Assessing Theory Errors from Residual Cutoff Dependence

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Providing reliable theoretical uncertainties,
testing non-perturbative EFTs.

hg: *Nucl. Phys.* **A744** (2004) 192;
hg: NNPSS 2008, Saclay workshop 04.03.2013, Benasque workshop 24.07.2014;
hg: forthcoming

1. Serious Theorists Have Error Bars

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication in *Physical Review A* without a detailed discussion of the uncertainties involved in the measurements. For example, a graphical presentation of data is always accompanied by error bars for the data points. The determination of these error bars is often the most difficult part of the measurement. Without them, it is impossible to tell whether or not bumps and irregularities in the data are real physical effects, or artifacts of the measurement. Even papers reporting the observation of entirely new phenomena need to contain enough information to convince the reader that the effect being reported is real. The standards become much more rigorous for papers claiming high accuracy.

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers can be broadly classified as follows:

Workshop *"Predictive Capabilities of Nuclear Theories"*, Krakow (Poland), 25 Aug 2012

Special Issue *J. Phys. G* (Feb 2015):

"Enhancing the Interaction between Nuclear Experiment and Theory through Information and Statistics"

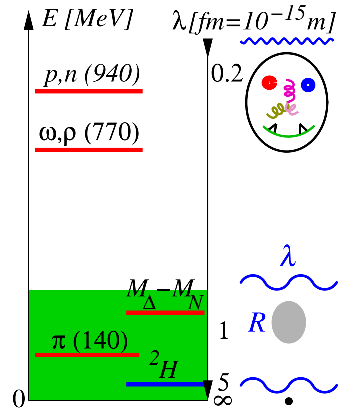
(a) Power-Counting Non-Perturbative EFTs

Correct long-range + symmetries: Chiral SSB, gauge, iso-spin,...
Short-range: ignorance into minimal parameter-set at given order.

Systematic ordering in $Q = \dfrac{\text{typ. momentum } p_{\text{typ}}}{\text{breakdown scale } \Lambda_{\text{EFT}}} \ll 1$

Controlled approximation: model-independent, error-estimate.

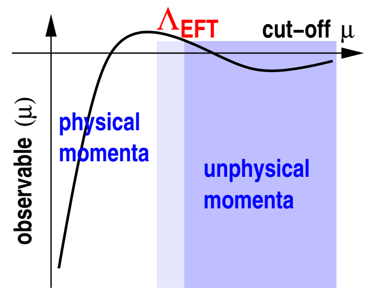
$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**
$\Longrightarrow$ **Pion-less Effective Field Theory EFT($\not\pi$) $\equiv$ low-energy χEFT**



Shallow real/virtual QCD bound states $\Longrightarrow$ Few-N non-perturbative!

$$T_{\text{LO}} = V_{\text{LO}} + V_{\text{LO}}\, G\, T_{\text{LO}}$$
$$T_{\text{NLO}} = (\mathbb{1} + T^{\dagger}_{\text{LO}})\, V_{\text{NLO}}\, (\mathbb{1} + T_{\text{LO}}) \qquad \text{strict perturbation about LO}$$

$\Longrightarrow$ Analytic results rare; regularisation by **cut-off** $\mu \overset{!!}{\neq} \Lambda_{\text{EFT}}$.



$\Longrightarrow$ saturated at $\Lambda_{\text{EFT}} \lesssim \mu$.

Some Ways to Estimate Errors: *a priori*; order-by-order convergence;
decreasing cut-off dependence; include selected higher-order effects,...

(b) NN χEFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.

Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

wave	order	**Yang/Long** PRC**86**(2012) 024001 etc.	**Pavon Valderrama** PRC**74** (2006) 054001 etc.	**Birse** PRC**74** (2006) 014003
$^1S_0$	LO	-1		
	NLO	0		
	N^{2}LO	1	2	
$^3S_1$	LO	-1		
	NLO	1	2	$\frac{1}{2}$
$^3SD_1$	LO	1	$-\frac{1}{2}$	-1
	NLO		2	$\frac{1}{2}$
$^3D_1$	LO		$-\frac{1}{2}$	-1
	NLO		2	$\frac{1}{2}$
$^3P_0$	LO	-1		$-\frac{1}{2}$
OPE	LO	-1		
TPE	LO	1	2	
# of param. at Q^{-1}		2	3	4
# of param. at Q^0		4	6	6
# of param. at Q^1		**8**	**6**	**9**

Weinberg: LO: 2; NLO: $+0$; N^{2}LO: $+7=9$ – different channels; consistency questioned Beane/...2002; Nogga/...2005

(a) Using Cut-Offs to Your Advantage

Observable $\mathcal{O}(k)$ at momentum k, order Q^n in EFT, cut-off μ:

$$\mathcal{O}_n(k;\mu) = \underbrace{\sum_i^n \left(\frac{k, p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^i \mathcal{O}_i}_{\text{renormalised, } \mu\text{-indep.}} + \underbrace{\mathcal{C}(\mu) \left(\frac{k, p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^{n+1}}_{\text{residual } \mu\text{-dependence}}$$

$\Longrightarrow$ **Difference between *any two* cut-offs:** $\dfrac{\mathcal{O}_n(k;\mu_1) - \mathcal{O}_n(k;\mu_2)}{\mathcal{O}_n(k;\mu_1)} = \left(\dfrac{k, p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^{n+1} \times \dfrac{\mathcal{C}(\mu_1) - \mathcal{C}(\mu_2)}{\mathcal{C}(\mu_1)}$

Isolate breakdown scale Λ_{EFT}, order n by double-ln plot of "derivative of observable w. r. t. cut-off".

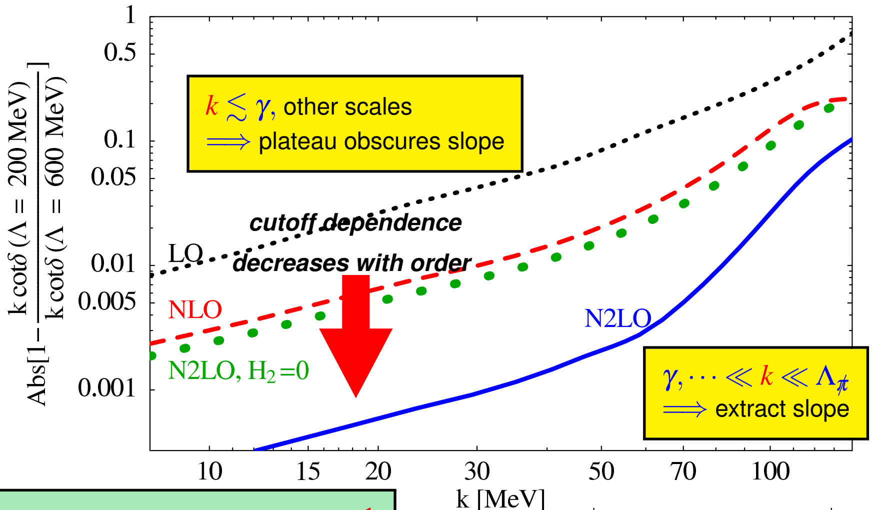
Complication: Several intrinsic low-energy scales in few-N EFT:

scattering momentum k, m_π, inverse NN scatt. lengths $\gamma({}^3\mathrm{S}_1) \approx 45\ \mathrm{MeV}$, $\gamma({}^1\mathrm{S}_0) \approx 8\ \mathrm{MeV}$,...

(b) Example: nd Doublet-S Wave in EFT($\not\pi$)

Bedaque/hg/Hammer/Rupak 2002, hg 2004

Does momentum-dependent 3NI H_2 enter at N^{2}LO hg/…2002-4 – or higher Platter/Phillips 2006?



$$\left|1 - \frac{k\cot\delta(\mu = 200\ \text{MeV})}{k\cot\delta(\mu=\infty)}\right| \sim \underbrace{\left(\frac{k, p_{\text{typ.}}}{\Lambda_{\not\pi}}\right)^{n+1}}_{Q^{n+1}}$$

	LO	NLO	N^{2}LO	N^{2}LO without H_2
$n+1$ fitted	~ 1.9	2.9	4.8	3.1
$n+1$ predicted	2	3	4	4!!!

$\Longrightarrow$ Fit to $k \in [70;100\dots130]$ MeV $\gg \gamma, \dots$: H_2 **is N^{2}LO**; re-confirmed by Ji/Phillips/Platter 2012

Slope Confirms Power Counting; Estimates $\Lambda_{\not\pi} \approx 140$ MeV; Determines Mom.-Dep. Uncertainties.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\mu_1)-\mathcal{O}_n(k;\mu_2)}{\mathcal{O}_n(k;\mu_1)} = \left(\frac{k, p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\mu_1)-\mathcal{C}(\mu_2)}{\mathcal{C}(\mu_1)}$$

What observable to choose?: Avoid Accidental Zeroes $\mathcal{O}(\mu_1)-\mathcal{O}(\mu_2)=0$ & Infinities $\mathcal{O}(\mu)=0$.

Best if unconstrained:

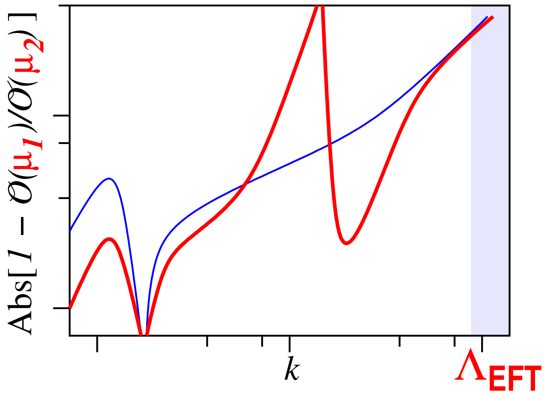
e.g. $k^{2l+1}\cot\delta_l(k)$ for lth scattering wave.

Not $\delta_l(k)$: $\delta_l(k\to 0)\propto k^{2l+1}$: constrained.

Best if same sign for all $k \lesssim \Lambda_{\text{EFT}} \implies$ Peruse μ_1, μ_2.

If LECs need fitting, do for small $k\to 0,\ k\sim p_{\text{typ}}$.

Slope may still emerge for $k \nearrow \Lambda_{\text{EFT}}$; larger LEC fit error.



Some Limitations:

– Cannot see LECs which do *not absorb cutoff-dependence*.

– Can be numerically indecisive.

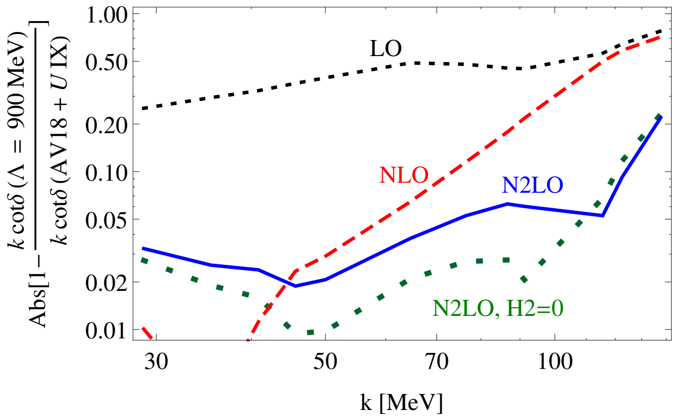
(c) Comments: It’s Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\mu_1)-\mathcal{O}_n(k;\mu_2)}{\mathcal{O}_n(k;\mu_1)} = \left(\frac{k,p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\mu_1)-\mathcal{C}(\mu_2)}{\mathcal{C}(\mu_1)}$$

These Are *Not* “Lepage-Plots” $\frac{\mathcal{O}_n(k;\mu)-\mathcal{O}(\textbf{data})}{\mathcal{O}(\textbf{data})}$.

Lepage: nucl-th/9706029; Steele/Furnstahl: nucl-th/9802069; . . .

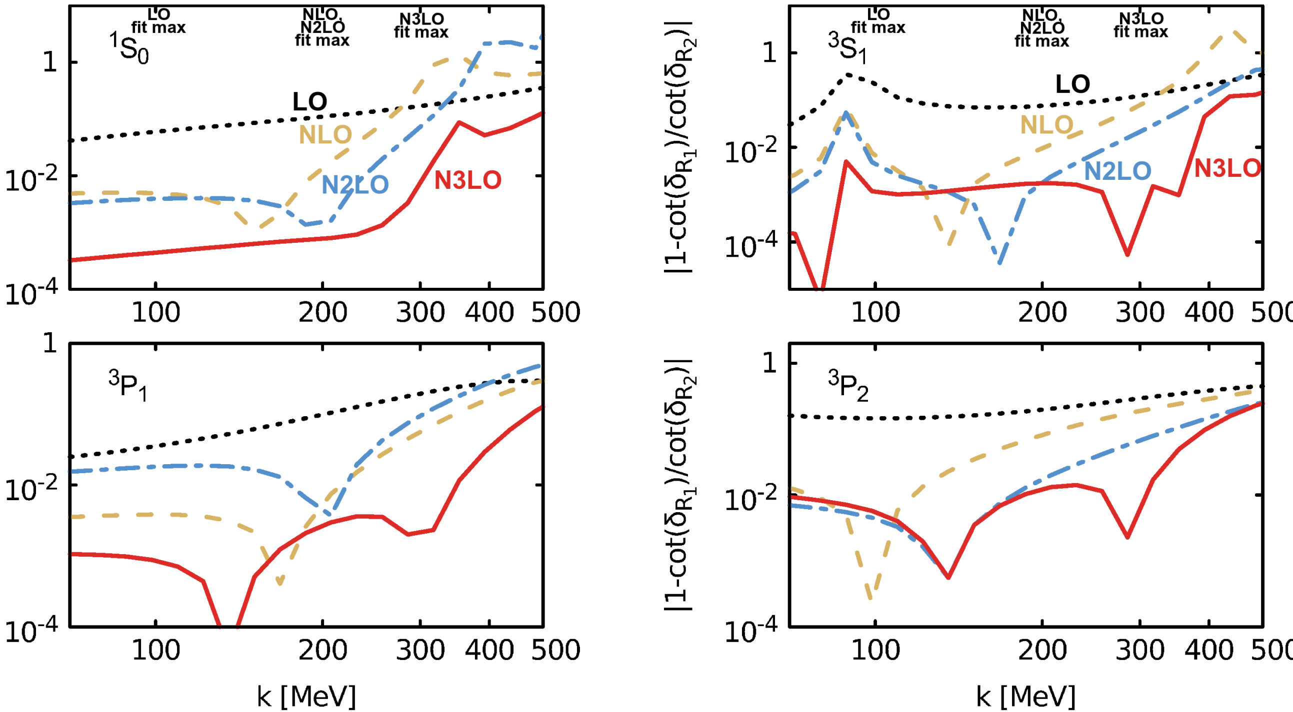
“Lepage” needs data/pseudo-data. $\Longrightarrow$ No consistency test; not double-blind; compromise predictive power.



EFT may converge by itself, but not to data. – Example χEFT without dynamical $\Delta(1232)$ at $k \sim 300$ MeV.

(d) What About NN?: Unsolicited Comments

Plot stolen from **Epelbaum/Krebs/Meißner EPJA51 (2015) 5, 53.**



Inconclusive: Breakdown scale $400 - 500$ MeV $\Longleftrightarrow \Delta(1232)$? NLO, N^{2}LO parallel? Slopes?

Coupled channels; attractive tensor? Fit- & slope-regions not clearly separated.

4. Concluding Questions

– EFTs make **quantitative, falsifiable predictions** how its uncertainties evolve with momentum.

– **Non-perturbative EFTs**: Power-counting not established analytically. Test by residual cut-off dependence:

"Momentum-dependent Renormalisation Group flow of observable with cut-off":

$$\frac{\mathcal{O}_n(k;\mu_1)-\mathcal{O}_n(k;\mu_2)}{\mathcal{O}_n(k;\mu_1)} \propto \left(\frac{k,p_{\text{typ.}}}{\Lambda_{\text{EFT}}}\right)^{n+1} \quad \textbf{for any two cut-offs } \mu_1,\mu_2 \gtrsim \Lambda_{\text{EFT}}.$$

- For ***order*** $\mathcal{O}(Q^n)$ ***to which result is complete***: slope at $k \gg$low scales;
- For ***breakdown scale*** Λ_{EFT}: k at which different orders show same-size variations;
- For lower bound on ***expansion parameter*** Q: vary μ_1,μ_2 over wide range.

– Using widely separate cutoffs μ_1,μ_2 increases leverage; decreases numerical noise.

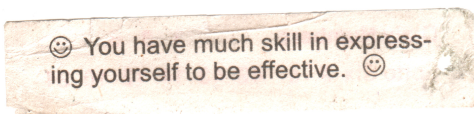
– Straightforward extension to include non-analytic running $\sim \ln[k,p_{\text{typ.}}],\dots$.

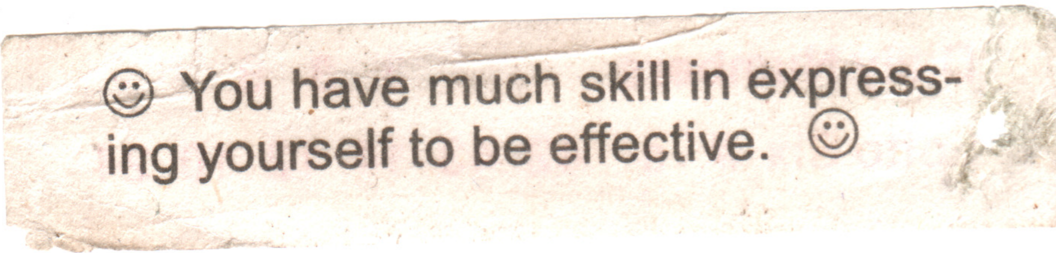
– **Not all observables equally suited (avoid constraints!).**

– **Self-consistency test of EFT: No resort to data.** — "Not A Lepage Plot".

– **Results may be inconclusive.** – One of hopefully many arrows in the quiver.

What will this look like for few-nucleon observables in χEFT?





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