Power Counting of Contact-Range Currents in EFT

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Physics is unique, but choice of theory depends on resolution $\Lambda$:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\frac{d}{d\Lambda} \langle \Psi | O | \Psi \rangle = 0$$
RG Evolution and Power Counting (I)

At $\Lambda \sim M$ there is no order

\[ \frac{d}{d\Lambda} \langle \Psi | O | \Psi \rangle = 0 \]

while at $\Lambda \sim Q$ there is order

\[ \begin{array}{cccccc}
\hline
\times & \times & \bullet & \times & \times & \cdots \\
\hline
\end{array} \]

LO  NLO  NNLO...
RG Evolution and Power Counting (II)

The intuitive picture about RG Evolution and Power Counting...

![Diagram showing RG Evolution and Power Counting](image-url)

- Infrared Enhanced
- Natural

- Variables: $C(\Lambda)$, $\Lambda$, $Q$, $M$
RG Evolution and Power Counting (III)

What is the exact relationship? EFT Operator: $\mathcal{O} = C(\Lambda) \times \mathcal{F}(Q)$

- $\Lambda = M$: coupling scales as powers of $M$
  \[ C(M) \sim \frac{1}{M^d} \]

- $M \geq \Lambda \geq Q$: coupling evolves
  \[ \frac{d}{d\Lambda} \left\langle \Psi | \mathcal{O} | \Psi \right\rangle = 0 \quad \Rightarrow \quad \frac{d}{d\Lambda} \left[ \Lambda^a C(\Lambda) \right] = 0 \]

- $\Lambda = Q$: coupling gets enhanced
  \[ C(Q) \sim \frac{1}{M^d} \times \left( \frac{M}{Q} \right)^a \]
Deriving the Power Counting

\[ \frac{d}{d\Lambda} \langle \Psi_{\text{EFT}} | \mathcal{O}_{\text{EFT}} | \Psi'_{\text{EFT}} \rangle = 0 \quad \Rightarrow \quad \frac{d}{d\Lambda} [\Lambda^a C(\Lambda)] = 0 \]

How to compute the enhancement of a coupling?

- Consider the LO wave functions and operators
  - \( |\Psi_{\text{EFT}}\rangle = |\Psi_{\text{LO}}\rangle + \Delta |\Psi\rangle \), with \( \Psi_{\text{LO}}(r) \sim r^b \) for \( Qr \leq 1 \)
  - \( \mathcal{O}_{\text{EFT}} = \mathcal{O}_{\text{LO}} + \Delta \mathcal{O} \), \( \mathcal{O}_{\text{LO}} = C(\Lambda) P(\vec{p}, \vec{p}', q, \ldots) \), \( P \) lowest order polynomial compatible with the process (e.g. \( P = 1 \)).
  - \( \langle \Psi_{\text{EFT}} | \mathcal{O}_{\text{EFT}} | \Psi'_{\text{EFT}} \rangle = C(\Lambda) \int_{\Lambda} \frac{dq}{q^{b+1}} \int_{\Lambda} \frac{dq'}{q'^{b'+1}} + \ldots = \frac{C(\Lambda)}{\Lambda^{b+b'}} + \ldots \)
  - \( a = -(b + b') \). The same for higher order couplings.
The Two- and Three-Nucleon System

- The two-nucleon system:
  - Pionless singlet and triplet / pionfull singlet ($\Psi_{LO}(r) \sim \frac{1}{r}$):
    \[
    \frac{d}{d\Lambda} \left[ \Lambda^2 C_{2n}(\Lambda) \right] = 0 \quad \Rightarrow \quad Q^2 \text{ enhancement}
    \]
  - Pionfull triplet ($\Psi_{LO}(r) \sim \frac{1}{r^{1/4}}$):
    \[
    \frac{d}{d\Lambda} \left[ \Lambda^{1/2} C_{2n}(\Lambda) \right] = 0 \quad \Rightarrow \quad Q^{1/2} \text{ enhancement}
    \]
  - The pionless three-nucleon system ($\Psi_{LO}(r, \rho) \sim \frac{1}{r} \frac{1}{\rho}$):
    \[
    \frac{d}{d\Lambda} \left[ \Lambda^4 C_3(\Lambda) \right] = 0 \quad \Rightarrow \quad Q^4 \text{ enhancement}
    \]

while the pionful running remains to be computed.
The Deuteron Form Factors

The RG evolution is identical to the pionful triplet:

\[
\frac{d}{d \Lambda} \left[ \Lambda^{1/2} C_{\text{em}}(\Lambda) \right] = 0 \quad \Rightarrow \quad Q^{1/2} \text{ enhancement}
\]

Why? Initial and final wave function are triplets.

We have to add the grade of polynomial:

- **Charge FF**: \( D(\Lambda) q^2 \Rightarrow Q^2 \times Q^{-1/2} = Q^{3/2} \)
- **Dipole FF**: \( M(\Lambda) \beta \times q \Rightarrow Q \times Q^{-1/2} = Q^{1/2} \)
- **Quadrupole FF**: \( Q(\Lambda) T_2(Q) \Rightarrow Q^2 \times Q^{-1/2} = Q^{3/2} \)

And the relative order with respect to leading:

- **1B Charge/Quadrupole at** \( Q^{-3} \Rightarrow 2B \text{ at } N^{9/2}\text{LO} \)
- **1B Dipole at** \( Q^{-2} \Rightarrow 2B \text{ at } N^{5/2}\text{LO} \)
Radiative Neutron Capture

The RG evolution enhanced from singlet \((Q^1)\) and triplet \((Q^{1/4})\) wfs:

\[
\frac{d}{d\Lambda} \left[ \Lambda^1 \times \Lambda^{1/4} C_{\text{em}}(\Lambda) \right] = 0 \quad \Rightarrow \quad Q^{5/4} \text{ enhancement}
\]

The lowest grade polynomial is \(M(\Lambda) \beta \times q\), so the order of 2B is \(Q \times Q^{-5/4} = Q^{-1/4}\). The 1B current enters at \(Q^{-2}\).

That is, the relative 2B order is then \(N^{7/4}\) LO.

Proton-proton fusion is analogous:

- \(Q^{5/4}\) enhancement
- \(A(\Lambda) \beta\) polynomial, \(Q^{-5/4}\) combined
- 1B current is \(Q^{-3}\) \(\Rightarrow\) 2B is then \(N^{7/4}\) LO relative to 1B.
To Summarize

A two-body contact-range current connecting the partial waves:

\[ \langle 2S + 1 L_J | C_{2B}(\vec{q}, \Lambda) | 2S' + 1 L'_J \rangle \]

will be enhanced (w.r.t. to NDA) if it connects the following waves:

- \(|^1 S_0 \rangle \implies Q^1 \) enhancement
- \(|^3 S_1 \rangle \implies Q^{1/4} \) enhancement
- \(|^3 P_0 \rangle \implies Q^{5/4} \) enhancement

while for other partial waves NDA will hold.
Conclusions

The RG Evolution of Operators:

\[
\frac{d}{d \Lambda} \left\langle \psi_{EFT}^{i} \right| O_{EFT} \left| \psi_{EFT}^{f} \right\rangle = 0
\]

determines the power counting of EFT operators.

- Two-nucleon system: pionless and pionfull counting.
- Three-nucleon system: pionless counting.
- Deuteron reactions:
  - Pionless: power counting of previous works reproduced
  - Pionful: moderate enhancement of contact two-body currents.

Large $Q^2$ enhancement for $^1S_0 \rightarrow ^1S_0$ transitions ($e + ^3\text{He}$)

Time to Finish

Thanks for your attention!

The End.