Studies of the K_{e4} decay at NA48/2

Michal Zamkovský on behalf of NA48/2 collaboration

Charles University in Prague

June 29, 2015

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Ø NA48 detectors

- Physics motivation
- Ourrent experimental & theoretical situation
- Ø Event selection
- Form factor measurement
- Ø Branching ratio measurement
- Ø Summary

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Outline

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Ø Detector performances and resolutions:

DCH
$$\sigma_x = \sigma_y = 90 \mu m$$

 $\sigma_p/p = (1.02 \oplus 0.044 \cdot p)\% p \text{ in GeV/c}$
HOD $\sigma_t \sim 150 \text{ ps}$
LKr $\sigma_E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\% E \text{ in GeV}$
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Wire chamber 4
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Wire chamber 7
Wire chamber 4
Wire chamber 2
Superimposed beam axes
within 1 mm

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Wire chamber 4
Wire chamber 4
Wire chamber 3
Wire chamber 2
• Focused at DCH1 with
~ 10 mm transverse size
• Superimposed beam axes
within 1 mm

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Kinematics

Ø Mandelstam variables:

$$s = (p_{\pi_1} + p_{\pi_2})^2 = (k - L)^2,$$

 $t = (p_{\pi_2} + L)^2 = (k - p_{\pi_1})^2, \quad u = (p_{\pi_1} + L)^2 = (k - p_{\pi_2})^2$
 $L = p_l + p_{\nu}, \quad P = p_{\pi_1} + p_{\pi_2}, \quad Q = p_{\pi_1} - p_{\pi_2}$

Winematic constrain: $s + t + u = m_K^2 + 2m_\pi^2 + L^2$

O Cabibbo-Maksymowicz formulation: $S_{\pi} = M_{\pi\pi}^2$, $S_e = M_{e\nu}^2$ and three angles: θ_{π} , θ_e , ϕ



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Cabibbo-Maksymowicz approach

 Differential decay rate summed over lepton spins in Cabibbo-Maksymowicz formulation:

$$d^{5}\Gamma = \frac{G_{F}^{2}|V_{us}|^{2}}{2(4\pi)^{6}m_{K}^{5}}\rho(S_{\pi},S_{e})J_{5}(S_{\pi},S_{e},\cos\theta_{\pi},\cos\theta_{e},\phi)$$
$$\times dS_{\pi}dS_{e}d\cos\theta_{\pi}d\cos\theta_{e}d\phi,$$

In Function J₅ depends on complex hadronic form factors:

$$F_{1} = \frac{1}{2} \lambda^{1/2} (m_{K}^{2}, S_{\pi}, S_{e}) \cdot F + \sigma(S_{\pi}) (PL) \cos \theta_{\pi} \cdot G,$$

$$F_{2} = \sigma(S_{\pi}) (S_{\pi}S_{l})^{1/2} G,$$

$$F_{3} = \sigma(S_{\pi}) (S_{\pi}S_{l})^{1/2} \frac{H}{m_{K}^{2}},$$

$$F_{4} = -(PL)F - S_{l}R - \sigma(S_{\pi}) \frac{1}{2} \lambda^{1/2} (m_{K}^{2}, S_{\pi}, S_{e}) \cos \theta_{\pi} \cdot G$$

In K_{e4} decays m_e can be neglected \Rightarrow F_4 will not contribute as is always multiplied by m_e^2

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Ø Partial wave expansion:

$$\frac{F_1}{m_K^2} = \sum_{l=0}^{\infty} P_l(\cos \theta_{\pi}) \ F_{1,l} e^{i\delta_l}, \quad \frac{F_{2(3)}}{m_K^2} = \sum_{l=0}^{\infty} P_l'(\cos \theta_{\pi}) \ F_{2(3),l} e^{i\delta_l}$$

Expression for F, G, H form factors:

$$F = F_s e^{i\delta_{fs}} + (F_p e^{i\delta_{fg}} \cos \theta_\pi + F_d e^{i\delta_{fd}} \cos^2 \theta_\pi + \dots),$$

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O S-wave term (only term for K^{00}_{e4} mode) contribution:

$$F_{s} = \left(f_{s} + f_{s}'q^{2} + f_{s}''q^{4} + f_{e}\left(\frac{S_{e}}{4m_{\pi^{+}}^{2}}\right)\right)e^{i\delta_{0}^{0}}, \quad q = \sqrt{\frac{S_{\pi} - 4m_{\pi^{+}}^{2}}{4m_{\pi^{+}}^{2}}}$$

Integration of J_5 over $\cos \theta_{\pi}$ and ϕ :

$$J_3 = |XF_s|^2 (1 - \cos 2\theta_e) = 2|XF_s|^2 \sin^2 \theta_e,$$

Differential rate:

$$d^{3}\Gamma = \frac{G_{F}^{2}|V_{us}|^{2}}{2(4\pi)^{6}m_{K}^{5}}\rho(S_{\pi},S_{e})J_{3}(S_{\pi},S_{e},\cos\theta_{e})\times dS_{\pi}dS_{e}d\cos\theta_{e}.$$

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Ø Measurements done by other experiments

- 37 events from three different experiments: $\mathrm{BR} = (2.2\pm0.4)\cdot10^{-5}$
- 214 events from KEK E470: ${\rm BR}=(2.29\pm0.34)\cdot10^{-5}$ error dominated by systematics
- No form factor determination so far, just a relation between partial rate and a constant form factor value : $\Gamma = 0.8 |V_{us} \cdot F|^2 \cdot 10^3 s^{-1}$
- Ø Theoretical predictions for BR
 - Isospin symmetry relates more precisely measured modes and predicts:

 ${\sf F}({
m K}^{+-}_{
m /4})=1/2{\sf F}({
m K}^{0\pm}_{
m /4})+2{\sf F}({
m K}^{
m 00}_{
m /4})$

Considering the different mean lifetimes $au_{K^+}, \; au_{K^0}$, this results in:

 $\mathrm{BR}(\mathrm{K}_{\mathrm{e4}}^{+-}) - 2\mathrm{BR}(\mathrm{K}_{\mathrm{e4}}^{00}) - \frac{1}{2}\mathrm{BR}(\mathrm{K}_{\mathrm{e4}}^{0\pm})\frac{\tau_{K^{\pm}}}{\tau_{K^{0}_{1}}} = (-0.772 \pm 0.801) \cdot 10^{-2},$

where the error is dominated by ${
m K}_{e4}^{00}$

• χ PT calculations $O(p^2, p^4, p^6)$ from Bijnens Colangelo Gasser (1994) using $K_{e^4}^{+-}$ form factors from 1977 predicts: $BR(K_{e^6}^{00}) = (2.01 \pm 0.11) \cdot 10^{-5}$

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Ø Measurements done by other experiments

• 37 events from three different experiments: $\mathrm{BR} = (2.2\pm0.4)\cdot10^{-5}$

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② Theoretical predictions for BR

Isospin symmetry relates more precisely measured modes and predicts:

 $\Gamma({
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m K}^{0\pm}_{\prime 4}) + 2\Gamma({
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Considering the different mean lifetimes $au_{K^+}, \ au_{K^0}$, this results in:

 $\mathrm{BR}(\mathrm{K}^{+-}_{e4}) - 2\mathrm{BR}(\mathrm{K}^{00}_{e4}) - \frac{1}{2}\mathrm{BR}(\mathrm{K}^{0\pm}_{e4})\frac{\tau_{K^{\pm}}}{\tau_{K^{0}_{t}}} = (-0.772 \pm 0.801) \cdot 10^{-2},$

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 $\Gamma(\mathrm{K}^{+-}_{I4}) = 1/2\Gamma(\mathrm{K}^{0\pm}_{I4}) + 2\Gamma(\mathrm{K}^{00}_{I4})$

Considering the different mean lifetimes τ_{K^+} , $\tau_{K^0_I}$, this results in:

$$\mathrm{BR}(\mathrm{K}_{e4}^{+-}) - 2\mathrm{BR}(\mathrm{K}_{e4}^{00}) - \frac{1}{2}\mathrm{BR}(\mathrm{K}_{e4}^{0\pm})\frac{\tau_{K^{\pm}}}{\tau_{K_{l}^{0}}} = (-0.772 \pm 0.801) \cdot 10^{-2},$$

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- **②** Final state with one charged track and four photons from π^0 's pointing to the same vertex
- ${}^{(0)}$ Common selection for signal ${
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Particle ID against background

- **③** Fit performed in 2D plane (S_{π}, S_e) after background subtraction
- The event density in the Dalitz plot is proportional to the S-wave axial vector form factor F_s^2
- Fitting ten equal population bins for S_{π} above $4m_{\pi^+}^2$ and two bins below



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Studies of the K_{e4} decay at NA48/2

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•
$$F_s = \left(1 + aq^2 + bq^4 + c\left(\frac{S_e}{4m_{\pi^+}^2}\right)\right)$$
 $q^2 \ge 0$
• $F_s = \left(1 + d\sqrt{|q^2/(1+q^2)|} + c\left(\frac{S_e}{4m_{\pi^+}^2}\right)\right)$ $q^2 < 0$
• cusp singularity at $q^2 = (S_{\pi}/4m_{\pi^+}^2 - 1) = 0$

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only empirical parameterization

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Fitting procedure for $K^{00}_{\bullet A}$

③ cusp singularity at $q^2 = (S_{\pi}/4m_{\pi^+}^2 - 1) = 0$



•
$$d = -0.256 \pm 0.049$$

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 F_s'/f_s
 $(r_s/f_s)^2/N$
 f_s'/f_s
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 $f_{ef}'f_s$
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$(\mathrm{K}^+ \to \pi^0 \pi^0 l^{\pm} \nu) \text{ decay amplitude: } \mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1$

The unperturbed amplitude \mathcal{M}_0 corresponds to the tree level:

$$\mathscr{M}_{0} = f_{s0} \left(1 + aq^{2} + bq^{4} + c \frac{S_{e}}{4m_{\pi^{+}}^{2}} \right)$$

One loop contribution through π⁺π⁻ → π⁰π⁰ charge exchange M₁:
 conserved isospin symmetry (m_{π+} = m_{π0}):

$$\mathscr{M}_1 = -2 \frac{a_0^0 - a_2^0}{3} f_s \sqrt{\left| \frac{q^2}{1 + q^2} \right|}, \ \ q^2 = \frac{S_\pi - 4 m_{\pi^+}^2}{4 m_{\pi^+}^2}$$

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• broken isospin symmetry $(m_{\pi^+} \neq m_{\pi^0})$: More elaborated calculation

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The amplitude \mathcal{M}_1 changes from real to imaginary at $2m_{\pi^+}$ with the consequence that \mathcal{M}_1 interferes destructively with \mathcal{M}_0 in the region below $2m_{\pi^+}$ threshold, while it adds quadratically above it:

$$\begin{split} |\mathscr{M}|^2 &= |\mathscr{M}_0 + i\mathscr{M}_1|^2 = |\mathscr{M}_0|^2 + |\mathscr{M}_1|^2, & \text{above threshold} \\ |\mathscr{M}|^2 &= |\mathscr{M}_0 + \mathscr{M}_1|^2 = |\mathscr{M}_0|^2 + |\mathscr{M}_1|^2 + 2|\mathscr{M}_0||\mathscr{M}_1|, & \text{below threshold} \end{split}$$



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Related measurements on NA48 experiment



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Related measurements on NA48 experiment



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Branching ratio measurement

Ø Branching ratio measurement:

$$\begin{array}{l} \mathrm{BR}\left(\mathrm{K}_{e4}^{00}\right)=\frac{N\left(\mathrm{K}_{e4}^{00}\right)}{N\left(\mathrm{K}_{3\pi}^{00}\right)}\cdot\frac{A\left(\mathrm{K}_{3\pi}^{00}\right)}{A\left(\mathrm{K}_{e4}^{00}\right)}\cdot\frac{\epsilon\left(\mathrm{K}_{3\pi}^{00}\right)}{\epsilon\left(\mathrm{K}_{e4}^{00}\right)}\cdot\mathrm{BR}\left(\mathrm{K}_{3\pi}^{00}\right)\\ \mathrm{BR}(\mathrm{K}_{3\pi}^{00})=\left(1.761\pm0.022\right)\times10^{-2}\end{array}$$

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 ${
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m K}^{00}_{e4}) = (2.552\pm0.010_{stat}\pm0.010_{syst}\pm0.032_{ext}) imes10^{-5}$

Branching ratio measurement

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$$BR\left(K_{e4}^{00}\right) = \frac{N\left(K_{e4}^{00}\right)}{N\left(K_{3\pi}^{00}\right)} \cdot \frac{A\left(K_{3\pi}^{00}\right)}{A\left(K_{e4}^{00}\right)} \cdot \frac{\epsilon\left(K_{3\pi}^{00}\right)}{\epsilon\left(K_{e4}^{00}\right)} \cdot BR\left(K_{3\pi}^{00}\right)$$

$$BR(K_{3\pi}^{00}) = (1.761 \pm 0.022) \times 10^{-2}$$

 $BR(K_{e4}^{00}) = (2.552 \pm 0.010_{stat} \pm 0.010_{syst} \pm 0.032_{ext}) \times 10^{-5}$

Source	$\delta {\sf BR}/{\sf BR} imes 10^2$
Background and electron-ID	0.25
Radiative events modeling	0.19
Form factor uncertainty	0.17
Acceptance stability	0.16
Level 2 Trigger cut	0.04
Simulation statistics	0.07
Trigger efficiency	0.03
Total systematics	0.40
External error from $BR(K_{3\pi}^{00})$	1.25
Statistical error	0.39
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Sample of 65000 K⁰⁰_{e4} reconstructed events has been studied (JHEP 08 (2014) 159)

- First measurement of form factor F_s parametrized by described a, b, c, d constants obtained from the fit in (S_{π}, S_e) plane
- Significantly improved precision of BR measurement
- Observation of the cusp singularity which can be related to $\pi \pi$ scattering consistent with a_0^0, a_2^0 values measured in K_{e4}^{+-} mode

Prospects

NA62 is starting data taking (see Augusto Ceccucci talk in plenary session)

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- Lots of kaon decays expected in the next years (2015-2018)
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Thank you for the attention!