# Nuclear Axial Current in $\chi$EFT

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# Overview

- Motivation / $\chi$EFT review
- From Lagrangians to currents
- Axial charge
- Axial current
- Outlook

# Introduction

- Previous work on axial current done by T.-S. Park, D.-P. Min, and M. Rho (1993)
- $\chi$EFT uses approximate chiral-symmetry (and Lorentz symmetry) of QCD to constrain the interactions of $\pi$ with $N$'s and other $\pi$'s
- Hard scale $\Lambda_\chi \simeq 1$ GeV $>> Q$ soft scale
- $\chi$EFT gives a perturbative expansion of $\mathcal{L}_{eff}$ in powers of $Q/\Lambda_\chi$

$$\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

- Unknown coefficients of the perturbative expansion are called LEC's and are fixed by comparison with the experiments

# Transition amplitude in TOPT

- Time-ordered perturbation theory (TOPT)

$$\langle f|T|i\rangle = \langle f|H_I \sum_{n=1}^{\infty} \left( \frac{1}{E_i - H_0 + i\eta} H_I \right)^{n-1} |i\rangle$$

- $H_0$ free Hamiltonian of $\pi$'s and $N$'s
- $H_I$ contains interactions among $\pi$'s and $N$'s and of these particles with external field
- Completeness: $\sum_{I_i} |I_i\rangle\langle I_i|$ between successive terms of $H_I$

$$\begin{aligned}\langle f|T|i\rangle &= \langle f|H_I|i\rangle + \sum_{I_1} \langle f|H_I|I_1\rangle \frac{1}{E_i - E_1 + i\eta} \langle I_1|H_I|i\rangle \\ &+ \sum_{I_1, I_2} \langle f|H_I|I_2\rangle \frac{1}{E_i - E_2 + i\eta} \langle I_2|H_I|I_1\rangle \frac{1}{E_i - E_1 + i\eta} \langle I_1|H_I|i\rangle \\ &+ \cdots\end{aligned}$$

- Power counting determined by

$$\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right) \times \left[Q^{-(N-N_K-1)}\, Q^{-2N_K}\right] \times Q^{3L}$$

  - $\beta_i$ =number of $\pi$'s at each vertex
  - $L$ =number of loops in the diagram ($Q^3$)
  - $\alpha_i = Q-$power associated to $i$ vertex
  - $N$ =number of energy denominators

- $\chi$EFT $T$-matrix has the following expansion

$$T = T^{\text{LO}} + T^{\text{NLO}} + T^{\text{N2LO}} + \cdots \quad T^{\text{NnLO}} \sim \left(\frac{Q}{\Lambda_\chi}\right)^n T^{\text{LO}}$$

# From Amplitudes to Potential

- Two-nucleon potential such that

$$v + vG_0 v + vG_0 vG_0 v + \cdots$$

  reproduces field theory $T$-matrix
- $v$ is assumed to have the same expansion as $T$

$$v = v^{LO} + v^{NLO} + \cdots \qquad v^{N^n LO} = \left(\frac{Q}{\Lambda_\chi}\right)^n v^{LO}$$

- Matching order by order

$$\begin{aligned} v^{LO} &= T^{LO} \\ v^{NLO} &= T^{NLO} - \left[v^{LO} G_0 v^{LO}\right] \\ &\cdots \end{aligned}$$

- In the presence of an external field $v_5 = A_a^0 \rho_{5,a} - \mathbf{A}_a \cdot \mathbf{j}_{5,a}$

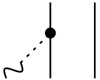
$$v \rightarrow v + v_5 \text{ (first order in v}_5\text{)}$$

$$\text{from } v_5^{(n)} \rightarrow \rho_{5,a}^{(n)} \ \mathbf{j}_{5,a}^{(n)}$$

# Axial charge

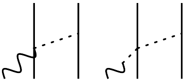
- Axial charge $\rho_{5,a}$
  - tree level contributions
  - loop contributions

# Tree Level up to order $Q$



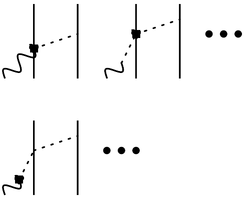
LO one-body axial charge

$$\rho_{5,a}^{(-2)} = -\frac{g_A}{4m_N}\,\tau_{1,a}\,\boldsymbol{\sigma}_1 \cdot (\mathbf{p}_1 + \mathbf{p}_1')$$



NLO two-body axial charge

$$\rho_{5,a}^{(-1)} = i\,\frac{g_A}{4f_\pi^2}\,(\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_a\,\frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{\omega_2^2}$$

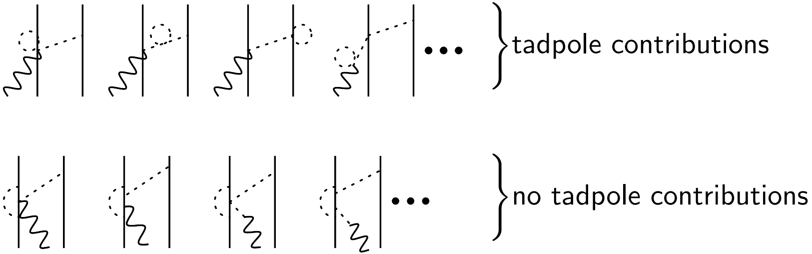


N3LO two-body axial charge dependent on LECs

- $d_i$'s $\rightarrow \mathcal{L}_{\pi N}^{(3)}$
- $l_i$'s $\rightarrow \mathcal{L}_{\pi\pi}^{(4)}$

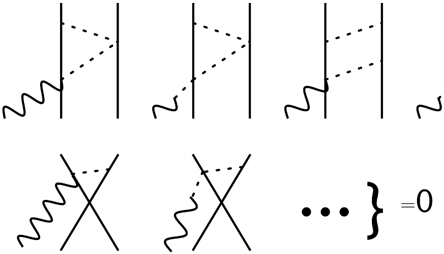
# Loop corrections to OPE at order $Q$

Vertices from $\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi\pi}^{(2)}$




- "reducible" (= reducible-iterated LS),
- UV divergences removed by renormalization of selected LEC's in $\mathcal{L}_{\pi N}^{(3)}$ and$\mathcal{L}_{\pi\pi}^{(4)}$

# Remaining loop corrections at order $Q$




- "reducible" and irreducible contributions taken into account
- Loop corrections to CT interactions vanish
- Loops are UV divergent and the divergences are reabsorbed by contact terms

# Axial charge contact terms at order $Q$

- Axial charge CT built requiring that $v_5$ is $\mathcal{T}$ and $\mathcal{P}$ invariant
  - $\rho_{5,a} \xrightarrow{\mathcal{T}} (-)^{a+1}\, \rho_{5,a}$
  - $\rho_{5,a} \xrightarrow{\mathcal{P}} -\rho_{5,a}$
- We obtain 9 terms of which 4 are independent (Fierz identities)
  - 2 LEC's needed to reabsorb the divergences of TPE diagrams

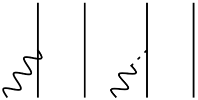
# Comparison with Park *et al.*

- Tree level
  - agreement for LO and OPE (NLO)
  - differences at N3LO, because Park *et al.* did not include all couplings from $\mathcal{L}_{\pi N}^{(3)}$ and $\mathcal{L}_{\pi\pi}^{(4)}$
- Differences in loop corrections due to different prescriptions used to construct $\rho_{5,a}$ (and $\mathbf{j}_{5,a}$)
- Park *et al.* only included the 2 contact terms needed to remove divergences

# Axial current

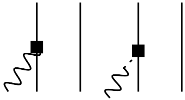
- Axial current $\mathbf{j}_{5,a}$
    - tree level contribution
    - loop corrections

# Tree Level up to order $Q$



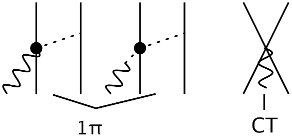
LO one-body axial current

$$\mathbf{j}_{5,a}^{(-3)} = -\frac{g_A}{2}\left(\boldsymbol{\sigma} - \mathbf{q}\,\frac{\mathbf{q}\cdot\boldsymbol{\sigma}}{q^2 + m_\pi^2}\right)\tau_a$$



N2LO relativistic correction to one-body axial current

$$\mathbf{j}_{5,a}^{(-1)} \to \mathcal{O}\left(\frac{1}{m_N^2}\right)$$



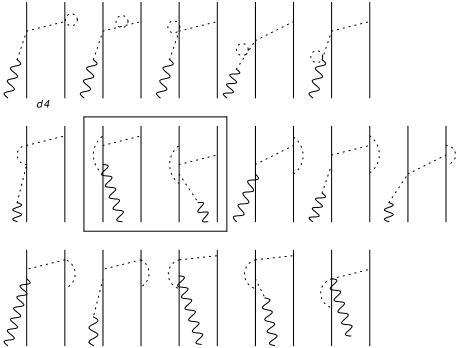

N3LO two-body axial current 3 LEC's

$$\mathbf{j}_{5,a}^{(0)}(1\pi) \to c_3,\ c_4,\ c_6 \ \text{ from } \mathcal{L}_{\pi N}^{(2)}$$

contact

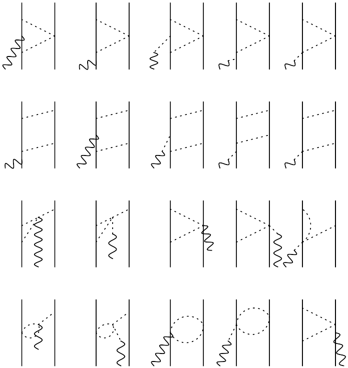
$$\mathbf{j}_{5,a}^{\mathrm{CT}(0)} \to 1\,\mathrm{LEC}$$

# Loop corrections to OPE at order Q



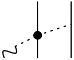

- OPE corrections vanish except for boxed diagrams
- Integrals finite in DR

# Remaining Loops at order $Q$



- integrals are finite in DR
- there are no contact terms at order $Q$ for $\mathbf{j}_{5,a}$
- freedom in $\pi$-field choice ( due to $3\pi A$, $4\pi$ and $NN3\pi$ vertices ) cancels out

# Comparison with Park *et al.*

- Park *et al.* pion-pole diagrams not considered 
- Agreement for tree level
- TPE
  - agreement for irreducible contribution
  - reducible diagrams not considered in Park *et al.*
- Remaining contributions
  - Park *et al.* dependent on the parametrization of the pion field
  - Our results independent of the parametrization of the pion field

$$U = 1 + \frac{i}{f_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{1}{2\,f_\pi^2}\,\boldsymbol{\pi}^2 - \frac{i\,\alpha}{f_\pi^3}\,\boldsymbol{\pi}^2\,\boldsymbol{\tau}\cdot\boldsymbol{\pi} + \frac{8\,\alpha - 1}{8\,f_\pi^4}\,\boldsymbol{\pi}^4 + \dots$$

# Outlook

- Weak transitions in few-nucleon systems only carried at N3LO (no loop corrections) so far

- Forthcoming applications
  - $\mu^-$ capture on $A = 2 - 4$
  - $\nu$-scattering and weak capture ($p\,{}^3$He)
  - weak transitions in $A > 4$, $\beta$ decays, $e^-$ and $\mu^-$ captures

# Bibliography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# Reducible Irreducible diagrams

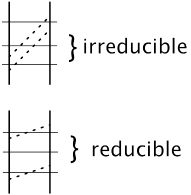


Figure : two kind of diagrams