Evolution of the $\bar{K}N - \pi\Sigma$ system with M_π^2 in a box from UχPT

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Outline

Introduction

- The $\Lambda(1405)$ has strong influence in the low energy $\bar{K}N$ scattering data [1,2].
- This resonance lies between the $\pi\Sigma$ and $\bar{K}N$ thresholds, therefore, a couple channel treatment is appropiated.
- All the unitary frameworks based in chiral Lagrangians for the study of the s wave meson baryon interaction lead to the generation of the $\Lambda(1405)$ [3-7...].
- Within the UχPT approach, two poles close to the mass of the $\Lambda(1405)$ appear. Experimental evidence of the two pole structure has been found [8-13].

Introduction

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The $\Lambda(1405)$ as dynamically generated resonance in the infinite volume

- In the chiral unitary approach, the $\Lambda(1405)$ resonance is dynamically generated in s-wave meson-baryon scattering. Channels: $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$ and $K\Xi$.

$$T = (1 - VG)^{-1}\, V \,, \qquad (1)$$

- V is is given by the lowest order of the chiral perturbation theory (the Weinberg-Tomozawa interaction) projected in s-wave:

$$V_{ij}(W) = -C_{ij}\frac{1}{4f_if_j}(2W - M_i - M_j)\sqrt{\frac{M_i + E_i}{2M_i}}\sqrt{\frac{M_j + E_j}{2M_j}} \qquad (2)$$

M_i, M_j, E_i, E_j, masses and energies of the baryons in the channels i, j: $P_iB_i \to P_jB_j$. W is the c.m. energy.

- The f_i, f_j for $P_i B_i \to P_j B_j$ are obtained from the SU(3) chiral extrapolation [14]:

m_π	m_K	m_η	m_N	m_Λ	m_Σ	m_Ξ	f_π	f_K	f_η
138.0	495.7	547.9	938.9	1115.7	1193.2	1318.3	92.4	112.7	122.4

Table : Masses and decay constants for the physical masses (PDG average masses).

- These a_i constants are fitted to obtain similar amplitudes for the $f_i's$ above than in [5] where a common value $f = 1.123\, f_\pi$ for all the reactions is used.

$$a_{\bar{K}N} = -2.2, \quad a_{\pi\Sigma} = -1.6, \quad a_{\eta\Lambda} = -2.5, \quad a_{K\Xi} = -2.9 \tag{3}$$

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The couplings of the three poles found are, (close to a pole, $T_{ij} \simeq g_i g_j/(\sqrt{s} - \sqrt{s_0})$):

$\sqrt{s_0}$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$1379 - 71i$	2.2	3.1	0.8	0.5
$1412 - 20i$	3.1	1.7	1.5	0.3
$1672 - 18i$	0.8	0.3	1.1	3.4

Table : Coupling constants $|g_i|$ to the meson-baryon channels obtained as the residua of the scattering amplitude at the pole position.

Formalism in the finite volume

This "V" is the kernel of the scattering equation:

$$T = T + VGT \longrightarrow T = [I - VG]^{-1} V, \text{with} \tag{4}$$

One can evaluate the loop function G with a cuttoff,

$$G = G^{co}(E) = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{2M_i}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \tag{5}$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i\,|^2}$ is the energy and $\vec{q}$ stands for the momentum of the meson in the channel i. In the finite volume, the momenta is quantized,

$$\vec{q}_i = \frac{2\pi}{L}\vec{n}_i; \qquad T \longrightarrow \tilde{T}; \qquad G(E) \longrightarrow \tilde{G}(E)\ , \tag{6}$$

where [15]

$$\tilde{G}(E) = \frac{2M_i}{L^3} \sum_{\vec{q}_i} I(E, \vec{q}_i) \ , \tag{7}$$

with

$$I(E, \vec{q}_i) = \frac{\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(E)^2 - (\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i))^2} \tag{8}$$

This form produces a degeneracy, $n_x^2 + n_y^2 + n_z^2 = m$, such that $q_i^2 = \frac{4\pi^2}{L^2} m_i$. The sum over the momenta is done till q_{max}.
As in the infinite volume, the formalism should also be made independent of q_{max} and related to α [16].

$$\tilde{G} = G^{DR} + \lim_{q_{max}\to\infty} \left(\frac{1}{L^3} \sum_{q<q_{max}} I(E, \vec{q}) - \int_{q<q_{max}} \frac{d^3q}{(2\pi^3)} I(E, \vec{q}) \right)$$
$$\equiv G^{DR} + \lim_{q_{max}\to\infty} \delta G \ , \tag{9}$$

where $\delta G \equiv \tilde{G} - G^{co}$, and G^{co} can be taken from [17]. Here $I(E, \vec{q})$ is the factor given in Eq. (8). δG converges as $q_{max} \to \infty$. The Bethe-Salpeter equation in finite volume, can be written as,

$$\tilde{T}^{-1} = V^{-1} - \tilde{G} \tag{10}$$

The energy levels in the box in the presence of interaction V correspond to the condition

$$det(I - V\tilde{G}) = 0. \tag{11}$$

For one channel, the amplitude in infinite volume T

$$T = (\tilde{G}(E_i) - G(E_i))^{-1}. \tag{12}$$

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Meson decay constants from SU(3) Uχ PT extrapolation

The physical masses can be expressed as a function the leading order masses (M_0), LEC's (L^r) and pseudoscalar decay constants (f).

$$M_\pi^2 = M_{0\,\pi}^2 \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16 M_{0\,K}^2}{f_0^2}\left(2L_6^r - L_4^r\right) + \frac{8 M_{0\,\pi}^2}{f_0^2}\left(2L_6^r + 2L_8^r - L_4^r - L_5^r\right)\right], \quad (13)$$

$$M_K^2 = M_{0\,K}^2 \left[1 + \frac{2\mu_\eta}{3} + \frac{8 M_{0\,\pi}^2}{f_0^2}\left(2L_6^r - L_4^r\right) + \frac{8 M_{0\,K}^2}{f_0^2}\left(4L_6^r + 2L_8^r - 2L_4^r - L_5^r\right)\right], \quad (14)$$

$$M_\eta^2 = M_{0\,\eta}^2 \left[1 + 2\mu_K - \frac{4}{3}\mu_\eta + \frac{8 M_{0\,\eta}^2}{f_0^2}(2L_8^r - L_5^r) + \frac{8}{f_0^2}(2M_{0\,K}^2 + M_{0\,\pi}^2)(2L_6^r - L_4^r)\right]$$
$$+ M_{0\,\pi}^2\left[-\mu_\pi + \frac{2}{3}\mu_K + \frac{1}{3}\mu_\eta\right] + \frac{128}{9 f_0^2}(M_{0\,K}^2 - M_{0\,\pi}^2)^2(3L_7 + L_8^r), \quad (15)$$

$$\mu_P = \frac{M_{0\,P}^2}{32\pi^2 f_0^2}\log\frac{M_{0\,P}^2}{\mu^2}, \qquad P = \pi, K, \eta\,, \quad (16)$$

where f_0 is the pion decay constant in the chiral limit, $4\pi f_0 \simeq 1.2$ GeV, μ is the regularization scale.

Meson decay constants from SU(3) $U\chi$ PT extrapolation

The decay constants evaluated to one loop in the SU(3) $U\chi PT$,

$$f_\pi = f_0\left[1 - 2\mu_\pi - \mu_K + \frac{4M_{0\,\pi}^2}{f_0^2}(L_4^r + L_5^r) + \frac{8M_{0\,K}^2}{f_0^2}L_4^r\right], \tag{17}$$

$$f_K = f_0\left[1 - \frac{3\mu_\pi}{4} - \frac{3\mu_K}{2} - \frac{3\mu_\eta}{4} + \frac{4M_{0\,\pi}^2}{f_0^2}L_4^r + \frac{4M_{0\,K}^2}{f_0^2}(2L_4^r + L_5^r)\right], \tag{18}$$

$$f_\eta = f_0\left[1 - 3\mu_K + \frac{4L_4^r}{f_0^2}\left(M_{0\,\pi}^2 + 2M_{0\,K}^2\right) + \frac{4M_{0\,\eta}^2}{f_0^2}L_5^r\right]. \tag{19}$$

[14] J. Nebreda and J. R. Pelaez., Phys. Rev. D **81**, 054035 (2010)

Results

Set	$L(fm)$	m_π	m_K	m_η	m_N	m_Λ	m_Σ	m_Ξ	f_π	f_K	f_η
1	2.99	170.29	495.78	563.97	962.2	1135.8	1181.5	1323.6	94.5	113.2	122.1
2	3.04	282.84	523.26	581.72	1058.7	1173.4	1235.5	1332.8	102.5	116.1	122.3
3	3.08	387.81	559.46	605.97	1150.1	1261.0	1292.4	1377.4	109.5	118.5	122.6
4	3.23	515.56	609.75	638.07	1274.5	1333.4	1353.5	1401.8	116.3	120.6	122.4
5	3.27	623.14	670.08	685.01	1420.3	1434.2	1449.8	1472.4	120.1	121.9	122.6

Table. Masses and decay constants for sets 1-5.

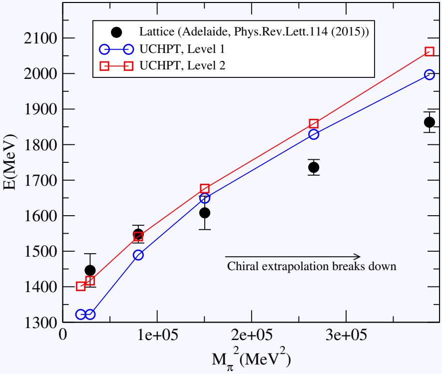

Figure. Comparison between the UχPT prediction and the Lattice data of [1] done for sets 1 to 5, and the physical set.

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Results

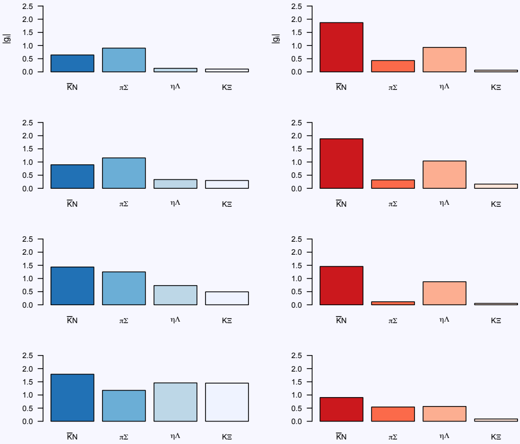

Figure : Couplings $|g_i|$ for the sets of parameters 1 to 4.

Results

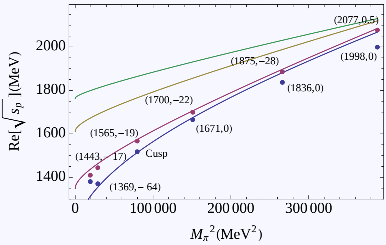

Figure : Behaviour of the the $\mathrm{Re}\sqrt{s_p}$ of the poles found from the T-matrix in the infinite volume with the M_π^2, for the physical, and 1 to 5 sets.

Results

Infinite volume

Pole	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$	$b_{\bar{K}N}$	$b_{\pi\Sigma}$
1379-i 71	2.20	3.1	0.8	0.5	56	-48
1412-i 19	3.1	1.7	1.5	0.3	23	-81
1369-i 64	1.9	2.9	0.6	0.5	89	-17
1443-i 17	2.6	1.35	1.32	0.3	15	-91
Cusp at 1518.34				64	0	
1565-i 19	2.5	1.5	1.4	0.5	17	-47
1671	2.0	1.3	1.1	0.6	39	9
1700-i 22	0.6	0.4	1.2	2.9	10	-20
1836	1.9	1.2	1.7	1.8	48	33
1875-i 28	1.3	1.8	1.6	1.7	9	-6
1998	0.9	0.8	1.9	2.9	92	75
2077-i 0.5	2.1	0.4	0.3	1.1	13	-4

Finite volume

Pole	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$	$b_{\bar{K}N}$	$b_{\pi\Sigma}$
1322	0.5	0.6	0.1	0.07	113	9
1401	2.2	1.0	1.0	0.2	34	-70
1322	0.6	0.9	0.1	0.1	136	30
1417	1.9	0.4	0.9	0.06	41	-65
1489	0.9	1.2	0.3	0.3	93	29
1541	1.9	0.3	1.0	0.2	41	-23
1649	1.4	1.3	0.7	0.5	61	31
1676	1.5	0.1	0.9	0.06	34	4
1829	1.8	1.2	1.5	1.5	55	40
1859	0.9	0.5	0.6	0.09	25	10
1997	1.0	0.9	1.9	2.9	93	76
2062	0.9	0.7	0.2	0.9	28	11

Table Pole positions and couplings of the states in the infinite (left) and finite (right) volume.

Conclusions

- For masses $m_\pi \lesssim 400$ MeV we find very good agreement between the Lattice data of J. M. M. Hall et al. and UχPT.
- The states found in the Lattice for the two first sets correspond to the second energy level of the $\bar{K}N - \pi\Sigma$ system in the box.
- For the third set, the energy level found in Lattice is consistent with the first state in the box from UχPT. For these m_π's, the $\bar{K}N$ component begins to be more dominant than the $\pi\Sigma$ component for the first energy level.
- For the first two sets ($m_\pi \lesssim 300$ MeV), we find similar properties between the state in the finite volume and the second pole of the $\Lambda(1405)$ from UχPT.