Chiral three-nuclear forces up to N^4LO

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RUB

LENPIC

Low Energy Nuclear Physics International Collaboration

TECHNISCHE UNIVERSITÄT DARMSTADT — Binder, Calci, Hebeler, Langhammer, Roth

IOWA STATE UNIVERSITY — Maris, Potter, Vary

RUB — Epelbaum, Krebs

IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY — Bernard

JAGIELLONIAN UNIVERSITY IN KRAKOW — Golak, Skibinski, Topolniki, Witala

THE OHIO STATE UNIVERSITY — Furnstahl

universitätbonn — Meißner

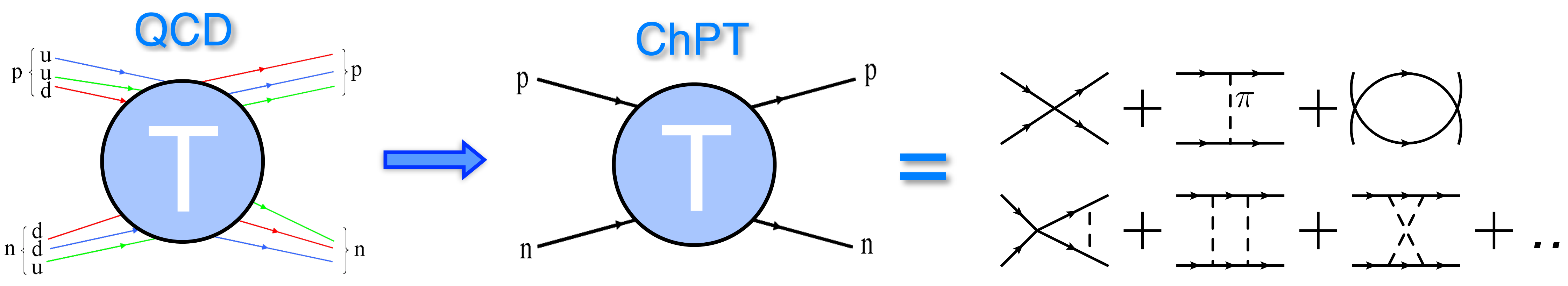
JÜLICH FORSCHUNGSZENTRUM — Nogga

Kyutech Kyushu Institute of Technology — Kamada

Outline

- Nuclear forces in chiral EFT
- 3NF’s upto N^4LO
- PWD of the three-nucleon forces
- Summary & Outlook

ChPT pros and cons



ChPT as an effective field theory of QCD

- ✓ is the most general field theory with pions, nucleons (deltas) as dofs in line with the symmetries of QCD
- ✓ is systematically improvable
- ✓ gives a unified description of ππ, πN, NN, (axial) vector currents etc.
- ✓ naturally explains the hierarchy $V_{2N} >> V_{3N} >> V_{4N}$
- ✓ predicts the long range behavior of nuclear forces
- ✓ allows doing precision physics with/from light nuclei

- ✖ number of free parameters (LEC) increases with increasing order in ChPT
- ✖ does not provide an explanation on the size of a particular LEC
- ✖ is only applicable in the low energy region
- ✖ convergence radius of ChPT is a priori unknown

ChPT nuclear forces

	V_{NN}	V_{3N}	V_{4N}
Worked out up to the order	N^4LO Evgeny's talk	N^3LO N^4LO in progress	N^3LO
Regularization used	Dim. Reg In combination with semi-local regularization in Schrödinger eq.	Dim. Reg.	——

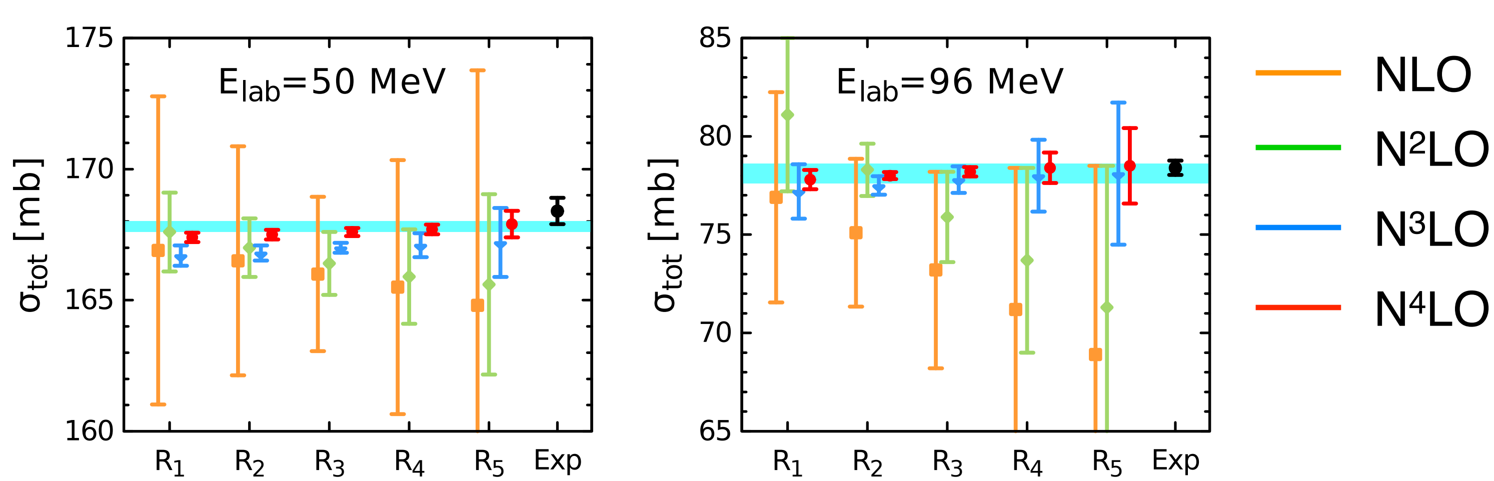
Partial N^5LO calculation → Ruprecht's talk

Novelties in NN sector (beside the construction of N^4LO NN)

- Local regularization in coordinate space: $V_{\rm long-range}(\vec{r}) \to V_{\rm long-range}(\vec{r}) \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$
  - ✓ By construction long - range physics is unaffected by this regulator
  - ✓ No additional SFR is needed
- Theoretical uncertainty estimation due to chiral expansion for every fixed cutoff R

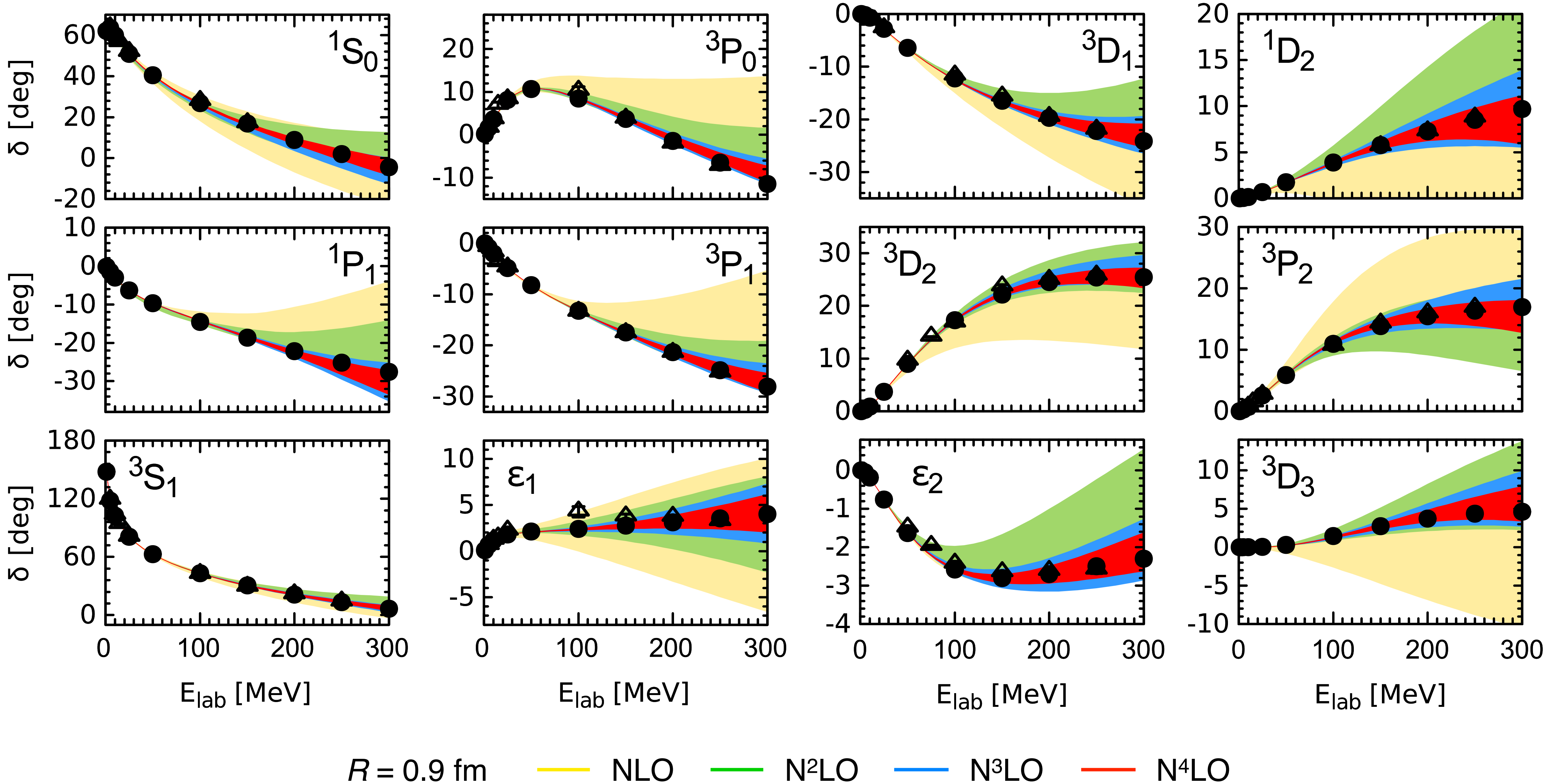
np total cross section at several cutoffs $R_i = (0.7 + i \times 0.1)$ fm

Epelbaum, HK, Meißner EPJA 51 (2015) 5



Phase shifts and mixing angles

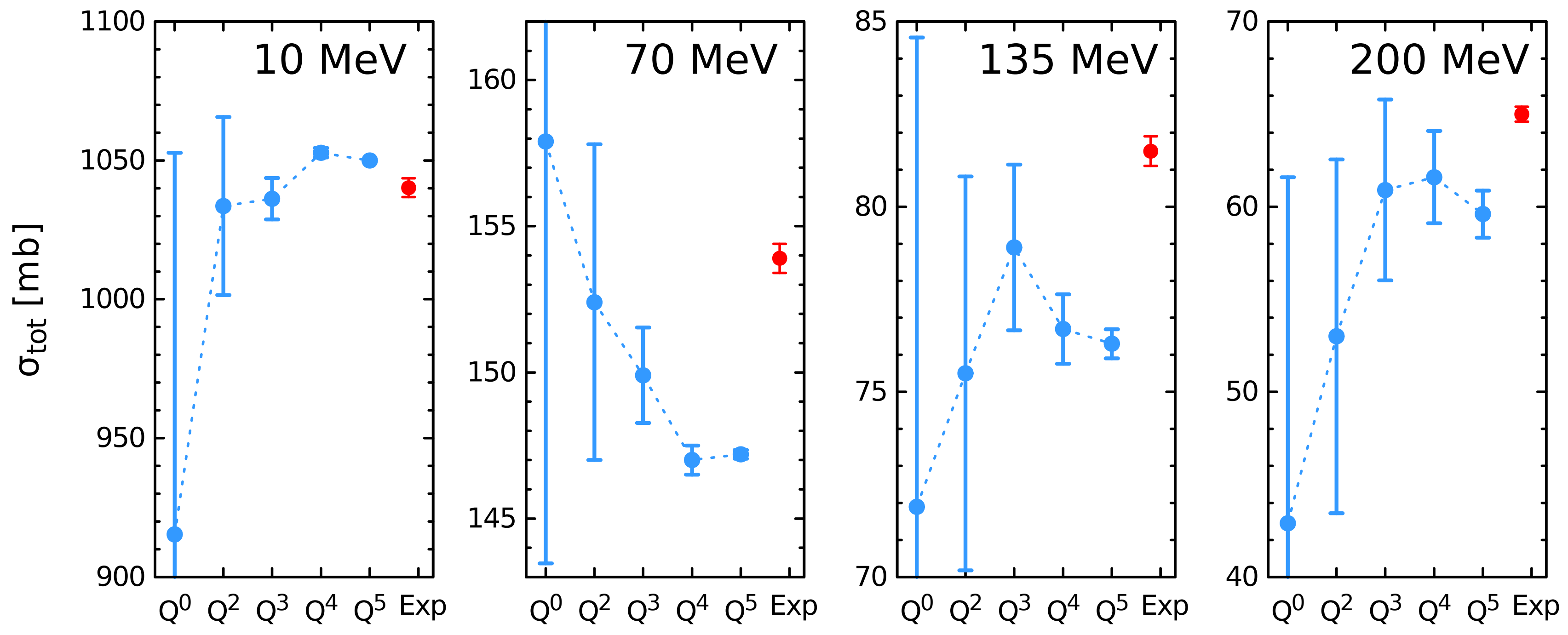
Epelbaum, HK, Meißner, arXiv: 1412.4623



- Good convergence of chiral expansion
- Error bands are consistent with each other → strong support of chiral uncertainty estimation
- Excellent agreement with NPWA data

Role of the 3NFs

LENPIC collaboration: Binder et al. arXiv:1505.07218



Total cross section for Nd scattering

chiral predictions without 3NF at R = 0.9 fm

- The discrepancy at 10 MeV is much lower than at other energies
- Significant discrepancy between experiment and theory

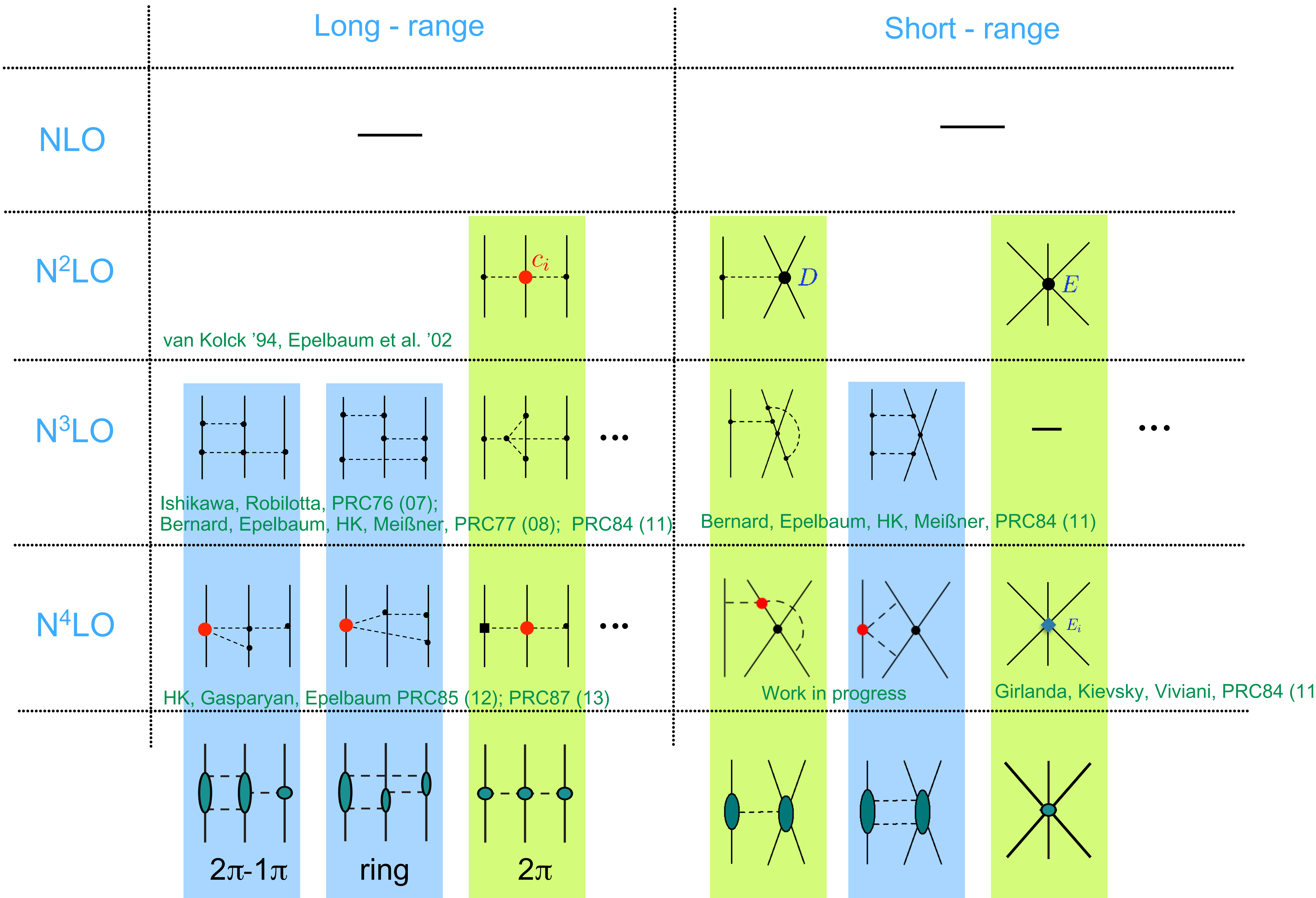
Cross section at low energy is governed by S-wave spin-doublet and spin-quartet Nd scattering lengths:

$^4a \gg {}^2a$ (one order of magnitude)

^4a is much less sensitive to 3NF (Pauli principle)

Clear evidence of missing 3NFs at higher energy

3NF up to N^4LO

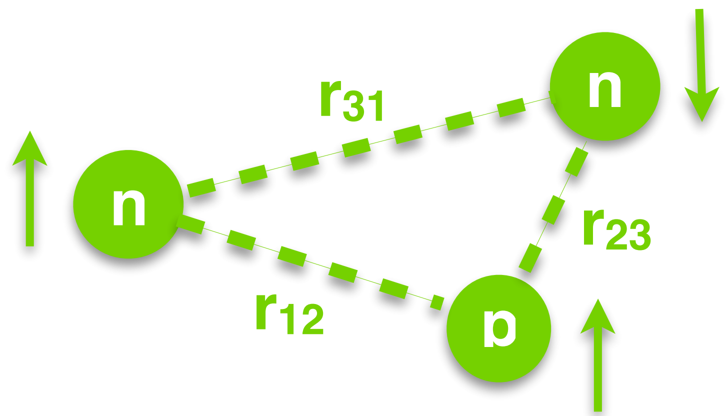


Most general structure of a local 3NF

Up to N^{4}LO, the computed contributions are local $\longrightarrow$ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1 \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3) \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2 \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2 \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{13} \cdot \vec{\sigma}_1 \, \hat{r}_{13} \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \, \hat{r}_{12} \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3 \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_3 \cdot \hat{r}_{23} \, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \hat{r}_{23} \, \vec{\sigma}_3 \cdot \hat{r}_{12} \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})
\end{aligned}
$$



Building blocks:

$$\boldsymbol{\tau}_1, \ \boldsymbol{\tau}_2, \ \boldsymbol{\tau}_3, \ \vec{\sigma}_1, \ \vec{\sigma}_2, \ \vec{\sigma}_3, \ \vec{r}_{12}, \ \vec{r}_{23}$$

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance

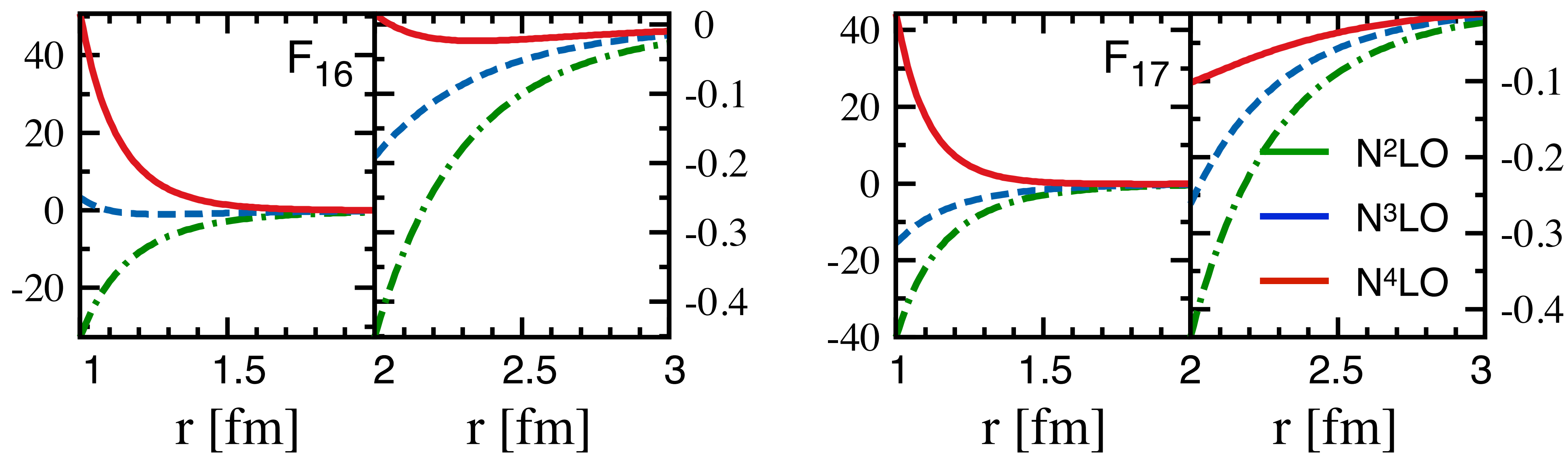
$$\longrightarrow \quad V_{3N} = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i F_i(r_{12}, r_{23}, r_{31}) + 5 \, \text{perm.}$$

Epelbaum, Gasparyan, HK, PRC87 (2013) 054007
Schat, Phillips, PRC88 (2013) 034002
Epelbaum, Gasparyan, HK, Schat, EPJA51 (2015) 3

Long-range 3NF up to N^4LO

Representative dominant contributions to profile functions

Epelbaum, Gasparyan, HK, Schat, EPJA51 (2015) 3



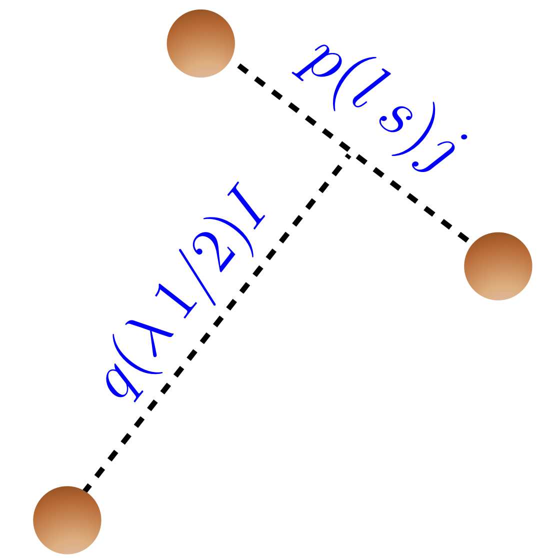
- All 22 profile functions start to contribute at N^4LO
- Large N^4LO contributions due to sizable c_i´s (hidden Δ dofs)
- No statement about convergence possible

  → explicit Δ treatment needed to clarify convergence issue

Quantitative statements are only possible once observables are calculated

Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



- Faddeev equation is solved in the partial wave basis

$$|p,q,\alpha\rangle \equiv |pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J\rangle\,|(t\frac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim\ 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\ldots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\,Y_{l',m_l'}(\hat{p}')\,\ldots\Big)\underbrace{\langle m_{s_1}'m_{s_2}'m_{s_3}'|V|m_{s_1}m_{s_2}m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Numerically expensive due to many channels and 5-dim. integration

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model

PWD for local forces

$$\langle m_s' | \vec{\sigma} \cdot \vec{p} | m_s \rangle = \sum_{\mu=-1}^{1} p \, Y_{1\mu}^*(\hat{p}) \sqrt{\frac{4\pi}{3}} \langle m_s' | \vec{\sigma} \cdot \vec{e}_\mu \, | m_s \rangle$$

← momentum-independent part

$$\langle m_{s_1}' m_{s_2}' m_{s_3}' | V | m_{s_1} m_{s_2} m_{s_3} \rangle = \sum_{\mu' s} \langle m_{s_1}' m_{s_2}' m_{s_3}' | \text{Spin matrices} \,\&\, \vec{e}_\mu\, '\text{s} | m_{s_1} m_{s_2} m_{s_3} \rangle (Y_{1\mu}' s\,)$$
$$\times \; V((\vec{p}\,' - \vec{p}\,)^2, (\vec{q}\,' - \vec{q}\,)^2, (\vec{p}\,' - \vec{p}\,) \cdot (\vec{q}\,' - \vec{q}\,))$$

can be reduced to 3 dim. integral

$$\langle p' q' \alpha' | V | p \, q \, \alpha \rangle = \sum_{m_l \ldots} (\, \text{CG coeffs.}) \int \overbrace{d\hat{p}' \, d\hat{q}' \, d\hat{p} \, d\hat{q}}\, Y_{l_1' m_1'}^*(\hat{p}'\,) Y_{l_2' m_2'}^*(\hat{q}'\,) Y_{l_1 m_1}^*(\hat{p}\,) Y_{l_2 m_2}^*(\hat{q}\,)$$
$$\times \; V((\vec{p}\,' - \vec{p}\,)^2, (\vec{q}\,' - \vec{q}\,)^2, (\vec{p}\,' - \vec{p}\,) \cdot (\vec{q}\,' - \vec{q}\,))$$ → Speed up factors > 1000

- Unregularized 3NF matrix elements can be used to generate locally regularized 3NFs

$\langle p' q' \alpha' | V | p \, q \, \alpha \rangle \to \sum_n \langle p' q' \alpha' | V | n \rangle \langle n | R | p \, q \, \alpha \rangle$ with $\langle p' q' \alpha' | R | p \, q \, \alpha \rangle$ matrix element of local regulator

Hebeler, HK, Epelbaum, Golak, Skibinski PRC91 (2015) 4

Summary

- Chiral 3NF’s are studied up to N^3LO / partly up to N^4LO
- Optimized version of PWD for local 3NF‘s
- Stored matrix elements can be used within local regularization

Outlook

- N^4LO Δ-less/N^3LO-Δ calc. of shorter range part of 3NF

  → Generation of matrix-elements for 3NF‘s up to N^4LO Δ-less/N^3LO-Δ

  Due to optimized PWD should not cost much