



# Progress in the quest for a realistic $3N$ force

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joint project with

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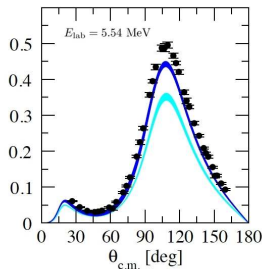
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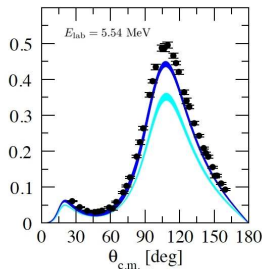
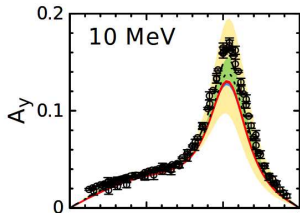
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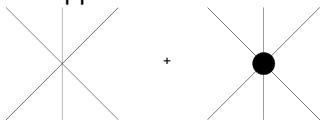
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- ▶ For  $Nd$ , possibly affected by large uncertainty [LENPIC, 1505.07218]

## Subleading 3N LECs

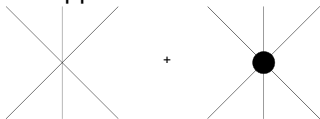
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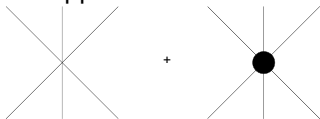
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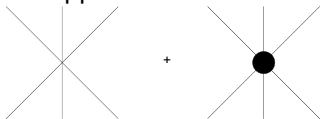
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$$\begin{aligned}
 V = & \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[ Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[ Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
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- ▶ to check the flexibility, we start with AV18 *NN* potential and no 3NF whatsoever
- ▶ we then fit the relevant LECs to reproduce the bound state b.e., the doublet and quartet  $n - d$  scattering lengths and accurate experimental data on  $p - d$  differential cross section and polarization observables at 3 MeV proton energy [Shimizu et al. PRC52 (1995) 1193]
- ▶ for so doing we use the HH method

## Numerical implementation

The N-d scattering wave function is written as

$$\Psi_{LSJJ_z} = \Psi_C + \Psi_A$$

with  $\Psi_C$  expanded in the HH basis

$$|\Psi_C\rangle = \sum_{\mu} c_{\mu} |\Phi_{\mu}\rangle$$

and  $\Psi_A$  describing the asymptotic relative motion

$$\Psi_A \sim \Omega_{LS}^R(k, r) + \sum_{L'S'} R_{LS, L'S'}(k) \Omega_{L'S'}^I(k, r)$$

with the unknown  $c_{\mu}$  and  $R$ -matrix elements (related to the  $S$ -matrix) to be determined so that the Kohn functional is stationary

$$[R_{LS, L'S'}] = R_{LS, L'S'} - \langle \Psi_C + \Psi_A | H - E | \Psi_C + \Psi_A \rangle$$



imposing the Kohn functional to be stationary leads to a *linear* system

$$\sum_{L''S''} R_{LS,L''S''} X_{L'S',L''S''} = Y_{LS,L'S'}$$

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11 set of matrices are calculated once for all, and only linear systems are solved for each choice of  $E_i$ 's

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- ▶ we exclude 1 LEC from the fits (e.g.  $E_8$ ) and absorb its effect in the remaining LECs

## Fit strategy

- ▶ we first rescale the LECs using naïve dimensional analysis

$$E_0 = \frac{c_E}{F_\pi^4 \Lambda} \quad (\text{LO}), \quad E_{i=1,\dots,10} = \frac{e_i^{NN}}{F_\pi^4 \Lambda^3} \quad (\text{NLO})$$



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- ▶ we find that  $E_3$  can be used with  $c_E$  to reproduce  $B(^3\text{H})$  and  $nd$  scattering lengths; moreover

$$T_{20} \rightarrow E_5, \quad A_y, T_{11} \rightarrow E_7, \quad T_{21} \rightarrow E_{10}, \quad T_{22} \rightarrow E_9$$

- ▶ this allows to establish starting points for the minimization procedure

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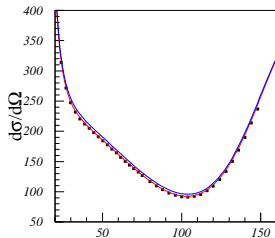
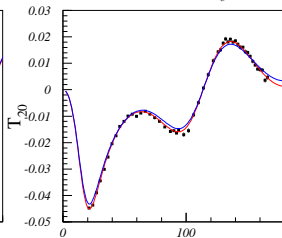
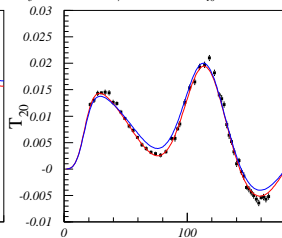
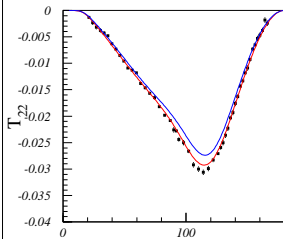
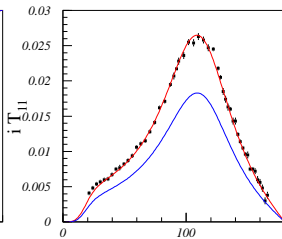
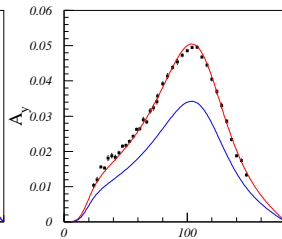
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- ▶ e.g. for  $\Lambda = 300$  MeV we obtain  $\chi^2/\text{d.o.f.} = 1.6$



$\Lambda=300$  MeV $\chi^2/\text{d.o.f.}=1.6$  $a_2 = 0.649$  fm $E_0=1.20$  $E_4=0.45$  $E_6=0.05$  $E_8=-10.9$  $B(^3\text{H}) = 8.482$  MeV $E_3=-1.75$  $E_5=-1.93$  $E_7=2.98$  $E_{10}=-1.76$  $\theta$  (degrees) $\theta$  (degrees) $\theta$  (degrees) $\theta$  (degrees) $\theta$  (degrees) $\theta$  (degrees)

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- ▶ needed theoretical uncertainty to make  $\chi^2 \sim 1$ : 1 %
- ▶ experimental errors given with 1 significant digit: a 10% larger error would imply  $\chi^2 \sim 1.6 \rightarrow 1.3$ , in line with realistic NN potentials

## Insight from the large- $N_c$ limit

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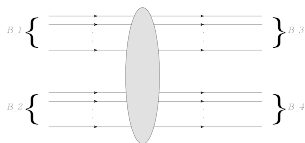
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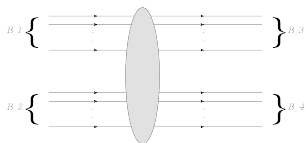
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- ▶ as a result, one finds e.g.

$$\mathbf{1} \sim \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \sim O(N_c)$$

while

$$\sigma_1 \cdot \sigma_2 \sim \tau_1 \cdot \tau_2 \sim O(1/N_c)$$

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- ▶ observable quantities will depend on two combinations of LECs,

$$\mathcal{L} = (c_1 - 2c_3 - 3c_4) N^\dagger N N^\dagger N + (c_2 - c_3) N^\dagger \sigma_i N N^\dagger \sigma_i N$$

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- ▶ in an effective theory one obtains that amplitude from

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- ▶ but from the identity of  $N$ ,  $o_3 = -o_2 - 2o_1$ ,  $o_4 = -3o_1$  which do not conform with the large- $N_c$  scaling
- ▶ one way to implement the Pauli principle is to start with a redundant set of operators, and declare, by tree-level matching,  $c_1 \sim c_4 \sim N_c$ ,  $c_2 \sim c_3 \sim 1/N_c$
- ▶ observable quantities will depend on two combinations of LECs,

$$\mathcal{L} = (c_1 - 2c_3 - 3c_4) N^\dagger N N^\dagger N + (c_2 - c_3) N^\dagger \sigma_i N N^\dagger \sigma_i N$$

reobtaining the well-established fact that  $C_S \gg C_T$

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- ▶ operators with different scaling properties in  $1/N_c$  get mixed

## large- $N_c$ constraints on subleading $3N$ contact interaction

- ▶ applying Phillips and Schat counting to our redundant operators we get 13 leading structures
- ▶ using Fierz identities we find 7 leading operators, out of 10
- ▶ we thus have predictions for some of the  $E_i$

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- ▶ after isospin  $1/2$  projection, working with  $E_8 = 0$ , large- $N_c$  constraints become

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not too far from the fits' outcome

## Summary and outlook

- ▶ We advocate a pragmatic approach, in which the subleading  $3N$  contact interaction is treated as a sort of remainder, to fine-tune existing realistic models
- ▶ we tested the feasibility of this approach by adopting the AV18 NN interaction. The  $\chi^2$  is drastically reduced, until 1.6 per d.o.f.
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It will be interesting,

- ▶ to repeat the analysis using a realistic pionless NN potential;
- ▶ to extend the analysis to other energies, and to include the breakup channel