Isospin-breaking effects in K^+_{e4} decays

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8th International Workshop on Chiral Dynamics – Pisa, June 29 - July 3, 2015

Based on work done in collaboration with V. Bernard and S. Descotes-Genon:

S. Descotes-Genon, M. K., Eur. Phys. J. C 72, 1962 (2012) [arXiv:1202.5886 [hep-ph]]
V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 73, 2478 (2013) [arXiv:1305.3843 [hep-ph]]
V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 75, 145 (2015) [arXiv:1501.07102 [hep-ph]]

OUTLINE

- Introduction - Motivation
- IB in the phases of the two-loop K_{e4} form factors
- Extraction of $\pi\pi$ scattering lengths a_0^0 and a_0^2
- Radiative corrections and cusp in the $K_{e4}^{\pm}(\pi^0\pi^0)$ mode
- Summary - Conclusion

Introduction - Motivation

Only a handful of processes provide precise information on $\pi\pi$ scattering lengths:
$K \to \pi\pi\pi$, pionic atoms, $K \to \pi\pi\ell\nu_\ell$ ($K_{\ell 4}$ decays),...

- Geneva-Saclay: $\sim 30\,000$ K_{e4}^{+-} events

[L. Rosselet et al., Phys. Rev. D 15, 574 (1977)]

- BNL-E865: $\sim 400\,000$ K_{e4}^{+-} events

[S. Pislak et al. (BNL-E865 Collaboration), Phys. Rev. Lett. 87, 221801 (2001)]

[Erratum-ibid. 105, 019901 (2010)]

[S. Pislak et al. (BNL-E865 Collaboration), Phys. Rev. D 67, 072004 (2003)]

[Erratum-ibid. D 81, 119903 (2010)]

- NA48/2: $\sim 1\,100\,000$ K_{e4}^{+-} events

[J.R. Batley et al. (NA48/2 Collaboration), Eur. Phys. J. C 54, 411 (2008)]

[J.R. Batley et al. (NA48/2 Collaboration), Eur. Phys. J. C 70, 635 (2010)]

- NA48/2: $\sim 65\,100$ K_{e4}^{00} events

[J.R. Batley et al. (NA48/2 Collaboration), JHEP 1408, 159 (2014)]

Standard angular analysis of the K_{e4}^{+-} form factors provides information on low-energy $\pi\pi$ scattering (Watson's theorem) through the phase difference

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}}$$

[N. Cabibbo, A. Maksymowicz, Phys. Rev. B 137, 438 (1965); Erratum-ibid 168, 1926 (1968)]

[F.A. Berends, A. Donnachie, G.C. Oades, Phys. Rev. 171, 1457(1968)]

measurable in the interference of the F^{+-} and G^{+-} form factors.

Comparison with solutions of the Roy equations

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2)$$

allows to extract the values of the $\pi\pi$ S-wave scattering lengths in the isospin channels $I = 0, 2$

$f_{\text{Roy}}(s; a_0^2, a_0^2)$ follows from:

- dispersion relations (analyticity, unitarity, crossing, Froissard bound)
- $\pi\pi$ data at energies $\sqrt{s} \geq 1$ GeV
- isospin symmetry

[S.M. Roy, Phys. Lett. B 36, 353 (1971)]

Solutions can be constructed for $(a_0^0, a_0^2) \in$ Universal Band

[B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rep. 353, 207 (2001)]

Once radiative corrections have been taken care of (see later), it is still important to take isospin-breaking corrections due to $M_\pi \neq M_{\pi^0}$ into account before analysing data

[J. Gasser, PoS KAON, 033 (2008), arXiv:0710.3048]

Evaluation of IB corrections in ChPT

[G. Colangelo, J. Gasser, A. Rusetsky, Eur. Phys. J. C 59, 777 (2009)]

$$\longrightarrow a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

However, IB corrections were evaluated at fixed values of the scattering lengths

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; (a_0^0)_{\text{ChPT}}, (a_0^2)_{\text{ChPT}})$$

Drawback shared by other studies devoted to isospin breaking in ChPT (QCD+QED)

[V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274]

[A. Nehme, Nucl. Phys. B 682, 289 (2004)]

[P. Stoffer, Eur. Phys. J. C 74, 2749 (2004)]

Is it possible to obtain

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; a_0^0, a_0^2) \quad ?$$

What is the quantitative effect in the determination of the scattering lengths?

• NA48/2: $\sim 65\,100\; K_{e4}^{00}$ events

[J.R. Batley et al. (NA48/2 Collaboration), JHEP 1408, 159 (2014)]

In the isospin limit, one form factor is common to K_{e4}^{+-} and K_{e4}^{00} ($F^{+-} = F^{00}$). This can be tested with the available data:

$$\begin{aligned} |V_{us}| f_s[K_{e4}^{+-}] &= 1.285 \pm 0.001_{\rm stat} \pm 0.004_{\rm syst} \pm 0.005_{\rm ext} \\ (1+\delta_{EM})|V_{us}| f_s[K_{e4}^{00}] &= 1.369 \pm 0.003_{\rm stat} \pm 0.006_{\rm syst} \pm 0.009_{\rm ext} \end{aligned}$$

$$\longrightarrow \quad (1+\delta_{EM}) \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

δ_{EM} not known (apart from the not very explicit [B. Morel, Quoc-Hung Do, Nuovo Cim. A 46, 253 (1978)])

Main issue: radiative corrections have been applied to K_{e4}^{+-} data. Computation of δ_{EM} should be carried out within the same framework as used there in order to make comparison meaningful

IB in the phases of the two-loop K_{e4} form factors

Goal: obtain a representation for K_{e4} form factors that is

a) valid at two loops in the low-energy expansion

b) where the $\pi\pi$ scattering lengths occur as free parameters

c) with IB effects included

Adapt the approach (“reconstruction theorem”) described in

[J. Stern, H. Sazdjian, N. H. Fuchs, Phys. Rev. D 47, 3814 (1993), arXiv:hep-ph/9301244]

for the $\pi\pi$ scattering amplitude, and implemented in

[M. Knecht, B. Moussallam, J. Stern, N.H. Fuchs, Nucl. Phys. B 457, 513 (1995), arXiv:hep-ph/9507319]

Rests on very general principle

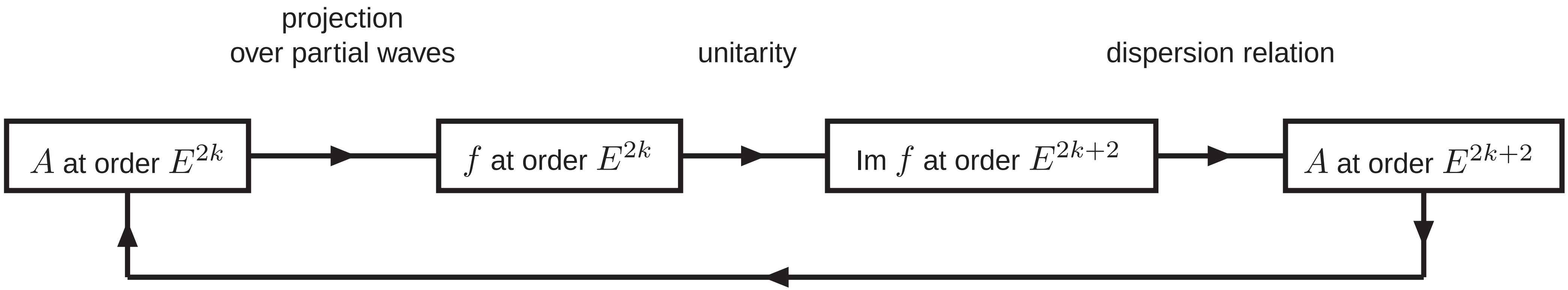
a) Relativistic invariance

b) Analyticity, unitarity, crossing

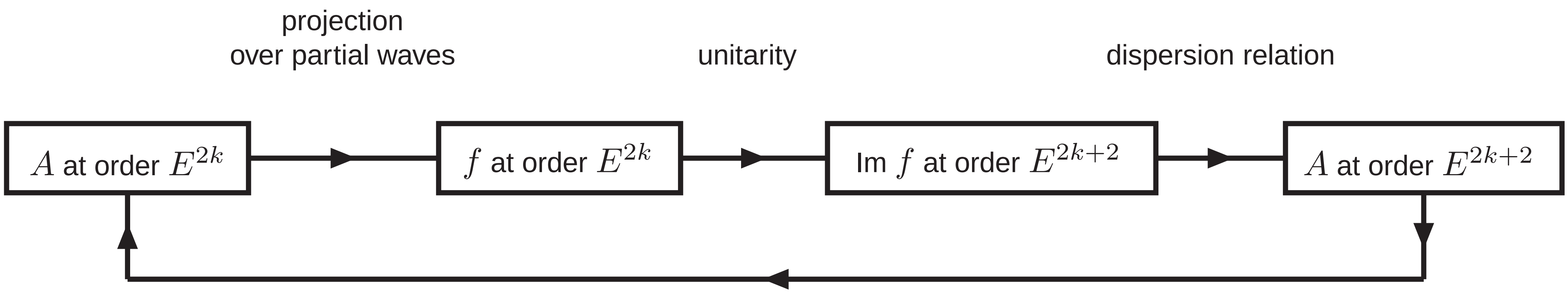
c) Chiral counting

Note: isospin symmetry not required

$\longrightarrow$ Iterative two-step construction of two-loop representation for meson scattering amplitudes and K_{e4} form factors



$\longrightarrow$ Iterative two-step construction of two-loop representation for meson scattering amplitudes and K_{e4} form factors



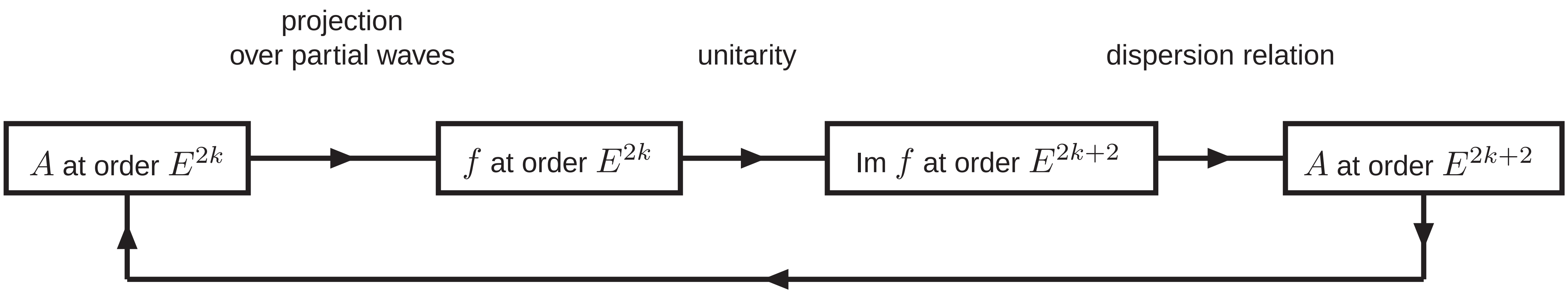
Partial-wave projections

$$\begin{aligned}
\mathcal{F}^{ab}(s,t,u) &= \sum_{l\geq 0} f_l^{ab}(s,s_\ell)P_l(\cos\theta_{ab}),\\
\mathcal{G}^{ab}(s,t,u) &= \sum_{l\geq 1} g_l^{ab}(s,s_\ell)P_l'(\cos\theta_{ab}),\\
\mathcal{R}^{ab}(s,t,u) &= \sum_{l\geq 0} r_l^{ab}(s,s_\ell)P_l(\cos\theta_{ab})
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}^{ab}(s,t,u) &= F^{ab}(s,t,u) + \left[\frac{M_a^2-M_b^2}{s} + \frac{M_c^2-s-s_\ell}{s}\,\frac{\lambda_{ab}^{\frac{1}{2}}(s)}{\lambda_{\ell c}^{\frac{1}{2}}(s)}\cos\theta_{ab}\right]G^{ab}(s,t,u),\\
\mathcal{G}^{ab}(s,t,u) &= G^{ab}(s,t,u),\\
\mathcal{R}^{ab}(s,t,u) &= R^{ab}(s,t,u) + \frac{M_c^2-s-s_\ell}{2s_\ell}F^{ab}(s,t,u)\\
&\quad + \frac{1}{2ss_\ell}\left[(M_a^2-M_b^2)(M_c^2-s-s_\ell) + \lambda_{ab}^{\frac{1}{2}}(s)\lambda_{\ell c}^{\frac{1}{2}}(s)\cos\theta_{ab}\right]G^{ab}(s,t,u)
\end{aligned}$$

[F.A. Berends, A. Donnachie, G.C. Oades, Phys. Rev. 171, 1457 (1968)]

$\longrightarrow$ Iterative two-step construction of two-loop representation for meson scattering amplitudes and K_{e4} form factors



Chiral counting

$$\text{Re}\, f_0^{ab}(s,s_\ell),\ \text{Re}\, f_1^{ab}(s,s_\ell),\ \text{Re}\, g_1^{ab}(s,s_\ell) \sim \mathcal{O}(E^0) \qquad \text{Im}\, f_0^{ab}(s,s_\ell),\ \text{Im}\, f_1^{ab}(s,s_\ell),\ \text{Im}\, g_1^{ab}(s,s_\ell) \sim \mathcal{O}(E^2)$$

$$\text{Re}\, f_l^{ab}(s,s_\ell),\ \text{Re}\, g_l^{ab}(s,s_\ell) \sim \mathcal{O}(E^2),\ l \geq 2 \qquad \text{Im}\, f_l^{ab}(s,s_\ell),\ \text{Im}\, g_l^{ab}(s,s_\ell) \sim \mathcal{O}(E^6),\ l \geq 2$$

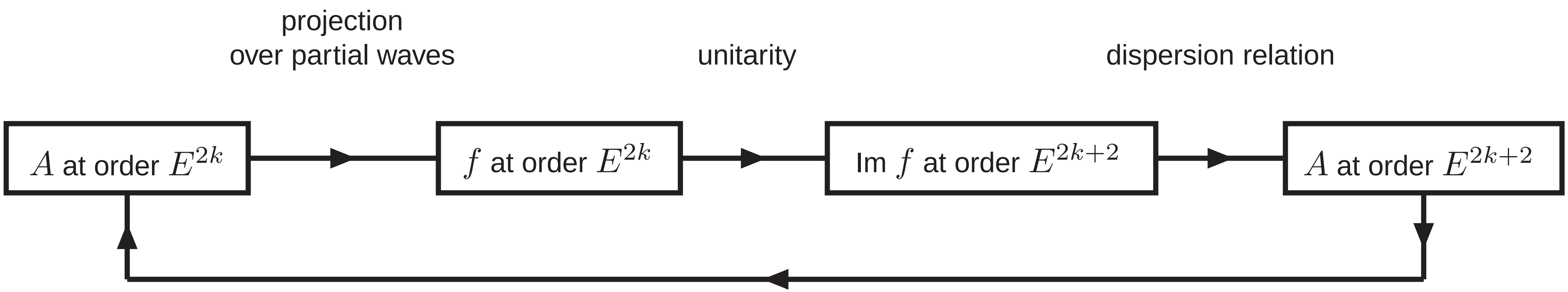
[G. Colangelo, M. Knecht, J. Stern, Phys. Lett. B 336, 543 (1994), arXiv:hep-ph/9406211]

$$\begin{aligned} F^{ab}(s,t,u) &= F_S^{ab}(s,s_\ell) + F_P^{ab}(s,s_\ell)\cos\theta_{ab} + F_>^{ab}(s,\cos\theta_{ab},s_\ell) \\ G^{ab}(s,t,u) &= G_P^{ab}(s,s_\ell) + G_>^{ab}(s,\cos\theta_{ab},s_\ell) \end{aligned}$$

$$\text{Re}F_>^{ab}(s,\cos\theta_{ab},s_\ell),\ \text{Re}G_>^{ab}(s,\cos\theta_{ab},s_\ell) \sim \mathcal{O}(E^2)$$
$$\text{Im}F_>^{ab}(s,\cos\theta_{ab},s_\ell),\ \text{Im}G_>^{ab}(s,\cos\theta_{ab},s_\ell) \sim \mathcal{O}(E^6)$$

$$\begin{aligned} F_S^{ab}(s,s_\ell) &= f_0^{ab}(s,s_\ell) - \frac{M_a^2 - M_b^2}{s}\, g_1^{ab}(s,s_\ell), \\ F_P^{ab}(s,s_\ell) &= f_1^{ab}(s,s_\ell) - \frac{M_c^2 - s - s_\ell}{s}\, \frac{\lambda_{ab}^{\frac{1}{2}}(s)}{\lambda_{\ell c}^{\frac{1}{2}}(s)}\, g_1^{ab}(s,s_\ell)\,, \quad G_P^{ab}(s,s_\ell) = g_1^{ab}(s,s_\ell) \end{aligned}$$

$\longrightarrow$ Iterative two-step construction of two-loop representation for meson scattering amplitudes and K_{e4} form factors



Analyticity, unitarity

$$\text{Im}\, f_l^{ab}(s, s_\ell) = \sum_{\{a',b'\}} \frac{1}{\mathcal{S}_{a'b'}} \frac{\lambda_{a'b'}^{\frac{1}{2}}(s)}{s} \,\text{Re}\, \left\{ t_l^{a'b';ab}(s) \left[f_l^{a'b'}(s, s_\ell) \right]^\star \right\} \theta(s - s_{a'b'}) + \mathcal{O}(E^8),$$

$$\text{Im}\, g_l^{ab}(s, s_\ell) = \sum_{\{a',b'\}} \frac{1}{\mathcal{S}_{a'b'}} \frac{\lambda_{a'b'}^{\frac{1}{2}}(s)}{s} \frac{\lambda_{a'b'}^{\frac{1}{2}}(s)}{\lambda_{ab}^{\frac{1}{2}}(s)} \,\text{Re}\, \left\{ t_l^{a'b';ab}(s) \left[g_l^{a'b'}(s, s_\ell) \right]^\star \right\} \theta(s - s_{a'b'}) + \mathcal{O}(E^8)$$

Involves also the mesonic scattering amplitudes $A^{a'b';ab}(s, \hat{t})$, $\hat{t} = (p_a - p_{a'})^2$

$$A^{a'b';ab}(s, \hat{t}) = 16\pi \sum_l (2l+1) t_l^{a'b';ab}(s) P_l(\cos\hat{\theta})$$

Partial waves $t_l^{a'b';ab}(s)$ parameterised in terms of the scattering lengths

Phases of the NNLO form factors

$$\begin{aligned} F(s,t,u) &= \widehat{F}_S(s,s_\ell)e^{i\delta_S(s,s_\ell)} + \widehat{F}_P(s,s_\ell)e^{i\delta_P(s,s_\ell)}\cos\theta + \text{Re}F_>(s,\cos\theta,s_\ell) + \mathcal{O}(E^6), \\ G(s,t,u) &= \widehat{G}_P(s,s_\ell)e^{i\delta_P(s,s_\ell)} + \text{Re}G_>(s,\cos\theta,s_\ell) + \mathcal{O}(E^6) \end{aligned}$$

$$\text{Re}\, F_S(s,s_\ell) = F_{S[0]} + F_{S[2]}(s,s_\ell) + \mathcal{O}(E^4), \qquad \text{Re}\, G_P(s,s_\ell) = G_{P[0]} + G_{P[2]}(s,s_\ell) + \mathcal{O}(E^4)$$

$$\text{Re}\, t_l^{a'b';+-}(s) = \varphi_l^{a'b';+-}(s) + \psi_l^{a'b';+-}(s) + \mathcal{O}(E^6)$$

$$\delta_S(s,s_\ell) = \sum_{\{a',b'\}} \frac{1}{\mathcal{S}_{a'b'}} \frac{\lambda^{\frac{1}{2}}_{a'b'}(s)}{s} \left[\varphi_0^{a'b';+-}(s)\, \frac{F^{a'b'}_{S[0]} + F^{a'b'}_{S[2]}(s,s_\ell)}{F_{S[0]} + F_{S[2]}(s,s_\ell)} + \psi_0^{a'b';+-}(s)\, \frac{F^{a'b'}_{S[0]}}{F_{S[0]}}\right] \theta(s - s_{a'b'}) + \mathcal{O}(E^6)$$

$$\delta_P(s,s_\ell) = \sum_{\{a',b'\}} \frac{\lambda^{\frac{1}{2}}_{a'b'}(s)}{s} \frac{\lambda^{\frac{1}{2}}_{a'b'}(s)}{\lambda^{\frac{1}{2}}_{ab}(s)} \left[\varphi_1^{a'b';+-}(s)\, \frac{G^{a'b'}_{P[0]} + G^{a'b'}_{P[2]}(s,s_\ell)}{G_{P[0]} + G_{P[2]}(s,s_\ell)} + \psi_1^{a'b';+-}(s)\, \frac{G^{a'b'}_{P[0]}}{G_{P[0]}}\right] \theta(s - s_{a'b'}) + \mathcal{O}(E^6)$$

IB in the phases of the NNLO form factors

$$
\begin{aligned}
\delta_S(s, s_\ell) - \delta_0(s) &= \sigma(s)\left\{\left[\varphi_0^{+-}(s) - \overset{o}{\varphi}{}_0^{+-}(s)\right] + \left[\psi_0^{+-}(s) - \overset{o}{\psi}{}_0^{+-}(s)\right]\right\} \\
&+ \frac{1}{2}\sigma_0(s)\left[\varphi_0^{x}(s)\,\frac{F_{S[0]}^{00} + F_{S[2]}^{00}(s, s_\ell)}{F_{S[0]}^{+-} + F_{S[2]}^{+-}(s, s_\ell)} + \psi_0^{x}(s)\,\frac{F_{S[0]}^{00}}{F_{S[0]}^{+-}}\right] \\
&+ \frac{1}{2}\sigma_0(s)\left[\overset{o}{\varphi}{}_0^{x}(s) + \overset{o}{\psi}{}_0^{x}(s)\right] + \mathcal{O}(E^6)
\end{aligned}
$$

$$
\delta_P(s) - \delta_1(s) = \sigma(s)\left\{\left[\varphi_1^{+-}(s) - \overset{o}{\varphi}{}_1^{+-}(s)\right] + \left[\psi_1^{+-}(s) - \overset{o}{\psi}{}_1^{+-}(s)\right]\right\} + \mathcal{O}(E^6)
$$

Note:

1) the dependence of the phase on the form factors (Watson's theorem no longer holds)

2) the dependence on s_ℓ in $\delta_S(s, s_\ell)$, resulting from IB effects
Numerically, it turns out to be negligible $\longrightarrow$ use $\delta_S(s) \equiv \delta_S(s, 0)$

Now we have

$$
[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; a_0^0, a_0^2)
$$

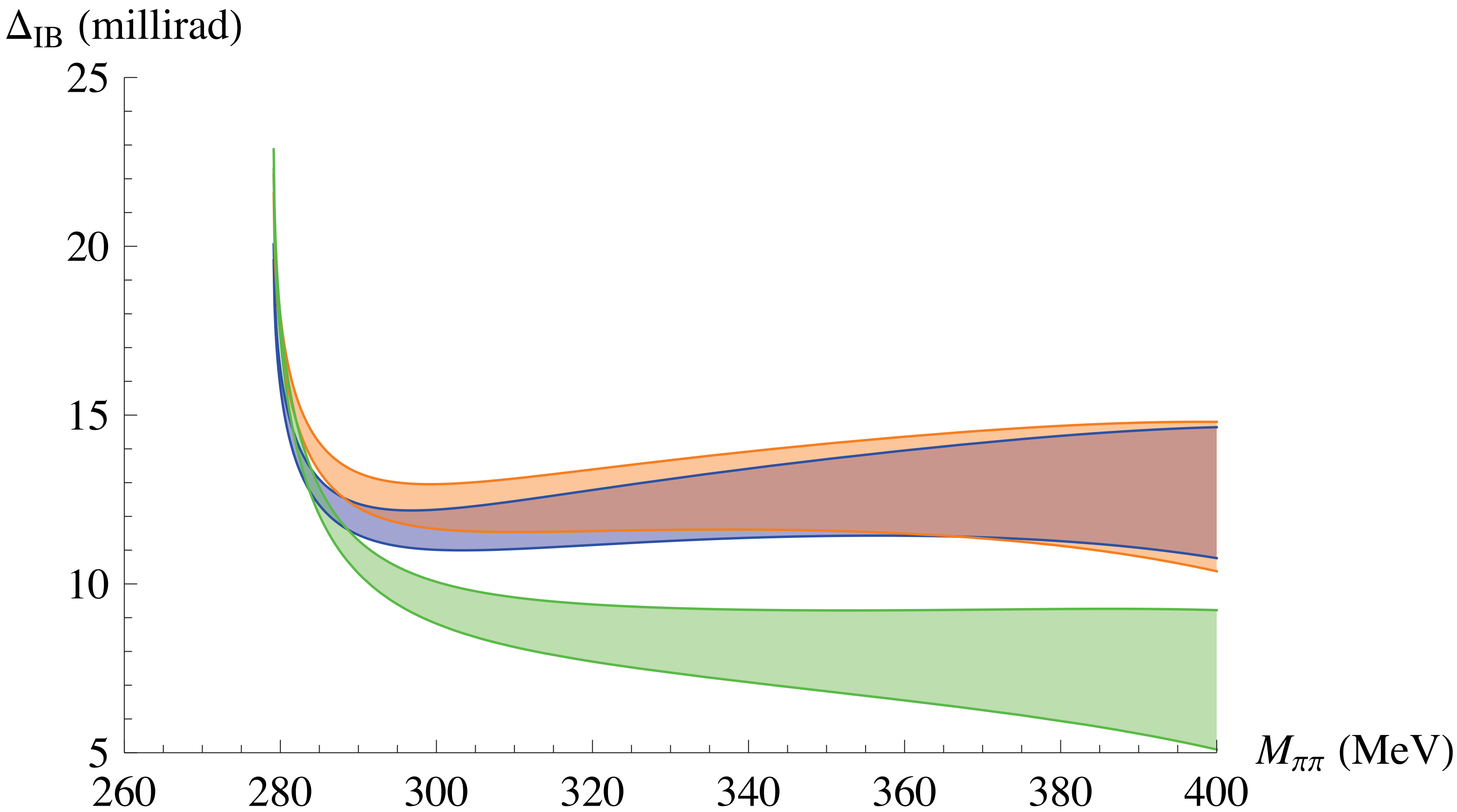


Figure 1: Isospin breaking in the phase of the two-loop form factors, $\Delta_{\rm IB}(s, s_\ell)$ as a function of the dipion invariant mass $M_{\pi\pi} = \sqrt{s}$, for $s_\ell = 0$. The middle (light-blue) band corresponds to the $(a_0^0, a_0^2) = (0.182, -0.052)$, whereas the other two cases shown correspond to $(a_0^0, a_0^2) = (0.205, -0.055)$ (upper orange band) and to $(a_0^0, a_0^2) = (0.24, -0.035)$ (lower green band). The widths of these bands result from the uncertainty on the various inputs needed at two loops.

Extraction of $\pi\pi$ scattering lengths a_0^0 and a_0^2

Re-analysis of NA48/2 data

Fit the data to ("$S-P$ fit")

$$[\delta_S(s)-\delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s;a_0^0,a_0^2)+\delta f_{\text{IB}}(s;a_0^0,a_0^2)$$

$$a_0^0 = 0.221 \pm 0.018 \qquad a_0^2 = -0.0453 \pm 0.0106$$

to be compared to

$$\longrightarrow a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}\,(37)_{\text{th}} \qquad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

NA48/2 data alone provide a strong correlation between a_0^0 and a_0^2, but a weaker constraint on each of them separately
$\longrightarrow$ supply additional information, either from

- $I=2$ data in S-wave ("extended fit")

[S. Descotes-Genon, N.H. Fuchs, L. Girlanda, J. Stern, Eur. Phys. J. C 24, 469 (2002)]

- $N_f=2$ ChPT and scalar radius of the pion ("scalar fit")

$$a_0^2 = -0.0444 + 0.236(a_0^0-0.22) - 0.61(a_0^0-0.22)^2 - 9.9(a_0^0-0.22)^3 \pm 0.0008$$

[G. Colangelo, J. Gasser, H. Leutwyler, Phys. Lett. B 488, 261 (2000)]

	With isospin-breaking corrections			Without isospin-breaking corrections		
	S-P	Extended	Scalar	S-P	Extended	Scalar
a_0^0	0.221 ± 0.018	0.232 ± 0.009	0.226 ± 0.007	0.247 ± 0.014	0.247 ± 0.008	0.242 ± 0.006
a_0^2	-0.0453 ± 0.0106	-0.0383 ± 0.0040	-0.0431 ± 0.0019	-0.0357 ± 0.0096	-0.0349 ± 0.0038	-0.0396 ± 0.0015
$\rho_{a_0^0,a_0^2}$	0.964	0.881	0.914	0.945	0.842	0.855
θ_0	$(82.3 \pm 3.4)^\circ$	$(82.3 \pm 3.4)^\circ$	82.3°	$(82.3 \pm 3.4)^\circ$	$(82.3 \pm 3.4)^\circ$	82.3°
θ_1	$(108.9 \pm 2)^\circ$	$(108.9 \pm 2)^\circ$	108.9°	$(108.9 \pm 2)^\circ$	$(108.9 \pm 2)^\circ$	108.9°
χ^2/N	7.6/6	16.6/16	7.8/8	7.2/6	15.7/16	7.3/8
α	1.043 ± 0.548	1.340 ± 0.231	1.179 ± 0.123	1.637 ± 0.472	1.672 ± 0.208	1.458 ± 0.098
β	1.124 ± 0.053	1.088 ± 0.020	1.116 ± 0.007	1.103 ± 0.055	1.098 ± 0.021	1.128 ± 0.008
$\rho_{\alpha\beta}$	0.47	0.31	0.02	0.47	0.32	0.00
$\lambda_1 \cdot 10^3$	-3.56 ± 0.68	-3.80 ± 0.58	-3.89 ± 0.10	-3.79 ± 0.68	-3.78 ± 0.57	-3.74 ± 0.11
$\lambda_2 \cdot 10^3$	9.08 ± 0.28	8.94 ± 0.10	9.14 ± 0.04	9.02 ± 0.23	9.02 ± 0.11	9.21 ± 0.42
$\lambda_3 \cdot 10^4$	2.38 ± 0.18	2.30 ± 0.14	2.32 ± 0.04	2.34 ± 0.18	2.34 ± 0.14	2.41 ± 3.67
$\lambda_4 \cdot 10^4$	-1.46 ± 0.10	-1.39 ± 0.04	-1.45 ± 0.02	-1.41 ± 0.10	-1.40 ± 0.04	-1.46 ± 0.02
$\bar{\ell}_3$	3.15 ± 9.9	-10.2 ± 5.7	-2.7 ± 6.6	-39.9 ± 20.3	-43.5 ± 19.1	-19.6 ± 7.8
$\bar{\ell}_4$	5.3 ± 0.8	4.4 ± 0.6	5.1 ± 0.3	5.2 ± 0.8	5.2 ± 0.7	6.0 ± 0.4
$X(2)$	0.88 ± 0.05	0.80 ± 0.06	0.82 ± 0.02	0.72 ± 0.05	0.71 ± 0.05	0.75 ± 0.03
$Z(2)$	0.87 ± 0.03	0.89 ± 0.02	0.86 ± 0.01	0.87 ± 0.02	0.87 ± 0.02	0.85 ± 0.01

Table 1: Scattering lengths, subthreshold parameters and chiral low-energy constants for the different fits considered, with and without the isospin-breaking correction.

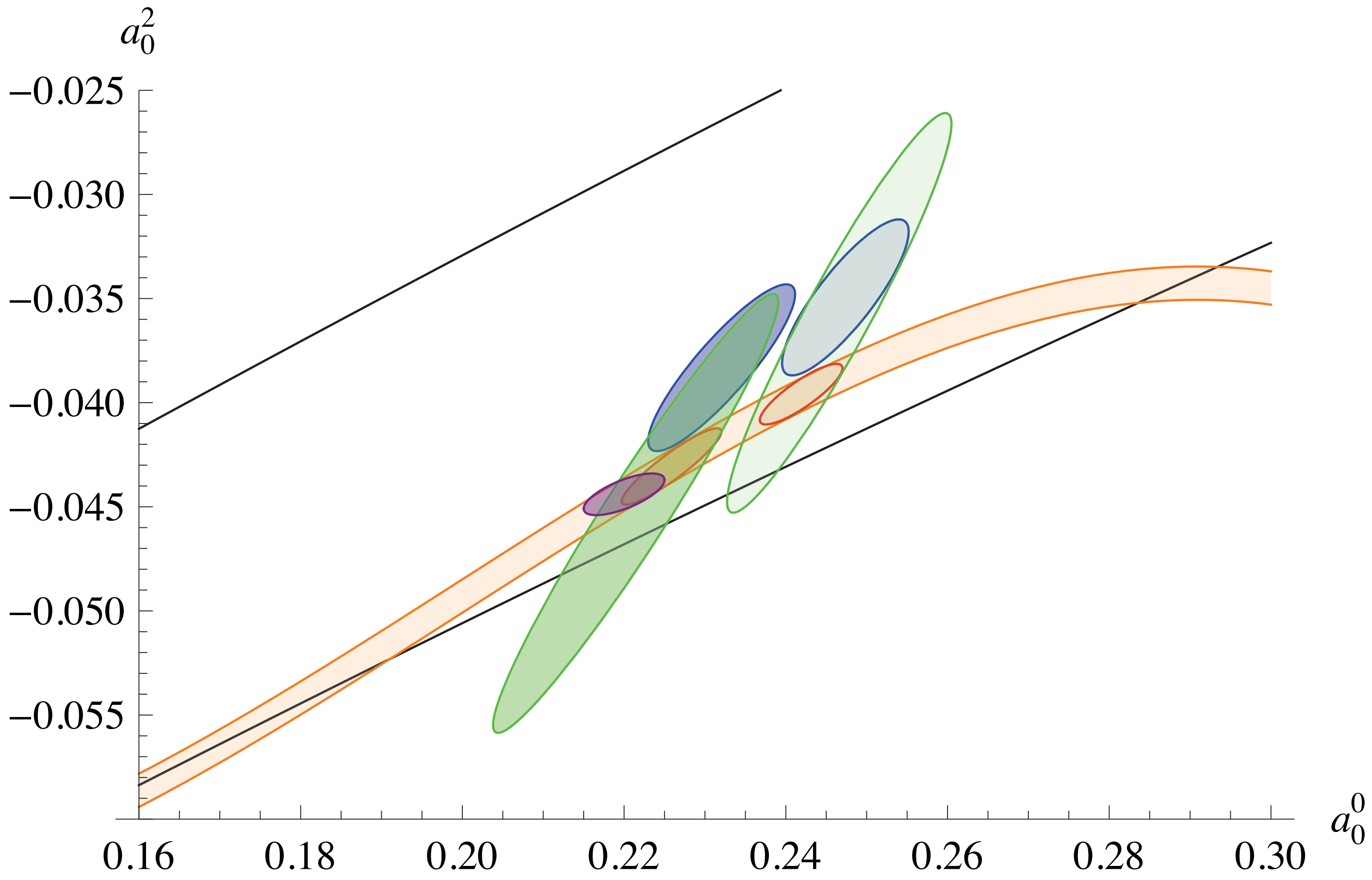


Figure 2: Results of the fits to the NA48/2 data in the (a_0^0, a_0^2) plane. The two black solid lines indicate the universal band where the two S-wave scattering lengths comply with dispersive constraints (Roy equations) and high-energy data on $\pi\pi$ scattering. The orange band is the constraint coming from the scalar radius of the pion. The small dark (purple) ellipse represents the prediction based on $N_f = 2$ chiral perturbation theory. The three other ellipses on the left represent, in order of increasing sizes, the 1-σ ellipses corresponding to the scalar (orange ellipse), extended (blue ellipse) and S-P (green ellipse), respectively, when isospin-breaking corrections are included. The light-shaded ellipses on the right represent the same outputs, but obtained without including isopin-breaking corrections.

Radiative corrections and cusp in K_{e4}^{00} mode

NA48/2: $\sim 65\,100\ K_{e4}^{00}$ events

$$\longrightarrow \quad (1+\delta_{EM})\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

[J.R. Batley et al. (NA48/2 Collaboration), JHEP 1408, 159 (2014)]

At lowest-order in ChPT

$$\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = \left(1+\frac{3}{2R}\right) \sim 1.040 \qquad R \equiv \frac{m_s - m_{ud}}{m_d - m_u} \sim 36$$

[V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274]

[A. Nehme, Nucl. Phys. B 682, 289 (2004)]

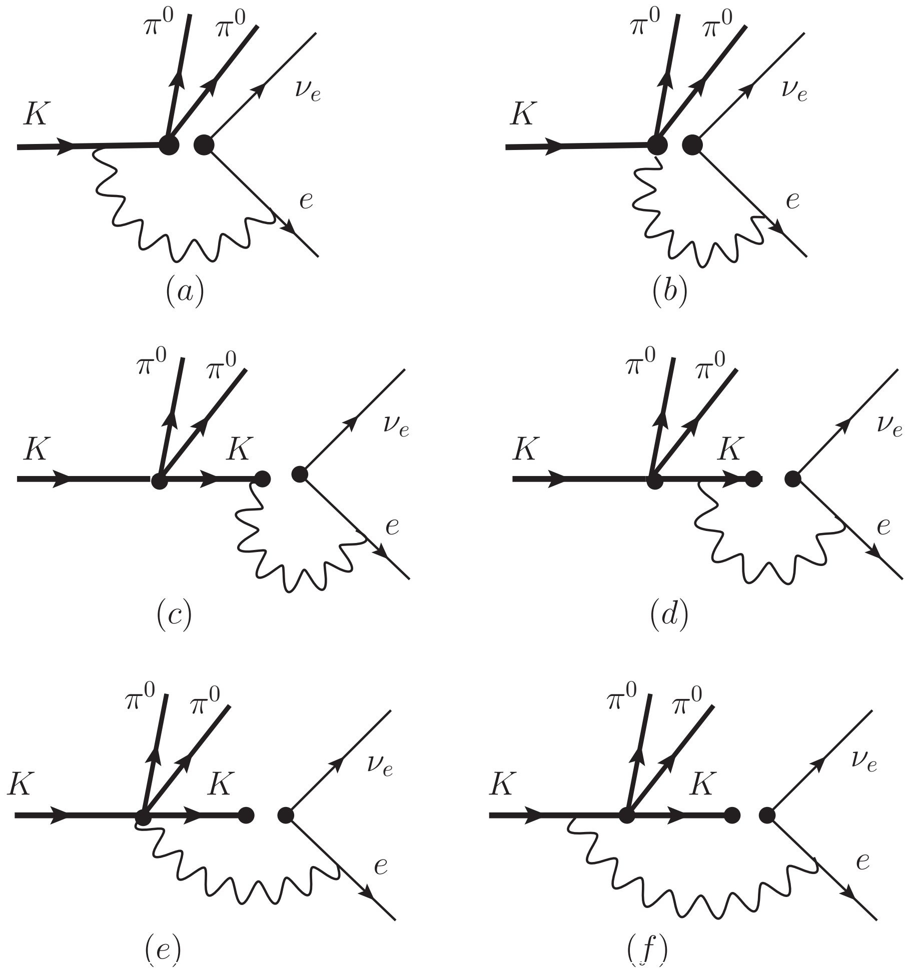
$\longrightarrow \delta_{EM}$ has to explain $\sim 1/3$ of the effect

Asymmetric treatment of the NA48/2 data as far as radiative corrections are concerned:

- $K_{e4}^{+-} \longrightarrow$ Sommerfeld-Gamow-Sakharov factors and PHOTOS for photon emission + w.f. factors of QED, treating the mesons as pointlike

- $K_{e4}^{00} \longrightarrow$ no radiative corrections applied (S-G-S factors not relevant)

Size of δ_{EM}? $\longrightarrow$ what does PHOTOS contain ?



Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for $m_e \to 0$]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\rm tot} = \Gamma(K_{e4}^{00}) + \bar{\Gamma}^{\rm soft}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM})$$

with $\delta_{EM} = 0.018 \longrightarrow \quad \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010 - 0.018 \sim \left(1 + \frac{3}{2R}\right)$

cusp $\longrightarrow |a_0^0 - a_0^2|$

with present statistics, the relative uncertainty varies from 40% to 80% depending on the parameterisation used

if statistical uncertainty is divided by 10, the relative error drops to 10% – 27% (DIRAC $\longrightarrow$ 4.3%)

Summary - Conclusion

• The high-precision data for $\delta_S(s) - \delta_P(s)$ obtained by the NA48/2 experiment require that isospin-breaking corrections be included

• Since the ultimate goal is to extract a_0^0 and a_0^2, the $\pi\pi$ scattering lengths in the isospin limit, the corrections should not be computed at fixed values of the scattering lengths, but should be parameterised in terms of them

• General properties (analyticity, unitarity, crossing, chiral counting) provide the necessary information to do this in a model independent way

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^2, a_0^2) + \delta f_{\text{IB}}(s; a_0^2, a_0^2)$$

with $\delta f_{\text{IB}}(s; a_0^2, a_0^2)$ worked out at NLO

• Fit to NA48/2 data have been redone. Results compatible with those published by NA48/2 within errors

• Radiative corrections provided for K_{e4}^{00} in the same framework as used for K_{e4}^{+-}. No apparent problem to explain remaining difference in form factors by $m_u - m_d$ effects

• A more quantitative statement would required a more involved treatment of radiative corrections (the quality of the data deserve it!), again taking into account the dependence on the scattering lengths