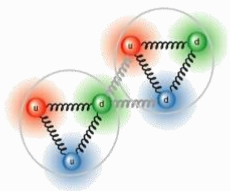
EFFECTIVE THEORY FOR LATTICE NUCLEI

Lorenzo Contessi

Nuclear challenge

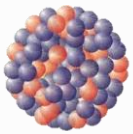
- Make precise predictions
- Understand the origin of the interaction:



From QCD
(Quarks)

EFT

To Nuclear physics
(Nucleons)



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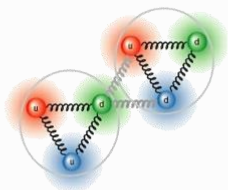

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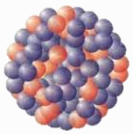

Keystone: Use the correct **powercounting** and **degrees of freedom**

Contact EFT

Start with a contact EFT (PionLess approach)

✓

- It is relatively **easy**:
  Only δ interactions
- **Simplest** theory under control:
  Not too many **scales** enter in the powerconting
- Best way to **understand** powercounting:
  That applies what we learn to **pionful** theories

✗

- **No pions included:** the theory will break at some point.
  (Not reliable for predictions)

Contact EFT

$$V_{2b} = C_0 + C_1(\sigma_1 \cdot \sigma_2) + C_2(P_1^2 + P_2^2) + C_3(P_1^2 + P_2^2)(\sigma_1 \cdot \sigma_2) + \cdots$$

$$+$$

$$V_{3b} = D_0 + \cdots$$

Contact EFT

$$V_{2b} = \overbrace{C_0 + C_1(\sigma_1 \cdot \sigma_2)}^{\text{LO}} + \overbrace{\underbrace{C_2(P_1^2 + P_2^2) + C_3(P_1^2 + P_2^2)(\sigma_1 \cdot \sigma_2) + \cdots}_{\text{Perturbation theory}}}^{\text{NLO}}$$

$$+$$

$$\underbrace{V_{3b}}_{\text{Avoid Thomas collapse}} = D_0 + \cdots$$

Contact EFT

$$V_{2b} = \underbrace{\overbrace{C_0 + C_1(\sigma_1 \cdot \sigma_2)}^{\text{LO}} + \overbrace{C_2(P_1^2 + P_2^2) + C_3(P_1^2 + P_2^2)(\sigma_1 \cdot \sigma_2) + \cdots}^{\text{NLO}}}_{\text{Perturbation theory}}$$

$$+$$

$$\underbrace{V_{3b} = D_0}_{\text{Avoid Thomas collapse}} + \cdots$$

After **regolarization** in the coordinate space:

$$V^{LO} = \sum_{ij} [C_0^\Lambda\, e^{-\frac{1}{2}|r_{ij}|^2\Lambda^2} + C_1^\Lambda\, e^{-\frac{1}{2}|r_{ij}|^2\Lambda^2}(\vec{\sigma_i} \cdot \vec{\sigma_j})\,]$$

$$+ D_0^\Lambda \sum_{ijk} \left[e^{-\frac{\Lambda^2}{2}\left(|r_{ij}|^2+|r_{ik}|^2\right)} + e^{-\frac{\Lambda^2}{2}\left(|r_{ij}|^2+|r_{jk}|^2\right)} + e^{-\frac{\Lambda^2}{2}\left(|r_{jk}|^2+|r_{ik}|^2\right)}\right]$$

Contact EFT - Two possible applications

Start from **LQCD** data:

- Input from $m_{\pi} = (500\ and\ 800)\ MeV$.
- ✓ **Extension of LQCD** in low bound systems.
- ✓ **Very heavy pions** (less severe computational problems).

Start from **Experimental** data:

- Pions can be neglected only for light nuclei (in LO at least).
- ✓ **Many experiments** to fit LECs.
- ✓ **Many observables** to guide powercounting.

LQCD: $M_\pi = 510\ and\ 805\ MeV$

Nucleus	Nature	$m_\pi = 510\ MeV$ [1]	$m_\pi = 805\ MeV$ [2]
N (mass)	939.6	1320.0	1634.0
P (mass)	938.3	1320.0	1634.0
$n-p$	2.224	11.5(1.3)	19.5(4.8)
$n-n$	---------	7.4(1.4)	15.9(3.8)
^3H	8.482	20.3(4.5)	53.9(10.7)
^3He	7.718	20.3(4.5)	53.9(10.7)
^4He	28.296	43.0(14.4)	107.0(24.2)

[1] - Takeshi Yamazaki, Ken-ichi Ishikawa and al. [Phys. rev. D 86, 074514 (2012)]
[2] - S. R. Beane, E. Chang and al. [Phys. rev. D 87, 034506 (2013)]

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2b LECs

3b LEC

Benchmark

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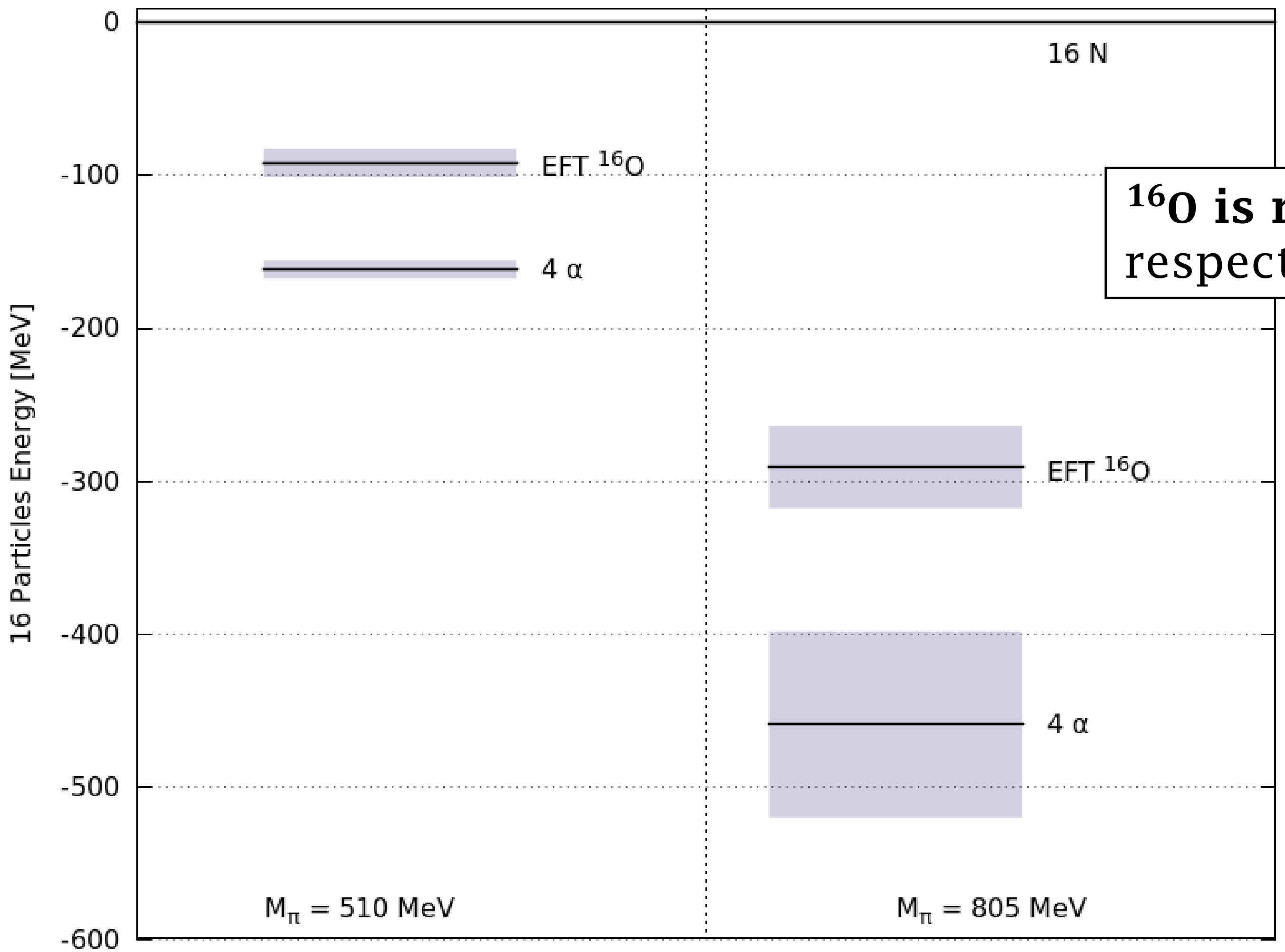
LQCD: $M_\pi = 510\ and\ 805\ MeV$

$m_\pi = 510\ MeV$		
Λ $[\mathrm{fm}^{-1}]$	$^4He\ Energy$ $[MeV]$	$^5He\ Energy$ $[MeV]$
2	-31,1(1)	-31,2(1)
4	-34,8(2)	-32,1(3)
6	-36,7(2)	-32,4(4)
8	-38,1(3)	-32,5(4)
∞	-40.4(1.5)	-30.0(1)
LQCD	-43(14)	

$m_\pi = 805\ MeV$		
Λ $[\mathrm{fm}^{-1}]$	$^4He\ Energy$ $[MeV]$	$^5He\ Energy$ $[MeV]$
2	-87,9(2)	-93,7(1)
4	-91,3(3)	-87,4(2)
6	-96,4(4)	-88,6(2)
8	-101,3(5)	-92,9(3)
∞	-114(15)	-120(25)
LQCD	-107(24)	

LQCD: $M_\pi = 510\ and\ 805\ MeV$



^{16}O is not bound with respect to 4α threshold

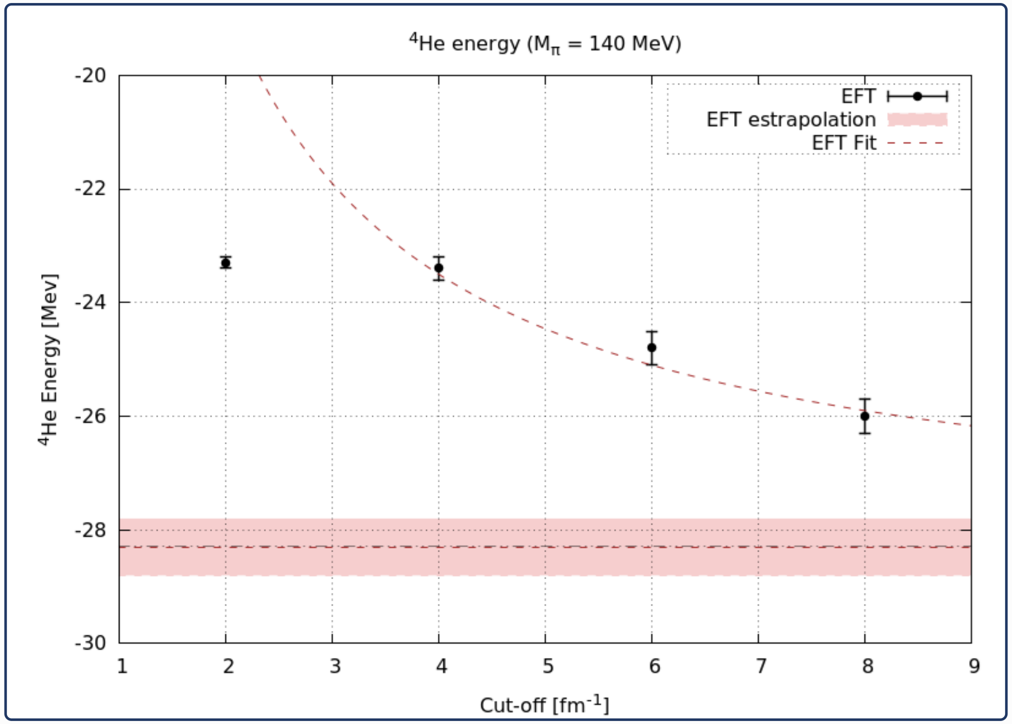
Physical M_π

$$V_{Lo} = \sum_{ij}[C_0 + C_1(\sigma_1 \cdot \sigma_2)]\delta_{r_i,r_j} + \sum_{ijk} D_0\, \delta_{r_i,r_j,r_k}$$

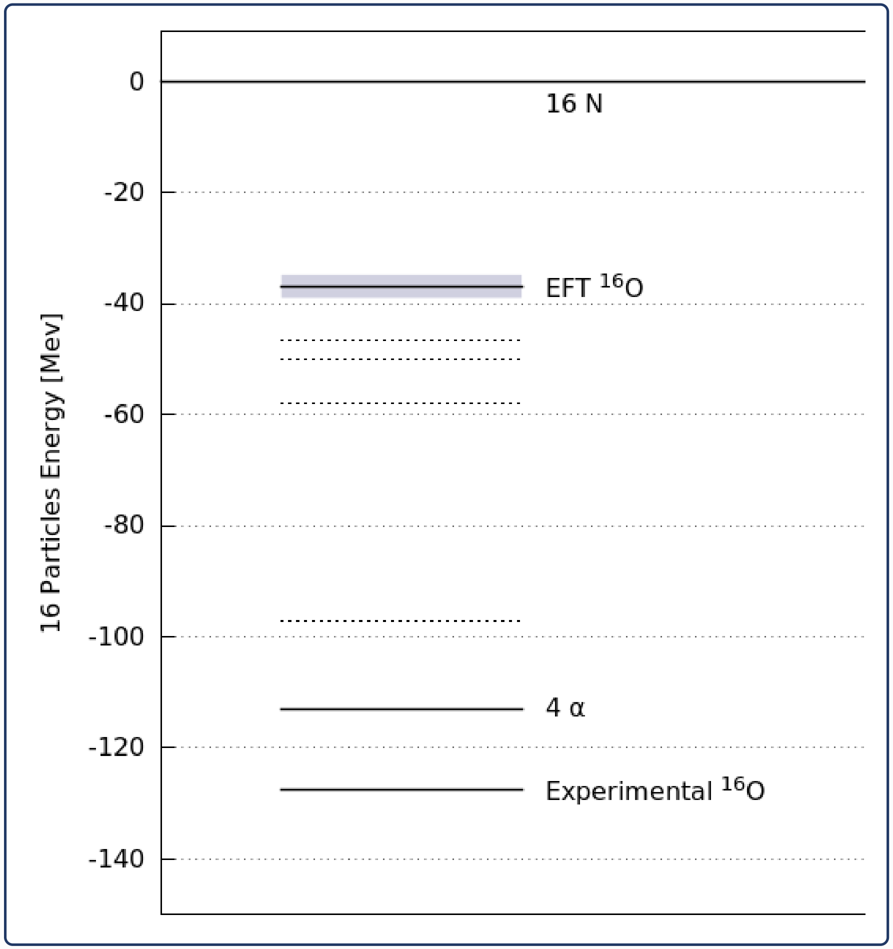
$n-p$
scattering lenght
and binding energy

^3He binding energy

Λ $[\mathrm{fm}^{-1}]$	$^4He\ Energy$ $[MeV]$
2	-23.3(1)
4	-23.4(2)
6	-24.8(3)
8	-26.0(3)
∞	-28.3(5)
Exp	-28.296



Physical M_π



Oxygen seems not bound.
There are many possible explanations:

- Not convergence of pionless theory:
  Pions become important.
- New relevant scale of the theory:
  Number of particles.
- Problem in the calculations:
  Ground state much different from expectations.

Overwiev

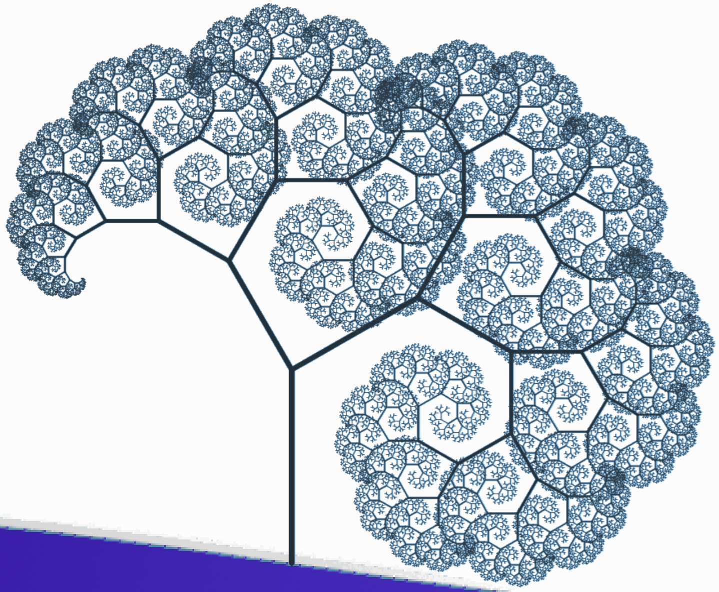
- ✓ Contact theory is very good for fewbody system.
- x Give wrong results in heavier nuclei.

  What happens in betweens?
  When does the theory break and why?

- o Oxygen is not bound for high M_π too.

  Powercount breaking.
  LQCD physics does not bind Oxygen.

Thanks for your attention



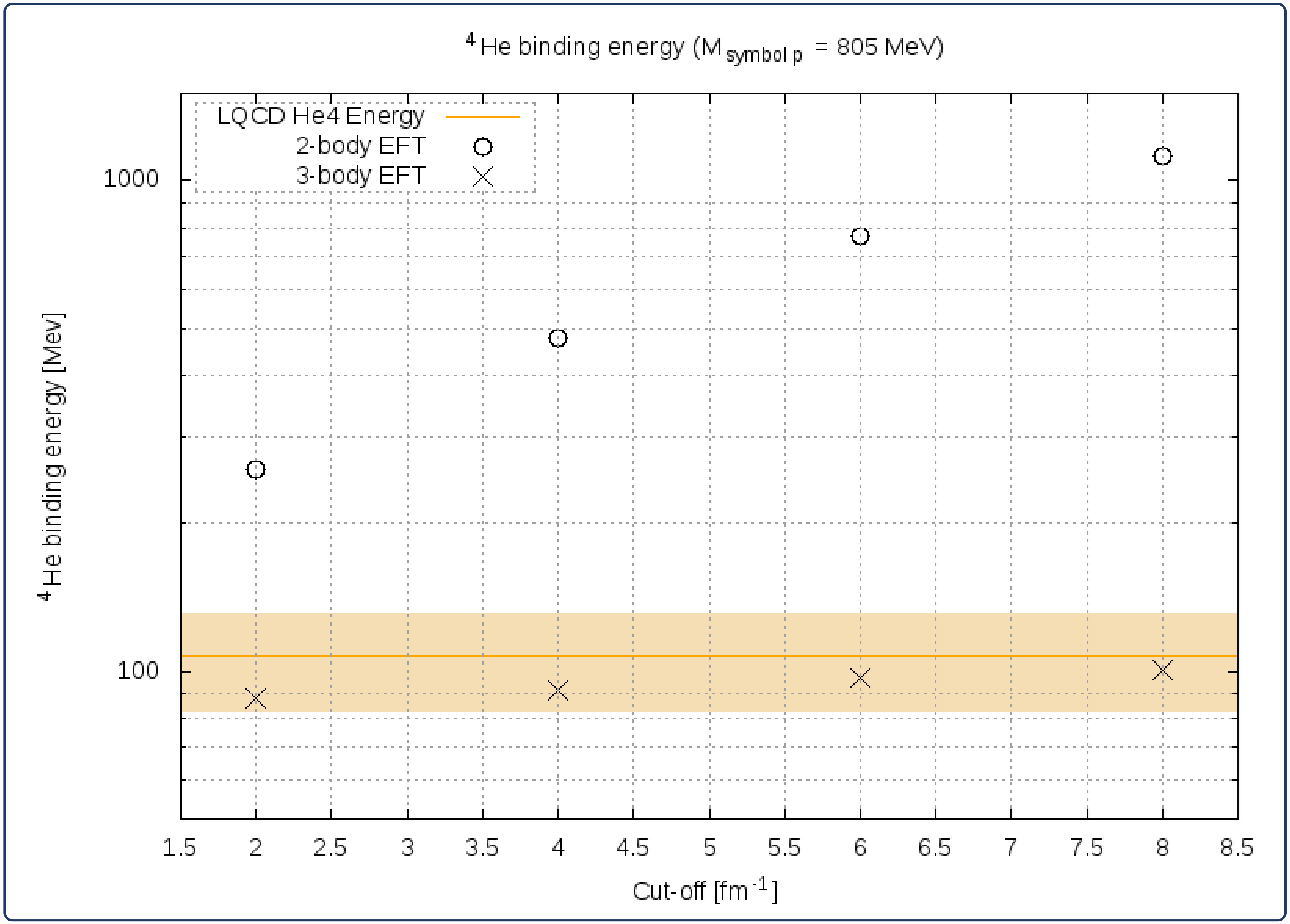
References

[3] A. Nogga, R. G. E. Timmermans and U. van Kolck, Phys. Rev. C 72, 054006 (2005) [arXiv:nucl-th/0506005].

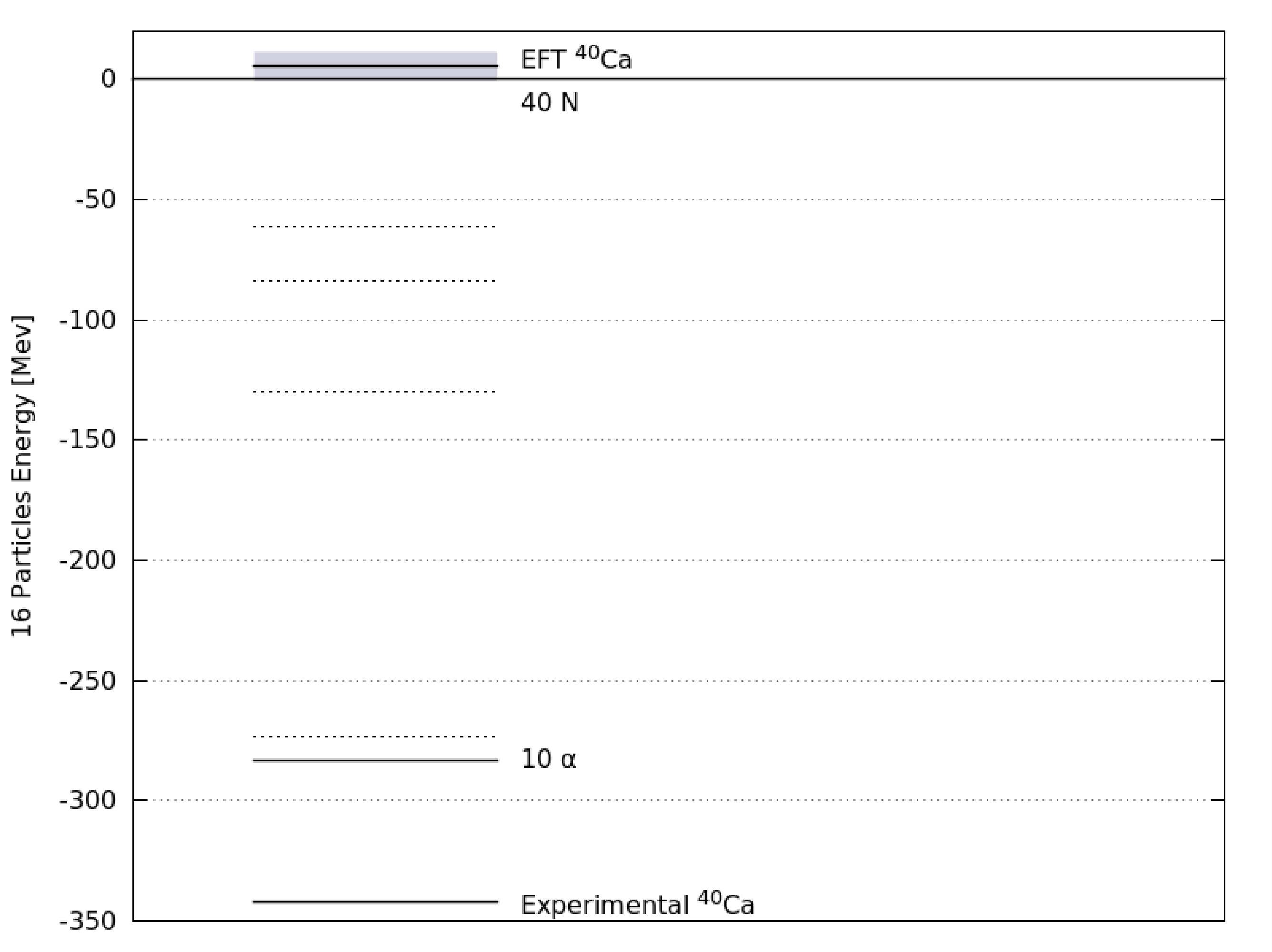
[4] S. Fleming, T. Mehen and I. W. Stewart, Nucl. Phys. A 677, 313 (2000) [arXiv:nucl-th/9911001].

[5] L.H.Thomas, Phys.Rev. 47 (1935) 903.

Contact EFT – 3b force contribution



Calcium



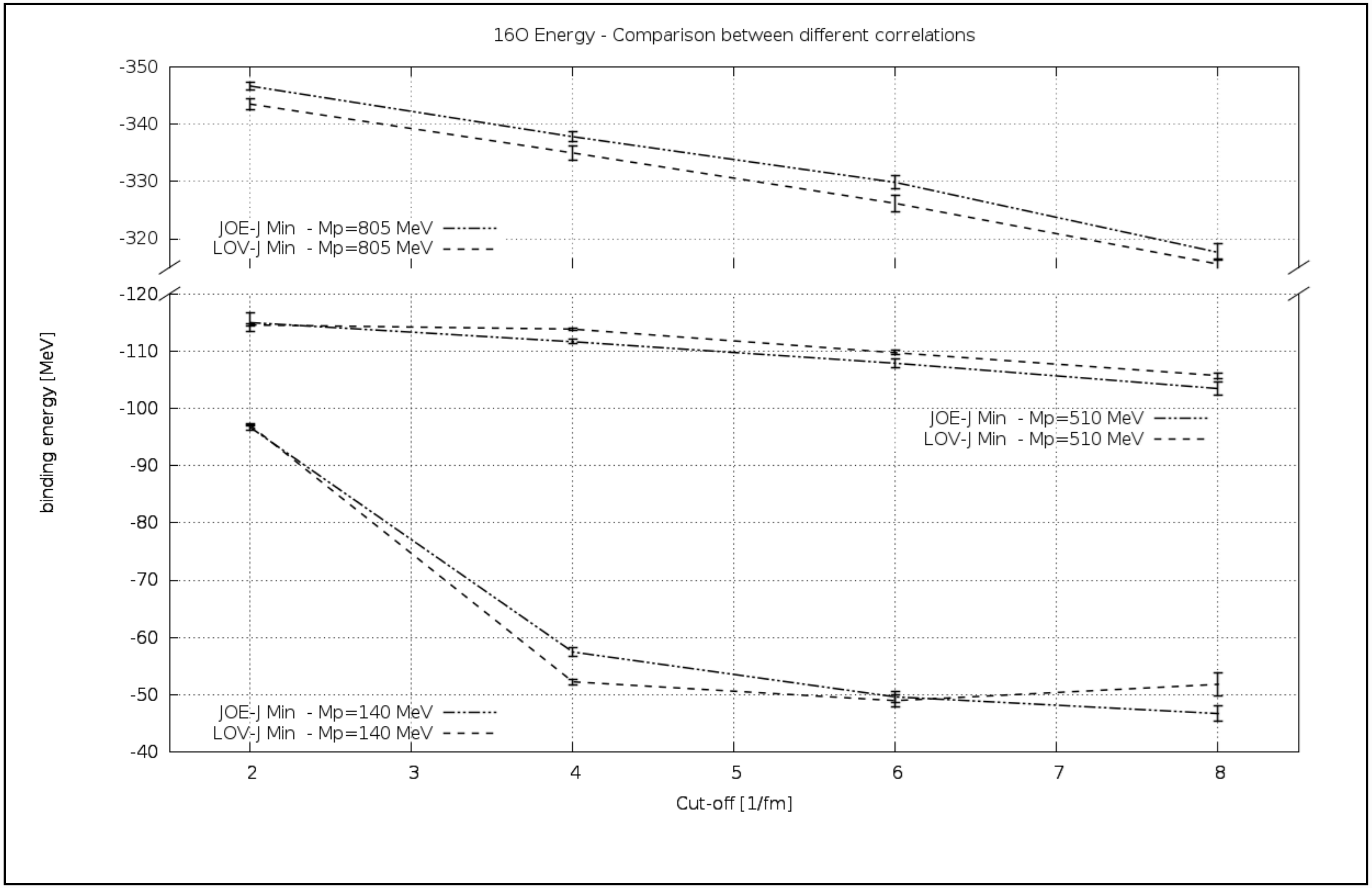
EFT ^{40}Ca
40 N
10 α
Experimental ^{40}Ca
16 Particles Energy [Mev]
0
-50
-100
-150
-200
-250
-300
-350

Helion

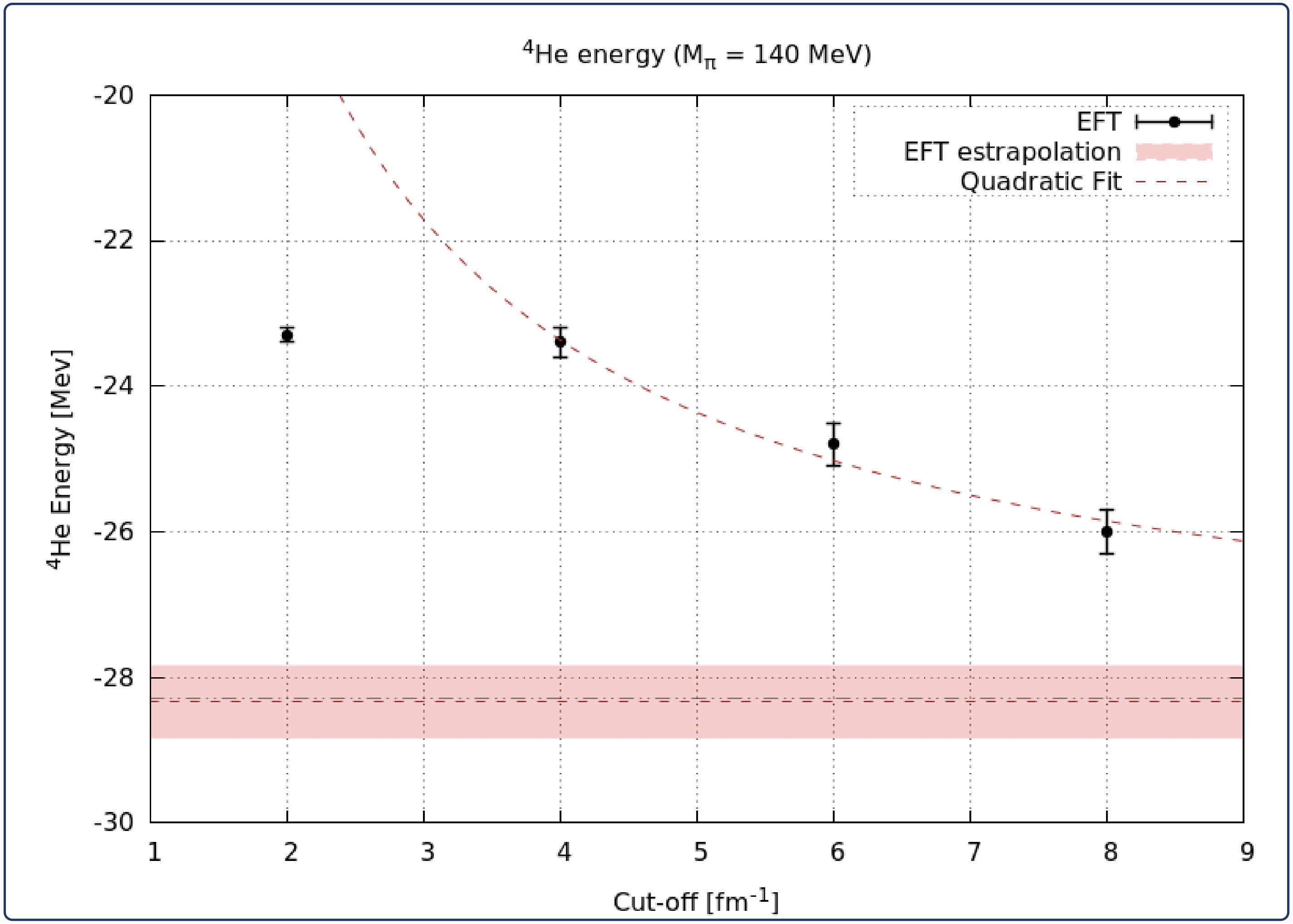
$$m_\pi = 140\ MeV$$

Λ $[\mathrm{fm}^{-1}]$	$^4He\ Energy$ $[MeV]$	$^5He\ Energy$ $[MeV]$
2	-23.3(1)	-19.8(2)
4	-23.4(2)	-17.8(4)
6	-24.8(3)	-17.9(4)
8	-26.0(3)	-18(1)
∞	-28.3(5)	-22(3)
Exp	**-28.296**	**27.406**

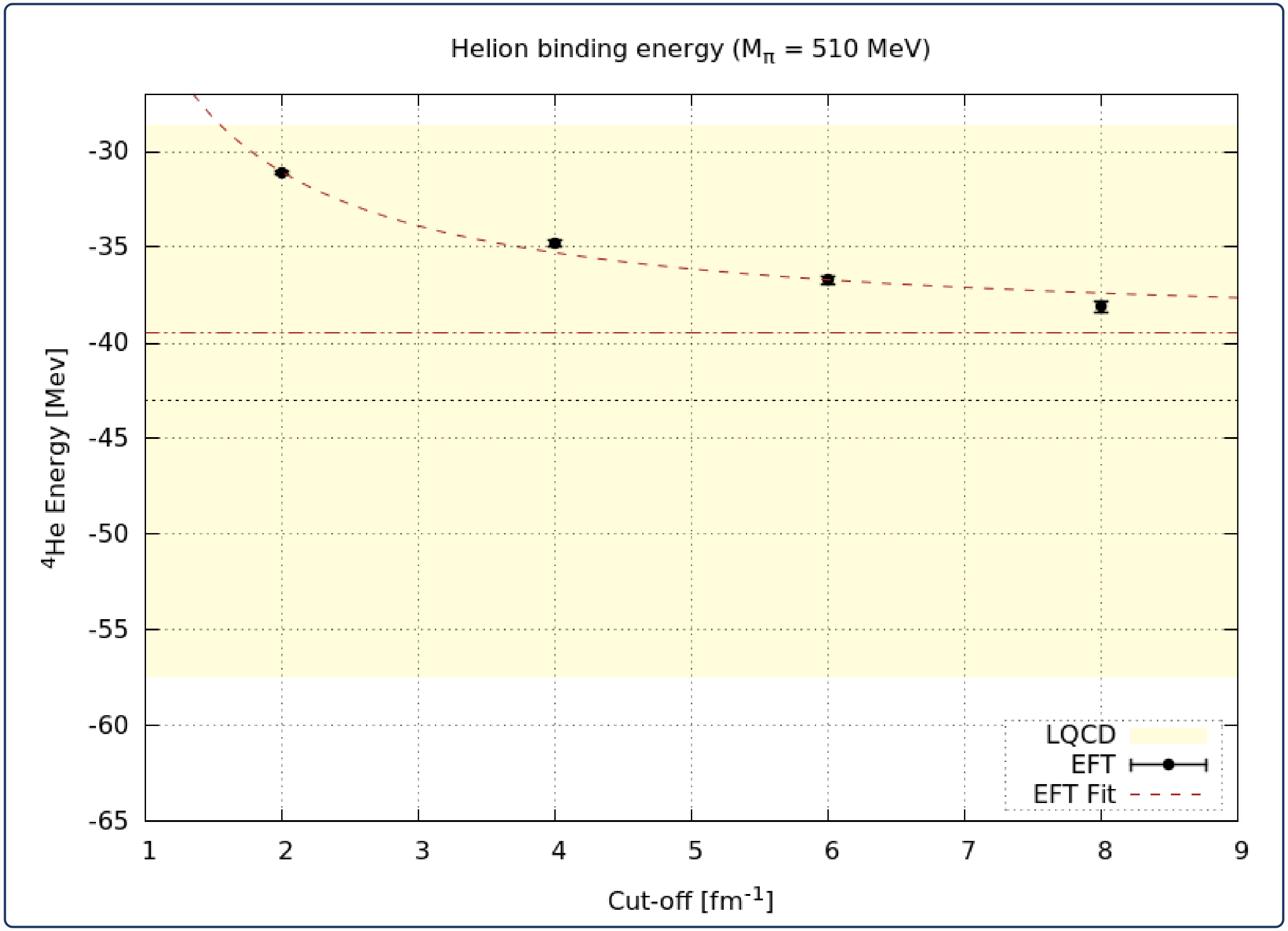
Oxygen convergence in LQCD



Physical M_π



LQCD: $M_\pi = 510\ and\ 805\ MeV$



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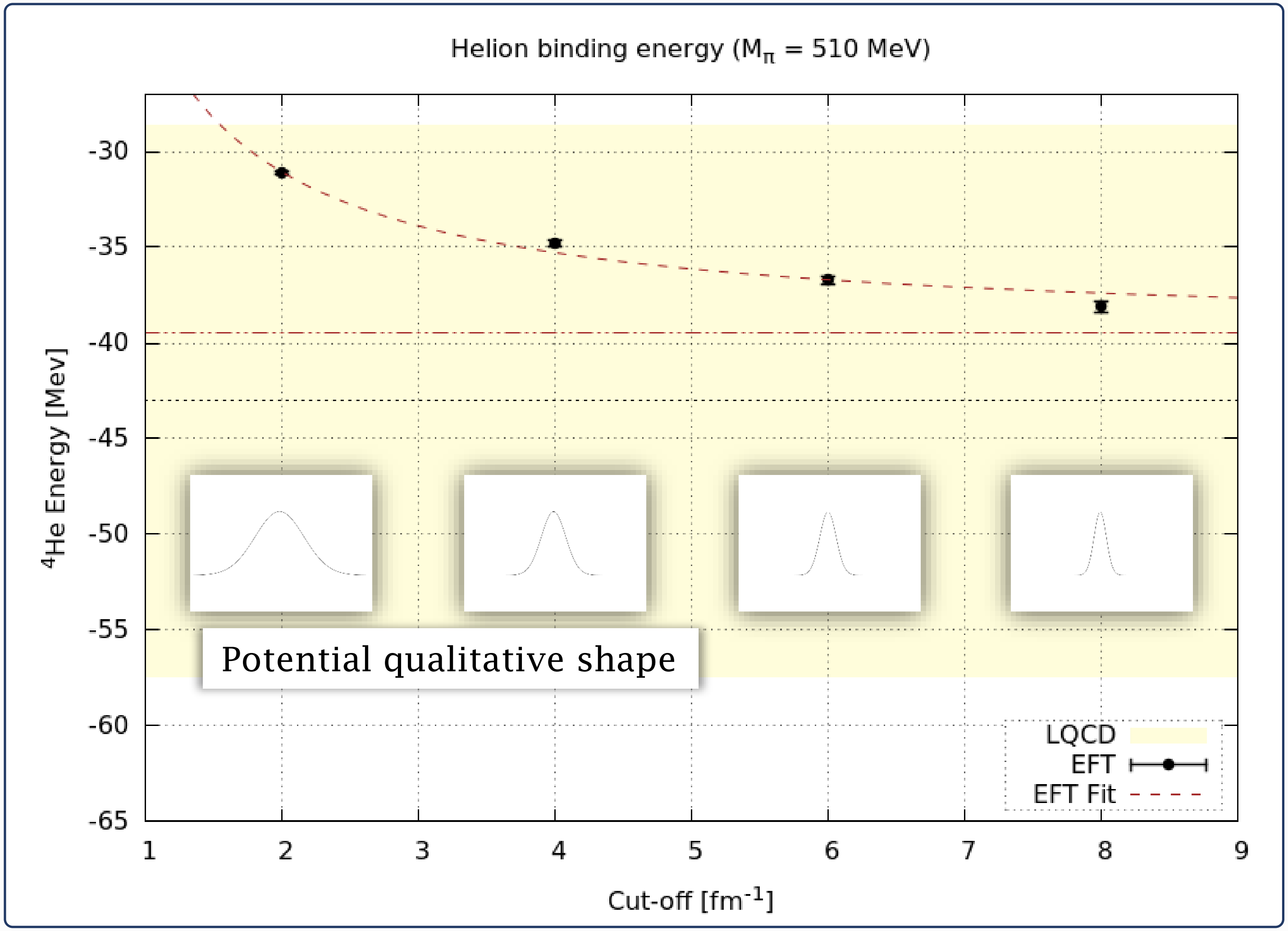
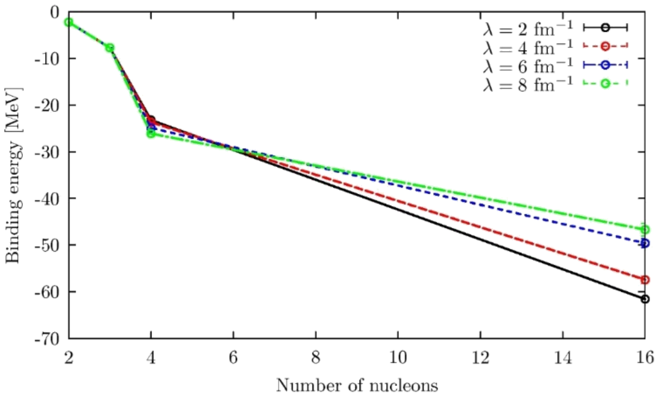
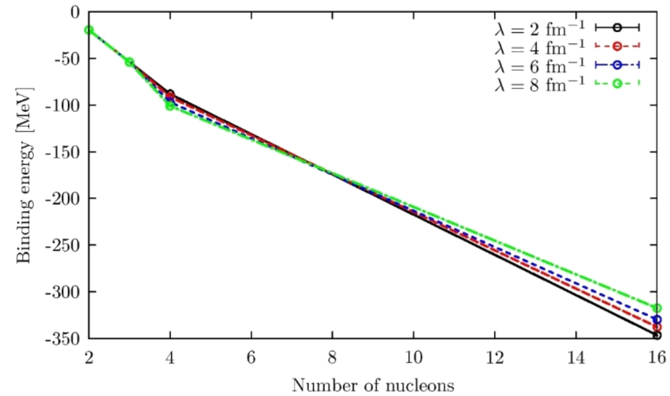


Results

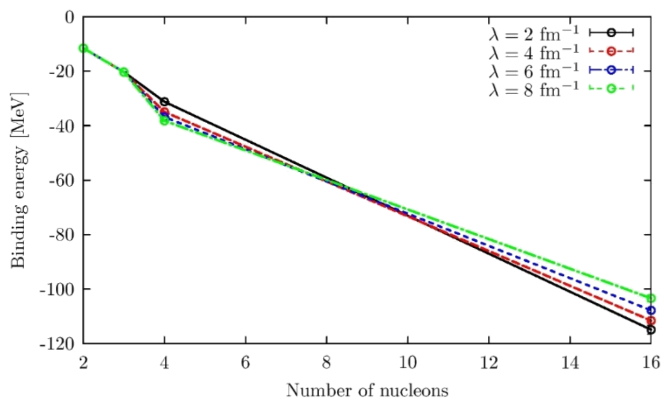
$m_\pi \sim 140\ MeV$



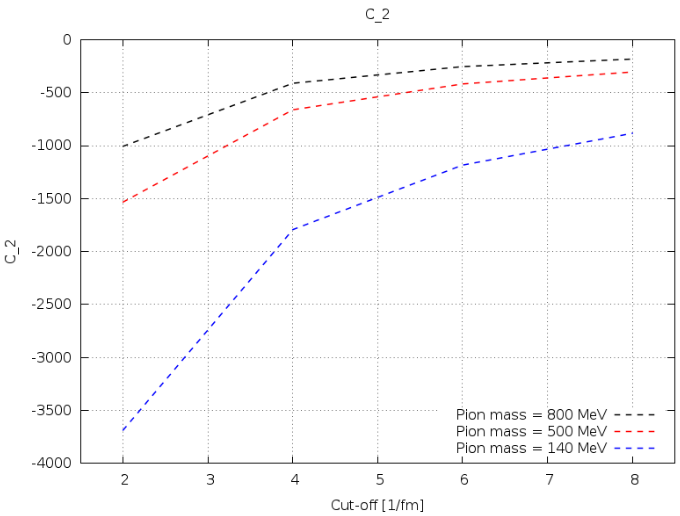
$m_\pi \sim 805\ MeV$

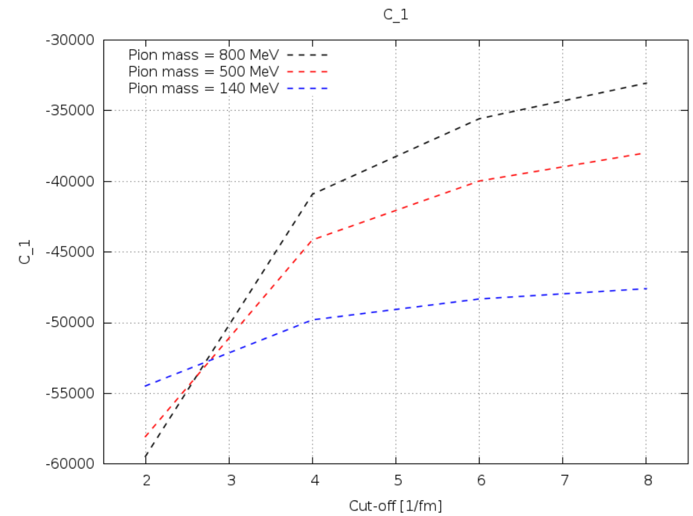


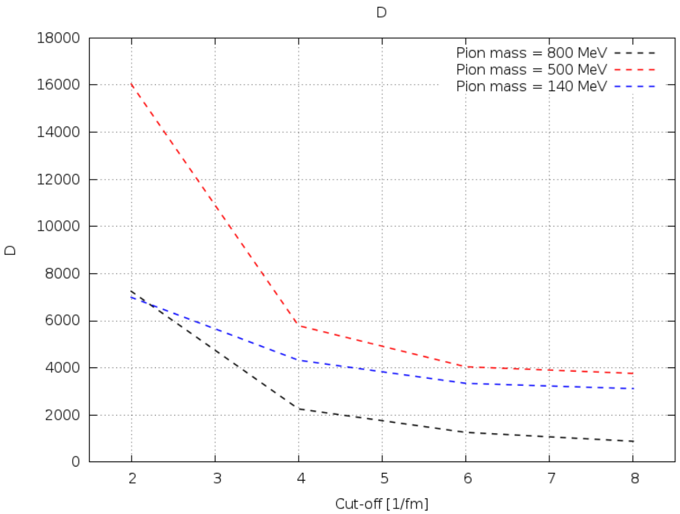
$m_\pi = 510\ MeV$



Data and parameters







The parameters shown are:

$$C_1^{\sim} = C_1 \cdot \frac{m_N}{\Lambda^2}$$

$$C_2^{\sim} = C_2 \cdot \frac{m_N}{\Lambda^2}$$

$$D^{\sim} = D \cdot \frac{m_N}{\Lambda^4}$$