Joint Physics Analysis Center Activities:

Pion-Nucleon Scattering and Other Reactions

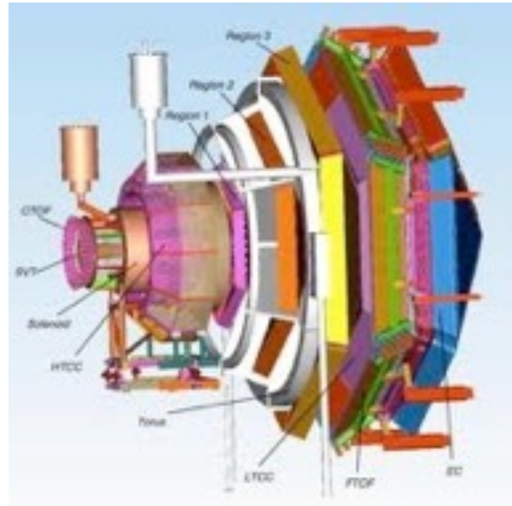
Vincent Mathieu
Indiana University - JPAC

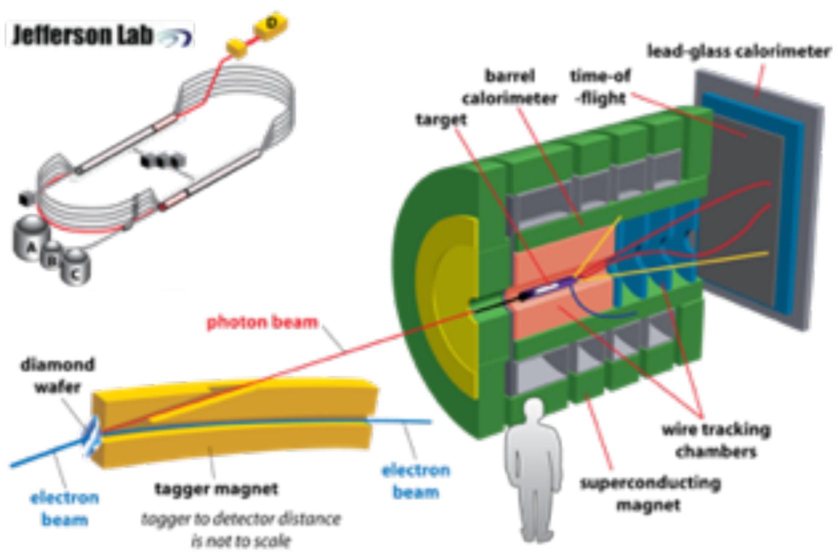
Chiral Dynamics
Pisa June 2015

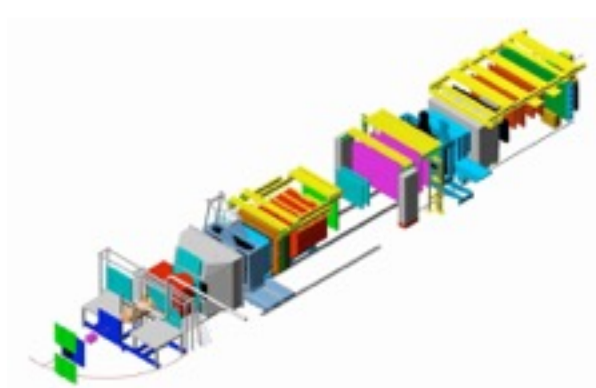
INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Motivations: Hadron Spectroscopy Experiments







CLAS, GlueX

MAMI, ELSA

COMPASS

BES, LHCb, PANDA,…

Aim to:

Complete understanding on the hadron spectrum and discover new resonances

HASPECT:

Genova, Roma, Torino, Glasgow, Edinburgh,…

Physics Analysis Center (IU/JLab):

Analyze data and provide tools for the analysis

Joint Physics Analysis Center (JPAC)

JPAC People

Mike Pennington (JLab)
Adam Szczepaniak (IU/JLab)
Peng Guo (IU/JLab)
Igor Danilkin (JLab)
Cesar Fernandez-Ramirez (JLab)
Emilie Passemar (IU/JLab)
Lingyun Dai (IU/JLab)
Andrew Jackura (IU)
Vincent Mathieu (IU)

Collaborators

Meng Shi (Beijing/JLab)
Ron Workman (GWU)
Mike Doering (GWU)

CLAS collaboration

Diane Schott (GWU/JLab)
Viktor Mokeev (JLab)
HASPECT (Italy)

Formalisms

Regge Theory
Dispersive Relations
Dual Models
Isobar Models
...

Projects

$J/\psi \to 3\pi$

$\eta \to 3\pi$

$\omega \to 3\pi$

$\omega \to \pi\gamma^*(e^+e^-)$

$\pi N \to \pi N$

$\pi N \to \eta N$

$KN \to KN$

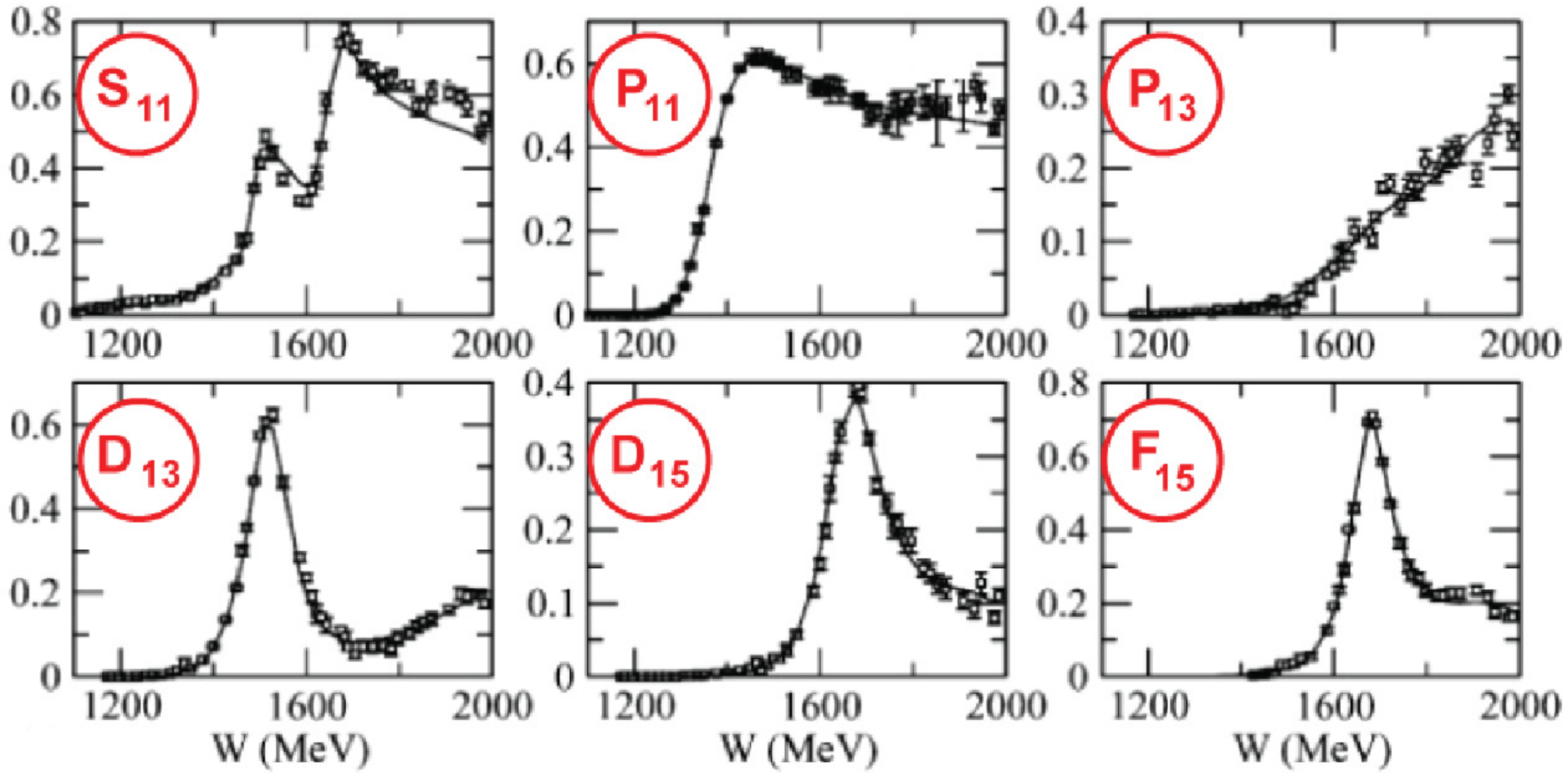
$\gamma N \to \pi N$

$\gamma p \to K^+K^-p$

$\gamma p \to \pi^0\eta p$

$\pi^-p \to \pi^-\eta p$

πN amplitudes



Isospin 1/2
Imaginary T

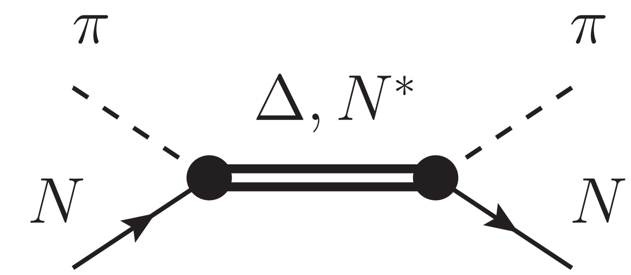
SAID: Workman *et al*

resonances W < 2 GeV → fit data W < 2 GeV

BUT going beyond is ‘easy’ and related to resonances !

Pion Nucleon Amplitudes

$$T = \bar{u}_2 \left[A(s,t) + \frac{1}{2}\gamma^\mu (p_1 + p_2)_\mu B(s,t) \right] u_1$$



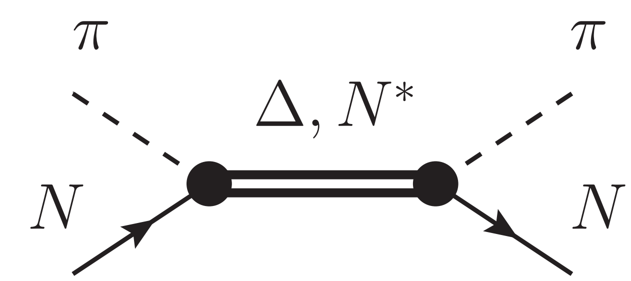
$$A' \equiv A + \frac{\nu B}{1 - t/4m_p^2}$$

$$\nu = \frac{s-u}{4m} = E_{\text{lab}} + t/4m_p$$

VM et al (JPAC) arXiv:1506.01764

Pion Nucleon Amplitudes

$$T = \bar{u}_2 \left[A(s,t) + \frac{1}{2}\gamma^\mu (p_1 + p_2)_\mu B(s,t) \right] u_1$$

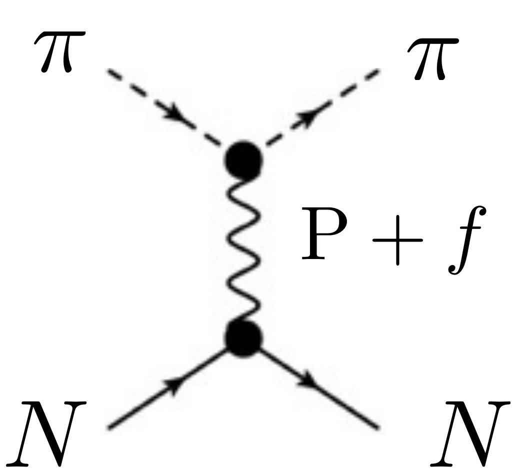


$$A' \equiv A + \frac{\nu B}{1 - t/4m_p^2}$$

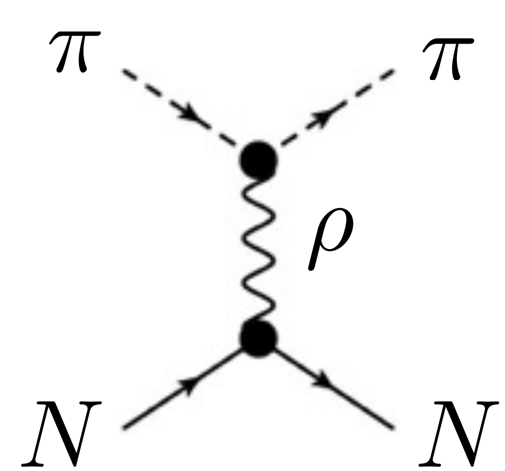
$$\nu = \frac{s-u}{4m} = E_{\text{lab}} + t/4m_p$$

2 amplitudes and 2 isospin combinations
Interpretation at high energies:

$$A'^{(+)},\ B^{(+)}$$



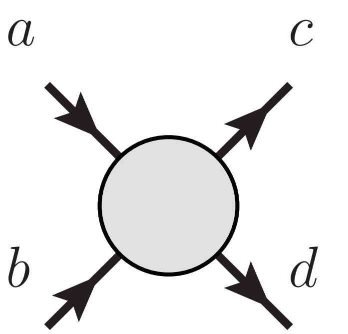
$$A'^{(-)},\ B^{(-)}$$



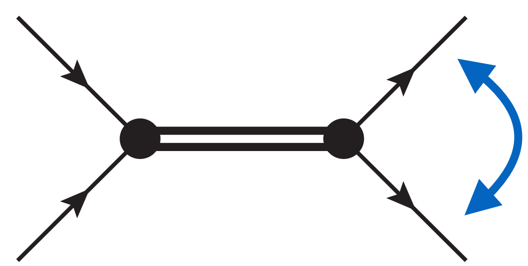
VM et al (JPAC) arXiv:1506.01764

Partial Waves vs Regge

Two different representations of amplitudes $A'^{(\pm)}(s,t),\ B^{(\pm)}(s,t)$



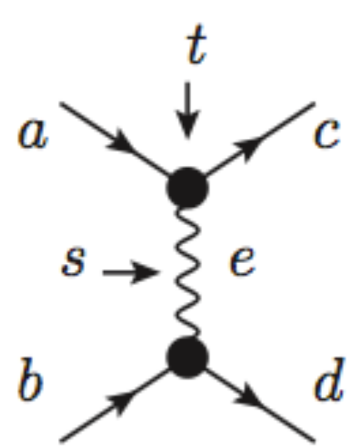
$= \sum_{\ell=0}^{\infty}$



$\ell \quad P_\ell(\cos\theta)$

Partial wave series

$=$



$+\mathcal{O}\left(\frac{1}{\sqrt{s}}\right)$

$\beta(t)s^{\alpha(t)}$

Sum over Regge poles
+ background integral

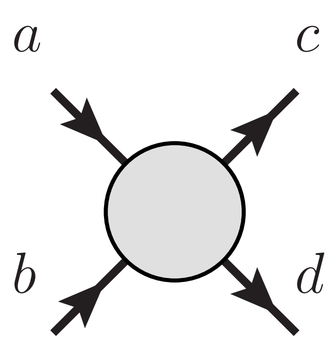
VM et al (JPAC) arXiv:1506.01764

Partial Waves vs Regge

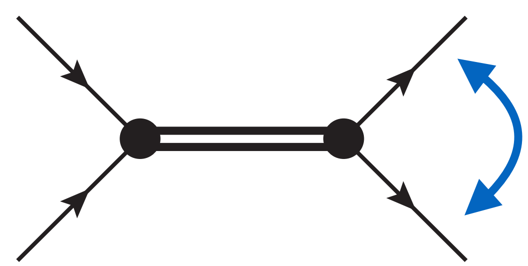
Two different representations of amplitudes $A'^{(\pm)}(s,t),\ B^{(\pm)}(s,t)$

Problems:

- **truncated sum**
- **Regge only at high energies**



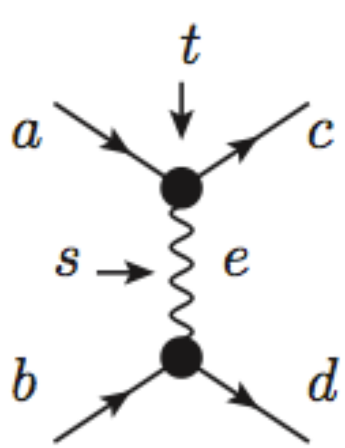
$$= \sum_{\ell=0}^{\infty} \quad L_{max}$$



$\ell \quad P_\ell(\cos\theta)$

Partial wave series

$=$



$$+\mathcal{O}\left(\frac{1}{\sqrt{s}}\right) \ ?$$

$\beta(t)s^{\alpha(t)}$

Sum over Regge poles
+ background integral

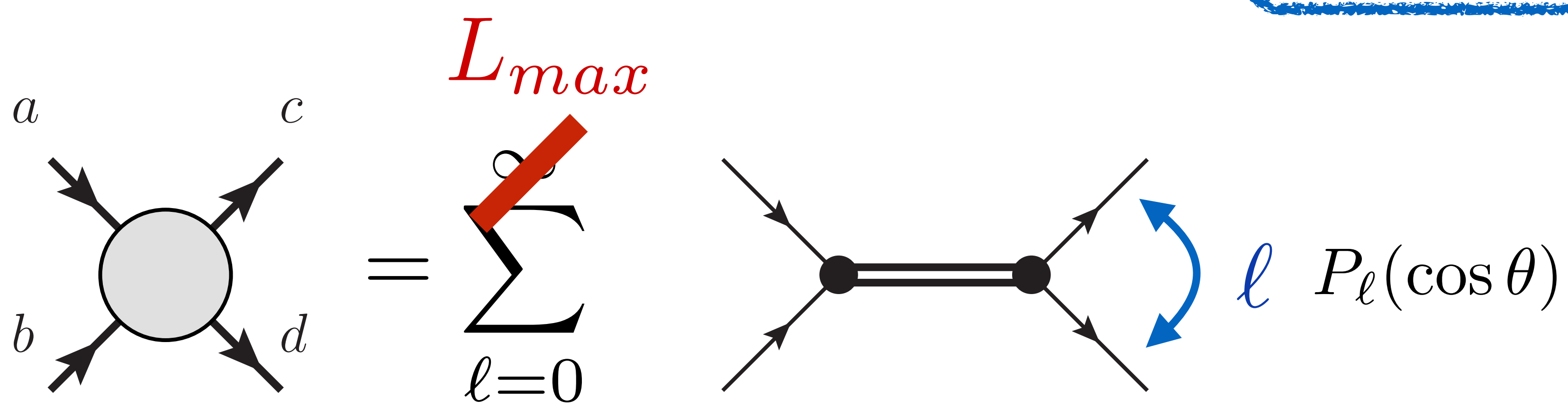
VM et al (JPAC) arXiv:1506.01764

Partial Waves vs Regge

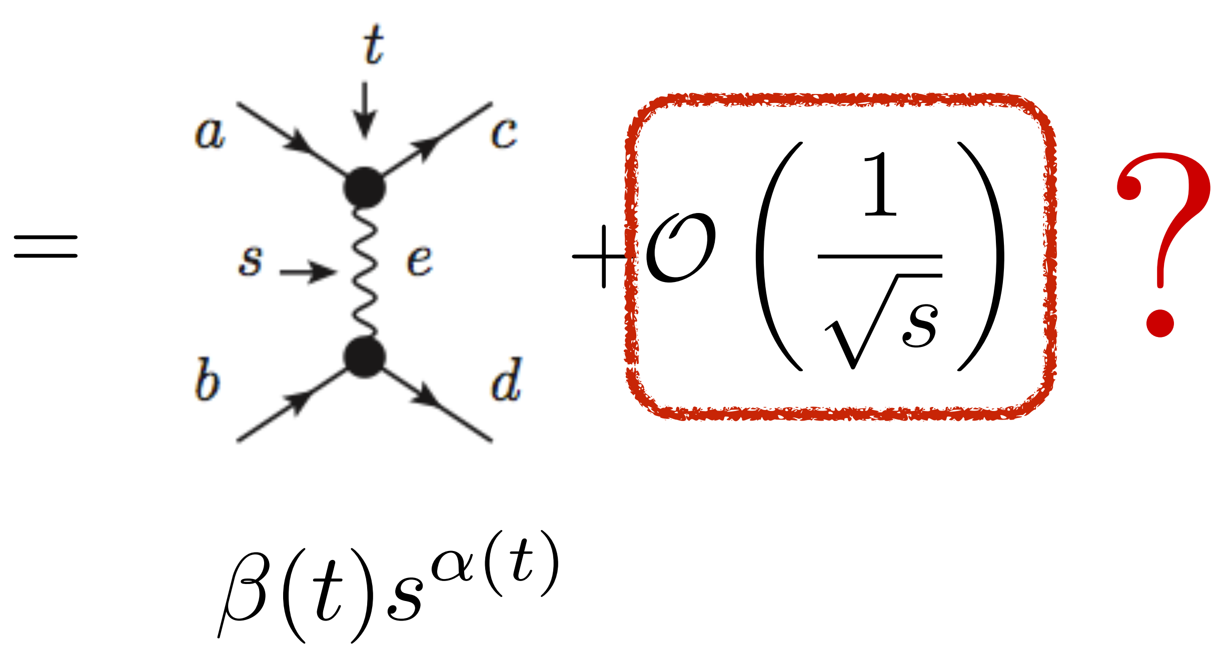
Two different representations of amplitudes $A'^{(\pm)}(s,t),\ B^{(\pm)}(s,t)$

Problems:
- **truncated sum**
- **Regge only at high energies**

How to take advantages of both ?



Partial wave series



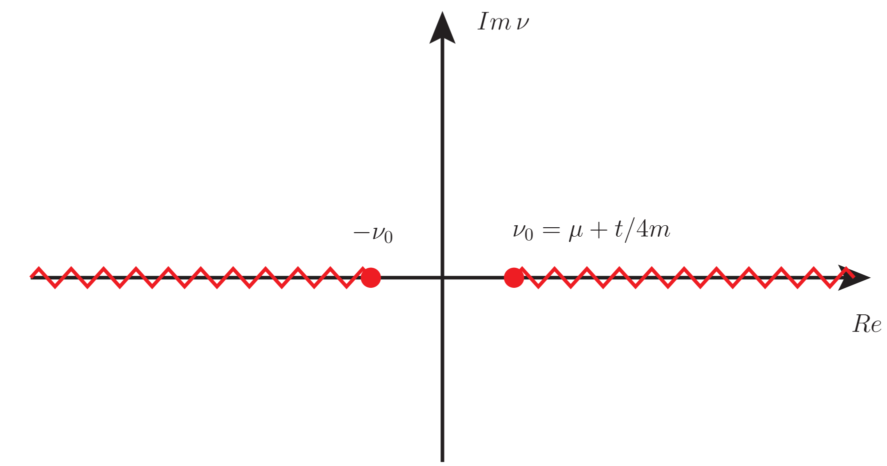
Sum over Regge poles
+ background integral

VM et al (JPAC) arXiv:1506.01764

Finite Energy Sum Rules

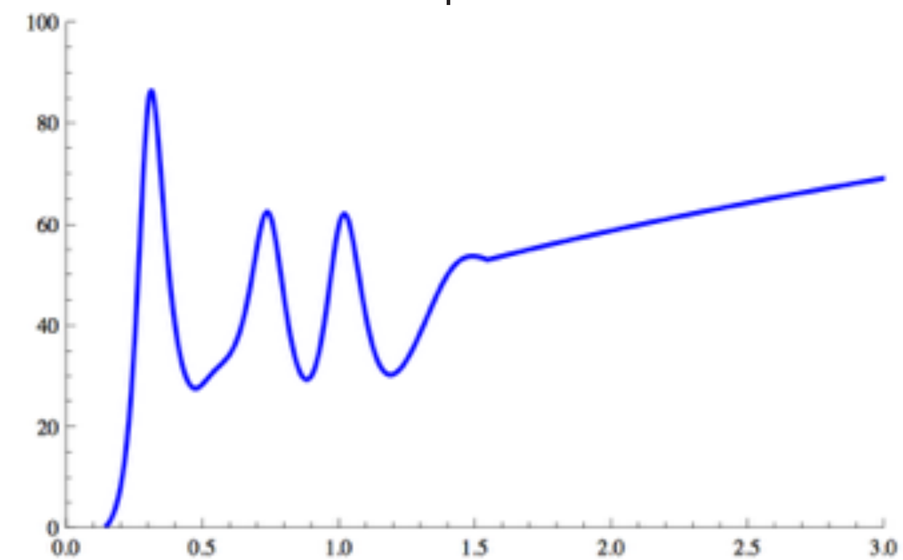
Satisfy dispersion relations

$$A^{(-)}(\nu, t) = \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\ A^{(-)}(\nu', t)}{\nu'^2 - \nu^2} d\nu'$$



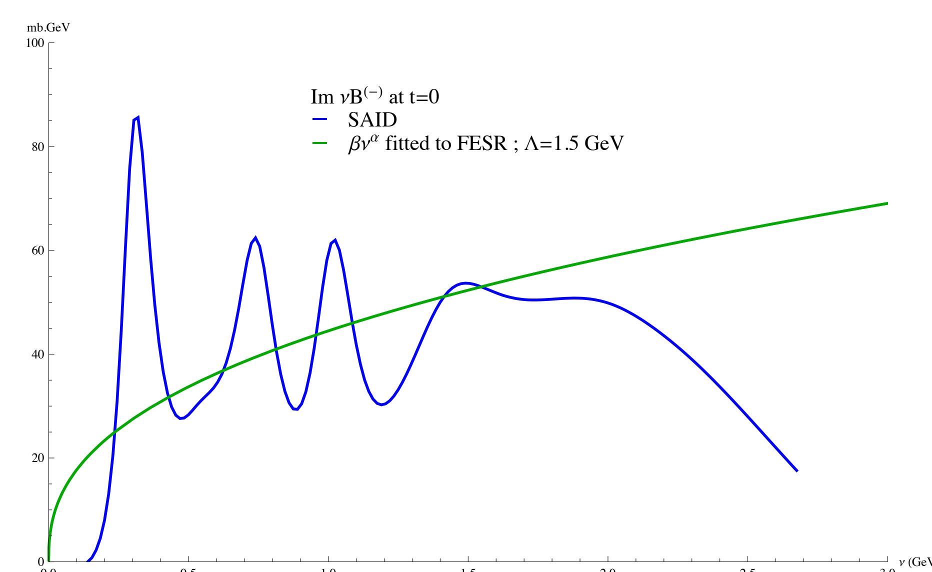
$$\nu > \Lambda$$

$$\mathrm{Im}\ A^{(-)}(\nu, t) \longrightarrow \beta(t)\nu^{\alpha_\rho(t)}$$



Analyticity implies FESR

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A^{(-)}(\nu', t)\nu'^{2k} d\nu' = \beta(t) \frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

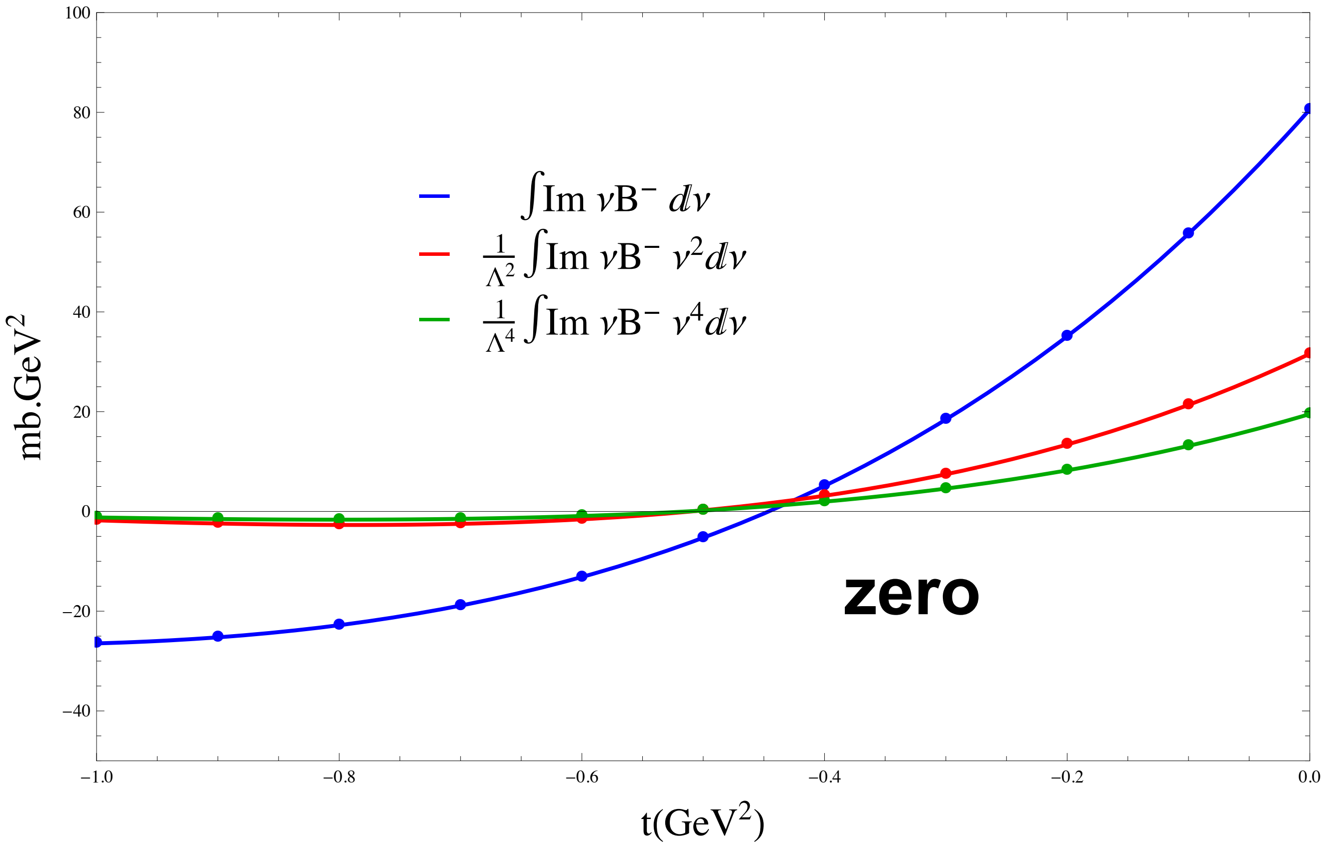


High energy data constrain RHS

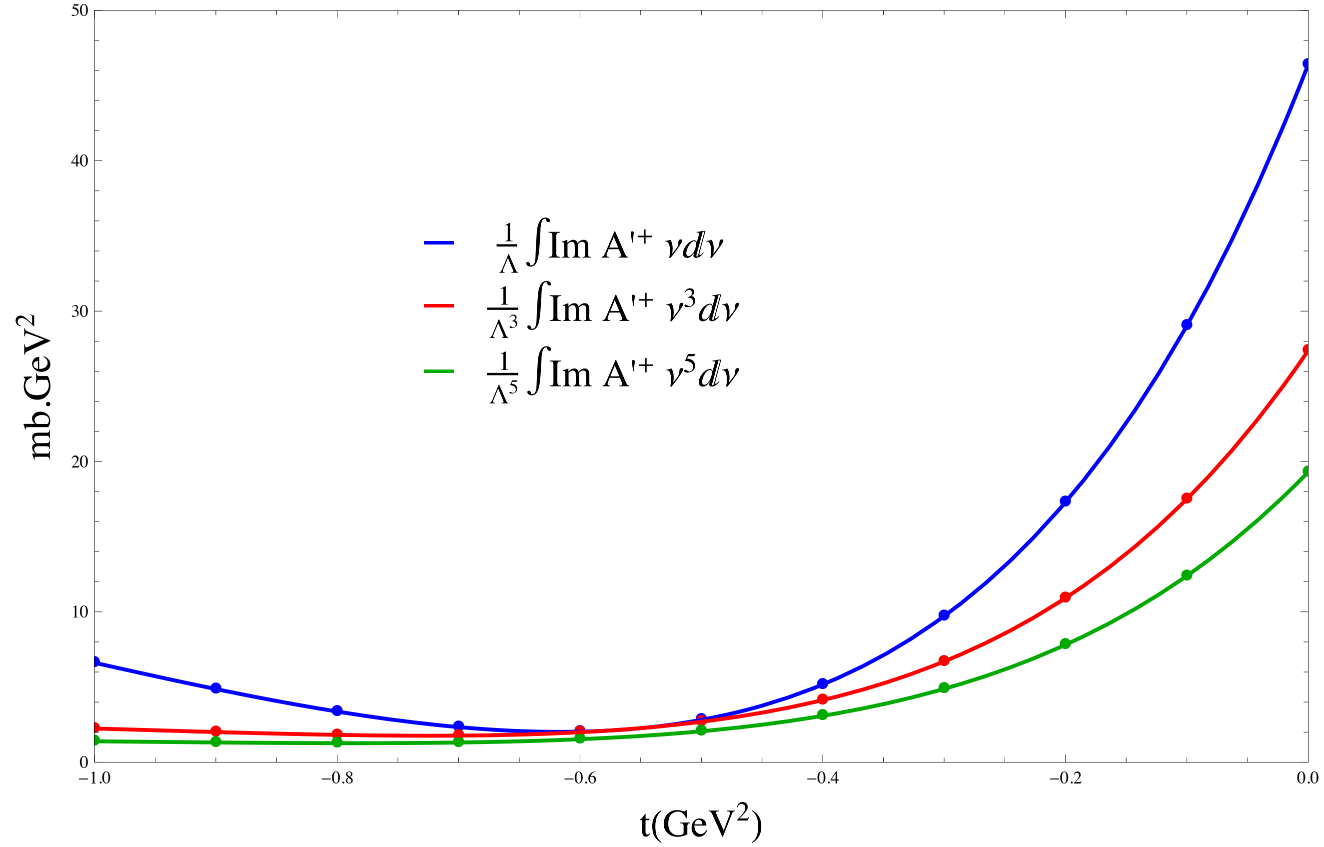
RHS constrains baryon resonances

VM et al (JPAC) arXiv:1506.01764

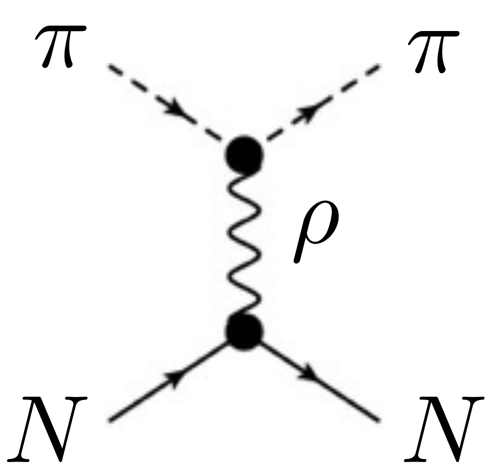
Isovector: ρ

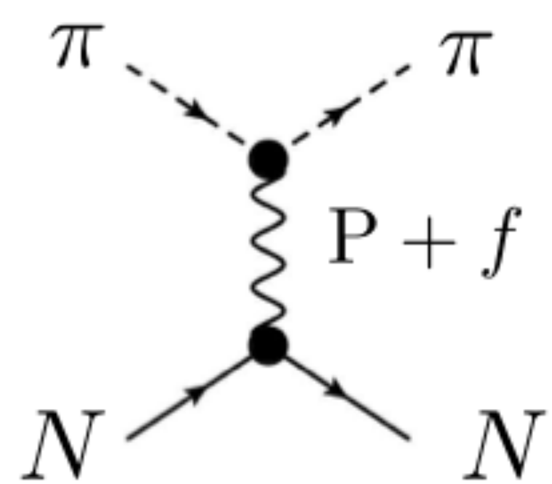


Isoscalar: $\mathbb{P} + f$



$$\int_{\nu_0}^{\Lambda} \text{Im } A^{(-)}(\nu', t)\nu'^{2k} d\nu' = \beta(t) \frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

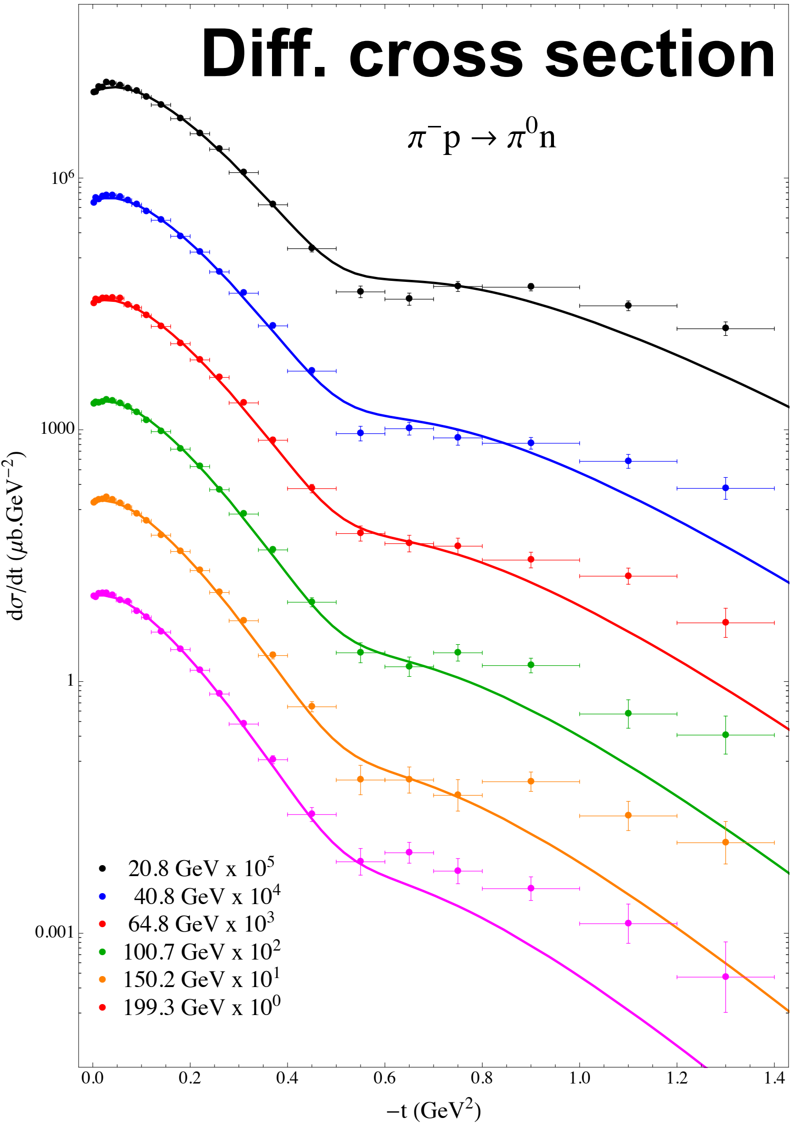


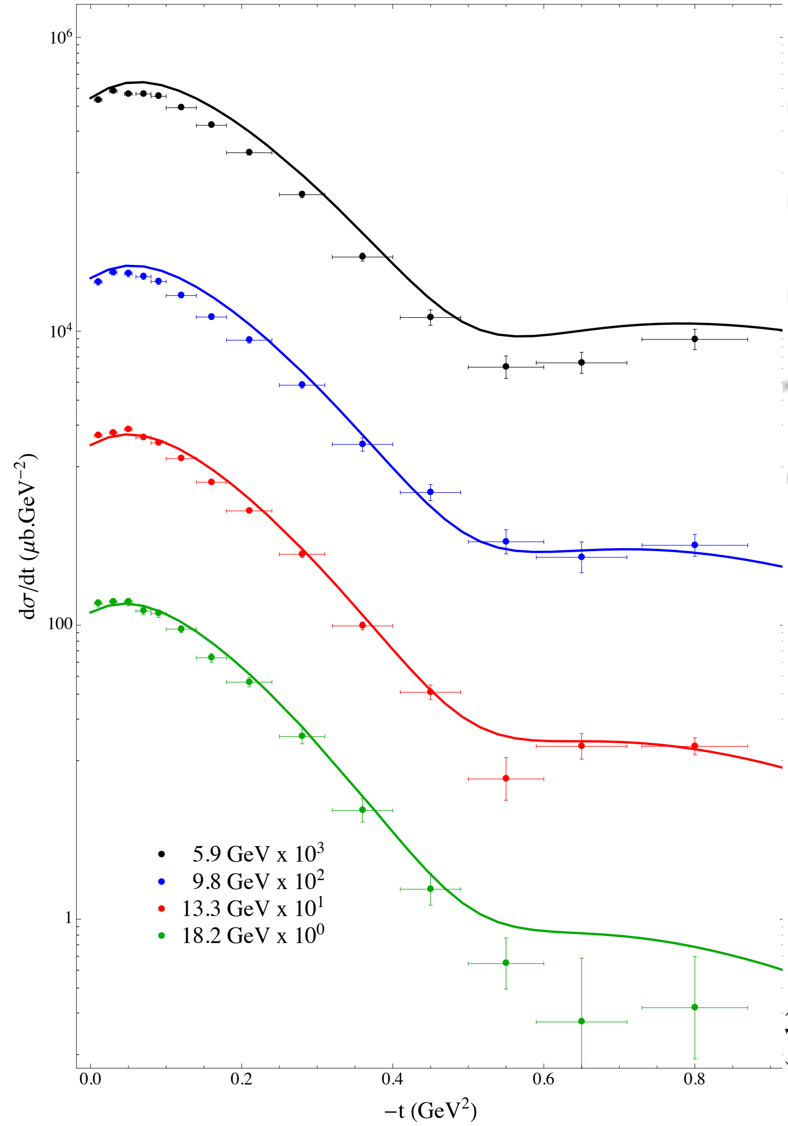


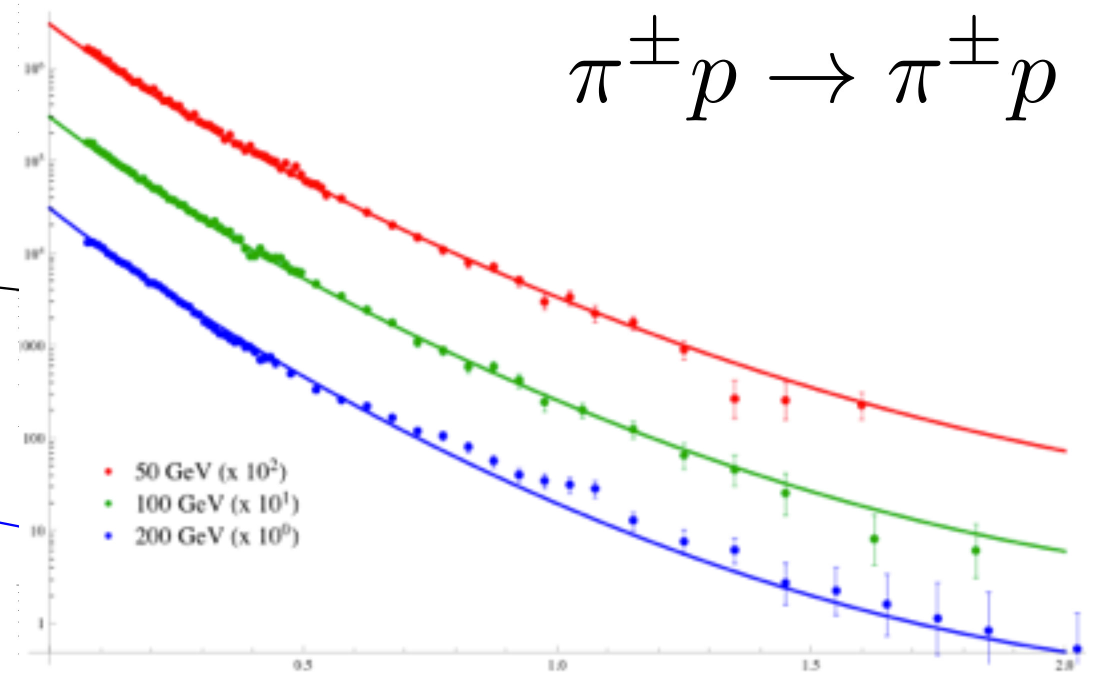
SAID: WI08 R. Workman et al. 2008

VM et al (JPAC) arXiv:1506.01764

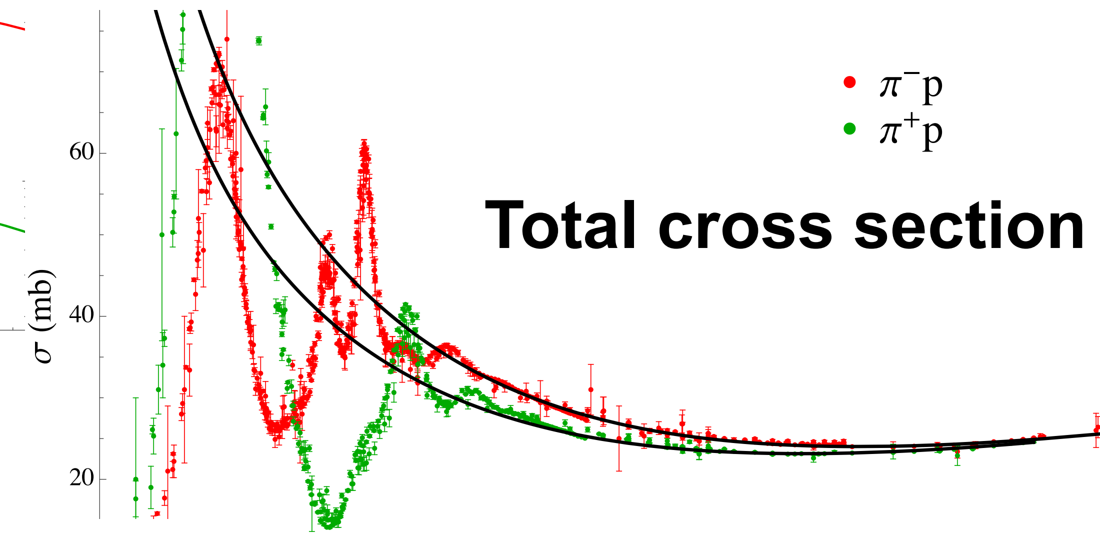
Diff. cross section

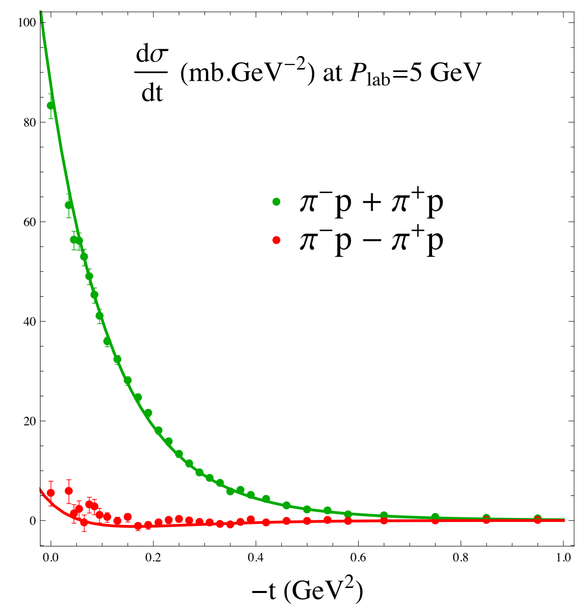


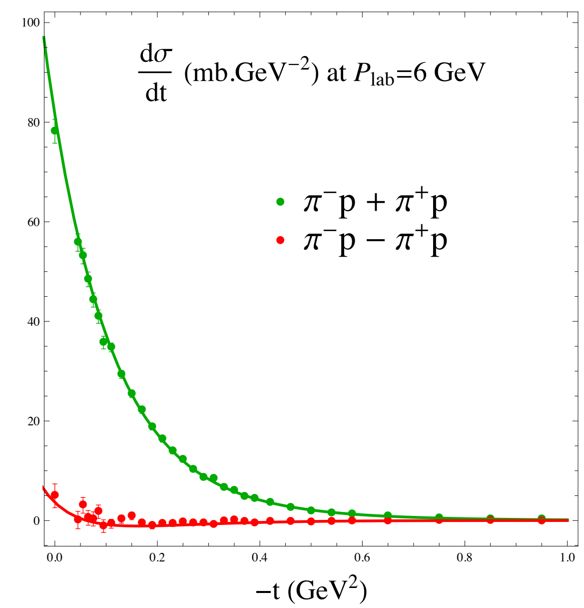


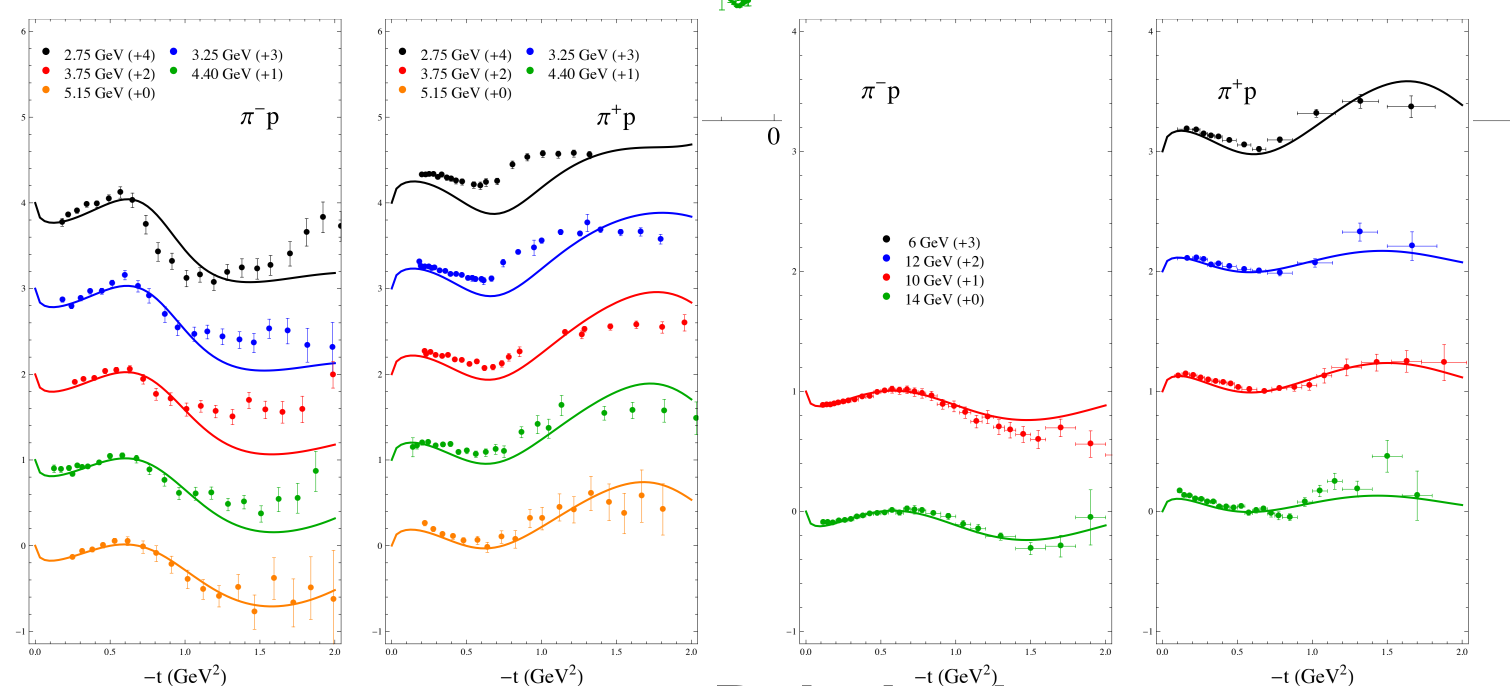


Total cross section





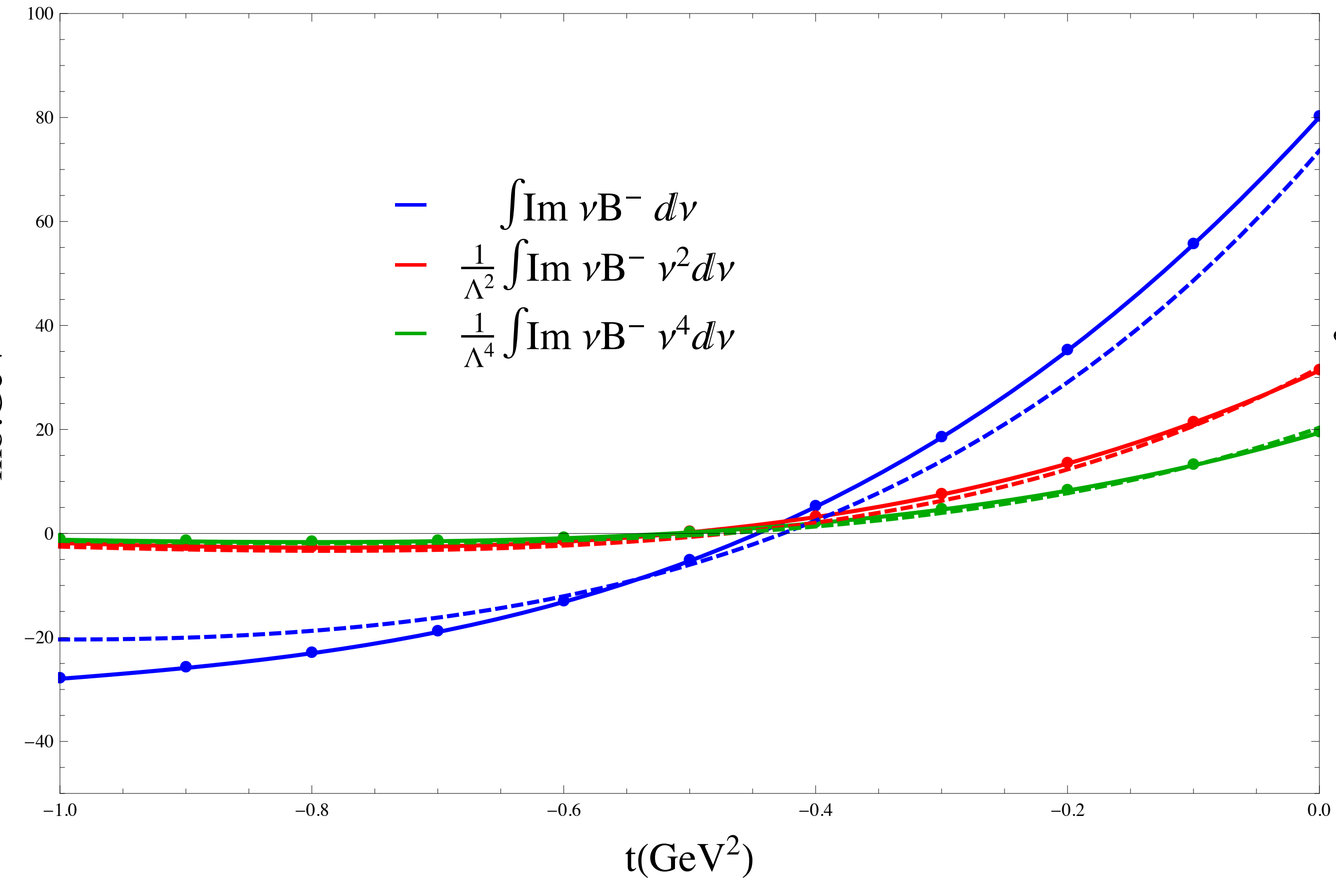




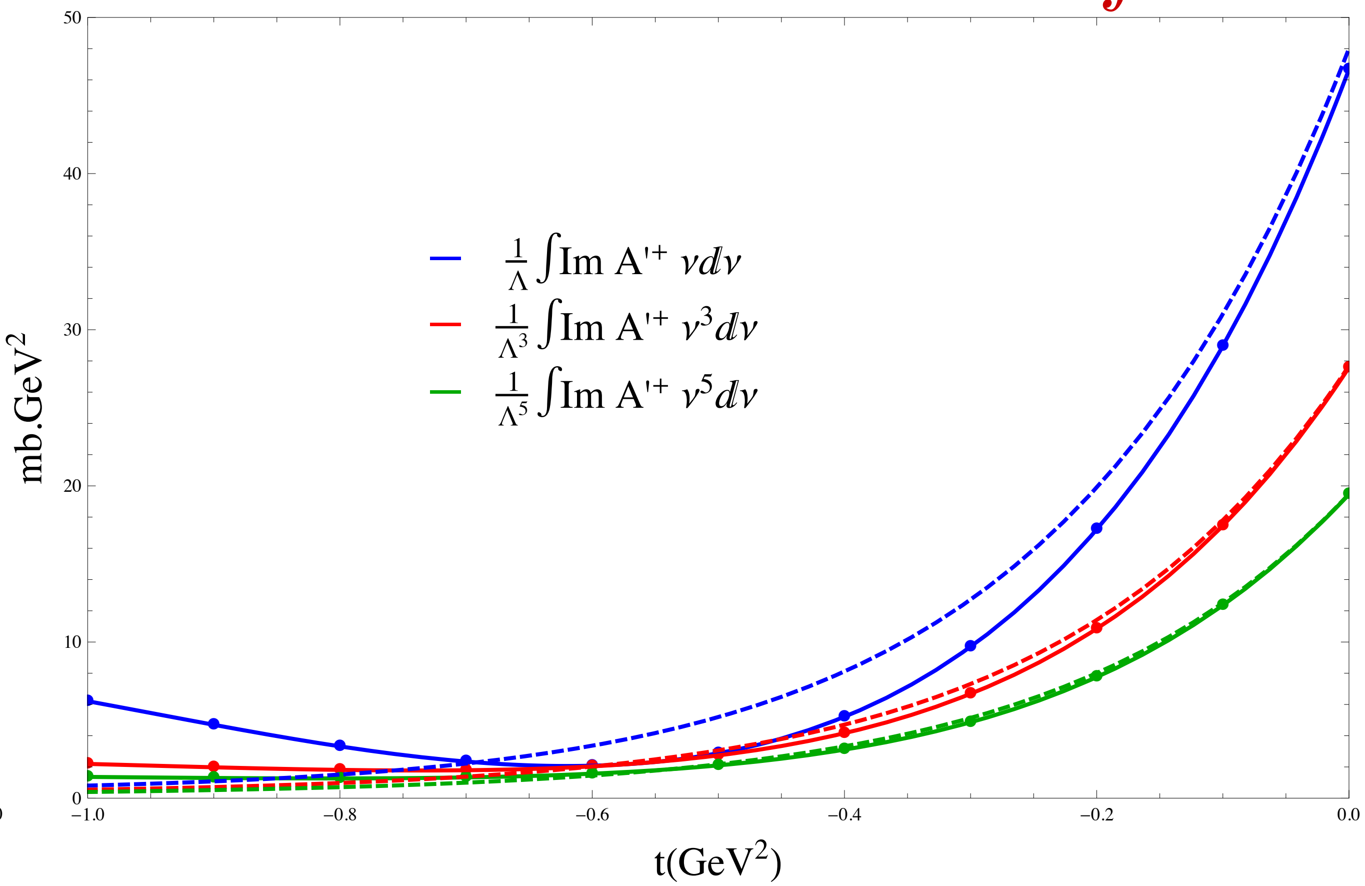
Polarization

VM et al (JPAC) arXiv:1506.01764

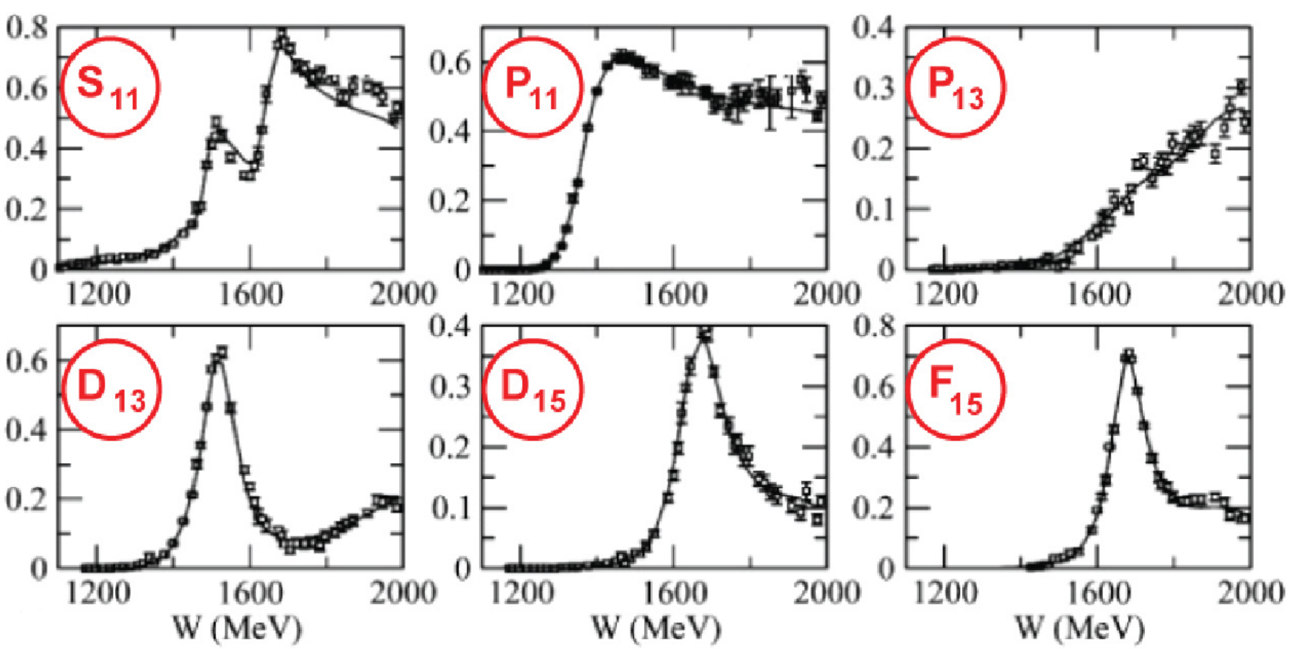
Isovector:



Isoscalar: $\mathbb{P} + f$



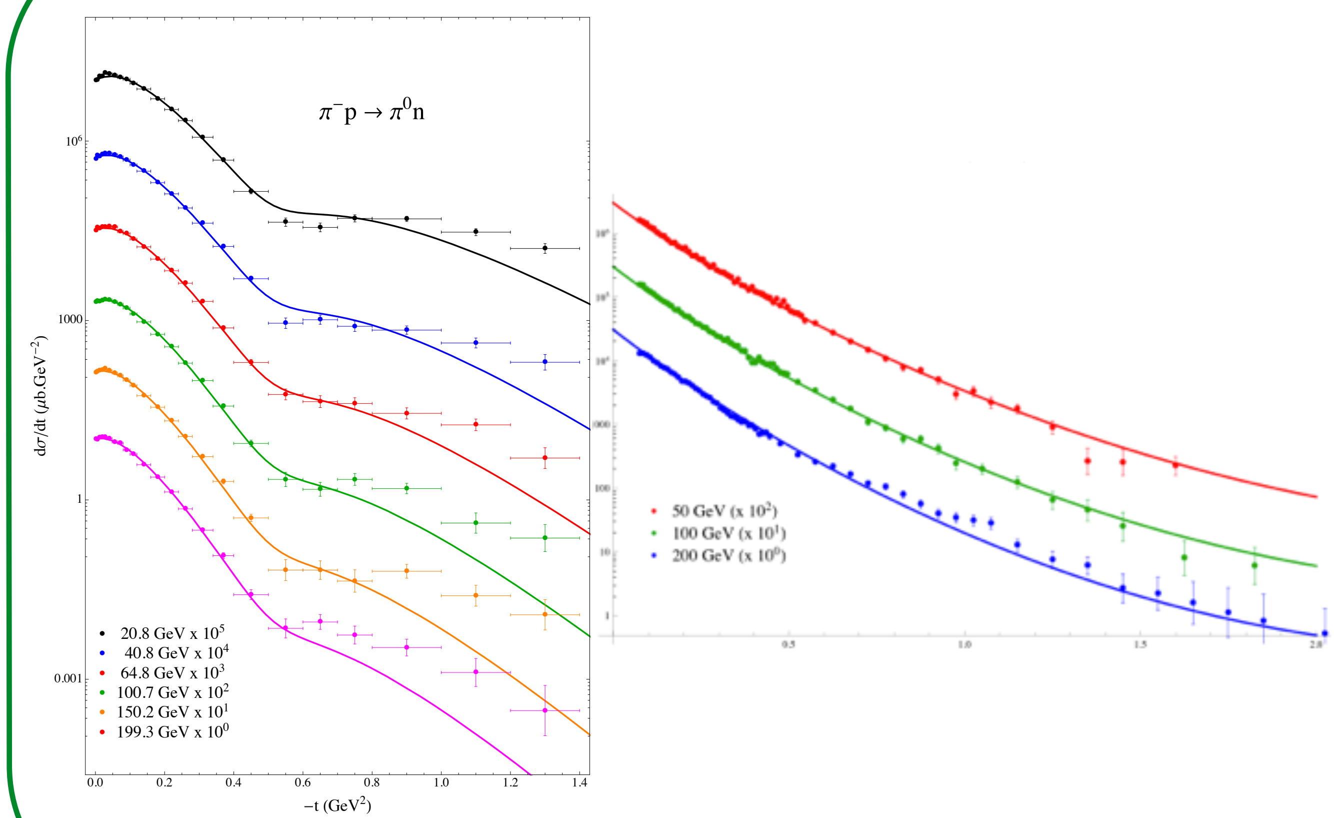
solid line = SAID



Isospin 1/2
Imaginary T

SAID: Workman *et al*

dashed line = Regge



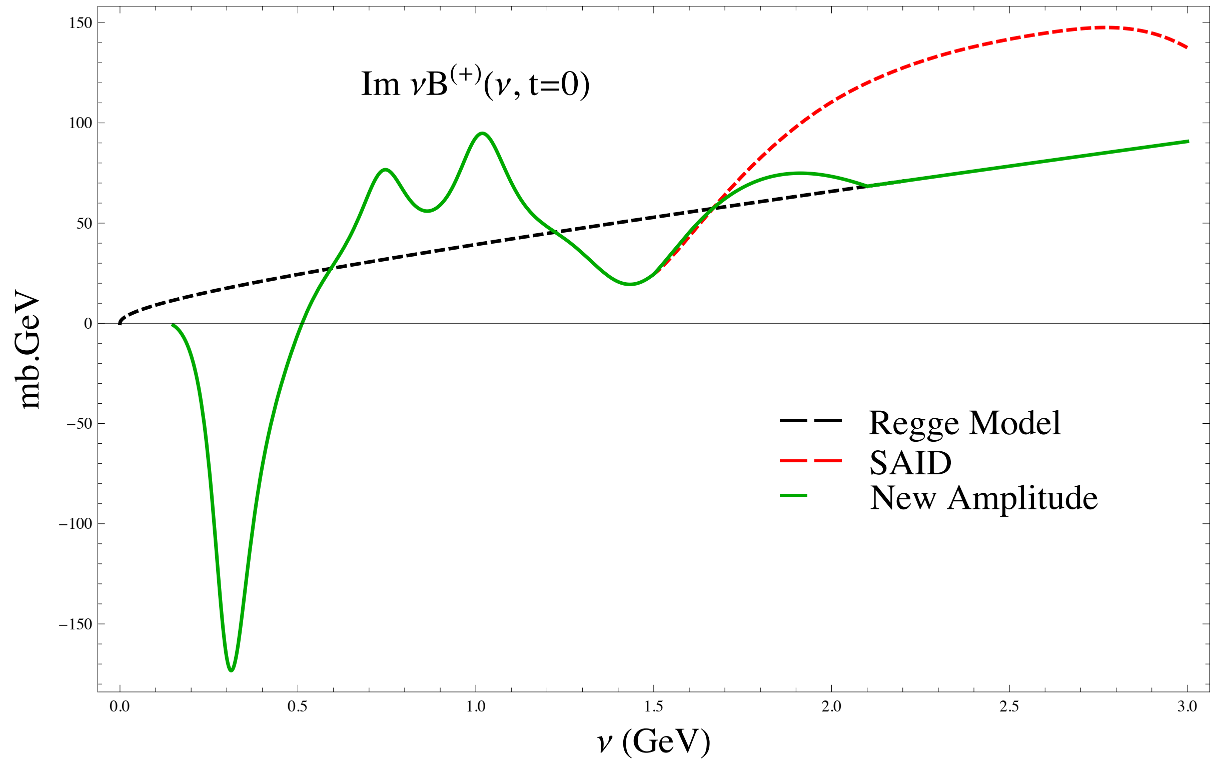
VM et al (JPAC) arXiv:1506.01764

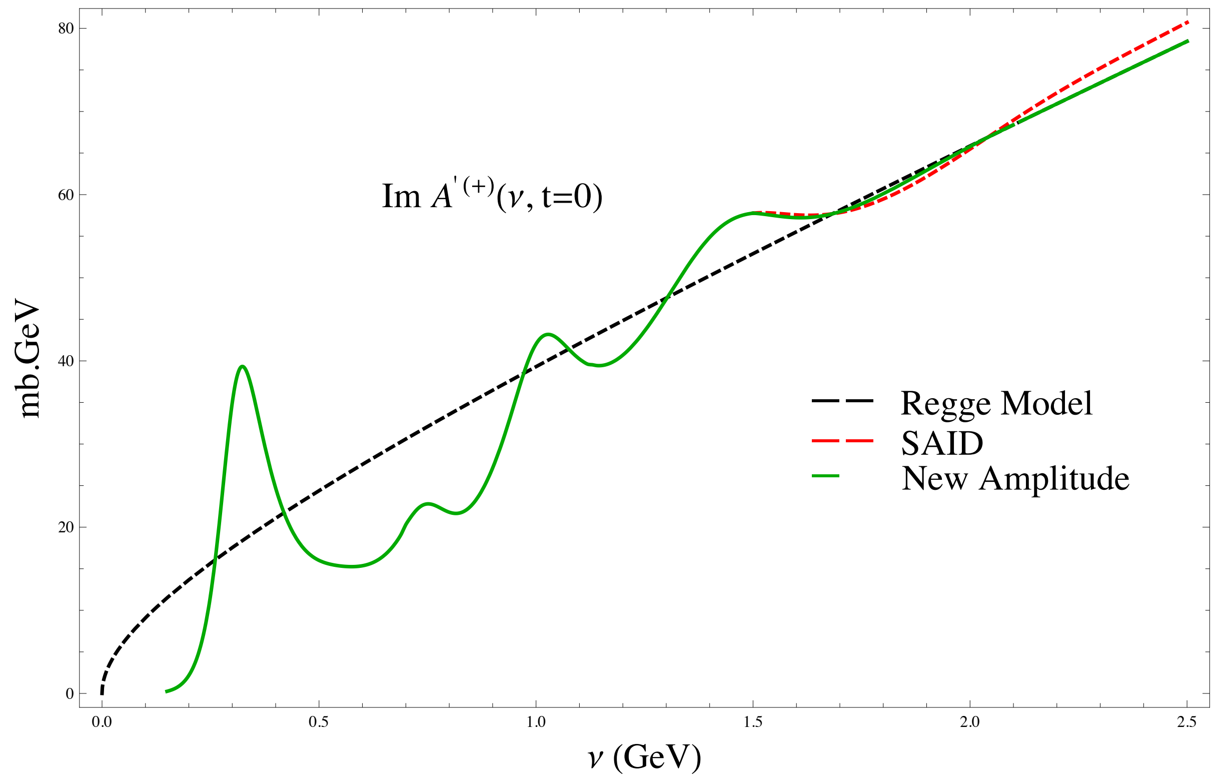
Finite Energy Sum Rules

$\pi N \to \pi N$

Construct Im(amplitude) from 0 to infinity via FESR
Reconstruct Re(amplitude) from dispersion relation

$$\mathrm{Re}\ \nu B^{(+)}(\nu,t) = \frac{g_r^2}{2m}\frac{2\nu^2}{\nu_m^2-\nu^2} + \frac{2\nu^2}{\pi}\mathrm{P}\int_{\nu_0}^{\infty}\frac{\mathrm{Im}\ B^{(+)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$

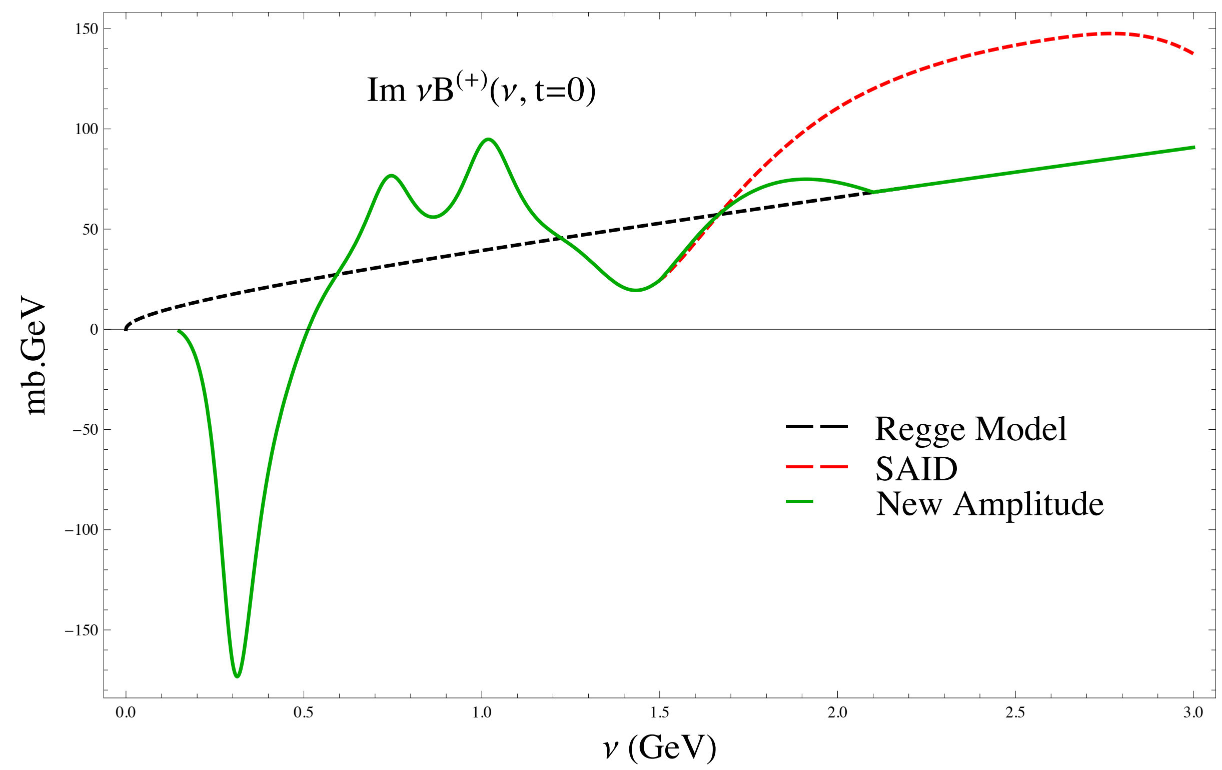




VM et al (JPAC) arXiv:1506.01764

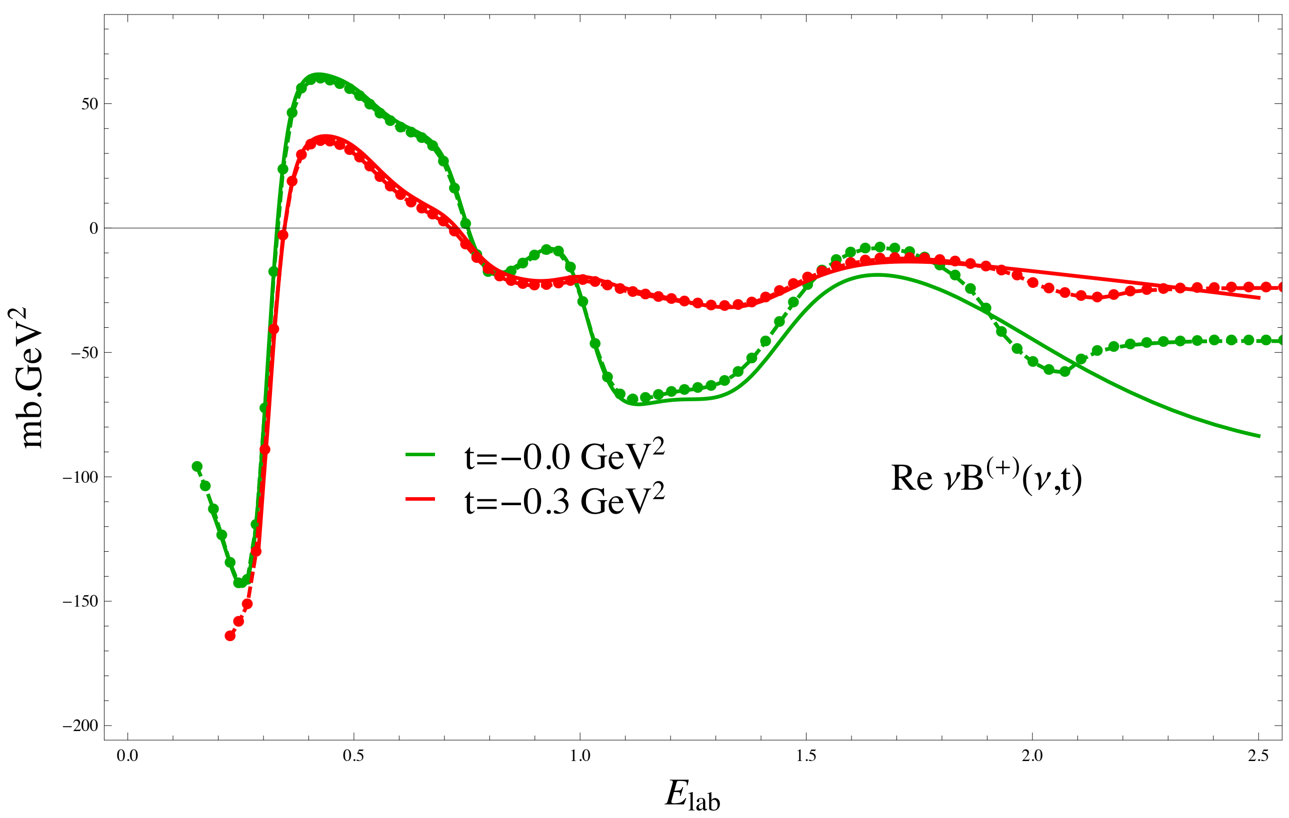
Finite Energy Sum Rules

$\pi N \to \pi N$



Construct Im(amplitude) from 0 to infinity via FESR
Reconstruct Re(amplitude) from dispersion relation

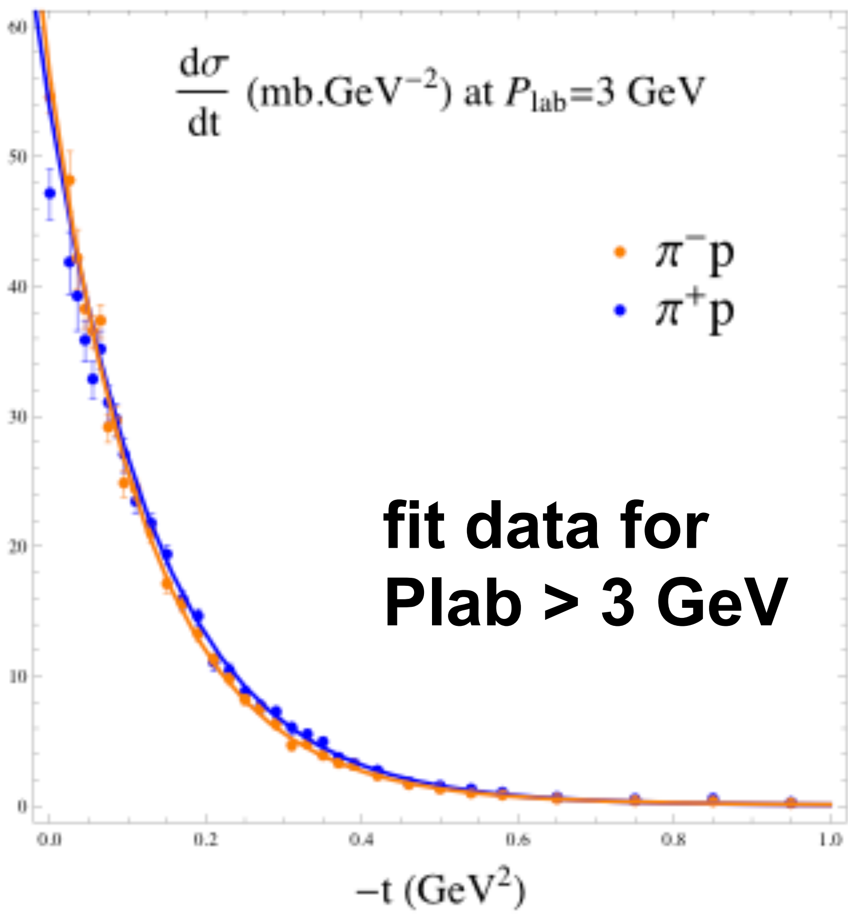
$$\text{Re }\nu B^{(+)}(\nu,t) = \frac{g_r^2}{2m}\frac{2\nu^2}{\nu_m^2-\nu^2} + \frac{2\nu^2}{\pi}\text{P}\int_{\nu_0}^{\infty}\frac{\text{Im }B^{(+)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$

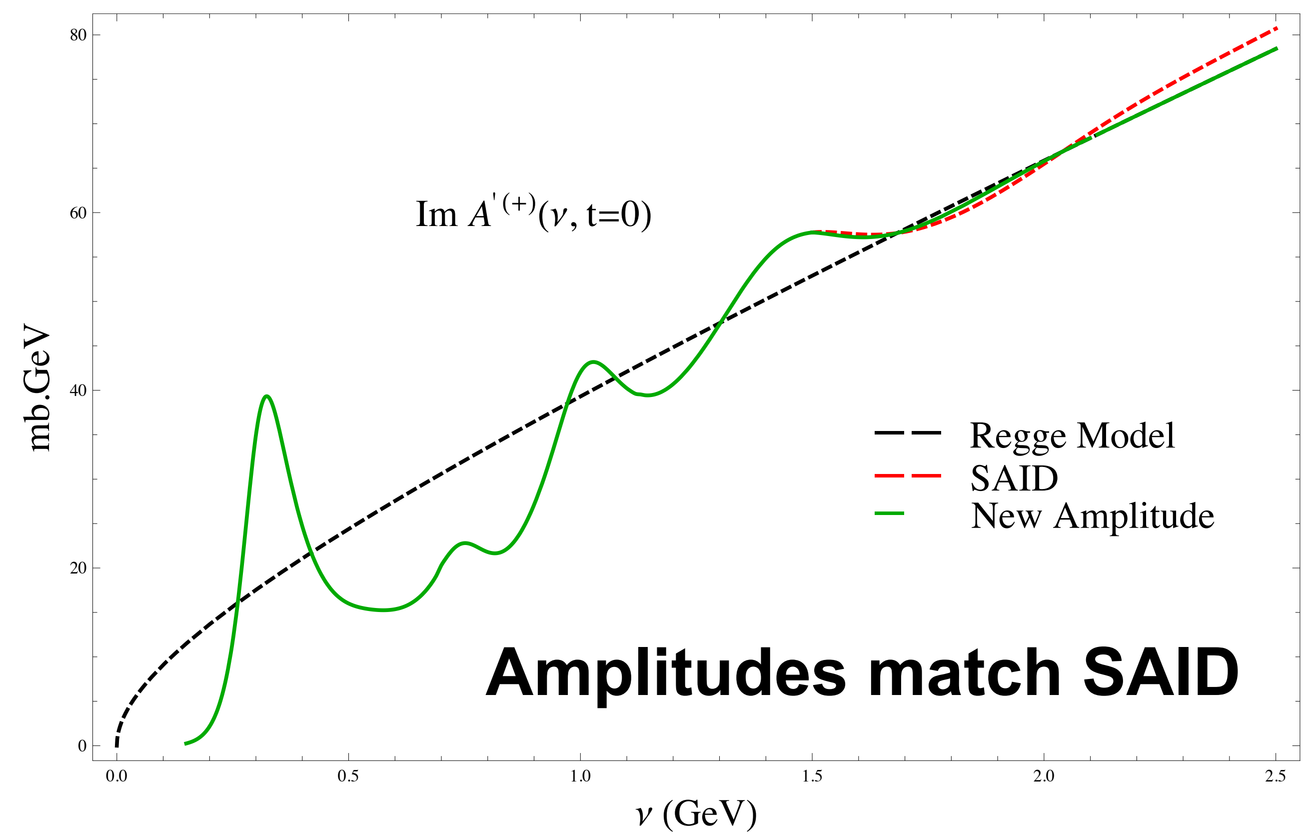


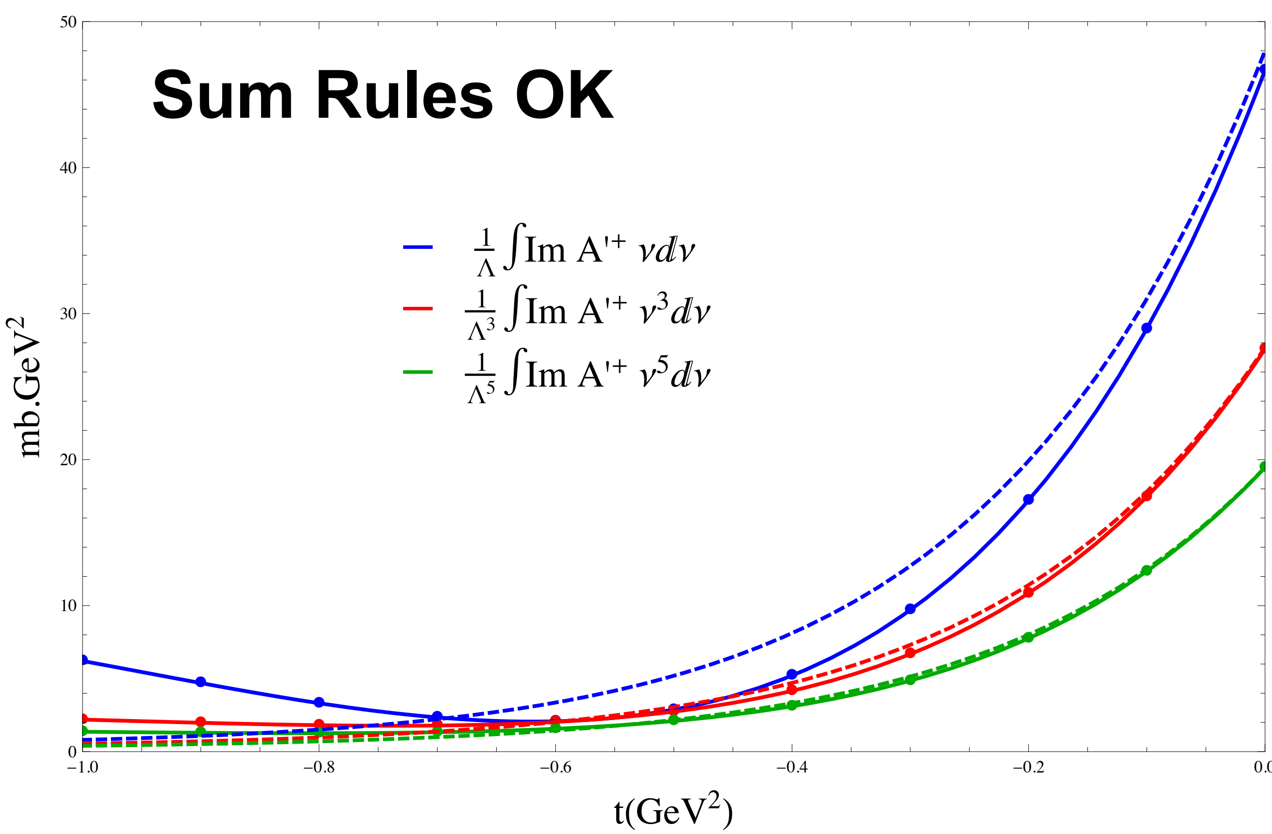
Excellent Match between Re(SAID) Solid lines and Re(Reconstructed) Dashed-Dotted line!

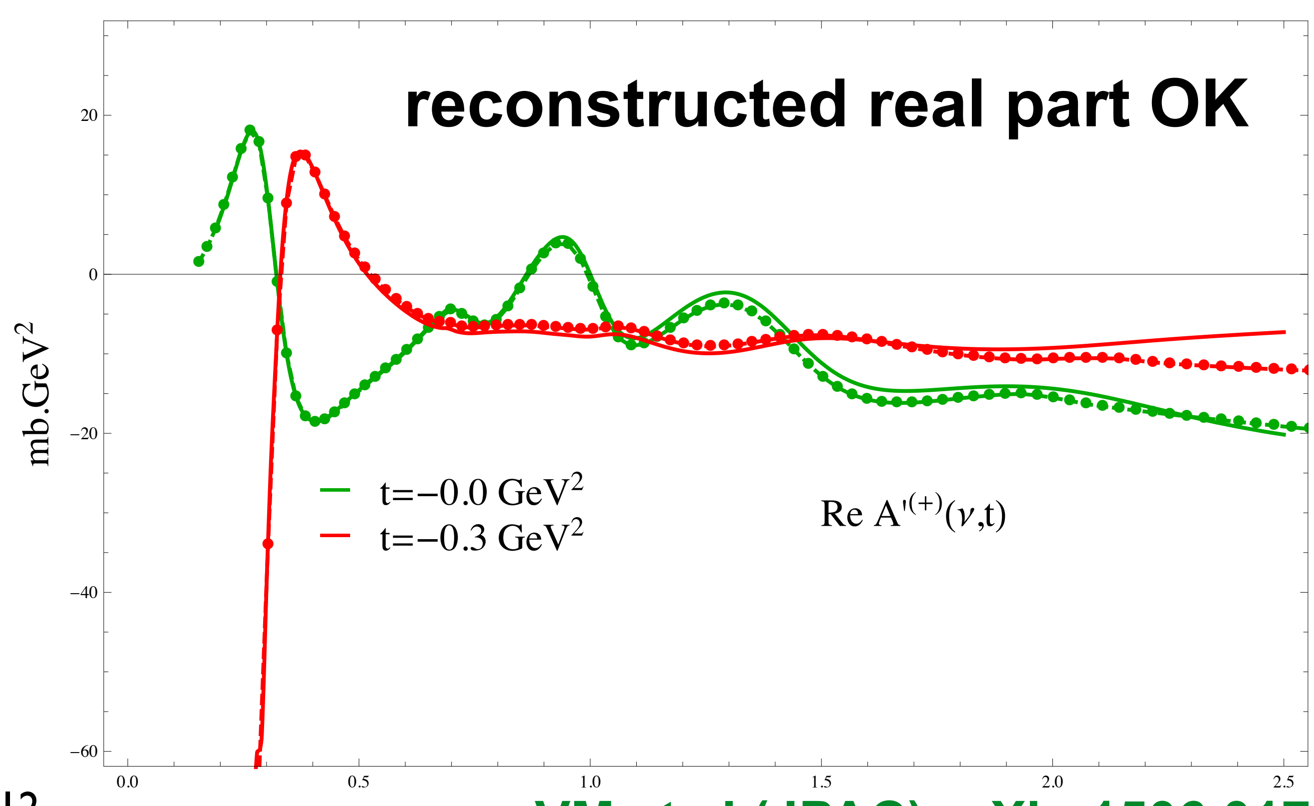
VM et al (JPAC) arXiv:1506.01764

Summary $\pi N \to \pi N$









VM et al (JPAC) arXiv:1506.01764

CGLN Phys. Rev. 106 1345

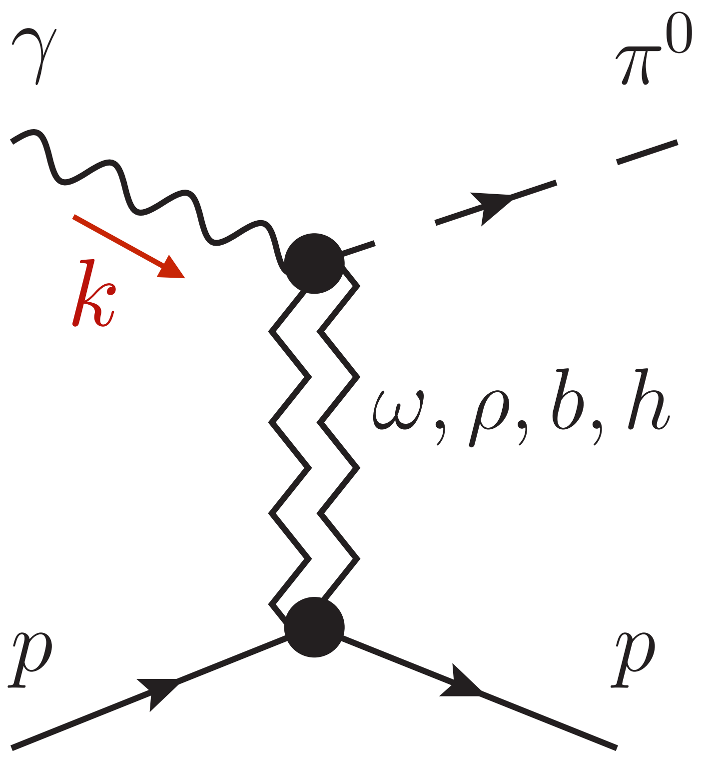
VM et al arXiv:1505.02321

$\gamma p \to \pi^0 p$

$$\bar{u}_1 \left[A_1 M_1 + A_2 M_2 + A_3 M_3 + A_4 M_4 \right] u_2$$

$$M_1 = \gamma_5 \gamma_\mu \gamma_\nu \ \epsilon^\mu(\lambda, k) k^\nu$$

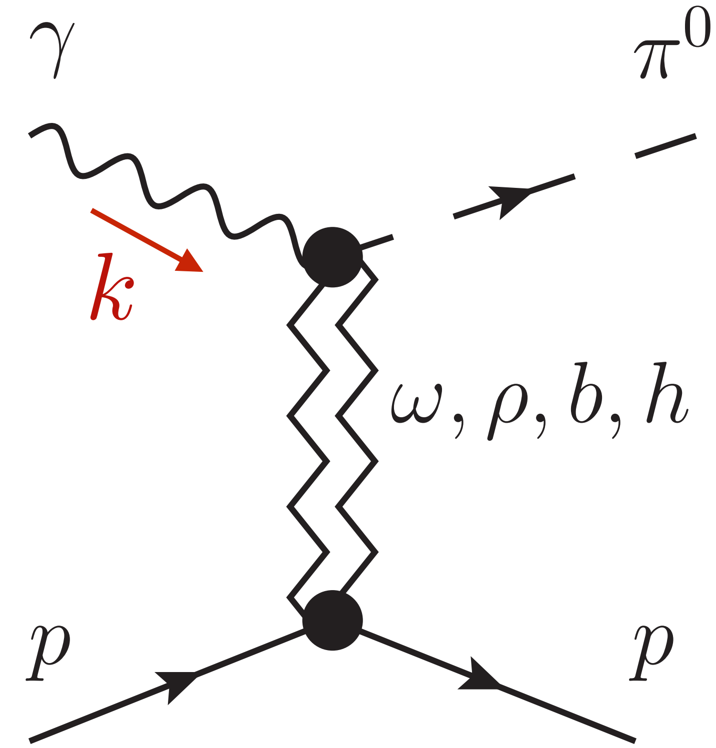
$$M_2 = \ldots$$



CGLN Phys. Rev. 106 1345
VM et al arXiv:1505.02321

$\gamma p \to \pi^0 p$

$$\bar{u}_1 \left[A_1 M_1 + A_2 M_2 + A_3 M_3 + A_4 M_4 \right] u_2$$



$$M_1 = \gamma_5 \gamma_\mu \gamma_\nu \ \epsilon^\mu(\lambda, k) k^\nu$$

$$M_2 = \ldots$$

J^{PC}

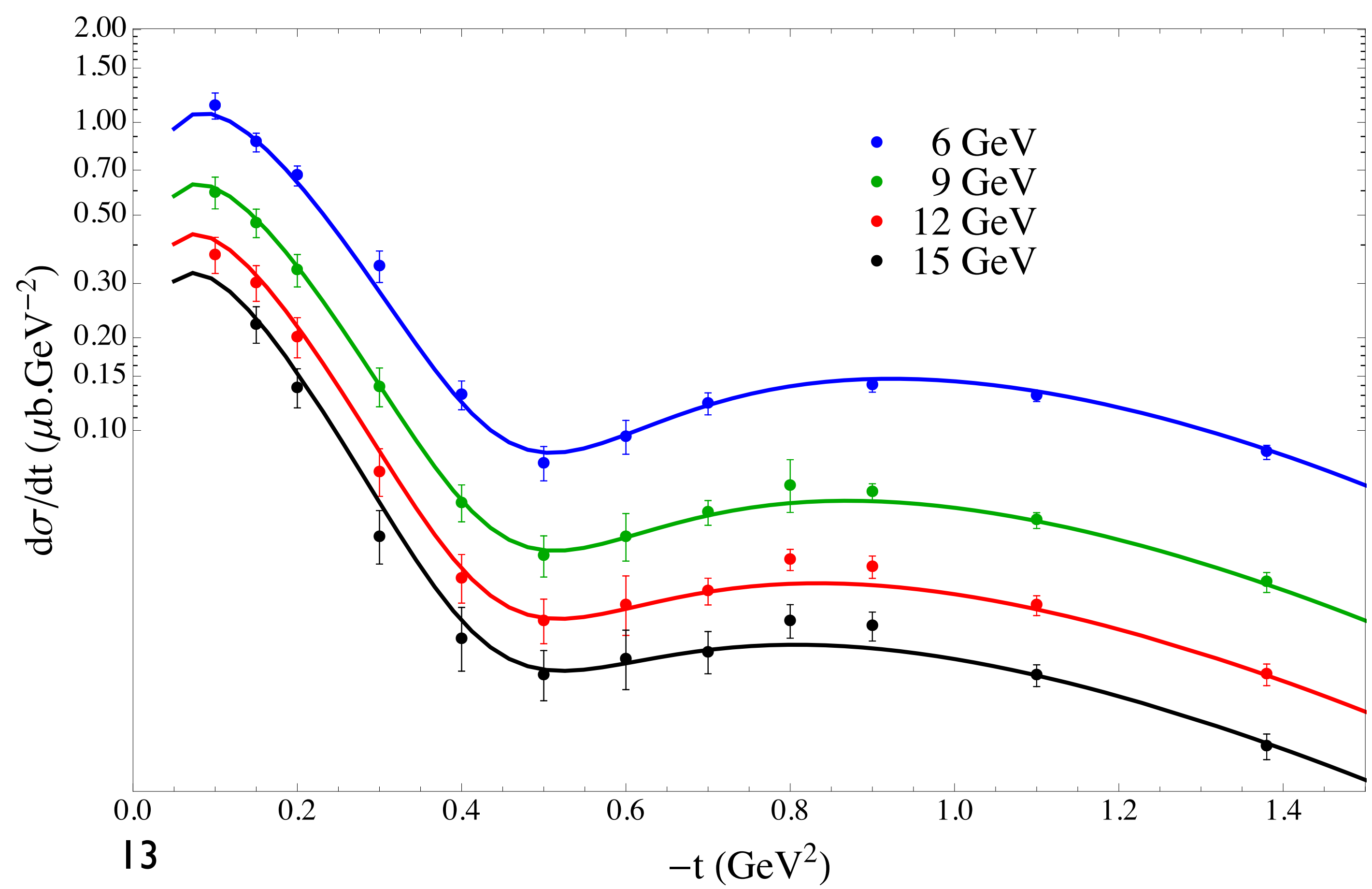
$A_1 : \{1, 3, 5, \ldots\}^{--}$ $\rho + \omega$

$A_2 : \{1, 3, 5, \ldots\}^{+-}$ $b + h$

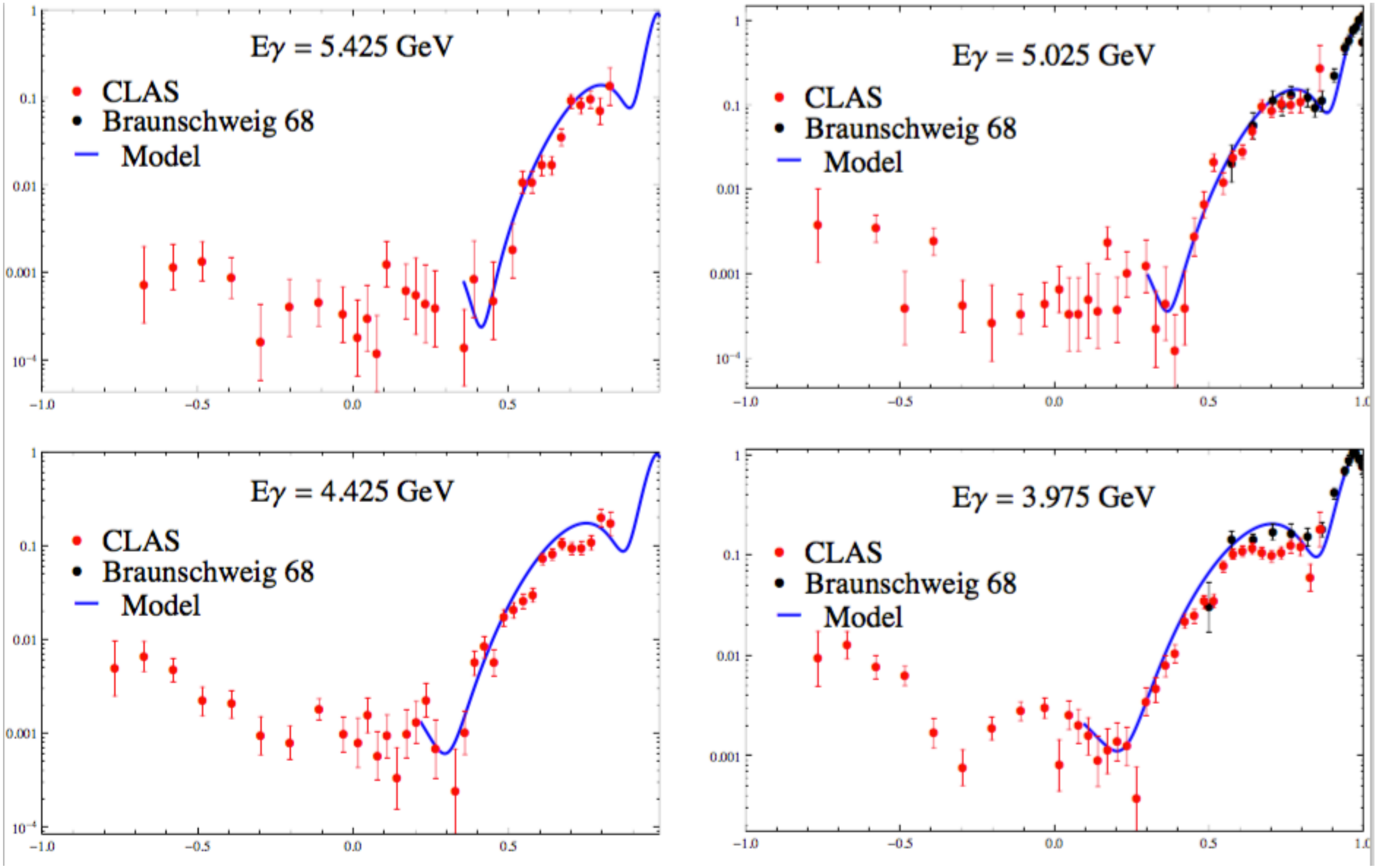
$A_3 : \{0, 2, 4, \ldots\}^{--}$

$A_4 : \{1, 3, 5, \ldots\}^{--}$ $\rho + \omega$

$$A_i(s, t) = \beta_{ij}(t) s^{\alpha_j(t)}$$



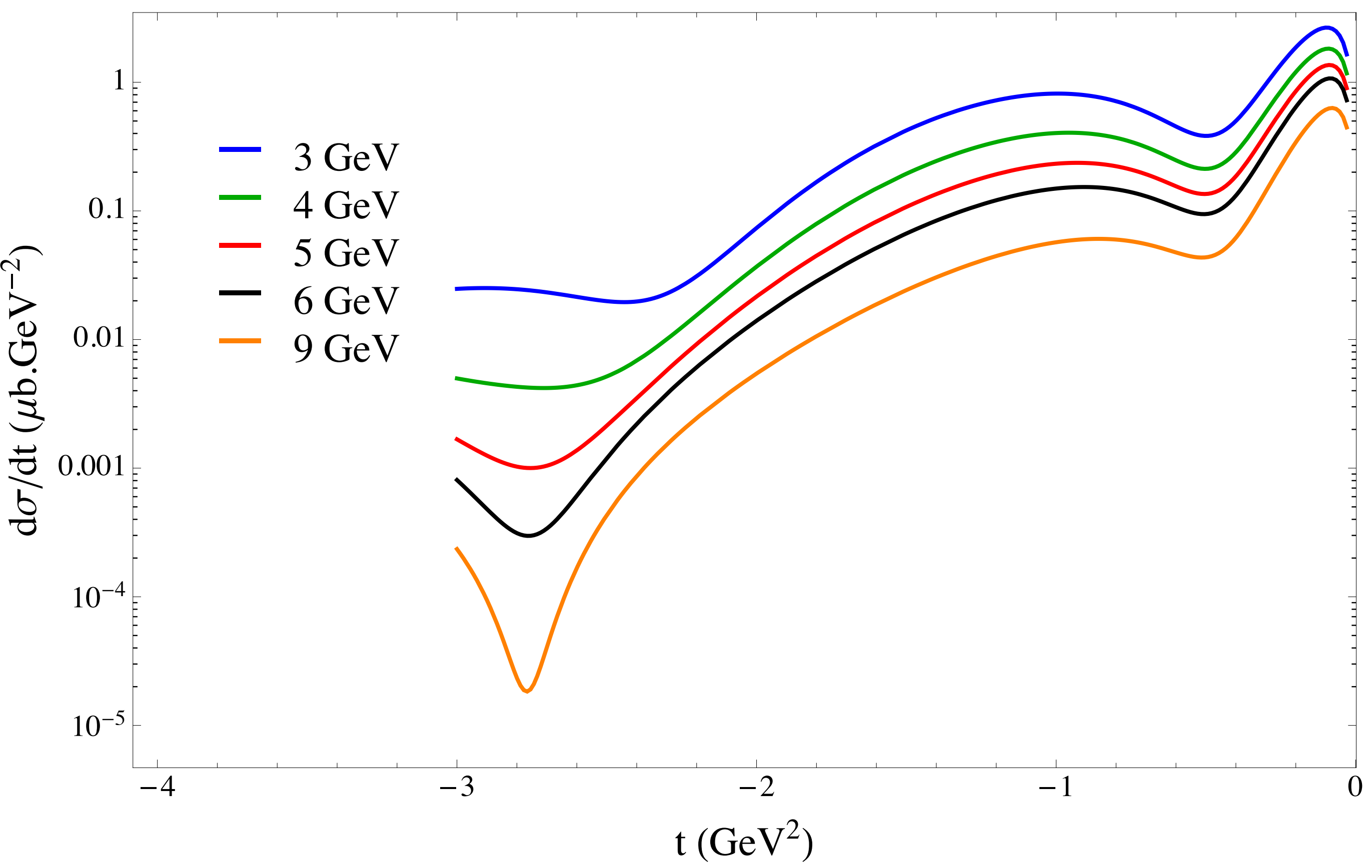
CLAS Preliminary Data

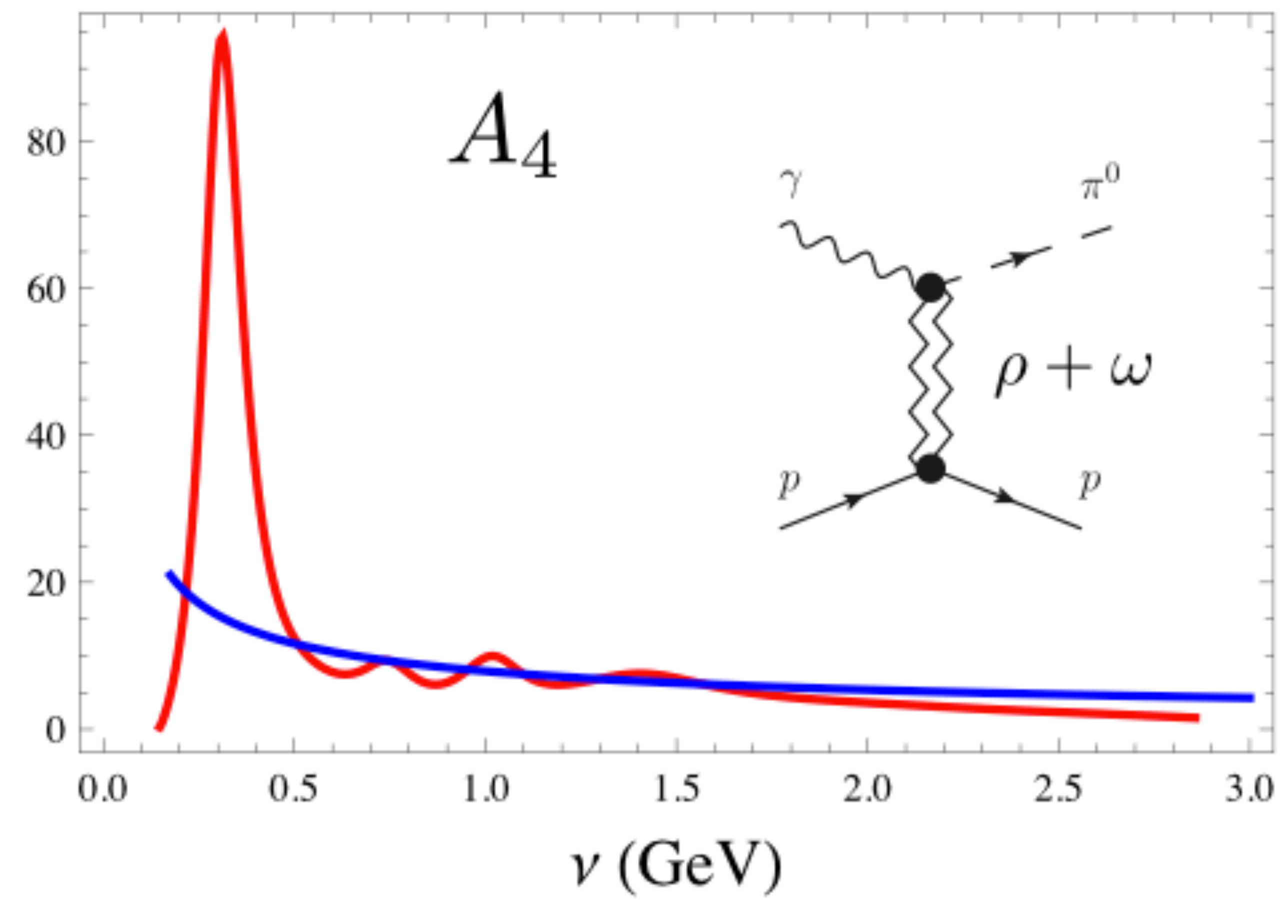


Data from M. C. Kunkel

Predictions for CLAS and GlueX

VM et al arXiv:1505.02321





Interactive webpage: http://www.indiana.edu/~jpac/index.html

INDIANA UNIVERSITY

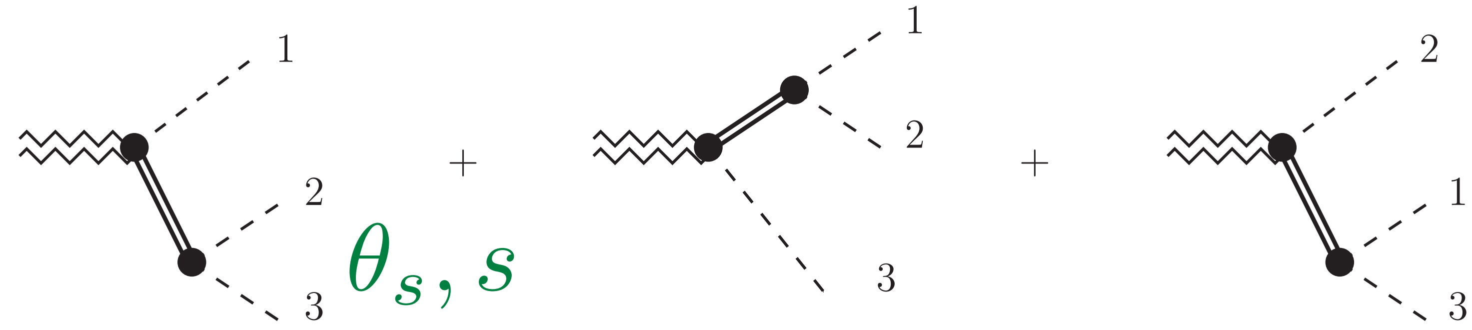
Jefferson Lab
Thomas Jefferson National Accelerator Facility

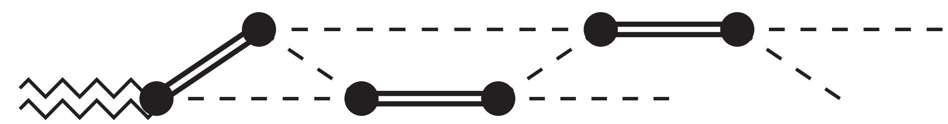
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

NSF

Three Pions

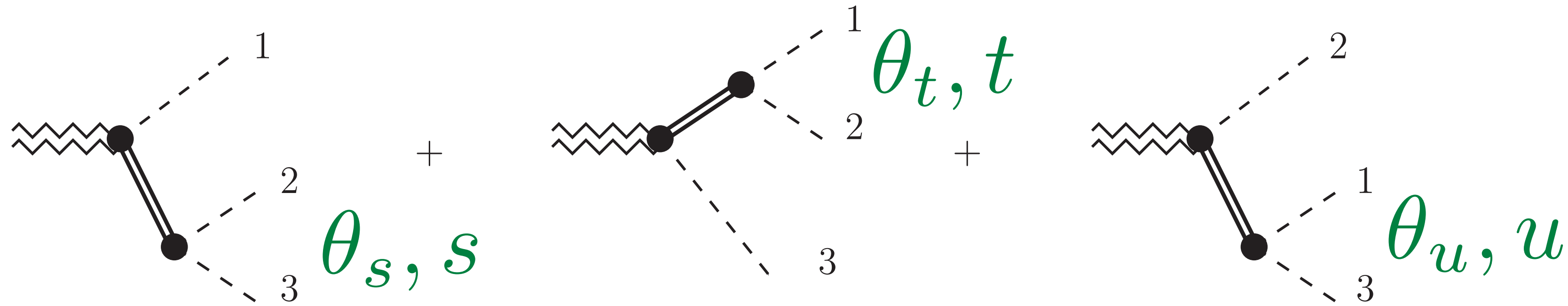
$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$



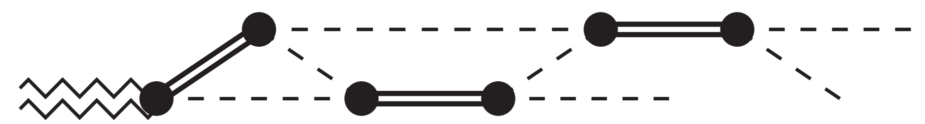


Three Pions

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$



$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$



Isobar approximation

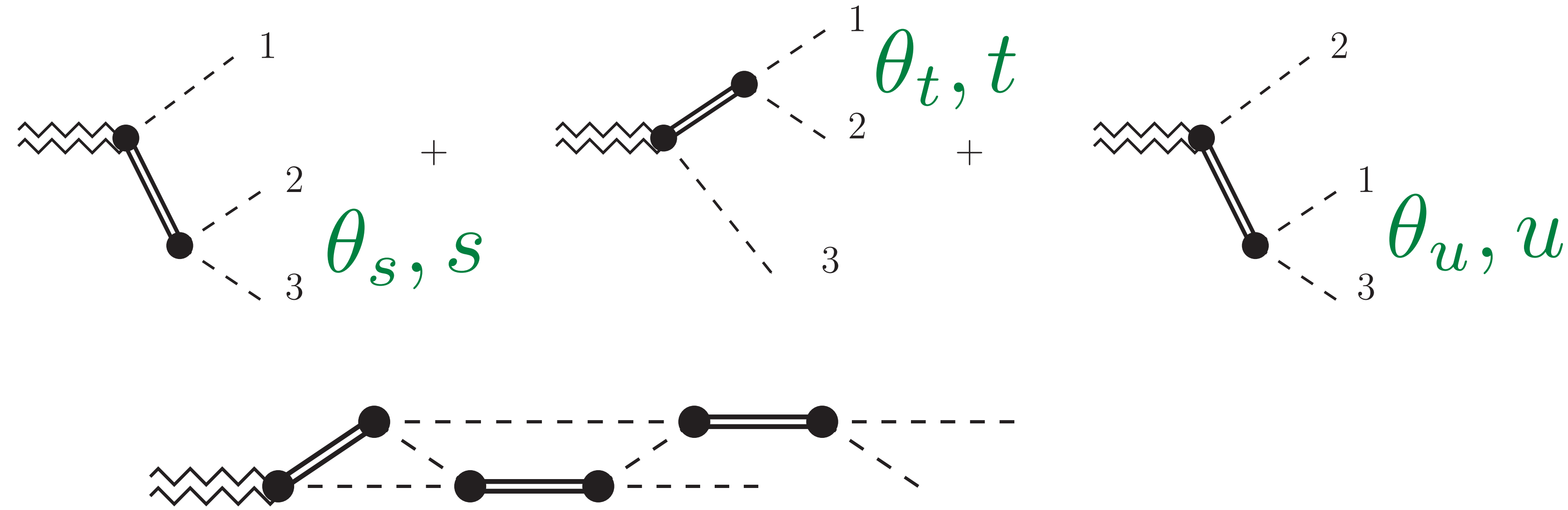
violation of unitarity

Rescattering effets

Khuri-Treiman equations

Three Pions

$$A_\lambda(s,t) = \sum_J^\infty (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$



$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$

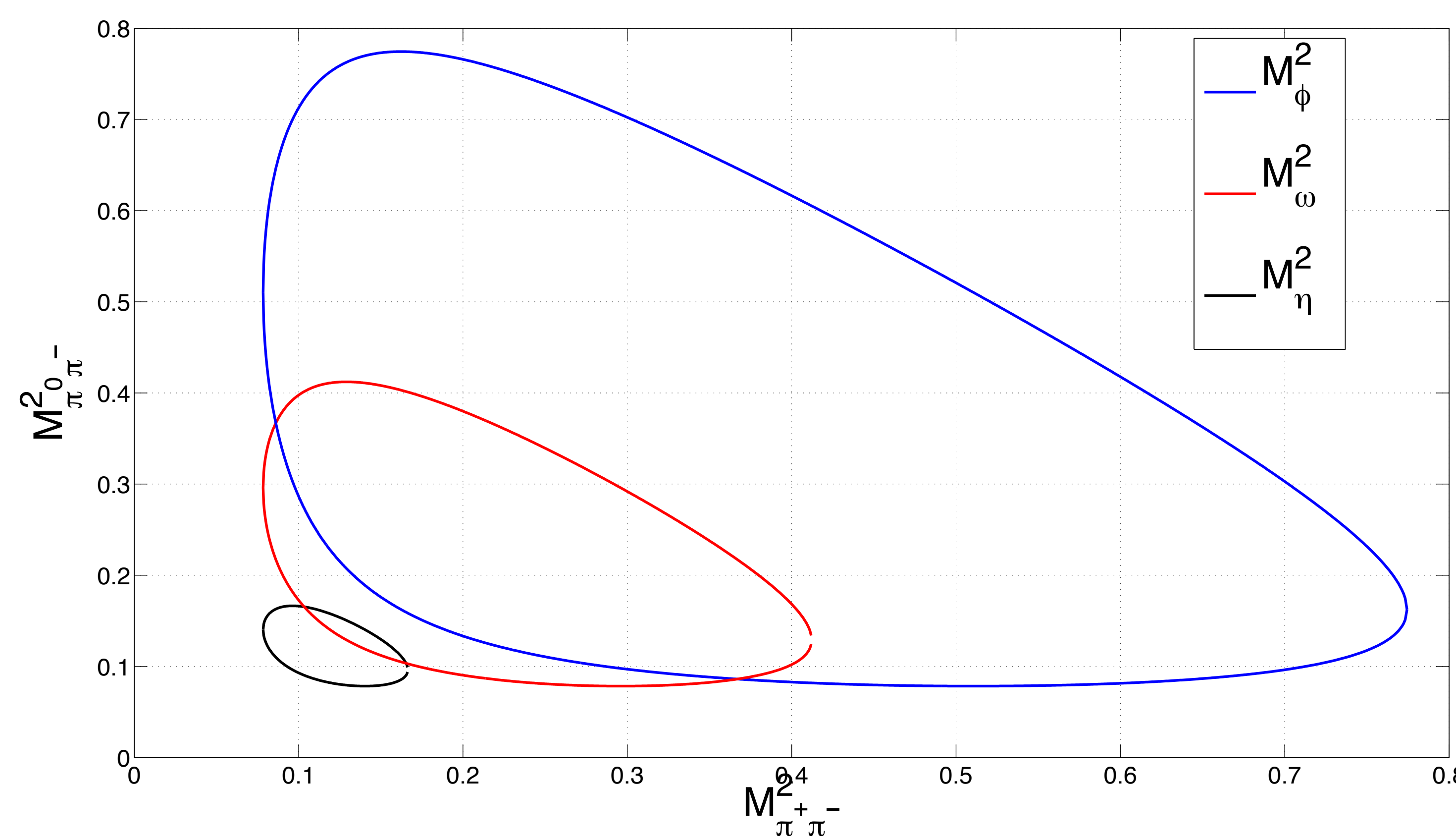
Isobar approximation
violation of unitarity

Rescattering effets
Khuri-Treiman equations

Application to

$$\eta \to \pi^+\pi^-\pi^0$$

$$\omega \to \pi^+\pi^-\pi^0$$



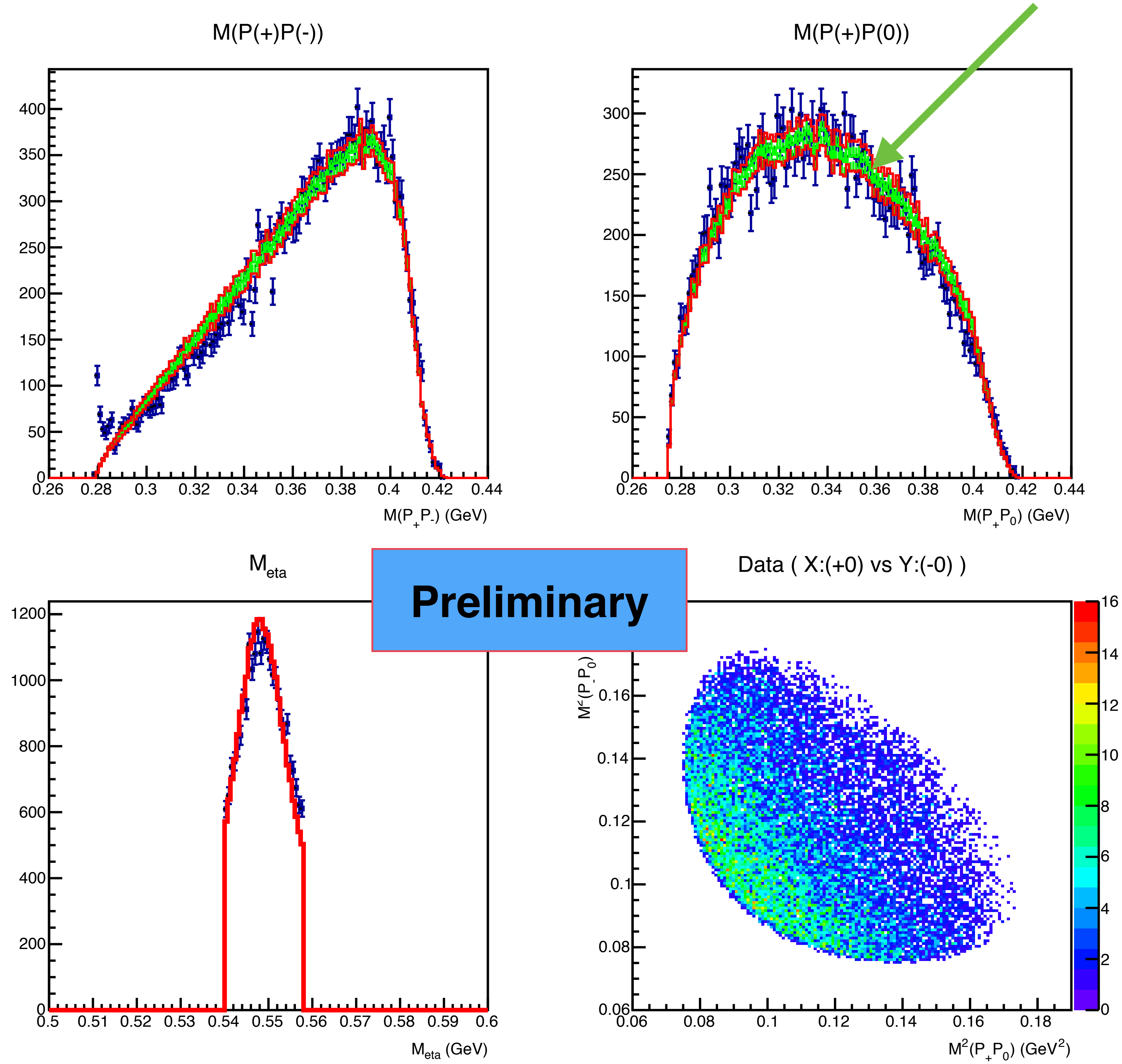
P. Guo, I. Danilkin, D. Schott (JPAC)

$$\eta \to \pi^+ \pi^- \pi^0$$

Truncated PW: S and P waves

g12 CLAS6 data

3 body effect



Toolbox for exp. (Amplitudes in C++)

P. Guo et al (JPAC)
arXiv:1505.01715

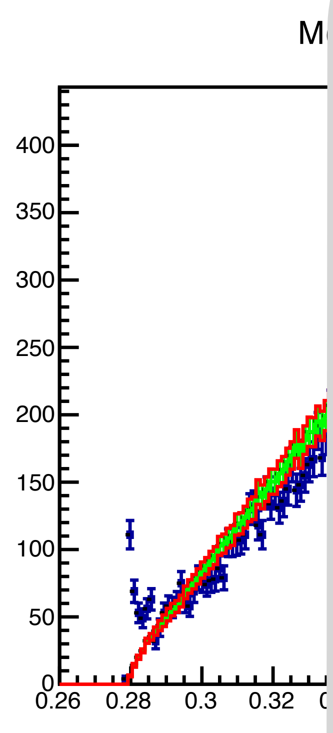
$$\eta \to \pi^+\pi^-\pi^0$$

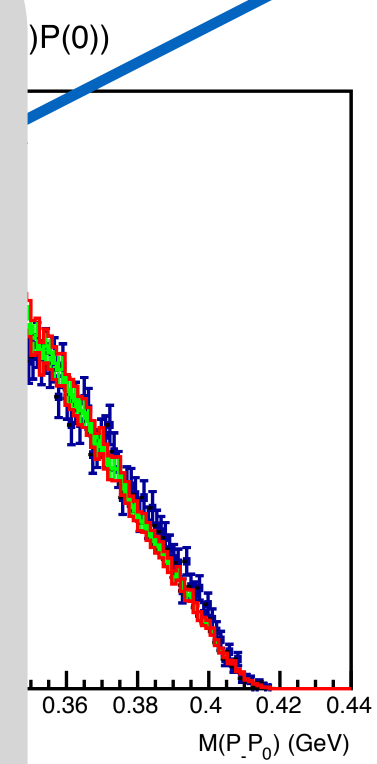
Truncated PW: S and P waves

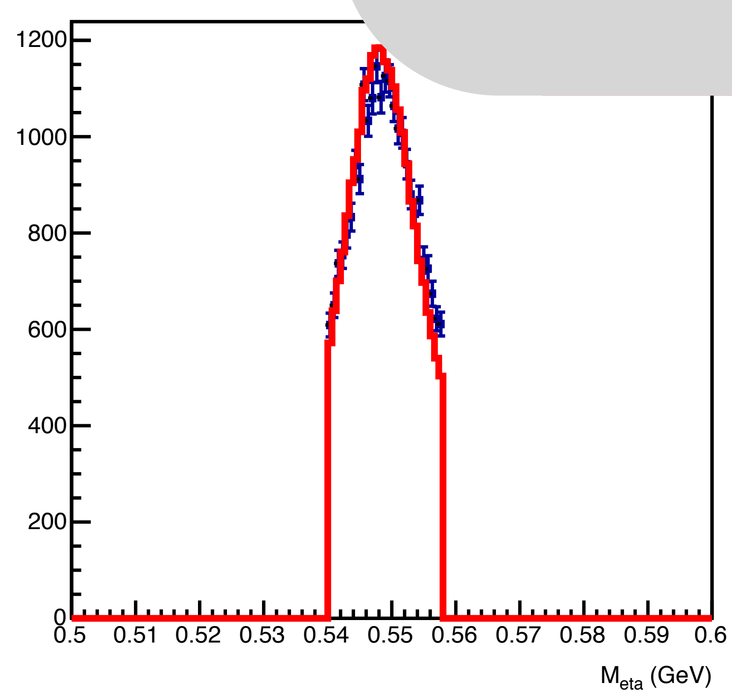
g12 CLAS6 data

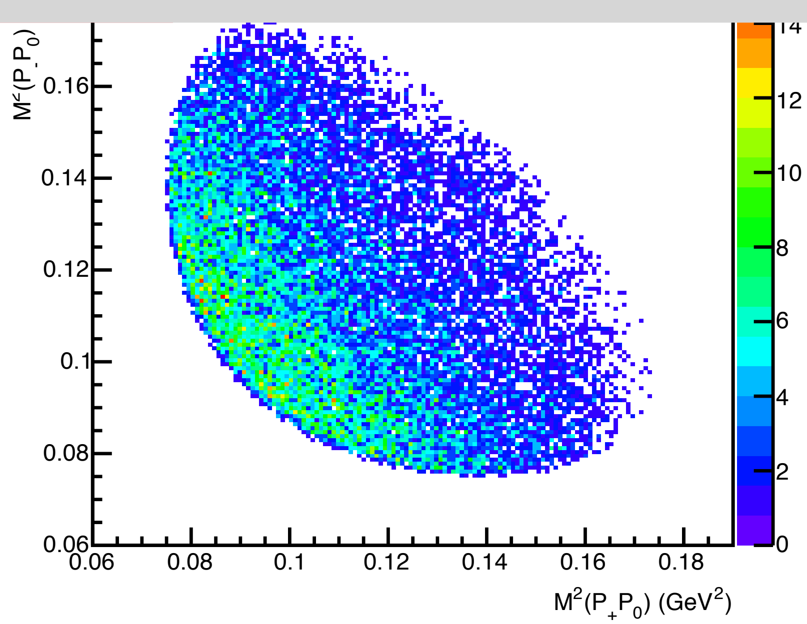
$$Q^2 = \frac{m_s^2 - (m_u + m_d)^2/4}{m_d^2 - m_u^2}$$

WASA@COSY	$Q = 21.4 \pm 0.4$
CLAS@CEBAF	**in preparation**
KLOE@DAPHNE	**in preparation**





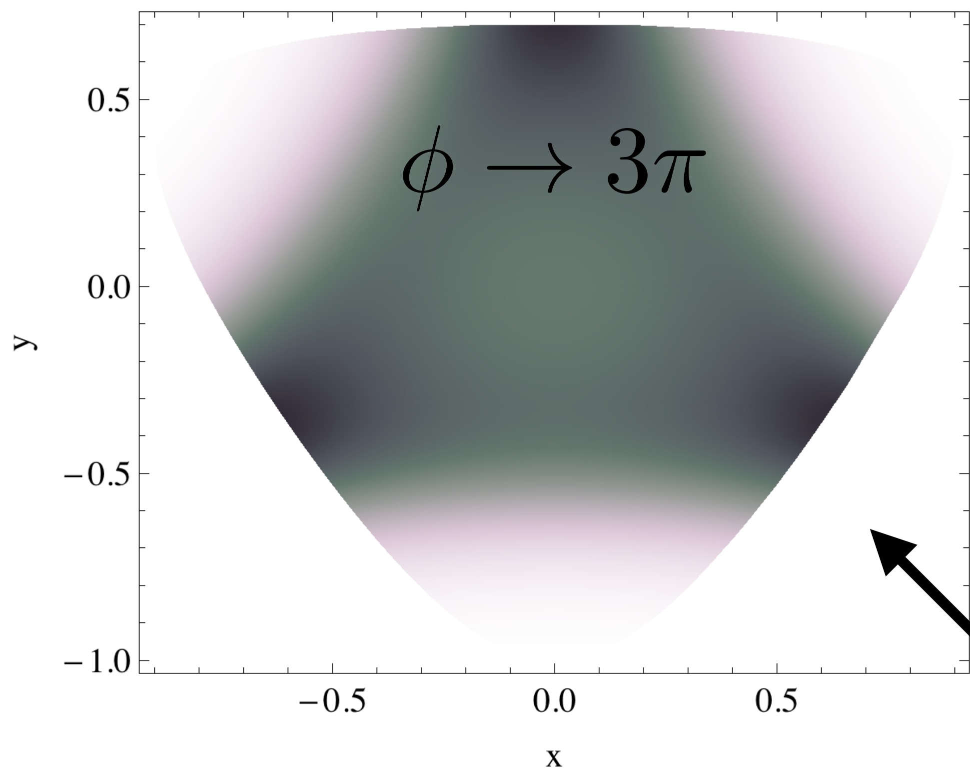


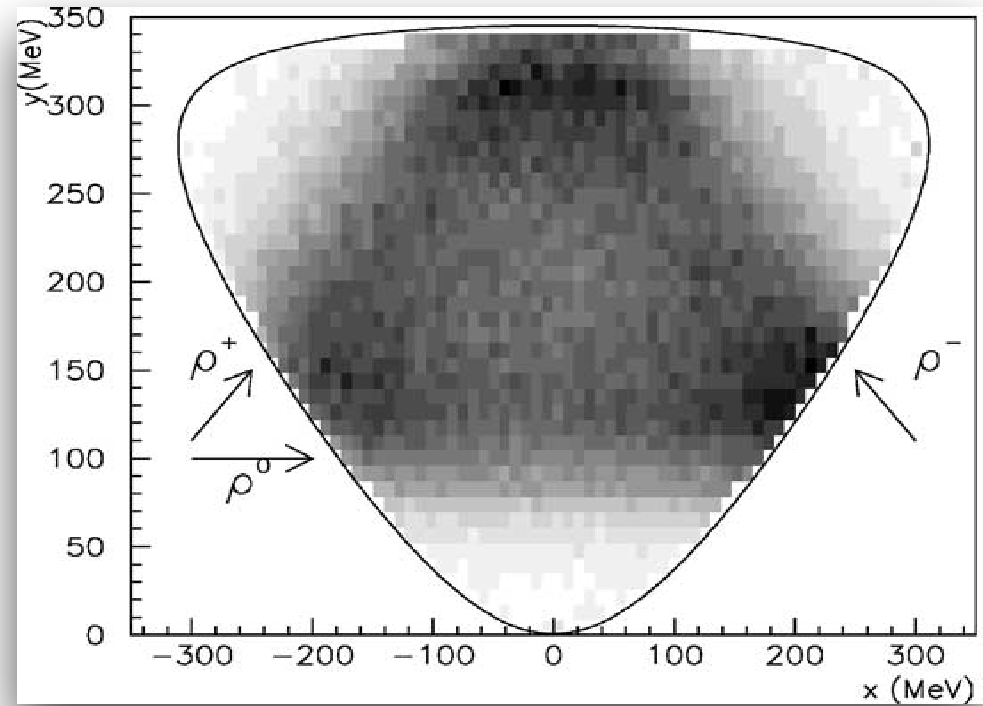


Toolbox for exp. (Amplitudes in C++)

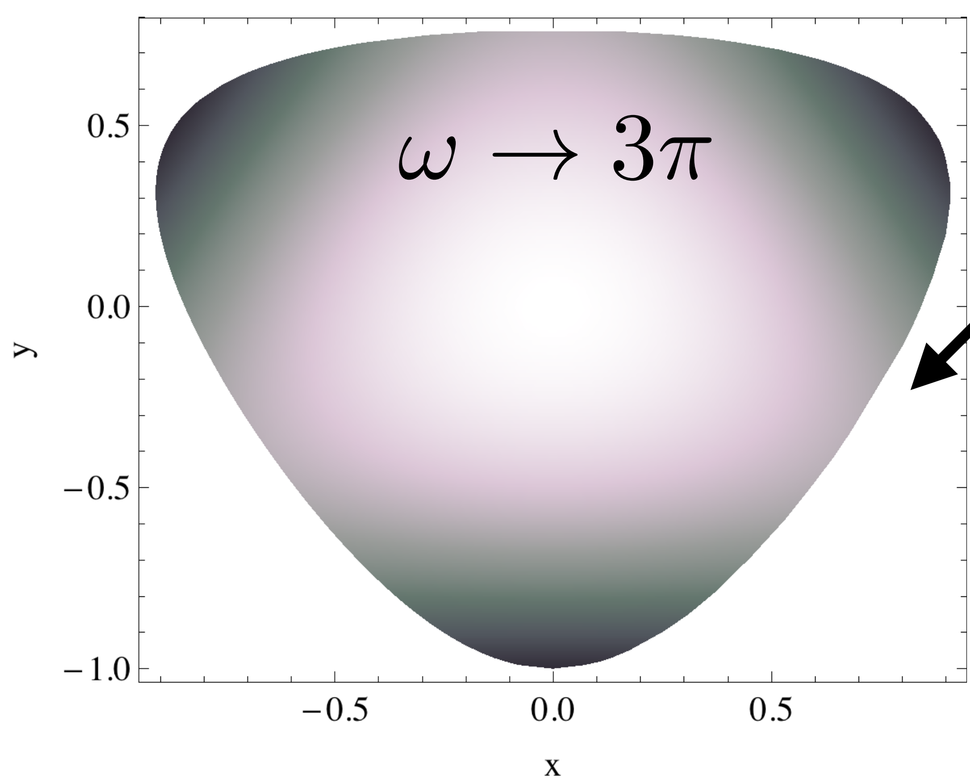
P. Guo et al (JPAC) arXiv:1505.01715

$\omega/\phi \to \pi^+\pi^-\pi^0$





KLOE (2003)



Theoretical predictions (only elastic cut)

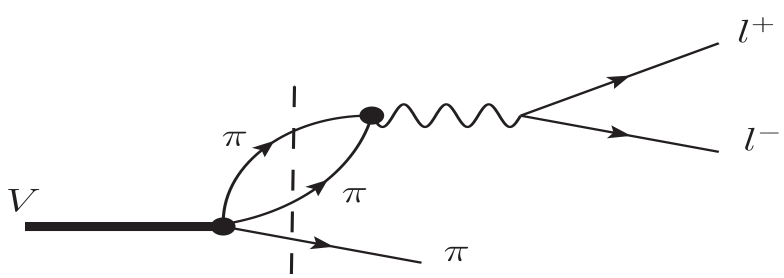
Next: flexible amplitudes parameters for inelasticities

Upcoming:
Toolbox for exp.
(Amplitudes in C++)

See B. Kubis, E. Passemar and others for similar approach

I. Danilkin et al (JPAC) PRD91 094029

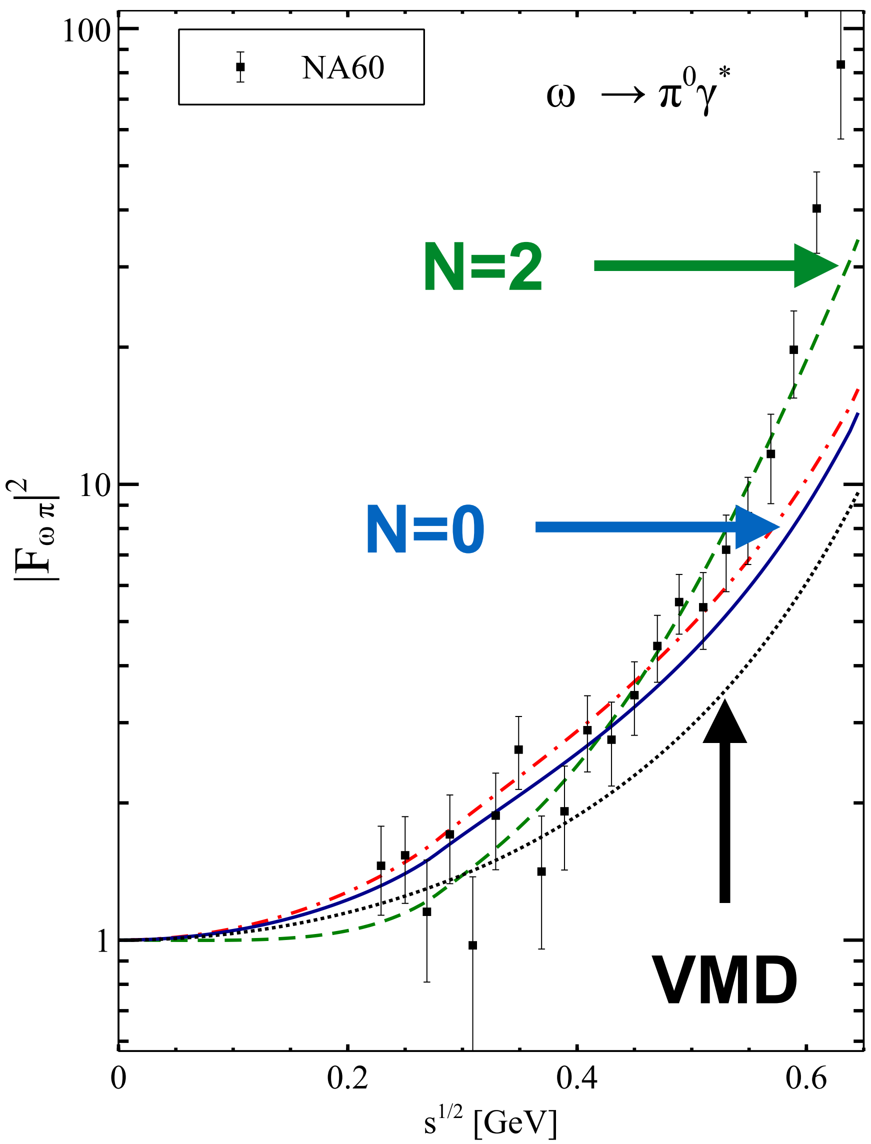
$\omega \to \gamma^* \pi^0$

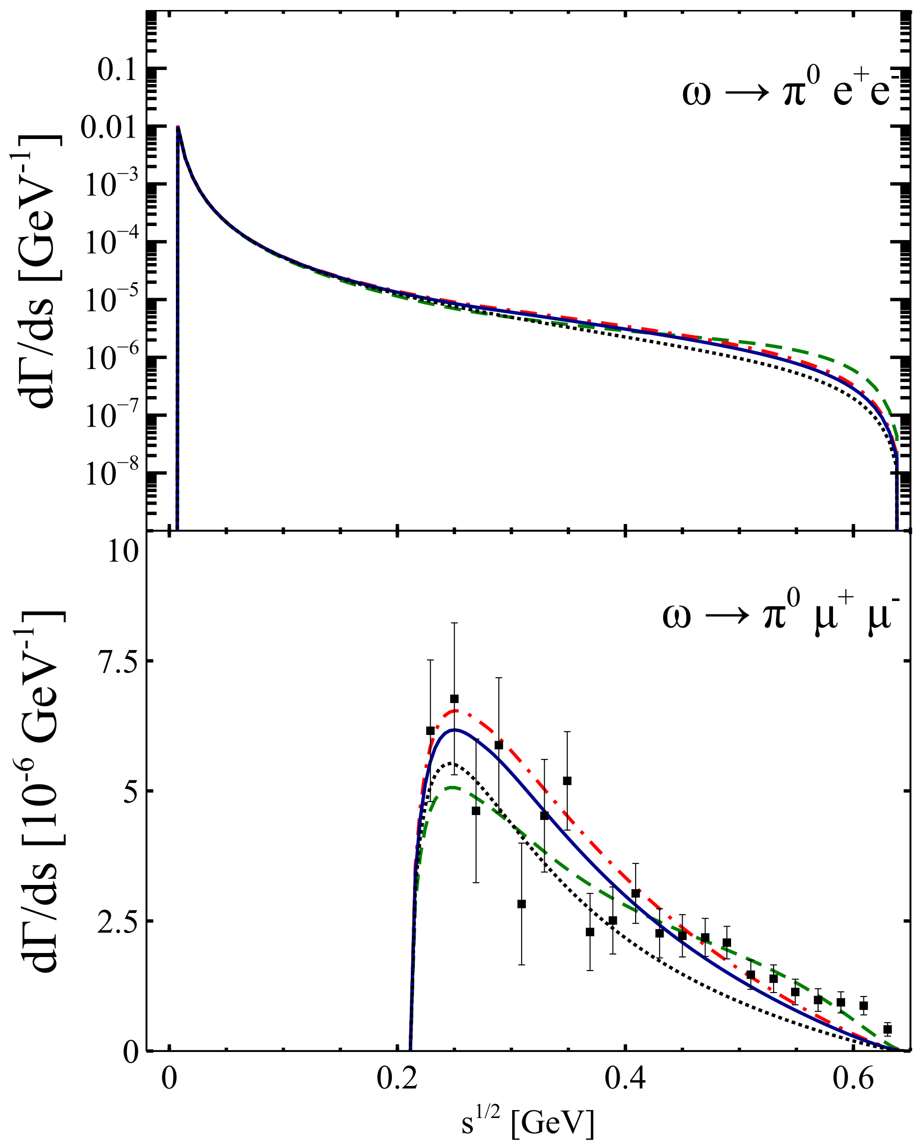


$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s'-s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$

s_i → $4M_K^2$

Inelasticities (conformal map.)



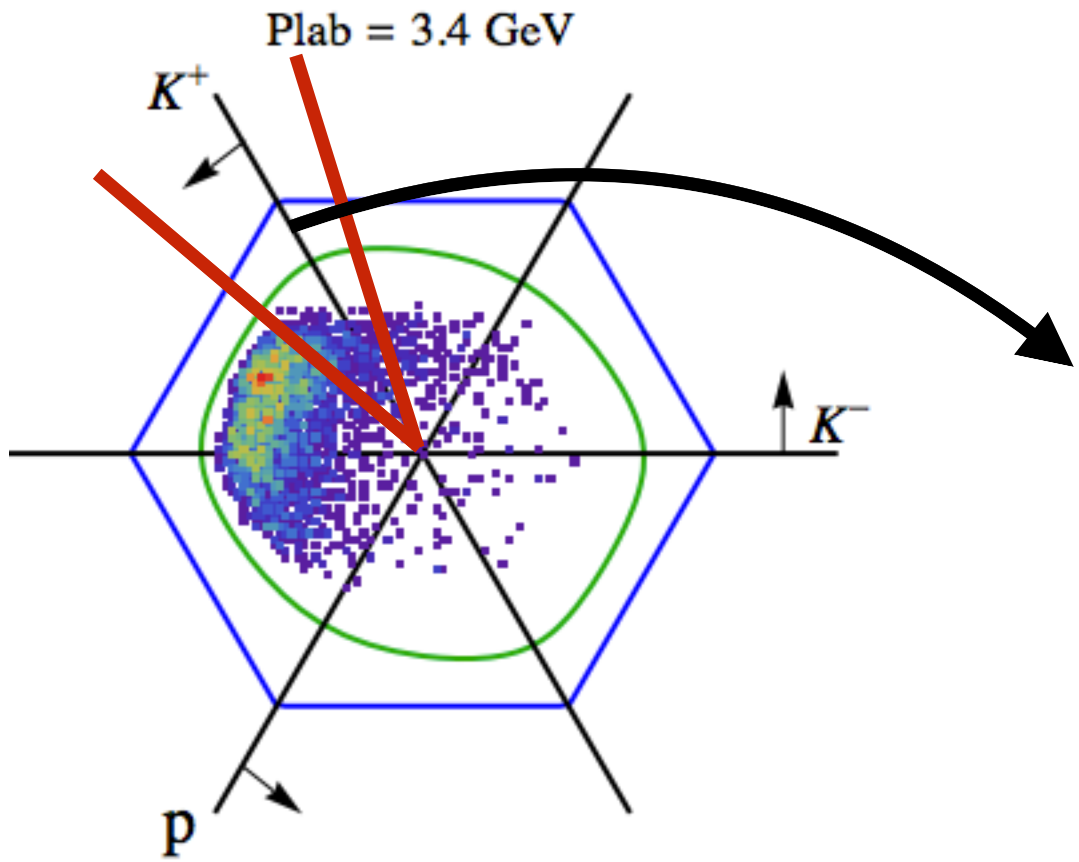


$K\bar{K}$ **cut important**

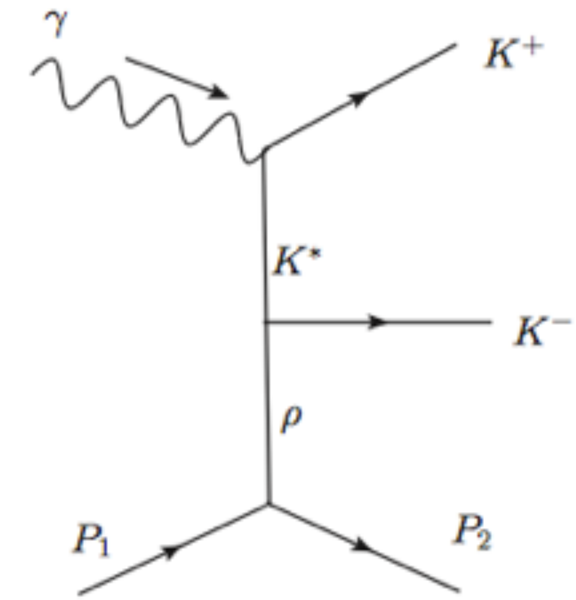
What about $\phi \to \gamma^* \pi^0$?

I. Danilkin et al (JPAC)
PRD91 094029

Veneziano Amplitudes



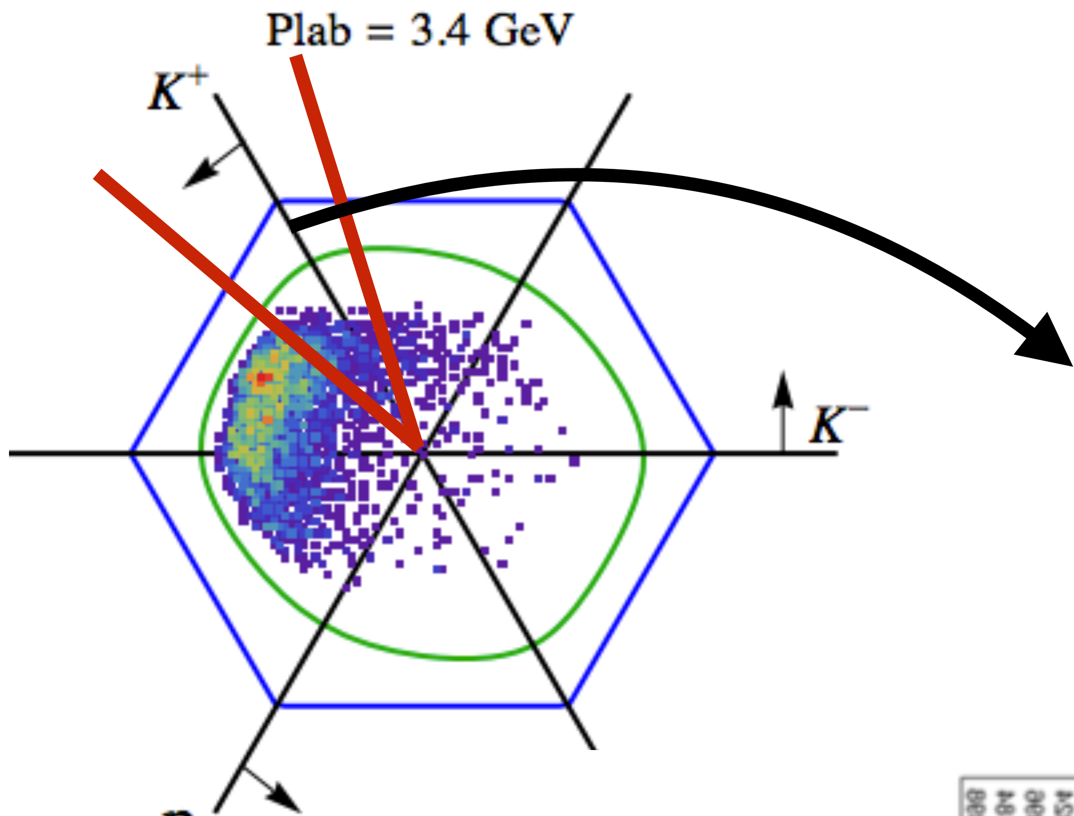
$$\gamma p \to K^+ K^- p$$



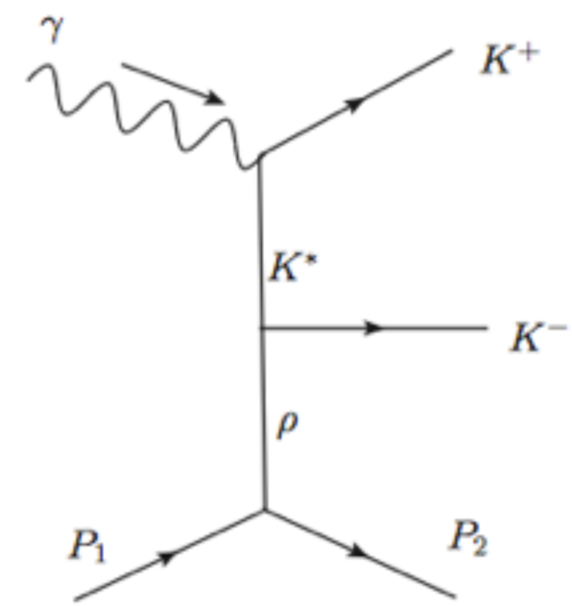
Double Regge limit

M. Shi et al (JPAC)
PRD 91, 034007

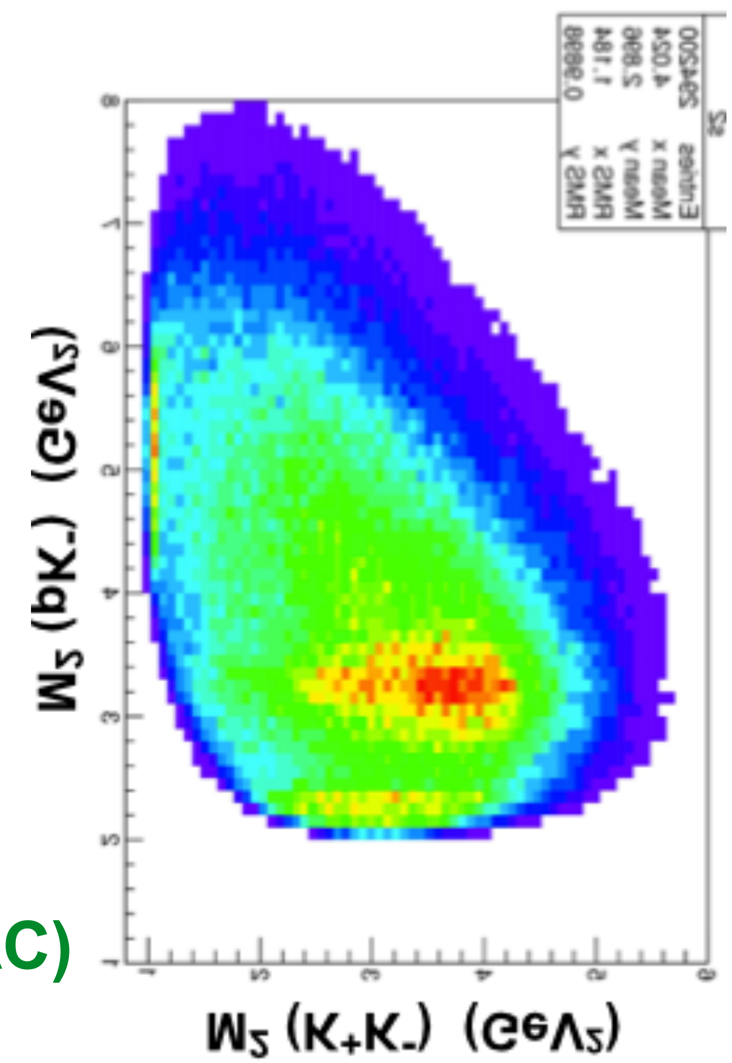
Veneziano Amplitudes

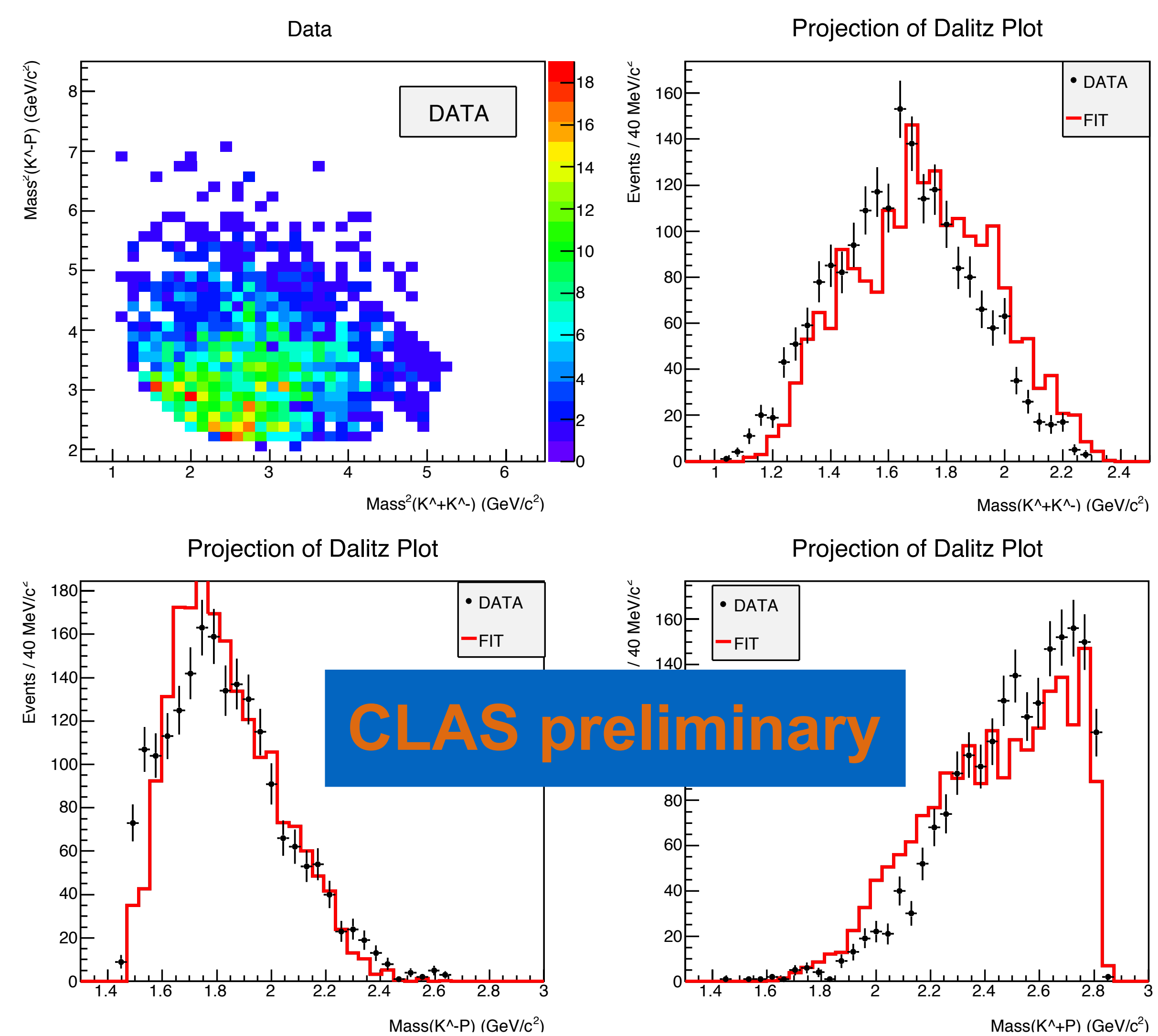


$$\gamma p \to K^+ K^- p$$



Double Regge limit





M. Shi et al (JPAC)
PRD 91, 034007

$\pi N \to \pi N$	VM et al (JPAC)	arXiv:1506.01764	
$\gamma p \to \pi^0 p$	VM et al	arXiv:1505.02321	
$\eta \to \pi^+ \pi^- \pi^0$	P. Guo et al (JPAC)	arXiv:1505.01715	
$\omega, \phi \to \pi^+ \pi^- \pi^0$ $\to \gamma^* \pi^0$	I. Danilkin et al (JPAC)	arXiv:1409.7708	PRD 91, 094029
$\gamma p \to K^+ K^- p$	M. Shi et al (JPAC)	arXiv:1411.6237	PRD 91, 034007
$KN \to KN$	C. Fernandez-Ramirez et al (JPAC)		in preparation

Interactive webpage: http://www.indiana.edu/~jpac/index.html

INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

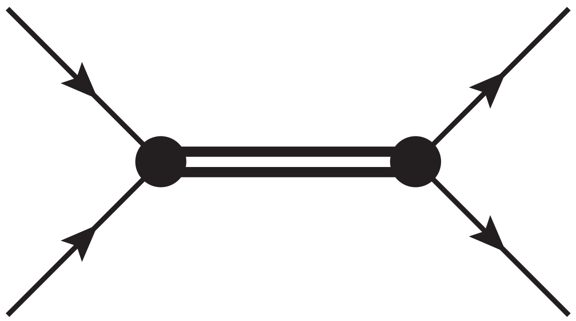
THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

NSF

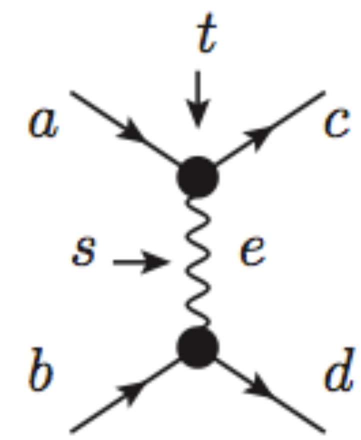
Backup Slides

$\pi^- p \to \pi^0 n$

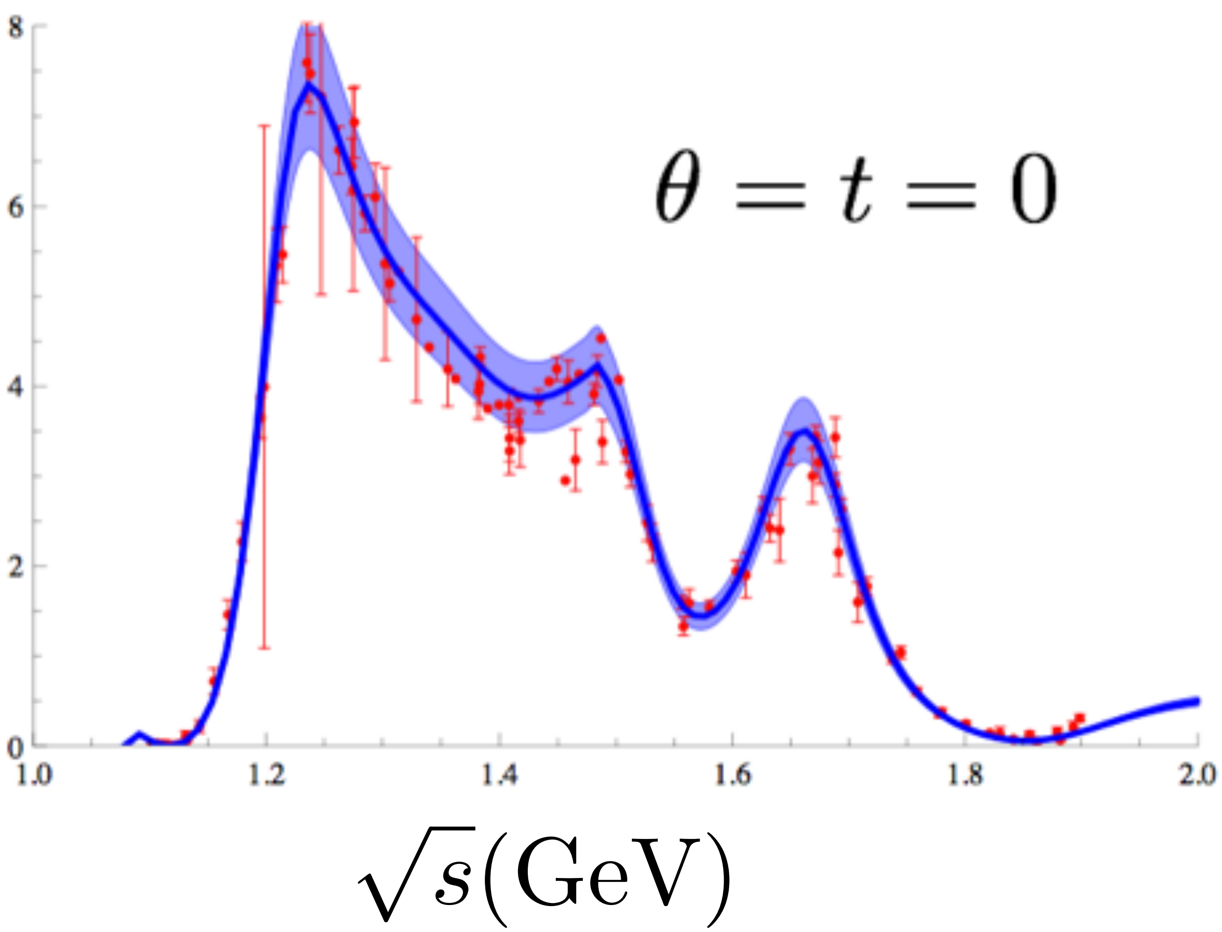
Low energy: **baryon resonances**

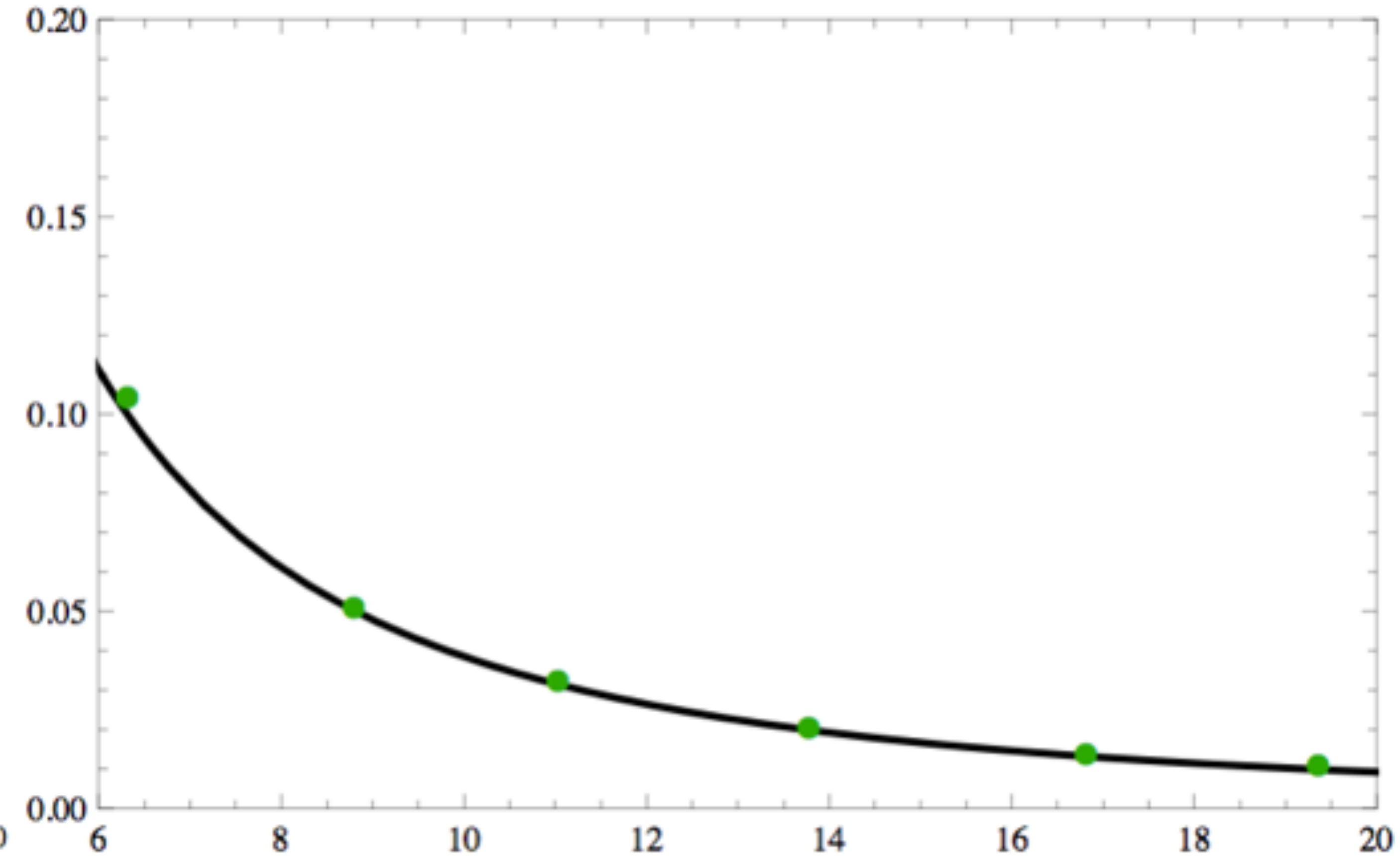


High energy: **Regge exchange**



Total cross section



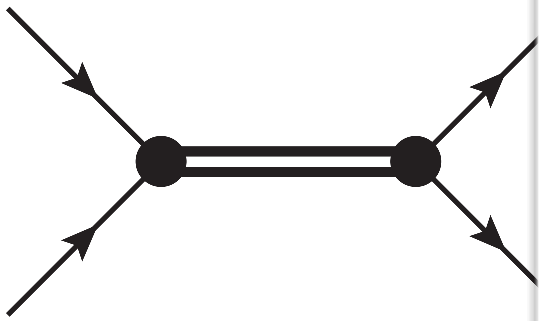


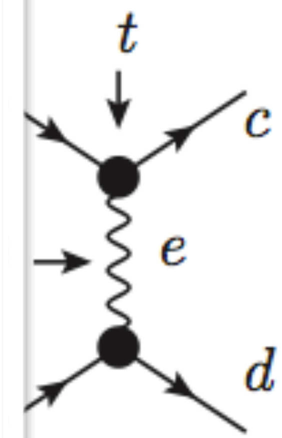
VM et al (JPAC) arXiv:1506.01764

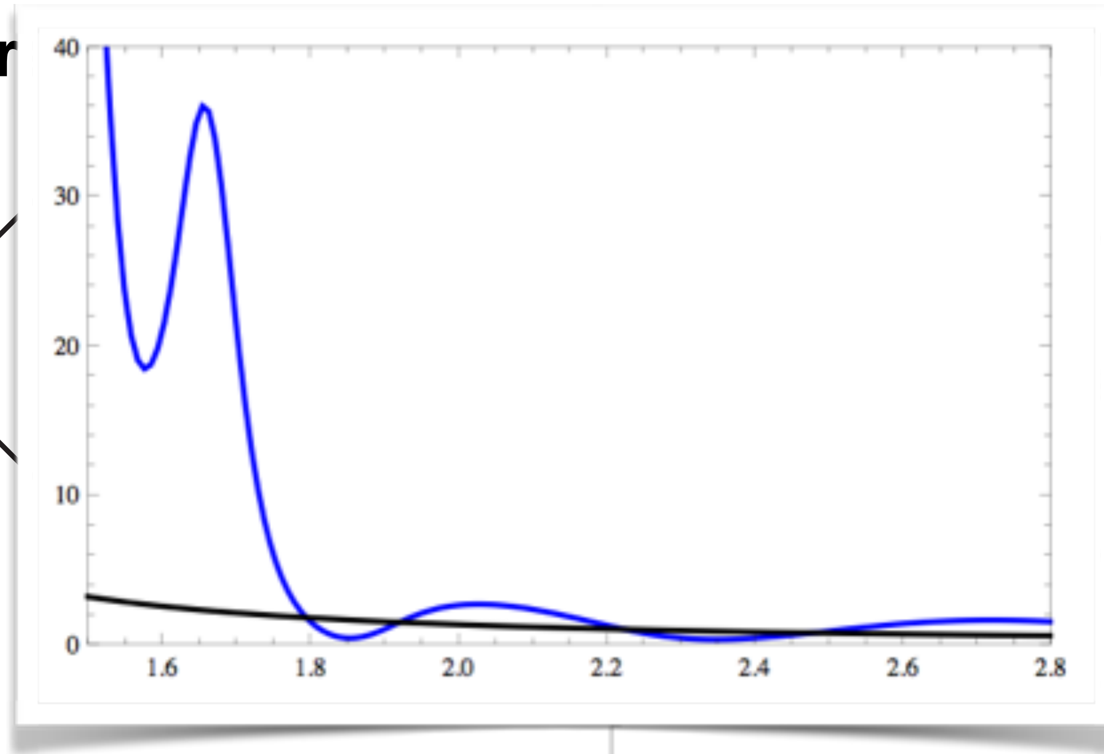
$\pi^- p \to \pi^0 n$

Low energy: **baryon r**

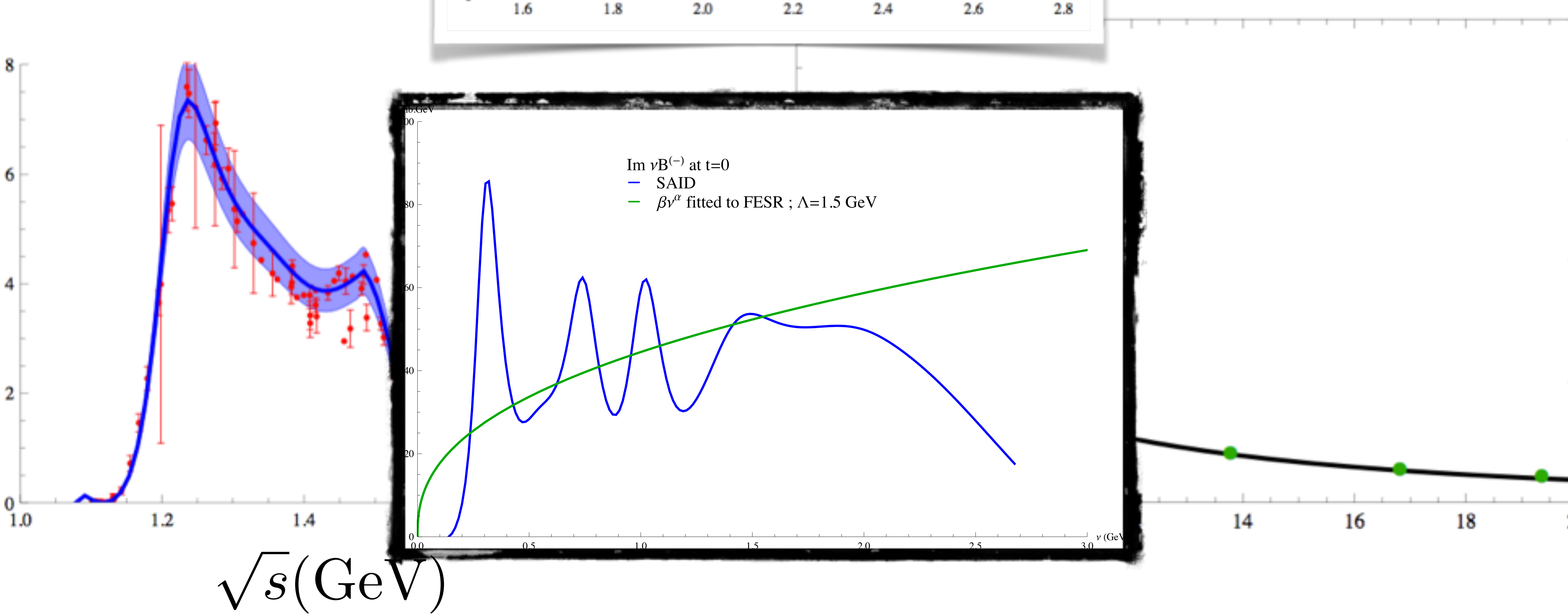
Regge exchange





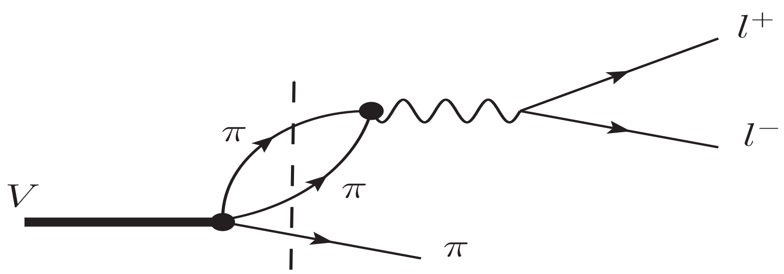


Total cross section

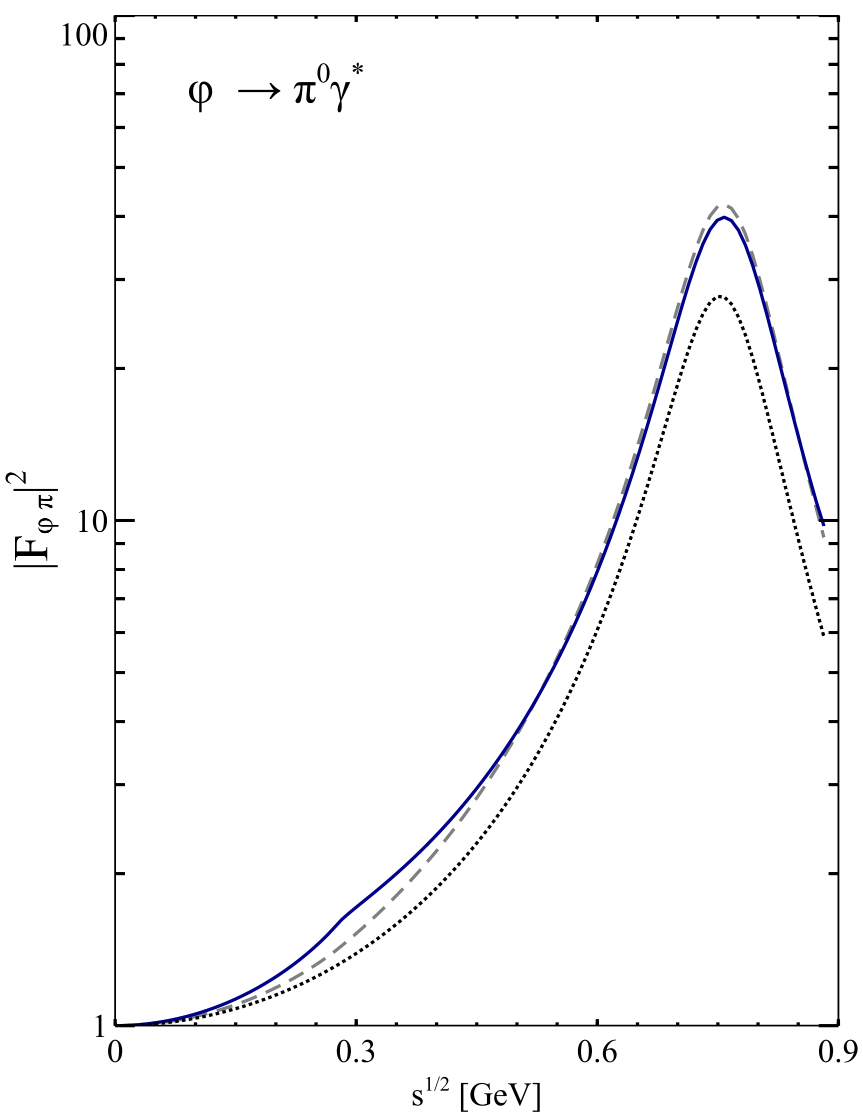


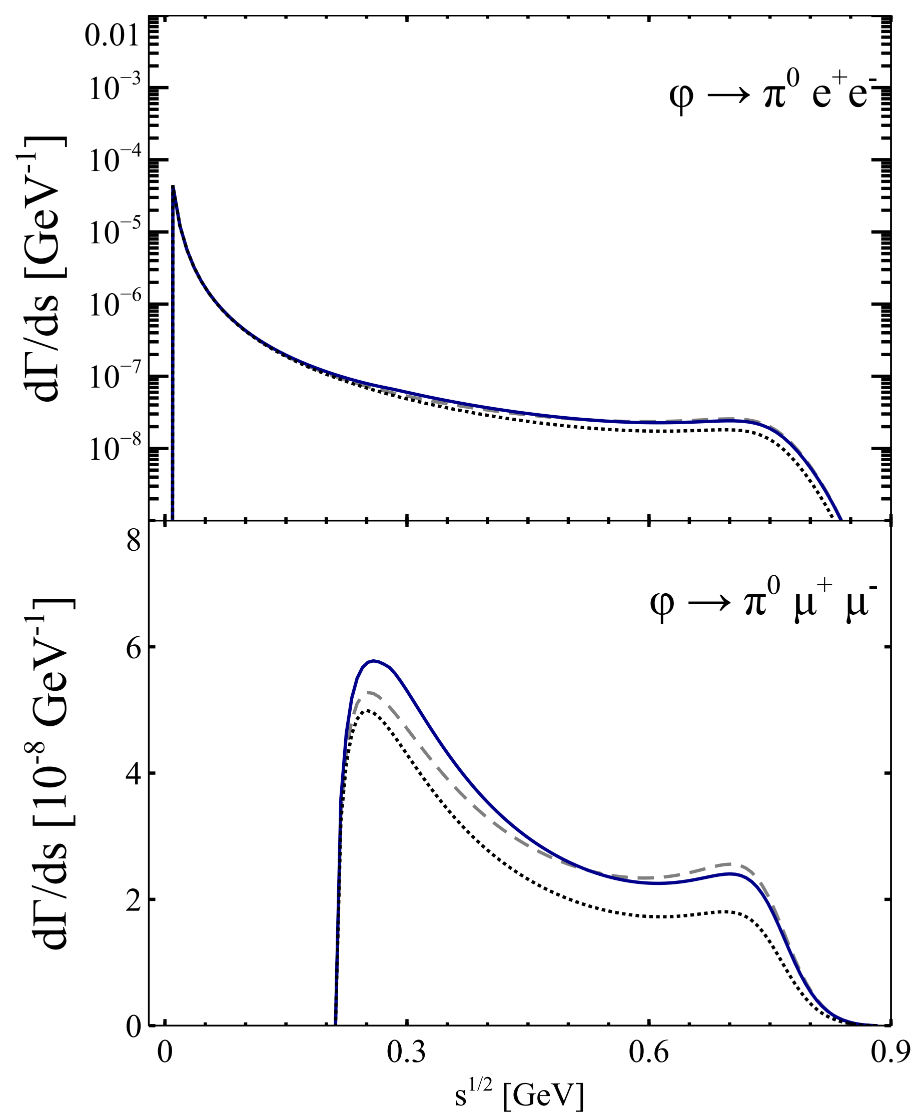
VM et al (JPAC) arXiv:1506.01764

$\phi \to \gamma^* \pi^0$



$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$





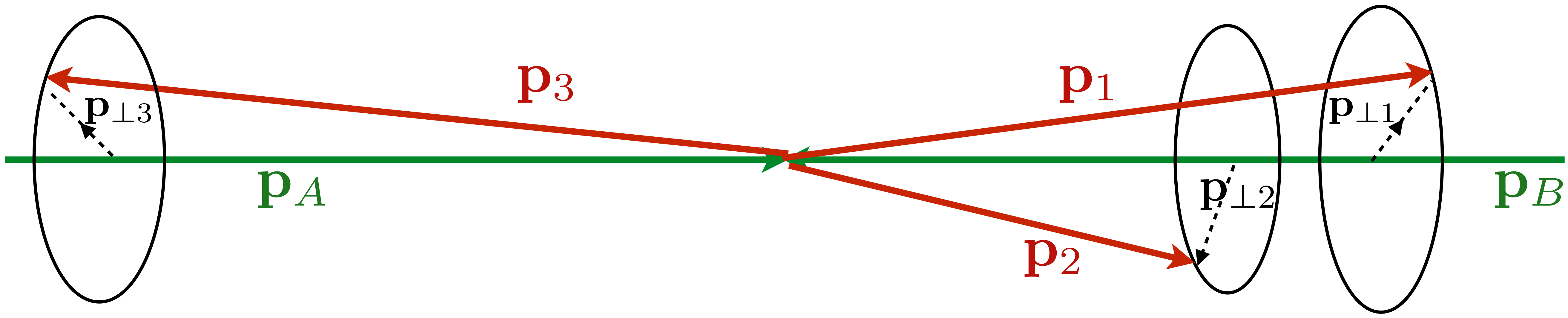
Data are welcome !

future data from VEPP-2000 thanks S. Eidelman !

I. Danilkin
A. Szczepaniak
(in preparation)

[Van Hove 1969]

Longitudinal Plot

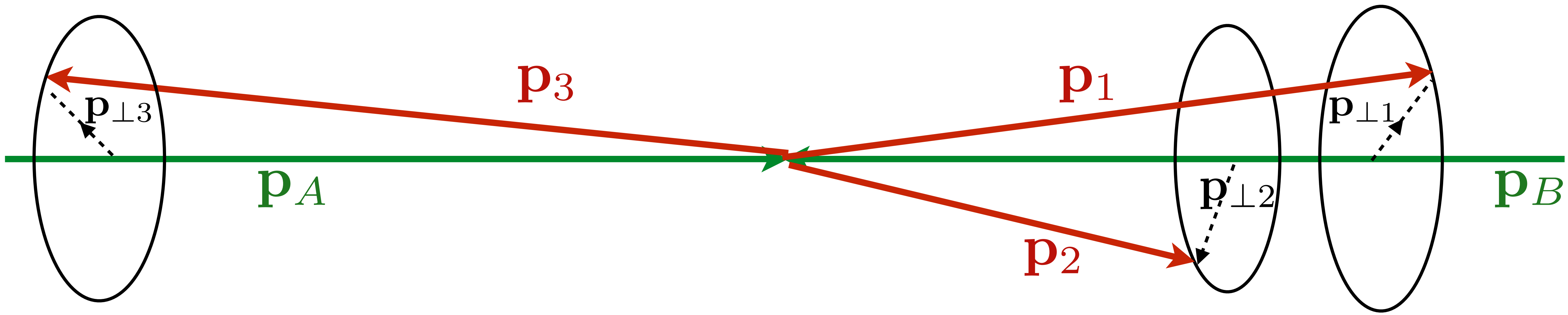


$$\mathbf{p}_i = q_i + \mathbf{p}_{\perp i}$$

$$\sum_{i=1}^{N} q_i = 0$$

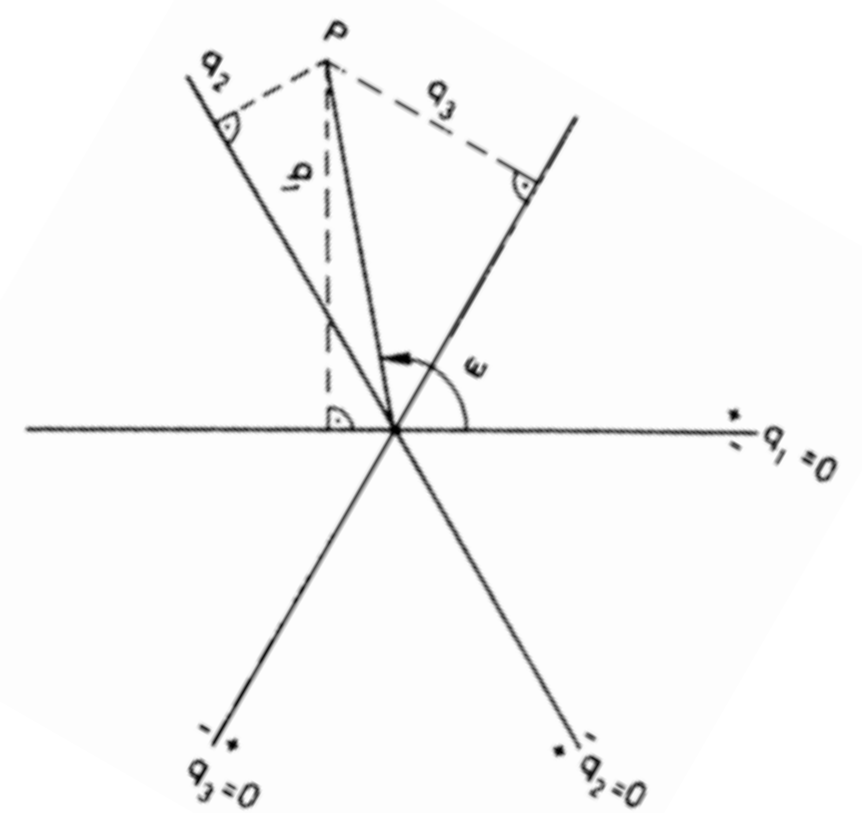
[Van Hove 1969]

Longitudinal Plot



$$\mathbf{p}_i = q_i + \mathbf{p}_{\perp i}$$

$$\sum_{i=1}^{N} q_i = 0$$



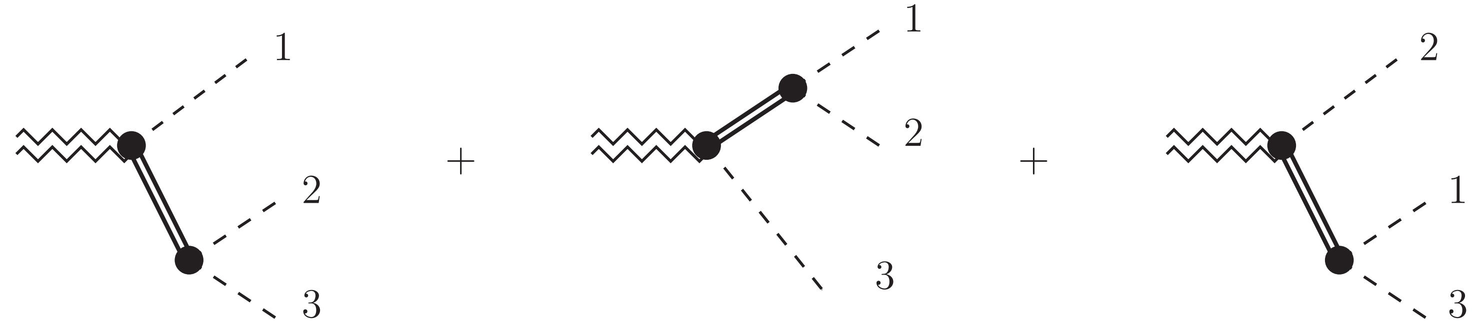
$$q_1 = r \sin \omega$$

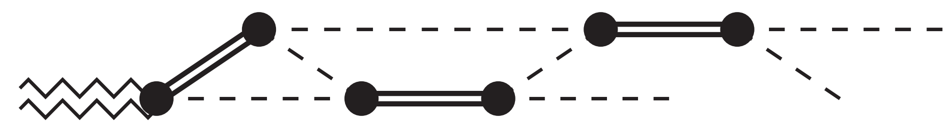
$$q_2 = r \sin (2\pi/3 + \omega)$$

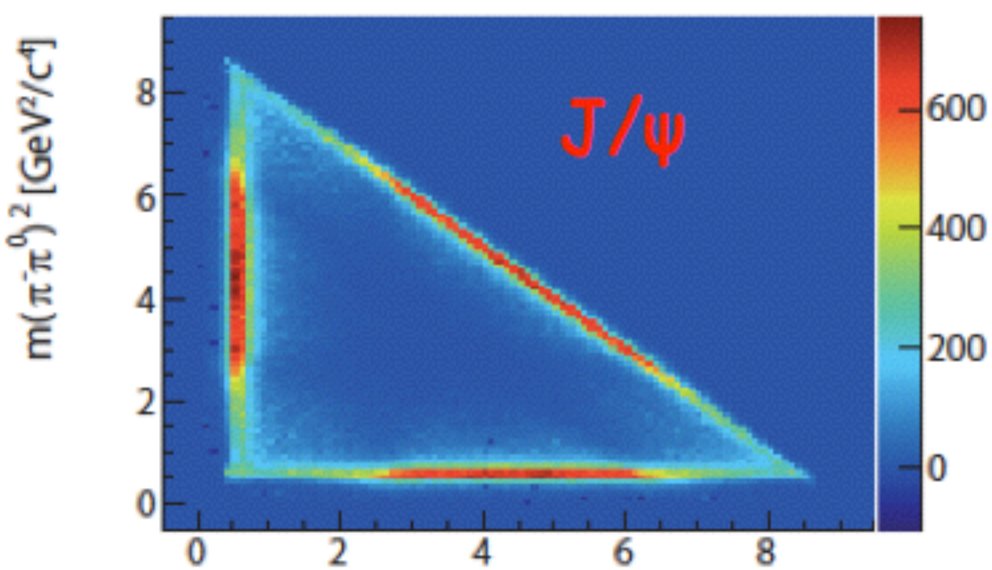
$$q_3 = r \sin (4\pi/3 + \omega)$$

Three Pions

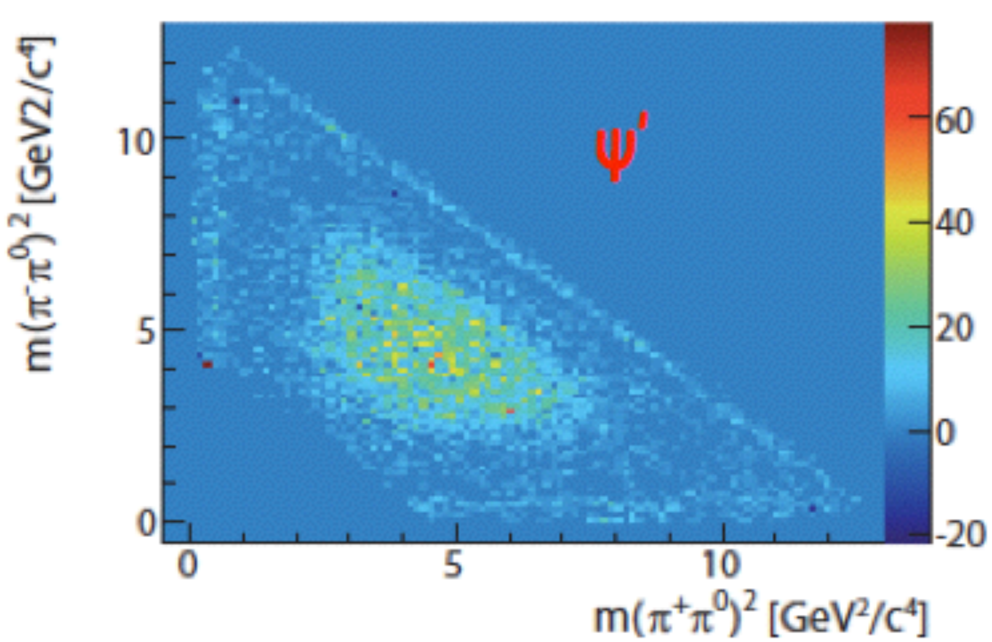
$$J/\psi, \psi' \to 3\pi$$

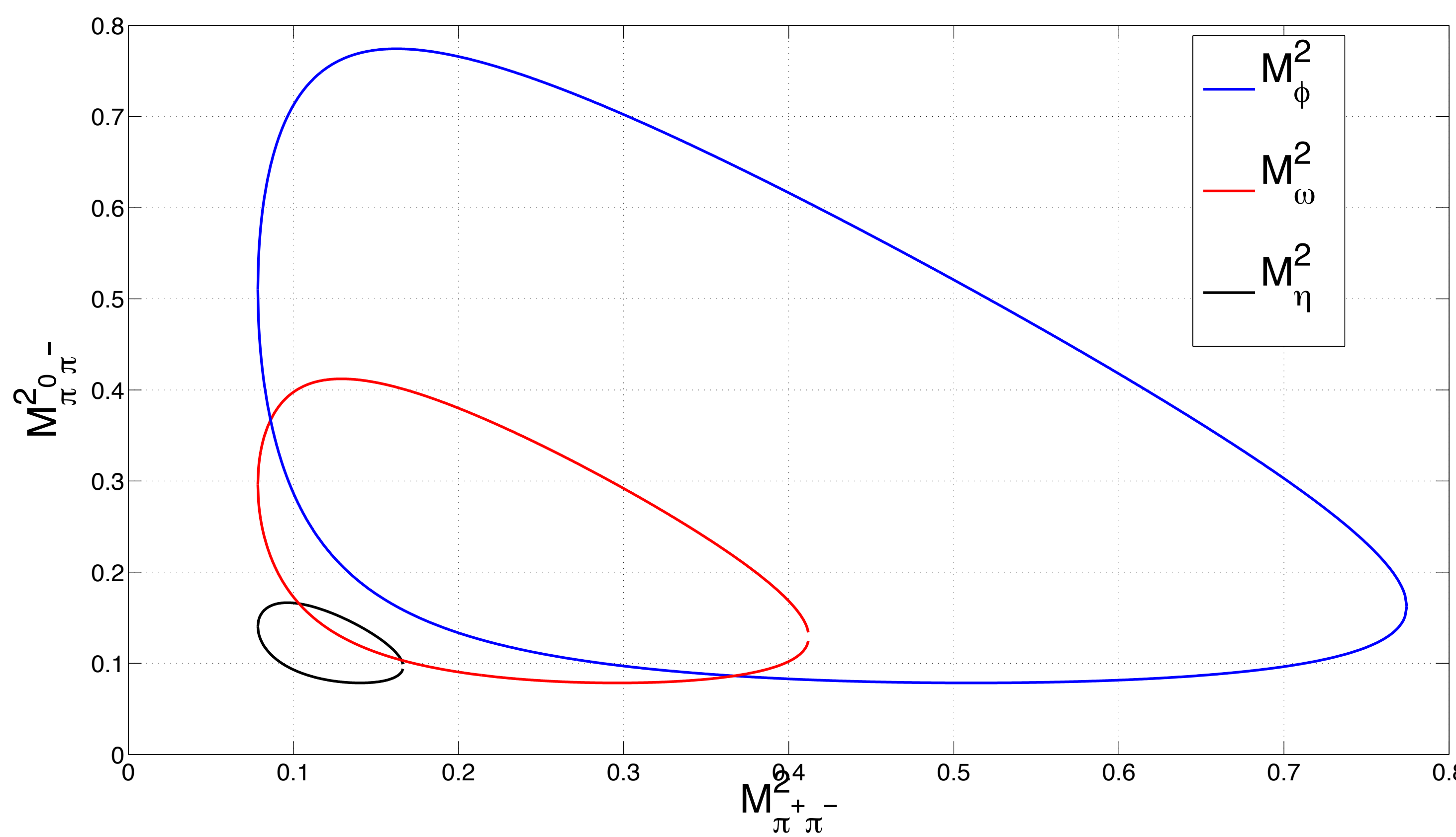






BESIII, Phys.Lett. B710 (2012) 594-599

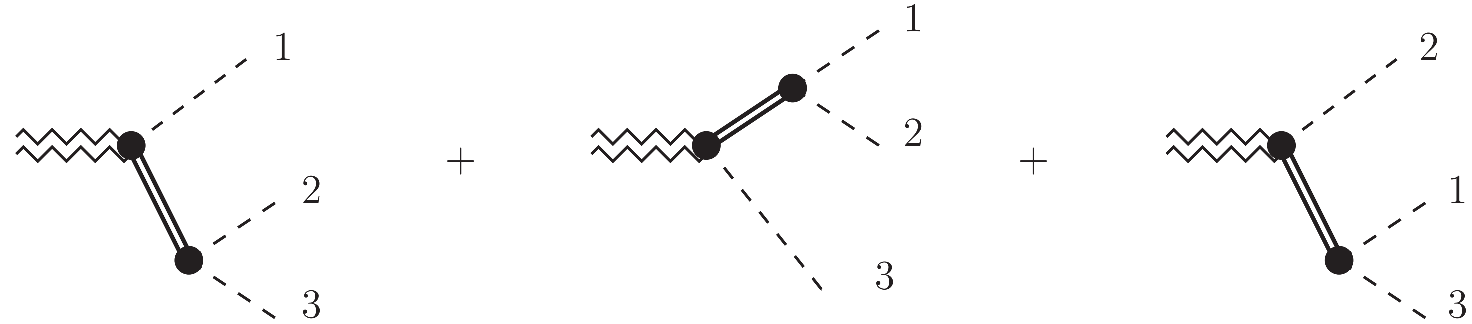


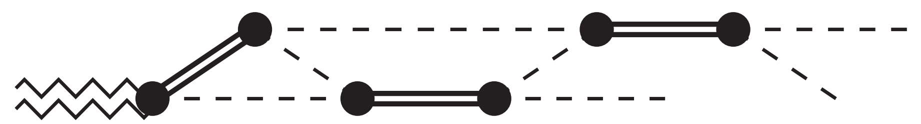


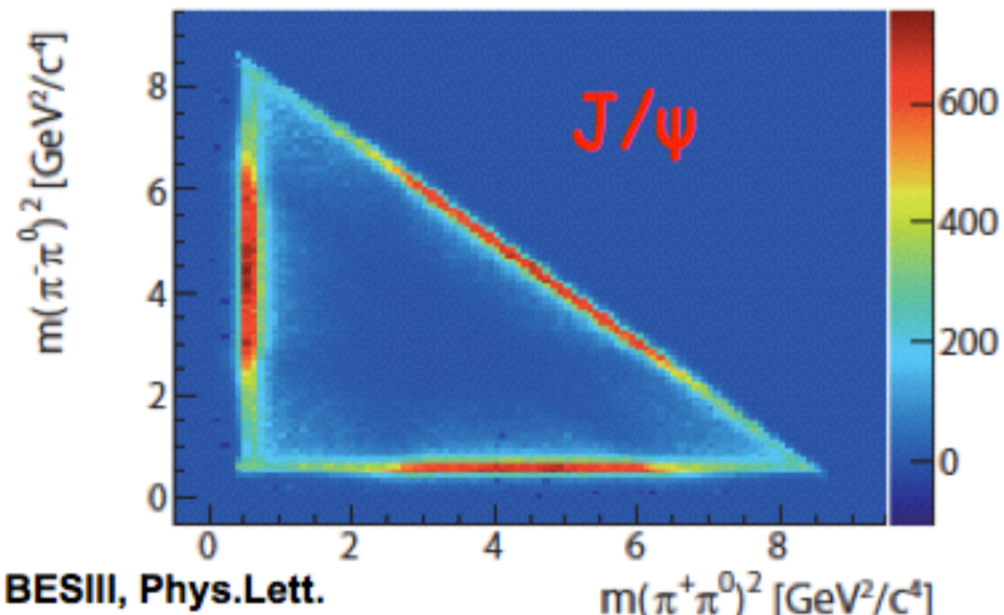
Three Pions

$$J/\psi, \psi' \to 3\pi$$

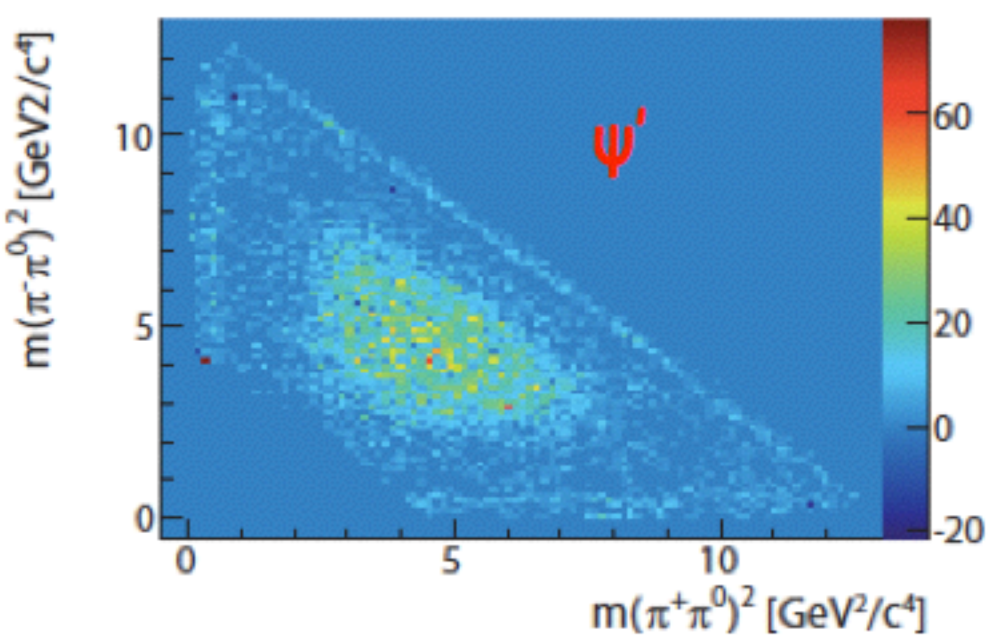
Good approximation:
Isobar model

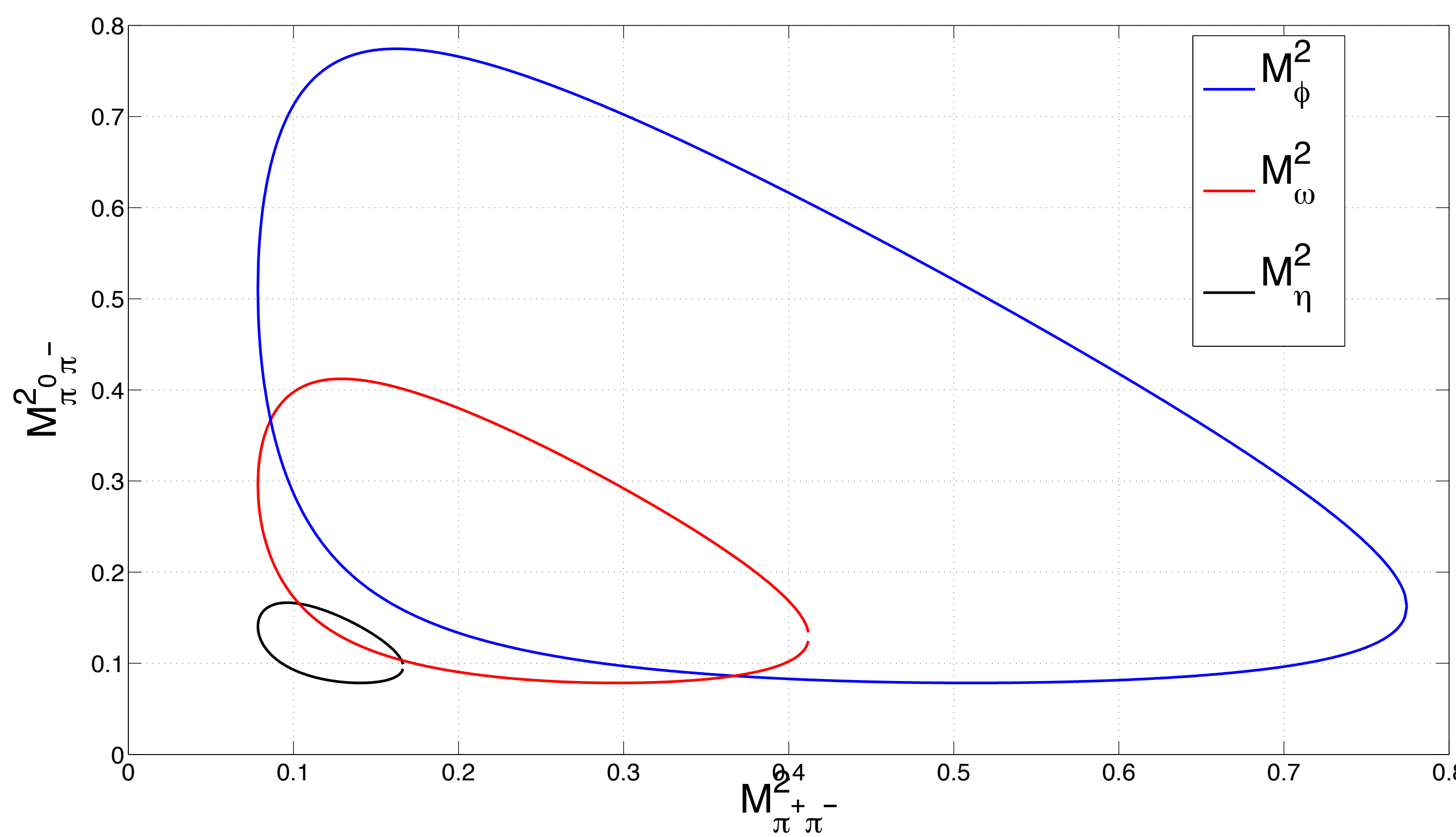






BESIII, Phys.Lett. B710 (2012) 594-599

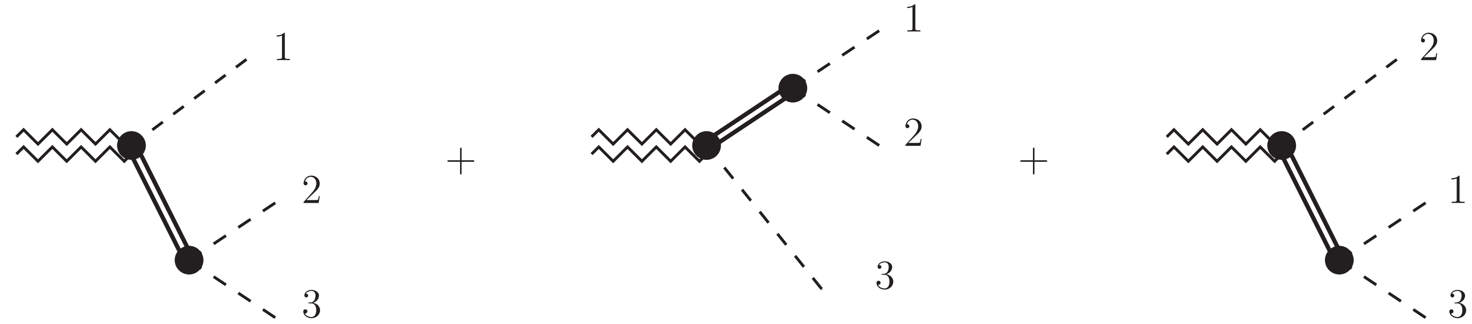


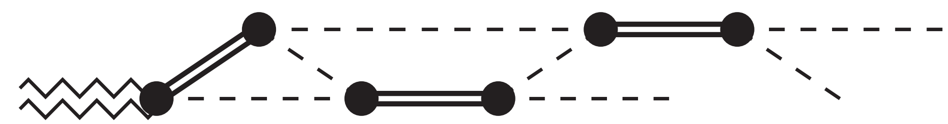


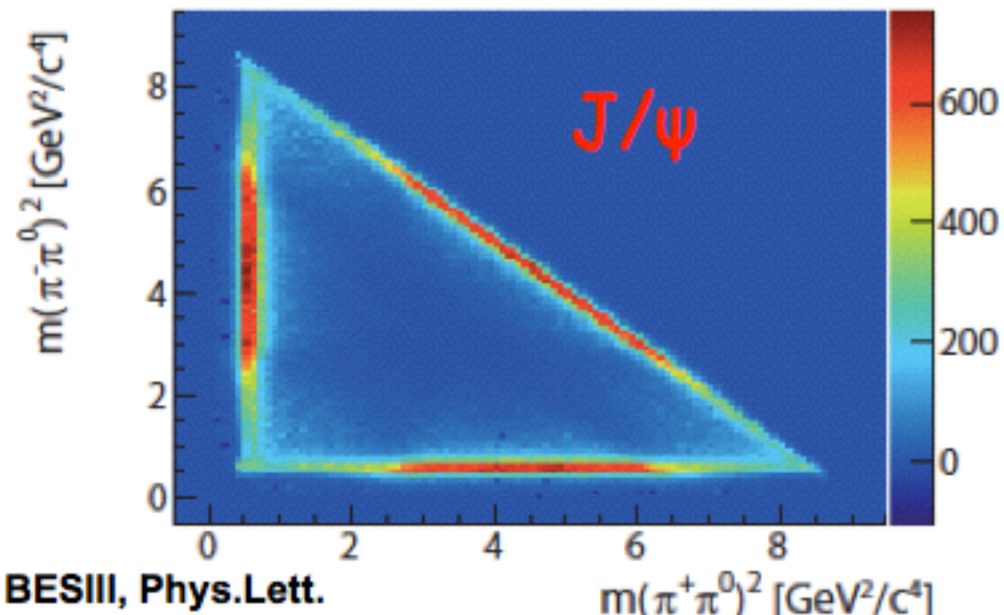
Three Pions

$J/\psi, \psi' \to 3\pi$

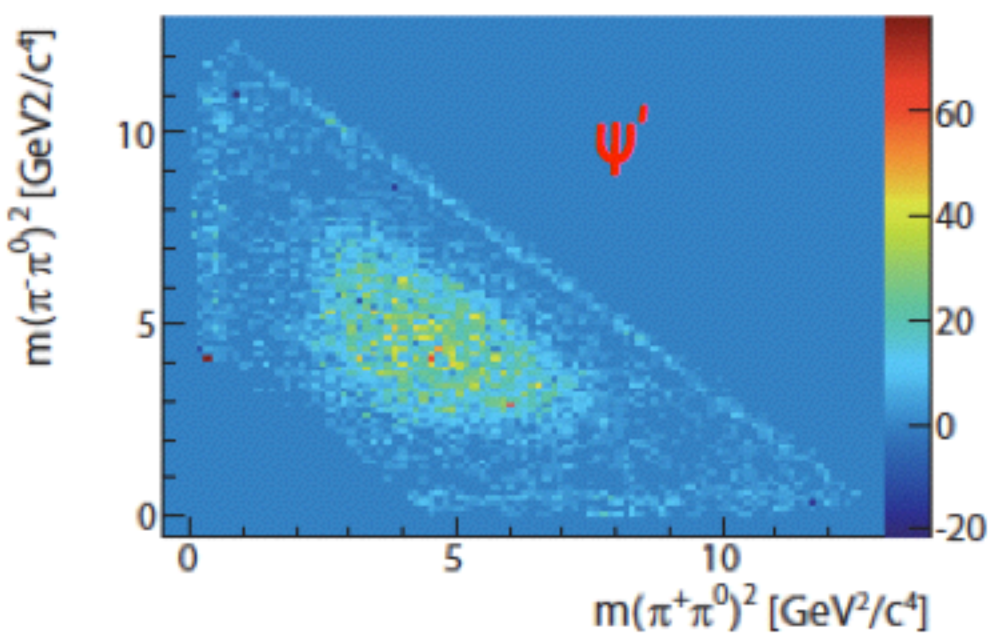
Good approximation: Isobar model



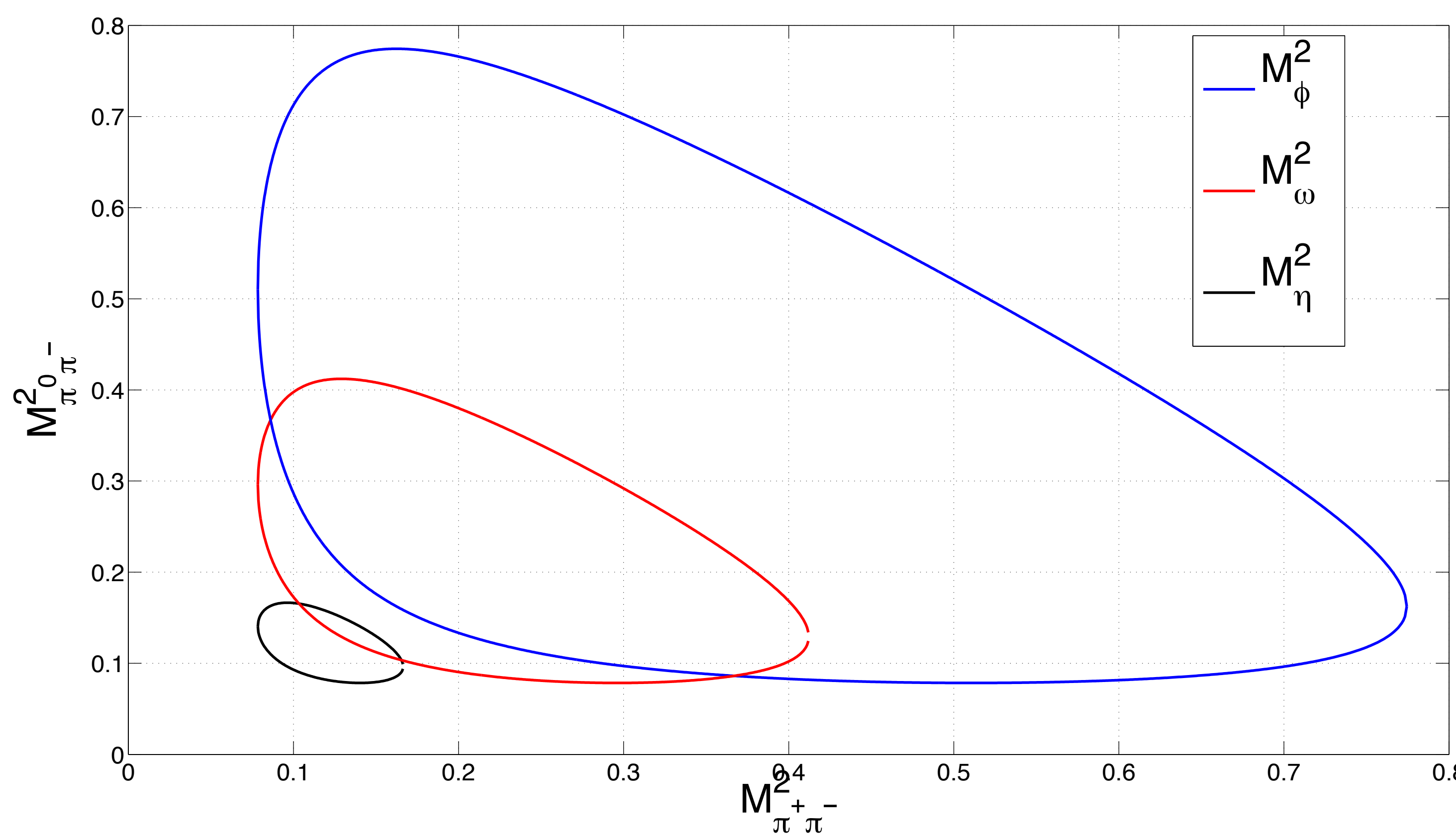




BESIII, Phys.Lett. B710 (2012) 594-599

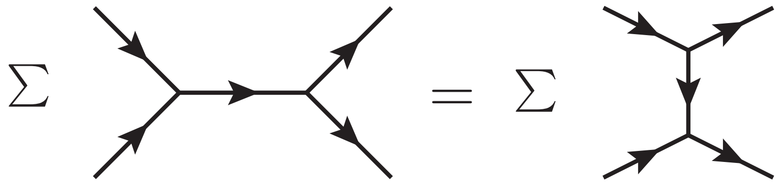


How to extract resonance parameters ?



Veneziano Amplitudes

Dual model



violation of unitarity

$$A(s,t) = \sum_{n,m} c_{n,m} \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$

Parameters:

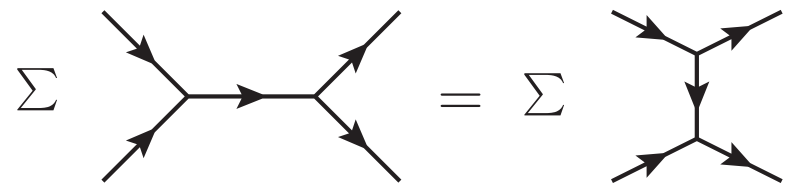
trajectory $\alpha(s)$

couplings $c_{n,m}$

A. Szczepaniak and M. Pennington
hep-ph/1403.5782

Veneziano Amplitudes

Dual model



violation of unitarity

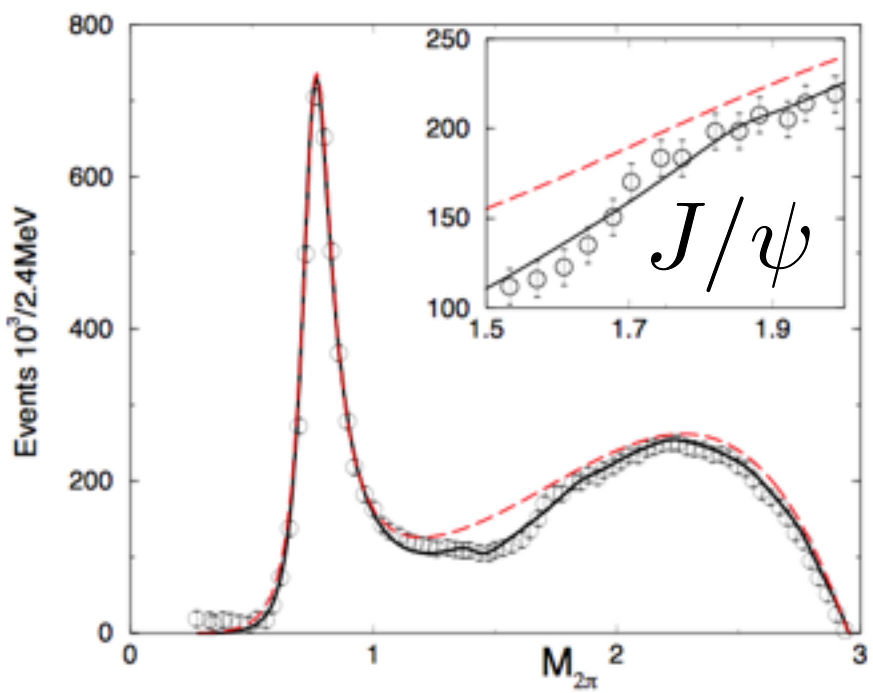
$$A(s,t) = \sum_{n,m} c_{n,m} \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$

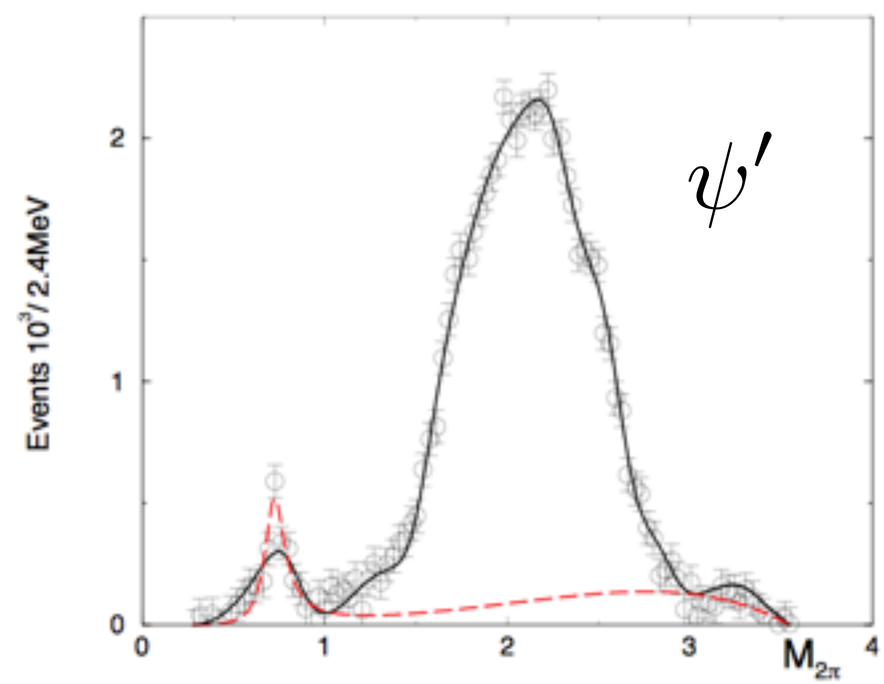
Parameters:

trajectory $\alpha(s)$

couplings $c_{n,m}$

$J/\psi, \psi' \to 3\pi$

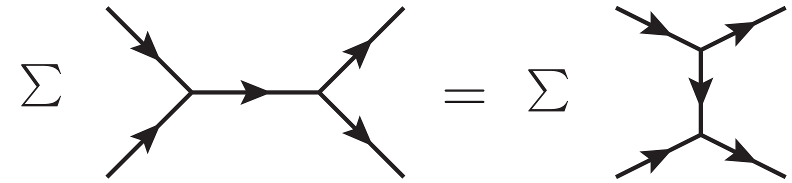




A. Szczepaniak and M. Pennington
hep-ph/1403.5782

Veneziano Amplitudes

Dual model



violation of unitarity

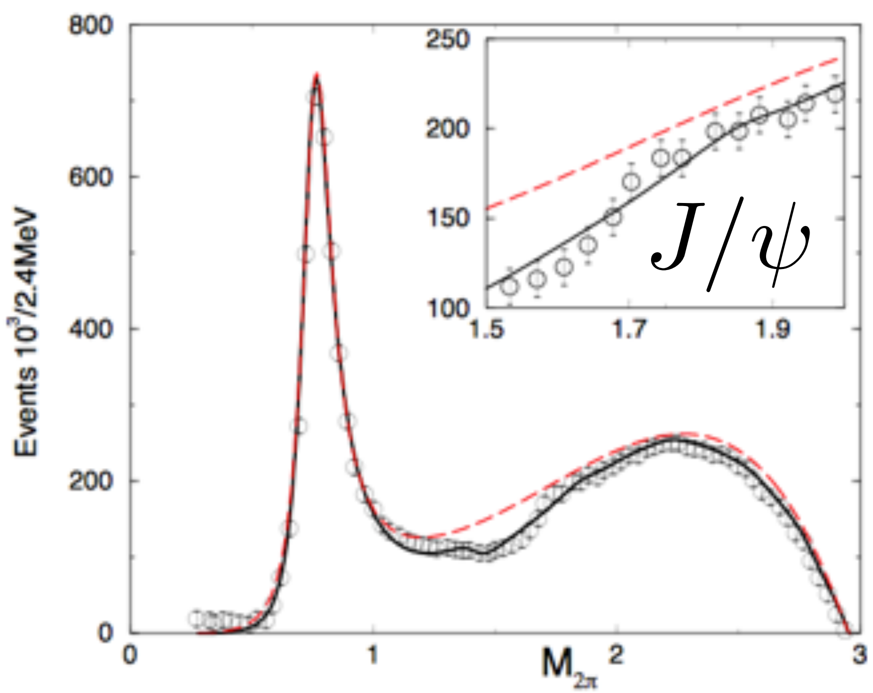
$$A(s,t)=\sum_{n,m} c_{n,m}\frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$

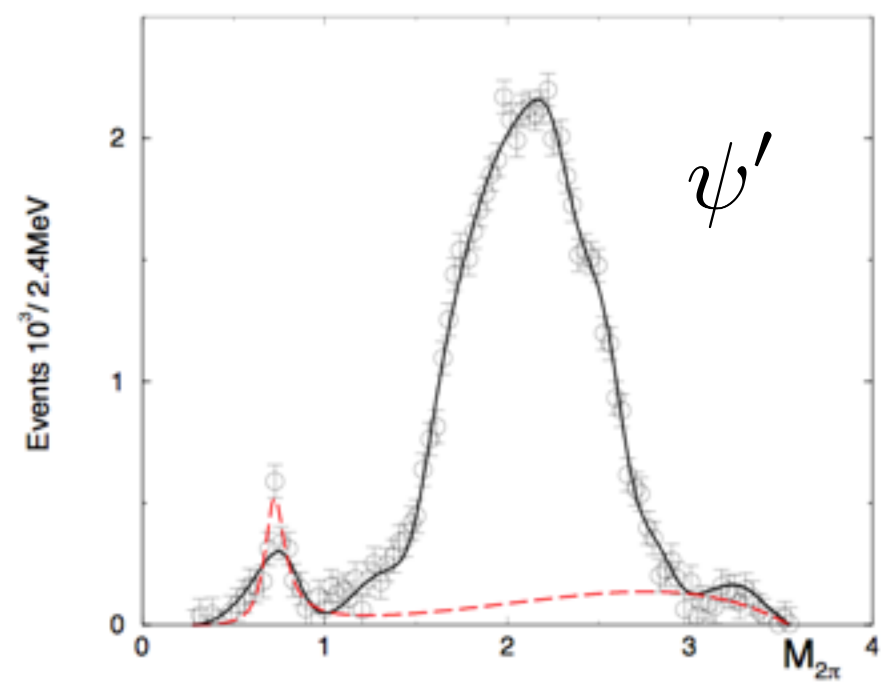
Parameters:

trajectory $\alpha(s)$

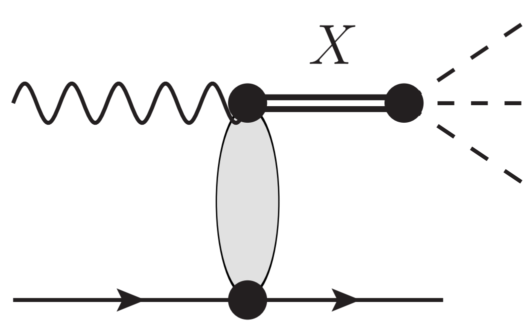
couplings $c_{n,m}$

$J/\psi, \psi' \to 3\pi$





Can be applied to



$\eta \to 3\pi$

$\eta' \to 3\pi$

$\omega \to 3\pi$

$\eta' \to \pi\pi\eta$

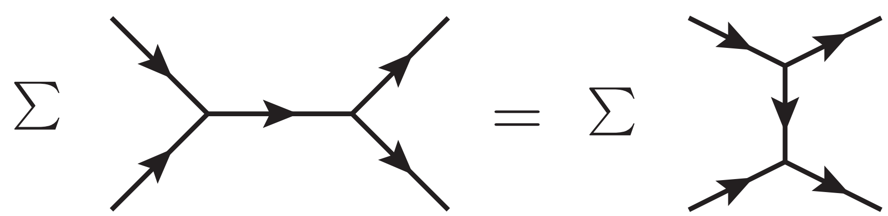
$\phi \to 3\pi$

A. Szczepaniak and M. Pennington
hep-ph/1403.5782

Veneziano Amplitudes

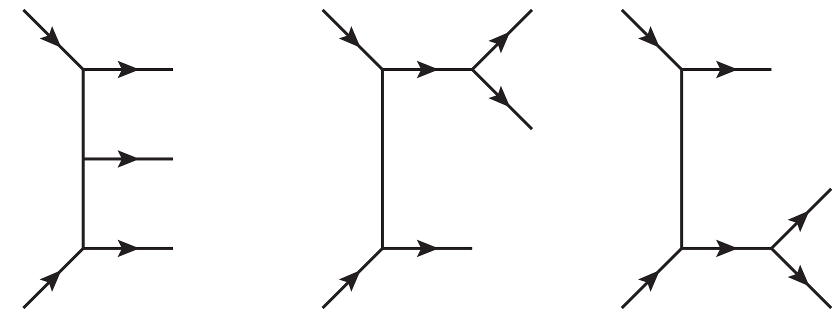
Dual model with 4 particules

$$A(s,t) = \sum_{n,m} c_{n,m} \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$



Extension to 5 particles

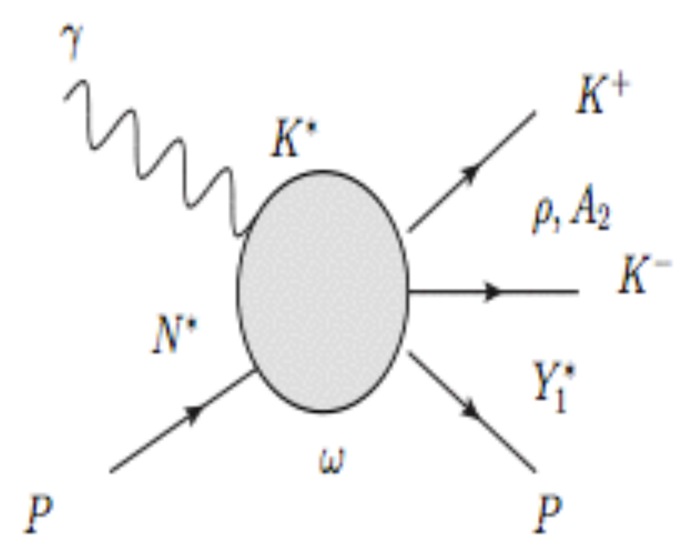
$${}_3F_2(a,b,c;x;y)$$



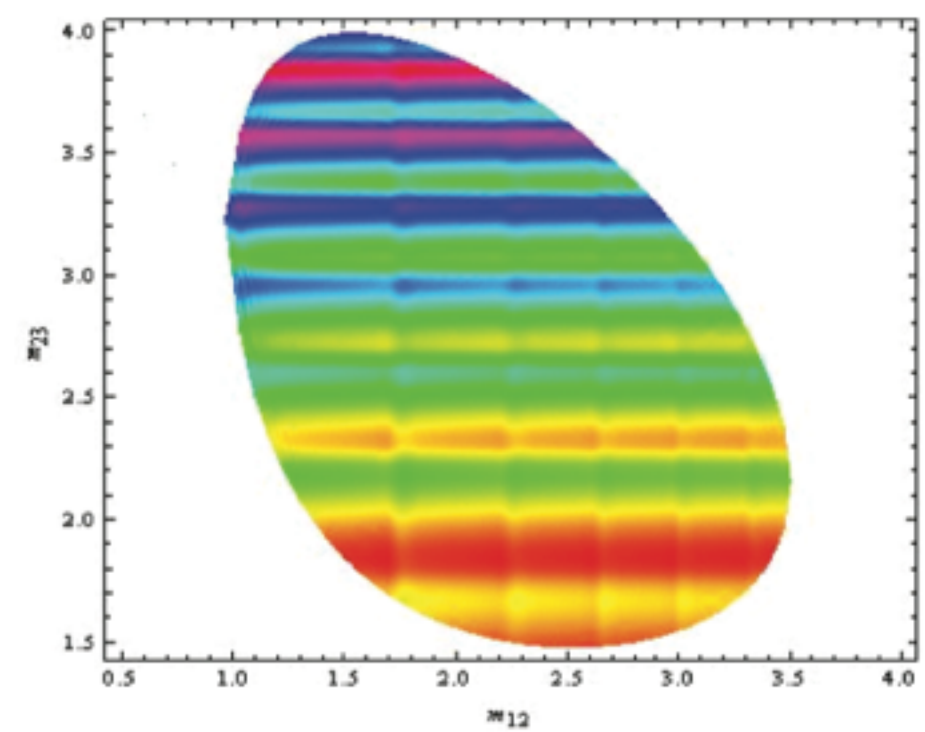
Application to $\gamma p \to K^+ K^- p$

Amplitude is a sum of narrow resonances

numerical evaluation of hypergeometric functions ?

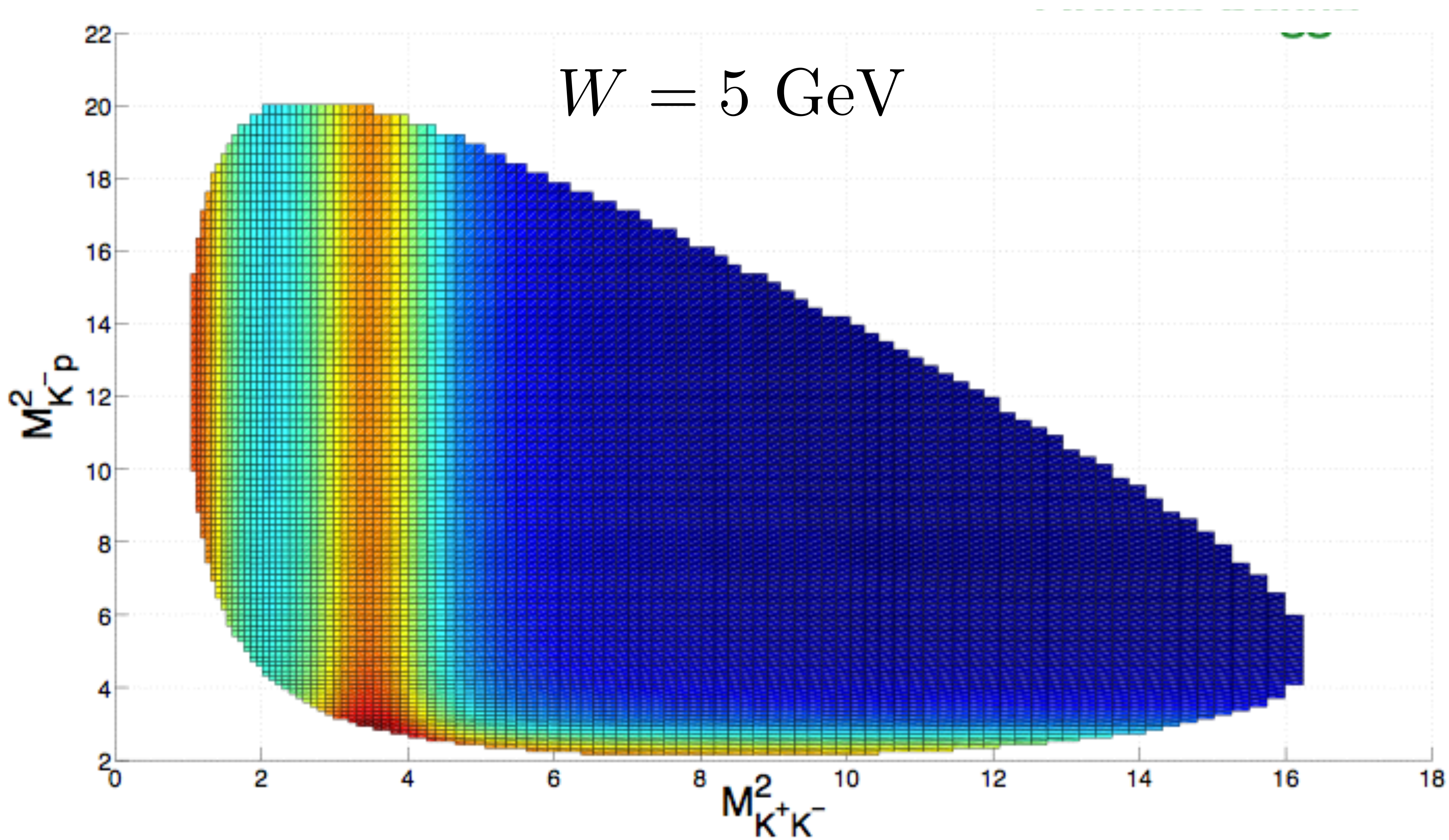


(a)



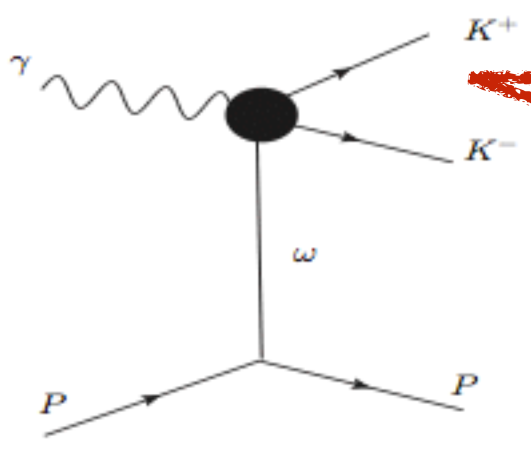
M. Shi and A. Szczepaniak (in preparation)

$$\gamma p \to K^+ K^- p$$

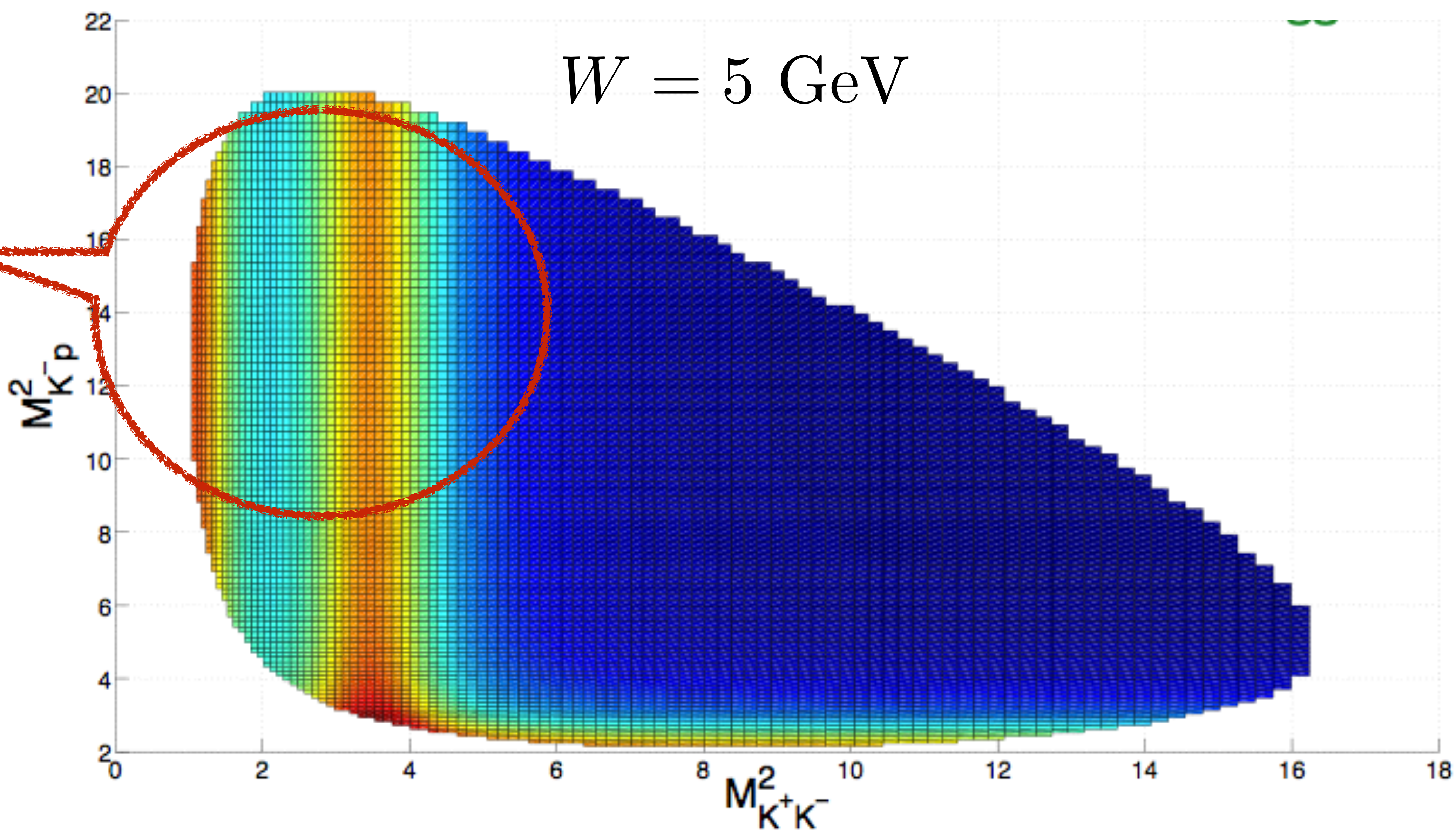


$$\gamma p \to K^+ K^- p$$

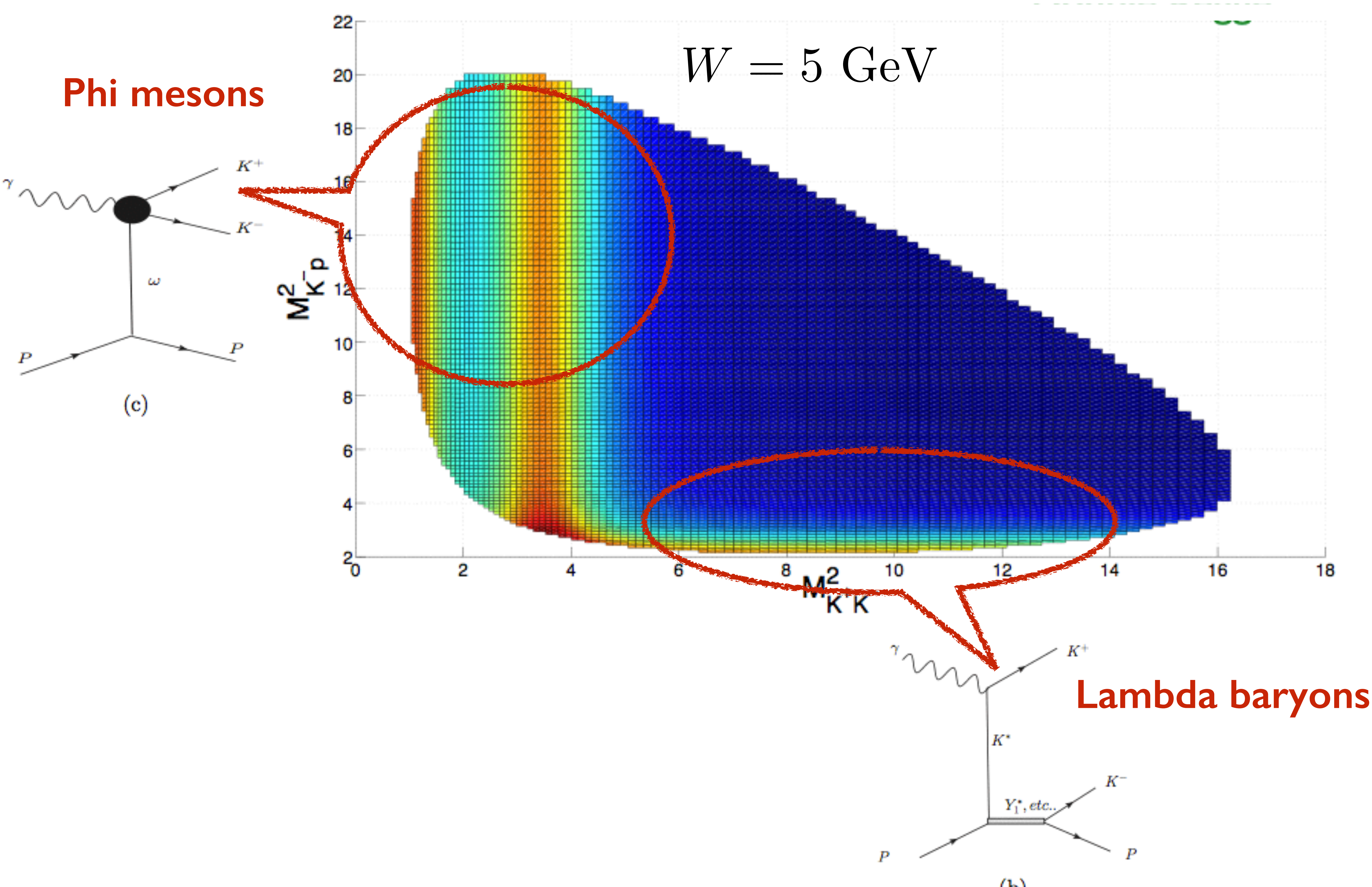
Phi mesons



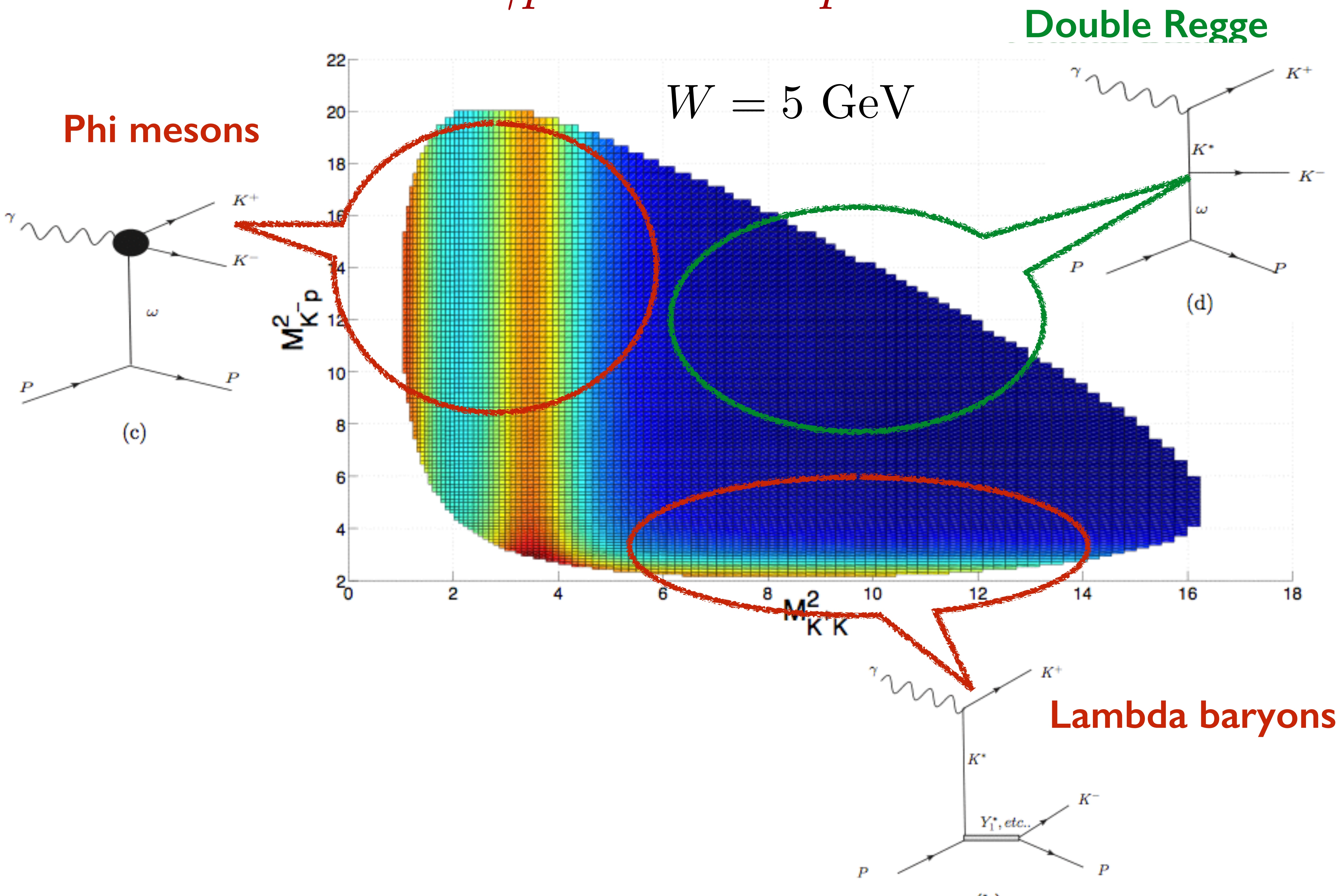
(c)



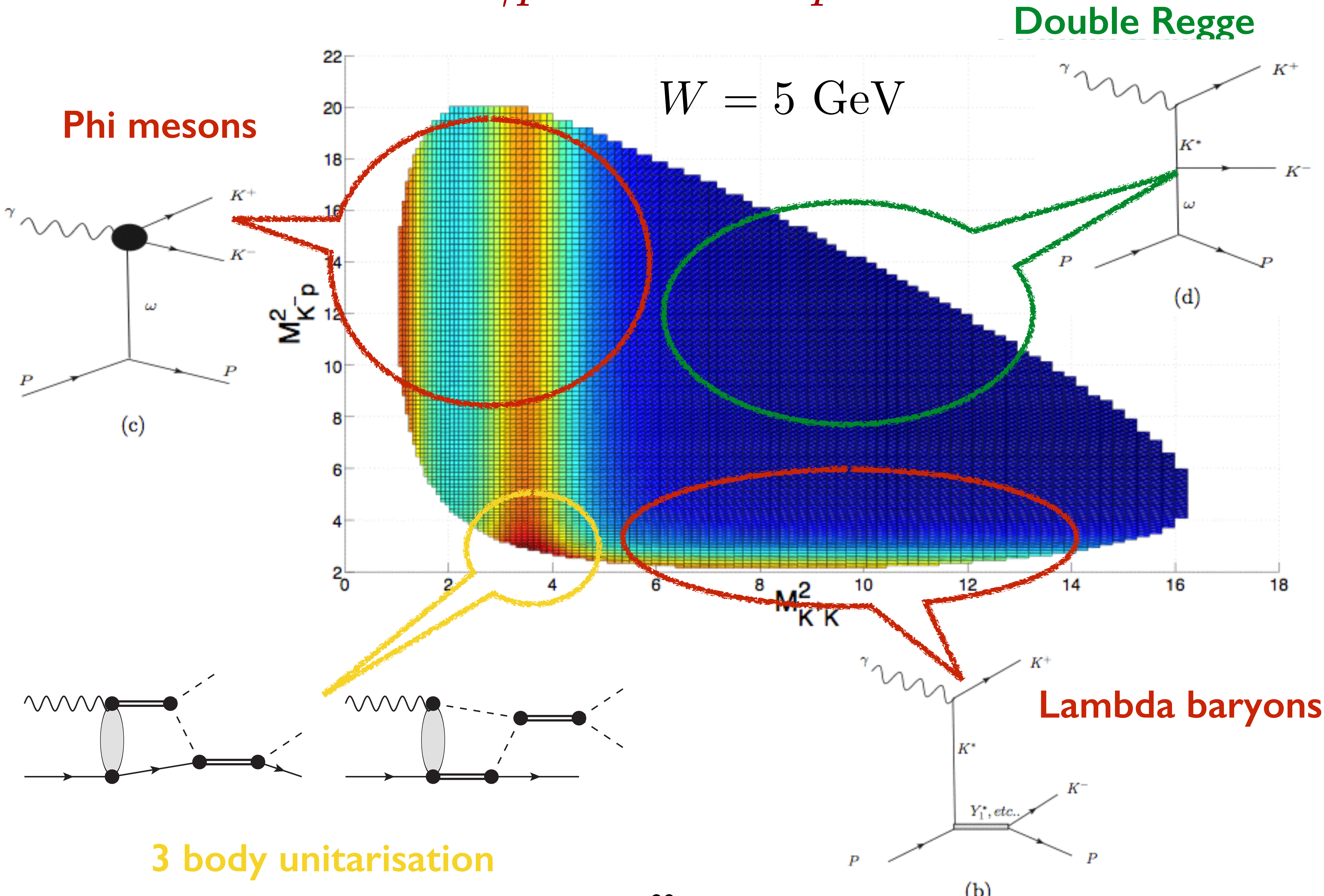
$$\gamma p \to K^+ K^- p$$



$$\gamma p \to K^+ K^- p$$

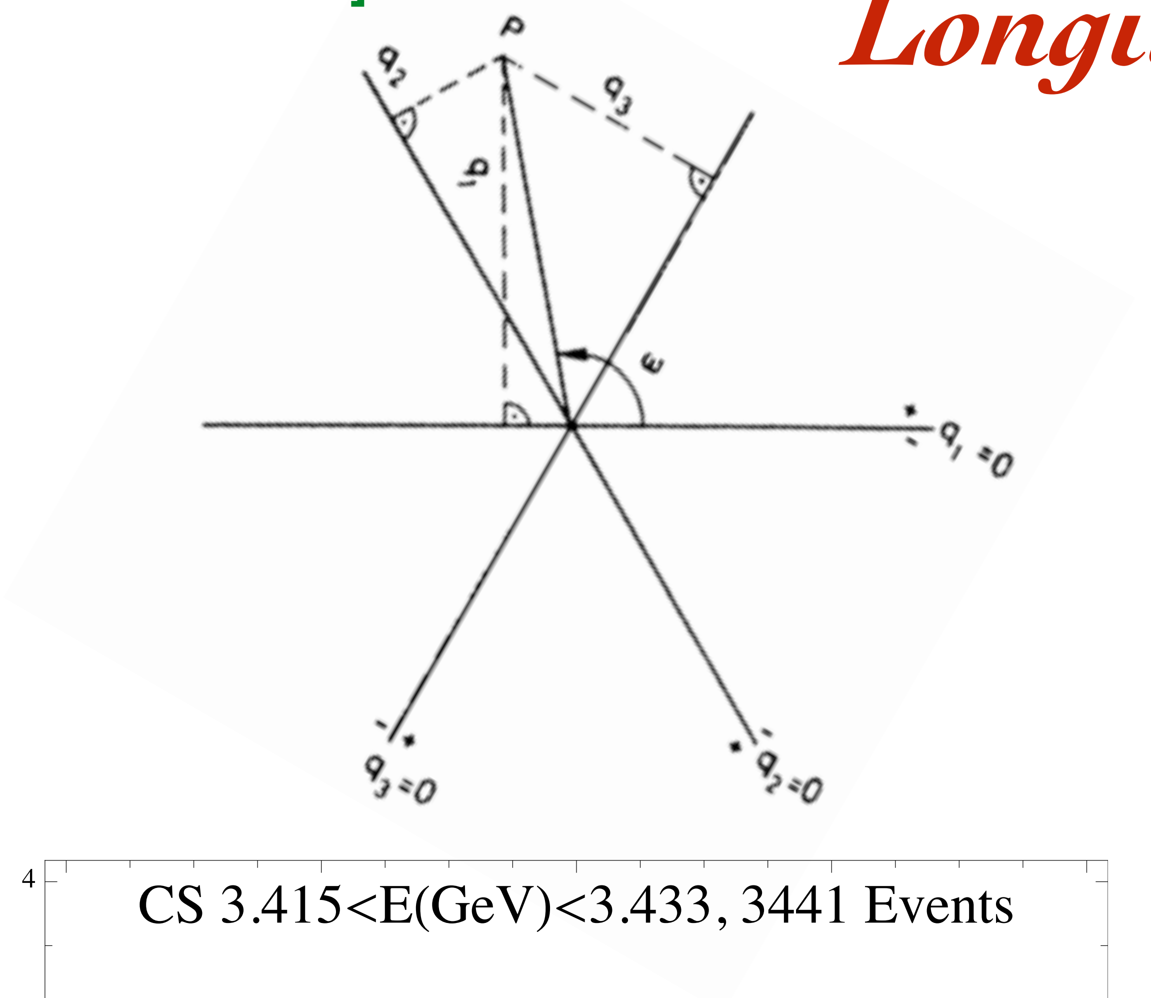


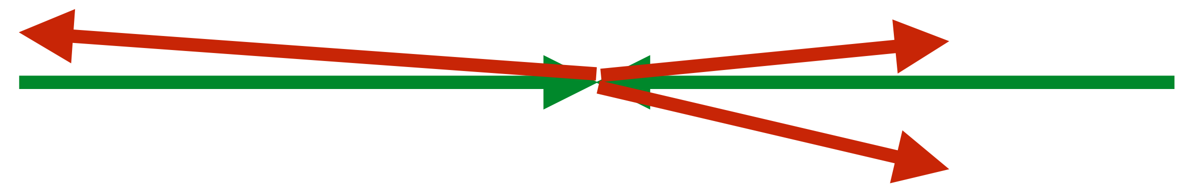
$$\gamma p \to K^+ K^- p$$

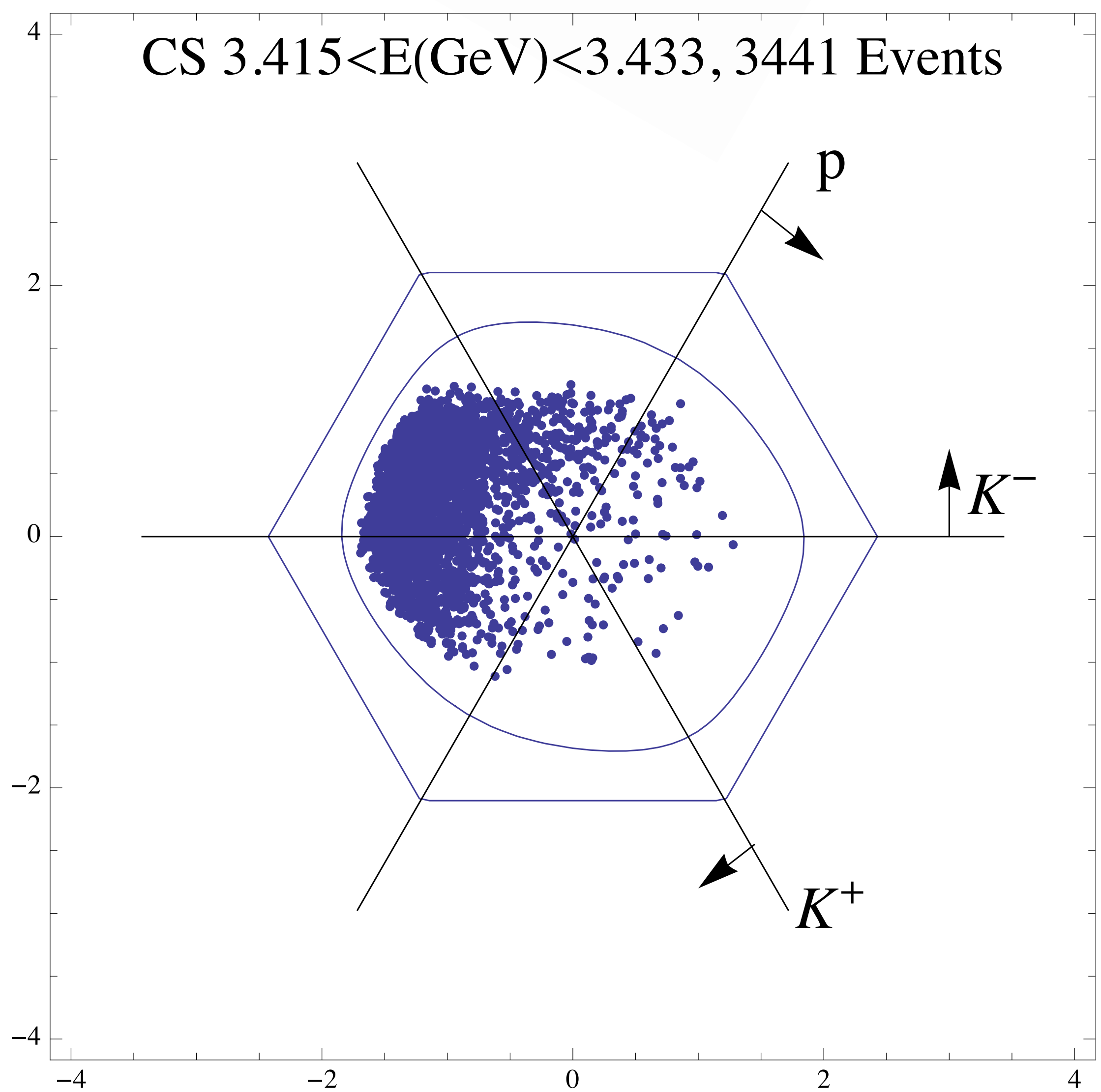


[Van Hove 1969]

Longitudinal Plot $\gamma p \to K^+ K^- p$

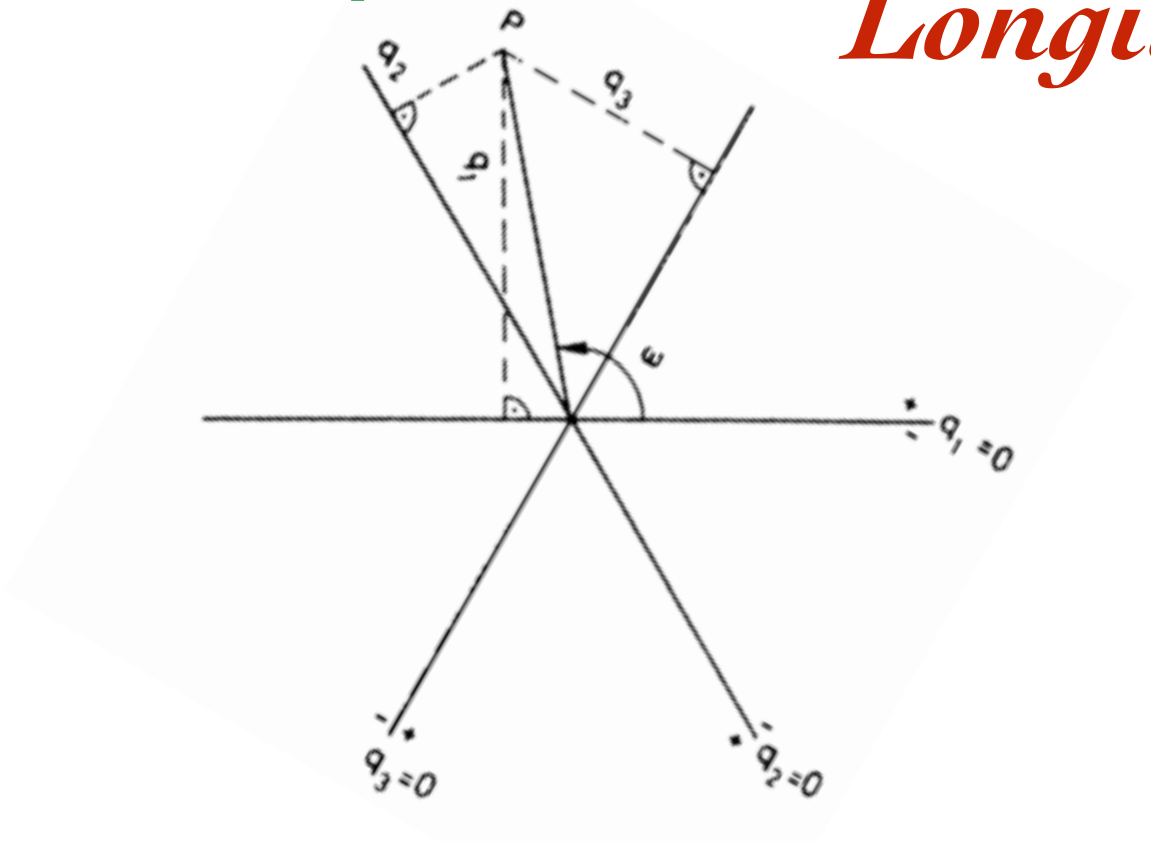


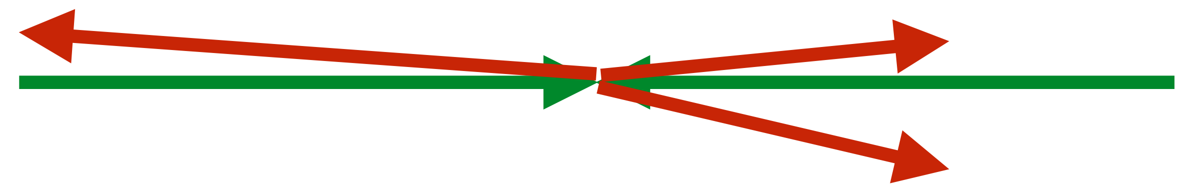


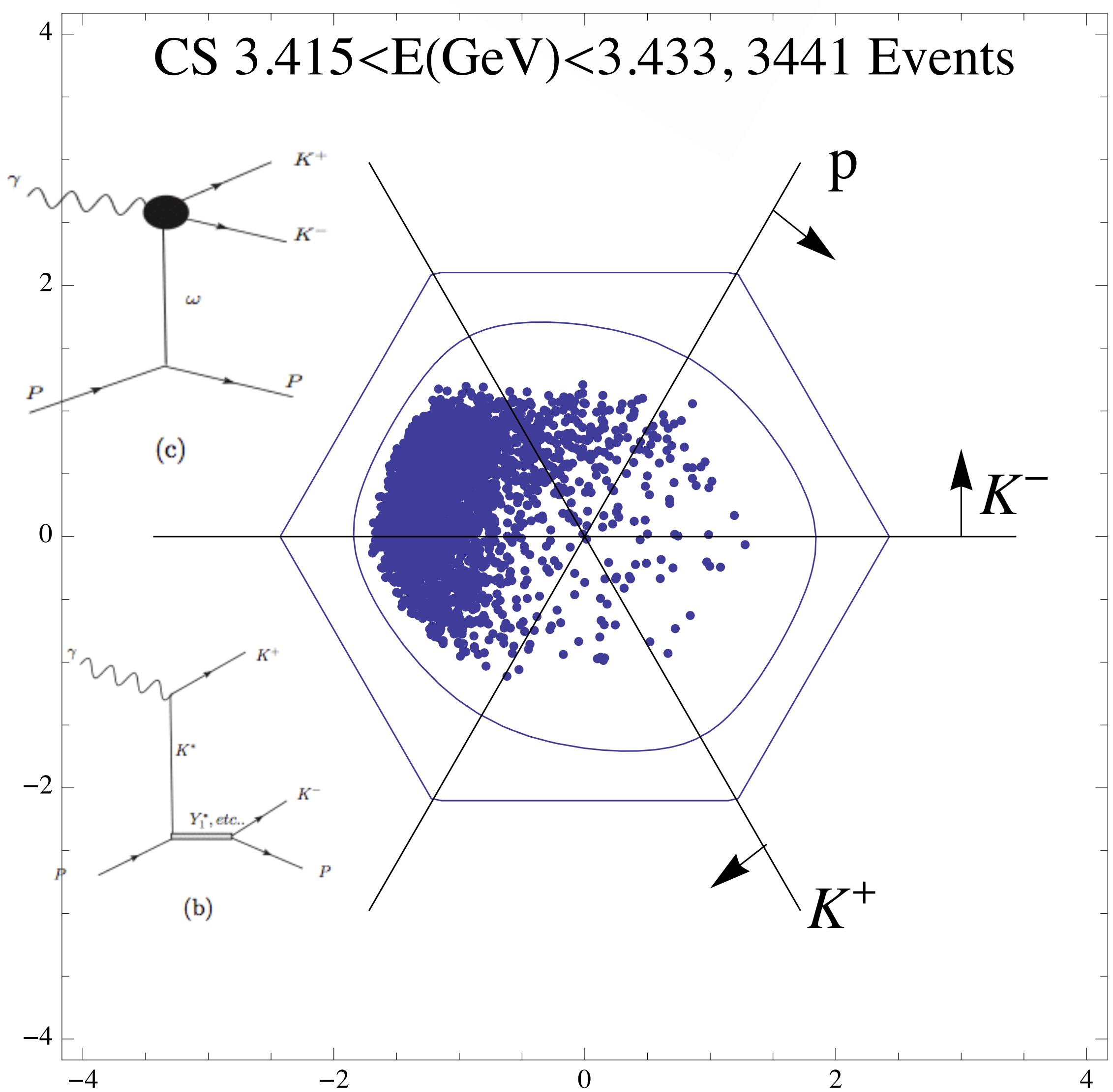


[Van Hove 1969]

Longitudinal Plot $\gamma p \to K^+ K^- p$

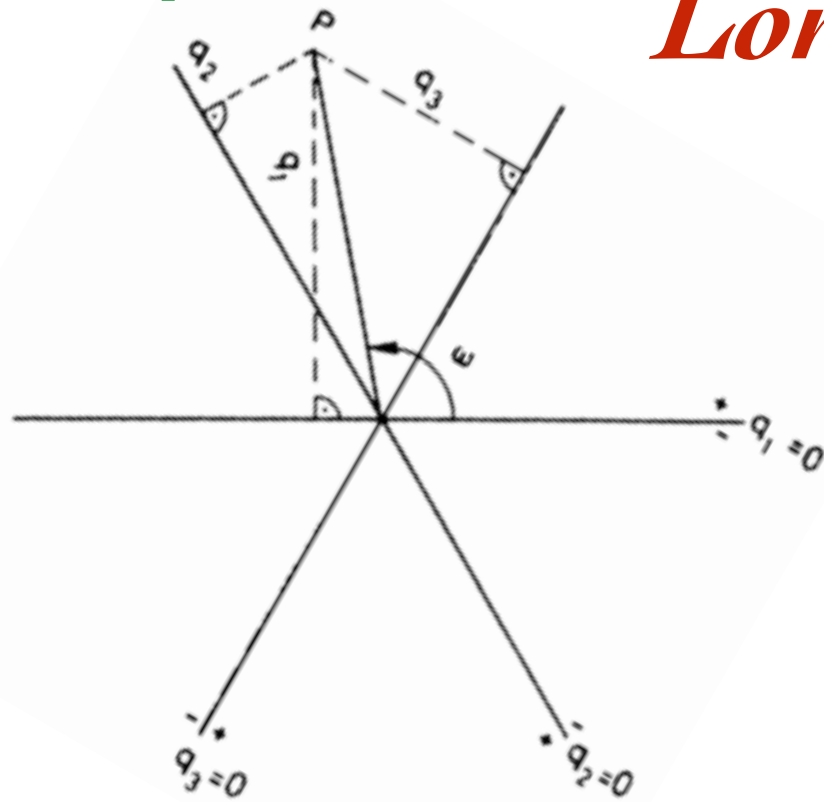


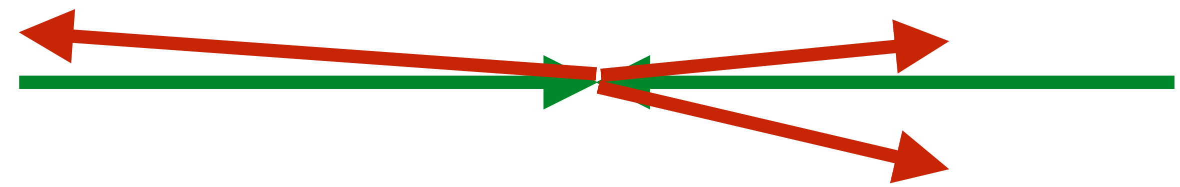


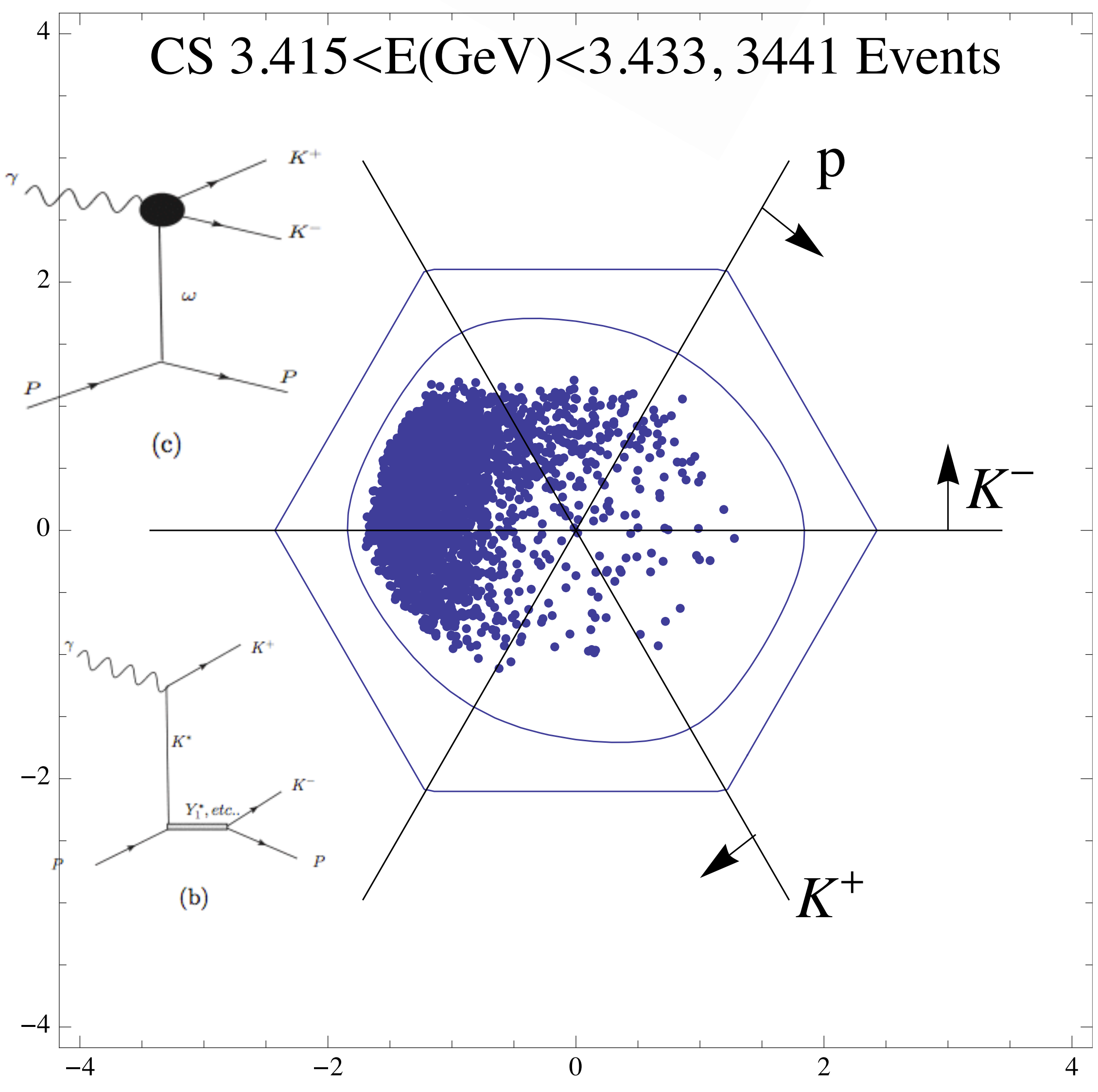


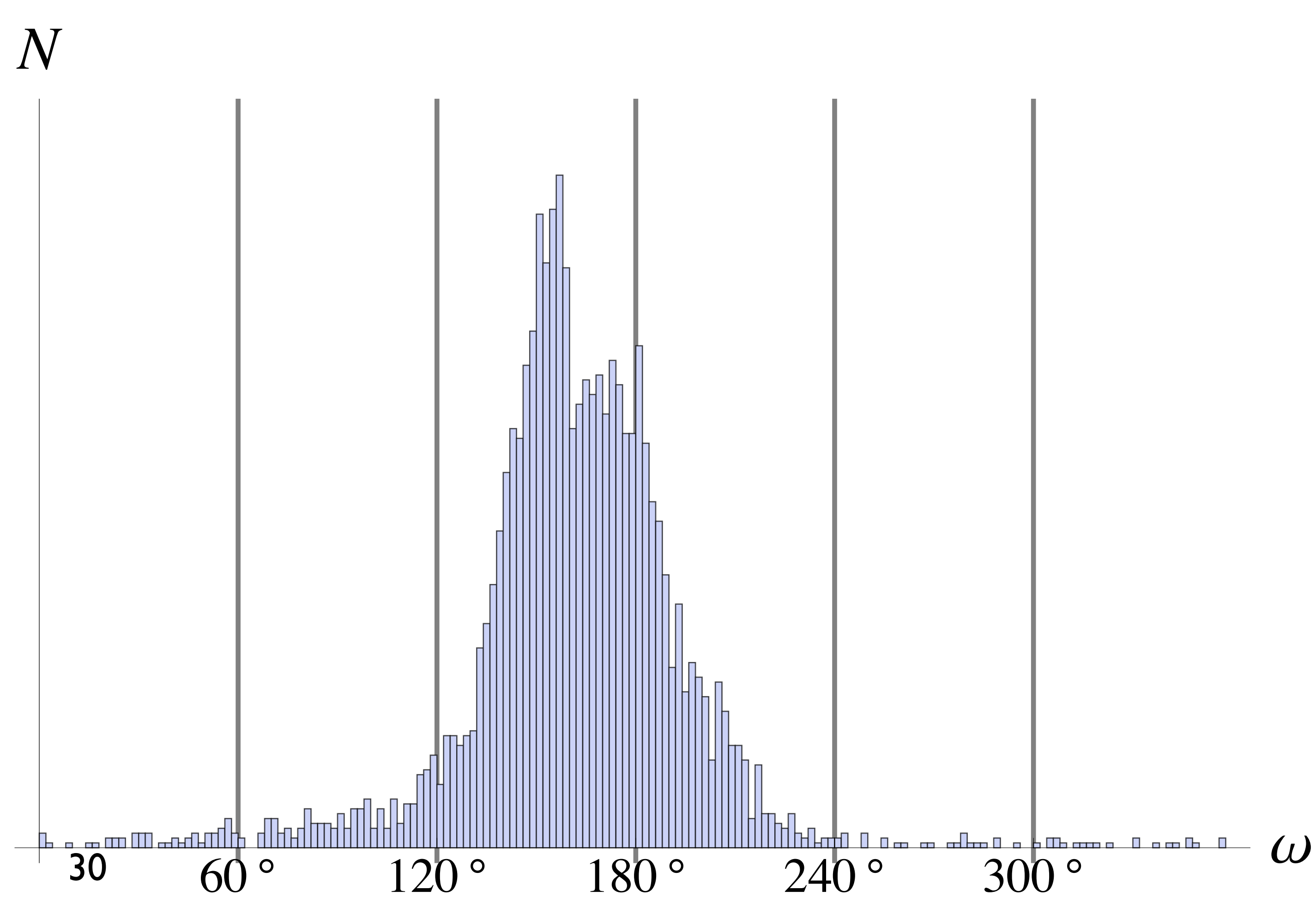
[Van Hove 1969]

Longitudinal Plot $\gamma p \to K^+ K^- p$

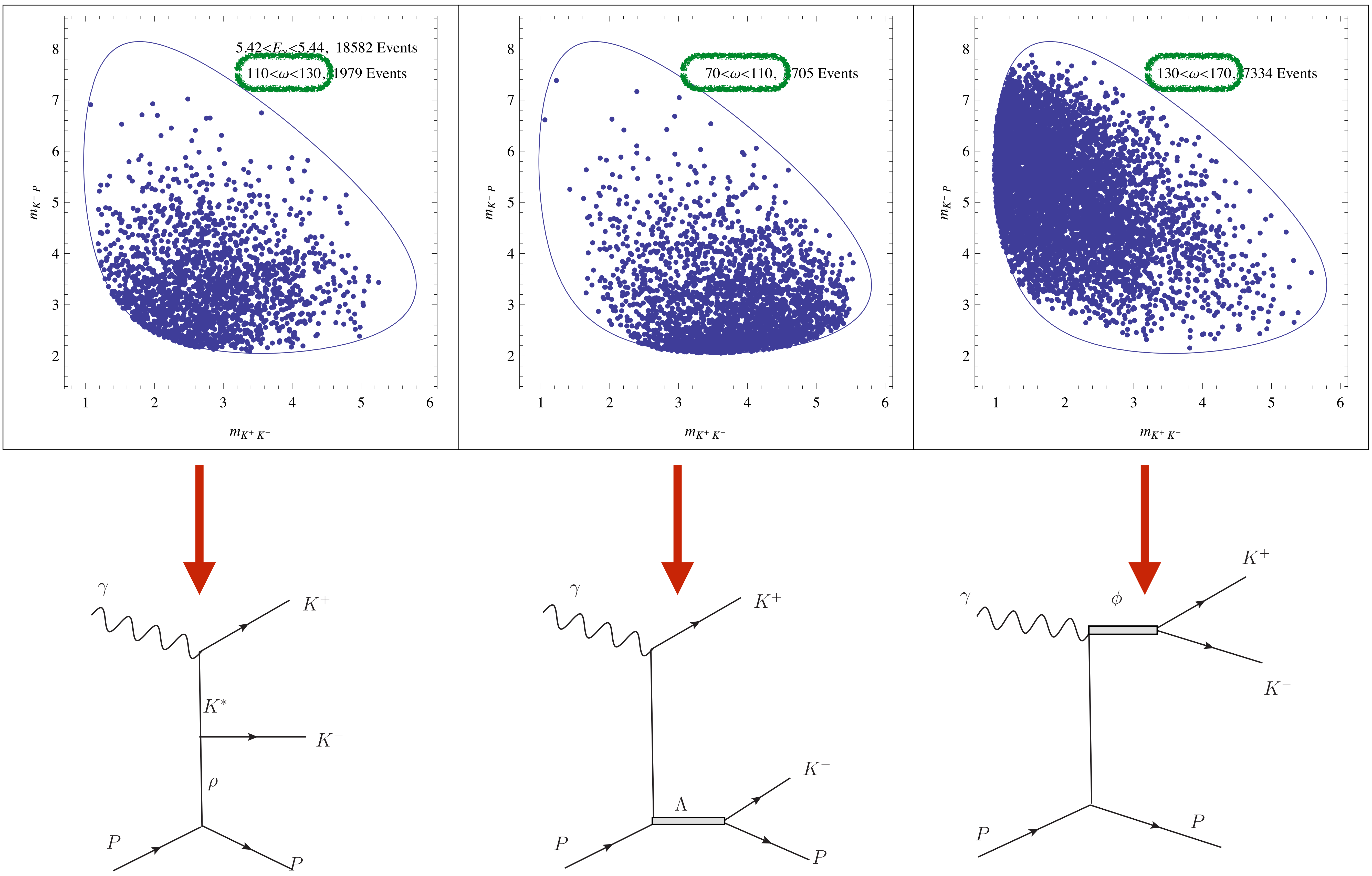




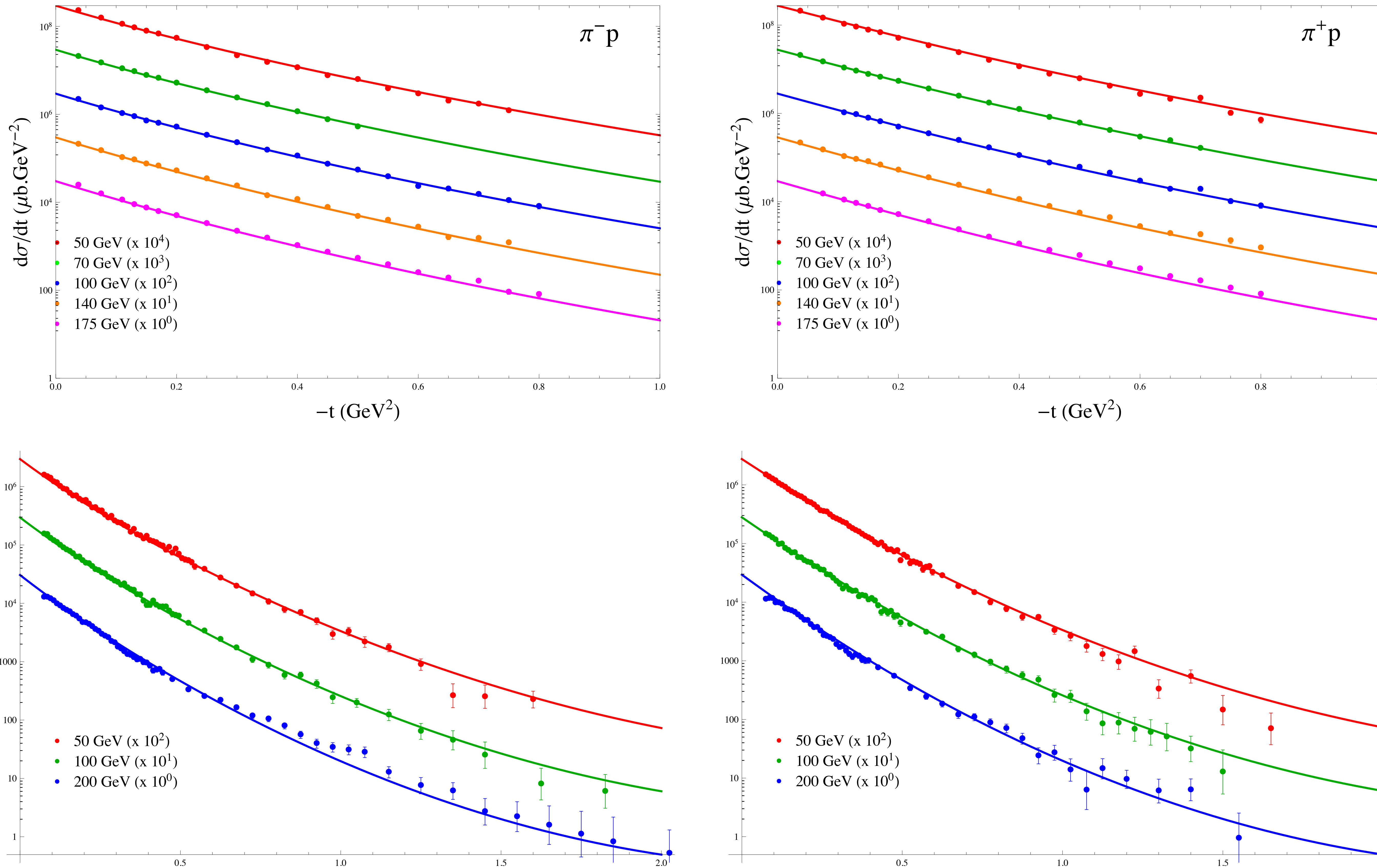




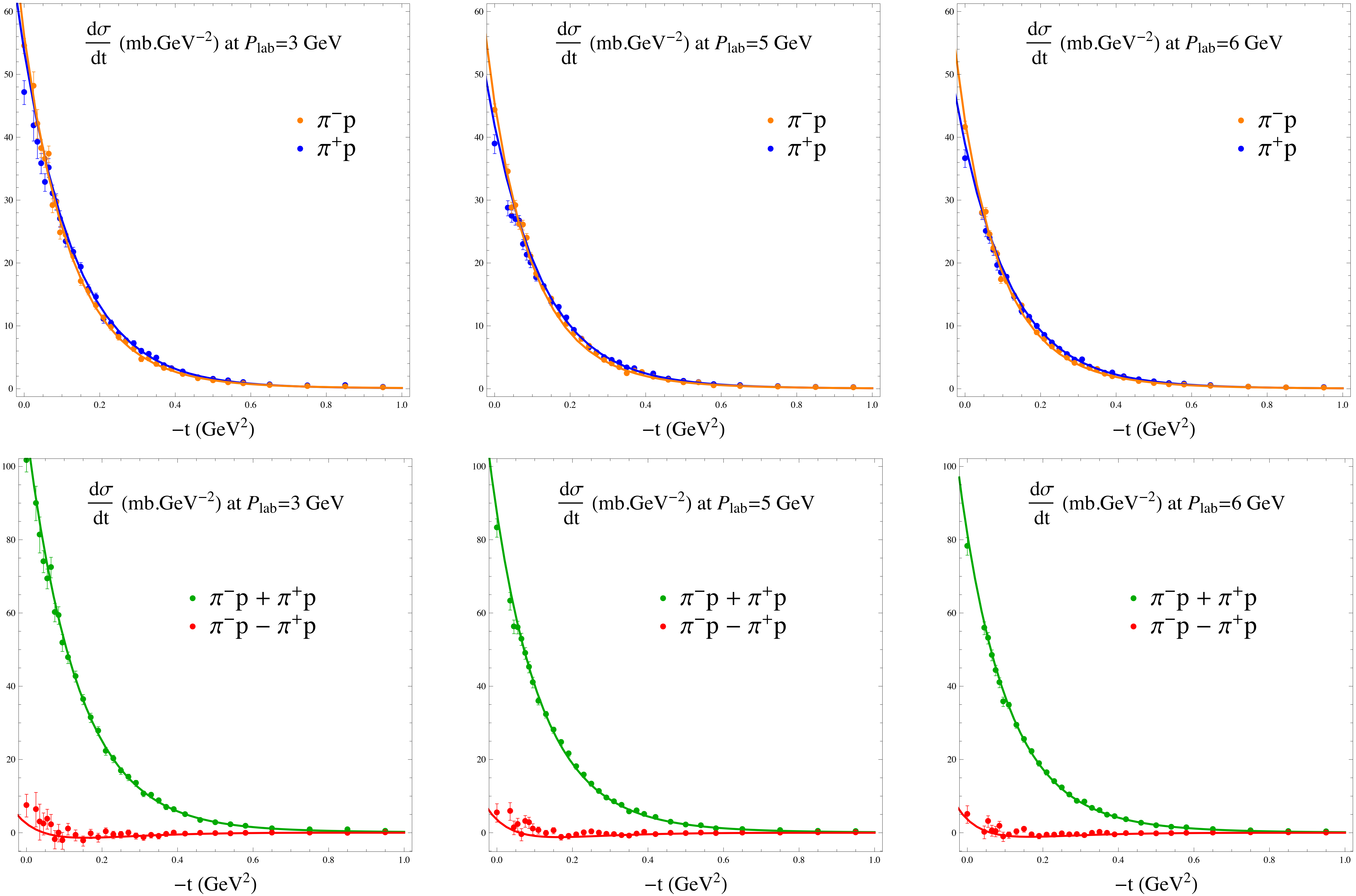
Longitudinal Plot



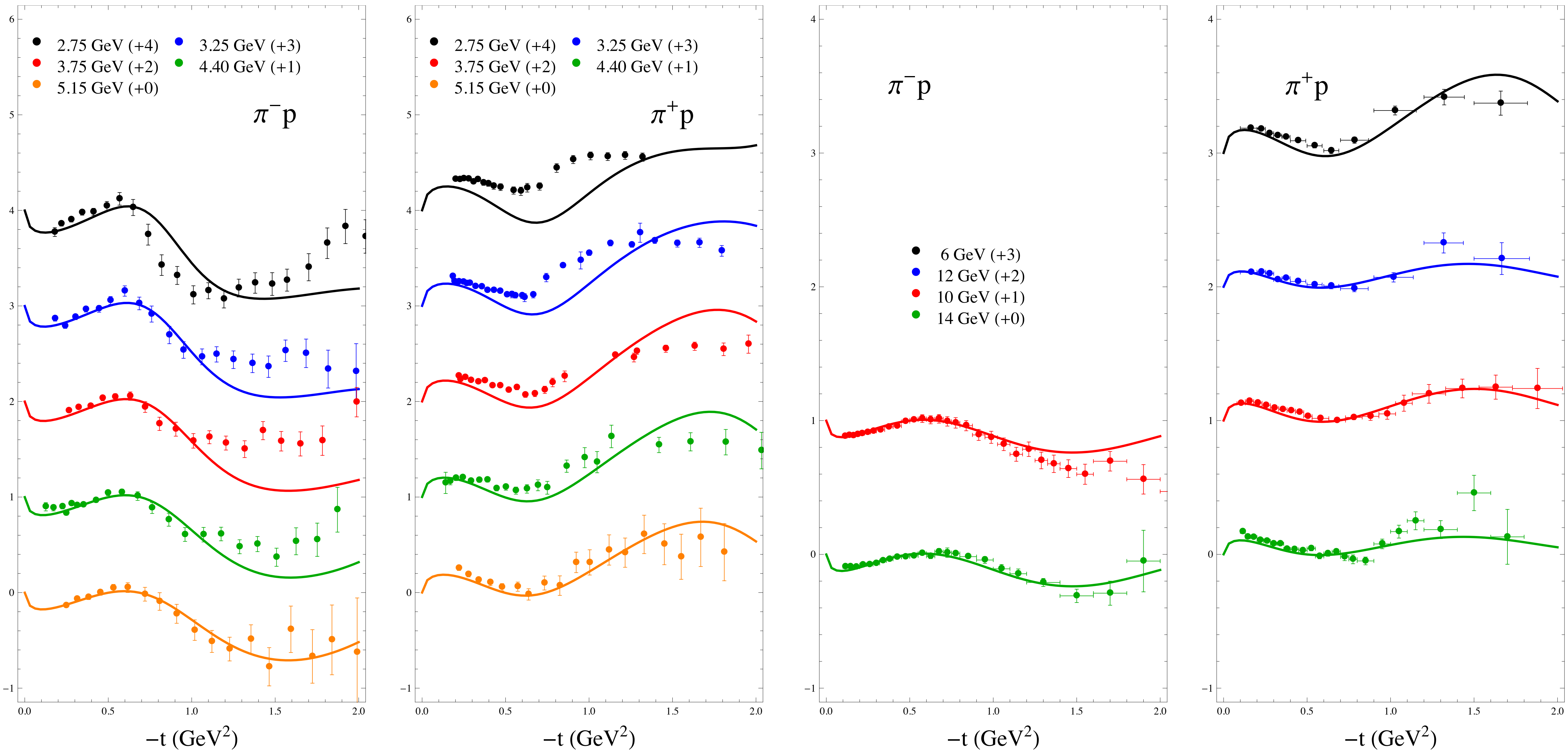
$\pi^{\pm}p \rightarrow \pi^{\pm}p$ **Diff. cross section**



$\pi^{\pm} p \to \pi^{\pm} p$ **Diff. cross section**

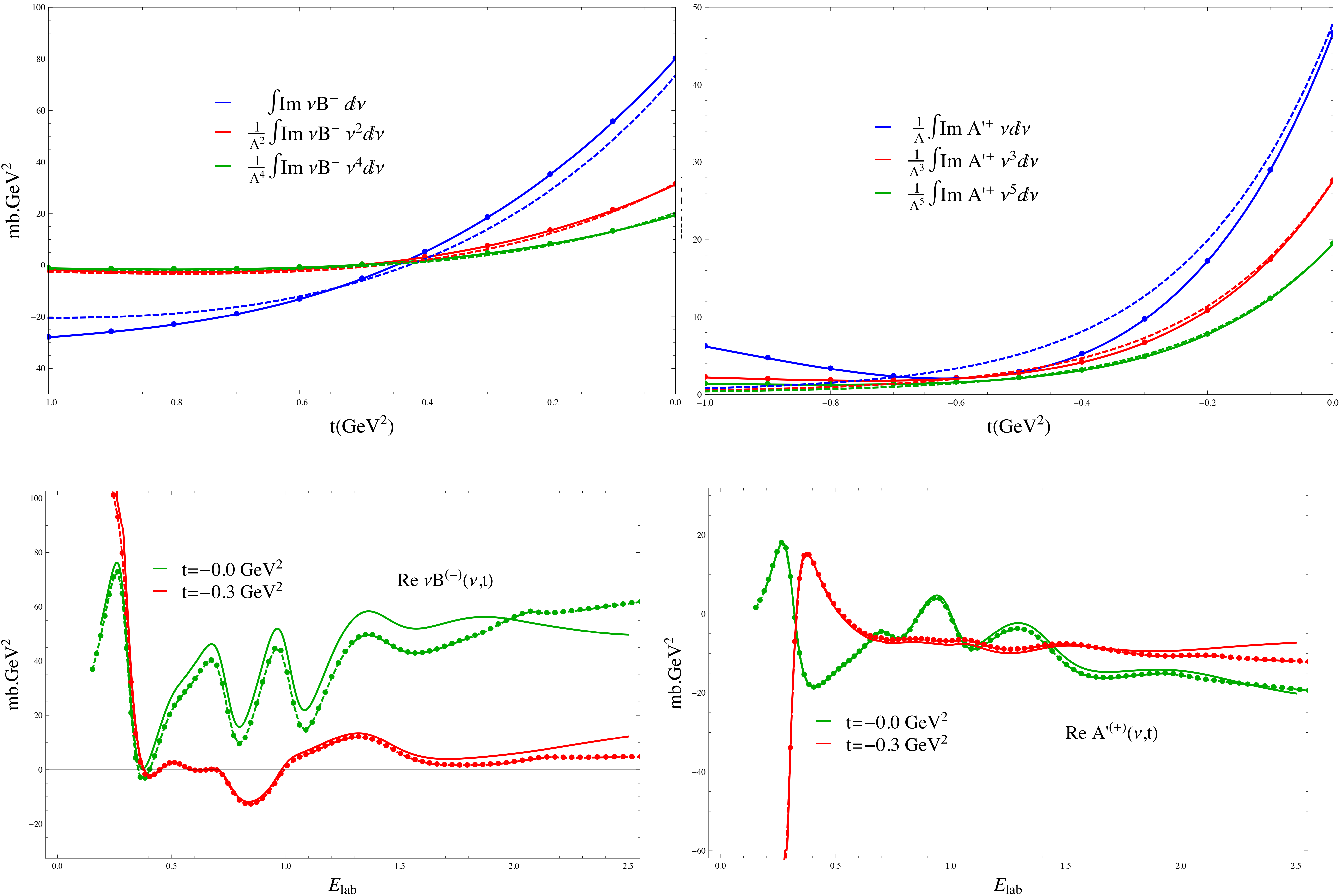


$\pi^{\pm} p \to \pi^{\pm} p$ **Polarization**

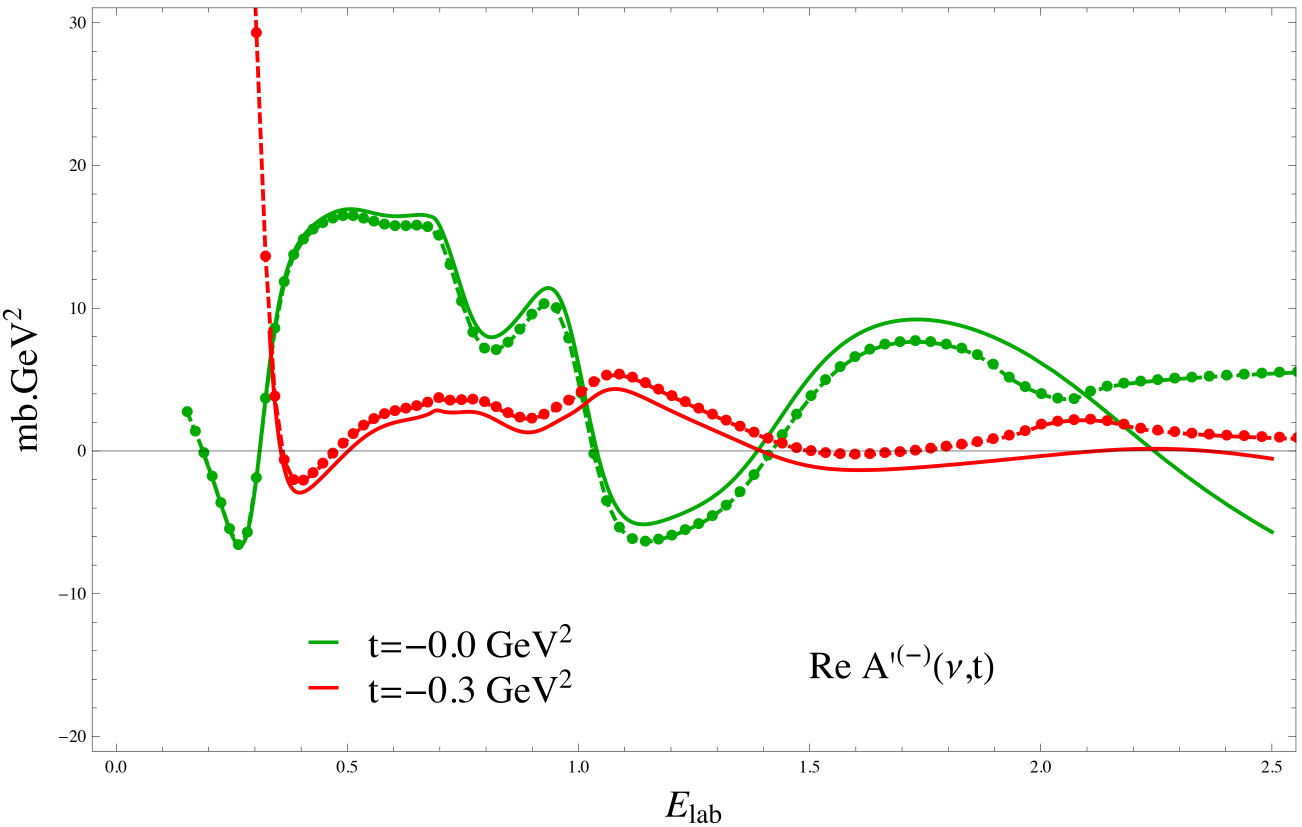


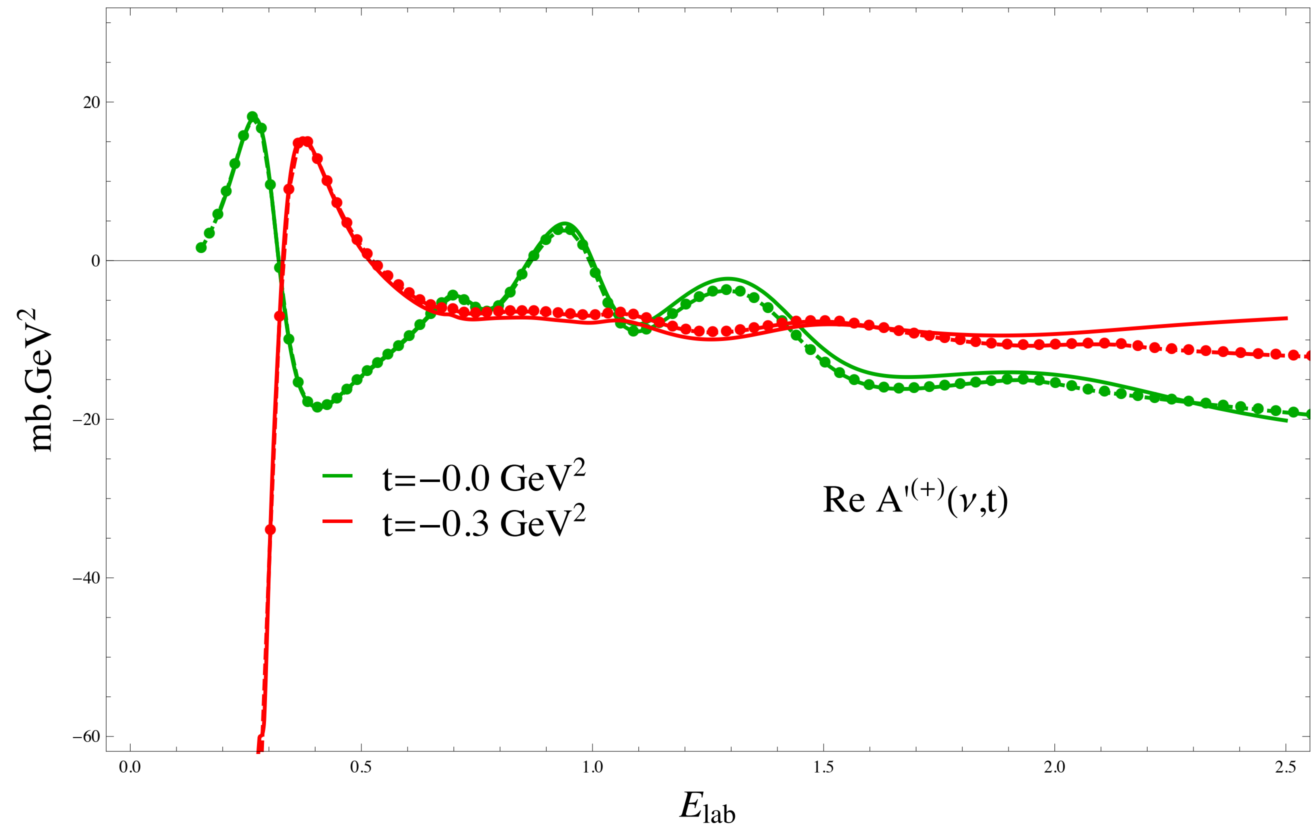
Finite Energy Sum Rules

$\pi N \to \pi N$

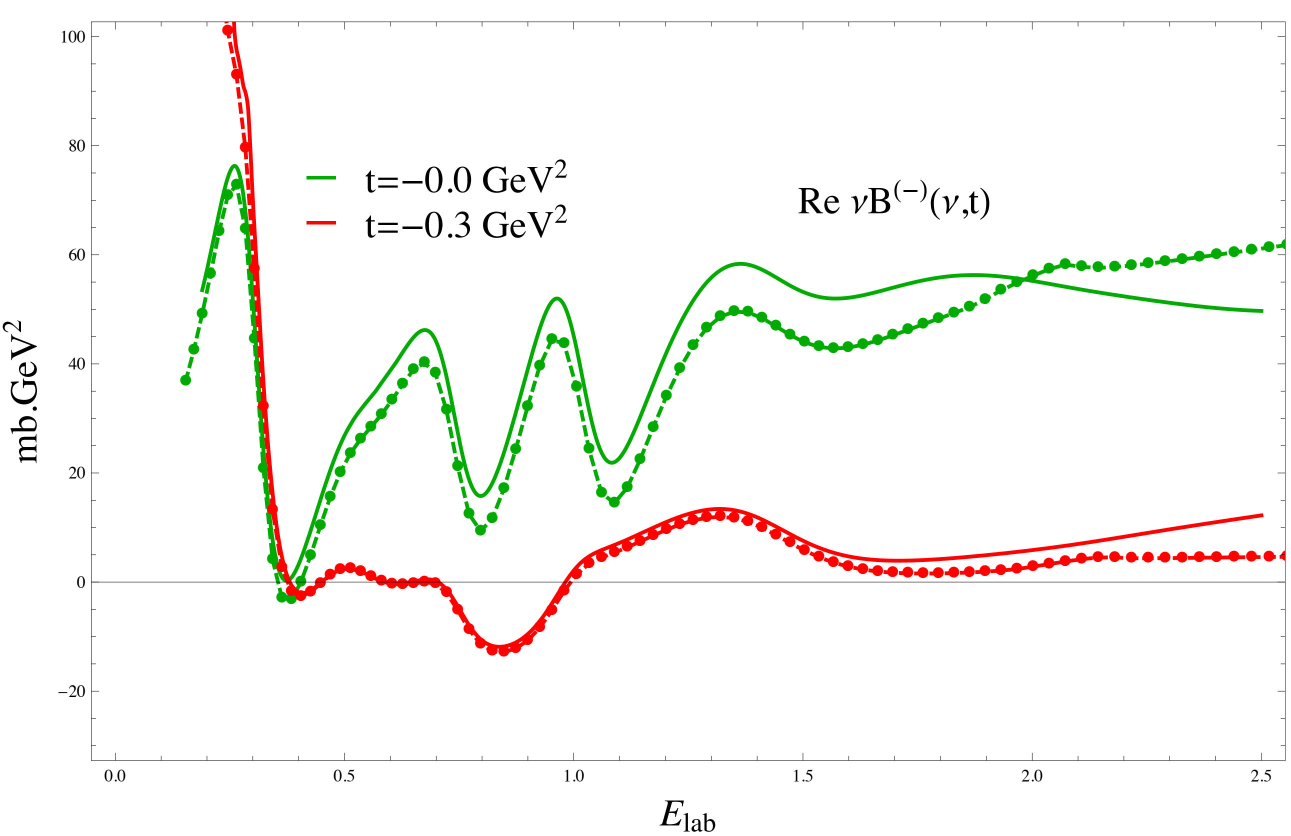


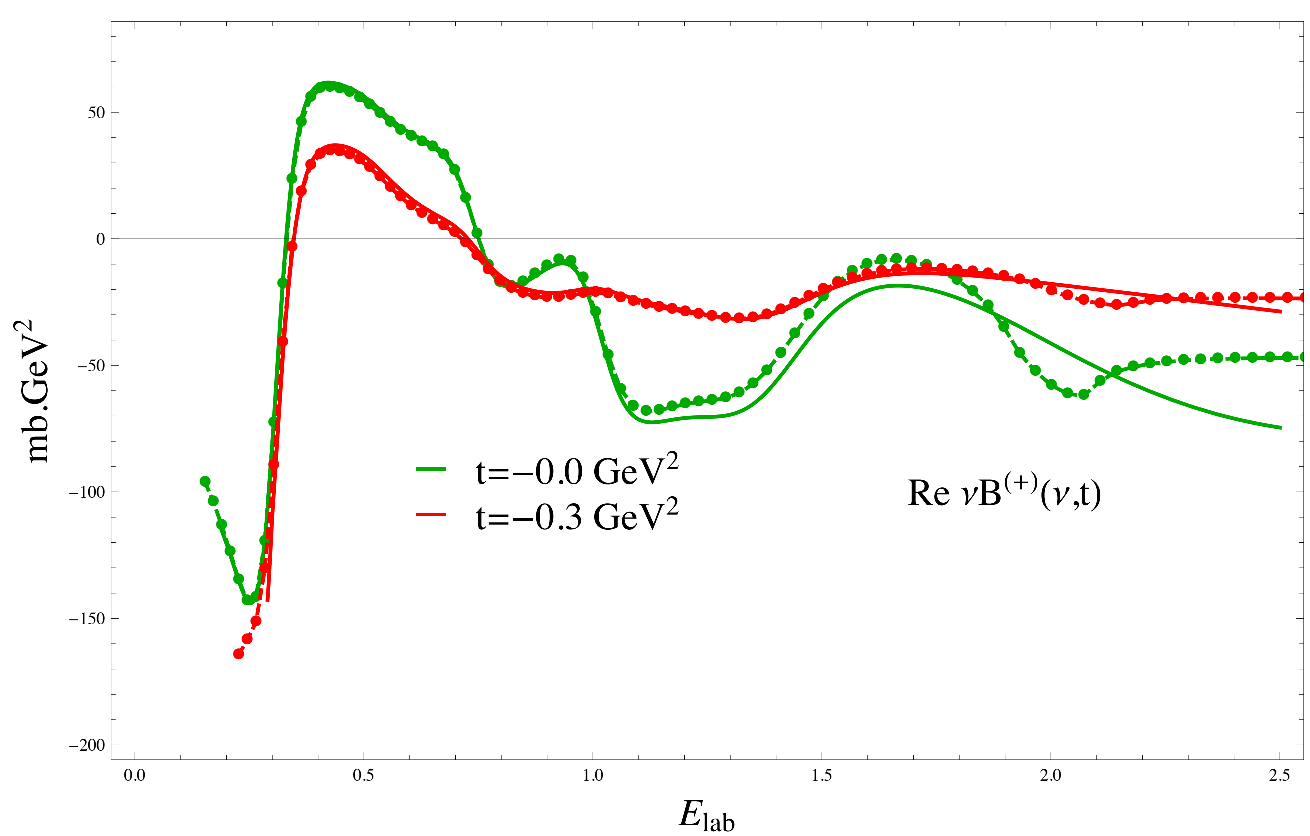
VM et al (JPAC) arXiv:1506.01764





dotted lines = reconstructed from dispersion relation
solid lines = SAID





VM et al (JPAC) arXiv:1506.01764

VM et al (JPAC) arXiv:1506.01764