The Nuclear Contact: From Nucleus Photodisintegration To Nucleons Momentum Distributions

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Outline

1. Tan's Relations

2. The Nuclear Contact(s)

3. Nuclear Photoabsorption

4. Experimental Evaluation of the np Contact

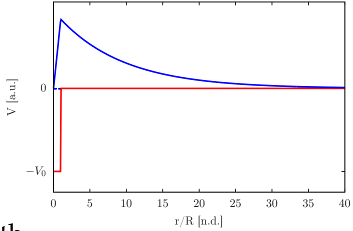
5. Momentum Distributions

6. Conclusions

References:

1. Nuclear Neutron-Proton Contact and the Photoabsorption Cross Section
   Ronen Weiss, BB, and Nir Barnea, PRL **114**, 012501 (2015).
2. Generalized nuclear contacts and the nucleon's momentum distributions
   Ronen Weiss, BB, and Nir Barnea, arXiv:1503.07047 (2015).

Universality



- Universality occur when a system is not sensitive to its microscopic details.
- Low energy: Only s-wave survives.
- Short range: Most of the wave function is outside the range of the potential.
- The wave function depends on a single length scale - the **scattering length a**
- The potential can be replaced by the Bethe-Peierles boundary condition

$$\boxed{\left.\frac{d\log(r\psi)}{dr}\right|_0 = \left.\frac{u'}{u}\right|_0 = -\frac{1}{a}}$$

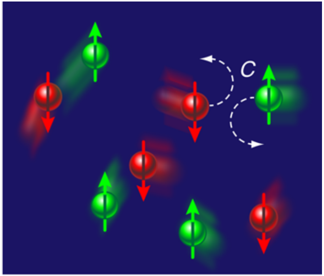
- Valid for **any** short range potential.

The Contact

A system of spin up - spin down fermions

The contact C measures the number of fermions pairs with small separations,

$$C = \int d\boldsymbol{R} \mathcal{C}(\boldsymbol{R})$$



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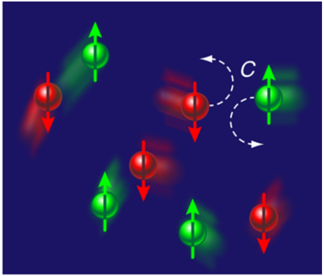

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The Contact - Tan's Relations

- **Tail of momentum distribution** $|a|^{-1} \ll k \ll r_0^{-1}$

$$\boxed{n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}}$$

- **Adiabatic relation**

$$\boxed{\left(\frac{dE}{da^{-1}}\right)_S = -\frac{\hbar^2}{4\pi m}C}$$

- **The energy relation**

$$\boxed{T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4}\right) + \frac{\hbar^2}{4\pi m a}C}$$

- **Density-Density correlator at short distances**

$$\boxed{\left\langle n1\left(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}\right) n2\left(\boldsymbol{R} - \frac{\boldsymbol{r}}{2}\right)\right\rangle \longrightarrow \frac{1}{16\pi^2}\left(\frac{1}{r^2} - \frac{2}{ar}\right)\mathcal{C}(\boldsymbol{R})}$$

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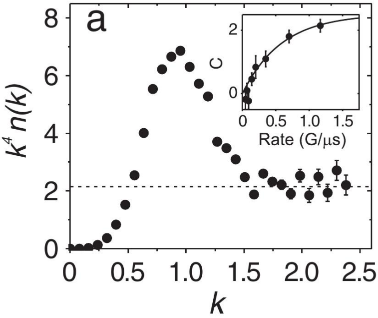
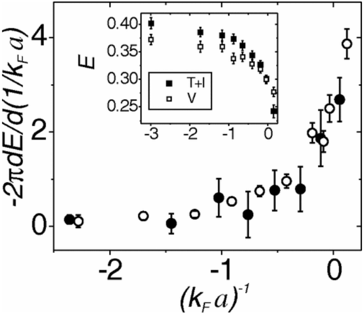
The Contact - Experimental Results

Momentum Distribution



Adiabatic relation



Ultra cold gas of fermionic ^{40}K

J. T. Stewart et al. PRL **104**, 235301 (2010)

The Contact - The Two Body Case

- Consider two particles interacting with short range interaction with large scattering length.
- The energy of a universal dimer,

$$E = -\frac{\hbar^2}{ma^2}$$

- Using the adiabatic relation,

$$C = -\frac{4\pi m}{\hbar^2}\frac{dE}{da^{-1}} = \frac{8\pi}{a}$$

- The wave function reads,

$$\psi(\mathbf{r}) = Y_{00}\sqrt{\frac{2}{a}}\frac{e^{-r/a}}{r} \approx Y_{00}\sqrt{\frac{2}{a}}\left(\frac{1}{r} - \frac{1}{a}\right)$$

- and therefore the tail of the momentum distribution

$$n(k) \longrightarrow \frac{8\pi/a}{k^4} = \frac{C}{k^4}$$

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The Contact - The Many Body Case

- When two particles approach each other

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k\neq i,j} d\boldsymbol{r}_k \, d\boldsymbol{R}_{ij} \, A^{\dagger}_{ij} \left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j} \right) \cdot A_{ij} \left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j} \right)$$

- For large k, the Fourier transform is dominated by the short-range divergences,

$$\int d^3 r_i e^{-ik\cdot r_i} \psi(r_1, \ldots, r_N) \approx \int d^3 r_i e^{-ik\cdot r_i} \sum_{j,j\neq i} \frac{1}{r_{ij}} A(R_{ij}, r_l)$$

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The Nuclear Contact(s)

Scales

- NN interaction range $\mu_\pi^{-1} = \hbar/m_\pi c \approx 1.4$ fm
- NN scattering lengths $a_t = 5.4$ fm , $a_s \approx 20$ fm thus $\mu_\pi |a| \geq 3.8$
- The nuclear radius is $R \approx 1.2 A^{1/3}$ fm
- The interparticle distance $d \approx 2.4$ fm thus $\mu_\pi d \approx 1.7$

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics.
- The interaction range is significant.
- There could be different interaction channels - not only s-wave.
- Therefore, we need to replace the asymptotic form

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The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

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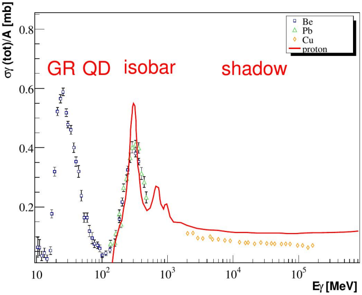
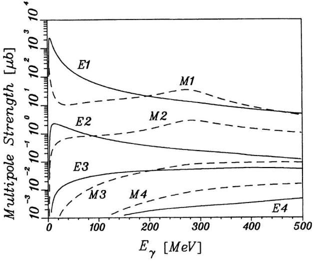


R. Al Jebali, PhD Thesis, U. Glasgow (2013)

The Deuteron cross-section



H. Arenhovel, and M. Sanzone, Few-Body Syst. (1991).

Up to $\hbar\omega \approx 200$ MeV the cross-section $\sigma_A(\omega)$ is dominated by the **dipole** operator

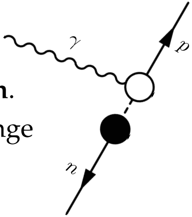
$$\sigma_A(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

R is the response function

$$R(\omega) = \sum_f \left| \langle \Psi_f \left| \boldsymbol{\epsilon} \cdot \hat{\boldsymbol{D}} \right| \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

The Quasi-Deuteron Picture

- The photon carries **energy** but (almost) **no momentum**
- It is captured by a single **proton.**
- The proton is ejected without any FSI.
- Momentum conservation ⇒ a nucleon with opposite momentum must be ejected $\boldsymbol{k} \approx -\boldsymbol{k}_p$.
- Dipole dominance ⇒ this partner must be a **neutron**.
- $\hbar\omega \longrightarrow \infty \Rightarrow \sigma(\omega)$ depends on a **universal** short range pn wave-function.
- The resulting cross-section is given by



$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- L is known as the Levinger Constant

J. S. Levinger, Phys. Rev. **84**, 43 (1951).

The Quasi-Deuteron Model Revised

- When a **pn** pair are close together Ψ_0 is factorized into

$$\Psi_0(\mathbf{r}_1,...,\mathbf{r}_A) = \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{pn}) A_{pn}^\alpha\left(\boldsymbol{R}_{pn},\{\boldsymbol{r}_j\}_{j\neq p,n}\right) + \ldots$$

$$\Psi_f^\alpha(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_A) = \frac{4\pi}{\sqrt{C_\alpha}}\hat{\mathcal{A}}\left\{\frac{1}{\sqrt{\Omega}}e^{-i\mathbf{k}\cdot\mathbf{r}_{pn}}\chi_S A_{pn}^\alpha(\boldsymbol{R}_{pn},\{\boldsymbol{r}_j\}_{j\neq p,n})\right\}$$

- Assuming s-wave dominance, α is either singlet or triplet and $\varphi_\alpha \approx \varphi_d$,

$$\langle\Psi_f^\alpha|\epsilon\cdot\hat{D}|\Psi_0\rangle \approx \sqrt{\frac{C_\alpha a_t}{8\pi}}\langle\psi_{d,f}|\epsilon\cdot\hat{D}|\psi_{d,0}\rangle$$

- The cross section,

$$\sigma_A(\omega) = \frac{a_t}{4\pi}\frac{(C_s+C_t)}{2}\sigma_d(\omega) = \frac{a_t}{4\pi}\bar{C}_{pn}\sigma_d(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

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The Levinger Constant and the Nuclear Contact

- In his original paper Levinger has estimated $L = 6.4$
- In view of the available data we can conclude

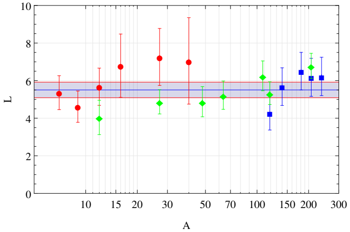
$$L = 5.50 \pm 0.21$$

- $N = Z = A/2$
- Normalize by the Fermi momentum

$$\frac{\bar{C}_{pn}}{k_F A} = \frac{\pi}{k_F a_t}\,(5.50 \pm 0.21)$$

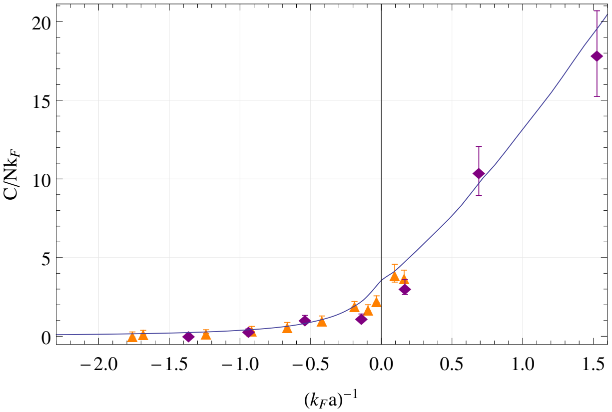
- $1/k_F a_t \approx 0.15$

$$\bar{C}_{pn}/k_F A \approx 2.55 \pm 0.10$$



O. A. P. Tavares and M. L. Terranova, J. Phys. G **18**, 521 (1992).

Comparison to Atomic Physics



Atomic data - ^{40}K - J. T. Stewart et al., PRL **104**, 235301 (2010)
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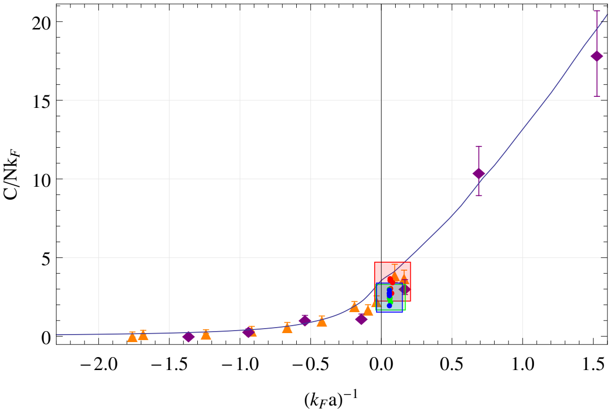

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Momentum distributions

1-body neutron and proton momentum distributions

$$n_n(\boldsymbol{k}), \ \ n_p(\boldsymbol{k})$$

2-body *nn*, *np*, *pp* momentum distributions

$$F_{nn}(\boldsymbol{k}), \ \ F_{pn}(\boldsymbol{k}), \ \ F_{pp}(\boldsymbol{k})$$

Momentum distributions

The proton momentum distribution

$$n_p^{JM}(\boldsymbol{k}) = Z \int \prod_{l\neq p} \frac{d^3k_l}{(2\pi)^3} \left|\tilde{\Psi}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_p = \boldsymbol{k},\ldots,\boldsymbol{k}_A)\right|^2$$

Using the asymptotic wave-function

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j})$$

we get

$$n_p(\boldsymbol{k}) = \frac{1}{2J+1}\sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) Z(Z-1)\langle A_{pp}^\alpha | A_{pp}^\beta\rangle + \frac{1}{2J+1}\sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) NZ\langle A_{pn}^\alpha | A_{pn}^\beta\rangle$$

Momentum distributions

The proton momentum distribution

$$n_p^{JM}(\boldsymbol{k}) = Z \int \prod_{l \neq p} \frac{d^3 k_l}{(2\pi)^3} \left| \tilde{\Psi}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_p = \boldsymbol{k}, \ldots, \boldsymbol{k}_A) \right|^2$$

Using the asymptotic wave-function

$$\Psi \xrightarrow[r_{ij} \to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

we get

$$n_p(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) \frac{2C_{pp}^{\alpha\beta}}{16\pi^2} + \sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) \frac{C_{pn}^{\alpha\beta}}{16\pi^2}$$

Momentum distributions

Furthermore, starting from the general assumption

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

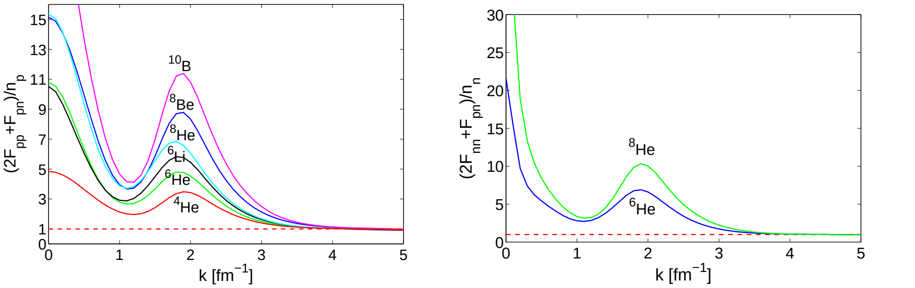
The following **asymptotic** relations between the 1-body and 2-body momentum distributions can be **proven**

$$n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

$$n_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

These relations hold regardless of the specific form of φ_α and without any assumptions on $\{\alpha\}$

Numerical verification of the momentum relations



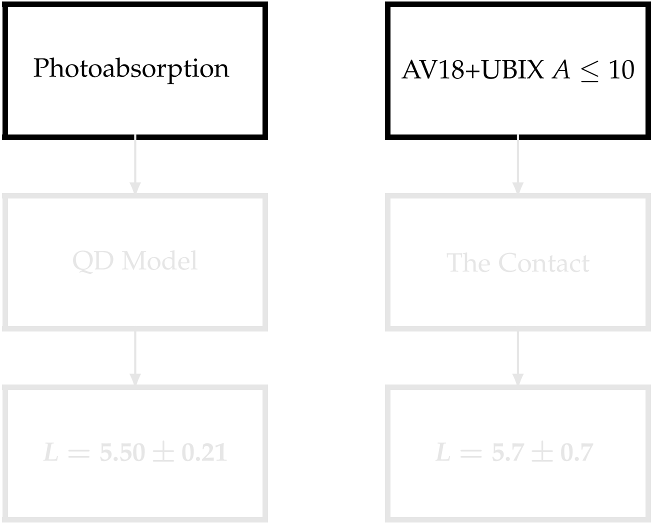
VMC calculations of light nuclei

- Wiringa et al. published a series of 1-body, 2-body momentum distributions

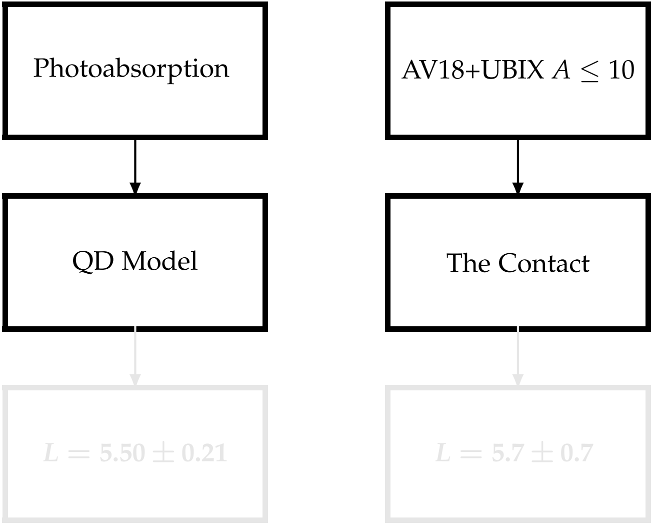
  R. B. Wiringa *et al.*, PRC **89**, 024305 (2014)
- The data is available for nuclei in the range $2 \leq A \leq 10$.
- The calculations were done with the VMC method
- For symmetric nuclei $n_p = n_n$

The momentum relations holds for $4\ \text{fm}^{-1} \leq k \leq 5\ \text{fm}^{-1}$

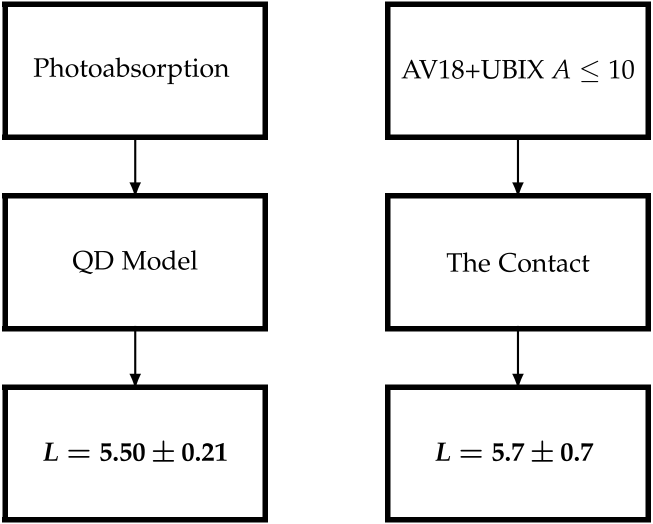
The nuclear contact



The nuclear contact



The nuclear contact



Conclusions and outlook

Generalizing Tan's contact to nuclear physics

- The Quasi-Deuteron model was revised.
- The Levinger constant and the nuclear contacts are close relatives.
- $\bar{C}_{pn}$ was deduced using previous evaluations of Levinger constant.
- $\bar{C}_{pn}/A$ seems to be constant throughout the nuclear chart.
- Its value stands in line with the universal curve measured in ultracold atomic systems.
- Momentum relations were derived, connecting one-body and two-body distributions.
- There relations were verified using VMC data.
- Levinger constant derived from this data is in agreement with that derived from photoabsorption experiments.

Outlook

- Electron scattering.
- Neutrino scattering.
- ...

We have only started to explore the usefulness of
the contact formalism in nuclear physics!

Backup slides

The contact: QFT perspective

- Quantum field theory formulation of the Zero-Range Model,

$$\mathcal{H} = \sum_\sigma \frac{\hbar^2}{2m} \nabla\psi_\sigma^\dagger \nabla\psi_\sigma(\boldsymbol{R}) + \frac{g(\Lambda)}{m} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1(\boldsymbol{R}) + \mathcal{V}$$

- Renormalization,

$$g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}$$

- Defining di-atomic field operator, $\Phi(\boldsymbol{R}) = g(\Lambda)\psi_2\psi_1(\boldsymbol{R})$

$$\mathcal{H} = \sum_\sigma \frac{\hbar^2}{2m} \nabla\psi_\sigma^\dagger \nabla\psi_\sigma(\boldsymbol{R}) - \frac{\Lambda}{2\pi^2 m}\Phi^\dagger\Phi + \frac{1}{4\pi m a}\Phi^\dagger\Phi + \mathcal{V}$$

- Identifying $C = \int d^3R \langle \Phi^\dagger \Phi(\boldsymbol{R}) \rangle$ we got the adiabatic relation.

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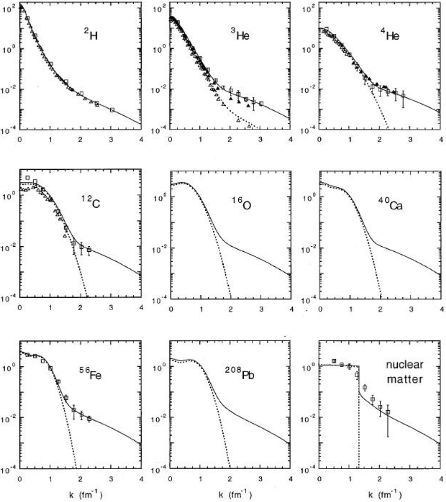
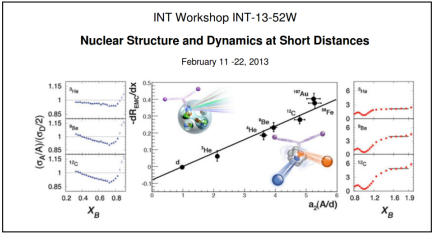


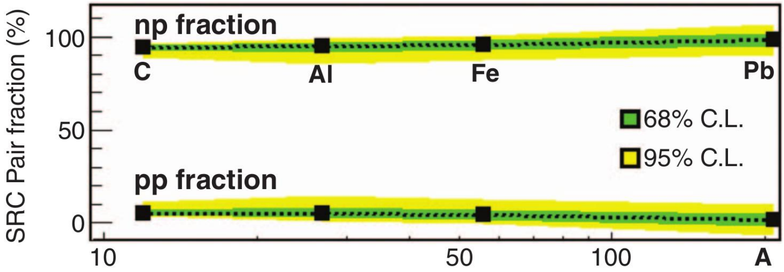
C. Ciofi degli Atti, and S. Simula, PRC 53, 1689 (1996)

Short range correlations and their universal nature is an intensive line of research in NP

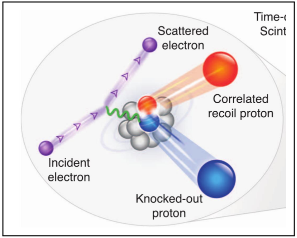
[Ciofi degli Atti, Frankfurt, Strikman, Sargasian, Piasetzky,..]



Comment II
Nuclear Short Range Correlations



O. Hen, et al., Science **346**, 614 (2014)

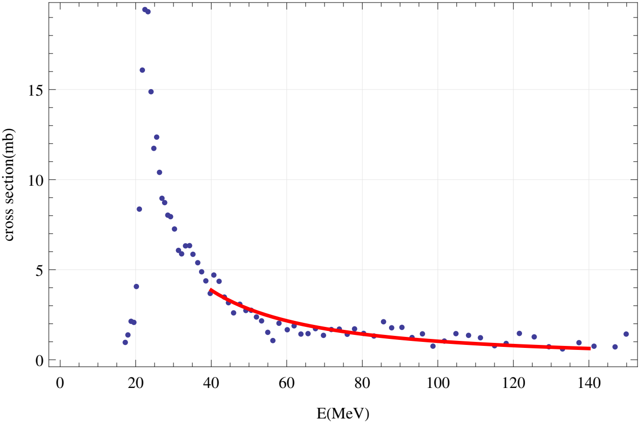


Short range correlations measured in (e,e′) experiments at JLAB.

A clear preference for correlated *np* pairs.

Experimental Results - fitting the Levinger Constant

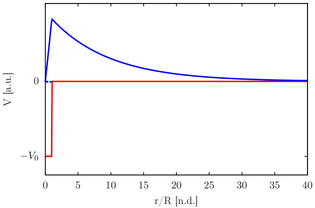
The ^{12}C photoabsorption cross-section



Points - data of Ahrens (Nucl. Phys. A **446**, 229 (1985))

Line - the Quasi-Deutron model $L = 5.8$

Short range interaction



We start with 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

At vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0$$

$$r\varphi(r) = 0|_{r=0}$$

- Valid for any A-body system.