The Nuclear Contact:
From Nucleus Photodisintegration
To Nucleons Momentum Distributions

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Outline

1. Tan’s Relations
2. The Nuclear Contact(s)
3. Nuclear Photoabsorption
4. Experimental Evaluation of the np Contact
5. Momentum Distributions
6. Conclusions

References:

1. Nuclear Neutron-Proton Contact and the Photoabsorption Cross Section

2. Generalized nuclear contacts and the nucleon’s momentum distributions
Universality occur when a system is not sensitive to its microscopic details.

- Low energy: Only $s$-wave survives.
- Short range: Most of the wave function is outside the range of the potential.
- The wave function depends on a single length scale - the **scattering length** $a$
- The potential can be replaced by the Bethe-Peierles boundary condition

$$\left.\frac{d \log(r\psi)}{dr}\right|_0 = \left.\frac{u'}{u}\right|_0 = -\frac{1}{a}$$

- Valid for any short range potential.
The contact $C$ measures the number of fermions pairs with small separations,

$$C = \int dR C(R)$$

- Naively, the number of pairs in a sphere of volume $V$ should scale as $V^2$.
- For large scattering length, it scales as $V^{4/3}$ due to strong correlations.
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The Contact - Tan’s Relations

Tail of momentum distribution \(|a|^{-1} \ll k \ll r_0^{-1}\)

\[
n_\sigma(k) \rightarrow \frac{C}{k^4}
\]

Adiabatic relation

\[
\left( \frac{dE}{da^{-1}} \right)_s = -\frac{\hbar^2}{4\pi m} C
\]

The energy relation

\[
T + U = \sum_\sigma \int \frac{dk}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left( n_\sigma(k) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi ma} C
\]

Density-Density correlator at short distances

\[
\langle n_1 \left( R + \frac{r}{2} \right) n_2 \left( R - \frac{r}{2} \right) \rangle \rightarrow \frac{1}{16\pi^2} \left( \frac{1}{r^2} - \frac{2}{ar} \right) C(R)
\]
The Contact - Tan’s Relations

1. **Tail of momentum distribution** $|a|^{-1} \ll k \ll r_0^{-1}$
   
   $n_\sigma(k) \rightarrow \frac{C}{k^4}$

2. **Adiabatic relation**
   
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\[\ldots\]
The Contact - Experimental Results

Momentum Distribution

Ultra cold gas of fermionic $^{40}\text{K}$

J. T. Stewart et al. PRL 104, 235301 (2010)

Adiabatic relation
Consider two particles interacting with short range interaction with large scattering length.

The energy of a universal dimer,

\[ E = -\frac{\hbar^2}{ma^2} \]

Using the adiabatic relation,

\[ C = -\frac{4\pi m}{\hbar^2} \frac{dE}{da^{-1}} = \frac{8\pi}{a} \]

The wave function reads,

\[ \psi(r) = Y_{00} \sqrt{\frac{2}{a}} e^{-r/a} \approx Y_{00} \sqrt{\frac{2}{a}} \left( \frac{1}{r} - \frac{1}{a} \right) \]

and therefore the tail of the momentum distribution

\[ n(k) \rightarrow \frac{8\pi/a}{k^4} = \frac{C}{k^4} \]
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and therefore the tail of the momentum distribution

\[ n(k) \xrightarrow{k^4} \frac{8\pi/a}{k^4} = \frac{C}{k^4} \]
The Contact - The Many Body Case

- When two particles approach each other

\[
\Psi \xrightarrow{r_{ij} \to 0} \left( \frac{1}{r_{ij}} - \frac{1}{a} \right) A_{ij}(R_{ij}, \{r_k\}_{k \neq i,j})
\]

\[
C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle
\]

- Where

\[
\langle A_{ij} | A_{ij} \rangle = \int \prod_{k \neq i,j} dr_k dR_{ij} A_{ij}^\dagger \left( R_{ij}, \{r_k\}_{k \neq i,j} \right) \cdot A_{ij} \left( R_{ij}, \{r_k\}_{k \neq i,j} \right)
\]

- For large \(k\), the Fourier transform is dominated by the short-range divergences,

\[
\int d^3 r_i e^{-ik \cdot r_i} \psi(r_1, ..., r_N) \approx \int d^3 r_i e^{-ik \cdot r_i} \sum_{i \neq j} \frac{1}{r_{ij}} A(R_{ij}, r_i)
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The Contact - The Many Body Case

- When two particles approach each other

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\Psi \xrightarrow{r_{ij} \to 0} (1/r_{ij} - 1/a) A_{ij}(R_{ij}, \{r_k\}_{k \neq i, j})
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For large k, the Fourier transform is dominated by the short-range divergences,

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Scales

- NN interaction range \( \mu_\pi^{-1} = \hbar / m_\pi c \approx 1.4 \text{ fm} \)
- NN scattering lengths \( a_t = 5.4 \text{ fm}, a_s \approx 20 \text{ fm} \) thus \( \mu_\pi |a| \geq 3.8 \)
- The nuclear radius is \( R \approx 1.2 A^{1/3} \text{ fm} \)
- The interparticle distance \( d \approx 2.4 \text{ fm} \) thus \( \mu_\pi d \approx 1.7 \)

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics.
- The interaction range is significant.
- There could be different interaction channels - not only s-wave.
- Therefore, we need to replace the asymptotic form

\[
\Psi \xrightarrow[r_{ij} \to 0]{} (1/r_{ij} - 1/a) A_{ij}(R_{ij}, \{r_k\}_{k \neq i,j})
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- Consequently, we don’t expect a \( 1/k^4 \) tail
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- The Tan conditions are not strictly applicable in nuclear physics.
- The interaction range is significant.
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- Therefore, we need to replace the asymptotic form

$$\Psi \rightarrow \sum_\alpha \varphi_\alpha (r_{ij}) A_{ij}^\alpha (R_{ij}, \{r_k\}_{k \neq i,j})$$

- Consequently we don’t expect a $1/k^4$ tail
In nuclear physics we have \(3\) possible particle pairs

\[ ij = \{pp, \; nn, \; pn\} \]

For each pair there are different channels

\[ \alpha = (s, \ell)jm \]

For each pair we define the contact matrix

\[ C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^\alpha | A_{ij}^\beta \rangle \]

For \(\ell = 0\) we need consider only \(4\) contacts

\[ P = \{(pp)_{S=0}, \; (nn)_{S=0}, \; (np)_{S=0}, \; (np)_{S=1}\} \]

Adding isospin symmetry the number of contacts is reduced to \(2\),

\[ C_s \leftrightarrow \{(pp)_{S=0}, \; (nn)_{S=0}, \; (np)_{S=0}\} \]

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Photoabsorption of Nuclei

Up to $\hbar \omega \approx 200$ MeV the cross-section $\sigma_A(\omega)$ is dominated by the dipole operator

$$\sigma_A(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

$R$ is the response function

$$R(\omega) = \sum_f \left| \langle \Psi_f | \mathbf{e} \cdot \hat{D} | \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

The Quasi-Deuteron Picture

- The photon carries energy but (almost) no momentum.
- It is captured by a single proton.
- The proton is ejected without any FSI.
- Momentum conservation $\Rightarrow$ a nucleon with opposite momentum must be ejected $k \approx -k_p$.
- Dipole dominance $\Rightarrow$ this partner must be a neutron.
- $\hbar \omega \rightarrow \infty \Rightarrow \sigma(\omega)$ depends on a universal short range $pn$ wave-function.
- The resulting cross-section is given by

$$\sigma_A(\omega) = L \frac{NZ}{A} \sigma_d(\omega)$$

- $L$ is known as the Levinger Constant

J. S. Levinger, Phys. Rev. 84, 43 (1951).
When a $pn$ pair are close together $\Psi_0$ is factorized into

$$\Psi_0(r_1, \ldots, r_A) = \sum_{\alpha} \varphi_\alpha(r_{pn}) A^\alpha_{pn}(R_{pn}, \{r_j\}_{j \neq p,n}) + \ldots$$

Assuming $s$-wave dominance, $\alpha$ is either singlet or triplet and $\varphi_\alpha \approx \varphi_d$.

$$\langle \Psi_\alpha^f | \epsilon \cdot \hat{D} | \Psi_0 \rangle \approx \sqrt{\frac{C_\alpha a_t}{8\pi}} \langle \psi_{d,f} | \epsilon \cdot \hat{D} | \psi_{d,0} \rangle$$

The cross section,

$$\sigma_A(\omega) = \frac{a_t}{4\pi} \frac{(C_s + C_t)}{2} \sigma_d(\omega) = \frac{a_t}{4\pi} \tilde{C}_{pn} \sigma_d(\omega) = L \frac{NZ}{A} \sigma_d(\omega)$$
When a pn pair are close together $\Psi_0$ is factorized into

$$\Psi_0(r_1, \ldots, r_A) = \sum_{\alpha} \varphi_{\alpha}(r_{pn}) A_{pn}^{\alpha}(R_{pn}, \{r_j \}_{j \neq p, n}) + \ldots$$

Assuming s-wave dominance, $\alpha$ is either singlet or triplet and $\varphi_{\alpha} \approx \varphi_d$,

$$\Psi_f^{\alpha}(r_1, \ldots, r_A) = \frac{4\pi}{\sqrt{C_{\alpha}}} \hat{A} \left\{ \frac{1}{\sqrt{\Omega}} e^{-ik \cdot r_{pn}} \chi_S A_{pn}^{\alpha}(R_{pn}, \{r_j \}_{j \neq p, n}) \right\}$$

The cross section,

$$\sigma_A(\omega) = \frac{a_t}{4\pi} \frac{(C_s + C_t)}{2} \sigma_d(\omega) = \frac{a_t}{4\pi} \tilde{C}_{pn} \sigma_d(\omega) = \frac{L}{A} \frac{NZ}{\sigma_d(\omega)}$$
In his original paper Levinger has estimated $L = 6.4$

In view of the available data we can conclude

$$L = 5.50 \pm 0.21$$

$N = Z = A/2$

Normalize by the Fermi momentum

$$\frac{\tilde{C}_{pn}}{k_F A} = \frac{\pi}{k_F a_t} (5.50 \pm 0.21)$$

$$1/k_F a_t \approx 0.15$$

$$\frac{\tilde{C}_{pn}}{k_F A} \approx 2.55 \pm 0.10$$

Comparison to Atomic Physics

Atomic data - $^{40}$K - J. T. Stewart et al., PRL 104, 235301 (2010)
$^6$Li - G.B. Partridge et al., PRL 95, 020404 (2005)

Nuclear data - The main source of the horizontal error bar is the range $(a_s, a_t)$
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Nuclear data - The main source of the horizontal error bar is the range $(a_s, a_t)$
Momentum distributions

1-body neutron and proton momentum distributions

\[ n_n(k), \ n_p(k) \]

2-body \( nn, np, pp \) momentum distributions

\[ F_{nn}(k), \ F_{pn}(k), \ F_{pp}(k) \]
Momentum distributions

The proton momentum distribution

\[ n_p^{JM}(k) = Z \int \prod_{l \neq p} \frac{d^3 k_l}{(2\pi)^3} |\tilde{\Psi}(k_1, \ldots, k_p = k, \ldots, k_A)|^2 \]

Using the asymptotic wave-function

\[ \Psi \xrightarrow{r_{ij} \to 0} \sum_{\alpha} \varphi_{\alpha}(r_{ij}) A_{ij}^\alpha(R_{ij}, \{ r_k \}_{k \neq i,j}) \]

we get

\[ n_p(k) = \frac{1}{2J + 1} \sum_{\alpha, \beta} \tilde{\varphi}_{pp}^\alpha(k) \tilde{\varphi}_{pp}^\beta(k) Z(Z - 1) \langle A_{pp}^{\alpha} | A_{pp}^{\beta} \rangle + \frac{1}{2J + 1} \sum_{\alpha, \beta} \tilde{\varphi}_{pn}^\alpha(k) \tilde{\varphi}_{pn}^\beta(k) NZ \langle A_{pn}^{\alpha} | A_{pn}^{\beta} \rangle \]
The proton momentum distribution

\[ n_p^{JM}(\mathbf{k}) = Z \int \prod_{l \neq p} \frac{d^3k_l}{(2\pi)^3} |\tilde{\Psi}(\mathbf{k}_1, \ldots, k_p = \mathbf{k}, \ldots, k_A)|^2 \]

Using the asymptotic wave-function

\[ \Psi \xrightarrow{r_{ij} \to 0} \sum_\alpha \varphi_\alpha(r_{ij}) A^\alpha_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k \neq i,j}) \]

we get

\[ n_p(\mathbf{k}) = \sum_{\alpha, \beta} \bar{\varphi}_{pp}^\alpha(\mathbf{k}) \bar{\varphi}_{pp}^\beta(\mathbf{k}) \frac{2C_{\alpha\beta}^{pp}}{16\pi^2} + \sum_{\alpha, \beta} \bar{\varphi}_{pn}^\alpha(\mathbf{k}) \bar{\varphi}_{pn}^\beta(\mathbf{k}) \frac{C_{\alpha\beta}^{pn}}{16\pi^2} \]
Furthermore, starting from the general assumption

\[ \Psi \xrightarrow{r_{ij} \to 0} \sum_{\alpha} \varphi_{\alpha}(r_{ij}) A_{ij}^{\alpha}(R_{ij}, \{r_k\}_{k \neq i,j}) \]

The following \textbf{asymptotic} relations between the 1-body and 2-body momentum distributions can be proven

\[ n_p(k) \xrightarrow{k \to \infty} 2F_{pp}(k) + F_{pn}(k) \]

\[ n_n(k) \xrightarrow{k \to \infty} 2F_{nn}(k) + F_{pn}(k) \]

These relations hold regardless of the specific form of \( \varphi_{\alpha} \) and without any assumptions on \( \{\alpha\} \).
Numerical verification of the momentum relations

VMC calculations of light nuclei

- Wiringa et al. published a series of 1-body, 2-body momentum distributions
  
  R. B. Wiringa et al., PRC 89, 024305 (2014)

- The data is available for nuclei in the range $2 \leq A \leq 10$.
- The calculations were done with the VMC method
- For symmetric nuclei $n_p = n_n$

The momentum relations holds for $4 \text{ fm}^{-1} \leq k \leq 5 \text{ fm}^{-1}$
The nuclear contact

Photoabsorption

AV18+UBIX $A \leq 10$

QD Model

$L = 5.50 \pm 0.21$

The Contact

$L = 5.7 \pm 0.7$
The nuclear contact

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Conclusions and outlook

Generalizing Tan’s contact to nuclear physics

- The Quasi-Deuteron model was revised.
- The Levinger constant and the nuclear contacts are close relatives.
- $\bar{C}_{pn}$ was deduced using previous evaluations of Levinger constant.
- $\bar{C}_{pn}/A$ seems to be constant throughout the nuclear chart.
- Its value stands in line with the universal curve measured in ultracold atomic systems.
- Momentum relations were derived, connecting one-body and two-body distributions.
- There relations were verified using VMC data.
- Levinger constant derived from this data is in agreement with that derived from photoabsorption experiments.

Outlook

- Electron scattering.
- Neutrino scattering.
- …

We have only started to explore the usefulness of the contact formalism in nuclear physics!
The contact: QFT perspective

- Quantum field theory formulation of the Zero-Range Model,

\[
\mathcal{H} = \sum_{\sigma} \frac{\hbar^2}{2m} \nabla \psi^\dagger_{\sigma} \nabla \psi_{\sigma}(R) + \frac{g(\Lambda)}{m} \psi^\dagger_1 \psi^\dagger_2 \psi_2 \psi_1(R) + \mathcal{V}
\]

- Renormalization,

\[
g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}
\]

- Defining di-atomic field operator, \( \Phi(R) = g(\Lambda) \psi_2 \psi_1(R) \)

\[
\mathcal{H} = \sum_{\sigma} \frac{\hbar^2}{2m} \nabla \psi^\dagger_{\sigma} \nabla \psi_{\sigma}(R) - \frac{\Lambda}{2\pi^2 m} \Phi^\dagger \Phi + \frac{1}{4\pi ma} \Phi^\dagger \Phi + \mathcal{V}
\]

- Identifying \( C = \int d^3R \langle \Phi^\dagger \Phi(R) \rangle \) we got the adiabatic relation.
The contact: QFT perspective

- Quantum field theory formulation of the Zero-Range Model,

\[ H = \sum_\sigma \frac{\hbar^2}{2m} \nabla \psi_\sigma^\dagger \nabla \psi_\sigma(R) + \frac{g(\Lambda)}{m} \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1(R) + \mathcal{V} \]

- Renormalization,

\[ g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda / \pi} \]

- Defining di-atomic field operator, \( \Phi(R) = g(\Lambda)\psi_2\psi_1(R) \)

\[ H = \sum_\sigma \frac{\hbar^2}{2m} \nabla \psi_\sigma^\dagger \nabla \psi_\sigma(R) - \frac{\Lambda}{2\pi^2 m} \Phi^\dagger \Phi + \frac{1}{4\pi ma} \Phi^\dagger \Phi + \mathcal{V} \]

- Identifying \( C = \int d^3R \langle \Phi^\dagger \Phi(R) \rangle \) we got the adiabatic relation.
The contact: QFT perspective

- Quantum field theory formulation of the Zero-Range Model,

\[
\mathcal{H} = \sum_{\sigma} \frac{\hbar^2}{2m} \nabla \psi_{\sigma}^\dagger \nabla \psi_{\sigma}(R) + \frac{g(\Lambda)}{m} \psi_{1}^\dagger \psi_{2}^\dagger \psi_{2} \psi_{1}(R) + \mathcal{V}
\]

- Renormalization,

\[
g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}
\]

- Defining di-atomic field operator, \( \Phi(R) = g(\Lambda) \psi_{2} \psi_{1}(R) \)

\[
\mathcal{H} = \sum_{\sigma} \frac{\hbar^2}{2m} \nabla \psi_{\sigma}^\dagger \nabla \psi_{\sigma}(R) - \frac{\Lambda}{2\pi^2 m} \Phi^\dagger \Phi + \frac{1}{4\pi ma} \Phi^\dagger \Phi + \mathcal{V}
\]

- Identifying \( C = \int d^3R \langle \Phi^\dagger \Phi(R) \rangle \) we got the adiabatic relation.
The contact: QFT perspective

- Quantum field theory formulation of the Zero-Range Model,

\[
\mathcal{H} = \sum_\sigma \frac{\hbar^2}{2m} \nabla \psi_\sigma^+ \nabla \psi_\sigma (\mathbf{R}) + \frac{g(\Lambda)}{m} \psi_1^+ \psi_2^+ \psi_2 \psi_1 (\mathbf{R}) + \mathcal{V}
\]

- Renormalization,

\[
g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}
\]

- Defining di-atomic field operator, \( \Phi (\mathbf{R}) = g(\Lambda) \psi_2 \psi_1 (\mathbf{R}) \)

\[
\mathcal{H} = \sum_\sigma \frac{\hbar^2}{2m} \nabla \psi_\sigma^+ \nabla \psi_\sigma (\mathbf{R}) - \frac{\Lambda}{2\pi^2 m} \Phi^+ \Phi + \frac{1}{4\pi ma} \Phi^+ \Phi + \mathcal{V}
\]

- Identifying \( C = \int d^3 R \langle \Phi^+ \Phi (\mathbf{R}) \rangle \) we got the adiabatic relation.
Short range correlations and their universal nature is an intensive line of research in NP

[Ciofi degli Atti, Frankfurt, Strikman, Sargasian, Piasetzky,..]

Short range correlations measured in (e,e') experiments at JLAB.
A clear preference for correlated $np$ pairs.

Experimental Results - fitting the Levinger Constant

The $^{12}\text{C}$ photoabsorption cross-section


Line - the Quasi-Deutron model $L = 5.8$
We start with 2-body Schrodinger ...

\[
\left[ -\frac{\hbar^2}{m} \nabla^2 + V(r) \right] \psi = E \psi
\]

At vanishing distance, \( r \to 0 \)

- The energy becomes negligible \( E \ll \hbar^2 / mr^2 \)
- The w.f. \( \psi \) assumes an asymptotic energy independent form \( \varphi \)

\[
\left[ -\frac{\hbar^2}{m} \nabla^2 + V(r) \right] \varphi(r) = 0
\]

\( r \varphi(r) = 0 |_{r=0} \)

- Valid for any \( A \)-body system.