Search for the H-Dibaryon in two flavor Lattice QCD

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Outline

- Brief Introduction
  - Experimental results and status of lattice calculations.
- Lattice methodology
  - Operators employed
  - Lattice set up
- Discussion of results

The H-dibaryon

Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

Predicted by R. L. Jaffe (1977) as a six quark bound state using MIT bag model as $[H \sim uuddss]$

$J = I = 0,\ S = -2,\ m_H < 2m_\Lambda \sim -80$ MeV

Experimental searches

Stongest Constraint comes from "Nagara" Event which found a double ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ double-hypernucleus with binding energy

$$B_{\Lambda\Lambda} = 6.91 \pm 0.16 \text{ MeV}$$

The absence of a strong decay ${}^{6}_{\Lambda\Lambda}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + H$ implies,

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda}$$

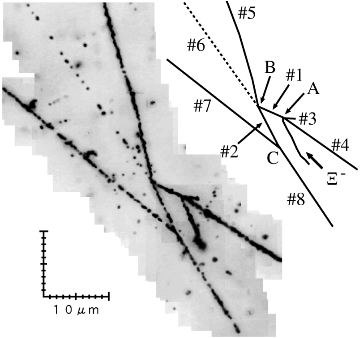
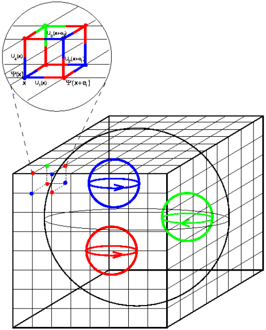


FIG. 2. Photograph and schematic drawing of NAGARA event. See text for detailed explanation.

KEK-E176, Nucl.Phys. A835(207-214)2010

Lattice Calculation of H-dibaryon

Lattice techinques offer first principle calculation of QCD observables. But ...,



Courtesy:USQCD

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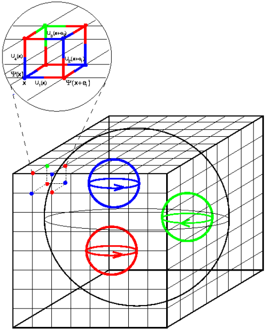

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- Unphysical quarks, Continuum limit, Infinite volume limit.

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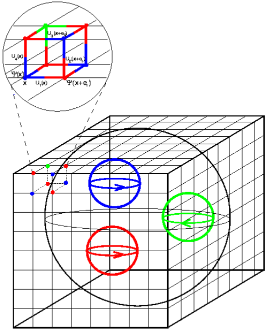

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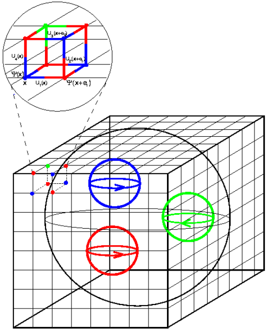

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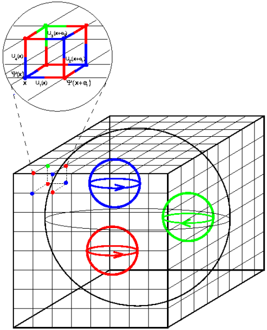

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- Unphysical quarks, Continuum limit, Infinite volume limit.
- Exponential Signal/noise problem for baryon systems
- No of contractions can be non-trivial $N = \prod_i^{N_f} N_{q_i}!$
- Multiple volumes necessary to get reliable results.

Systems with shallow binding energy, for eg maybe : $H^{\mathbf{1}}$? combined with the issues mentioned .. presents a formidable challenge !

Status of Lattice results

Early attempts (1985 $\sim$ 2003) on quenched lattices gave mixed results.

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Group	Method	N_f	Action	N_{Vol}	m_π(MeV)	B_H(MeV)
NPLQCD	2pt	3	clover	3	806	74.6(3.3)(3.4)
		2+1	aclover	4	390	13.2(1.8)(4.0)
			aclover	1	230	-0.6(8.9)(10.3)
HALQCD	B-B Potential	3	clover	1	1171	84(4)
				3	1015	32.9(4.5)(6.6)
				1	837	37.4(4.4)(7.3)
				1	672	35.6(7.4)(4.0)
				1	469	26(4)
Mainz	**2pt**	**2**	**clover**	**1**	**1000**	**92(10)(7)**
					450	**77(11)(7)**

Our methodology

Interpolating operators

- Positive parity projected six quark operators at source and sink

$$[abcdef] = \varepsilon^{ijk}\varepsilon^{lmn}(b_i^T C\gamma_5 P_+ c_j)(e_l^T C\gamma_5 P_+ f_m)(a_k^T C\gamma_5 P_+ d_n)$$

$$H^{\mathbf{1}} = \frac{1}{48}\big(\,[sudsud] - [udusds] - [dudsus]\,\big)$$

$$H^{\mathbf{27}} = \frac{1}{48\sqrt{3}}\big(2\,[sudsud] + [udusds] + [dudsus]\,\big)$$

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- Momentum projected two-baryon operators at the sink

$$B_\alpha = [abc]_\alpha = \varepsilon^{ijk}(b_i^T C\gamma_5 P_+ c_j)a_{k\alpha}$$

$$B_1 B_2(\vec{p_1},\vec{p_2}) = \sum_{\vec{x},\vec{y}} e^{i\vec{p_1}\cdot\vec{x}} e^{i\vec{p_2}\cdot\vec{y}}\; B_1^T(\vec{x})\; C\gamma_5 P_+\; B_2(\vec{y})$$

Operators belonging to $BB^{\mathbf{1}}, BB^{\mathbf{8}}, BB^{\mathbf{27}}$

Contractions

An efficient way to contract the six-quark operators into correlation functions is to use a blocking algorithm:

- Form blocks of three propagators contracted into a color-singlet at the sink

$$B(\alpha_1, \xi'_1, \xi'_2, \xi'_3) = \epsilon_{c_1,c_2,c_3}(C\gamma_5 P_+)_{\alpha_2\alpha_3} \\ S_l(\xi_1, \xi'_1) S_l(\xi_2, \xi'_2) S_s(\xi_3, \xi'_3)$$

- Then sum over all permutations as,

$$[sudsud] = (C\gamma_5 P_+)_{\alpha\beta} \times \epsilon_{c'_1,c'_2,c'_3}\epsilon_{c'_4,c'_5,c'_6}(C\gamma_5 P_+)_{\alpha'_2\alpha'_3}(C\gamma_5 P_+)_{\alpha'_5\alpha'_6} \\ \sum_{\sigma_u,\sigma_d,\sigma_s} B(\alpha, \xi'_{\sigma_u(1)}, \xi'_{\sigma_d(2)}, \xi'_{\sigma_s(3)}) B(\beta, \xi'_{\sigma_u(4)}, \xi'_{\sigma_d(5)}, \xi'_{\sigma_s(6)})$$

All mode Averaging

Employ low precision propagator solves over multiple sources and compute observable as,

$$\mathcal{O}^{\text{AMA}} = \mathcal{O}_{\vec{x}_0}^{\text{high prec}} - \mathcal{O}_{\vec{x}_0}^{\text{low prec}} + \frac{1}{N_{\vec{x}}} \sum_{N_{\vec{x}}} \mathcal{O}_{N_{\vec{x}}}^{\text{low prec}}$$

Variance with AMA :

$$\sigma_{\text{AMA}}^2 = \sigma^2 \left(2(1-r) + \frac{1}{N_{\vec{x}}} \right) \quad , \quad r = \text{Corr}(\mathcal{O}_{\vec{x}_0}^{\text{high prec}}, \mathcal{O}_{\vec{x}_0}^{\text{low prec}})$$

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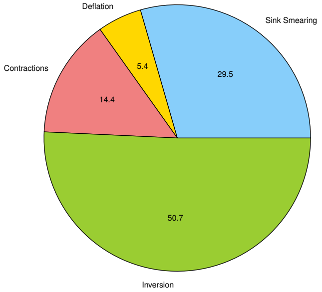
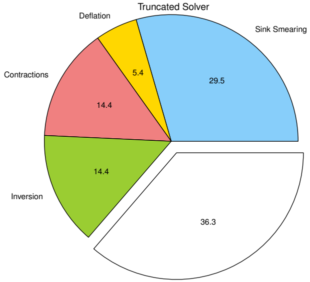




Ensemble E1

- $m_\pi = 1000$ MeV
- $\mathrm{L}^3 = (2\ \mathrm{fm})^3$
- $m_\pi L = 10$
- 1 high/low precision solve for AMA bias
- $N_{\mathrm{srcs}} = 128$ with low precision solves.
- Double statistics using P_+ and P_- for forward/backward propagating states.
- Total measurements

$$168 \times 128 \times 2 \sim 43000$$

- $\kappa_s = \kappa_{ud}$ implies no mixing between **1** and **27**
- Two sets of smearing provide independent operators for GEVP.

Operators on Ensemble E1

- Operators at the source $\mathrm{H}^{\mathbf{1}}$(N) and $\mathrm{H}^{\mathbf{1}}$(M)
- Operators choices at sink
  - Choice of smearings : Narrow and Medium (medium is noisy)
  - Choice of six-quark and two-baryon operators at different kinematics.
- Construct various 2×2 correlator matrices to explore ground state.
- Estimate systematic uncertainty as,

$$\chi^2 = \sum_{t_i,t_j}^{N} (\overline{G}(t_i) - F(t_i, A)) C_{ij}^{-1} (\overline{G}(t_j) - F(t_j, A))$$

Generalized EigenValue Problem

We compute matrix of two point functions as,

$$C_{ij} = \sum_{\vec{x}} \langle \mathcal{O}_i(t_0 + t, \vec{x}) \mathcal{O}^\dagger(t_0, \vec{x}_0) \rangle,$$

and solve the generalized eigenvalue problem (GEVP),

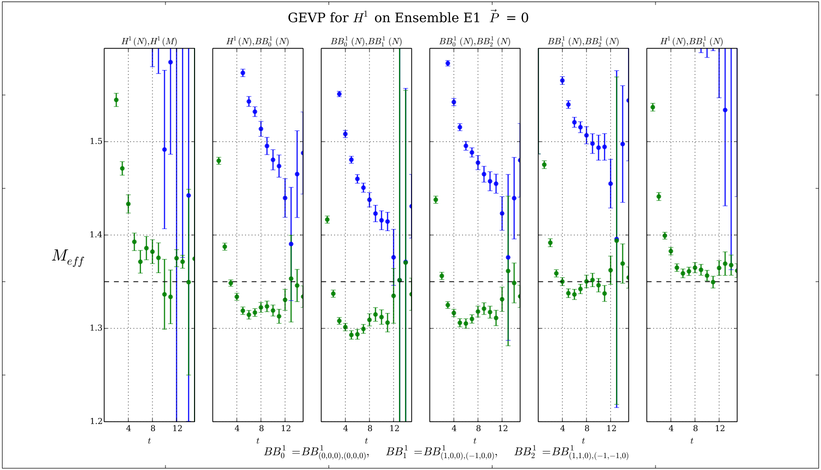
$$C_{ij}(t + \Delta t) v_j(t) = \lambda(t) C_{ij}(t) v_j(t)$$

and compute effective masses as,

$$m_{\text{eff}} = \frac{-\log \lambda(t)}{\Delta t}$$

Asymptotically dominated by a single exponential

GEVP on E1



GEVP for H^1 on Ensemble E1 $\vec{P} = 0$
$H^1(N), H^1(M)$
$H^1(N), BB^1_0(N)$
$BB^1_0(N), BB^1_1(N)$
$BB^1_0(N), BB^1_2(N)$
$BB^1_1(N), BB^1_2(N)$
$H^1(N), BB^1_1(N)$
M_{eff}
t
$BB^1_0 = BB^1_{(0,0,0),(0,0,0)}$, $BB^1_1 = BB^1_{(1,0,0),(-1,0,0)}$, $BB^1_2 = BB^1_{(1,1,0),(-1,-1,0)}$

Scattering phase shift from Energy levels

The two particle scattering/binding momenta,

$$p^2 = \frac{1}{4}\left(E^2 - \vec{P}\cdot\vec{P}\right) - M_\Lambda^2$$

is related to scattering phases in the continuum via,

$$p\ \cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}^d_{0,0}(1,q^2) \quad q = \frac{pL}{2\pi}$$

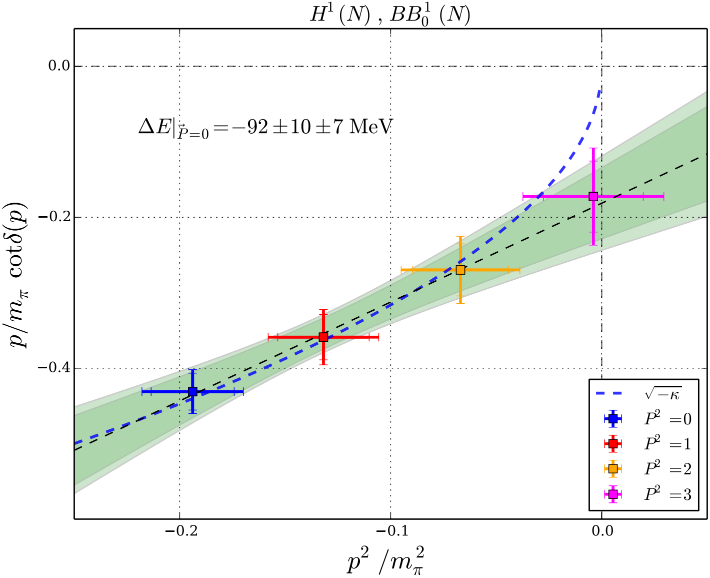
[Lüscher (1991), Rummukainen Gottlieb (1995)]

$$\mathcal{Z}^d_{0,0}(1,q^2) = \frac{1}{\sqrt{4\pi}}\left\{\sum_{q^2\neq n^2}^{\Lambda}\frac{1}{q^2-n^2} - 4\pi\Lambda\right\}$$

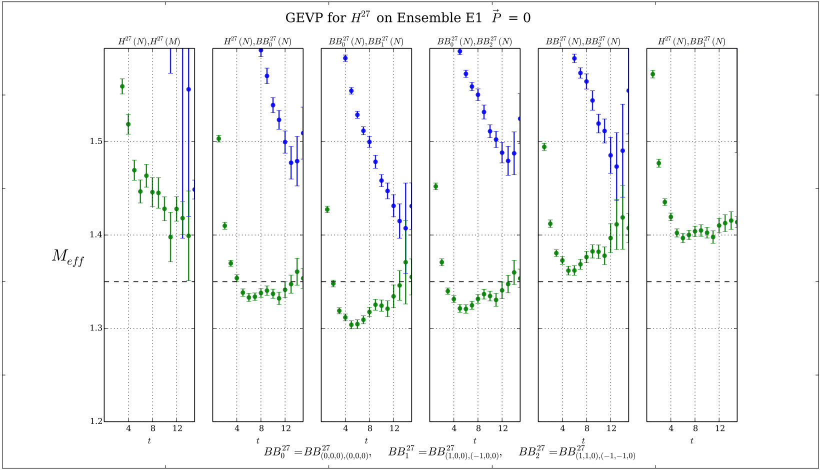
Use scattering information to locate the pole in the scattering amplitude,

$$\mathcal{A} \propto \frac{1}{p\ \cot\delta_0(p) - ip} \qquad p\ \cot\delta_0(p) = -\frac{1}{a} + \frac{1}{2}r_0p^2 + \dots$$

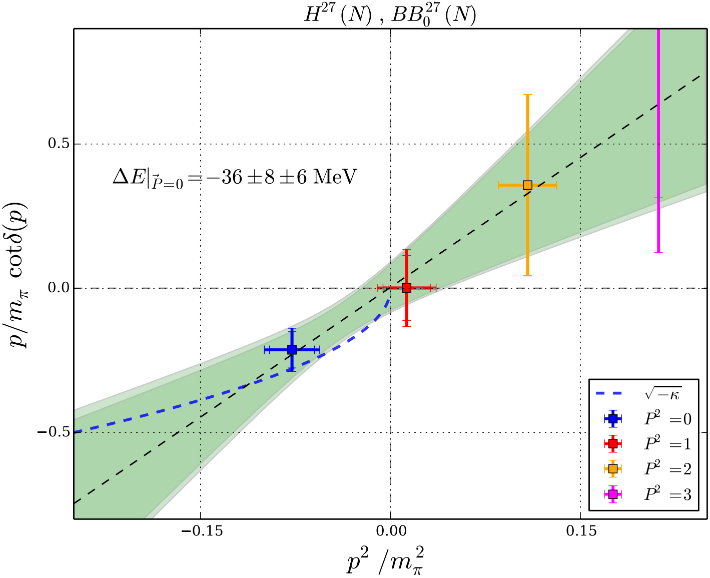
Scattering phase shift of H¹ - Ensemble E1 (Preliminary)



GEVP for $\mathbf{H^{27}}$ - Ensemble E1



Scattering phase shift of $\mathbf{H^{27}}$ - Ensemble E1 (Preliminary)



Ensemble E5

- $m_\pi = 451$ MeV
- $\mathrm{L}^3 = (2\ \mathrm{fm})^3$
- $m_\pi L = 4.6$
- $N_{\mathrm{cfgs}} = 1990$ gauge configurations.
- 1 high/low precision solve for AMA bias
- $N_{\mathrm{srcs}} = 32$ with low precision solves.
- Double statistics using P_+ and P_- for forward/backward propagating states.
- Total measurements

$$1990 \times 32 \times 2 \sim 125000$$

- $\kappa_s > \kappa_{ud}$ implies mixing between **1** and **27**

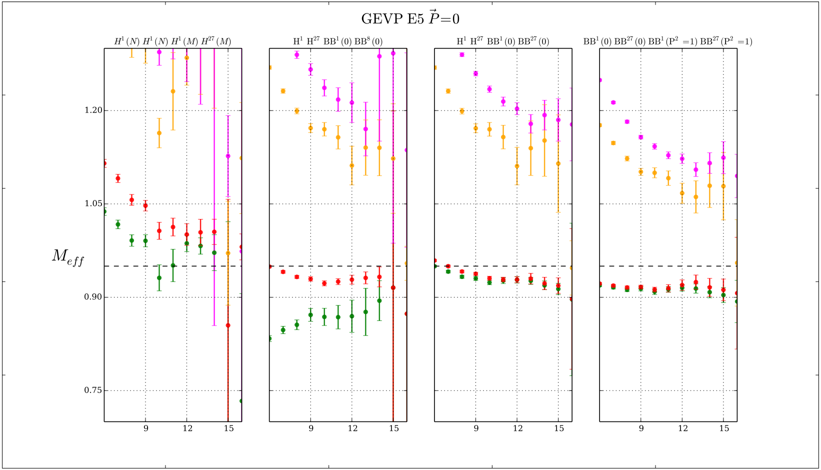
Operators on Ensemble E5

Solve a GEVP with the available operators:

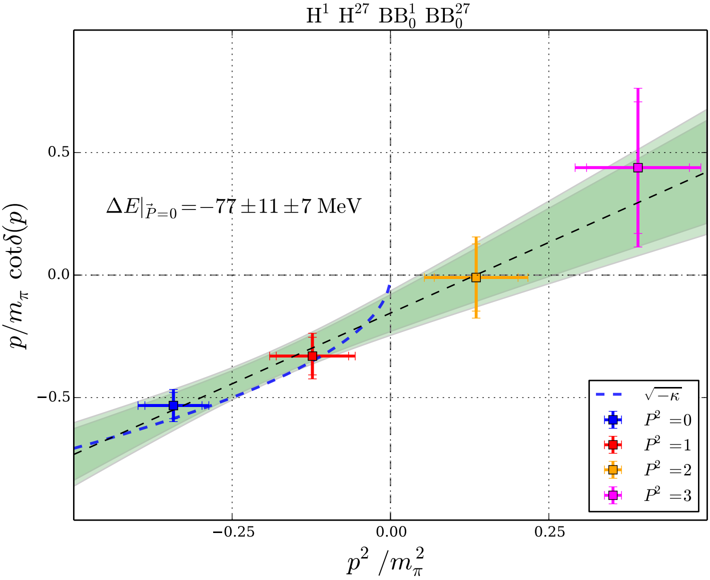
- Four source operators Narrow(N) and Medium (M) smeared $H^{\mathbf{1}}$ and $H^{\mathbf{27}}$
- Choice of six-quark operators $H^{\mathbf{1}}$ & $H^{\mathbf{27}}$ and $BB^{\mathbf{1}}$,$BB^{\mathbf{8}}$ & $BB^{\mathbf{27}}$ with different kinematic combinations. Employ only narrow smeared operators
- Construct various 4×4 correlator matrices to explore the ground state.

For scattering studies, this is coupled channel scattering problem requiring total 3 parameters.

GEVP on Ensemble E5



Ground state scattering phase shift on E5 (Preliminary)



Conclusions and Outlook

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- At $m_\pi = 451$ MeV, $\mathrm{H}^{\mathbf{1}}$ is bound in finite volume at $\vec{P} = 0$ with $B_H = 77(11)(7)$ MeV.
- In both cases, the existence of the pole in the scattering ampltude is unclear.

Things to pursue...

- Understand the ground state contibutions from $BB^{\mathbf{8}}$.
- Perform a systematic study of finite volume effects for a reliable determination on the fate of $H^{\mathbf{1}}$....