Baryon ChPT extended beyond the low-energy region

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Chiral Dynamics, June 29 - July 3, 2015, Pisa, Italy

Outline

- πN scattering at tree-order and effective Lagrangian of BChPT;
- EOMS scheme with sliding scale;
- Pion-nucleon scattering at leading one-loop order;
- Summary and Outlook;

Inclusion of nucleons in manifestly Lorentz invariant chiral EFT proved to be complicated due to the non-vanishing nucleon mass in chiral limit:

J. Gasser, M. E. Sainio and A. Svarc, Nucl. Phys. B **307**, 779 (1988).

Encountered problem of power counting (PC) has been resolved by applying the heavy baryon approach

E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991); **259**, 353 (1991).
V. Bernard, N. Kaiser, J. Kambor, and U.-G. Meißner, Nucl. Phys. **B388**, 315 (1992).

Later it has been realised that PC can be respected within the original manifestly Lorentz invariant formulation.

H. Tang, hep-ph/9607436.
P. J. Ellis and H. B. Tang, Phys. Rev. C **57**, 3356 (1998).
T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).
J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).
T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

PC violating parts of loop diagrams are analytic and can be subtracted by renormalizing the parameters of the Lagrangian.

Here we present an extension of applicability of BChPT beyond the low-energy region for small scattering angles.

This is achieved by:

- Re-arranging the chirally invariant terms of the standard low-energy effective Lagrangian
- Introducing a generalization of the extended on-mass-shell (EOMS) scheme.

In particular:

- Re-arranging terms in the standard effective Lagrangian we obtain an EFT with well-defined PC for tree diagrams in the vicinity of a new expansion point.
- PC violating pieces of loop diagrams are absorbed in the redefinition of parameters of the re-arranged Lagrangian.
- As the subtractions are made above the threshold, the renormalized parameters become complex:
- The applied renormalization scheme belongs to the class of complex mass schemes (CMS) first considered in
  R. G. Stuart, in Z^0 *Physics*, ed. J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1990), p.41.)
  A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B560**, 33 (1999).

Consider $\pi^a(q)N(p) \to \pi^{a'}(q')N(p')$ assuming exact isospin-symmetry.

Standard parametrization of the pion-nucleon scattering amplitude:

$$
\begin{aligned}
T_{aa'} &= \delta_{aa'} T^+ + \frac{1}{2}\left[\tau_a, \tau_{a'}\right] T^-, \\
T^\pm &= \bar{u}(p', \sigma') \left[D^\pm(t, \nu) - \frac{1}{4m_N}[\not{q}', \not{q}] B^\pm(t, \nu) \right] u(p, \sigma).
\end{aligned}
$$

It is convenient to present D and B as functions of t and $\nu = \frac{s-u}{4m_N}$.

$X \in \{D^+,\ D^-/\nu,\ B^+/\nu,\ B^-\}$ are even functions of ν.

The difference between the full amplitude and the pseudovector Born term can be expanded around $\nu = t = 0$

$$X(\nu, t) = X_{pv}(\nu, t) + x_{00} + x_{10}\nu^2 + x_{01}t + x_{20}\nu^4 + x_{11}\nu^2 t + x_{02}t^2 + \cdots$$

where $X_{pv}(\nu, t)$ are the pseudovector Born terms.

Spontaneously broken chiral symmetry predicts that $d^+_{00} = 0$ and $d^-_{00} = 1/(2F^2)$ in chiral limit.

G. Höhler, in: H. Schopper (Ed.), Landolt-Börnstein, Vol. 9b2, Springer, Berlin, 1983.
T. Becher and H. Leutwyler, JHEP **0106**, 017 (2001).

Non-Born tree order contributions of the effective Lagrangian can be parameterized as

$$\begin{aligned}
D^{+} &= d_0^{+}(t,M) + d_2^{+}(t,M)\nu^2 + d_4^{+}(t,M)\nu^4 + \cdots \,, \\
D^{-} &= d_1^{-}(t,M)\nu + d_3^{-}(t,M)\nu^3 + \cdots \,, \\
B^{+} &= b_1^{+}(t,M)\nu + b_3^{+}(t,M)\nu^3 + \cdots \,, \\
B^{-} &= b_0^{-}(t,M) + b_2^{-}(t,M)\nu^2 + \cdots \,,
\end{aligned} \tag{1}$$

where $d_j^{\pm}(t,M)$ and $b_j^{\pm}(t,M)$ are Taylor series of t and M. Coefficients of these series also contain $\ln M$.

In threshold region amplitudes are organized according to a PC assigning various orders of q to contributions of various diagrams.

To consider the tree-order amplitudes beyond the threshold region we expand the amplitudes at $\nu^2 = \mu_0^2 \sim q^0$ as follows

$$
\begin{aligned}
D^+ &= \tilde{d}_0^+(t,M) + \nu^2 \left[\tilde{d}_2^+(t,M) + \tilde{d}_4^+(t,M)(\nu^2 - \mu_0^2) + \cdots \right], \\
D^- &= d_1^-(t,M)\nu + \nu^3 \left[\tilde{d}_3^-(t,M) + \tilde{d}_5^-(t,M)(\nu^2 - \mu_0^2) + \cdots \right], \\
B^+ &= \nu \left[\tilde{b}_1^+(t,M) + b_3^+(t,M)(\nu^2 - \mu_0^2) + \cdots \right], \\
B^- &= \tilde{b}_0^-(t,M) + \tilde{b}_2^-(t,M)(\nu^2 - \mu_0^2) + \cdots .
\end{aligned}
$$

These power series are generated by an effective Lagrangian with the same structures as in the standard effective Lagrangian, however the terms re-arranged according to new PC rules.

New PC (Q as a small parameter):

t counts as of order Q^2,
M and $\nu^2 - \mu_0^2$ count as of order Q^1.

Terms of the effective Lagrangian, which generate contributions of order Q^N at tree level count as of order Q^N.

The Lagrangian contains a finite number of chirally invariant structures at any finite order.

Resonances have to be included explicitly.

Tree-order $\mathcal{O}(q^4)$ amplitudes generated by OPI diagrams:

$$
\begin{aligned}
D^{+}_{tree} &= \frac{16\,c_2\,m_N^2\,\nu^2}{8F^2m^2} - \frac{4\,c_1\,M^2}{F^2} + \frac{c_3(2M_\pi^2 - t)}{F^2} + \frac{16\,e_{16}\,\nu^4}{F^2} \\
&+ \frac{8e_{15}(2M_\pi^2 - t)\nu^2}{F^2} + \frac{16m_N^2\,M^2\nu^2(e_{20} + e_{35})}{F^2\,m^2} - \frac{c_2\,t^2}{8F^2\,m^2} \\
&+ \frac{8M^4}{F^2}(e_{22} - 4e_{38}) + \frac{4M^2 e_x(2M_\pi^2 - t)}{F^2} + \frac{4e_{14}(2M_\pi^2 - t)^2}{F^2}, \\
D^{-}_{tree} &= \frac{\nu}{2F^2} + \frac{4d_3\nu^3}{F^2} + \frac{2\nu\left[2M_\pi^2(2d_5 + d_1 + d_2) - (d_1 + d_2)t\right]}{F^2}, \\
B^{+}_{tree} &= \frac{4\,(d_{14} - d_{15})\,m_N\,\nu}{F^2}, \\
B^{-}_{tree} &= \frac{1}{2F^2} + \frac{2c_4\,m_N}{F^2} + \frac{16\,m_N\,e_{18}\nu^2}{F^2} - \frac{8\,m_N\,e_{17}\,t}{F^2} \\
&+ \frac{8\,m_N[M^2(2e_{21} - e_{37}) + 2M_\pi^2 e_{17}]}{F^2}.
\end{aligned}
$$

$e_x = 2e_{19} - e_{22} - e_{36}$ and c_i, d_i and e_i are LECs.

Re-arranged tree-order amplitudes:

$$
\begin{aligned}
D^{+}_{tree} &= \frac{16\,\tilde{c}_2\,\nu^2}{8F^2} + \frac{16\,\tilde{e}_{16}\,\nu^2(\nu^2-\mu_0^2)}{8F^2} + \frac{8\tilde{e}_{15}(2M^2-t)(\nu^2-\mu_0^2)}{F^2} + \\
D^{-}_{tree} &= \frac{\nu}{2F^2} + \frac{4\tilde{d}_3\nu^3}{F^2} + \frac{2\nu\left[2M^2(2\tilde{d}_5+\tilde{d}_1+\tilde{d}_2)-(\tilde{d}_1+\tilde{d}_2)t\right]}{F^2} + \cdots \\
B^{+}_{tree} &= \frac{4\left(\tilde{d}_{14}-\tilde{d}_{15}\right)m\,\nu}{F^2} + \cdots\,, \\
B^{-}_{tree} &= \frac{1}{2F^2} + \frac{2\tilde{c}_4 m_N}{F^2} + \frac{16\,m_N\,\tilde{e}_{18}(\nu^2-\mu_0^2)}{F^2} + \cdots\,.
\end{aligned}
$$

New constants $\tilde{c}_i$, $\tilde{d}_i$ and $\tilde{e}_i$ depend on μ_0 and they are related to the original low-energy constants:

$$
\begin{aligned}
c_2 - \Delta c_2 &= \tilde{c}_2 + \tilde{e}_{16}\mu_0^2 + \cdots\,,\\
c_3 - \Delta c_3 &= \tilde{c}_3 + 8\,\tilde{e}_{15}\mu_0^2 + \cdots\,,\\
c_4 - \Delta c_4 &= \tilde{c}_4 + 8\,\tilde{e}_{18}\mu_0^2 + \cdots\,,\\
&\cdots\,,
\end{aligned}
$$

where Δc_i are the contributions of resonances which need to be included dynamically in extended effective theory and the dots stand for terms with increasing powers of μ_0.

LO re-arranged effective Lagrangian:

$$\tilde{\mathcal{L}}_{\pi N}^{(0)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m\right)\Psi - \frac{\tilde{c}_2}{4m^2}\langle u_\mu u_\nu\rangle\,\bar{\Psi}\left(D^\mu D^\nu + h.c.\right)\Psi + \frac{\tilde{d}_3}{12m^3}\,\bar{\Psi}\left\{[u_\mu,[D_\nu,u_\lambda]]\left(D^\mu D^\nu D^\lambda + sym.\right) + h.c.\right\}\Psi,$$

where Ψ denotes the nucleon field, m_0 stands for the bare mass of the nucleon, $D_\mu\Psi = (\partial_\mu + \Gamma_\mu)\Psi$ is the covariant derivative and

$$u^2 = U, \quad u_\mu = iu^\dagger\partial_\mu U u^\dagger, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u],$$

where U is a (2×2) matrix of the Goldstone boson fields.

One of the NLO terms of the re-arranged effective Lagrangian

$$\tilde{\mathcal{L}}_{\pi N}^{(1)} = \frac{\tilde{e}_{16}}{48 m^4}\Big\{\bar{\psi}\left[\langle h_{\lambda\mu} h_{\nu\rho}\rangle D^{\lambda\mu\nu\rho} + h.c.\right]\psi + 12\, m^2 \mu_0^2 \langle u_\mu u_\nu\rangle \bar{\psi}\,(D^\mu D^\nu + h.c.)\,\psi\Big\}.$$

For loop diagrams we use the EOMS scheme with sliding scale.

PC is also applicable to renormalised loop diagrams.

It is convenient to consider rest frame of the initial nucleon.

In this frame the three-momenta of external nucleons are small;

The four-momenta of external pions are large;

The four-momentum of the loop-integration is considered small so that a loop integration in n dimensions counts as of order Q^n.

Pion and fermion propagators and vertices count variously depending on the momenta flowing through them.

Let us demonstrate on a simple loop integral:

$$B_0(p^2, M^2, m^2) = \frac{(2\pi)^{4-n}}{i\,\pi^2} \int \frac{d^n k}{\left[k^2 - M^2 + i\delta\right]\left[(p+k)^2 - m^2 + i\delta\right]},$$

We assign order $\mathcal{O}(Q^2)$ to $B_0(p^2, M^2, m^2)$ for $p^2 - m^2 \gg M^2$.

By direct expansion we obtain the following subtraction terms:

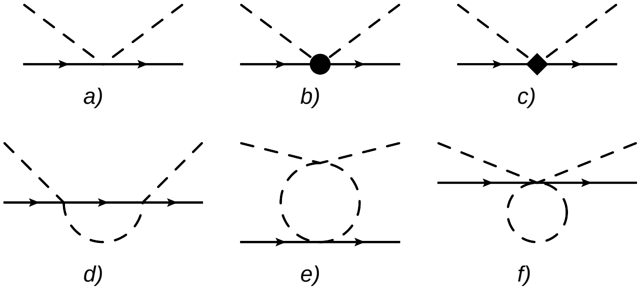
$$\begin{aligned} B_0^{ST} &= -32\,\pi^2\bar{\lambda} - 2\,\ln\frac{m}{\mu_d} + 1 + \frac{\left(m^2 - \mu^2\right)\left[\ln\left(\frac{\mu^2}{m^2} - 1\right) - i\pi\right]}{\mu^2} \\ &+ \frac{\left(\mu^2 - p^2\right)\left[m^2 \ln\left(\frac{\mu^2}{m^2} - 1\right) - i\pi m^2 + \mu^2\right]}{\mu^4}. \end{aligned}$$

It can be easily seen by expanding in M and $p^2 - \mu^2$ that the subtracted integral B_0^R is indeed of order $\mathcal{O}(Q^2)$.

Pion-nucleon scattering at leading one-loop order

Consider one-loop diagrams contributing to pion-nucleon scattering in a simplified model of EFT by taking $g_A = 0$.

Diagrams contributing to πN scattering at $\mathcal{O}(Q^2)$. The crossed diagram is not shown.



Subtraction terms of loop diagrams are canceled by counter terms generated by the re-arranged effective Lagrangian.

A bare coupling expressed in terms of renormalized ones:

$$\begin{aligned}\tilde{c}_4 &= \tilde{c}_4^R \\ &+ \frac{1}{64\pi^2 F^2\left(m^3-4m\mu^2\right)}\left\{m\mu^2\left[(m+2\mu)B_0\left(m(m-2\mu),0,m^2\right)\right.\right. \\ &+ \left.\left.(m-2\mu)B_0\left(m(m+2\mu),0,m^2\right)\right]+\left(2\mu^2-m^2\right)A_0\left(m^2\right)\right\}.\end{aligned}$$

Note that $B_0(x^2, 0, m^2)$ has an imaginary part for $x^2 > m^2$ and therefore the renormalized couplings are complex.

Summary

- We generalized BChPT to energies beyond the low-energy threshold region and small scattering angles.
- The new chirally invariant re-arranged effective Lagrangian contains a finite number of terms at any finite order.
- Resonances need to be included explicitly.
- For loop diagrams we apply EOMS scheme with sliding scale.
- Within this scheme we shift the renormalization point in the physical region beyond the threshold.
- Thus the renormalized coupling constants of the re-arranged effective Lagrangian become complex.
- Pion photo and electro-production processes as well as the Compton scattering can be treated analogously to $\pi N \to \pi N$.

Backup slides

CMS and unitarity

Unitarity is guaranteed by the Hermitian bare Lagrangian and the fact that the renormalization is an identical transformation.

Still, order-by-order unitarity within the CMS is a non-trivial issue. It has been probed in
T. Bauer, J. Gegelia, G. Japaridze and S. Scherer, "CMS and perturbative unitarity," Int. J. Mod. Phys. A **27**, 1250178 (2012)
and thoroughly investigated recently in
A. Denner and J. N. Lang, "The CMS and Unitarity in perturbative Quantum Field Theory," arXiv:1406.6280 [hep-ph].

An intuitive demonstrating argument:

Consider the scalar ϕ^4 theory in 4 dimensions.
The Lagrangian depends on the bare mass m_0 and the bare coupling λ_0.
Using dimensional regularization and the MS scheme we get rid off divergences and express physical quantities in terms of parameters of the MS scheme m_{MS} and λ_{MS} as power series of the renormalized coupling constant

$$M_i = F_i(m_{MS}(\mu), \lambda_{MS}(\mu), p, \mu),$$

where p stands for kinematical variables and μ is the renormalization scale.
Physical amplitudes are unitary up to the given order of accuracy.

We can switching to another renormalization.
For example, we calculate the pole mass of the scalar particle and the two-particle scattering amplitude $M(s,t,u)$ at symmetric non-physical kinematical point

$$\begin{aligned} m &= \phi_1(m_{MS}(\mu), \lambda_{MS}(\mu), p, \mu), \\ \lambda(\nu) &= M(-\nu^2/3, -\nu^2/3, -\nu^2/3) = \phi_2(m_{MS}(\mu), \lambda_{MS}(\mu), \nu, \mu), \end{aligned}$$

express $m_{MS}(\mu)$ and $\lambda_{MS}(\mu)$ in terms of m and $\lambda(\nu)$ and substitute in M_i. This way we obtain

$$M_i = \tilde{F}_i(m, \lambda(\nu), p, \nu),$$

Where $\tilde{F}_i$ are some functions (different from F_i).
By doing this identical transformation one does not violate the unitarity.

Although convenient, it is not necessary to choose the new renormalized coupling at non-physical kinematical point. Taking e.g.

$$\begin{aligned} m &= \phi_1(m_{MS}(\mu), \lambda_{MS}(\mu), p, \mu), \\ \lambda_C(\nu) &= M(2m^2 + \nu^2, 0, 2m^2 - \nu^2) = \phi_3(m_{MS}(\mu), \lambda_{MS}(\mu), \nu, \mu), \end{aligned}$$

expressing $m_{MS}(\mu)$ and $\lambda_{MS}(\mu)$ in terms of m and $\lambda_C(\nu)$ and substituting in M_i we obtain

$$M_i = \bar{F}_i(m, \lambda_C(\nu), p, \nu),$$

with $\bar{F}_i$ some functions (different from $\tilde{F}_i$ and F_i). Once more, by doing this identical transformation one does not violate the unitarity. However, as the unitarity condition is only satisfied up to higher orders of perturbation theory, the relevant issue is of course the convergence of the obtained perturbative series.

Expansion of B_0^R in M and $p^2 - \mu^2$:

$$
\begin{aligned}
B_0^R &= \frac{\left(p^2-\mu^2\right)^2}{2\mu^6\left(\mu^2-m^2\right)}\Big[2i\pi m^4 - 2i(\pi - i)m^2\mu^2 \\
&- 2\left(m^4 - m^2\mu^2\right)\ln\left(\frac{\mu^2}{m^2}-1\right) + \mu^4\Big] \\
&- \frac{M^2\left[\left(m^2+\mu^2\right)\ln\left(\frac{\mu^2}{m^2}-1\right) - i\pi m^2 - 2\mu^2\ln\frac{M}{m} - i\pi\mu^2 + \mu^2\right]}{\mu^2\left(m^2-\mu^2\right)} \\
&+ \mathcal{O}(Q^3).
\end{aligned}
$$

"... quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry."

S. Weinberg, Physica A **96**, 327 (1979).