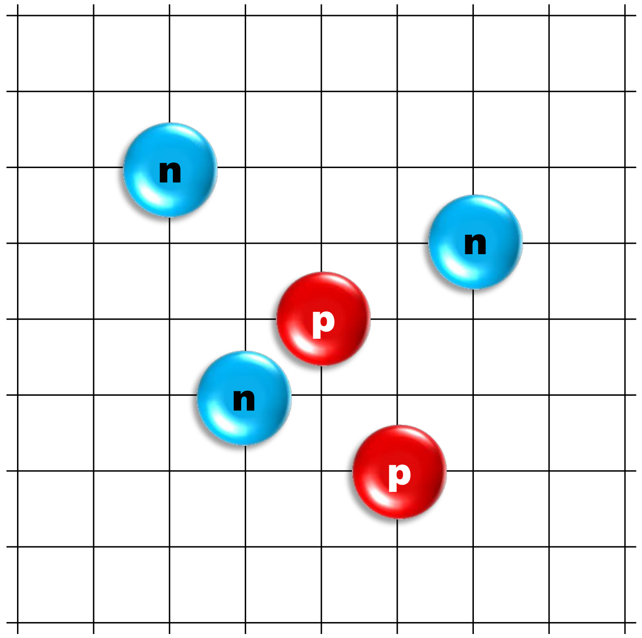

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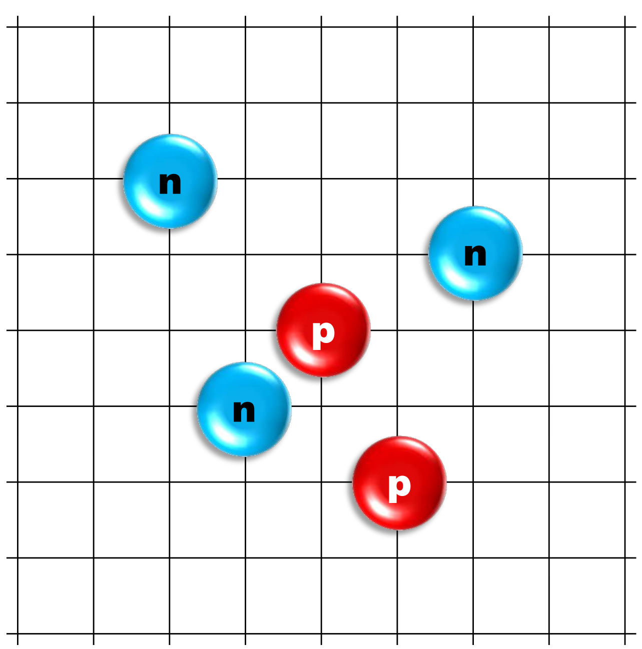

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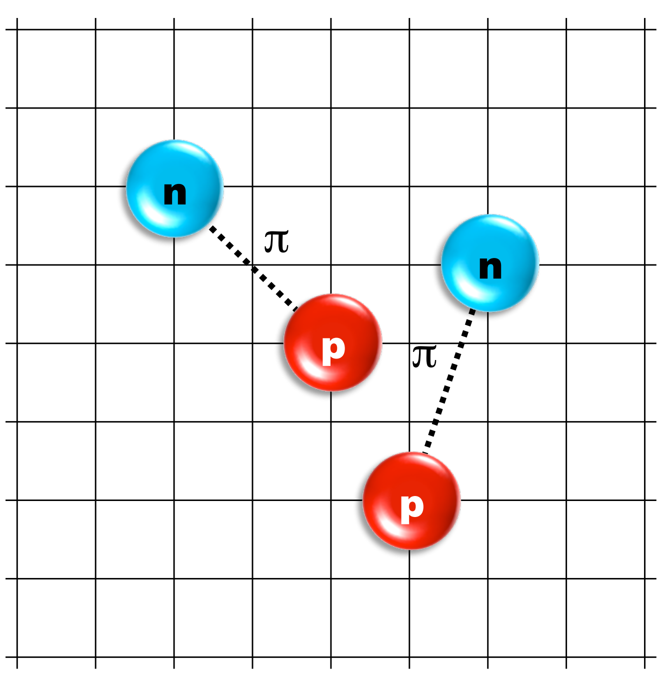

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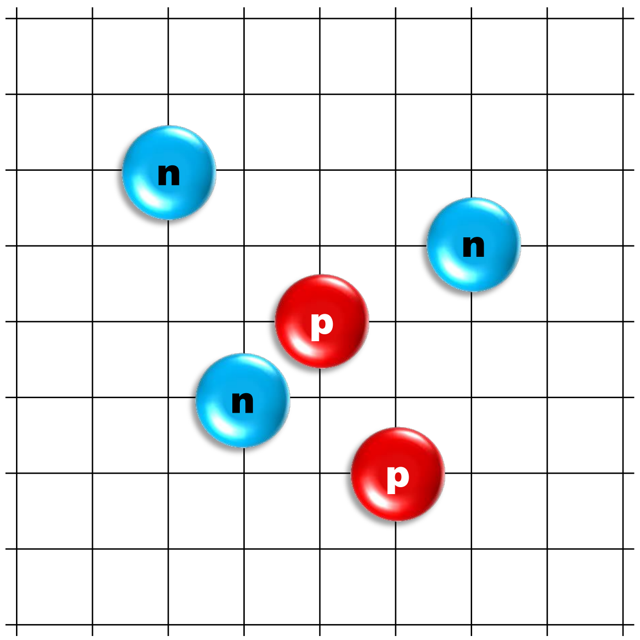

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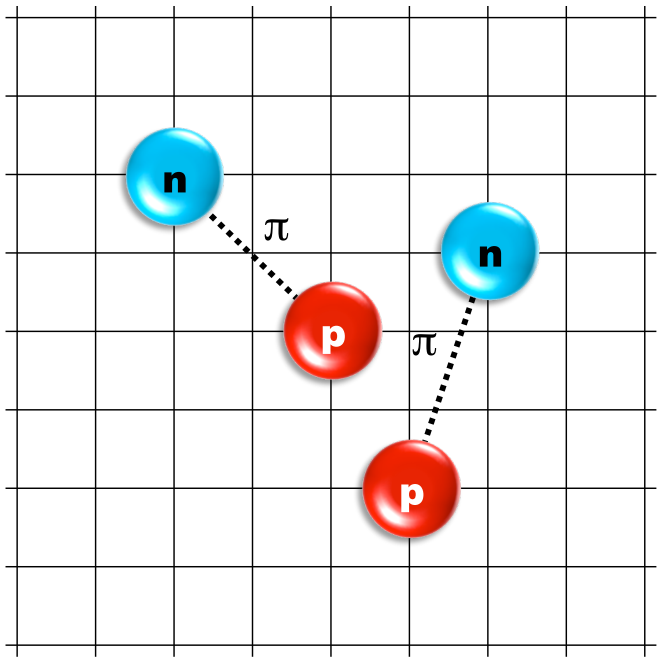

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  - Adiabatic projection method: L^d x L^d

M. Pine, D.Lee and G.Rupak: Eur. Phys. J. A (2013) 49: 151
G. Rupak and D. Lee, Phys. Rev. Lett. 111, no. 3 (2013), 032502
S. Elhatisari and D. Lee: Phys. Rev. C 90, no. 6 (2014), 064001

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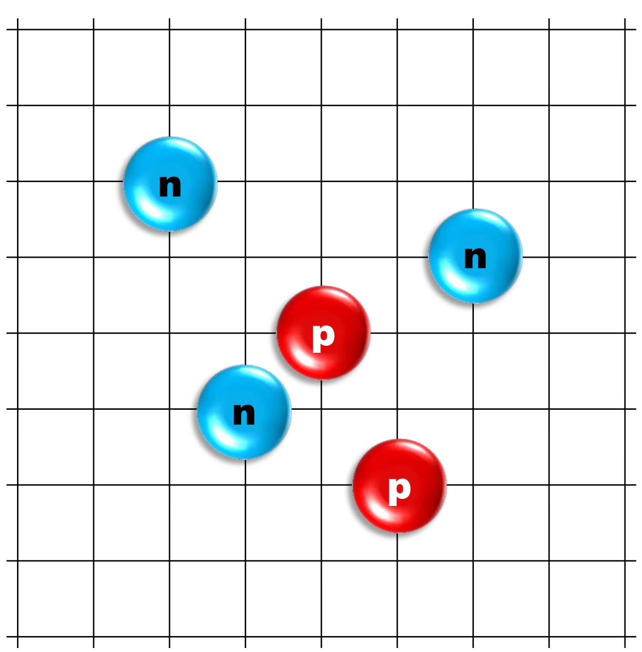

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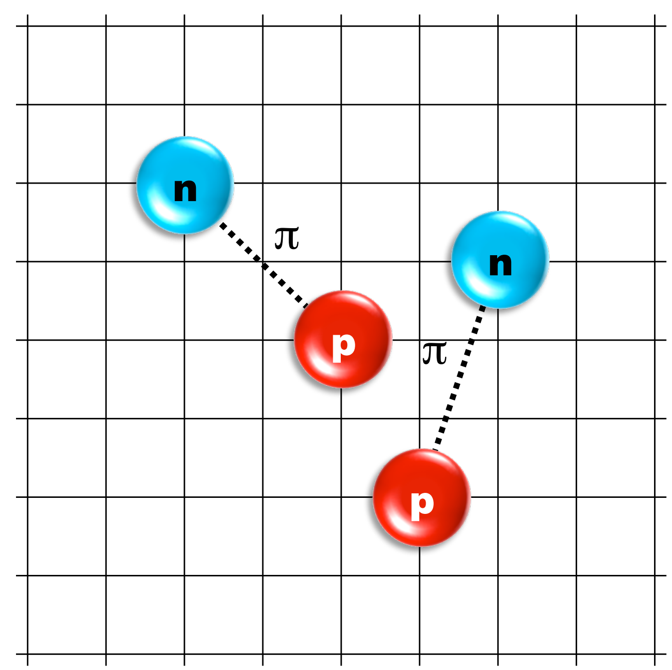

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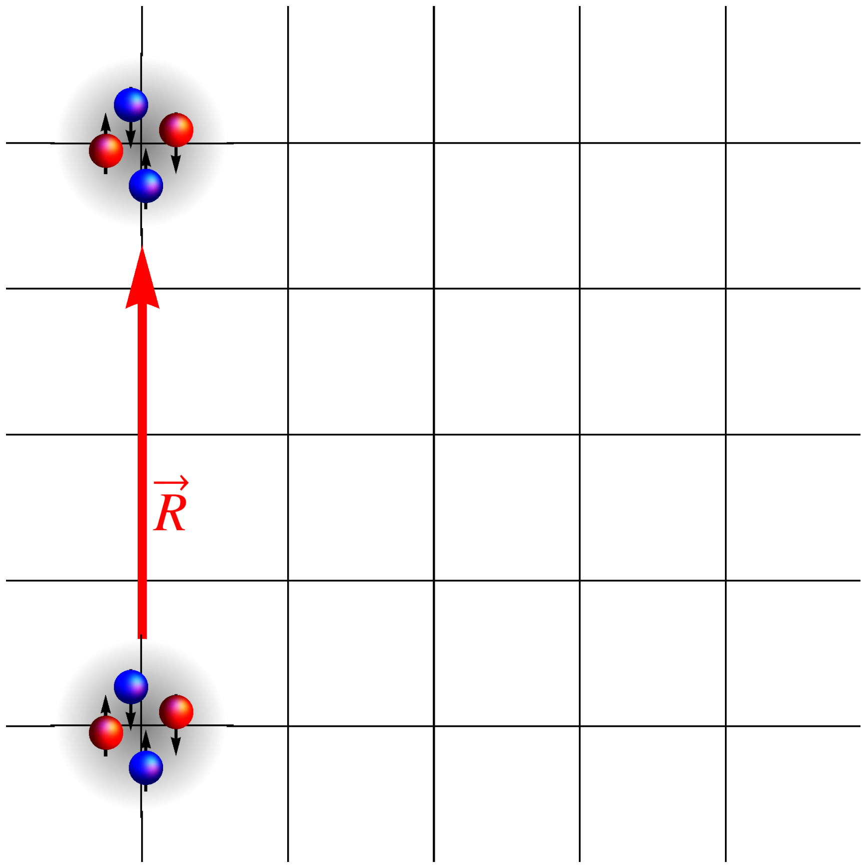

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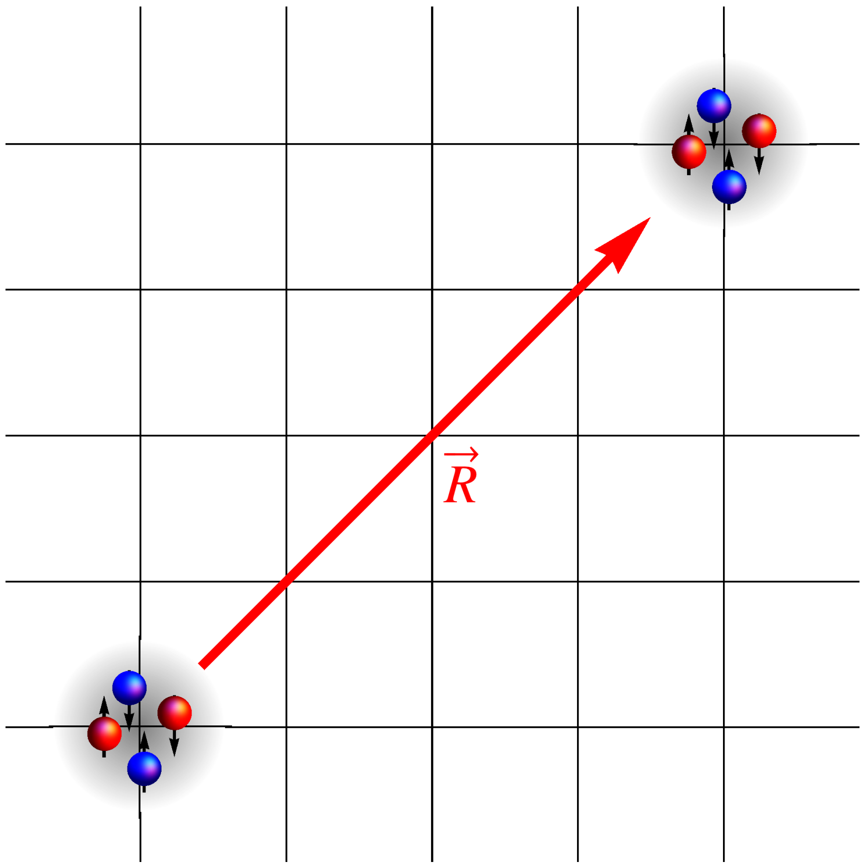

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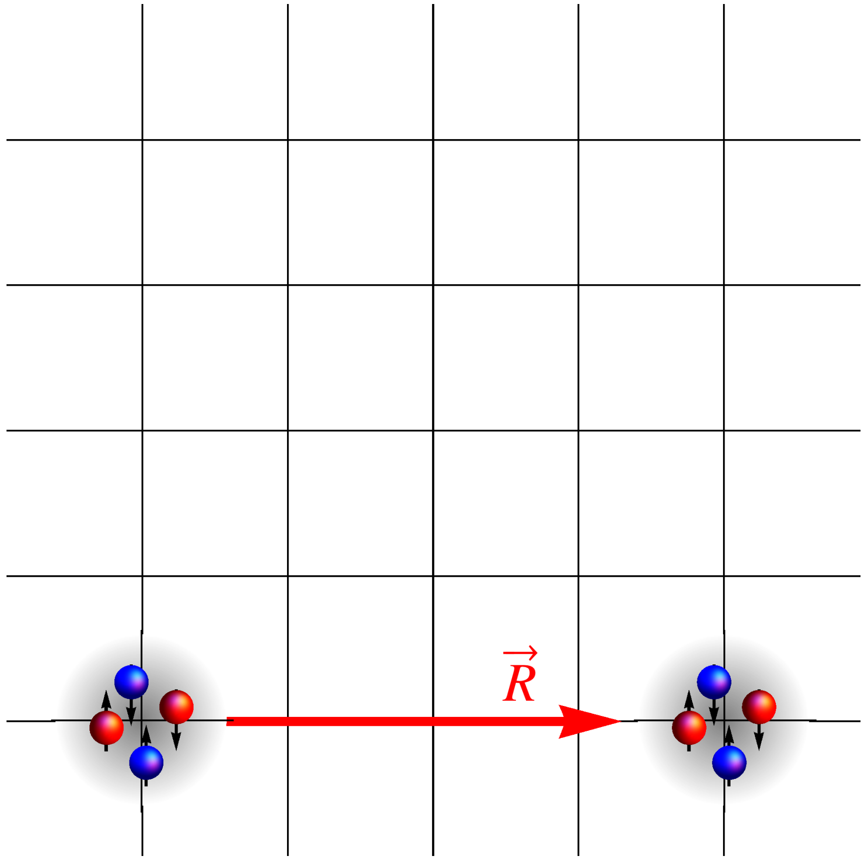

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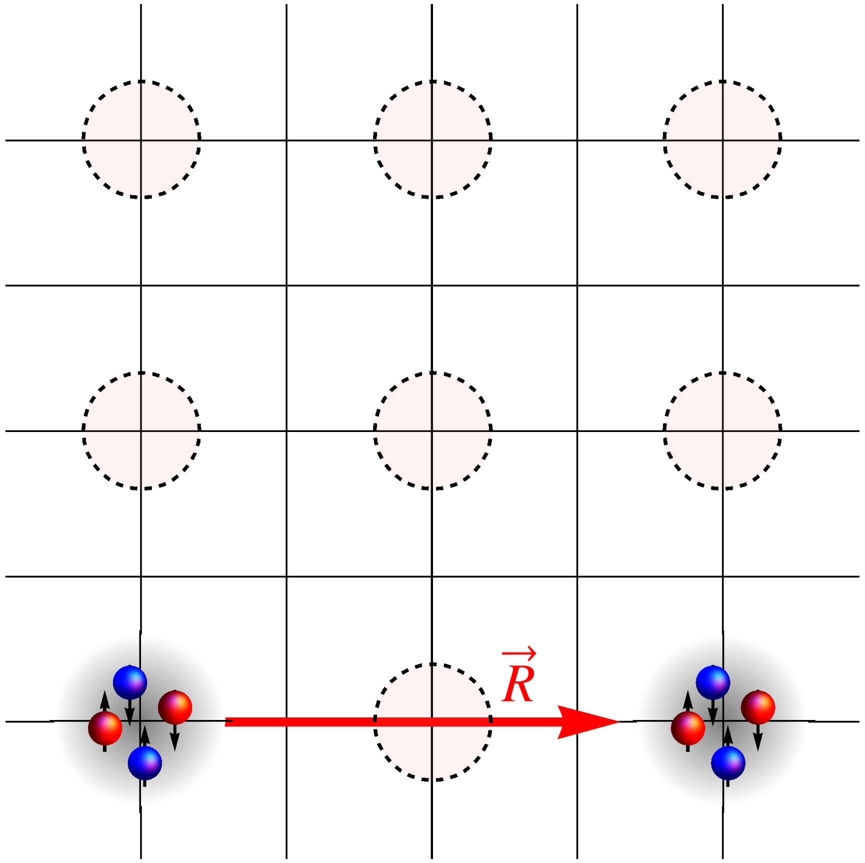

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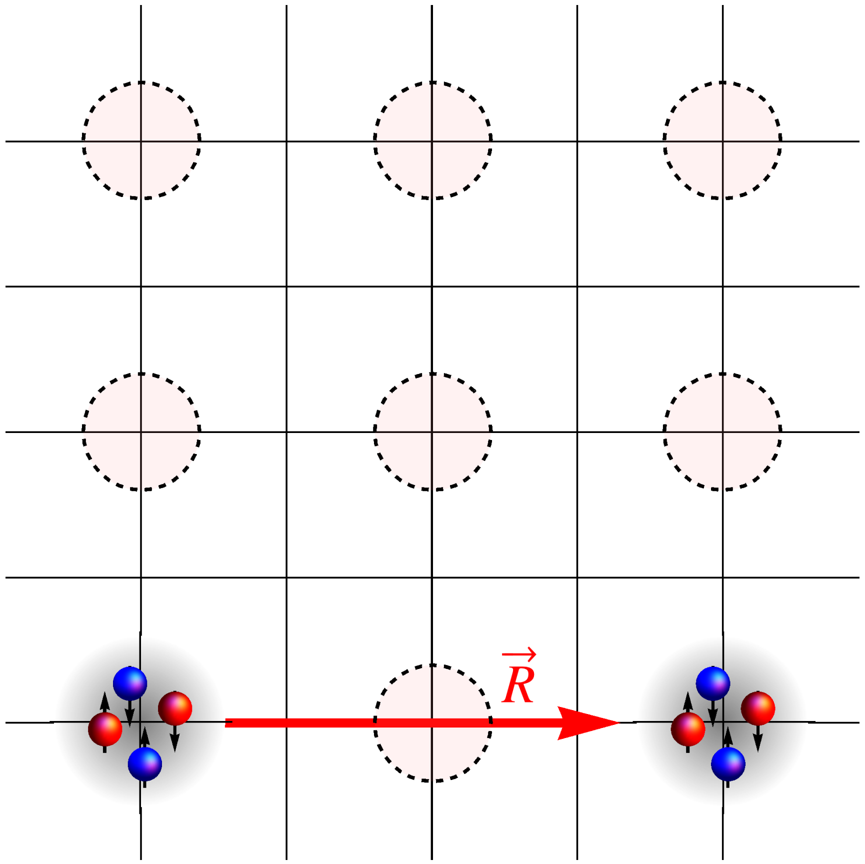
$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$



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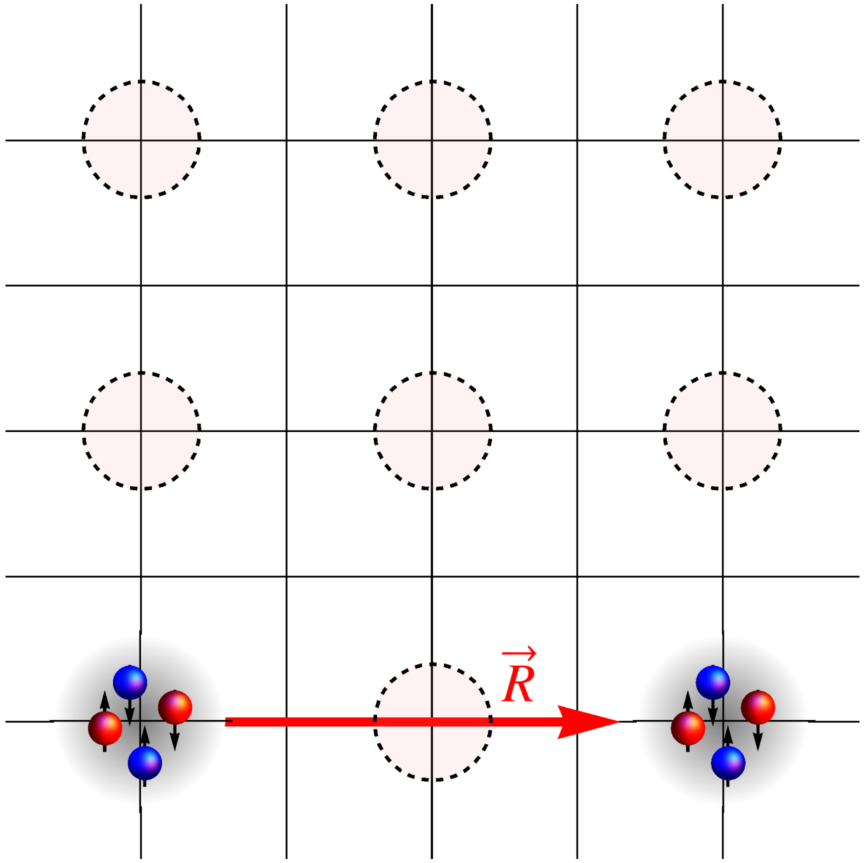

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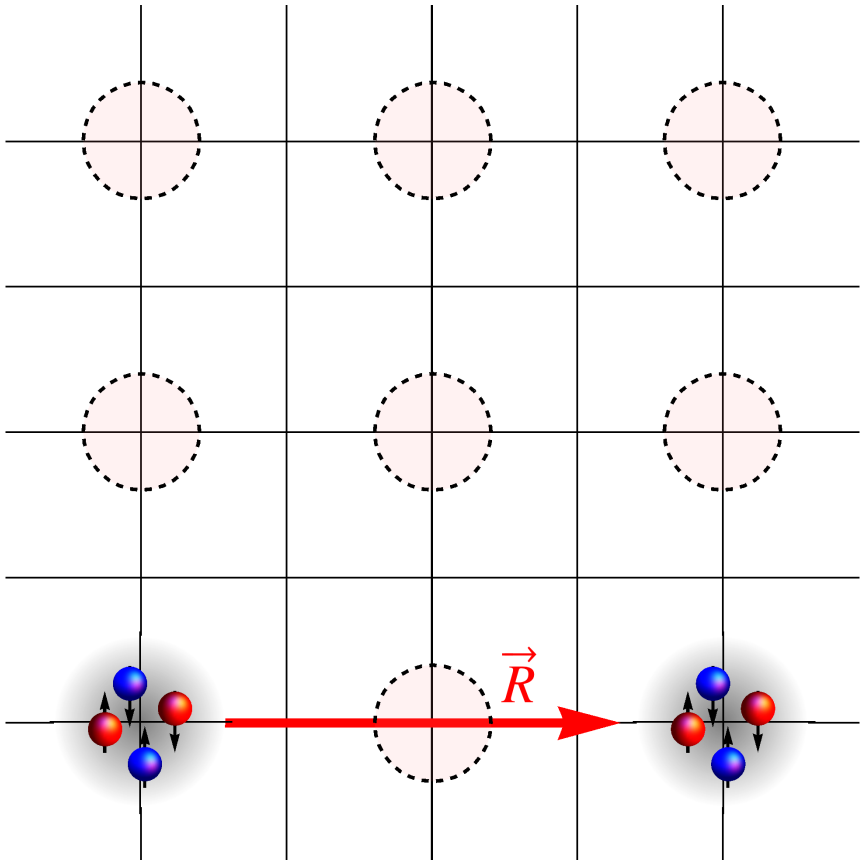

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Matrix elements of dressed cluster states

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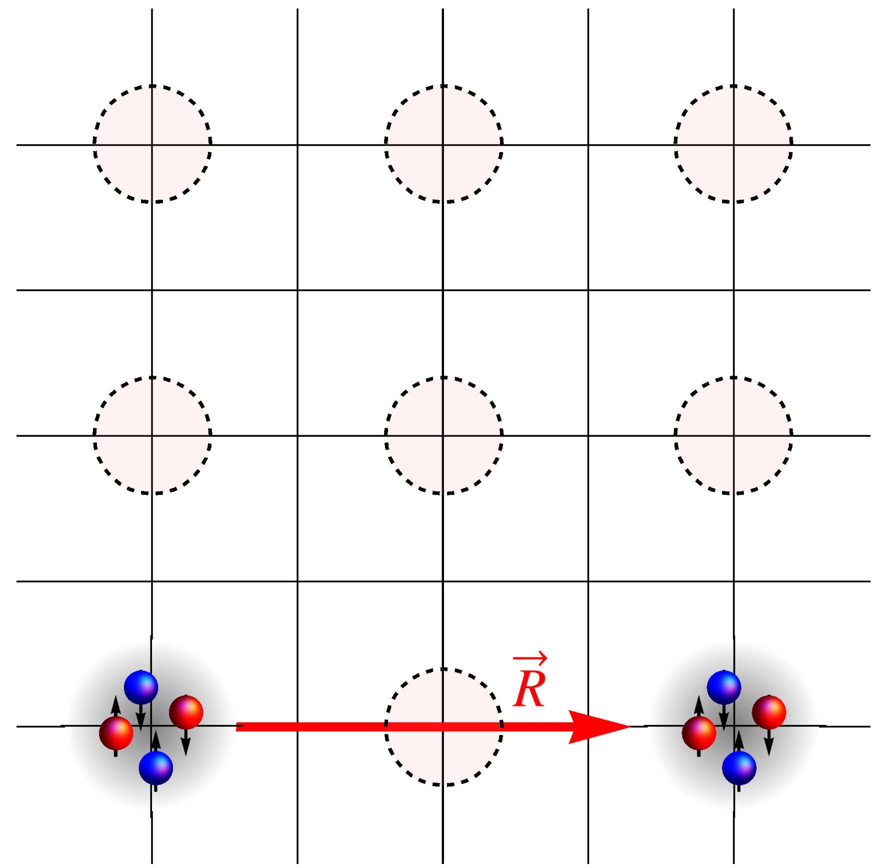

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Norm matrix

Matrix elements of dressed cluster states

RUB

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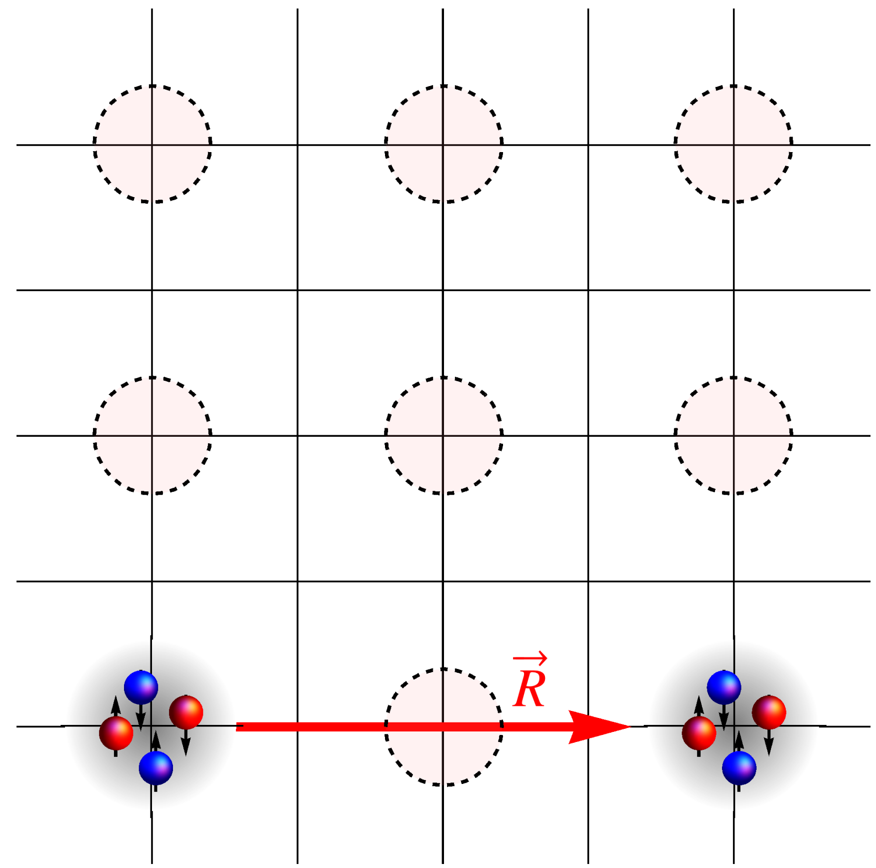

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similar to NCSM

Navratil, Quaglioni, Phys. Rev. C 83, 044609 (2011).

Adiabatic projection method (II)

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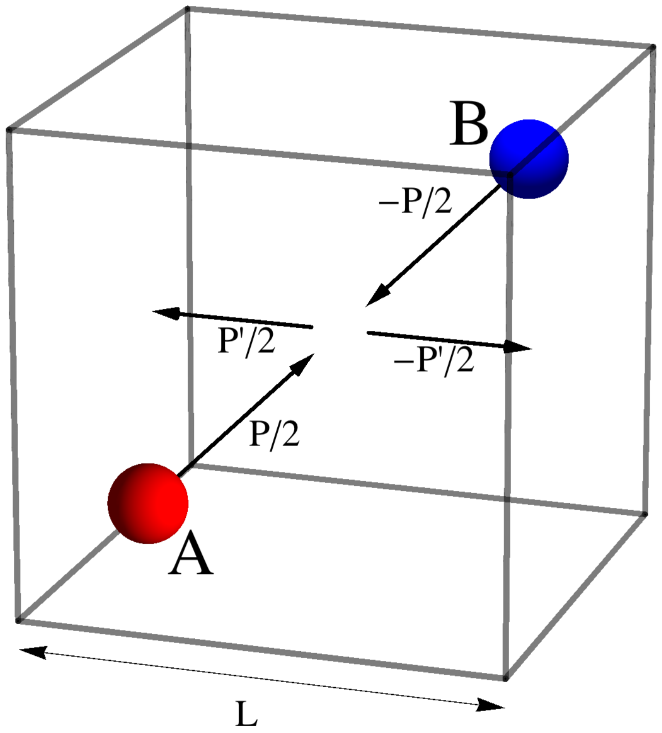
  - Lüscher's method: Relation between energy levels in a finite periodic box and the infinite volume scattering phase shifts

M.Lüscher, Commun. Math. Phys. 105 (1986), 153

$$p\cot\delta_0(p) = \frac{1}{\pi L}S(\eta), \quad \eta = \frac{p(L)^2 L^2}{4\pi^2}$$

$$S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\vec{k}\in\mathbb{Z}^3}\frac{\theta(\Lambda^2-\vec{k}^2)}{\vec{k}^2-\eta} - 4\pi\Lambda\right]$$



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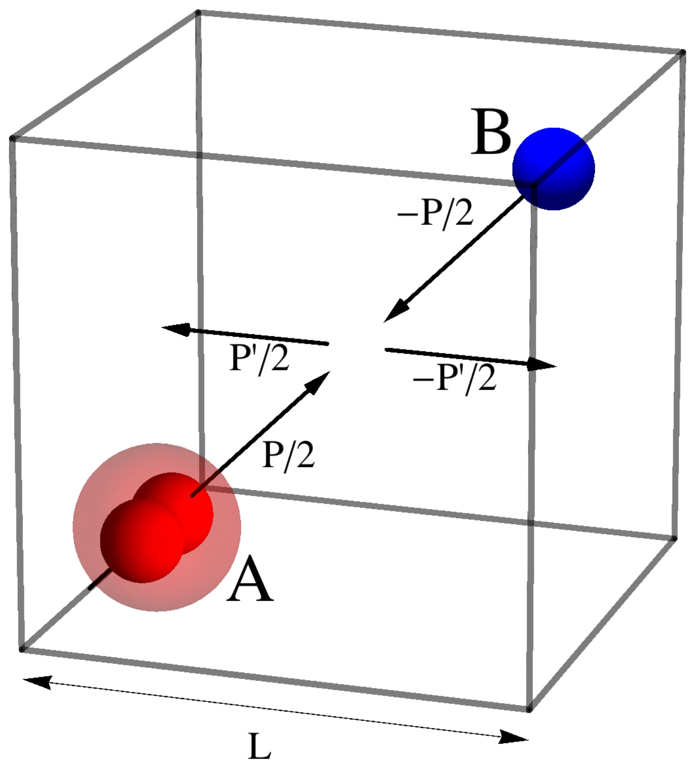

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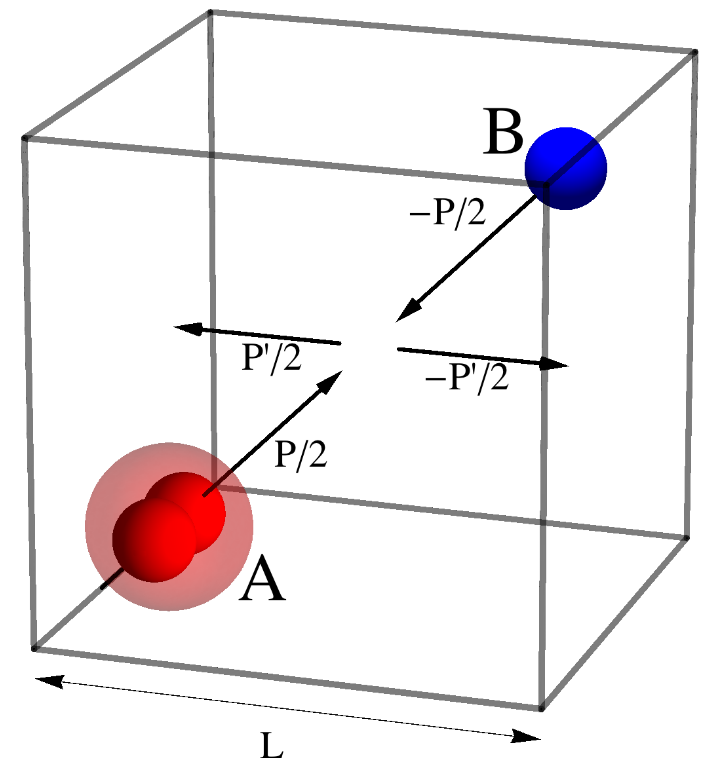


  - Topological corrections due to cluster character

$$E(p, L) = \frac{p^2}{2\mu} - B_1 - B_2 + \tau_1(\eta)\Delta E_1(L) + \tau_2(\eta)\Delta E_2(L)$$

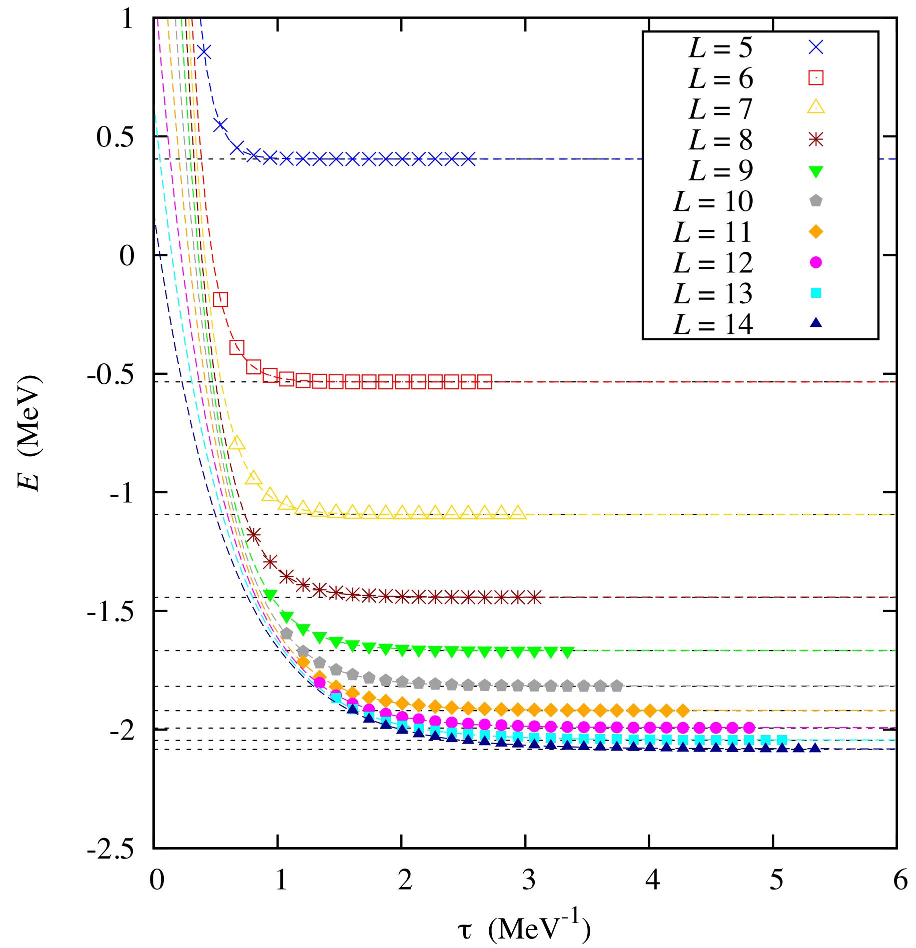
$$\tau(\eta) = \frac{1}{\sum_{\vec{k}} \left(\vec{k}^2 - \eta\right)^{-2}} \sum_{\vec{k}} \frac{\sum_{i=1}^{3} \cos\left(2\pi k_i\, \alpha\right)}{3\left(\vec{k}^2 - \eta\right)^2}$$

S. Bour, S. König, D. Lee, H.-W. Hammer and U.-G. Meißner, Phys. Rev. D 84 (2011), 091503

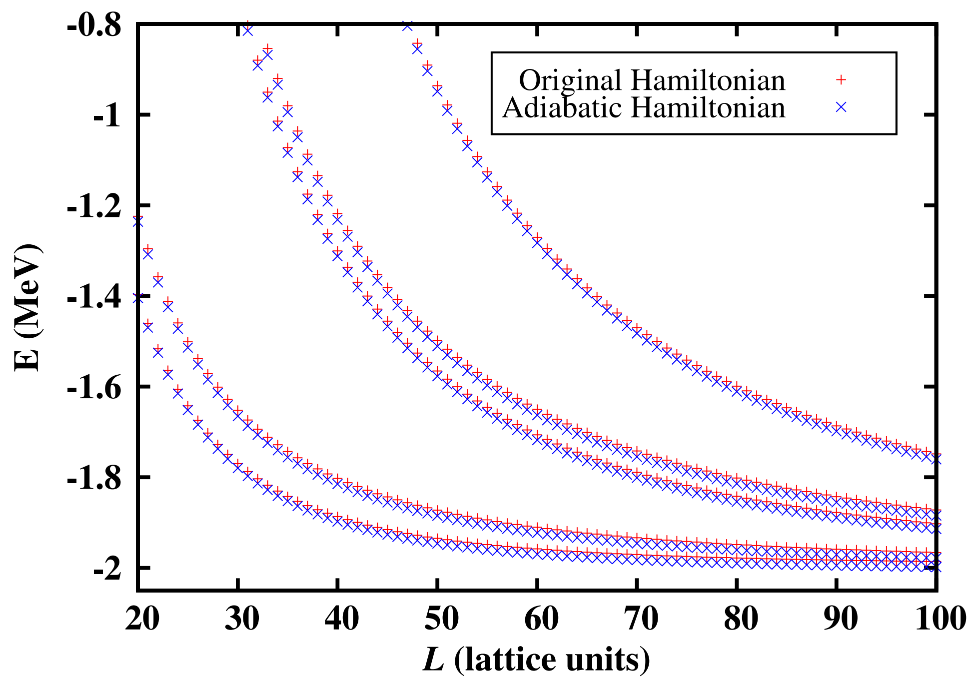
S. Bour, H.-W. Hammer, D. Lee and U.-G. Meißner, Phys. Rev. C 86 (2012), 034003

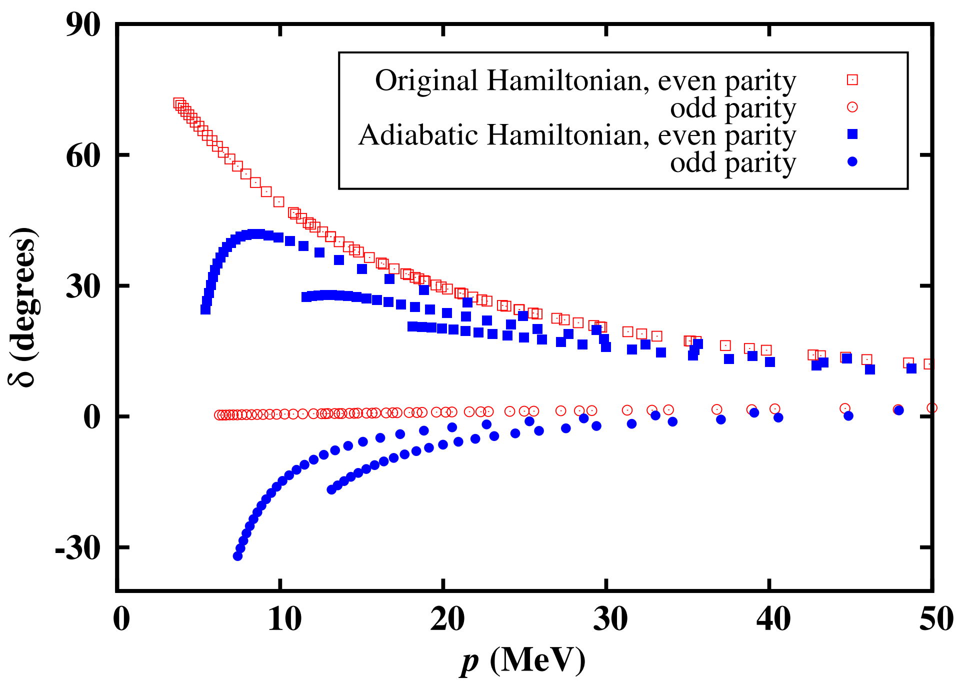
Adiabatic projection method (III)

- There is an exponentially small error in energy levels due to Euclidean time projection
- In larger systems there is a statistical error due to Monte Carlo methods

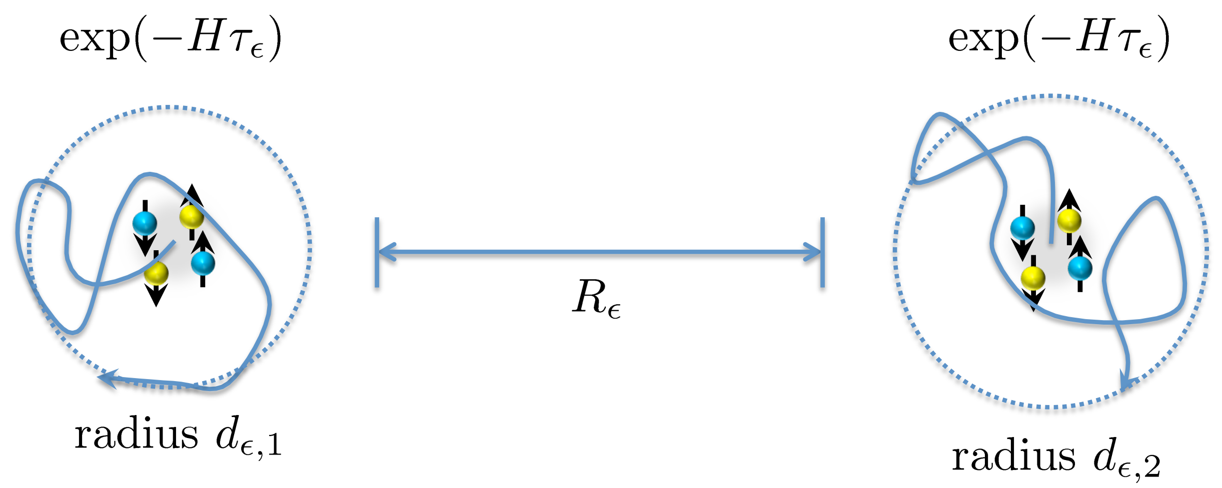


Lüscher's method is unfortunately very sensitive to small errors in energy levels!





Asymptotic cluster wave functions (I)



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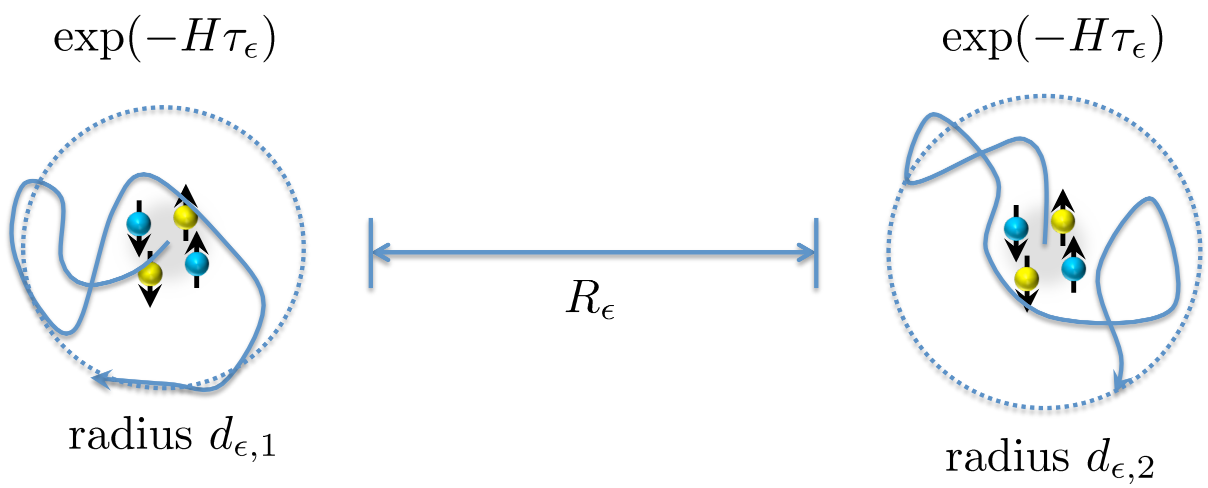
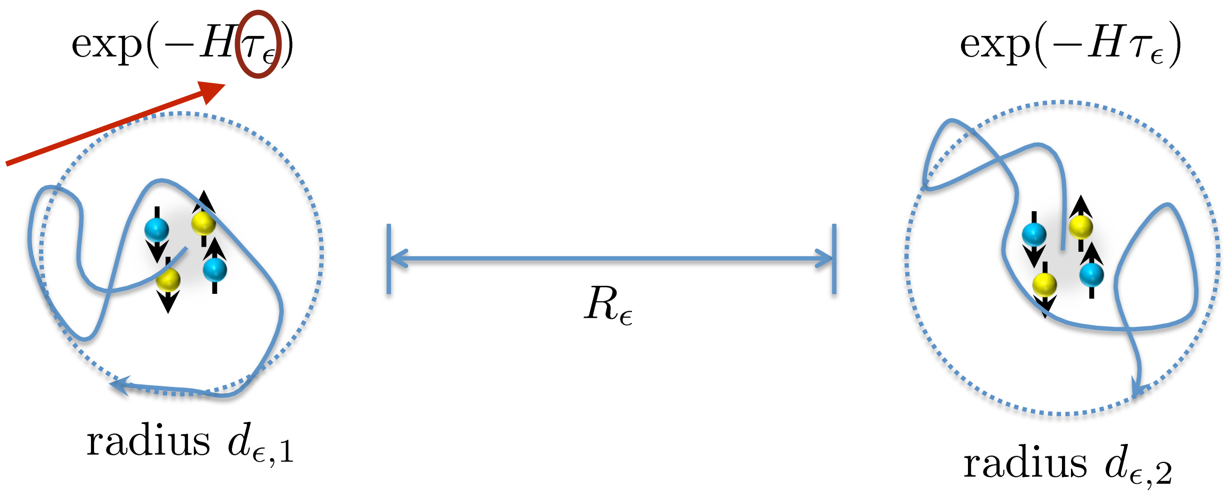


ϵ: relative error

Asymptotic cluster wave functions (I)

time interval such that the relative contamination from exited cluster states is less than ϵ



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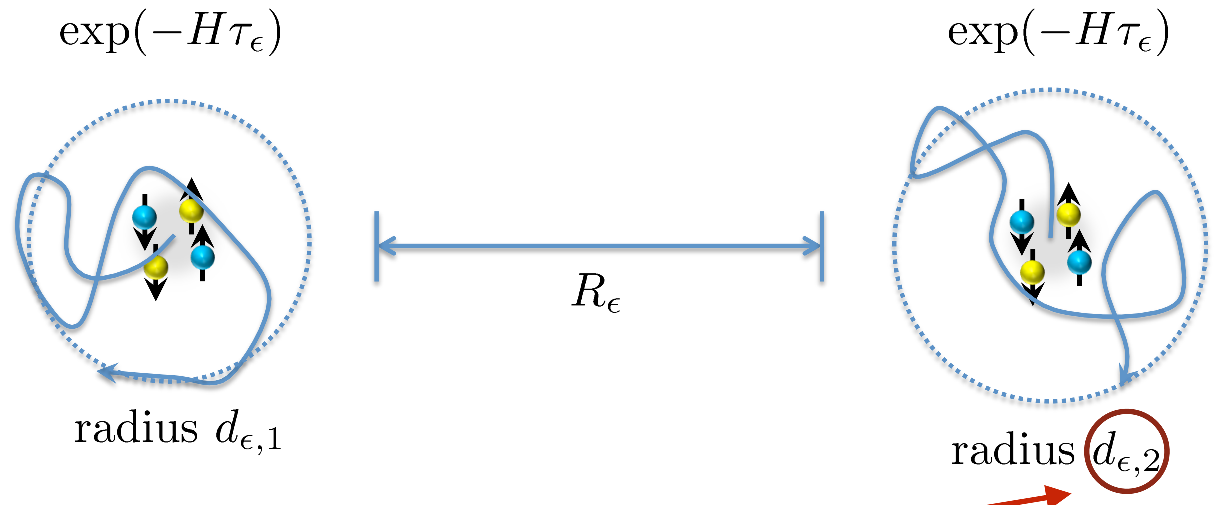

spatial diffusion that
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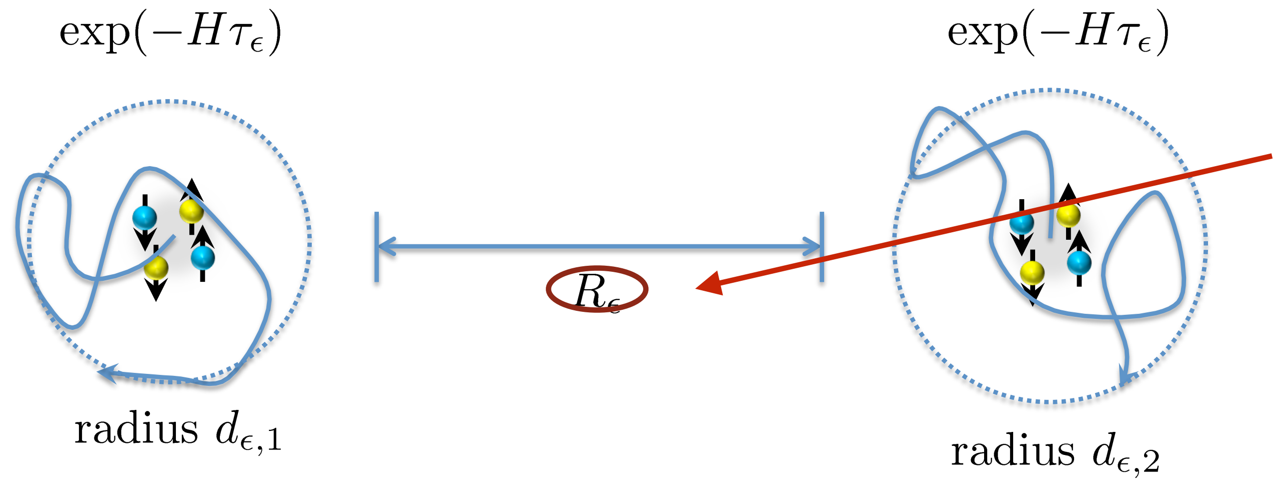

asymptotic distance such that the overlap between clusters is less than ϵ

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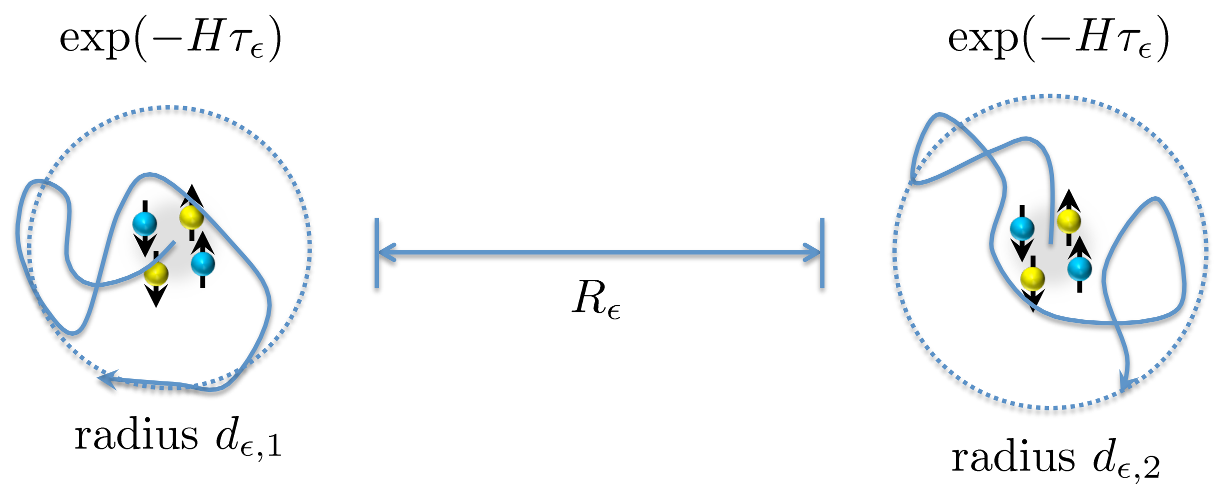

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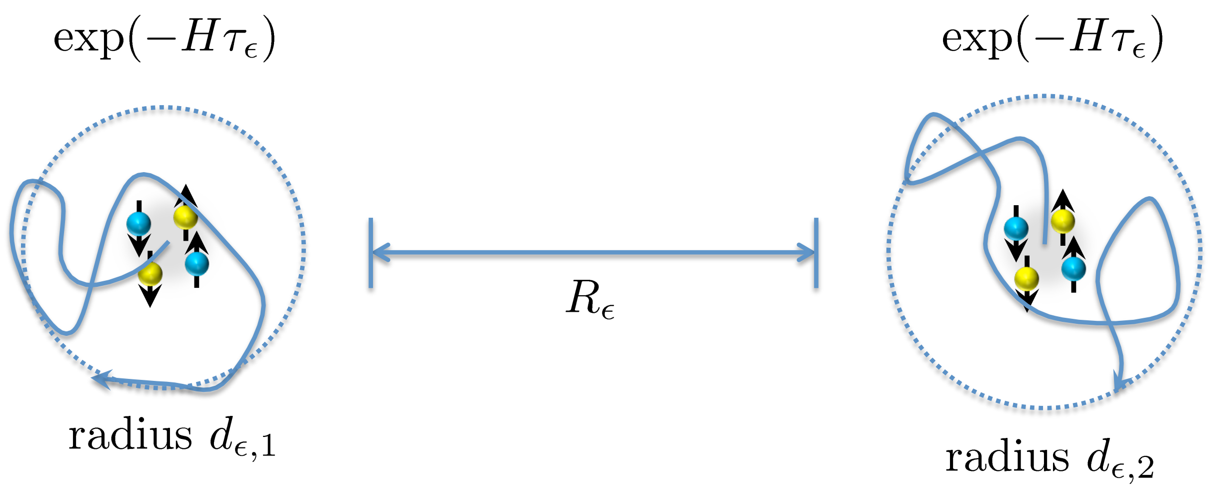

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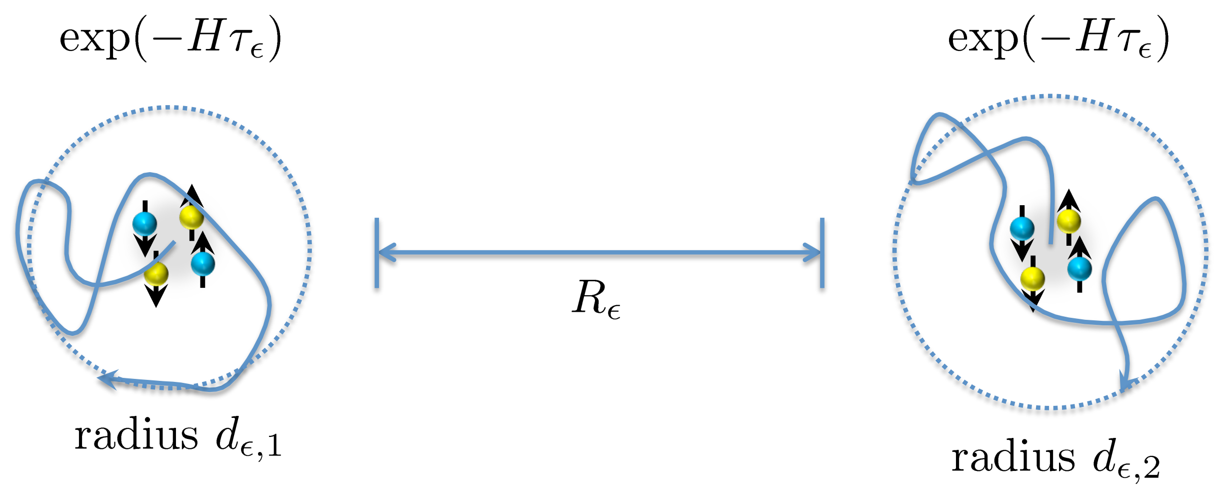

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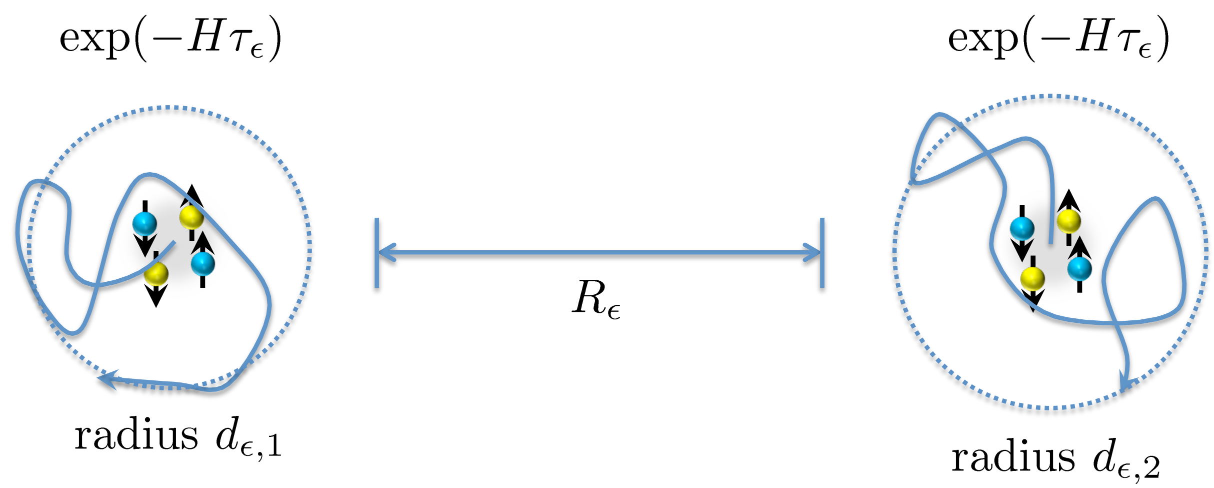

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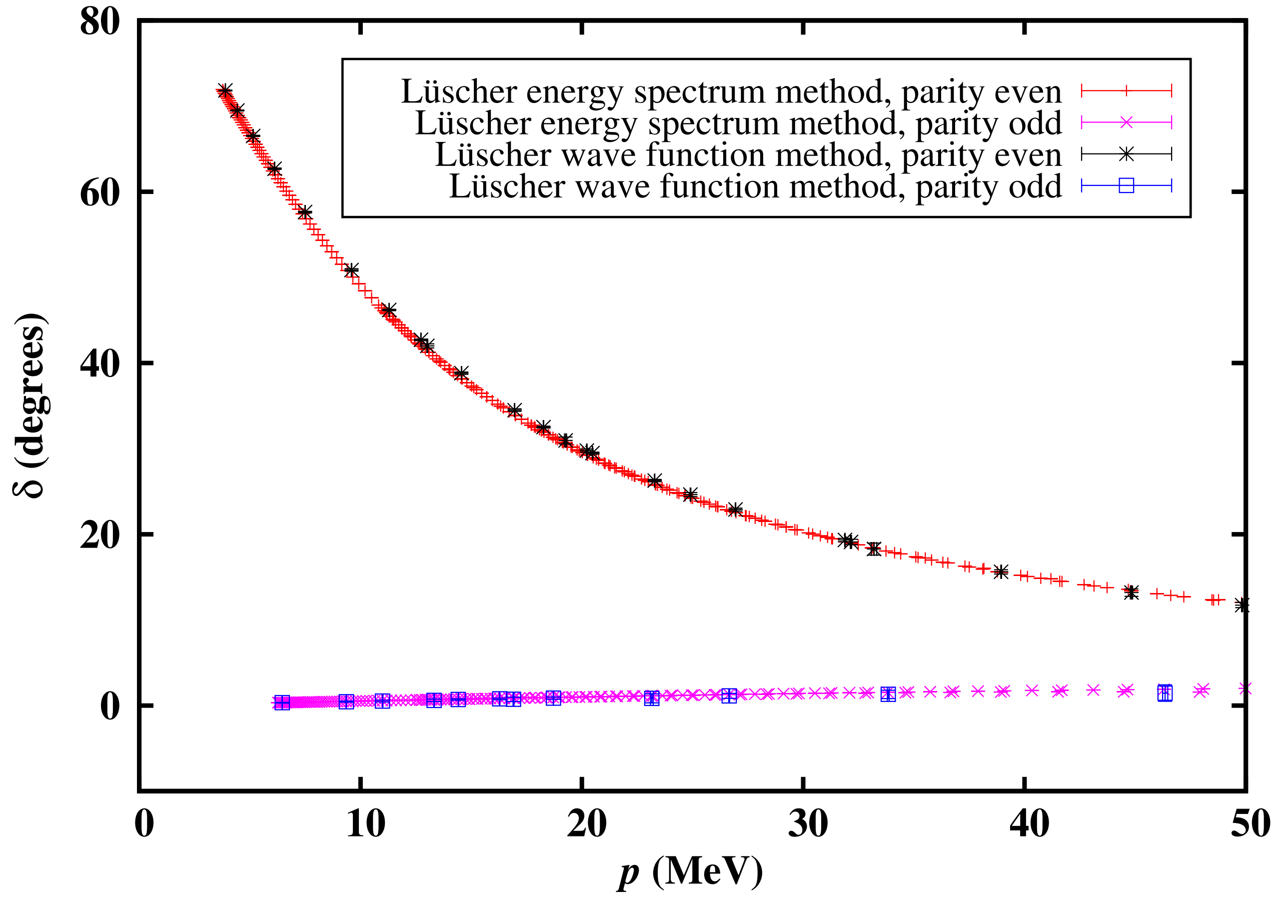
check with Lüscher's method applied to full microscopic Hamiltonian



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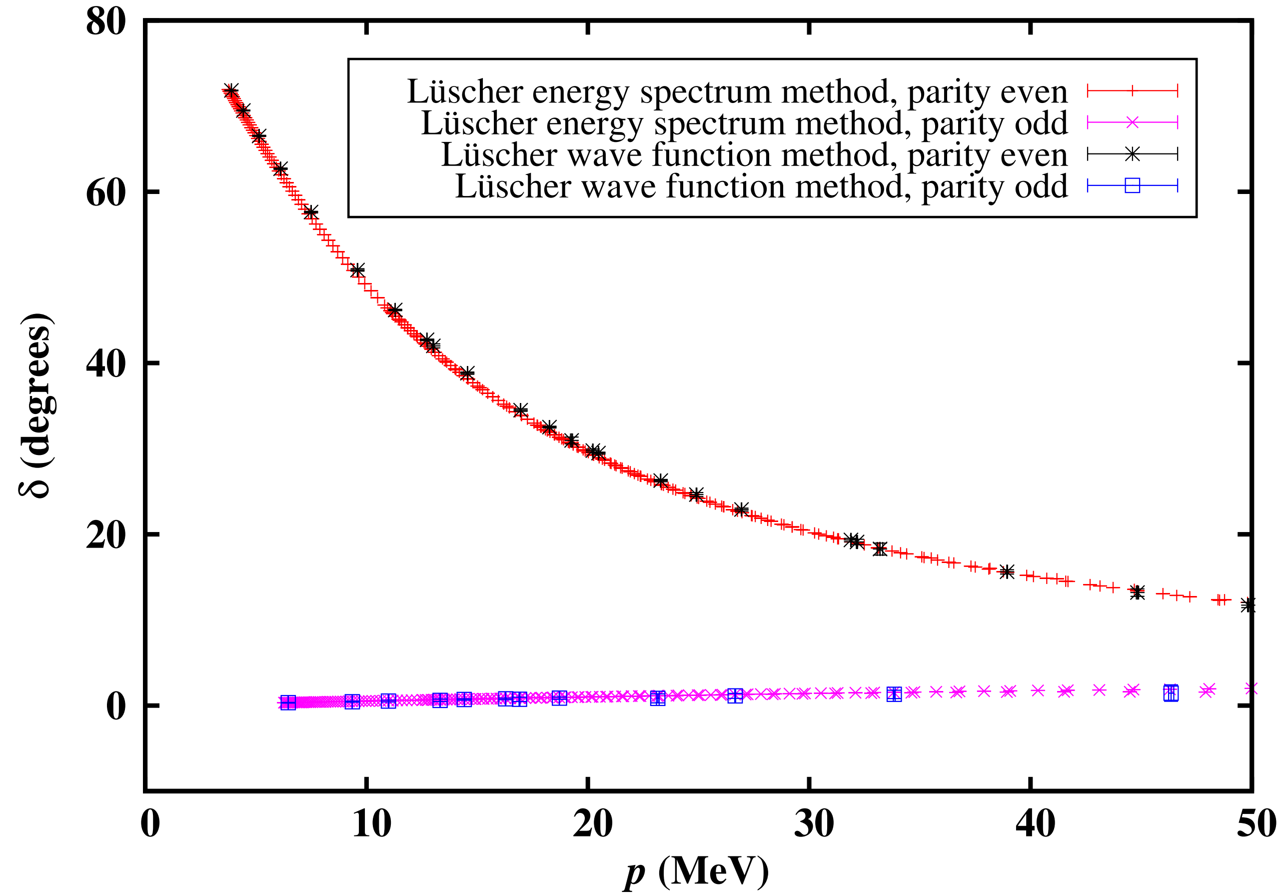
- The asymptotic cluster wave function can also be used to extract phase shifts

  - 1-dim asymptotic form:

$$\Psi_\ell^{(p)}(r) = A_\ell \cos(pr + \delta_\ell - \ell\pi/2)$$

check with Lüscher's method applied to full microscopic Hamiltonian



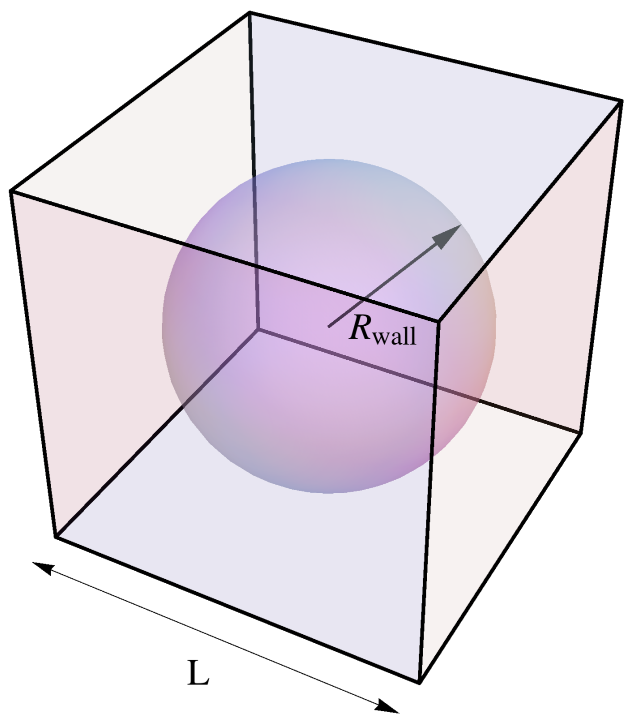
  - 3-dim asymptotic form:

$$\Psi_{\ell,m_\ell}^{(p)}(\boldsymbol{r}) = A_\ell Y_{\ell,m_\ell}(\theta,\phi)\,[\cos\delta_\ell(p)\,j_\ell(pr) - \sin\delta_\ell(p)\,n_\ell(pr)]$$

Spherical wall method (I)

- Impose a hard wall on the relative separation of two point-like particles and fit it to the asymptotic form

Borasoy, Epelbaum, Krebs, Lee, Meißner, EPJA 34 (2007) 185



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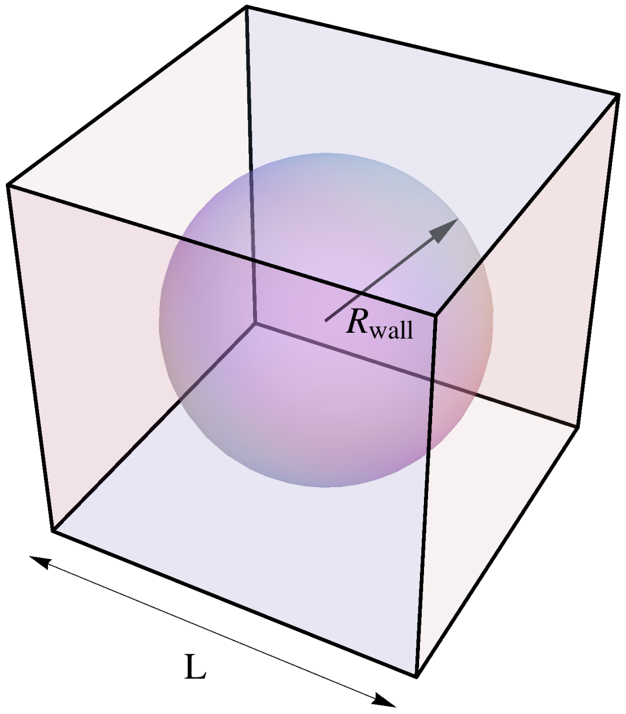

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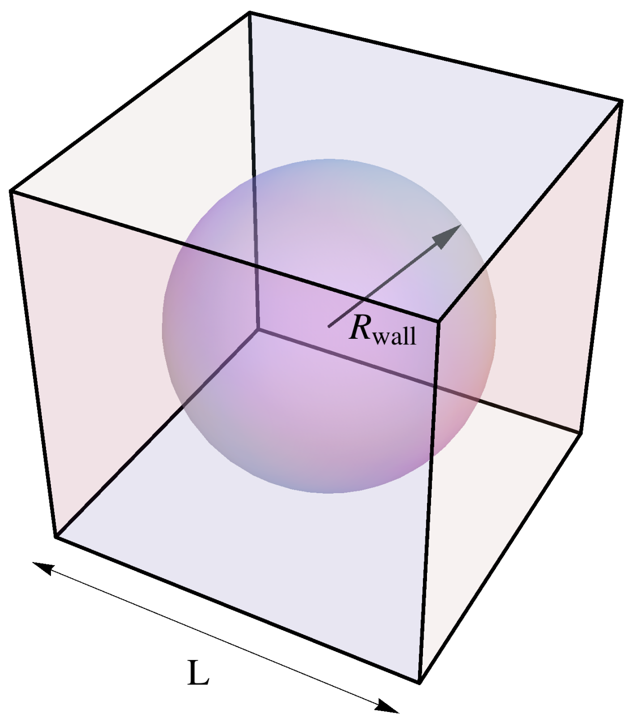

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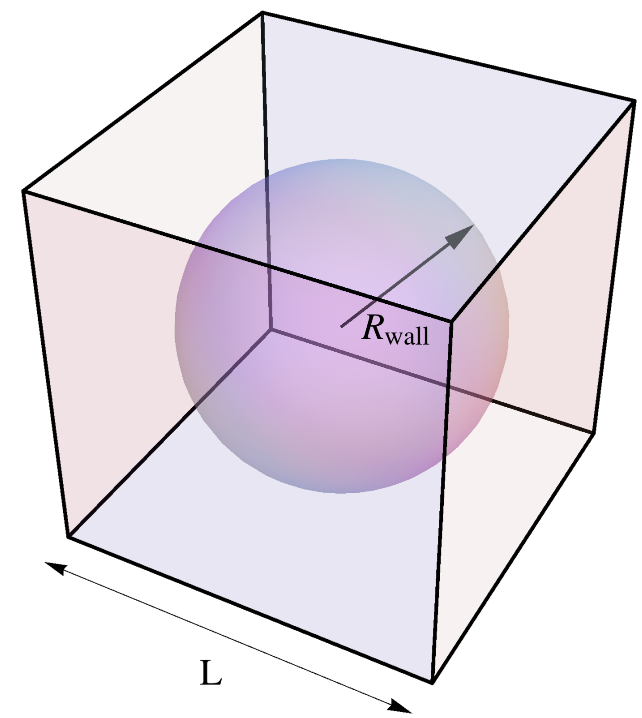

Borasoy, Epelbaum, Krebs, Lee, Meißner, EPJA 34 (2007) 185

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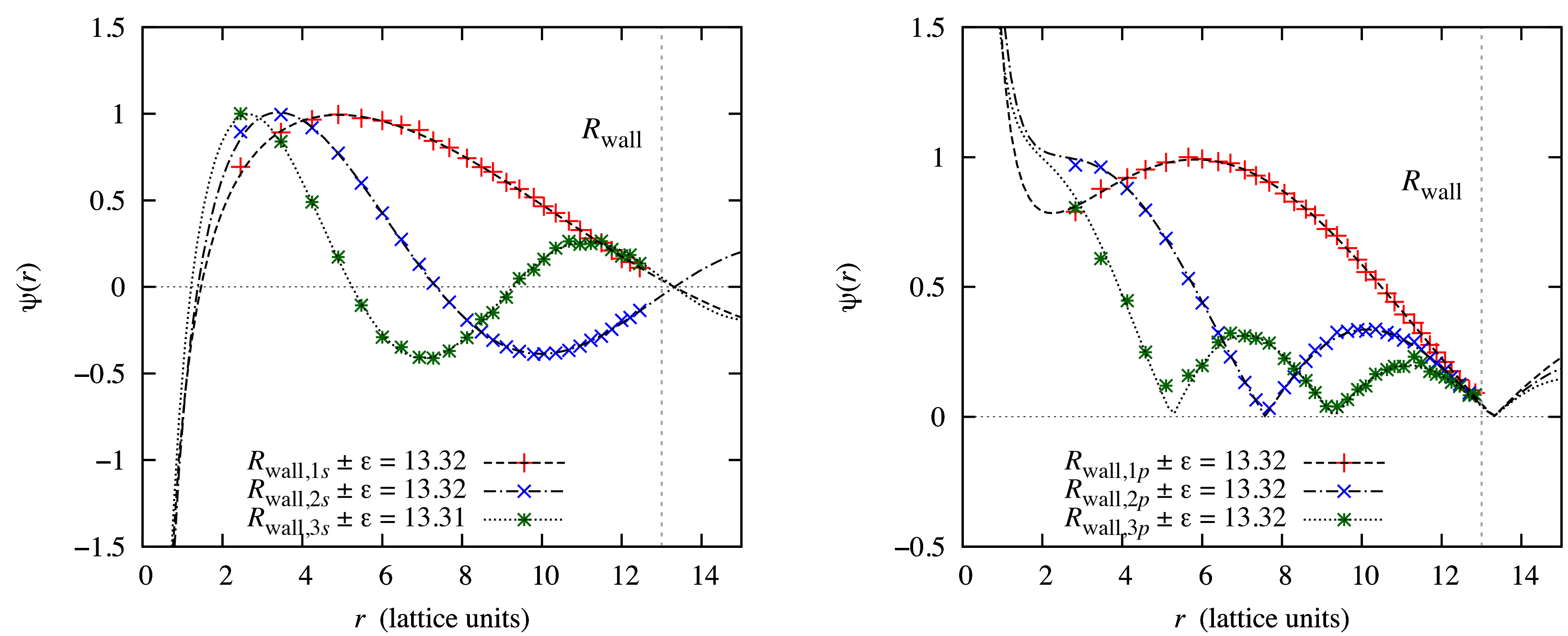
$$\Psi_\ell^{(p)}(r) = A_\ell \cos(pr + \delta_\ell - \ell\pi/2)$$

  - Observation: $R_{\text{wall}}' = R_{\text{wall}} + \varepsilon$ changes very little



RUB

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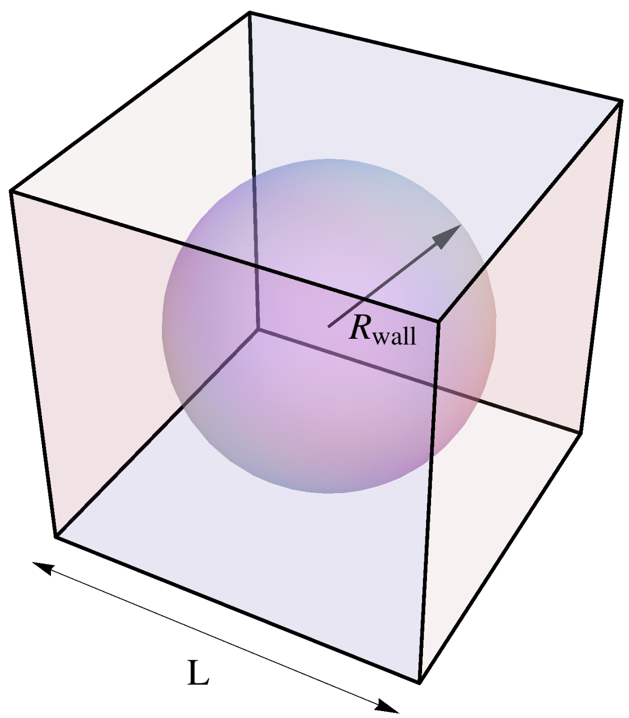

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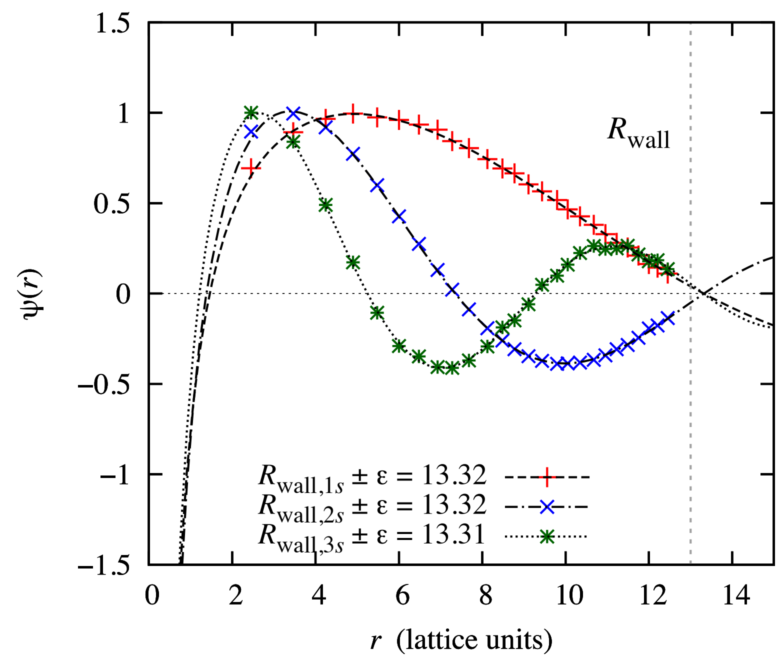

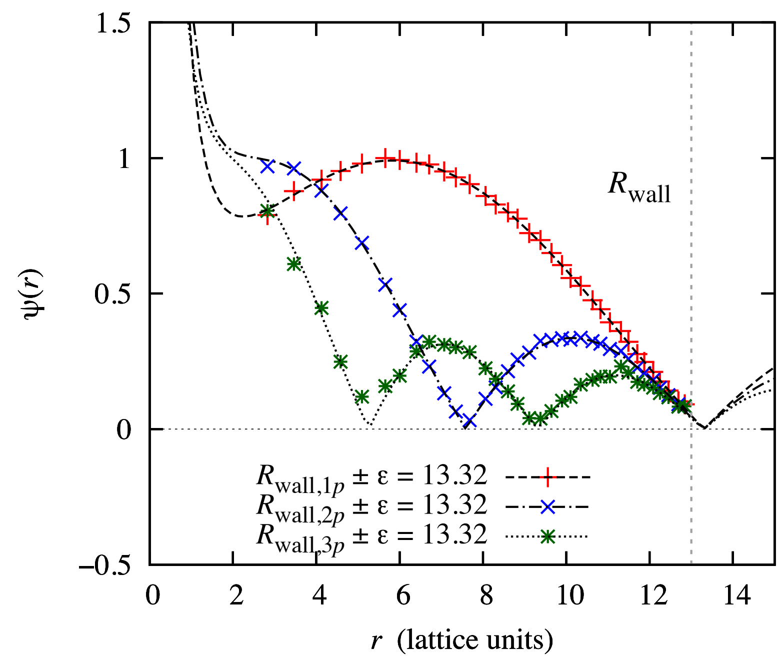

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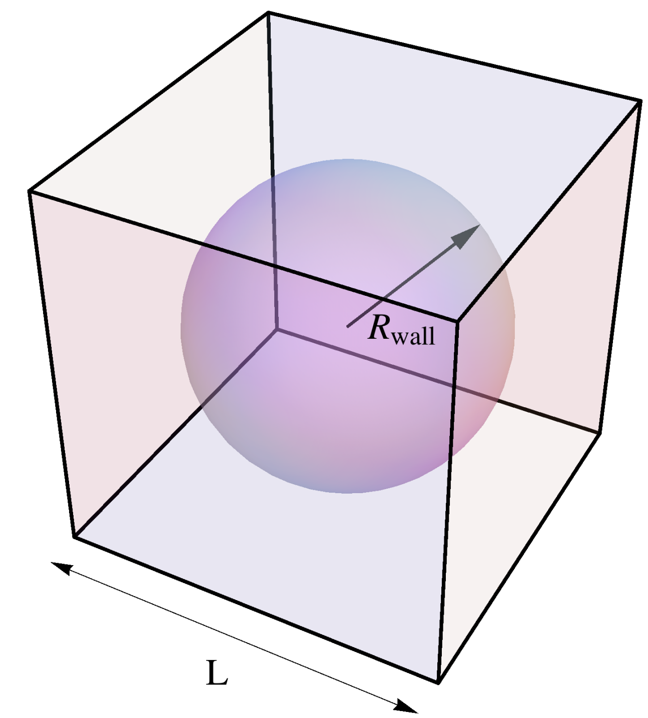

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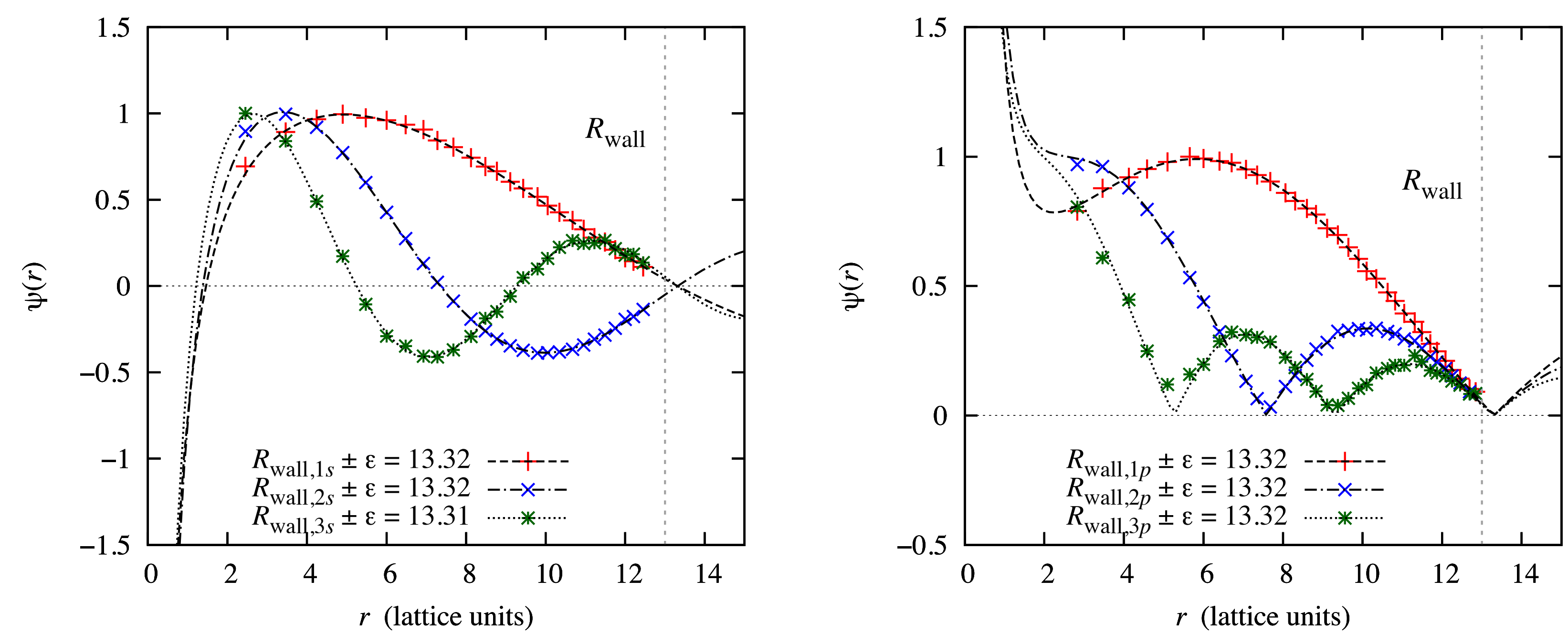

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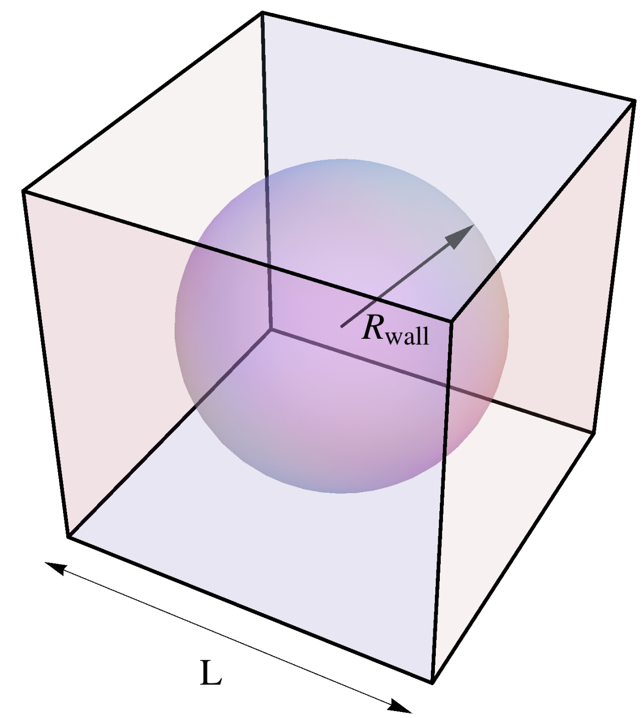

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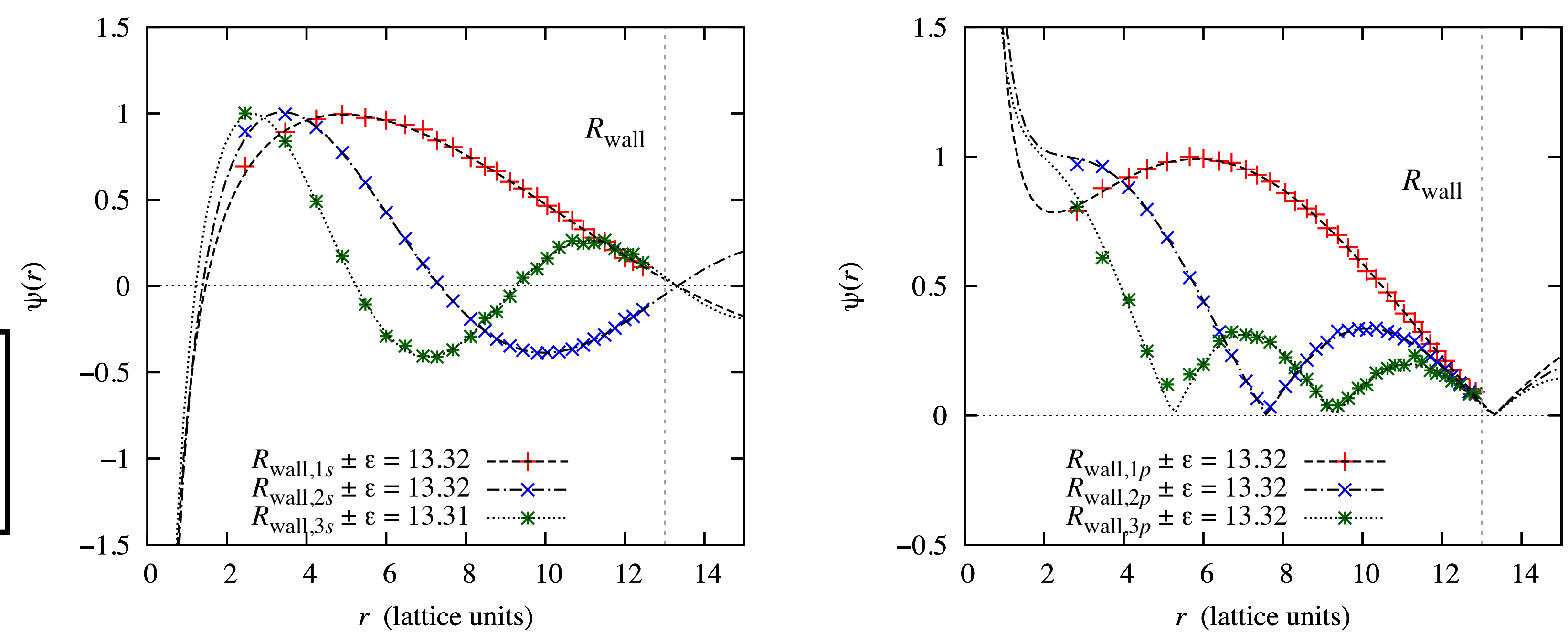
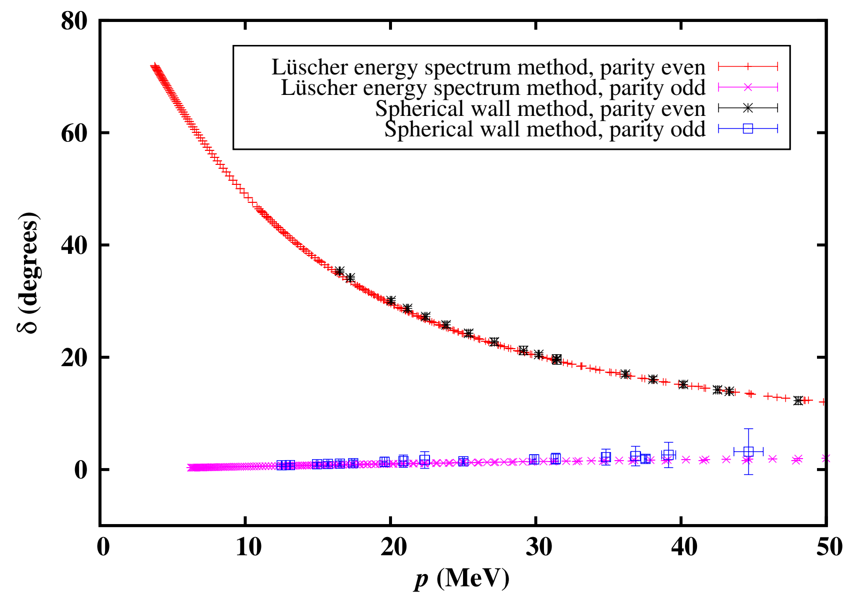
$$\delta_\ell(p) = \begin{cases} -pR'_{\rm wall} + \frac{\pi(\ell+1)}{2} \mod \pi \\ \tan^{-1}\left[\frac{j_\ell(pR'_{\rm wall}/a)}{n_\ell(pR'_{\rm wall}/a)}\right] \end{cases}$$

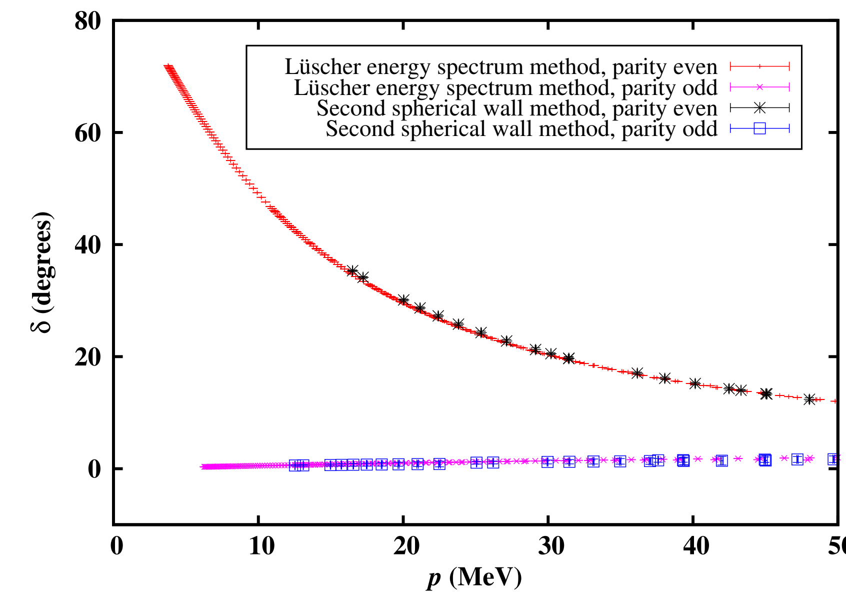


Spherical wall method (II)

- particle-dimer in one dimension



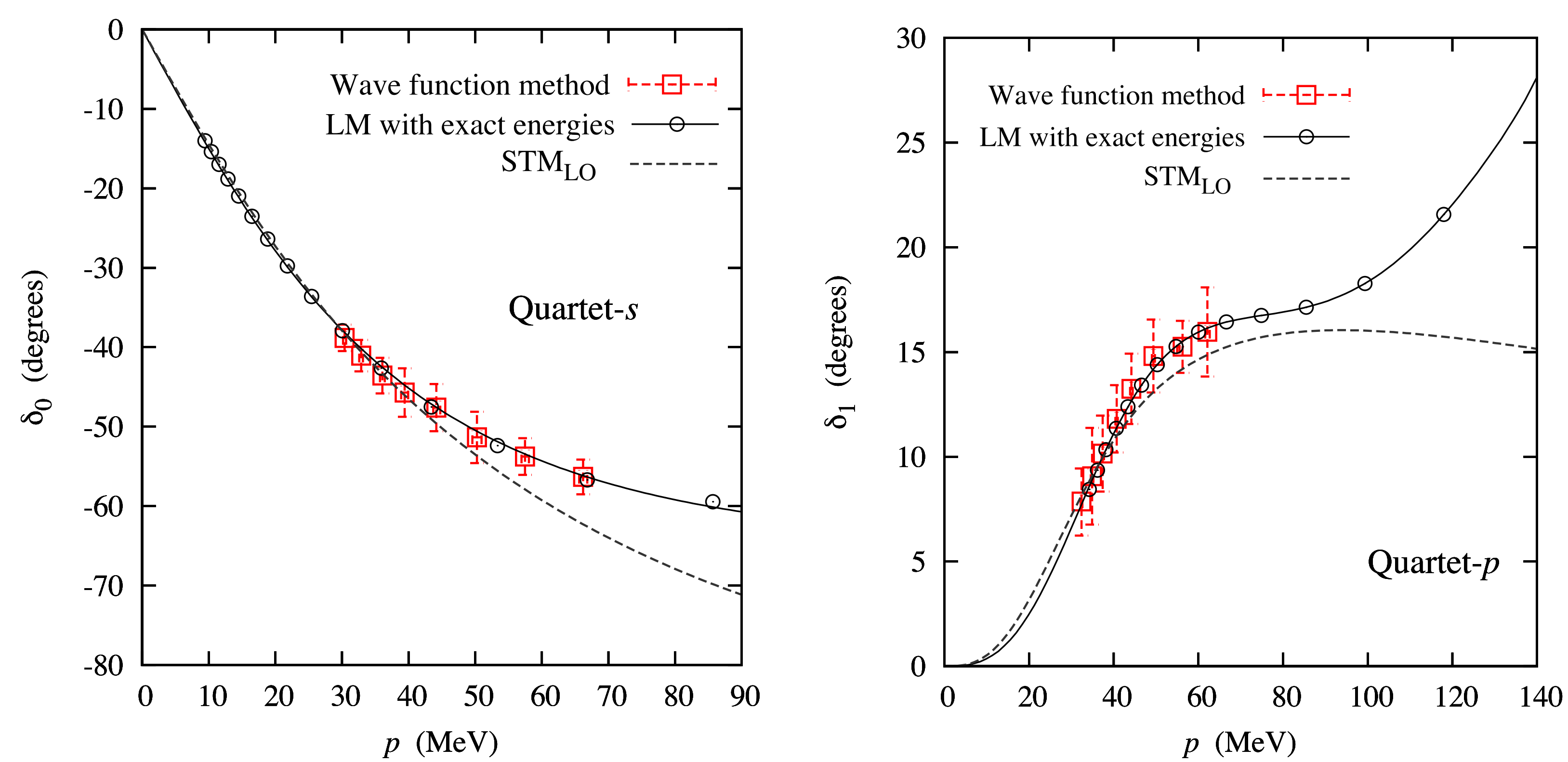
3-parameter fit



2-parameter fit

- fermion-dimer in three dimensions

Gabbiani, Bedaque, Grießhammer, NPA 675 (2000) 601



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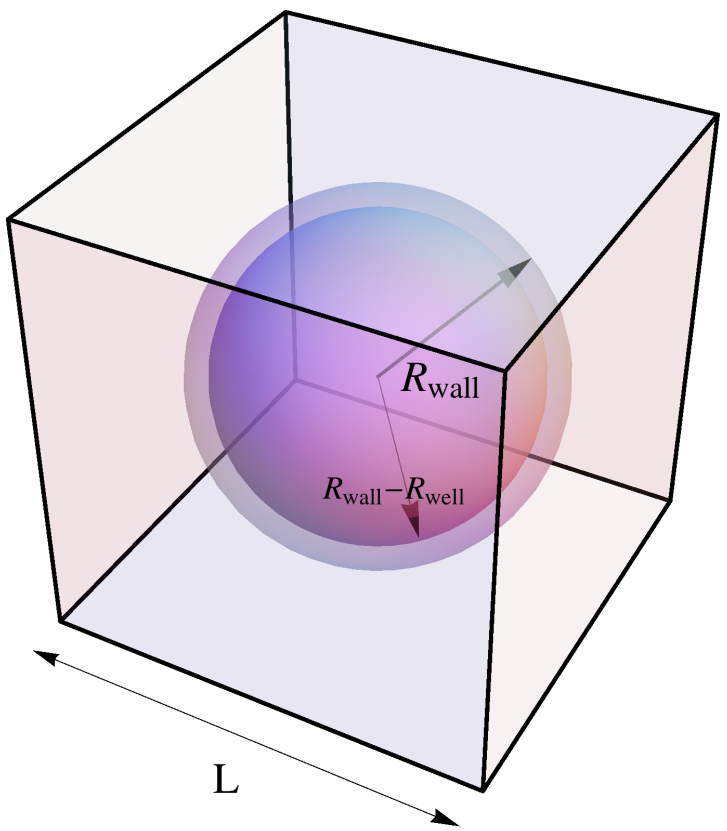

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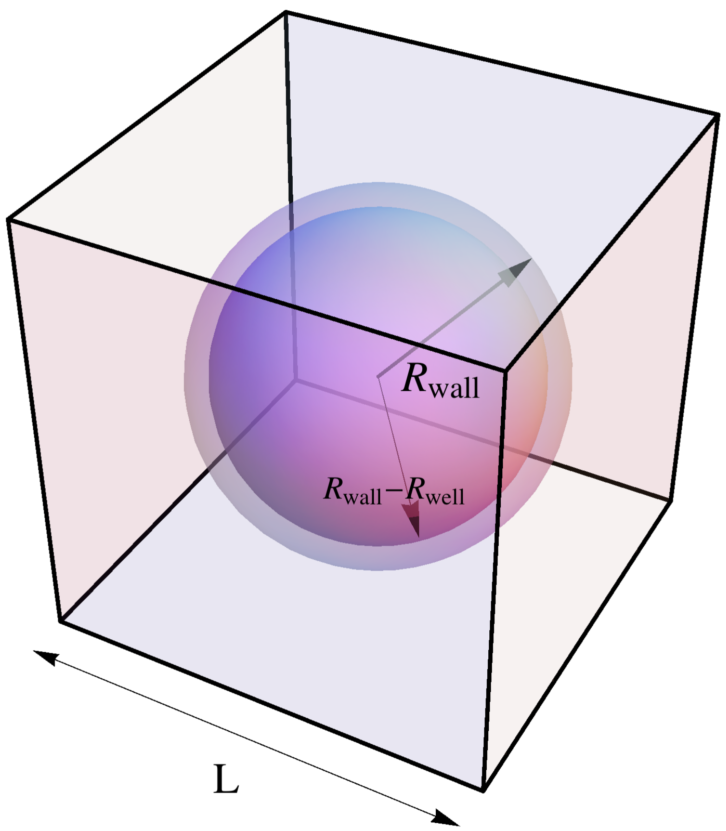

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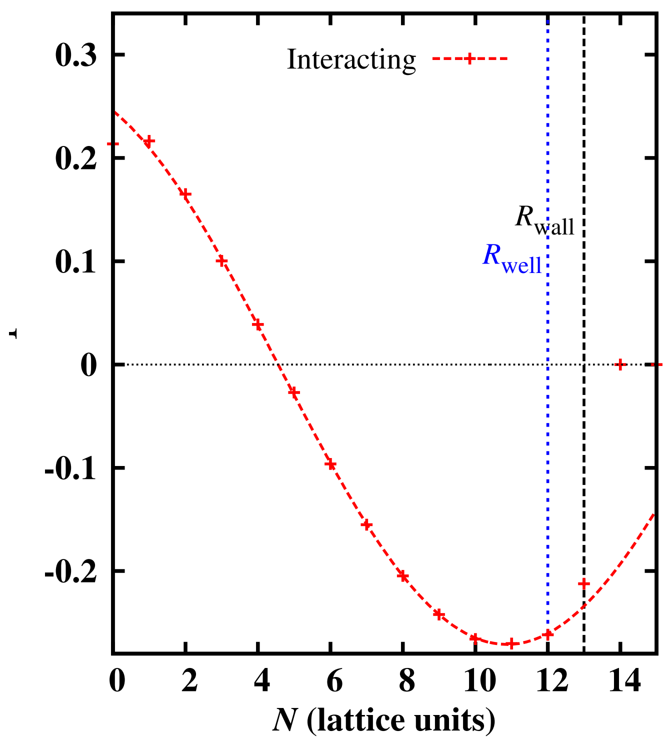

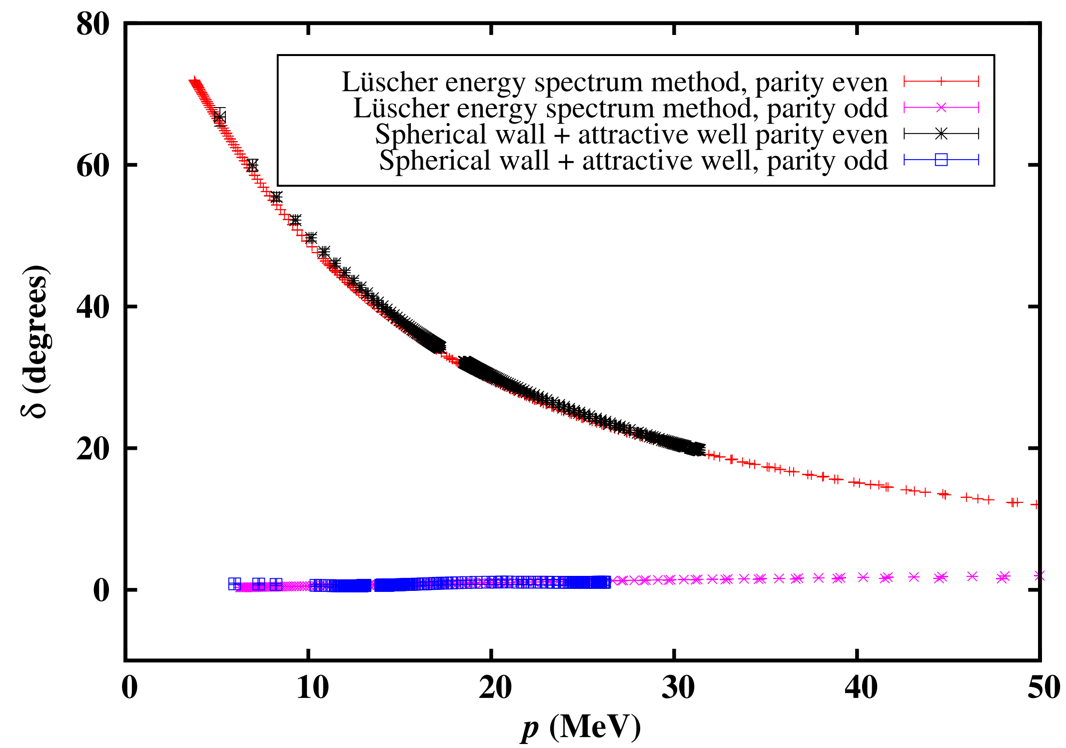

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Thank you for your attention!