SU(3) flavor breaking in baryon octet light-cone distribution amplitudes

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DAs: Definition

Schematically:

$$|B\rangle = |qqq\rangle + |qqqg\rangle + |qqq\bar{q}q\rangle + \dots$$

- **Distribution Amplitudes** (DAs) describe the distribution of the lightcone-momentum within a specific Fock state
- in **hard exclusive processes** Fock states are increasingly power-suppressed with a rising number of partons

$\Rightarrow$ at high momentum transfer the 3-quark contribution plays the most important role

DAs: Definition

Three quark DAs are defined via baryon-to-vacuum matrix elements of 3-quark operators:

$$\langle 0 | q^a_\alpha(a_1 n) q^b_\beta(a_2 n) q^c_\gamma(a_3 n) | B(p,s) \rangle$$
$$= \int [dx] e^{-i n\cdot p \sum_i a_i x_i} \Big[V_1^B(x_i) (\not{p} C)_{\alpha\beta} (\gamma_5 B^+(p,s))_\gamma + \ldots \Big]$$

- color antisymmetrization and Wilson-lines are not written out explicitly
- a, b and c are flavor indices; α, β and γ are Dirac indices; n is a light-like vector
- on the r.h.s. one has 24 different structures and the same number of different DAs:

	twist-3	twist-4	twist-5	twist-6
scalar		S_1^B	S_2^B	
pseudoscalar		P_1^B	P_2^B	
vector	V_1^B	V_2^B, V_3^B	V_4^B, V_5^B	V_6^B
axialvector	A_1^B	A_2^B, A_3^B	A_4^B, A_5^B	A_6^B
tensor	T_1^B	T_2^B, T_3^B, T_7^B	T_4^B, T_5^B, T_8^B	T_6^B

$\rightarrow$ ME decomposition by BFMS: **Nucl.Phys. B589 (2000) 381**

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- on the l.h.s. one has to **choose** the **correct flavor content**
- the order of flavors is relevant for the symmetry properties of the DAs
- a convenient choice is:

$$p \mathrel{\hat{=}} uud\ , \qquad n \mathrel{\hat{=}} ddu\ , \qquad \Sigma^+ \mathrel{\hat{=}} uus\ , \qquad \Sigma^0 \mathrel{\hat{=}} uds\ ,$$
$$\Sigma^- \mathrel{\hat{=}} dds\ , \qquad \Xi^0 \mathrel{\hat{=}} ssu\ , \qquad \Xi^- \mathrel{\hat{=}} ssd\ , \qquad \Lambda \mathrel{\hat{=}} uds\ .$$

Three quark operators

- split the quark fields in left- and righthanded parts to make use of chiral symmetry

$$
\begin{aligned}
q^a_\alpha(a_1 n) q^b_\beta(a_2 n) q^c_\gamma(a_3 n) = {} & \mathcal{O}^{abc}_{RR,\alpha\beta\gamma}(a_1,a_2,a_3) + \mathcal{O}^{abc}_{LL,\alpha\beta\gamma}(a_1,a_2,a_3) \\
& + \mathcal{O}^{abc}_{RL,\alpha\beta\gamma}(a_1,a_2,a_3) + \mathcal{O}^{abc}_{LR,\alpha\beta\gamma}(a_1,a_2,a_3) \\
& + \mathcal{O}^{cab}_{RL,\gamma\alpha\beta}(a_3,a_1,a_2) + \mathcal{O}^{cab}_{LR,\gamma\alpha\beta}(a_3,a_1,a_2) \\
& + \mathcal{O}^{bca}_{RL,\beta\gamma\alpha}(a_2,a_3,a_1) + \mathcal{O}^{bca}_{LR,\beta\gamma\alpha}(a_2,a_3,a_1)
\end{aligned}
$$

- i.e. there are **only two** different **types of operators**: chiral-even ($\mathcal{O}_{RR}/\mathcal{O}_{LL}$) and chiral-odd ($\mathcal{O}_{RL}/\mathcal{O}_{LR}$)

$$
\mathcal{O}^{abc}_{XY,\alpha\beta\gamma}(a_1 n, a_2 n, a_3 n) = q^a_{X,\alpha}(a_1 n) q^b_{X,\beta}(a_2 n) q^c_{Y,\gamma}(a_3 n)
$$

Ansatz for the effective operator

$$\mathcal{O}^{abc}_{XY,\alpha\beta\gamma}(a_1 n, a_2 n, a_3 n) = \int [dx] \sum_{i,j} \sum_{k=1}^{k_j} \mathcal{F}^{i,j,k}_{XY}(x_1,x_2,x_3)\, \Gamma^{i,XXY}_{\alpha\beta\gamma\delta}\, B^{j,k,XXY}_{\delta,abc}(z)$$

- the functions $\mathcal{F}$ are distribution amplitudes

  $\rightarrow$ play the role of **LEC**s

- Γ: i labels the possible Dirac structures (6 for chiral-even and 6 for chiral-odd)

  $\rightarrow$ Lorentz indices are contracted with n or derivatives acting on B

- B: contains hadron fields; e.g. $B^{1,1,RRL}_{\delta,abc} = (u\, B_\delta)_{aa'}(u)_{bb'}(u^\dagger)_{cc'}\varepsilon_{a'b'c'}$

  $j = 2, 3$ $\rightarrow$ structures with quark mass matrix (contained in χ^+)
  $k = 2, 3, \ldots$ $\rightarrow$ different positions of baryon-octet field B_δ and χ^+

Ansatz for the effective operator

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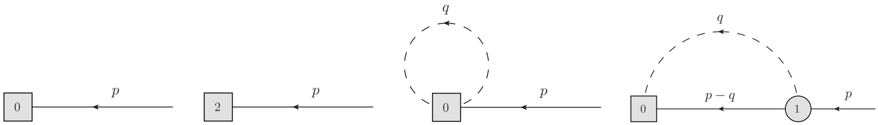
- $z = n \sum_i x_i a_i$ and $[dx] = dx_1\, dx_2\, dx_3\; \delta(1 - x_1 - x_2 - x_3)$
  $\rightarrow$ correct behaviour under translations in n direction
- **Parity:** relates left- and right-handed operators $\rightarrow \mathcal{F}_{RR} = -\mathcal{F}_{LL}$ and $\mathcal{F}_{LR} = -\mathcal{F}_{RL}$
- **CP** or **Time-reversal:** yield "reality condition" $\rightarrow \mathcal{F}^\dagger = \mathcal{F}$ up to an overall phase
- symmetry under **quark exchange**: relates different $\mathcal{F}$ to each other

$\rightarrow$ detailed description: **JHEP 1505 (2015) 073**

Leading one-loop calculation



(a) (b) (c) (d)

- loop integrals are (more or less) straightforward
- multiply with $\sqrt{Z}$ to take into account wave function renormalization
- we use IR regularization scheme

  $\rightarrow$ Becher, Leutwyler: **Eur. Phys. J. C9 (1999) 643**
- **main challenge:** handling the large number of different structures
- **last step:** obtain results for the 24 standard DAs $S^B_{1,2}$, $P^B_{1,2}$, V^B_{1-6}, A^B_{1-6}, T^B_{1-8} by matching to the general matrix element decomposition

Results: Overview

(a posteriori) we can define specific combinations of DAs with **remarkable features**:

1. **the Λ fits in** with the other octet baryons: same value for DAs at the flavor symmetric point (at the cost of a different definition for the Λ)
2. up to the fit parameters **extrapolation formulas** are **twist independent**
3. complete non-analytic structure contained in prefactors
4. constraints for $SU(3)_f$ symmetry breaking terms: e.g. $\Delta\Pi_3^\Sigma = -\frac{1}{2}\Delta\Phi^\Sigma_{+,3} - \frac{3}{2}\Delta\Phi^\Lambda_{+,3}$

Example: Leading twist DAs

$$\Phi^B_{\pm,3}(x_1,x_2,x_3) = \frac{c_B^\pm}{2}\Big(\big[V_1^B - A_1^B\big](x_1,x_2,x_3) \pm \big[V_1^B - A_1^B\big](x_3,x_2,x_1)\Big)$$

$$\Pi_3^B(x_1,x_2,x_3) = c_B^-(-1)_B\big[T_1^B\big](x_1,x_3,x_2)$$

$$c_B^+ = \begin{cases} 1 & \text{if } B \neq \Lambda \\ \sqrt{\frac{2}{3}} & \text{if } B = \Lambda \end{cases}, \quad c_B^- = \begin{cases} 1 & \text{if } B \neq \Lambda \\ -\sqrt{6} & \text{if } B = \Lambda \end{cases}, \quad (-1)_B \equiv \begin{cases} +1 & \text{for } B \neq \Lambda \\ -1 & \text{for } B = \Lambda \end{cases}$$

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Example: Extrapolation for chiral-odd DAs

here: $\bar{m} = \bar{m}^{\text{phys}}$, $\delta m \propto m_s - m_l \propto m_K^2 - m_\pi^2$

$$\Phi_{\pm,i}^{B} = \sqrt{\frac{Z_B}{Z^\star}}\left(1 + \Delta g_{\Phi\pm}^{B}\right)\left(\Phi_{\pm,i}^{\star} + \delta m \; \Delta\Phi_{\pm,i}^{B}\right)$$

$$\Pi_i^{B} = \sqrt{\frac{Z_B}{Z^\star}}\left(1 + \Delta g_{\Pi}^{B}\right) \times \begin{cases} \Phi_{+,i}^{\star} + \delta m \; \Delta\Pi_i^{B} & , \text{ if } B \neq \Lambda \\ \Phi_{-,i}^{\star} + \delta m \; \Delta\Pi_i^{B} & , \text{ if } B = \Lambda \end{cases}$$

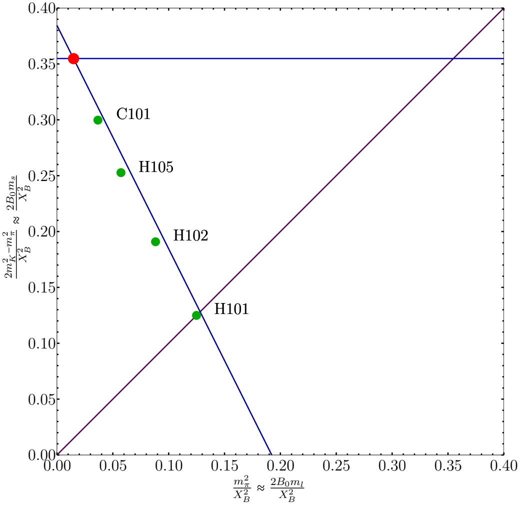
$\rightarrow$ all results: **JHEP 1505 (2015) 073**

Results: Lattice QCD

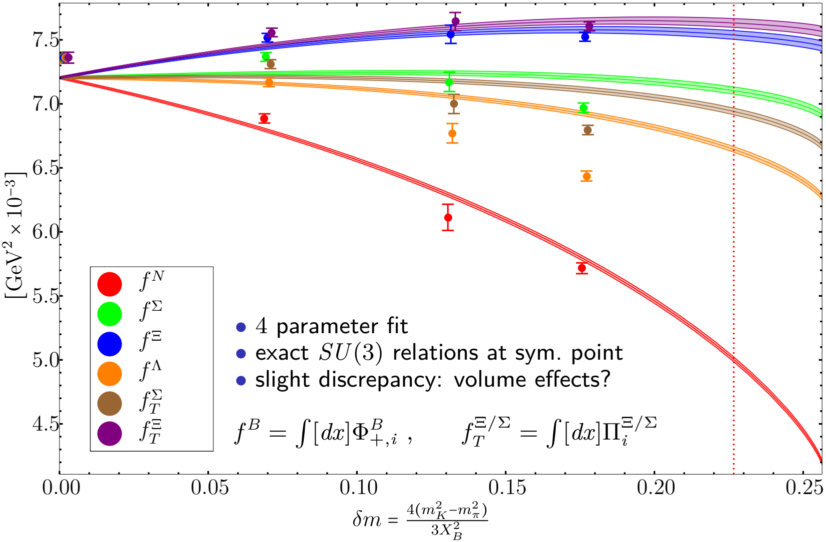
- Normalization and moments of DAs can be extracted from 2-point-correlation functions
- data points from **C**oordinated **L**attice **S**imulations effort:
  - 2+1 dynamical Wilson fermions
  - lattice spacing: $a \approx 0.086\,\mathrm{fm}$
  - volume: H: $(2.75\,\mathrm{fm})^3$, C: $(4.13\,\mathrm{fm})^3$

Disclaimer: preliminary!

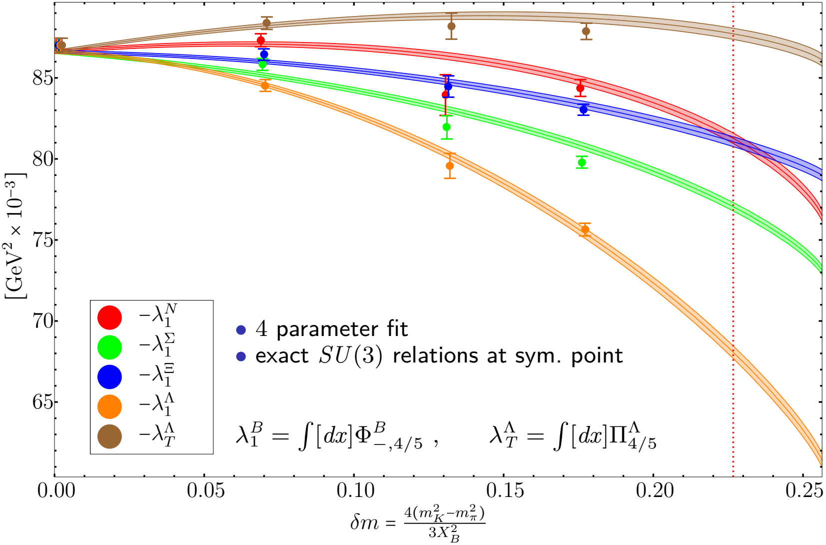
- data not renormalized yet
- purely statistical errors



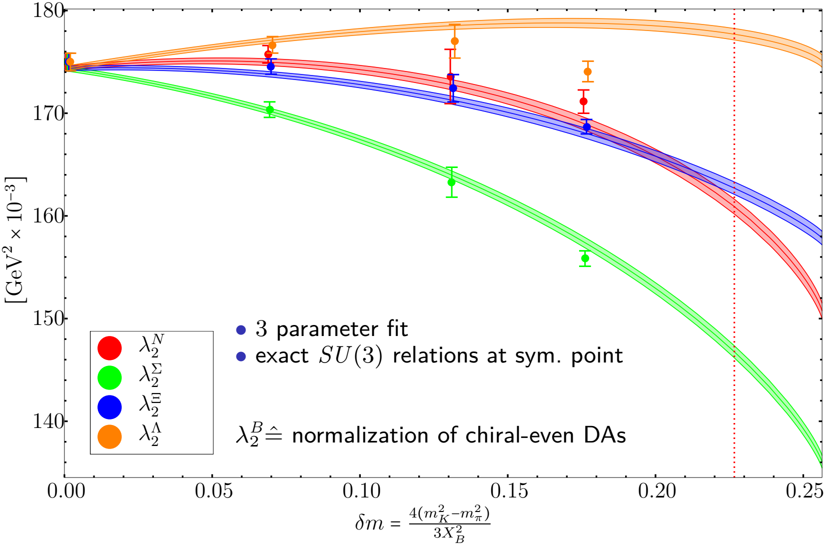
Leading twist normalization constants f^B and f_T^B (unrenormalized)



Higher twist normalization constants λ_1^B and λ_T^Λ (unrenormalized)



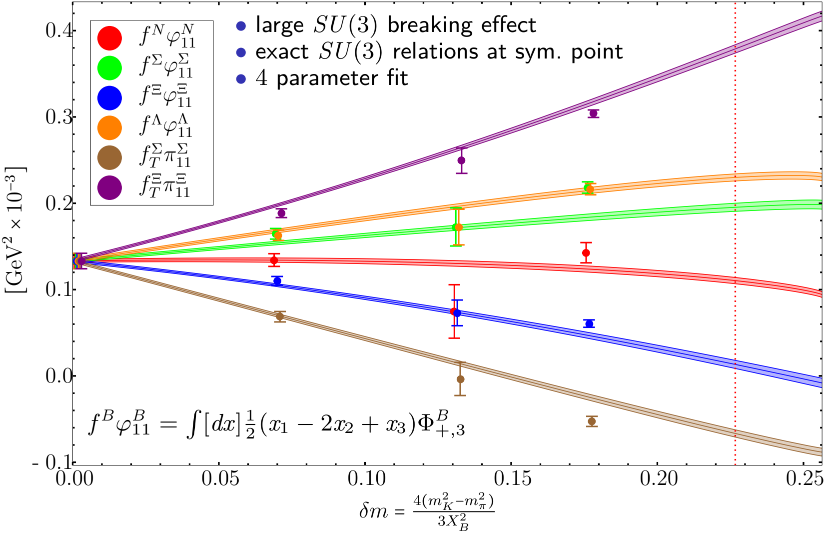
Higher twist normalization constants λ_2^B (unrenormalized)



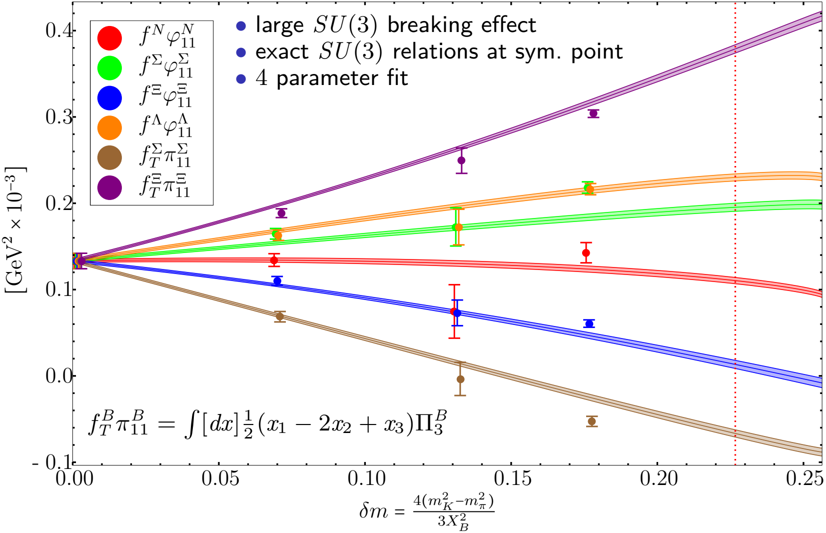
- 3 parameter fit
- exact $SU(3)$ relations at sym. point

$\lambda_2^B \hat{=}$ normalization of chiral-even DAs

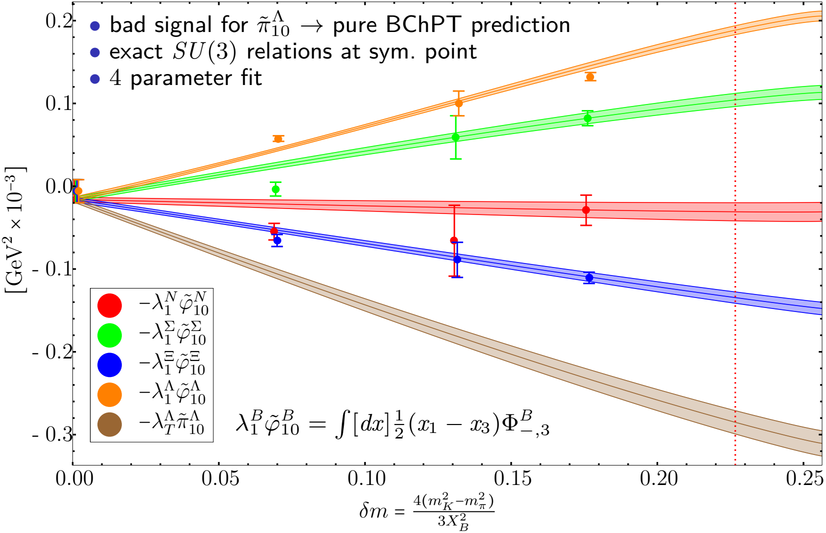
First moments of the leading twist DA φ_{11}^B and π_{11}^B (unrenormalized)



First moments of the leading twist DA φ_{11}^B and π_{11}^B (unrenormalized)

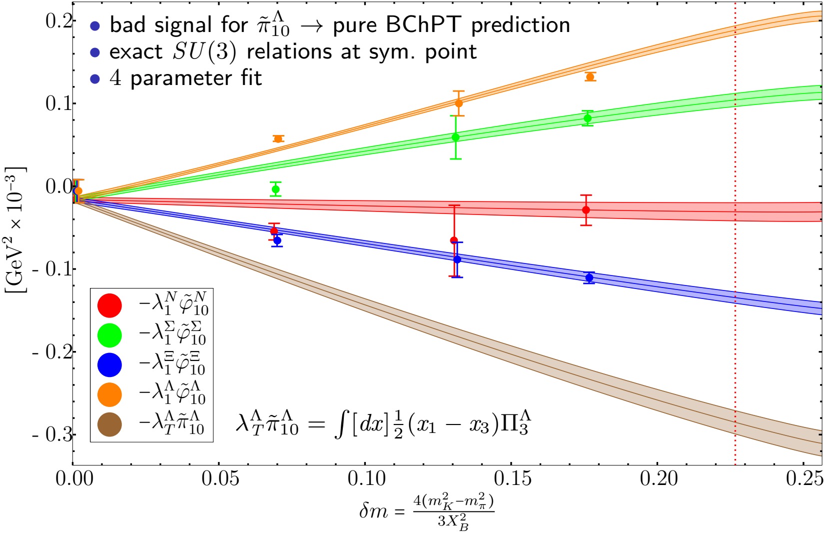


First moments of the leading twist DA $\tilde{\varphi}_{10}^{B}$ and $\tilde{\pi}_{10}^{\Lambda}$ (unrenormalized)



First moments of the leading twist DA $\tilde{\varphi}_{10}^{B}$ and $\tilde{\pi}_{10}^{\Lambda}$ (unrenormalized)

- bad signal for $\tilde{\pi}_{10}^{\Lambda}$ → pure BChPT prediction
- exact $SU(3)$ relations at sym. point
- 4 parameter fit



Summary

- we have performed a **NLO 3 flavor BChPT** calculation for baryon DAs
- $\rightarrow$ quark-mass dependence for all baryon octet DAs (including **higher twist**)
- we used non-local operators $\rightarrow$ **all moments** adressed simultaneously

investigation of preliminary lattice data:

- reasonable description of lattice QCD results at physical average quark mass (slight discrepancies due to volume effects?)
- normalization constants: $SU(3)$ breaking effects $\sim 15\%$
- deviations from the asymptotic shape exhibit large $SU(3)$ breaking effects