$K \to \pi\pi$ Decays and the $\Delta I = 1/2$ rule

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A warning: not a review talk

- This topic has long history, a lot of work has been done (effective theories, lattice and more)
- I only present here the work done by RBC and UKQCD collaborations
- Apologies if I don't mention your work or your favorite computation

Outline

- Introduction: $K \to \pi\pi$ and CP violation
- Overview of the computation
- $K \to (\pi\pi)_{I=2}$ channel
- $K \to (\pi\pi)_{I=0}$ channel
- Emerging understanding of the $\Delta I = 1/2$ rule

$K \to \pi\pi$ and CP violation

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)
- Noble prize in 1980 (Cronin and Fitch)
- Direct CP violation discovered in kaon decays [NA31, KTeV, NA48, '90-99]
- Very nice measurements of both direct and indirect CP violation (numbers from [PDG 2011])

$$\begin{cases} \textit{Indirect} \quad |\varepsilon| = (2.228 \pm 0.011) \times 10^{-3} \\ \textit{Direct} \quad Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3} \end{cases}$$

- Theoretically:

  Relate indirect CP violation parameter (ε) to neutral kaon mixing (B_K)

  Still lacking a quantitative description of direct CP violation (ε')
- Sensitivity to new physics expected

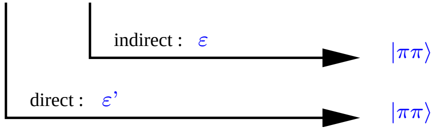
Background: Kaon decays and CP violation

Flavour eigenstates $\begin{pmatrix} K^0 = \bar{s}\gamma_5 d \\ \overline{K}^0 = \bar{d}\gamma_5 s \end{pmatrix} \neq$ CP eigenstates $|K^0_{\pm}\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\overline{K}^0\rangle\}$

They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle & \sim & |K^0_-\rangle + \overline{\varepsilon}|K^0_+\rangle \\ |K_S\rangle & \sim & |K^0_+\rangle + \overline{\varepsilon}|K^0_-\rangle \end{cases}$

Direct and indirect CP violation in $K \to \pi\pi$

$$|K_L\rangle \propto |K_-\rangle + \varepsilon|K_+\rangle$$



$$\varepsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = |\varepsilon| e^{i\phi_\varepsilon} \sim \bar{\varepsilon}$$

$K \to \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \to (\pi\pi)_{\mathrm{I}=0,2}$$

Corresponding amplitudes defined as

$$A[K \to (\pi\pi)_{\mathrm{I}}] = A_{\mathrm{I}} \exp(i\delta_{\mathrm{I}}) \qquad \text{/w } \mathrm{I} = 0, 2 \qquad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_o} \sim 1/22 \qquad \text{(experimental number)}$$

Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$\varepsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

$$\varepsilon = e^{i\phi_\varepsilon} \left[\frac{\mathrm{Im}\langle \bar{K}^0 | H_{\mathrm{eff}}^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

$\Rightarrow$ Related to $K^0 - \bar{K}^0$ mixing

The $\Delta I = 1/2$ rule

- In $K \to \pi\pi$ decays, the final state can have isospin 0 or 2
- Experimentally we observe that

$$\mathbb{P}[K \to (\pi\pi)_{\mathrm{I}=0}] \sim 450 \times \mathbb{P}[K \to (\pi\pi)_{\mathrm{I}=2}]$$

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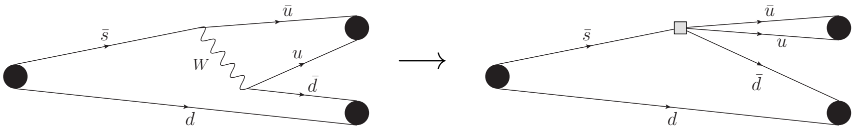
  $\Rightarrow$ Can we extract an explanation for this phenomena ?

Computation of $K \to \pi\pi$ amplitudes

Overview of the computation

- Operator Product expansion



- Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian [Buchalla, Buras, Lautenbacher '96]

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{10} \big(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \big) Q_i(\mu) \Big\}$$

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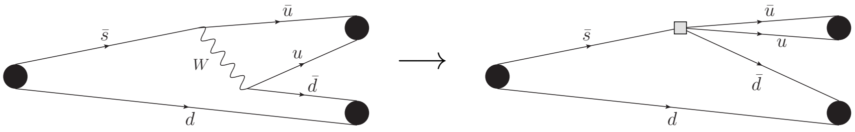

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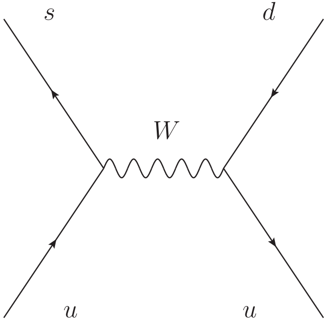
- Amplitude given by $A \propto \langle \pi\pi | H^{\Delta s=1} | K \rangle$
- Short distance effects factorized in the Wilson coefficients y_i, z_i , computed at NLO in [BBL '96]
- Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i(\mu) | K \rangle \longrightarrow \text{task for the Lattice}$$

See reviews by [Christ @ Kaon'09, Lellouch @ Les Houches'09, Sachrajda @ Lattice '10], ...

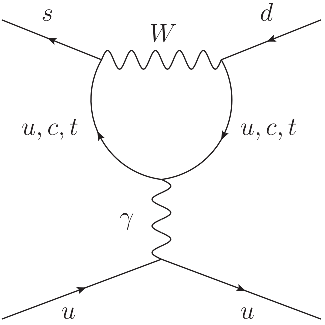
Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $\qquad$ $Q_2 =$ color mixed

4-quark operators

Electroweak penguins

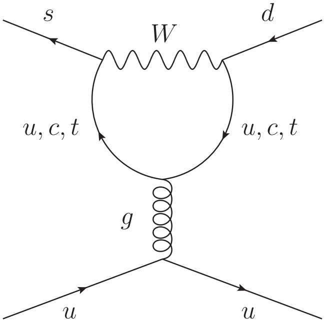


$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

4-quark operators

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$\begin{aligned} \bar{3} \otimes 3 &= 8 + 1 \\ \bar{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1 \end{aligned}$$

Relevant operators transform under $(27, 1), (8, 8)$ and $(8, 1)$ of $SU(3)_L \otimes SU(3)_R$

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Relevant operators transform under $(27,1), (8,8)$ and $(8,1)$ of $SU(3)_L \otimes SU(3)_R$

Decomposition of the 4-quark operators gives

$$\begin{aligned} Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\ Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\ Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\ Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\ Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2} \end{aligned}$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Only 7 are independent: one $(27,1)$ four $(8,1)$, and two $(8,8)$, $\Rightarrow$ we call them Q'

$$
\begin{array}{llcl}
(27,1) & Q'_1 & = & Q_1'^{(27,1),\Delta I=3/2} + Q_1'^{(27,1),\Delta I=1/2} \\
 & & & \\
(8,1) & Q'_2 & = & Q_2'^{(8,1),\Delta I=1/2} \\
 & Q'_3 & = & Q_3'^{(8,1),\Delta I=1/2} \\
 & Q'_5 & = & Q_5'^{(8,1),\Delta I=1/2} \\
 & Q'_6 & = & Q_6'^{(8,1),\Delta I=1/2} \\
 & & & \\
(8,8) & Q'_7 & = & Q_7'^{(8,8),\Delta I=3/2} + Q_7'^{(8,8),\Delta I=1/2} \\
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\end{array}
$$

Only 3 operators contribute to the $\Delta I = 3/2$ chanel

A challenge !

Many obstacles:

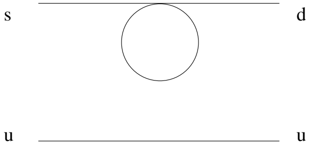
- Final state with two pions
- Many operators that mix under renormalisation
- Require the evaluation of disconnected graphs

Need to preserve chiral-flavour symetry at finite lattice spacing

Plus the usual difficulties: light dynamical quarks, large volume, ...

Isospin channels

- Only 3 of these operators contribute to the $\Delta I = 3/2$ channel
  - A tree-level operator
  - 2 electroweak penguins
- No disconnect graphs contribute to the $\Delta I = 3/2$ channel



$\Rightarrow$ A_2 is much simpler than A_0

Still highly non-trivial, but perfect challenge for lattice QCD with chiral fermions

Lattice computation of A_2

A_2 from RBC-UKQCD, Overview of the computation

- First problem: the two-pion state

  $\Rightarrow$ Lellouch-Lüscher method [Lellouch, Lüscher '00] to obtain the physical matrix element from the finite-volume Euclidean amplitude and the derivative of the phase shift

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- Unfortunately this implies that the desired physical state is an excited one (difficult to extract)

  $\Rightarrow$ For A_2, combine

  - Wigner-Eckart theorem (Exact up to isospin symmetry breaking)

$$\langle \pi^+(p_1)\pi^0(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=1/2}|K^+\rangle = 3/2\langle \pi^+(p_1)\pi^+(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=3/2}|K^+\rangle$$

  and then compute the unphysical process $K^+ \to \pi^+\pi^+$

  - Use Anti-periodic B.C. to eliminate the unwanted (wrong-kinematic) state

  [Kim '04, Sachrajda & Villadoro '05]

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- A popular way is the Rome-Southampton technique [Martinelli, Pittori, Sachrajda, Testa, Vladikas '94]

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- On the other hand, dealing with a 2-pion state requires a large physical volume

  $\Rightarrow$ Our first computation of A_2 was performed on coarse lattice ($a \sim 0.14\ \mathrm{fm},\ L \sim 4.5\ \mathrm{fm}$)

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- Solution

  Renormalise at low energy $\mu_0 \sim 1.1$ GeV on and run non-perturbatively using finer lattices to $\mu = 3$ GeV and match to $\overline{\mathrm{MS}}$ [Arthur, Boyle '10, Arthur, Boyle, N.G. , Kelly, Lytle '11]

$$\underbrace{\lim_{a_1 \to 0} \left[Z(\mu_1, a_1) Z^{-1}(\mu_0, a_1) \right]}_{\text{fine lattice}} \times \underbrace{Z(\mu_0, a_0)}_{\text{coarse lattice}} = Z(\mu_1, a_0)$$

A_2 from RBC-UKQCD (2012)

- Very challenging both theoretically and numerically
- Computation performed with state-of-the-art algorithm and large-scale computer resources
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- $2+1$ chiral fermions (Domain-Wall on IDSDR $a \sim 0.14\ \mathrm{fm}$)
- lightest unitary pion mass $\sim 170\ \mathrm{MeV}$ (partially quenched $140\ \mathrm{MeV}$)
- Physical kinematics
- Non-perturbative-renormalization through RI-SMOM schemes

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- Find $\mathrm{Re}A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}} 10^{-8}$ GeV, experimental value is $1.479(4)\, 10^{-8}$ GeV
- And $\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}$ GeV

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- And $\mathrm{Im}A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}}$ GeV
- Important computation in the field: first realistic computation of a hadronic decay
- Main limitation: single (and rather coarse) lattice spacing

2014-2015 update

- Main limitation on the previous computation : only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37\ \mathrm{GeV} \Rightarrow a \sim 0.14\ \mathrm{fm}$, $L \sim 4.6\ \mathrm{fm}$

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- New computation:

  two lattice spacing, $n_f = 2 + 1$, large volume at the physical point

  New discretisation of the Domain-Wall fermion forumlation: Möbius Brower, Neff, Orginos '12

  - $48^3 \times 96$, with $a^{-1} \sim 1.729$ GeV $\Rightarrow a \sim 0.11$ fm, $L \sim 5.5$ fm
  - $64^3 \times 128$ with $a^{-1} \sim 2.358$ GeV $\Rightarrow a \sim 0.084$ fm, $L \sim 5.4$ fm
  - $am_{res} \sim 10^{-4}$

$K \to (\pi\pi)_{I=2}$ 2015 update

2012 Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*

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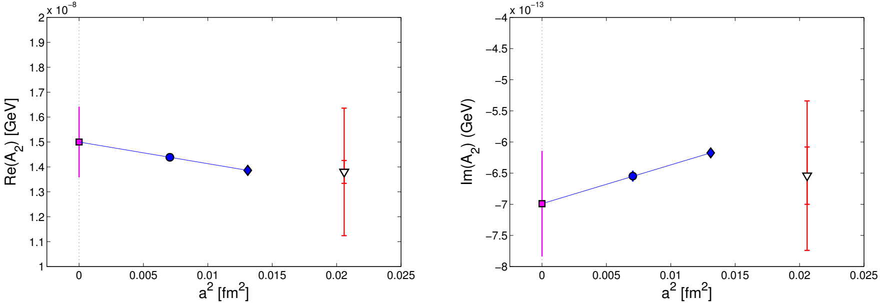
2015 Blum, Boyle, Christ, Frison, N.G., Janowski, Jung, Kelly, Lehner, Lytle, Mawhinney, Sachrajda, Soni, Hin, Zhang, *PRD'15*

$\text{Re}\, A_2 = 1.50(4)_{\rm stat}(14)_{\rm syst}\, 10^{-8}\, \text{GeV}$ $\qquad$ $\text{Im}\, A_2 = -6.99(20)_{\rm stat}(84)_{\rm syst} 10^{-13}\, \text{GeV}$



see also talk by T.Janowski @ lat'13

$K \to (\pi\pi)_0$ and the $\Delta I = 1/2$ rule

A_0 from RBC-UKQCD (2011)

"Pilot" computation of the full process

T. Blum, Boyle, Christ, N.G., Goode, Izubuchi, Lehner, Liu, Mawhinney, Sachrajda, Soni, Sturm, Yin, Zhou, PRD'11.

Unphysical:

- "Heavy" pions (lightest $\sim m_\pi \sim 300\ \mathrm{MeV}$), small volume
- Non-physical kinematics: pions at rest

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- Two-pion state
- All the contractions of the 7 fourk-operators are computed
- Renormalisation done non-perturbatively

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obtain

$$\begin{aligned} \mathrm{Re}\, A_0 &= 3.80(82) \times 10^{-7}\mathrm{GeV} \\ \mathrm{Im}\, A_0 &= -2.5(2.2) \times 10^{-11}\mathrm{GeV} \end{aligned}$$

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

	$1/a$ [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8}GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	kinematics
16^3 **IW**	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold
24^3 **IW**	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold
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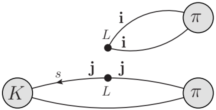
Exp	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

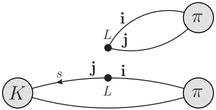
Pattern which could explain the $\Delta I = 1/2$ enhancement

Boyle, Christ, N.G., Goode, Izubuchi, Janowski, Lehner, Liu, Lytle, Sachrajda, Soni, Zhang, PRL'13

Two kinds of contraction for each $\Delta I = 3/2$ operator



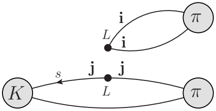
Contraction ①



Contraction ②

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



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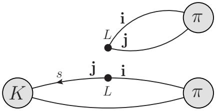

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$$ReA_2 \sim ① + ②$$

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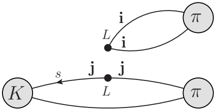

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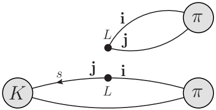

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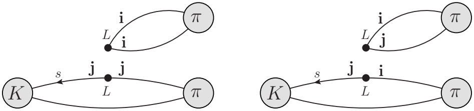

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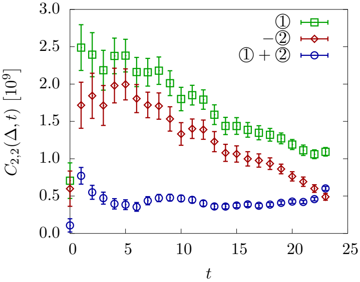
- Our computation: ② $\sim -0.7$①

$\Rightarrow$ large cancellation in ReA_2



Toward an quantitative understanding of the $\Delta I = 1/2$ rule

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Dominant contribution to Q_2^{lat} is $\propto$ $(\,2②-①\,) \Rightarrow$ Enhancement in $\mathrm{Re}A_0$

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2② - ①}{① + ②}$$

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With this unphysical computation (kinematics, masses) we find

$$\begin{aligned}\frac{{\rm Re}A_0}{{\rm Re}A_2} &= 9.1(2.1) \text{ for } m_K = 878 \text{ MeV } m_\pi = 422 \text{ MeV}\\ &= 12.0(1.7) \text{ for } m_K = 662 \text{ MeV } m_\pi = 329 \text{ MeV}\end{aligned}$$

Emerging understanding of the $\Delta I = 1/2$ rule

- Relative sign between ① and ② implies both a cancellation in $\mathrm{Re}A_2$ and an enhancement in $\mathrm{Re}A_0$
- Analytic work in that direction, e.g. Pich, de Rafael '96, Bardeen, Buras, Gerard '87
- See also discussion in Lellouch @ Les Houches '09

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- See also discussion in Lellouch @ Les Houches '09
- Similar observation done by another very recent lattice computation Ishizuka, Ishikawa, Ukawa, Yoshié '15

  $K \to \pi\pi$ amplitudes with unphysical kinematics (and Wilson fermions)

A_0, 2015 update

- First complete computation of $K \to \pi\pi$ (both isospin channel) with physical kinematics

  Bai, Blum, Boyle, Christ, Frison, N.G., Izubuchi, Jung, Kelly, Lehner, Mawhinney, Sachrajda, Soni, Zhang

- Pion mass $m_\pi = 143.1(2.0)\ \mathrm{MeV}$, single lattice spacing $a \sim 0.14\ \mathrm{fm}$
- Physical kinematics achieved with G-Parity boundary conditions

  Kim, Christ, '03 and '09

- Requires algorithmic development, dedicated generation of gauge configurations, ...
- See talk by C.Kelly and proceeding from Lattice'14

A_0, 2015 update

After renormalisation at $\mu \sim 1.5\ \mathrm{GeV}$, we combine with the Wilson coefficients and find

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$
Exp	$3.3201(18) \times 10^{-7}$	-

Standard model prediction for ε'/ε

ε'/ε can be computed from

$$Re(\varepsilon'/\varepsilon) = Re\left\{ \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}\varepsilon} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right] \right\}$$

Combining our new value of $\mathrm{Im}A_0$ and δ_0 with

- our continuum value for $\mathrm{Im}A_2$
- the experimental value for $\mathrm{ReA_0}$, $\mathrm{ReA_2}$ and their ratio ω

we find

$$Re(\varepsilon'/\varepsilon) = 1.38(5.15)(4.43) \times 10^{-4}$$

whereas the experimental value is

$$Re(\varepsilon'/\varepsilon) = 16.6(2.3) \times 10^{-4}$$

Conclusions, outlook

Finally, a complete computation for $K \to \pi\pi$, with physical quark masses and physical kinematics

- A_2 now extrapolated to the continuum limit
- Very recent computation of A_0 with physical setup at single lattice spacing
- Only approximate agreement for ε'/ε
- Observe a mechanism which contributes to a large enhancement in A_0/A_2
- Clearly shows the need for a non-perturbative method
- Precision still far from the experimental one, but provide a value for $\mathrm{Im}A_2$ and $\mathrm{Im}A_0$

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This was possible because of important hardware and algorithmic improvement,

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For A_0, it is only the beginning:

- statistical error should be reduced
- Renormalisation at higher energy (now $\mu \sim 1.5\ \mathrm{GeV}$)
- Finer lattice and continuum limit ...

Backup

Standard model prediction for ε'/ε

	$1/a$ [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8}GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	kinematics
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cont.	-	139	497-507	1.50(4)(14)	-	-	physical
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RBC-UKQCD setup - History- Present

$2+1$ Domain-Wall fermions

Chiral-Flavour symmetry (almost) exact at finite lattice spacing

Finite fith dimension L_s $\rightarrow$ small additive quark mass renormalisation m_{res}

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  Unitary pion masses $m_\pi = 331, 419, (557)$ MeV

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- 2014: Möbius, Unitary pion mass 139 MeV
  - $a^{-1} = 1.730(4)$ GeV $\leftrightarrow$ $a \sim 0.1145$ fm, on $48^3 \times 96 \times 24$ ie $L \sim 4.62$ fm
  - $a^{-1} = 2.359(7)$ GeV $\leftrightarrow$ $a \sim 0.0839$ fm, on $64^3 \times 128 \times 12$ ie $L \sim 5.475$ fm