Low energy QCD parameters from $\eta \to 3\pi$ and beyond

Marián Kolesár*

(in collaboration with J.Novotný*)

update of arXiv:1409.3380

A) QCD parameters at low energies

B) Low energy EFT

C) The $\eta \to 3\pi$ decays

D) $\pi\pi$ scattering

E) Bayesian statistical analysis

F) Results

G) Conclusions

Chiral Dynamics 2015, Pisa, June 30, 2015

*Institute of particle and nuclear physics, Charles University, Prague

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

$\rightarrow$ pseudo-Goldstone bosons $\pi^{\pm}$,π^0 (N_f=2) or π,K,η (N_f=3)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q \rightarrow 0}, \ \ ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q \rightarrow 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F, \ \Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

Convenient reparameterization: ($\hat{m}$=$(m_u+m_d)/2$)

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\,\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\,\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0,1)$

Standard assumption: $Z(N_f) \sim 1, \ X(N_f) \sim 1$

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

$\to$ pseudo-Goldstone bosons $\pi^\pm, \pi^0$ ($N_f=2$) or π, K, η ($N_f=3$)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q \to 0}, \quad ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q \to 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F, \;\; \Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

Convenient reparameterization: $(\hat{m} = (m_u + m_d)/2)$

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0,1)$

Standard assumption: $Z(N_f) \sim 1, \; X(N_f) \sim 1$

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

$\to$ pseudo-Goldstone bosons $\pi^\pm$,π^0 (N_f=2) or π,K,η (N_f=3)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q\to 0}, \ \ ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q\to 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F, \ \ \Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

(Stern et al.2000,2002)

Convenient reparameterization: $(\hat{m}=(m_u+m_d)/2)$

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\,\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\,\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0,1)$

Standard assumption: $Z(N_f) \sim 1, \ X(N_f) \sim 1$

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

$\to$ pseudo-Goldstone bosons $\pi^{\pm}, \pi^0$ (N_f=2) or π, K, η (N_f=3)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q \to 0}, \quad ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q \to 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F, \ \Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

(Stern et al.2000,2002)

Convenient reparameterization: $(\hat{m} = (m_u + m_d)/2)$

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0, 1)$

Standard assumption: $Z(N_f) \sim 1, \ X(N_f) \sim 1$

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

$\to$ pseudo-Goldstone bosons $\pi^\pm,\pi^0$ ($N_f=2$) or π,K,η ($N_f=3$)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q\to 0}, \;\; ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q\to 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F$, $\Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

(Stern et al.2000,2002)

Convenient reparameterization: ($\hat{m}=(m_u+m_d)/2$)

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\,\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\,\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0,1)$

Standard assumption: $Z(N_f)\sim 1$, $X(N_f)\sim 1$

A) QCD parameters at low energies

Spontaneous chiral symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

$\to$ pseudo-Goldstone bosons $\pi^{\pm}, \pi^0$ (N_f=2) or π, K, η (N_f=3)

SBχS order parameters:

$F(N_f)$ - pseudoscalar decay constant in the chiral limit:

$$F(N_f) = F_P^a|_{m_q \to 0}, \quad ip_\mu F_P^a = \langle 0|A_\mu^a|P\rangle$$

$\Sigma(N_f)$ - quark condensate in the chiral limit:

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q \to 0}$$

Paramagnetic inequality: $F_0 \equiv F(3) < F(2) \equiv F, \; \Sigma_0 \equiv \Sigma(3) < \Sigma(2) \equiv \Sigma$

(Stern et al.2000,2002)

Convenient reparameterization: $(\hat{m} = (m_u + m_d)/2)$

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\,\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \quad Y(N_f) = \frac{X}{Z} = \frac{2\,\hat{m}B(N_f)}{M_\pi^2}$$

Allowed range: $X(N_f), Z(N_f) \in (0,1)$

Standard assumption: $Z(N_f) \sim 1, \; X(N_f) \sim 1$

Two flavour values:

		$Z(2)$	$X(2)$
Descotes-Genon et al. 2001	phenomenology	0.89±0.03	0.81±0.07
Bernard et al. 2012	lattice+ReχPT	0.86±0.01	0.89±0.01
FLAG 2013 N_f=2	lattice	0.87±0.01	0.86±0.09
FLAG 2013 N_f=2+1	lattice	0.886±0.004	0.84±0.14

Three flavour values:

phenomenology		$Z(3)$	$X(3)$
Bijnens, Ecker 2014	NNLO χPT (main fit)	0.59	0.63
Bijnens, Ecker 2014	NNLO χPT (free fit)	0.51	0.48
Amoros et al. 2001	NNLO χPT ("fit 10")	0.89	0.66
Descotes-Genon 2007	ReχPT $\pi\pi+\pi K$	>0.2	<0.8
lattice			
Bernard et al. 2012	RBC/UKQCD+ReχPT	0.54±0.06	0.38±0.05
Ecker et al. 2013	RBC/UKQCD+large N_c	0.91±0.08	
MILC 2009	MILC 09A	0.72±0.06	0.62±0.07

A) QCD parameters at low energies

Mass parameters (3 flavour):

$\widehat{m}$ - light quark mass average

r - strange to light quark mass ratio

R - isospin violation (light quark mass difference)

$$\widehat{m} = \frac{m_u + m_d}{2}, \qquad r = \frac{m_s}{\widehat{m}}, \qquad R = \frac{(m_s - \widehat{m})}{(m_d - m_u)}$$

Isospin breaking parameter R:

phenomenology		R
Dashen's theorem	LO	44
Dashen's theorem	NNLO	37
Bijnens et al. 2007	$\eta \to 3\pi$ NNLO χPT	41.3
Kampf et al. 2012	$\eta \to 3\pi$ dispersive	37.8±3.3
lattice		
FLAG 2013 N_f=2	lattice average	40.7±4.3
FLAG 2013 N_f=2+1	lattice average	35.8±2.6

B) Low energy EFT

Chiral perturbation theory (N_f=3) *(Gasser,Leutwyler 1985)*

Generating functional:

$$e^{iZ_{eff}[\pi,v,a,s,p]} = \int \mathcal{D}\pi \; e^{i\int \mathrm{d}^4x \; \mathcal{L}_{eff}[\pi,v,a,s,p]}$$

SBχS: $SU(3)_L \times SU(3)_R \to SU(3)_V$ expansion in momenta and quark masses

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \exp\frac{i}{F_0}\pi^a(x)\lambda^a, \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian: $\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B_0\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(L_1 \ldots L_{10}) + \mathcal{L}^{(4)}_{WZ}$$

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(C_1 \ldots C_{90}) + \mathcal{L}^{(6)}_{WZ}(C_1^W \ldots C_{23}^W)$$

B) Low energy EFT

Chiral perturbation theory (N_f=3) *(Gasser,Leutwyler 1985)*

Generating functional:

$$e^{iZ_{eff}[\pi,v,a,s,p]} = \int \mathcal{D}\pi \; e^{i\int d^4x \; \mathcal{L}_{eff}[\pi,v,a,s,p]}$$

SBχS: $SU(3)_L \times SU(3)_R \to SU(3)_V$ expansion in momenta and quark masses

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \exp \frac{i}{F_0} \pi^a(x)\lambda^a, \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian: $\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \dots$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B_0\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(L_1 \dots L_{10}) + \mathcal{L}^{(4)}_{WZ}$$

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(C_1 \dots C_{90}) + \mathcal{L}^{(6)}_{WZ}(C_1^W \dots C_{23}^W)$$

B) Low energy EFT

Chiral perturbation theory (N_f=3) *(Gasser,Leutwyler 1985)*

Generating functional:

$$e^{iZ_{eff}[\pi,v,a,s,p]} = \int \mathcal{D}\pi \; e^{i\int \mathrm{d}^4x \; \mathcal{L}_{eff}[\pi,v,a,s,p]}$$

SBχS: $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ expansion in momenta and quark masses

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \exp\frac{i}{F_0}\pi^a(x)\lambda^a, \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian: $\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B_0\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(L_1 \ldots L_{10}) + \mathcal{L}^{(4)}_{WZ}$$

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(C_1 \ldots C_{90}) + \mathcal{L}^{(6)}_{WZ}(C_1^W \ldots C_{23}^W)$$

B) Low energy EFT

Chiral perturbation theory (N_f=3) *(Gasser,Leutwyler 1985)*

Generating functional:

$$e^{iZ_{eff}[\pi,v,a,s,p]} = \int \mathcal{D}\pi \; e^{i\int \mathrm{d}^4x \; \mathcal{L}_{eff}[\pi,v,a,s,p]}$$

SBχS: $SU(3)_L \times SU(3)_R \to SU(3)_V$ expansion in momenta and quark masses

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \exp\frac{i}{F_0}\pi^a(x)\lambda^a, \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian: $\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B_0\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(L_1 \ldots L_{10}) + \mathcal{L}^{(4)}_{WZ}$$

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(C_1 \ldots C_{90}) + \mathcal{L}^{(6)}_{WZ}(C_1^W \ldots C_{23}^W)$$

B) Low energy EFT

Chiral perturbation theory (N_f=3) *(Gasser,Leutwyler 1985)*

Generating functional:

$$e^{iZ_{eff}[\pi,v,a,s,p]} = \int \mathcal{D}\pi \; e^{i\int \mathrm{d}^4x \; \mathcal{L}_{eff}[\pi,v,a,s,p]}$$

SBχS: $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ expansion in momenta and quark masses

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \exp\frac{i}{F_0}\pi^a(x)\lambda^a, \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian: $\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B_0\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(L_1 \ldots L_{10}) + \mathcal{L}^{(4)}_{WZ}$$

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(C_1 \ldots C_{90}) + \mathcal{L}^{(6)}_{WZ}(C_1^W \ldots C_{23}^W)$$

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting *(Gasser,Leutwyler 1984,1985)*
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting *(Gasser,Leutwyler 1984,1985)*
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting *(Gasser,Leutwyler 1984,1985)*
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion
(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting *(Gasser,Leutwyler 1984,1985)*
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Low energy EFT

Resummed χPT - a special treatment of the chiral expansion

(Descotes-Genon, Fuchs, Girlanda, Stern 2004)

Motivation and aim

- possibly slow or irregular convergence of 3 flavour chiral series
- traditional approach to χPT implicitly assumes good convergence, hides uncertainties

Summary of the method

- Standard χPT Lagrangian and power counting *(Gasser,Leutwyler 1984,1985)*
- only expansions derived directly from the generating functional trusted
- manipulations done in non-perturbative algebraic way
- explicitly to NLO, higher orders collected in remainders
- remainders not neglected, estimated and treated as sources of error

B) Treatment of low energy constants

$\mathcal{O}(p^2)$: F_0 - pseudoscalar decay constant in the chiral limit **left free**

Σ_0 - quark condensate in the chiral limit ($\Sigma_0 = B_0 F_0^2$)

r - strange to light quark ratio

R - isospin violation (light quark mass difference)

$$Z = \frac{F_0^2}{F_\pi^2}, \quad X = \frac{2\,\hat{m}\Sigma_0}{F_\pi^2 M_\pi^2}, \quad r = \frac{m_s}{\hat{m}}, \quad R = \frac{(m_s - \hat{m})}{(m_d - m_u)}$$

where $\hat{m} = (m_u + m_d)/2$

$\mathcal{O}(p^4)$: L_4-L_8 - in terms of F_P^2, M_P^2 reparametrized

- algebraically, indirect remainders generated

$\mathcal{O}(p^6)$ **and higher:** C_i's etc. implicit in remainders

B) Treatment of low energy constants

$\mathcal{O}(p^2)$: F_0 - pseudoscalar decay constant in the chiral limit **left free**

Σ_0 - quark condensate in the chiral limit ($\Sigma_0 = B_0 F_0^2$)

r - strange to light quark ratio

R - isospin violation (light quark mass difference)

$$Z = \frac{F_0^2}{F_\pi^2}, \quad X = \frac{2\,\hat{m}\Sigma_0}{F_\pi^2 M_\pi^2}, \quad r = \frac{m_s}{\hat{m}}, \quad R = \frac{(m_s - \hat{m})}{(m_d - m_u)}$$

where $\hat{m} = (m_u + m_d)/2$

$\mathcal{O}(p^4)$: L_4-L_8 - in terms of F_P^2, M_P^2 **reparametrized**

- algebraically, indirect remainders generated

$\mathcal{O}(p^6)$ and higher: C_i's etc. implicit in remainders

B) Treatment of low energy constants

$\mathcal{O}(p^2)$: F_0 - pseudoscalar decay constant in the chiral limit **left free**

Σ_0 - quark condensate in the chiral limit ($\Sigma_0 = B_0 F_0^2$)

r - strange to light quark ratio

R - isospin violation (light quark mass difference)

$$Z = \frac{F_0^2}{F_\pi^2}, \quad X = \frac{2\,\hat{m}\Sigma_0}{F_\pi^2 M_\pi^2}, \quad r = \frac{m_s}{\hat{m}}, \quad R = \frac{(m_s - \hat{m})}{(m_d - m_u)}$$

where $\hat{m} = (m_u + m_d)/2$

$\mathcal{O}(p^4)$: L_4-L_8 - in terms of F_P^2, M_P^2 **reparametrized**

- algebraically, indirect remainders generated

$\mathcal{O}(p^6)$ **and higher:** C_i's etc. **implicit in remainders**

C) The $\eta \to 3\pi$ decays

Decay widths: $\Gamma^{+}_{\text{exp}} = 300 \pm 12$ eV, $\Gamma^{0}_{\text{exp}} = 428 \pm 17$ eV *(PDG 2014)*

Dalitz plot parameters: $a_{\text{exp}} = -1.09 \pm 0.02$ *(KLOE 2008)*

The charged decay amplitude in terms of 4-point Green functions:

$$F_\pi^3 F_\eta A(s,t,u) = G^{(4)}_{+-83} - \varepsilon_\pi G^{(4)}_{+-33} + \varepsilon_\eta G^{(4)}_{+-88} + \Delta^{(6)}_{G_D}$$

- to first order in isospin breaking, EM effects neglected
- physical mixing angles to all chiral orders and first in $1/R$

Direct remainder expansion around the Dalitz plot center

$$\Delta_{G_D} = \Delta_A + \Delta_B(s-s_0) + \Delta_C(s-s_0)^2 + \Delta_D[(t-s_0)^2 + (u-s_0)^2]$$

19 parameters:

- LO: X, Z, r, R
- NLO: L_1, L_2, L_3
- direct rem.: Δ_A, Δ_B, Δ_C, Δ_D
- indirect rem.: Δ_{M_π}, Δ_{F_π}, Δ_{M_K}, Δ_{F_K}, Δ_{M_η}, Δ_{F_η}, $\Delta_{M_{38}}$, $\Delta_{Z_{38}}$

C) The $\eta \to 3\pi$ decays

Decay widths: $\Gamma^{+}_{\text{exp}} = 300 \pm 12$ eV, $\Gamma^{0}_{\text{exp}} = 428 \pm 17$ eV *(PDG 2014)*

Dalitz plot parameters: $a_{\text{exp}} = -1.09 \pm 0.02$ *(KLOE 2008)*

The charged decay amplitude in terms of 4-point Green functions:

$$F_\pi^3 F_\eta \, A(s,t,u) = G^{(4)}_{+-83} - \varepsilon_\pi G^{(4)}_{+-33} + \varepsilon_\eta G^{(4)}_{+-88} + \Delta^{(6)}_{G_D}$$

- to first order in isospin breaking, EM effects neglected *(Ditsche et al. 2009)*
- physical mixing angles to all chiral orders and first in $1/R$

Direct remainder expansion around the Dalitz plot center

$$\Delta_{G_D} = \Delta_A + \Delta_B(s-s_0) + \Delta_C(s-s_0)^2 + \Delta_D[(t-s_0)^2 + (u-s_0)^2]$$

19 parameters:

- LO: X, Z, r, R
- NLO: L_1, L_2, L_3
- direct rem.: Δ_A, Δ_B, Δ_C, Δ_D
- indirect rem.: Δ_{M_π}, Δ_{F_π}, Δ_{M_K}, Δ_{F_K}, Δ_{M_η}, Δ_{F_η}, $\Delta_{M_{38}}$, $\Delta_{Z_{38}}$

C) The $\eta \to 3\pi$ decays

Decay widths: $\Gamma^{+}_{\text{exp}} = 300 \pm 12$ eV, $\Gamma^{0}_{\text{exp}} = 428 \pm 17$ eV *(PDG 2014)*

Dalitz plot parameters: $a_{\text{exp}} = -1.09 \pm 0.02$ *(KLOE 2008)*

The charged decay amplitude in terms of 4-point Green functions:

$$F_\pi^3 F_\eta \, A(s,t,u) = G^{(4)}_{+-83} - \varepsilon_\pi G^{(4)}_{+-33} + \varepsilon_\eta G^{(4)}_{+-88} + \Delta^{(6)}_{G_D}$$

- to first order in isospin breaking, EM effects neglected *(Ditsche et al. 2009)*
- physical mixing angles to all chiral orders and first in $1/R$

Direct remainder expansion around the Dalitz plot center

$$\Delta_{G_D} = \Delta_A + \Delta_B(s-s_0) + \Delta_C(s-s_0)^2 + \Delta_D[(t-s_0)^2 + (u-s_0)^2]$$

19 parameters:

- LO: X, Z, r, R
- NLO: L_1, L_2, L_3
- direct rem.: Δ_A, Δ_B, Δ_C, Δ_D
- indirect rem.: Δ_{M_π}, Δ_{F_π}, Δ_{M_K}, Δ_{F_K}, Δ_{M_η}, Δ_{F_η}, $\Delta_{M_{38}}$, $\Delta_{Z_{38}}$

C) The $\eta \to 3\pi$ decays

Decay widths: $\Gamma^{+}_{\text{exp}} = 300 \pm 12$ eV, $\Gamma^{0}_{\text{exp}} = 428 \pm 17$ eV *(PDG 2014)*

Dalitz plot parameters: $a_{\text{exp}} = -1.09 \pm 0.02$ *(KLOE 2008)*

The charged decay amplitude in terms of 4-point Green functions:

$$F_\pi^3 F_\eta\, A(s,t,u) = G^{(4)}_{+-83} - \varepsilon_\pi G^{(4)}_{+-33} + \varepsilon_\eta G^{(4)}_{+-88} + \Delta^{(6)}_{G_D}$$

- to first order in isospin breaking, EM effects neglected *(Ditsche et al. 2009)*
- physical mixing angles to all chiral orders and first in $1/R$

Direct remainder expansion around the Dalitz plot center

$$\Delta_{G_D} = \Delta_A + \Delta_B(s-s_0) + \Delta_C(s-s_0)^2 + \Delta_D[(t-s_0)^2 + (u-s_0)^2]$$

19 parameters:

- LO: X, Z, r, R
- NLO: L_1, L_2, L_3
- direct rem.: Δ_A, Δ_B, Δ_C, Δ_D
- indirect rem.: Δ_{M_π}, Δ_{F_π}, Δ_{M_K}, Δ_{F_K}, Δ_{M_η}, Δ_{F_η}, $\Delta_{M_{38}}$, $\Delta_{Z_{38}}$

D) $\pi\pi$ scattering (preliminary)

Subthreshold parameters:

$$A_{\pi\pi}(s,t,u) = \frac{\alpha_{\pi\pi}}{F_\pi^2}\frac{M_\pi^2}{3} + \frac{\beta_{\pi\pi}}{F_\pi^2}\left(s - \frac{4M_\pi^2}{3}\right) +$$

$$+\frac{\lambda_1}{F_\pi^4}\left(s-2M_\pi^2\right)^2 + \frac{\lambda_2}{F_\pi^4}\left[(t-2M_\pi^2)^2+(u-2M_\pi^2)^2\right] +$$

$$+\frac{\lambda_3}{F_\pi^6}\left(s-2M_\pi^2\right)^3 + \frac{\lambda_4}{F_\pi^6}\left[(t-2M_\pi^2)^3+(u-2M_\pi^2)^3\right] +$$

$$+U^{(2+4+6)}(s|t,u) + \mathcal{O}(p^8)$$

Values: $\alpha_{\pi\pi}^{\text{exp}} = 1.381 \pm 0.242$, $\beta_{\pi\pi}^{\text{exp}} = 1.081 \pm 0.023$ *(Stern et al.2002)*

2 additional parameters: $\Delta_{\alpha_{\pi\pi}}$, $\Delta_{\beta_{\pi\pi}}$

E) Statistical analysis

Bayes' theorem

(Stern et al. 2004)

$$P(X_i|\text{data}) = \frac{P(\text{data}|X_i)P(X_i)}{\int \mathrm{d}X_i\, P(\text{data}|X_i)P(X_i)}$$

$P(X_i|\text{data})$ - probability density of X_i being true given data

$P(\text{data}|X_i) = \prod_k \frac{1}{\sigma_k\sqrt{2\pi}} \exp\left[-\frac{(\mathrm{Q}_k^{\text{exp}} - \mathrm{Q}_k(X_i))^2}{\sigma_k}\right]$ - experimental distribution

$P(X_i)$ - probability distribution of X_i (prior)

- theoretical assumptions explicit and under control
- various assumptions testable

num.integration $\rightarrow$ **Monte Carlo sampling**

- 10000 samples per grid element, $\sim 5 \cdot 10^6$ total samples
- stability tested with smaller samples (1000 per grid element)
- larger samples and in depth statistical stability test in progress

E) Statistical analysis

Bayes' theorem

(Stern et al. 2004)

$$P(X_i|\mathrm{data}) = \frac{P(\mathrm{data}|X_i)P(X_i)}{\int \mathrm{d}X_i\, P(\mathrm{data}|X_i)P(X_i)}$$

$P(X_i|\mathrm{data})$ - probability density of X_i being true given data

$P(\mathrm{data}|X_i) = \prod_k \frac{1}{\sigma_k\sqrt{2\pi}} \exp\left[-\frac{(\mathrm{Q}_k^{\mathrm{exp}} - \mathrm{Q}_k(X_i))^2}{\sigma_k}\right]$ - experimental distribution

$P(X_i)$ - probability distribution of X_i (prior)

- theoretical assumptions explicit and under control
- various assumptions testable

num.integration $\rightarrow$ **Monte Carlo sampling**

- 10000 samples per grid element, $\sim 5 \cdot 10^6$ total samples
- stability tested with smaller samples (1000 per grid element)
- larger samples and in depth statistical stability test in progress

E) Assumptions

$r = 27.5 \pm 0.4$: from lattice *(FLAG 2013)*

L_{1-3}: mean and spread of a set of standard χPT fits:

$$L_1^r(M_\rho) = (0.57 \pm 0.18) \cdot 10^{-3}$$
$$L_2^r(M_\rho) = (0.82 \pm 0.28) \cdot 10^{-3}$$
$$L_3^r(M_\rho) = (-2.95 \pm 0.38) \cdot 10^{-3}$$

weak dependence of the amplitude on L_{1-3}

Δ_k: based on general arguments about the convergence of chiral series

$$\Delta_G^{(4)} \approx \pm 0.3G, \quad \Delta_G^{(6)} \approx \pm 0.1G, \quad \Delta_G^{(\pi\pi)} \approx \pm 0.03G$$

implementation - normal distribution μ=0, σ=0.1G or σ=0.3G

X and Z: $0 < X < X(2) = 0.89 \pm 0.01$, $\quad 0 < Z < Z(2) = 0.86 \pm 0.01$

R: free $\in (0, 60)$ or $R = 37.8 \pm 3.3$

E) Assumptions

$r = 27.5 \pm 0.4$: from lattice (FLAG 2013)

L_{1-3}: mean and spread of a set of standard χPT fits:

$$L_1^r(M_\rho) = (0.57 \pm 0.18) \cdot 10^{-3}$$
$$L_2^r(M_\rho) = (0.82 \pm 0.28) \cdot 10^{-3}$$
$$L_3^r(M_\rho) = (-2.95 \pm 0.38) \cdot 10^{-3}$$

(M.K. 2014)

weak dependence of the amplitude on L_{1-3}

Δ_k: based on general arguments about the convergence of chiral series

$$\Delta_G^{(4)} \approx \pm 0.3G, \quad \Delta_G^{(6)} \approx \pm 0.1G, \quad \Delta_G^{(\pi\pi)} \approx \pm 0.03G$$

implementation - normal distribution μ=0, σ=0.1G or σ=0.3G

X and Z: $0 < X < X(2) = 0.89 \pm 0.01$, $0 < Z < Z(2) = 0.86 \pm 0.01$

R: free $\in (0, 60)$ or $R = 37.8 \pm 3.3$

E) Assumptions

$r = 27.5 \pm 0.4$: from lattice *(FLAG 2013)*

L_{1-3}: mean and spread of a set of standard χPT fits:

$$L_1^r(M_\rho) = (0.57 \pm 0.18) \cdot 10^{-3}$$
$$L_2^r(M_\rho) = (0.82 \pm 0.28) \cdot 10^{-3}$$
$$L_3^r(M_\rho) = (-2.95 \pm 0.38) \cdot 10^{-3}$$

(M.K. 2014)

weak dependence of the amplitude on L_{1-3}

Δ_k: based on general arguments about the convergence of chiral series

(Stern et al. 2004)

$$\Delta_G^{(4)} \approx \pm 0.3G, \quad \Delta_G^{(6)} \approx \pm 0.1G, \quad \Delta_G^{(\pi\pi)} \approx \pm 0.03G$$

implementation - normal distribution μ=0, σ=0.1G or σ=0.3G

X and Z: $0 < X < X(2) = 0.89 \pm 0.01, \quad 0 < Z < Z(2) = 0.86 \pm 0.01$

R: free $\in (0, 60)$ or $R = 37.8 \pm 3.3$

E) Assumptions

$\boldsymbol{r = 27.5 \pm 0.4}$**:** from lattice (FLAG 2013)

$\boldsymbol{L_{1-3}}$**:** mean and spread of a set of standard χPT fits:

$$L_1^r(M_\rho) = (0.57 \pm 0.18) \cdot 10^{-3}$$
$$L_2^r(M_\rho) = (0.82 \pm 0.28) \cdot 10^{-3}$$
$$L_3^r(M_\rho) = (-2.95 \pm 0.38) \cdot 10^{-3}$$

(M.K. 2014)

weak dependence of the amplitude on L_{1-3}

$\boldsymbol{\Delta_k}$**:** based on general arguments about the convergence of chiral series

(Stern et al. 2004)

$$\Delta_G^{(4)} \approx \pm 0.3G, \quad \Delta_G^{(6)} \approx \pm 0.1G, \quad \Delta_G^{(\pi\pi)} \approx \pm 0.03G$$

implementation - normal distribution μ=0, σ=0.1G or σ=0.3G

X and Z: $0 < X < X(2) = 0.89 \pm 0.01, \quad 0 < Z < Z(2) = 0.86 \pm 0.01$

(Bernard et al. 2012)

R: free $\in (0, 60)$ or $R = 37.8 \pm 3.3$

E) Assumptions

$r = 27.5 \pm 0.4$: from lattice *(FLAG 2013)*

L_{1-3}: mean and spread of a set of standard χPT fits:

$$L_1^r(M_\rho) = (0.57 \pm 0.18) \cdot 10^{-3}$$
$$L_2^r(M_\rho) = (0.82 \pm 0.28) \cdot 10^{-3}$$
$$L_3^r(M_\rho) = (-2.95 \pm 0.38) \cdot 10^{-3}$$

(M.K. 2014)

weak dependence of the amplitude on L_{1-3}

Δ_k: based on general arguments about the convergence of chiral series *(Stern et al. 2004)*

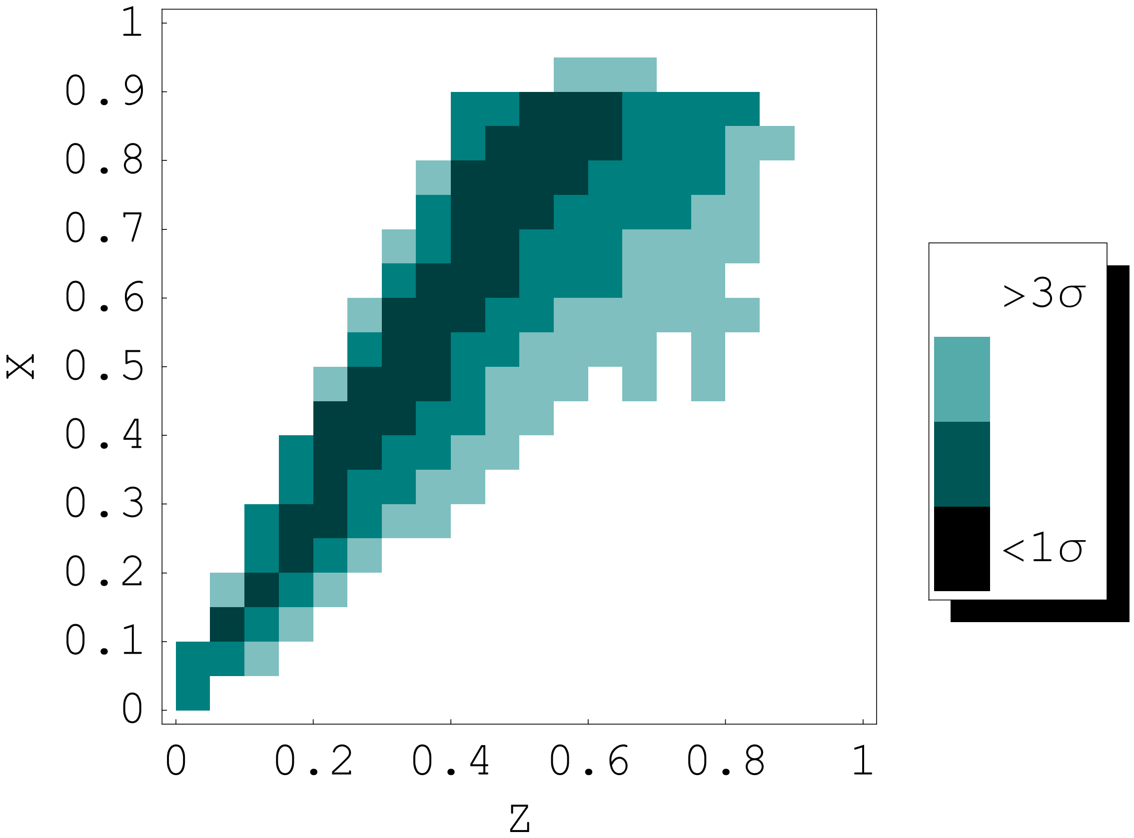
$$\Delta_G^{(4)} \approx \pm 0.3G, \quad \Delta_G^{(6)} \approx \pm 0.1G, \quad \Delta_G^{(\pi\pi)} \approx \pm 0.03G$$

implementation - normal distribution $\mu = 0$, $\sigma = 0.1$G or $\sigma = 0.3$G

X and Z: $0 < X < X(2) = 0.89 \pm 0.01$, $0 < Z < Z(2) = 0.86 \pm 0.01$ *(Bernard et al. 2012)*

R: free $\in (0, 60)$ or $R = 37.8 \pm 3.3$ *(Kampf et al. 2011)*

Constraints on X and Z: $R = 37.8 \pm 3.3$



$\eta \to 3\pi$:

$X = 0.57 \pm 0.21$

$Z = 0.40 \pm 0.17$

$Y = X/Z = 1.45 \pm 0.25$

$\eta \to 3\pi$ and $\pi\pi$ scattering:

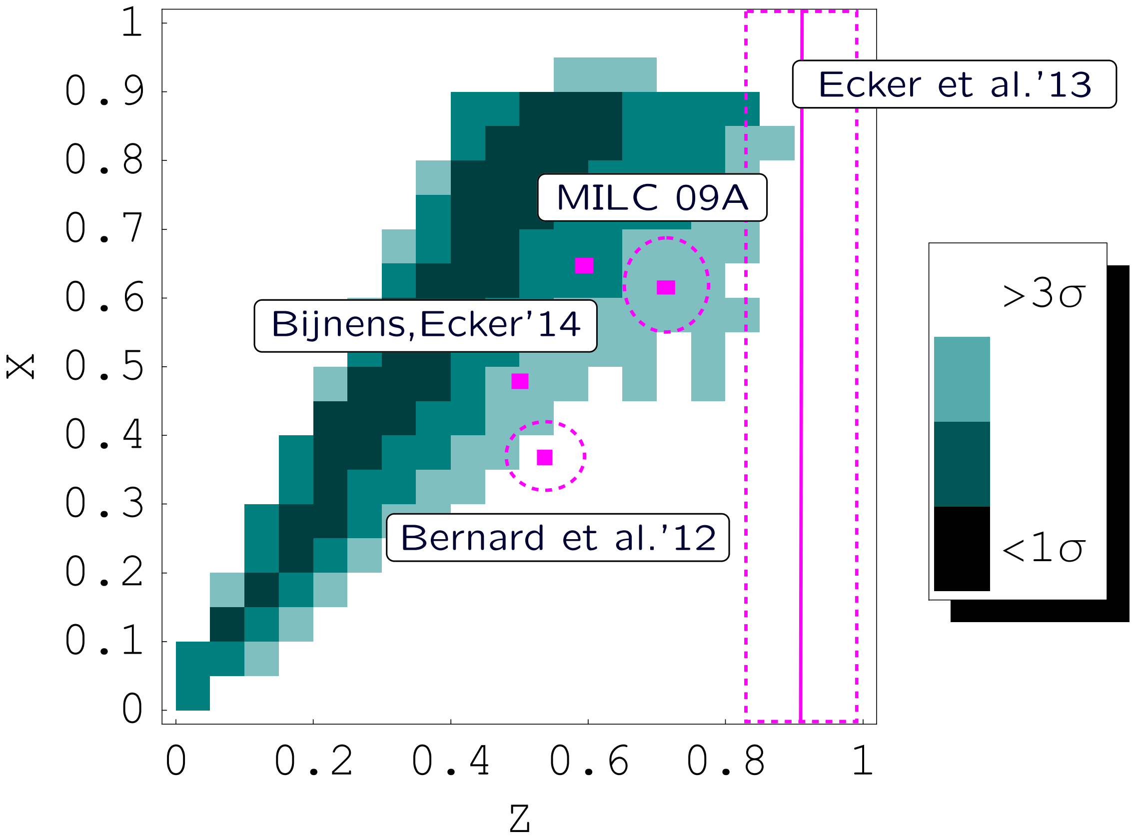
(preliminary)

$X = 0.59 \pm 0.21$

$Z = 0.46 \pm 0.17$

$Y = X/Z = 1.30 \pm 0.25$

Constraints on X and Z: $R = 37.8 \pm 3.3$



$\eta \to 3\pi$:

$X = 0.57 \pm 0.21$

$Z = 0.40 \pm 0.17$

$Y = X/Z = 1.45 \pm 0.25$

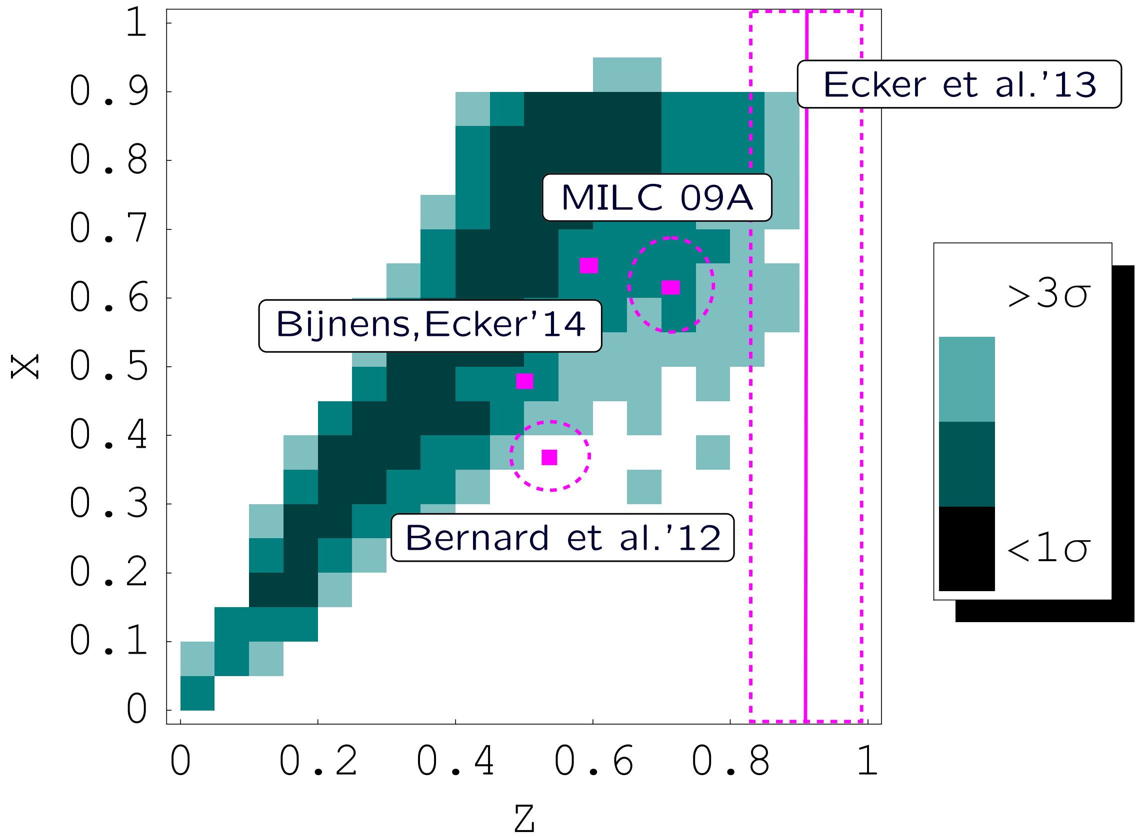
$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$X = 0.59 \pm 0.21$

$Z = 0.46 \pm 0.17$

$Y = X/Z = 1.30 \pm 0.25$

Constraints on X and Z: $R = 37.8 \pm 3.3$



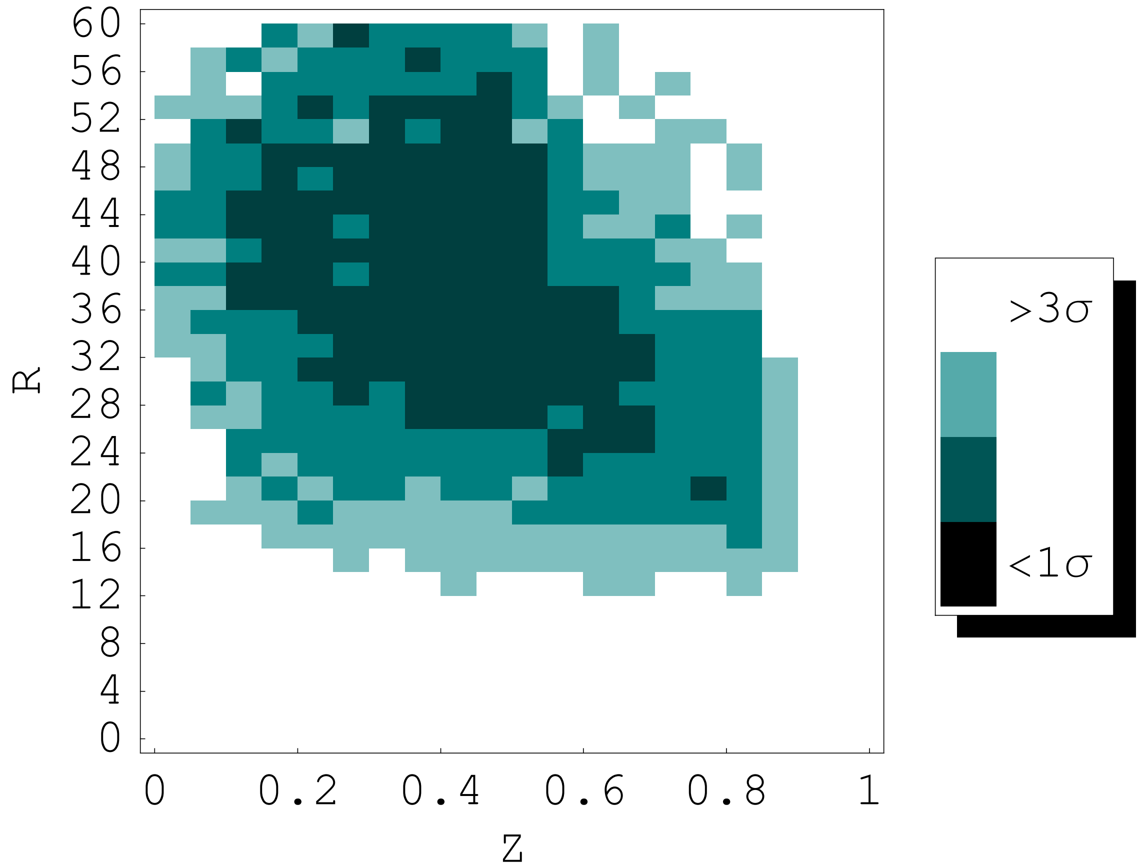
$\eta \to 3\pi$:

$X = 0.57 \pm 0.21$
$Z = 0.40 \pm 0.17$
$Y = X/Z = 1.45 \pm 0.25$

$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$X = 0.59 \pm 0.21$
$Z = 0.46 \pm 0.17$
$Y = X/Z = 1.30 \pm 0.25$

Constraints on R and Z: R free



$\eta \to 3\pi$:

$R = 38 \pm 10$

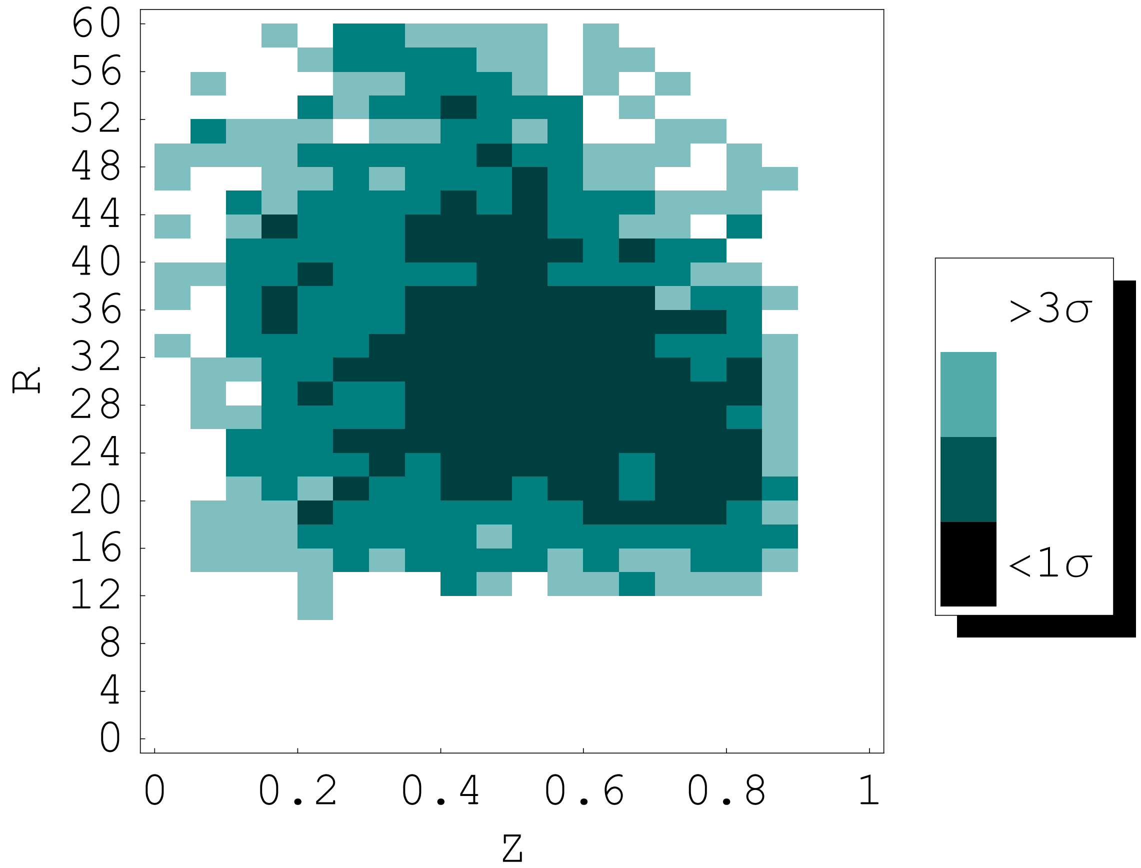
$Z = 0.42 \pm 0.18$

$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$R = 32 \pm 10$

$Z = 0.50 \pm 0.18$

Constraints on R and Z: R free



$\eta \to 3\pi$:

$R = 38 \pm 10$

$Z = 0.42 \pm 0.18$

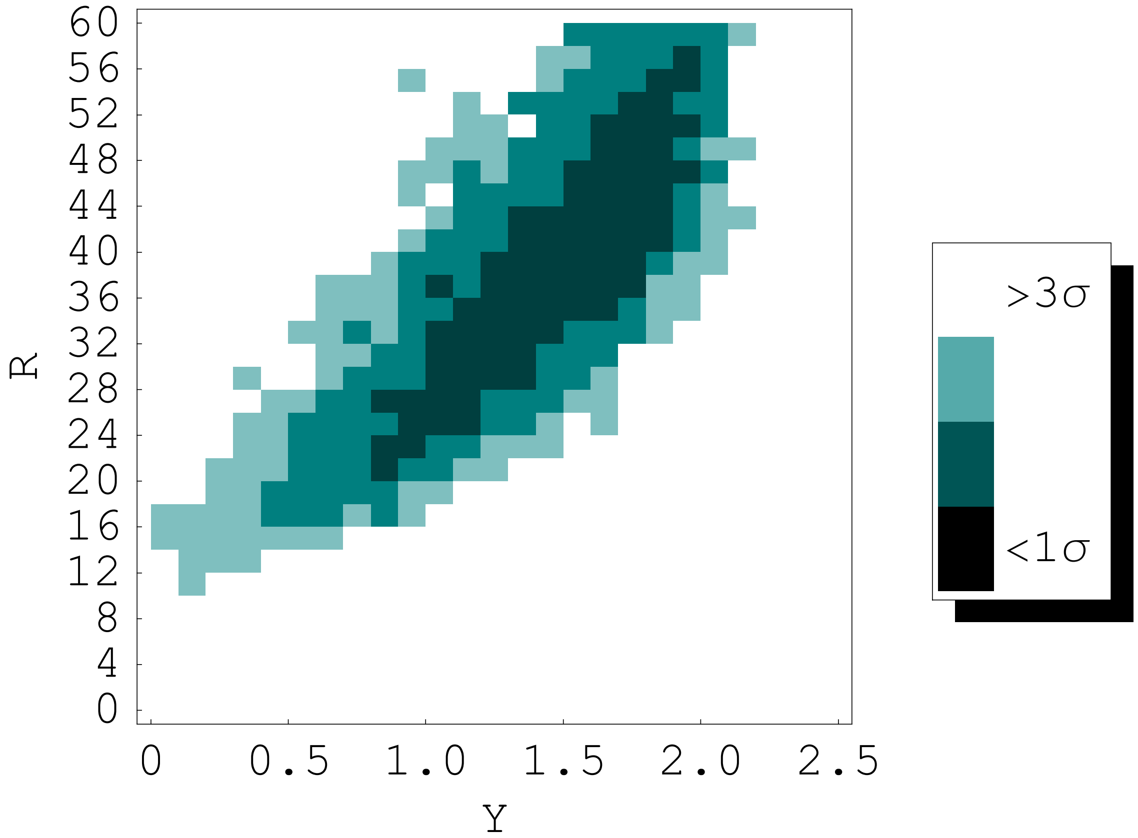
$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$R = 32 \pm 10$

$Z = 0.50 \pm 0.19$

F) Results - R free

Constraints on R and Y: R free



$\eta \to 3\pi$:

$R = 38 \pm 10$

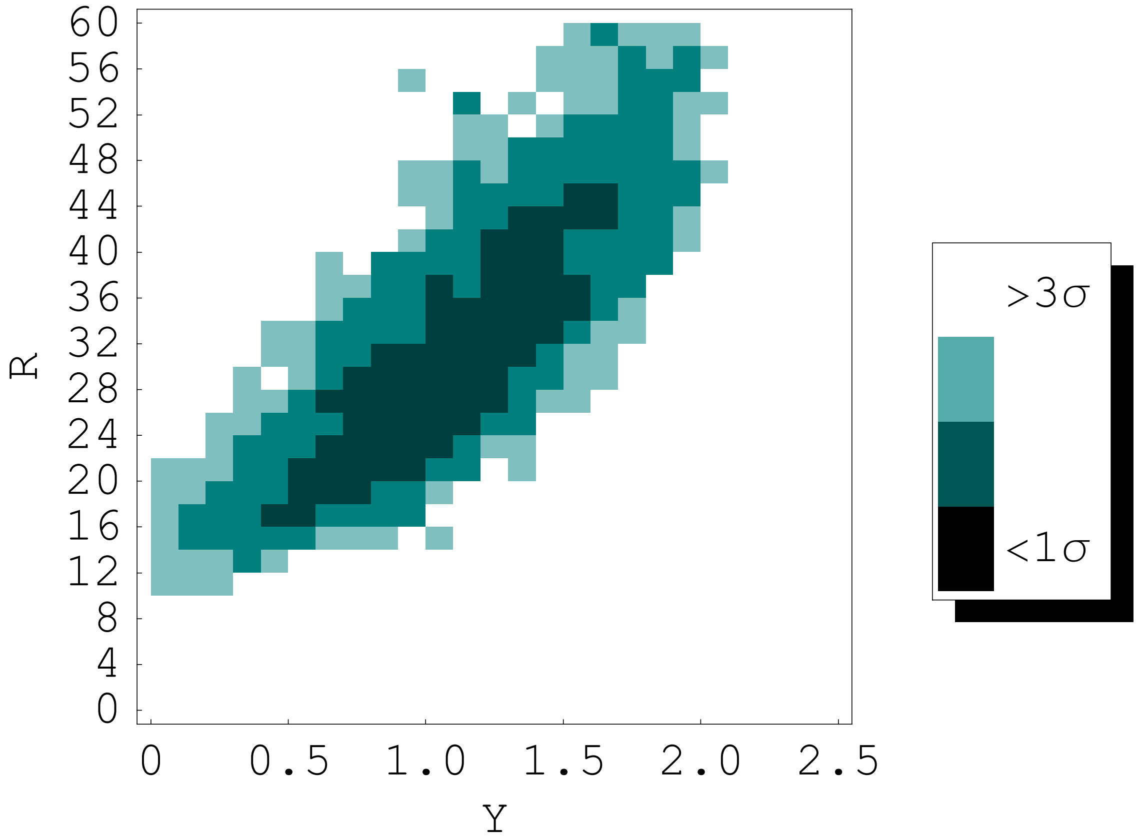
$Y = 1.41 \pm 0.37$

$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$R = 32 \pm 9$

$Y = 1.10 \pm 0.38$

Constraints on R and Y: R free



$\eta \to 3\pi$:

$R = 38 \pm 10$

$Y = 1.41 \pm 0.37$

$\eta \to 3\pi$ and $\pi\pi$ scattering:
(preliminary)

$R = 32 \pm 9$

$Y = 1.10 \pm 0.38$

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\mathrm{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\text{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\text{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79\text{MeV}$
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\text{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79\text{MeV}$
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\text{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\mathrm{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 >$79MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\mathrm{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\mathrm{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

G) Conclusions

Our results have shown:

- the $\eta \to 3\pi$ decays are sensitive to the value of the leading order low energy parameters F_0 and Σ_0
- a large portion of the parameter space can be excluded at 2σ C.L., given information about R, including $F_0 > 79$MeV
- in those regions, a reasonable convergence of the chiral series might fail
- $Y = X/Z = (M_\pi^{\mathrm{LO}}/M_\pi)^2 \sim 1.5$ seems to be preferred, in contrast to other results
- constraints are weaker when releasing the assumption on R
- a low value of F_0 is still preferred and we found a strong correlation between R and Y
- $\pi\pi$ scattering does not seem to constrain the parameters very much

Thank you for your attention!