Power Counting of Contact-Range Currents in EFT

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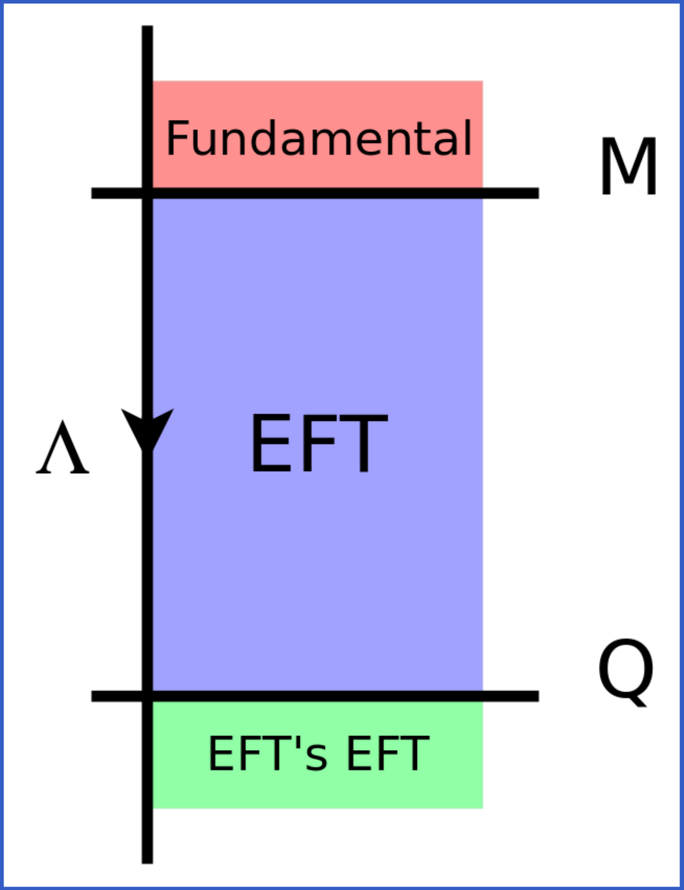
D.R. Phillips and MPV, Phys. Rev. 114 (2015) 8, 082502, arXiv:1407.0437.

RG Evolution



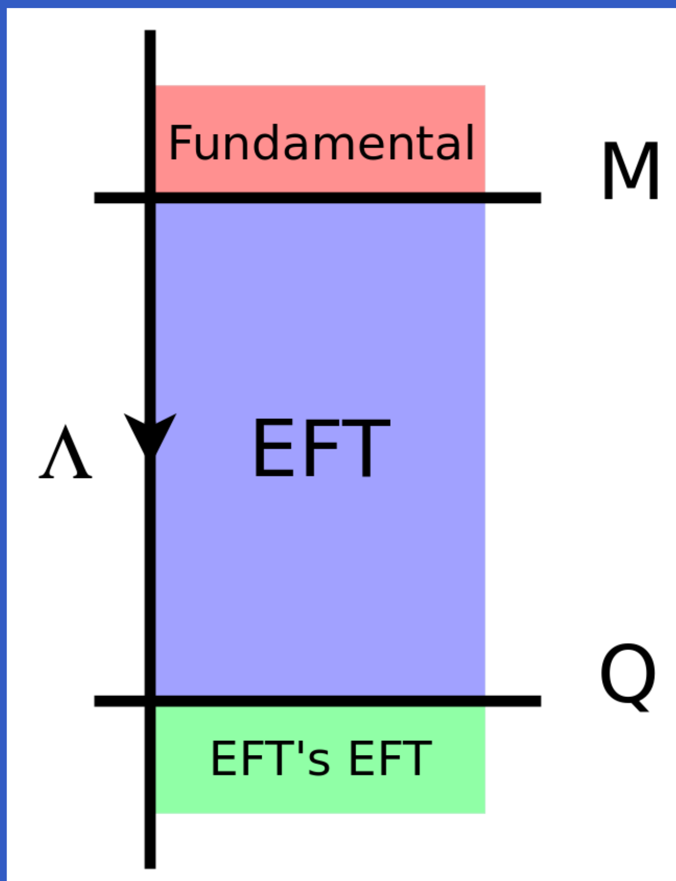
Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

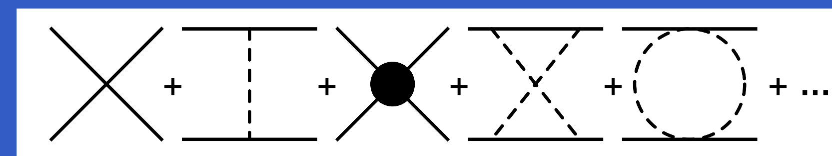
For equivalent descriptions:

$$\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0$$

RG Evolution and Power Counting (I)

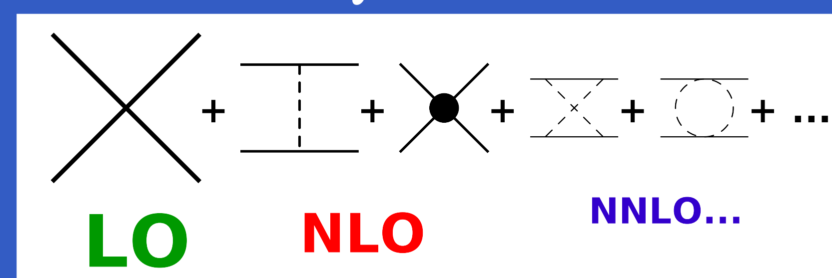


At $\Lambda \sim M$ there is no order



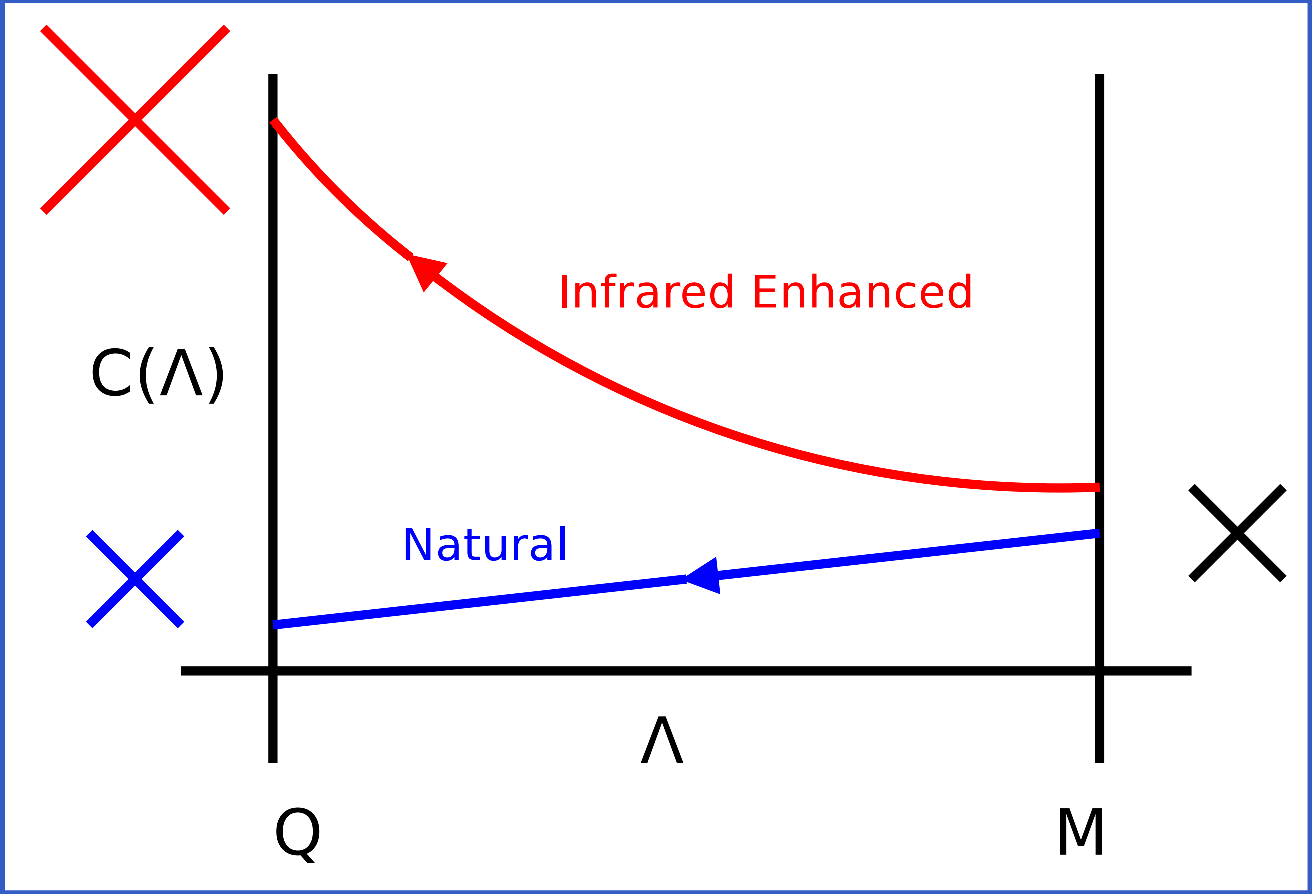
$$\frac{d}{d\,\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0$$

while at $\Lambda \sim Q$ there is order



RG Evolution and Power Counting (II)

The intuitive picture about RG Evolution and Power Counting...



RG Evolution and Power Counting (III)

What is the exact relationship? EFT Operator: $\mathcal{O} = C(\Lambda) \times \mathcal{F}(Q)$

- $\Lambda = M$: coupling scales as powers of M

$$C(M) \sim \frac{1}{M^d}$$

- $M \geq \Lambda \geq Q$: coupling evolves

$$\frac{d}{d\,\Lambda} \langle \Psi | \mathcal{O} | \Psi \rangle = 0 \quad \Rightarrow \quad \frac{d}{d\,\Lambda} \left[\Lambda^a C(\Lambda) \right] = 0$$

- $\Lambda = Q$: coupling gets enhanced

$$\boxed{C(Q) \sim \frac{1}{M^d} \times \left(\frac{M}{Q} \right)^a}$$

Deriving the Power Counting

$$\frac{d}{d\Lambda}\langle \Psi_{\rm EFT}|\mathcal{O}_{\rm EFT}|\Psi'_{\rm EFT}\rangle = 0 \quad \Rightarrow \quad \frac{d}{d\Lambda}\left[\Lambda^a C(\Lambda)\right] = 0$$

How to compute the enhancement of a coupling?

- Consider the LO wave functions and operators
  - $|\Psi_{\rm EFT}\rangle = |\Psi_{\rm LO}\rangle + \Delta|\Psi\rangle$, with $\Psi_{\rm LO}(r) \sim r^b$ for $Qr \leq 1$
  - $O_{\rm EFT} = O_{\rm LO} + \Delta O$, $O_{\rm LO} = C(\Lambda)\, P(\vec{p}, \vec{p}', q, \ldots)$, P lowest order polynomial compatible with the process (e.g. $P = 1$).
- $\langle \Psi_{\rm EFT}|\mathcal{O}_{\rm EFT}|\Psi'_{\rm EFT}\rangle = C(\Lambda) \int_\Lambda \frac{dq}{q^{b+1}} \int_\Lambda \frac{dq}{q^{b'+1}} + \ldots = \frac{C(\Lambda)}{\Lambda^{b+b'}} + \ldots$
- $a = -(b+b')$. The same for higher order couplings.

The Two- and Three-Nucleon System

- The two-nucleon system:
  - Pionless singlet and triplet / pionfull singlet ($\Psi_{\mathrm{LO}}(r) \sim \frac{1}{r}$):

$$\frac{d}{d\,\Lambda}\left[\Lambda^2 C_{2n}(\Lambda)\right] = 0 \quad \Rightarrow \quad Q^2 \text{ enhancement}$$

  - Pionfull triplet ($\Psi_{\mathrm{LO}}(r) \sim \frac{1}{r^{1/4}}$):

$$\frac{d}{d\,\Lambda}\left[\Lambda^{1/2} C_{2n}(\Lambda)\right] = 0 \quad \Rightarrow \quad Q^{1/2} \text{ enhancement}$$

- The pionless three-nucleon system ($\Psi_{\mathrm{LO}}(r,\rho) \sim \frac{1}{r}\,\frac{1}{\rho}$):

$$\frac{d}{d\,\Lambda}\left[\Lambda^4 C_3(\Lambda)\right] = 0 \quad \Rightarrow \quad Q^4 \text{ enhancement}$$

while the pionful running remains to be computed.

The Deuteron Form Factors

The RG evolution is identical to the pionful triplet:

$$\frac{d}{d\,\Lambda}\left[\Lambda^{1/2}C_{\rm em}(\Lambda)\right]=0 \quad\Rightarrow\quad Q^{1/2} \text{ enhancement}$$

Why? Initial and final wave function are triplets.

We have to add the grade of polynomial:

- Charge FF: $D(\Lambda)\,q^2 \Rightarrow Q^2\times Q^{-1/2}=Q^{3/2}$
- Dipole FF: $M(\Lambda)\,\beta\times q \Rightarrow Q\times Q^{-1/2}=Q^{1/2}$
- Quadrupole FF: $Q(\Lambda)\,T_2(Q) \Rightarrow Q^2\times Q^{-1/2}=Q^{3/2}$

And the relative order with respect to leading:

- 1B Charge/Quadrupole at $Q^{-3} \Rightarrow$ 2B at $\mathrm{N}^{9/2}\mathrm{LO}$
- 1B Dipole at $Q^{-2} \Rightarrow$ 2B at $\mathrm{N}^{5/2}\mathrm{LO}$

Radiative Neutron Capture

The RG evolution enhanced from singlet (Q^1) and triplet ($Q^{1/4}$) wfs:

$$\frac{d}{d\,\Lambda}\left[\Lambda^1 \times \Lambda^{1/4} C_{\rm em}(\Lambda)\right] = 0 \quad \Rightarrow \quad Q^{5/4} \text{ enhancement}$$

The lowest grade polynomial is $M(\Lambda)\,\beta \times q$, so the order of 2B is $Q \times Q^{-5/4} = Q^{-1/4}$. The 1B current enters at Q^{-2}.

That is, the relative 2B order is then $\mathrm{N}^{7/4}\mathrm{LO}$.

Proton-proton fusion is analogous:

- $Q^{5/4}$ enhancement
- $A(\Lambda)\,\beta$ polynomial, $Q^{-5/4}$ combined
- 1B current is $Q^{-3} \Rightarrow$ 2B is then $\mathrm{N}^{7/4}\mathrm{LO}$ relative to 1B.

To Summarize

A two-body contact-range current connecting the partial waves:

$$\langle {}^{2S+1}L_J | C_{2\mathrm{B}}(\vec{q}, \Lambda) | {}^{2S'+1}L'_{J'} \rangle$$

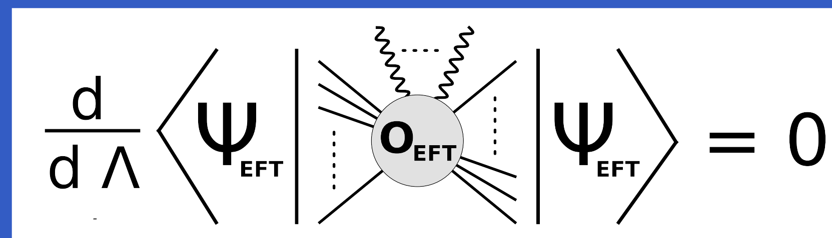
will be enhanced (w.r.t. to NDA) if it connects the following waves:

- $|{}^1S_0\rangle \implies Q^1$ enhancement
- $|{}^3S_1\rangle \implies Q^{1/4}$ enhancement
- $|{}^3P_0\rangle \implies Q^{5/4}$ enhancement

while for other partial waves NDA will hold.

Conclusions

The RG Evolution of Operators:



determines the power counting of EFT operators.

- Two-nucleon system: pionless and pionfull counting.
- Three-nucleon system: pionless counting.
- Deuteron reactions:
  - Pionless: power counting of previous works reproduced
  - Pionful: moderate enhancement of contact two-body currents. Large Q^2 enhancement for ${}^1S_0 \to {}^1S_0$ transtions ($e + {}^3\mathrm{He}$)

(Further details in Valderrama & Phillips, Phys. Rev. 114 (2015) 8)

Time to Finish

Thanks for your attention!

The End.