Theory Overview of the Proton’s Charge Radius Puzzle

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The puzzle

• Measure charge radius of the proton different ways, get different answers

• Difference is 7 s.d.
  (was 5 s.d. when first announced, 2010)

• Why? Don’t yet know.
1. The measurements: where the differences came from

2. Suggested explanations
   A. Ordinary explanations
      • Somebody screwed up
   B. Exotic explanations
      • Physics Beyond the Standard Model (BSM)

3. Highlight: List of coming relevant data
Measuring proton radius

- Use lepton-proton scattering or use atomic spectroscopy
- Use electrons or muons

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<th>lepton scattering</th>
<th>atomic energy splittings</th>
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<td>electrons</td>
<td>done—but more coming</td>
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<td>muons</td>
<td>not done—but coming</td>
<td>done—one experiment</td>
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**e-p scattering**

- Measure differential cross section, fit results to form factors,
  \[
  \frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2)
  \]
  \[
  \left[ \tau = Q^2 / 4m_p^2 ; \quad 1/\epsilon = 1 + 2(1 + \tau) \tan^2(\theta_e/2) \right]
  \]

- Low $Q^2$, mainly sensitive to $G_E$.

- Extrapolate to $Q^2 = 0$, whence
  \[
  R_E^2 = -6 \left( \frac{dG_E}{dQ^2} \right)_{Q^2=0}
  \]
Low-$Q^2$ scattering data

• Mainz has Gutenberg plus an electron accelerator

• Great data, Jan Bernauer et al., PRL 2010 (and later articles).

• $Q^2$ range 0.004 to 1 GeV$^2$

• From their analysis,

\[ R_E = 0.879(8) \text{ fm} \]
Atomic energy level splittings

- Basic: Schrödinger equation, H-atom, point protons
  \[ E = -\frac{\text{Ryd}}{n^2}, \quad \text{where} \quad \text{Ryd} = \frac{1}{2} m_e \alpha^2 \approx 13.6\text{eV} \]
- plus QED corrections
- plus finite size proton, pushing energy upward a bit.

\[ \Delta E_{\text{finite size}} = \frac{2\pi \alpha}{3} \phi_{nS}(0) R_E^2 \]

fine print: \[ \phi_{nS}(0) = (m_r \alpha)^3 / (n^3 \pi) \]

I.e., \[ E_{\text{total}} = -\frac{\text{Ryd}}{n^2} + \frac{\text{finite size}}{n^3} \delta_{\ell,0} \]
measure energy accurately
\[\iff \text{measure radius}\]

• Reminder, H-atom energy levels (diagram not to scale)
Atomic results

ep : 0.8758 (77) fm
(spectroscopic data only)
All electron results

- Consistent
- Combined by Committee on Data in Science and Technology (CODATA),

\[ R_E = 0.8775(51) \text{ fm} \]
Then in 2010

• CREMA = Charge Radius Experiment with Muonic Atoms

• Did atomic physics, specifically Lamb shift, with muons (muon = electron, but weighs 200 times more).

• Orbits 200 times closer: proton looks 200 times bigger

• Goal: measure proton radius with factor 10 smaller uncertainty
CREMA

• 2S-2P Lamb shift in $\mu$-H.
• Measured two lines,

\[
\begin{align*}
F=0 & \quad \text{2P}_{1/2} \\
F=1 & \quad \text{2P}_{3/2} \\
F=2
\end{align*}
\]

\[
\begin{align*}
\text{FS} & \quad 8.4 \text{ meV} \\
\text{HFS} & \quad 23 \text{ meV}
\end{align*}
\]

• pubs:
  upper line, Pohl et al., Nature 2010
  lower line Antognini et al., Science 2013

• Interpreting finite size effect in terms of proton radius,
  \[ R_E = 0.84087(39) \text{ fm} \]

• Whoops: result 4% or 7σ small
Other data-deuteron

• Reported at conferences 2013
• 2015 experimenters circulate draft of theory paper!
• Measured three lines

- $2S_{1/2}$
- $2P_{1/2}$
- $2P_{3/2}$

F=1/2, F=3/2, F=5/2

ca. 215 meV

• Quick summary: if proton radius is shrunken, the deuteron radius is also.
Other data — Helium

• New 2013/2014 data

• $\mu^{-4}\text{He}$ at Mainz Proton Radius Workshop, 2014

• $\mu^{-3}\text{He}$ at Gordon Conference, N.H., 2014

• Quick summary: He radii from $\mu$ Lamb shift in accord with electron scattering radii.
Explanations?

• Hard to see problems with $\mu$ experiment
  • Hard to get working
  • But once working, easy to analyze

• Problems with analysis of electron experiments? Theorists are chipping in here! But there are a lot of experiments.

• BSM explanations?
  • If so, further tests?
Review $e-p$ scattering data

• Point: Measurements at finite $Q^2$. Need to extrapolate to $Q^2 = 0$ to obtain charge radius. (Mainz group itself: $R_E = 0.879(8)$ fm.)

• Others have tried different analyses regarding the extrapolation

• Graczyk & Juszczak (2014), using Bayesian ideas and pre-Mainz world data, obtained
  $$R_E = 0.899(3) \text{ fm}.$$  

• Lee, Arrington, & Hill (2015) using Mainz data and neat mapping ideas to ensure convergence of expansions, obtained
  $$R_E = 0.895(20) \text{ fm}.$$  

• Arrington & Sick found
  $$R_E = 0.879(11) \text{ fm}.$$
Contrarian view

• Can also obtain “low” values of $R_E$ from $e-p$ scattering data

• Lorenz, Meißner, Hammer, & Dong (2015), used dispersive ideas to obtain their fit functions, and also used timelike data, and obtained
  $R_E = 0.840(15)$ fm.

• Griffioen, Maddox, and me (coming) believe that one should be able to obtain accurate $R_E$ from just lower-$Q^2$ data, finding
  $R_E = 0.840(5)$ fm.
More scattering coming

• Further experiments lower lowest $Q^2$, and will do $\mu$ scattering

• PRad at JLab: Just target and detector screen, allowing very small scattering angles. Anticipate $Q^2|_{\text{low}} \approx 0.0002 \text{ GeV}^2$. Hope running soon.

• ISR (Initial State Radiation) at Mainz. Photon radiation takes energy out of electron, allowing lower $Q$ at given scattering angle. Anticipate $Q^2|_{\text{low}} \approx 0.0001 \text{ GeV}^2$. Data for preliminary experiment taken; under analysis; will obtain further data

• MUSE = Muon scattering experiment at the PSI. Anticipate $Q^2|_{\text{low}} \approx 0.002 \text{ GeV}^2$. Production runs 2017/2018.
Back to atomic spectroscopy

• Same plot, but $\mu$-H value added

• Possible: correlated systematic errors. There are more measurements than independent expt’l groups.
Short term future

- Independent groups are doing more precise experiments that will individually get the proton radius to under 1%.

- York University (Canada): Ordinary hydrogen $2S$-$2P$ Lamb shift

- MPI Quantum Optics (Garching): $2S$-$4P$ transition

- Laboratoire Kastler Brossel (Paris): $1S$-$3S$ transition

- Under way, may see results soon. (All had hoped for delivery before end of 2014.)
Exotic possibilities

• Breakdown of Lorentz invariance? (Gomes, Kostelecky, & Vargas, 2014)

• Unanticipated QCD corrections? (G. Miller, 2013)

• Will consider breakdown of muon-electron universality. New particle coupling to muons and protons. Small or no coupling to other particles.


• References (less positive): Barger, Chiang, Keung, Marfatia (2011, 2012), Karshenboim, McKeen, & Pospelov (2014)
$\mu$-H Lamb shift

- Point: Experimenters do not directly measure proton radius. Measure energy deficit, 320 $\mu$eV. Interpret as proton radius deficit.


- New particle is scalar or vector. Pseudoscalar or axial vector have little effect on Lamb shift for similar couplings.
Energy shift

• e.g., scalar case

\[ V(r) = -\frac{C_S^\mu C_S^p}{4\pi r} e^{-Mr} \]

• Pick \( C_S^\mu C_S^p \) to give 320 \( \mu eV \) for given \( m_\phi \).

(Plot for \( C_S^\mu = C_S^p \).)
Other muon processes

• Worry about other processes where new particle couples to muons. *First:*

  • Loop corrections to $\mu$ magnetic moment

  • (Reminder: 3 discrepancy between measured and standard model calculated $(g-2)_\mu$. But only at ppm level.)

  • If new exchange particle light, effect on $(g-2)_\mu$ small enough (Tucker-Smith & Yavin). Otherwise, need to fix adding second new particle and fine tuning. Well understood: Batell, McKeen, Pospelov, or Rislow and me. Couplings still fixed.
New force seen elsewhere?

- Recent suggestion: $\mu$-$p$ scattering at JLab or Mainz a.k.a., lepton pair photoproduction, $\gamma \ p \rightarrow \ell^+ \ \ell^- \ p$.
  (Pauk & Vanderhaeghen, 2015)

- Extra force, even coupling only to $\mu$ and $p$ will affect muons production. Get normalization by comparing $\mu^+\mu^-$ to $e^+e^-$ production.

- Believe 2% measurement will show effect of extra force consistent with proton radius conflict.
\[ \gamma p \rightarrow \ell^+ \ell^- p \]

- Gap between lines corresponds to difference in \( G_{Ep} \) suggested by electron- and muon-measured charge radii at \( Q^2 \) of 0.02 GeV^2.

- Contribution from timelike Compton process small at this kinematics
Helium Lamb shift

- $^3$He & $^4$He give non-contradictory results.

- He radii measured in electron scattering, to about 1/4%. These radii go into prediction for Lamb shift.

- Preliminary data on $\mu$-He Lamb shift agrees with prediction, to about 1\(\sigma\). If due to heavy BSM particle exchange, should disagree by about 5\(\sigma\).

- Mass problem!

- How does mass creep in?
Heavy atom Lamb shift

- Physics: Range of potential is controlled by mass. Light mass, long range, like Coulomb potential, does not split S and P states.

- Application: Z=2 helium has orbital muons closer to nucleus than Z=1 hydrogen. What looks like long range to helium is short range to hydrogen, if mass chosen correctly.

- Quick bottom line: Get result for proton big enough and for He small enough if $m\phi \approx 1$ MeV.
High energy decay problem

- New particle coupling to $\mu$ gives radiative correction to $W$ decay: $W \rightarrow \mu \nu V$ or $W \rightarrow \mu \nu \phi$.
- Given mass, couplings known. Hence calculate.
- Ugh: Result larger than uncertainty in width of $W$
- Plot from Karshenboim et al.,
Hope

• But new particle not yet part of complete renormalizable model.

• Analog from past: with just massive vector boson, \( \nu \bar{\nu} \rightarrow WW \) has excessive high energy growth

• Fixed by complete (Weinberg-Salam) theory

• Similar here: needs further Weinberg-Salam-like \( W \)
Lots of new data coming

- New CREMA measurements (out at conferences, 2013/14)
- MUSE (2017/2018)
- PRad (run 2015)
- ISR form factor meas. (data taken)
- Electron deuteron scattering (Griffioen et al., Mainz) (data taken)

- High precision Lamb shift in e-H (York, 2014)
- $2P-4P$ e-H splitting at Garching (2014)
- $1S-3S$ e-H splitting at LKB, Paris (2014)
- TREK at JPARC
- Alternative measurements of the Rydberg (NIST, 2018)
- Trumuonium ($\mu^+\mu^-$) at JLab
5 years after the first announcement, the problem persists.

Interestingly little discussion of the correctness of the $\mu$-H Lamb shift data.

Extrapolations that obtain the charge radius from scattering data are unsettled. Theorists should settle. Additionally, more data coming.

Serious new Hydrogen energy level splitting experiments are in progress.

Exotic or beyond the standard model explanations face serious constraints, particularly mass contains. But windows are still open.

One impact: the theory for $(g-2)_\mu$ cannot be considered settled until the proton radius problem is settled. Further, there may be striking corrections to other processes that involve muons.

The end—for this talk!
Extras
New force seen elsewhere?

- Older suggestion: correction to $K$-decay, *viz.*, $K \rightarrow \mu \nu e^+e^-$ as correction to $K \rightarrow \mu \nu$.

- Of course, QED gives same final state, with smooth (calculable) spectrum of $e^+e^-$. 
A’ visible?

• A’ (name of new particle here) will give bump. Size calculable.

• Is it observable? Wow, Yes. (If it exists.)

• Note: TREK experiment (E36) at JPARC (Japan) will observe $10^{10}$ kaon decays, or about 200,000 $K \rightarrow \mu \nu e^+e^-$ events, about 1000 per MeV bin in the mass range we are considering. (Thanks to M. Kohl)

Plots from Rislow and me (2014)
Fixing $(g-2)_\mu$

- Will need extra particle and fine tuning
  - Lucky break: corrections to $(g-2)$ from regular vector and axial vector have opposite sign. Same is true of scalar and pseudoscalar.
- With extra particle, have new coupling, say $C_P^j$. Choose coupling to cancel in $(g-2)_\mu$. Does not much affect Lamb shift.
- Couplings now fixed, albeit mass sensitive. Hence predictions for other processes fixed.
Mass problems

- Recall measured HFS of 2S state of $\mu$-H measured in agreement with standard theory. Data number was,
  \[ \Delta E_{\text{HFS}}^{\text{exp}} = 22.8089(51) \text{ meV} \]
- Any HFS from exotics must be small, say below 5.1 $\mu$eV.
- Axial vector—if needed for fine tuning (g-2)—now a problem. Gives contribution to HFS in leading order in NR expansion.
- Straightforward atomic physics calculation
HFS and mass limits

• Conclude: HFS agreement with SM calculation compatible with new particle exchange if new particle light enough.

• Axial case o.k. if mass below about 13 MeV.

• Analogous pseudoscalar/scalar case has mass limit 35 MeV.

See Marfatia & Keung (2015), Carlson & Rislow (in prep.)
Numbers note

• Take 1S-3S as example (the LKB measurement)

• splitting about $2.9 \times 10^{12}$ kHz

• difference due to CODATA vs. $\mu$-H proton radii difference about 7.2 kHz

• $\therefore$ need ppt accuracy. Wow.

• Already have (2010) measurement with 13 kHz error bar.
Lamb shift

- Measure well

- Calculate point proton QED part well

- Difference is due to proton size

- Need to know Rydberg well enough. Do know it.

- Get proton radius to few %.
Details

• Splittings between different principal QN (e.g., $2S-3P$)
  • finite size term about $10^{-10}$ of Ryd term
  • need to know Rydberg very well. Don’t.
  • no problem: Take pairs of splittings instead

• Splittings within one energy level (i.e., Lamb shift),
  finite size term about $10^{-6}$ of Lamb shift, no problem
  with knowing value of Rydberg accurately enough.
Electron scattering data

• Mainz 2010 measures differential cross section, has 1422 data points, about 0.3% relative error, about or below 2% absolute error.

• Want slope of $G_E$ at $Q^2 = 0$. Cannot measure to $Q^2 = 0$, so extrapolate.

• Mainz data has $0.004 < Q^2 < 1$ GeV$^2$. 
Mainz’s own fit

- The experimenters fit $G_E$ and $G_M$ to their data using polynomials or modified polynomials in $Q^2$.

- Results have small error limits compared to other data.

- Extrapolation to $Q^2 = 0$ gave “big” result quoted already.
On the other hand

- There is reason to believe polynomial expansion don’t converge for $Q^2$ beyond $4m_{\pi}^2 \approx 0.08 \text{ GeV}^2$.
- Lorenz and Meissner did a conformal transformation to a new variable in terms of which a polynomial expansion would be convergent.
- They fit the Mainz data and got
  \[ R_E = 0.84(1) \text{ fm} \]
- hmm
MUSE

- Muon scattering experiment at the PSI.
- Proton radius measurement table

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<th>atomic spectroscopy</th>
<th>scattering</th>
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<tr>
<td>electron</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>muon</td>
<td>yes</td>
<td>no</td>
</tr>
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MUSE will fill in table. Anticipate $Q^2|_{\text{low}} \approx 0.002$ GeV$^2$. Production runs 2017/2018.
But still

- Hill and Paz also did a fit over a wide range of $Q^2$ using the variable that should allow convergence.

$$R_E = 0.870(23)(12) \text{ fm}$$

- But they did not use the Mainz 2010 data, only a collection of older data.
And then there is

- A fit using only low $Q^2$ data, where convergence of a polynomial expansion should not be a problem.

- Low $Q^2$, but still a long enough range to well determine the charge radius upon extrapolation.

$$R_E = 0.840(5) \text{ fm}$$

- Local product: Griffioen, Maddox, me.

- Conclusion: a bit up in the air.