Chiral three-nuclear forces up to N⁴LO

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Low Energy Nuclear Physics International Collaboration

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Outline

- Nuclear forces in chiral EFT
- 3NF’s upto N^4LO
- PWD of the three-nucleon forces
- Summary & Outlook
ChPT pros and cons

ChPT as an effective field theory of QCD

- is the most general field theory with pions, nucleons (deltas) as dofs in line with the symmetries of QCD
- is systematically improvable
- gives a unified description of $\pi\pi$, $\pi N$, $NN$, (axial) vector currents etc.
- naturally explains the hierarchy $V_{2N} \gg V_{3N} \gg V_{4N}$
- predicts the long range behavior of nuclear forces
- allows doing precision physics with/from light nuclei

- number of free parameters (LEC) increases with increasing order in ChPT
- does not provide an explanation on the size of a particular LEC
- is only applicable in the low energy region
- convergence radius of ChPT is a priori unknown
ChPT nuclear forces

<table>
<thead>
<tr>
<th>Worked out up to the order</th>
<th>(V_{\text{NN}})</th>
<th>(V_{\text{3N}})</th>
<th>(V_{\text{4N}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>N^4LO</td>
<td>N^3LO</td>
<td>N^3LO</td>
<td></td>
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<tr>
<td>Evgeny's talk</td>
<td>N^4LO in progress</td>
<td></td>
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<td>In combination with semi-local regularization in Schrödinger eq.</td>
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</table>

Partial N^5LO calculation \(\rightarrow\) Ruprecht’s talk

Novelties in NN sector (beside the construction of N^4LO NN)

- **Local regularization in coordinate space:**
  \[ V_{\text{long-range}}(\vec{r}') \rightarrow V_{\text{long-range}}(\vec{r}') \left[ 1 - \exp \left( -\frac{r'^2}{R^2} \right) \right]^n \]
  - By construction long-range physics is unaffected by this regulator
  - No additional SFR is needed

- Theoretical uncertainty estimation due to chiral expansion for every fixed cutoff \(R\)

*Epelbaum, HK, Meißner EPJA 51 (2015) 5*
Phase shifts and mixing angles

Epelbaum, HK, Meißner, arXiv: 1412.4623

$R = 0.9$ fm

\( \text{NLO} \quad \text{N}^2\text{LO} \quad \text{N}^3\text{LO} \quad \text{N}^4\text{LO} \)

- ☑️ Good convergence of chiral expansion
- ☑️ Error bands are consistent with each other ➡️ strong support of chiral uncertainty estimation
- ☑️ Excellent agreement with NPWA data
Role of the 3NFs


The discrepancy at 10 MeV is much lower than at other energies

Significant discrepancy between experiment and theory

Cross section at low energy is governed by S-wave spin-doublet and spin-quartet Nd scattering lengths:

\( ^4a \gg ^2a \) (one order of magnitude)

\( ^4a \) is much less sensitive to 3NF (Pauli principle)

Clear evidence of missing 3NFs at higher energy
### 3NF up to N⁴LO

<table>
<thead>
<tr>
<th></th>
<th>Long - range</th>
<th>Short - range</th>
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<tbody>
<tr>
<td>NLO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N²LO</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C_i)</td>
<td>(D)</td>
</tr>
<tr>
<td>van Kolck '94, Epelbaum et al. '02</td>
<td>Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)</td>
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</tr>
<tr>
<td>N³LO</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>Ishikawa, Robilotta, PRC76 (07); Bernard, Epelbaum, HK, Meißner, PRC77 (08); PRC84 (11)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>N⁴LO</td>
<td>(\ldots)</td>
<td>(E_i)</td>
</tr>
<tr>
<td>HK, Gasparyan, Epelbaum PRC85 (12); PRC87 (13)</td>
<td>Girlanda, Kievsky, Viviani, PRC84 (11)</td>
<td>Work in progress</td>
</tr>
</tbody>
</table>

- \(2\pi-1\pi\)  
- ring  
- \(2\pi\)
Most general structure of a local 3NF

Up to N^4LO, the computed contributions are local \rightarrow it is natural to switch to r-space.
A meaningful comparison requires a complete set of independent operators

\[
\begin{align*}
\tilde{G}_1 &= 1 \\
\tilde{G}_2 &= \tau_1 \cdot \tau_3 \\
\tilde{G}_3 &= \bar{\sigma}_1 \cdot \bar{\sigma}_3 \\
\tilde{G}_4 &= \tau_1 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_3 \\
\tilde{G}_5 &= \tau_2 \cdot \tau_3 \bar{\sigma}_1 \cdot \bar{\sigma}_2 \\
\tilde{G}_6 &= \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot (\bar{\sigma}_2 \times \bar{\sigma}_3) \\
\tilde{G}_7 &= \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
\tilde{G}_8 &= \hat{r}_{23} \cdot \bar{\sigma}_1 \hat{r}_{23} \cdot \bar{\sigma}_3 \\
\tilde{G}_9 &= \hat{r}_{23} \cdot \bar{\sigma}_3 \hat{r}_{12} \cdot \bar{\sigma}_1 \\
\tilde{G}_{10} &= \hat{r}_{23} \cdot \bar{\sigma}_1 \hat{r}_{12} \cdot \bar{\sigma}_3 \\
\tilde{G}_{11} &= \tau_2 \cdot \tau_3 \hat{r}_{23} \cdot \bar{\sigma}_1 \hat{r}_{23} \cdot \bar{\sigma}_2 \\
\tilde{G}_{12} &= \tau_2 \cdot \tau_3 \hat{r}_{23} \cdot \bar{\sigma}_1 \hat{r}_{12} \cdot \bar{\sigma}_2 \\
\tilde{G}_{13} &= \tau_2 \cdot \tau_3 \hat{r}_{12} \cdot \bar{\sigma}_1 \hat{r}_{23} \cdot \bar{\sigma}_2 \\
\tilde{G}_{14} &= \tau_2 \cdot \tau_3 \hat{r}_{12} \cdot \bar{\sigma}_1 \hat{r}_{12} \cdot \bar{\sigma}_2 \\
\tilde{G}_{15} &= \tau_1 \cdot \tau_3 \hat{r}_{13} \cdot \bar{\sigma}_1 \hat{r}_{13} \cdot \bar{\sigma}_3 \\
\tilde{G}_{16} &= \tau_2 \cdot \tau_3 \hat{r}_{12} \cdot \bar{\sigma}_2 \hat{r}_{12} \cdot \bar{\sigma}_3 \\
\tilde{G}_{17} &= \tau_1 \cdot \tau_3 \hat{r}_{23} \cdot \bar{\sigma}_1 \hat{r}_{12} \cdot \bar{\sigma}_3 \\
\tilde{G}_{18} &= \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot \bar{\sigma}_3 \bar{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
\tilde{G}_{19} &= \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_3 \cdot \hat{r}_{23} \hat{r}_{23} \cdot (\bar{\sigma}_1 \times \bar{\sigma}_2) \\
\tilde{G}_{20} &= \tau_1 \cdot (\tau_2 \times \tau_3) \bar{\sigma}_1 \cdot \hat{r}_{23} \bar{\sigma}_3 \cdot \hat{r}_{12} \bar{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})
\end{align*}
\]

Building blocks:
\[\tau_1, \tau_2, \tau_3, \bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3, \hat{r}_{12}, \hat{r}_{23}\]

Constraints:
- Locality
- Isospin symmetry
- Parity and time-reversal invariance

\[\rightarrow V_{3N} = \sum_{i=1}^{20} \tilde{G}_i F_i(r_{12}, r_{23}, r_{31}) + 5 \text{ perm.}\]

Epelbaum, Gasparyan, HK, PRC87 (2013) 054007
Schat, Phillips, PRC88 (2013) 034002
Epelbaum, Gasparyan, HK, Schat, EPJA51 (2015) 3
All 22 profile functions start to contribute at $N^4$LO

Large $N^4$LO contributions due to sizable $c_i$'s (hidden $\Delta$ dofs)

No statement about convergence possible

$\rightarrow$ explicit $\Delta$ treatment needed to clarify convergence issue

Quantitative statements are only possible once observables are calculated
Generators

of 89 independent operators

Too many terms for doing PWD by hand

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model
**PWD for local forces**

\[
\langle m'_S | \vec{\sigma} \cdot \vec{p'} | m_S \rangle = \sum_{\mu=-1}^{1} p Y^*_1(\hat{p}) \sqrt{\frac{4\pi}{3}} \langle m'_S | \vec{\sigma} \cdot \vec{e}_\mu | m_S \rangle
\]

\[
\langle m'_{s_1} m'_{s_2} m'_{s_3} | V | m_{s_1} m_{s_2} m_{s_3} \rangle = \sum_{\mu'_{s}} \langle m'_{s_1} m'_{s_2} m'_{s_3} | \text{Spin matrices & } \vec{e}_\mu\text{'s} | m_{s_1} m_{s_2} m_{s_3} \rangle (Y'_{1\mu}) (Y'_{1\mu'})
\times V((\vec{p}' - \vec{p})^2, (\vec{q}' - \vec{q})^2, (\vec{p}' - \vec{p}) \cdot (\vec{q}' - \vec{q}))
\]

- Can be reduced to 3 dim. integral
- Speed up factors > 1000

- Unregularized 3NF matrix elements can be used to generate locally regularized 3NFs

\[
\langle p' q' \alpha' | V | p q \alpha \rangle \rightarrow \sum_n \langle p' q' \alpha' | V | n \rangle \langle n | R | p q \alpha \rangle \text{ with } \langle p' q' \alpha' | R | p q \alpha \rangle \text{ matrix element of local regulator}
\]

Hebeler, HK, Epelbaum, Golak, Skibinski PRC91 (2015) 4
Summary

- Chiral 3NF’s are studied up to N³LO / partly up to N⁴LO
- Optimized version of PWD for local 3NF’s
- Stored matrix elements can be used within local regularization

Outlook

- N⁴LO Δ-less/N³LO-Δ calc. of shorter range part of 3NF
  - Generation of matrix-elements for 3NF’s up to N⁴LO Δ-less/N³LO-Δ
  - Due to optimized PWD should not cost much