Nuclear Axial Current in $\chi$EFT

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June 29, 2015

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Overview

- Motivation / $\chi$EFT review
- From Lagrangians to currents
- Axial charge
- Axial current
- Outlook
Previous work on axial current done by T.-S. Park, D.-P. Min, and M. Rho (1993)

χEFT uses approximate chiral-symmetry (and Lorentz symmetry) of QCD to constrain the interactions of π with N’s and other π’s

Hard scale $\Lambda_\chi \simeq 1 \text{ GeV} \gg Q$ soft scale

χEFT gives a perturbative expansion of $\mathcal{L}_{\text{eff}}$ in powers of $Q/\Lambda_\chi$

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots
$$

Unknown coefficients of the perturbative expansion are called LEC’s and are fixed by comparison with the experiments
Transition amplitude in TOPT

- Time-ordered perturbation theory (TOPT)

\[
\langle f | T | i \rangle = \langle f | H_I \sum_{n=1}^{\infty} \left( \frac{1}{E_i - H_0 + i\eta} H_I \right)^{n-1} | i \rangle
\]

- \(H_0\) free Hamiltonian of \(\pi\)'s and \(N\)'s
- \(H_I\) contains interactions among \(\pi\)'s and \(N\)'s and of these particles with external field
- Completeness: \(\sum_{l_i} |l_i\rangle \langle l_i|\) between successive terms of \(H_I\)

\[
\langle f | T | i \rangle = \langle f | H_I | i \rangle + \sum_{l_1} \langle f | H_I | l_1 \rangle \frac{1}{E_i - E_1 + i\eta} \langle l_1 | H_I | i \rangle \\
+ \sum_{l_1, l_2} \langle f | H_I | l_2 \rangle \frac{1}{E_i - E_2 + i\eta} \langle l_2 | H_I | l_1 \rangle \frac{1}{E_i - E_1 + i\eta} \langle l_1 | H_I | i \rangle \\
+ \cdots
\]
Power Counting

- Power counting determined by

\[
\left( \prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right) \times \left[ Q^{-(N-N_K-1)} Q^{-2N_K} \right] \times Q^{3L}
\]

- $\beta_i =$ number of $\pi$'s at each vertex
- $L =$ number of loops in the diagram ($Q^3$)
- $\alpha_i = Q$—power associated to $i$ vertex
- $N =$ number of energy denominators

- $\chiEFT$ $T$-matrix has the following expansion

\[
T = T^{\text{LO}} + T^{\text{NLO}} + T^{\text{N2LO}} + \ldots \quad T^{\text{NnLO}} \sim \left( \frac{Q}{\Lambda_\chi} \right)^n T^{\text{LO}}
\]
Two-nucleon potential such that

\[ \nu + \nu G_0 \nu + \nu G_0 \nu G_0 \nu + \cdots \]

reproduces field theory \( T \)-matrix

\[ \nu \text{ is assumed to have the same expansion as } T \]

\[ \nu = \nu^{LO} + \nu^{NLO} + \cdots \]

\[ \nu^{N^2LO} = \left( \frac{Q}{\Lambda^2} \right)^n \nu^{LO} \]

Matching order by order

\[ \nu^{LO} = T^{LO} \]

\[ \nu^{NLO} = T^{NLO} - [\nu^{LO} G_0 \nu^{LO}] \]

\[ \cdots \]

In the presence of an external field \( \nu_5 = A_0^0 \rho_{5,a} - A_a \cdot j_{5,a} \)

\[ \nu \rightarrow \nu + \nu_5 \text{ (first order in } \nu_5) \]

from \( \nu_5^{(n)} \rightarrow \rho_{5,a}^{(n)} j_{5,a}^{(n)} \)
Axial charge

- Axial charge $\rho_{5,a}$
  - tree level contributions
  - loop contributions
Tree Level up to order $Q$

**LO one-body axial charge**

$$\rho_{5,a}^{(-2)} = -\frac{g_A}{4m_N} \tau_{1,a} \sigma \cdot (p_1 + p'_1)$$

**NLO two-body axial charge**

$$\rho_{5,a}^{(-1)} = i \frac{g_A}{4f^2_\pi} \left( \tau_1 \times \tau_2 \right)_a \frac{\sigma \cdot k_2}{\omega_2^2}$$

**N3LO two-body axial charge dependent on LECs**

- $d_i$'s $\rightarrow \mathcal{L}_{\pi N}^{(3)}$
- $l_i$'s $\rightarrow \mathcal{L}_{\pi \pi}^{(4)}$
Loop corrections to OPE at order $Q$

Vertices from $\mathcal{L}^{(1)}_{\pi N}$ and $\mathcal{L}^{(2)}_{\pi\pi}$

\[
\begin{align*}
\text{tadpole contributions} & \quad \cdots \\
\text{no tadpole contributions} & \quad \cdots
\end{align*}
\]

- "reducible" (= reducible-iterated LS),
- UV divergences removed by renormalization of selected LEC's in $\mathcal{L}^{(3)}_{\pi N}$ and $\mathcal{L}^{(4)}_{\pi\pi}$
Remaining loop corrections at order $Q$

"reducible" and irreducible contributions taken into account

Loop corrections to CT interactions vanish

Loops are UV divergent and the divergences are reabsorbed by contact terms
Axial charge contact terms at order $Q$

- Axial charge CT built requiring that $v_5$ is $\mathcal{T}$ and $\mathcal{P}$ invariant
  
  \[ \rho_{5,a} \xrightarrow{\mathcal{T}} (-)^{a+1} \rho_{5,a} \]
  
  \[ \rho_{5,a} \xrightarrow{\mathcal{P}} -\rho_{5,a} \]

- We obtain 9 terms of which 4 are independent (Fierz identities)

- 2 LEC’s needed to reabsorb the divergences of TPE diagrams
Comparison with Park et al.

- Tree level
  - agreement for LO and OPE (NLO)
  - differences at N3LO, because Park et al. did not include all couplings from $\mathcal{L}^{(3)}_{\pi N}$ and $\mathcal{L}^{(4)}_{\pi\pi}$

- Differences in loop corrections due to different prescriptions used to construct $\rho_{5,a}$ (and $j_{5,a}$)

- Park et al. only included the 2 contact terms needed to remove divergences
Axial current

- Axial current $j_{5,a}$
  - tree level contribution
  - loop corrections
Tree Level up to order $Q$

LO one-body axial current

$$j^{(-3)}_{5,a} = -\frac{g_A}{2} \left( \sigma - q \frac{\mathbf{q} \cdot \mathbf{\sigma}}{q^2 + m^2_{\pi}} \right) \tau_a$$

N2LO relativistic correction to one-body axial current

$$j^{(-1)}_{5,a} \rightarrow \mathcal{O} \left( \frac{1}{m^2_N} \right)$$

N3LO two-body axial current 3 LEC’s

$$j^{(0)}_{5,a}(1\pi) \rightarrow c_3, c_4, c_6 \text{ from } \mathcal{L}^{(2)}_{\pi N}$$

contact

$$j^{CT(0)}_{5,a} \rightarrow 1 \text{ LEC}$$
Loop corrections to OPE at order $Q$

- OPE corrections vanish except for boxed diagrams
- Integrals finite in DR
Remaining Loops at order $Q$

- Integrals are finite in DR.
- There are no contact terms at order $Q$ for $j_{5,a}$.
- Freedom in $\pi$-field choice (due to $3\pi A$, $4\pi$ and $NN3\pi$ vertices) cancels out.
Comparison with Park \textit{et al.}

- Park \textit{et al.} pion-pole diagrams not considered
- Agreement for tree level
- TPE
  - agreement for irreducible contribution
  - reducible diagrams not considered in Park \textit{et al.}
- Remaining contributions
  - Park \textit{et al.} dependent on the parametrization of the pion field
  - Our results independent of the parametrization of the pion field

\[ U = 1 + \frac{i}{f_\pi} \tau \cdot \pi - \frac{1}{2 f_\pi^2} \pi^2 - \frac{i \alpha}{f_\pi^3} \pi^2 \tau \cdot \pi + \frac{8 \alpha - 1}{8 f_\pi^4} \pi^4 + \ldots \]
Outlook

- Weak transitions in few-nucleon systems only carried at N3LO (no loop corrections) so far

- Forthcoming applications
  - $\mu^-$ capture on $A = 2 - 4$
  - $\nu$-scattering and weak capture ($p^3\text{He}$)
  - weak transitions in $A > 4$, $\beta$ decays, $e^-$ and $\mu^-$ captures


Reducible Irreducible diagrams

\[ \text{Reducible} \]

\[ \text{Irreducible} \]

**Figure**: two kind of diagrams