Effective Field Theories and Lattice QCD

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Overview

- ✦ Brief introductions to effective field theories (EFTs) and to lattice systematics.
- ✦ Uses of EFTs in lattice QCD.
  - ChPT, Symanzik effective theory, HQET.
  - Discretization errors.
  - Partial quenching.
  - Finite volume effects and twisted boundary conditions; heavy quarks; ....
- ✦ The payback: ChPT results from the lattice.
  - Mesons, mainly SU(2).
  - Nucleons (a little).
  - Preliminary results: SU(3) and 3-flavor chiral limit.

Effective Field Theories

✦ Powerful tool to describe physics in some limited range of scales.

- Useful when the fundamental theory is too difficult to handle (or unknown).
- Typically:
  1. "Integrate out" high energy modes of a theory (those above a cutoff Λ).
  2. Expand the resulting non-local theory in inverse powers of Λ times local operators (an OPE).
  3. Left with a local effective field theory (EFT) at low energy.
     - In rare cases (*e.g*. heavy quark effective theory), steps can actually be carried out (perturbatively).
     - Usually just imagine performing steps 1-3; use symmetries to constrain EFT.

Effective Field Theories

- ✦ These days, often said that all field theories are effective theories.
  - Unknown new physics must kick in at some higher scale.
  - E.g., QCD could be supplemented by higher dimension terms, such as:

$$\frac{1}{M}\,\bar{q}\,\sigma_{\mu\nu}G^{\mu\nu}q$$

    - M is mass scale of new physics.
  - Distinction between renormalizable and unrenormalizable theories is less important than we used to think.
  - Still, an important distinction:
    - If LO effective theory is nonrenormalizable (e.g. ChPT), it tells you the scale at which new physics must enter ($4\pi f_\pi$), so sets natural scale for NLO terms.
    - If the LO effective theory is renormalizable (e.g., QCD), then scale of new physics undetermined.
      - must be found/bound by experiment,
      - or by knowing/guessing the more fundamental underlying theory.

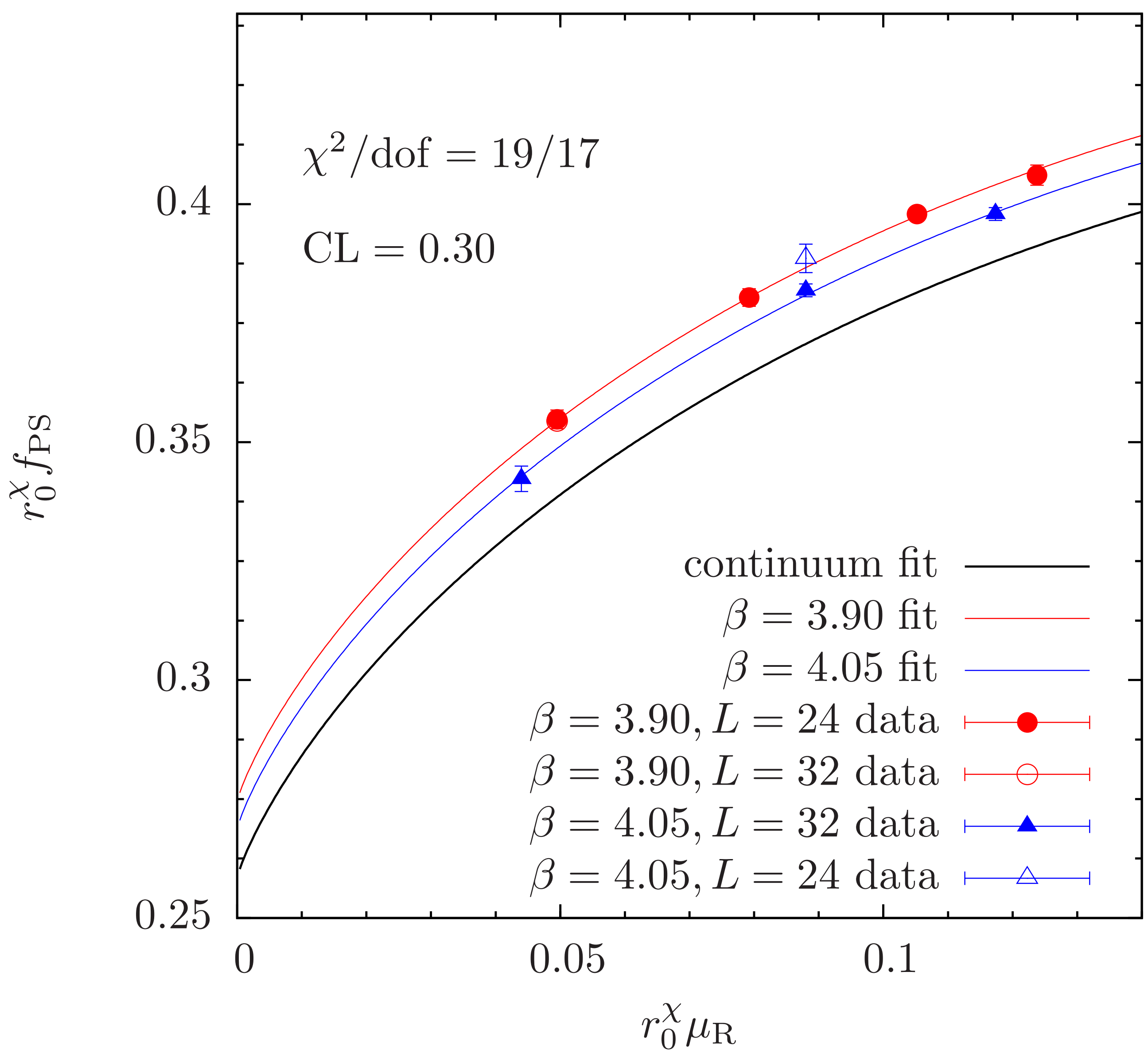
Lattice QCD Systematic Errors

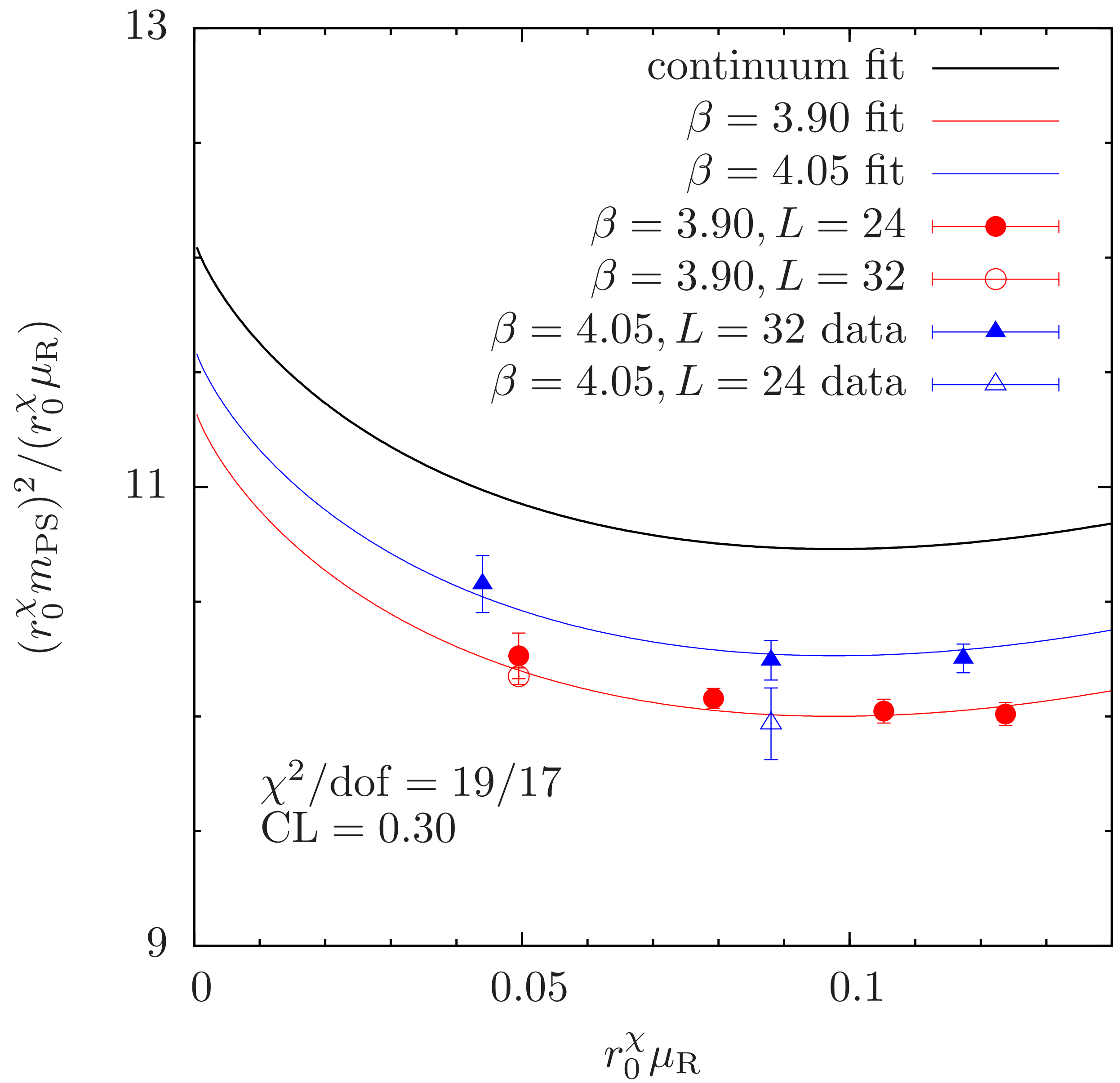
- Lattice computation of QCD path integral inherently includes systematic errors.
  - Continuum extrapolation error: need to take lattice spacing $a \to 0$.
  - (Residual) finite-volume errors: need to take space & time extent L, $T \to \infty$.
  - Chiral extrapolation error: for practical reasons may choose m_u, m_d larger than physical; need to extrapolate to physical values.
    - Even if near-physical values chosen (now possible), need to interpolate to precise physical values (can only be found *a posteriori*): chiral interpolation error.

Use of EFTs in Lattice QCD

- ✦ EFTs provide functional forms for relevant extrapolations/ interpolations.
  - thereby reduce systematic errors.
- ✦ First use: ChPT, to guide quark mass extrapolations.
  - ChPT gives functional form of expansion in quark masses (and momenta).
    - all dependence explicit.
    - exactly as needed for extrapolations.
  - Soon realized that ChPT also gives leading finite volume corrections.
    - from pions, looping around the finite volume. [Gasser & Leutwyler, 1987, 1988; Neuberger, 1988; Hasenfratz & Leutwyler, 1990].

Example of mass extrapolation





ETM Collaboration [R Baron, et al.], JHEP 1008 (2010) 097, arXiv:0911.5061

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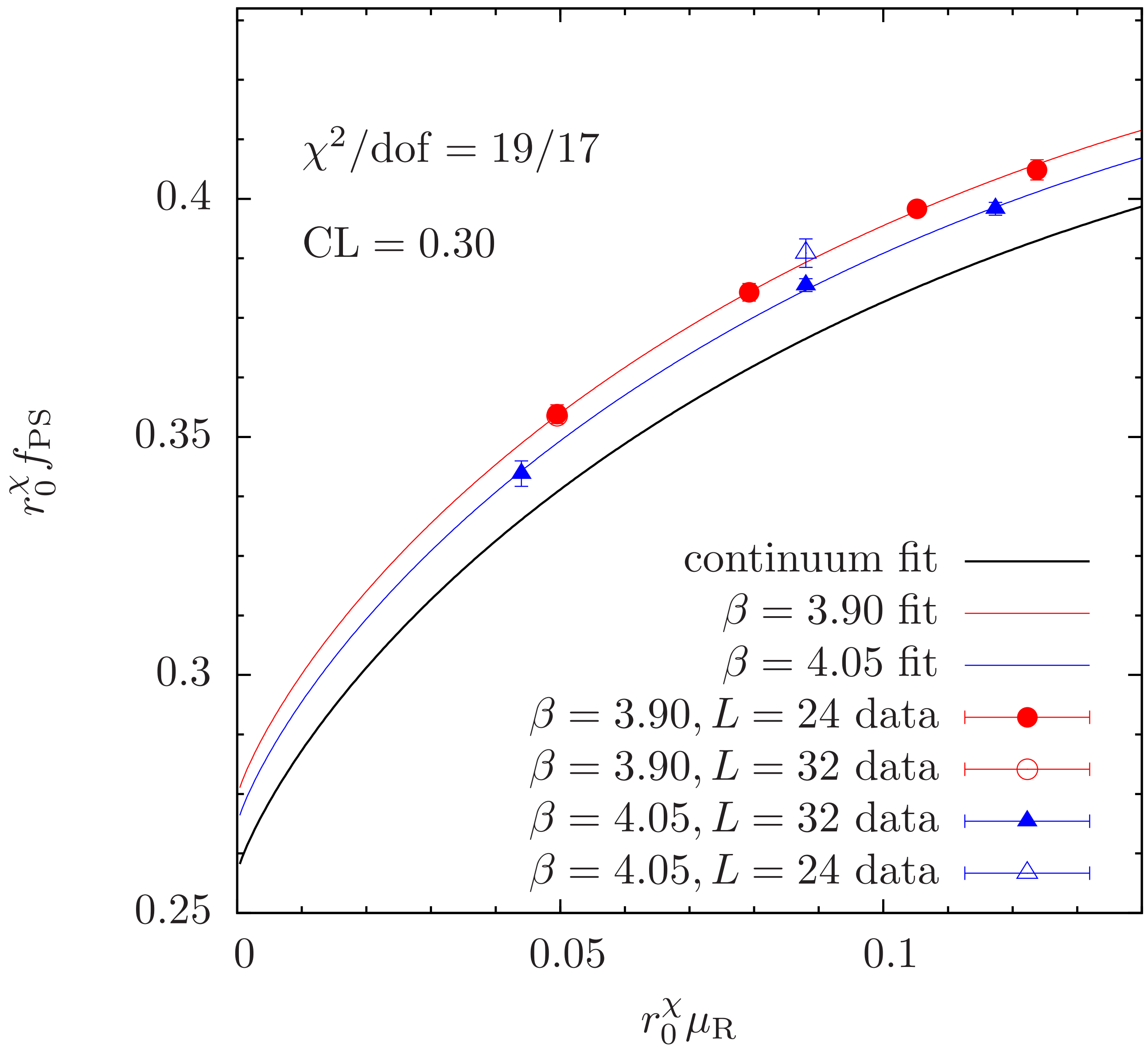

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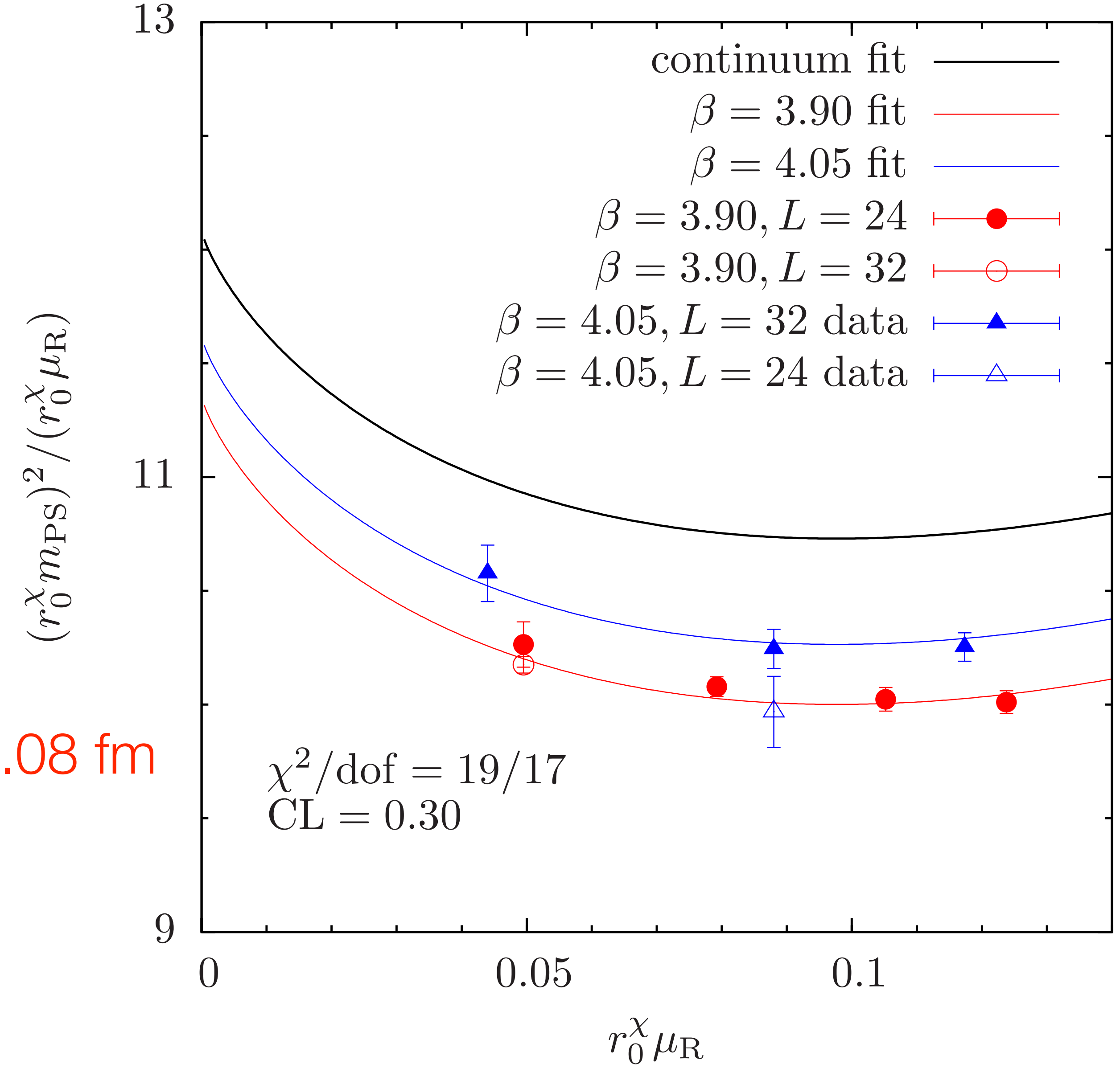

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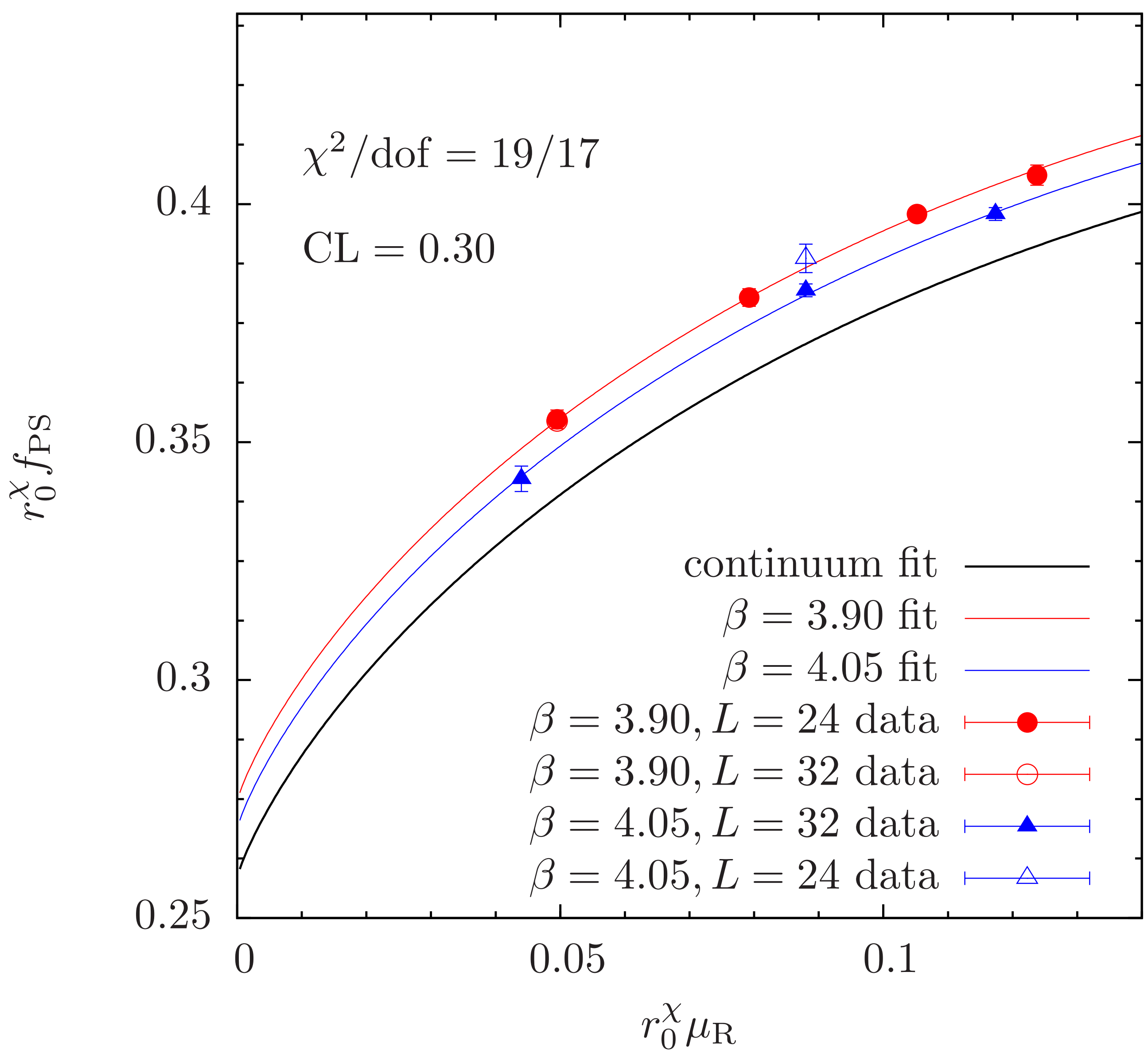

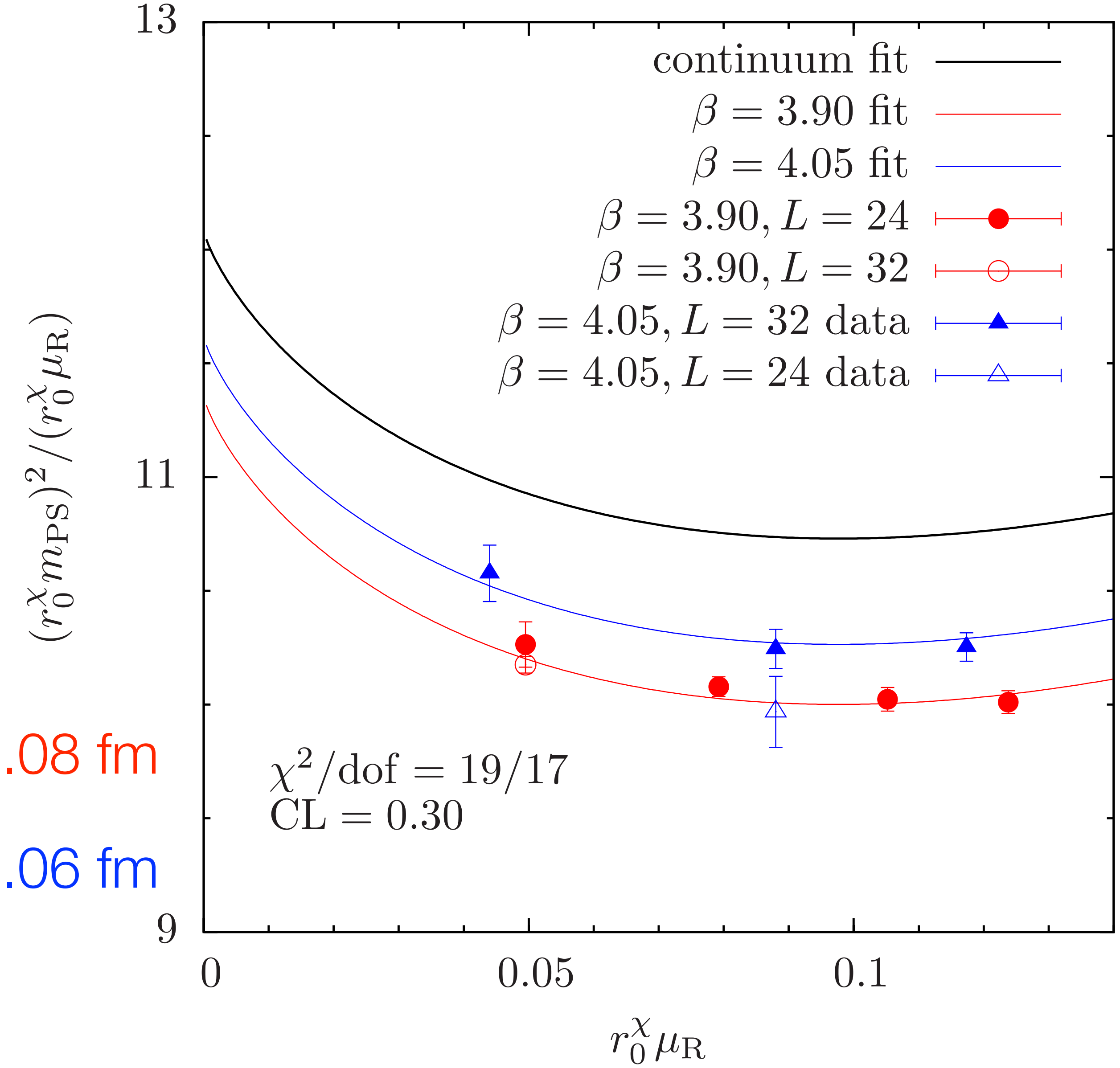

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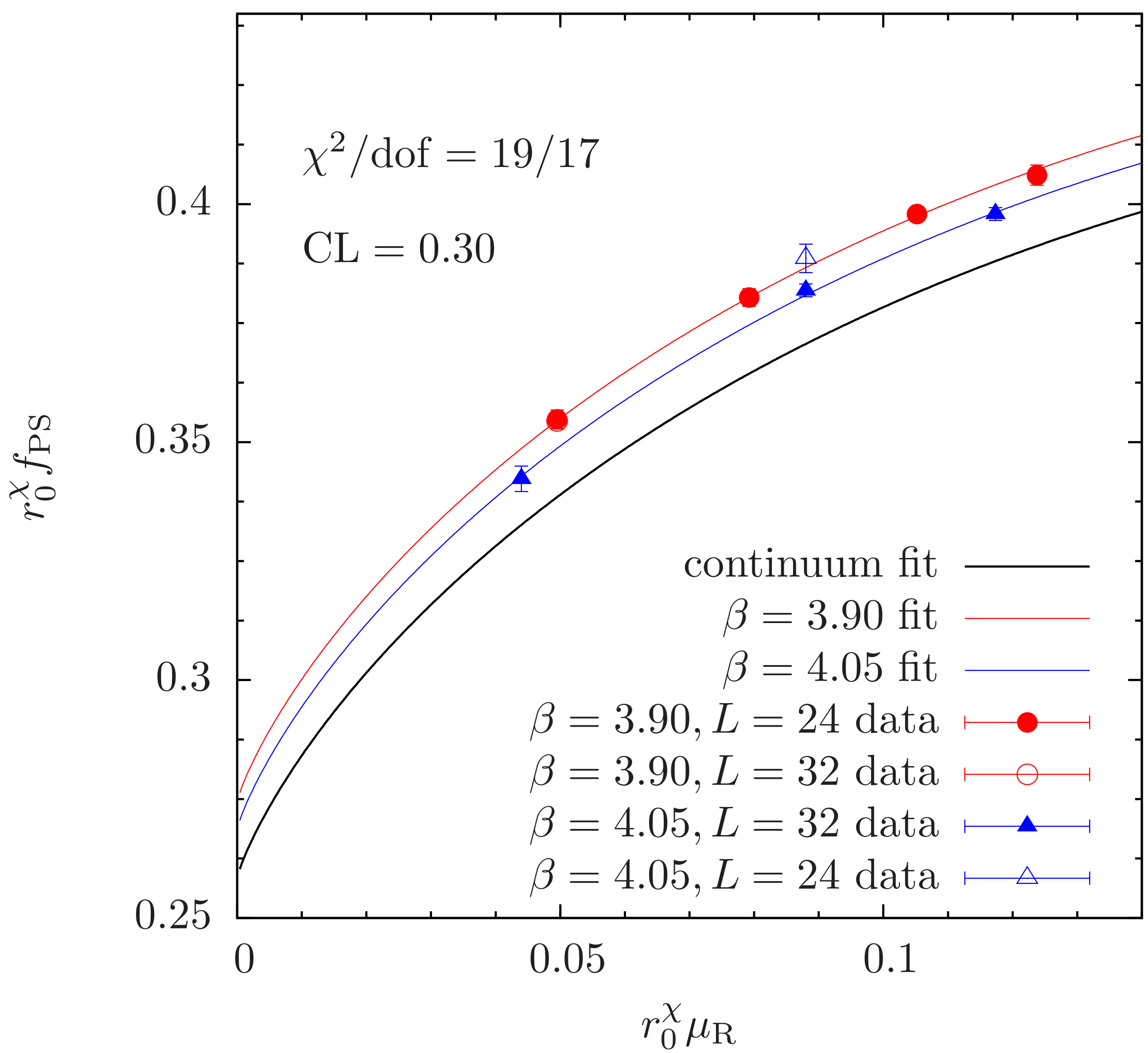

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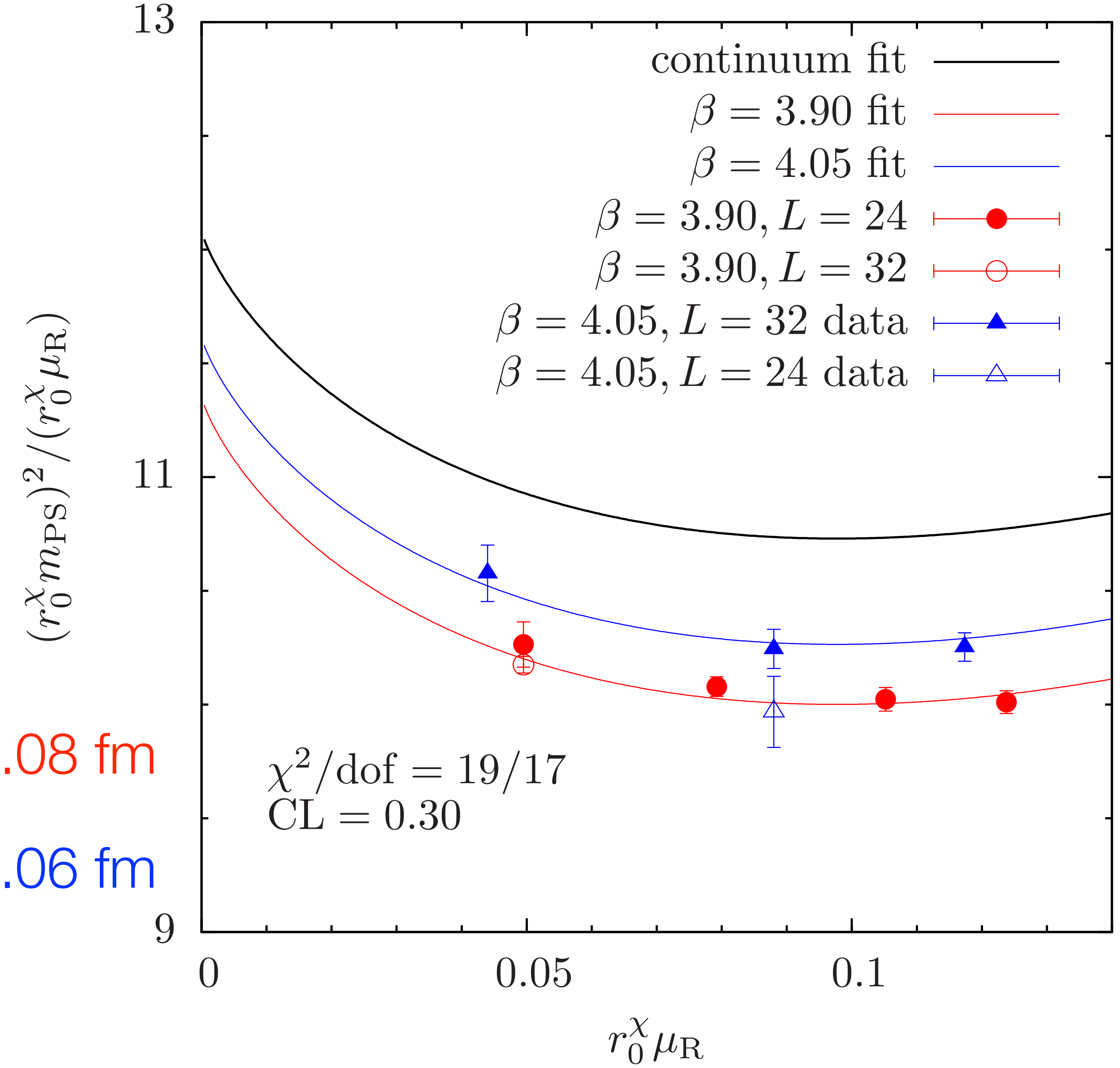

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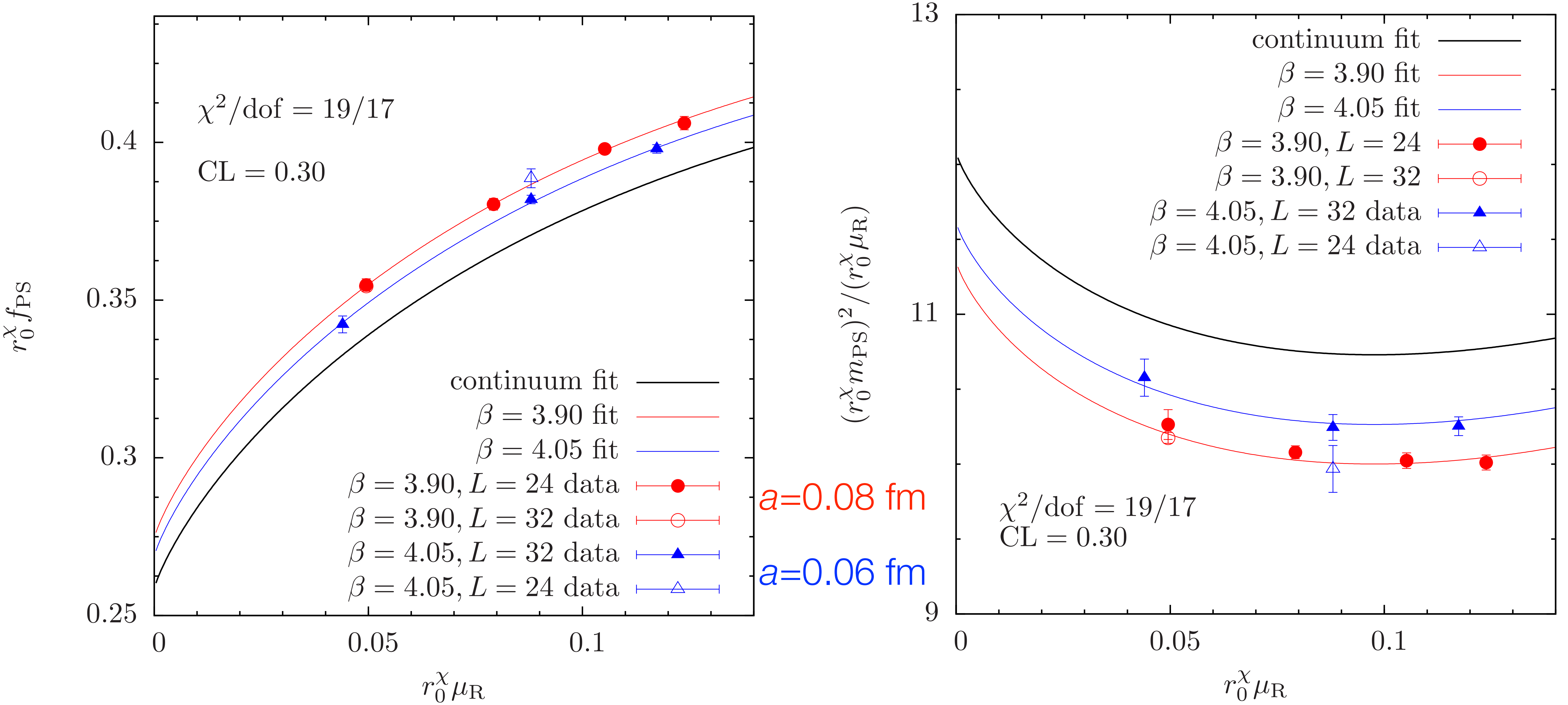

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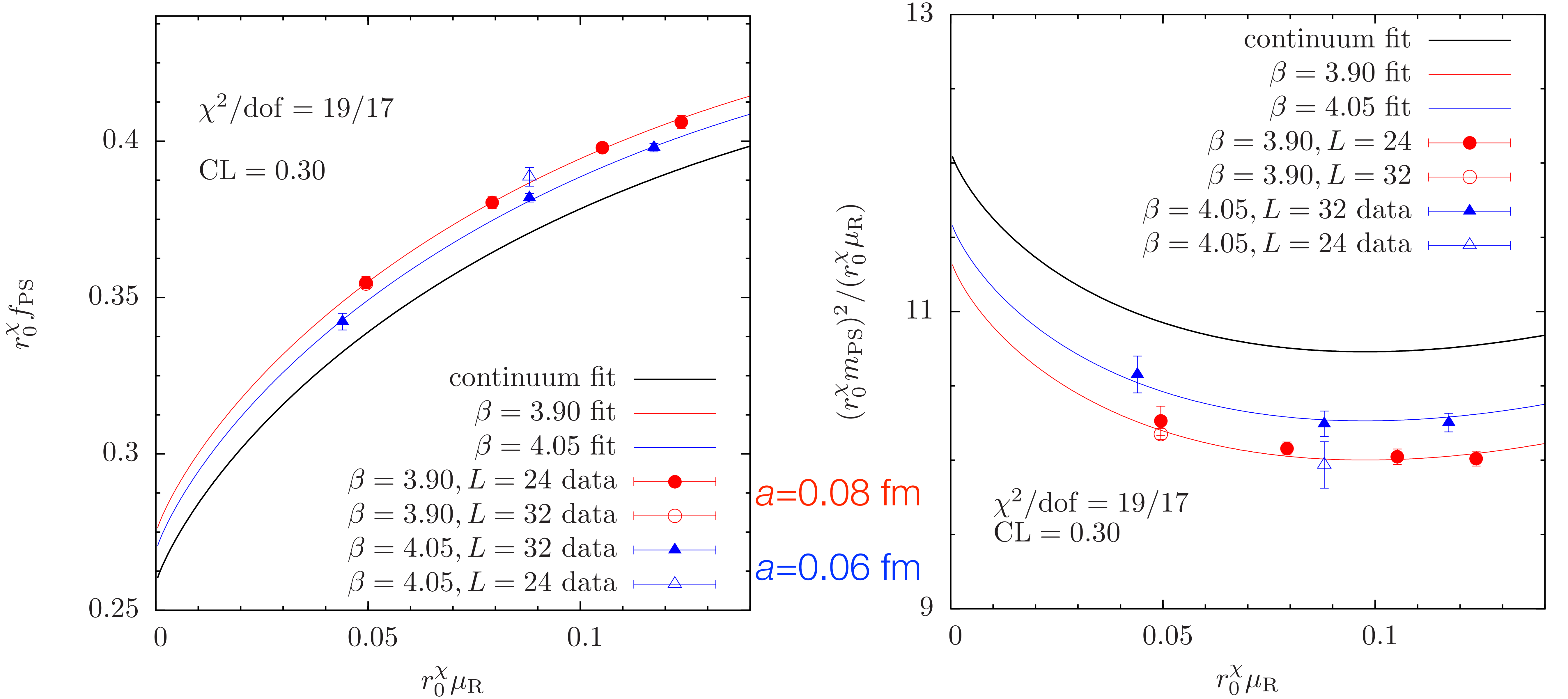

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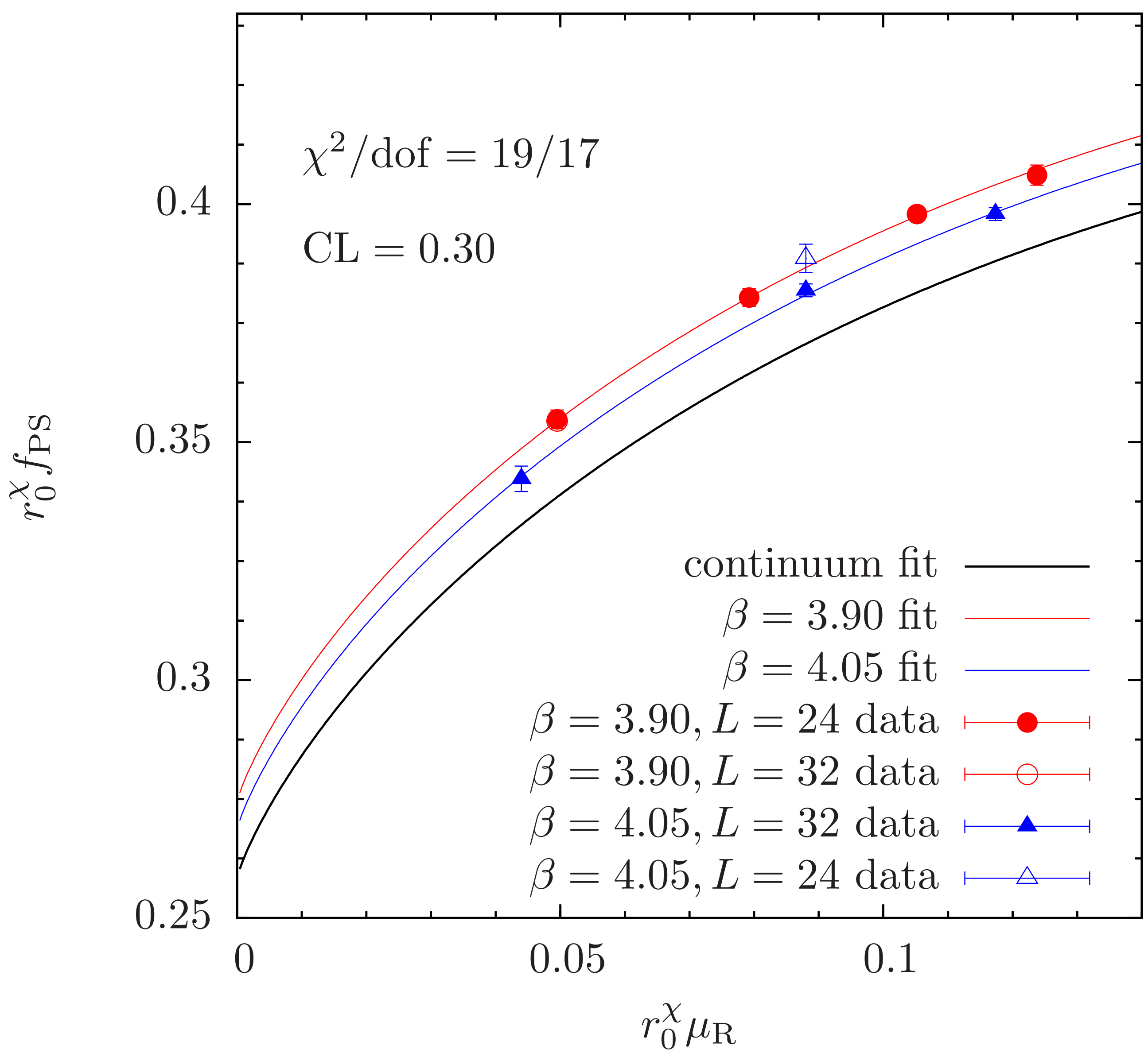

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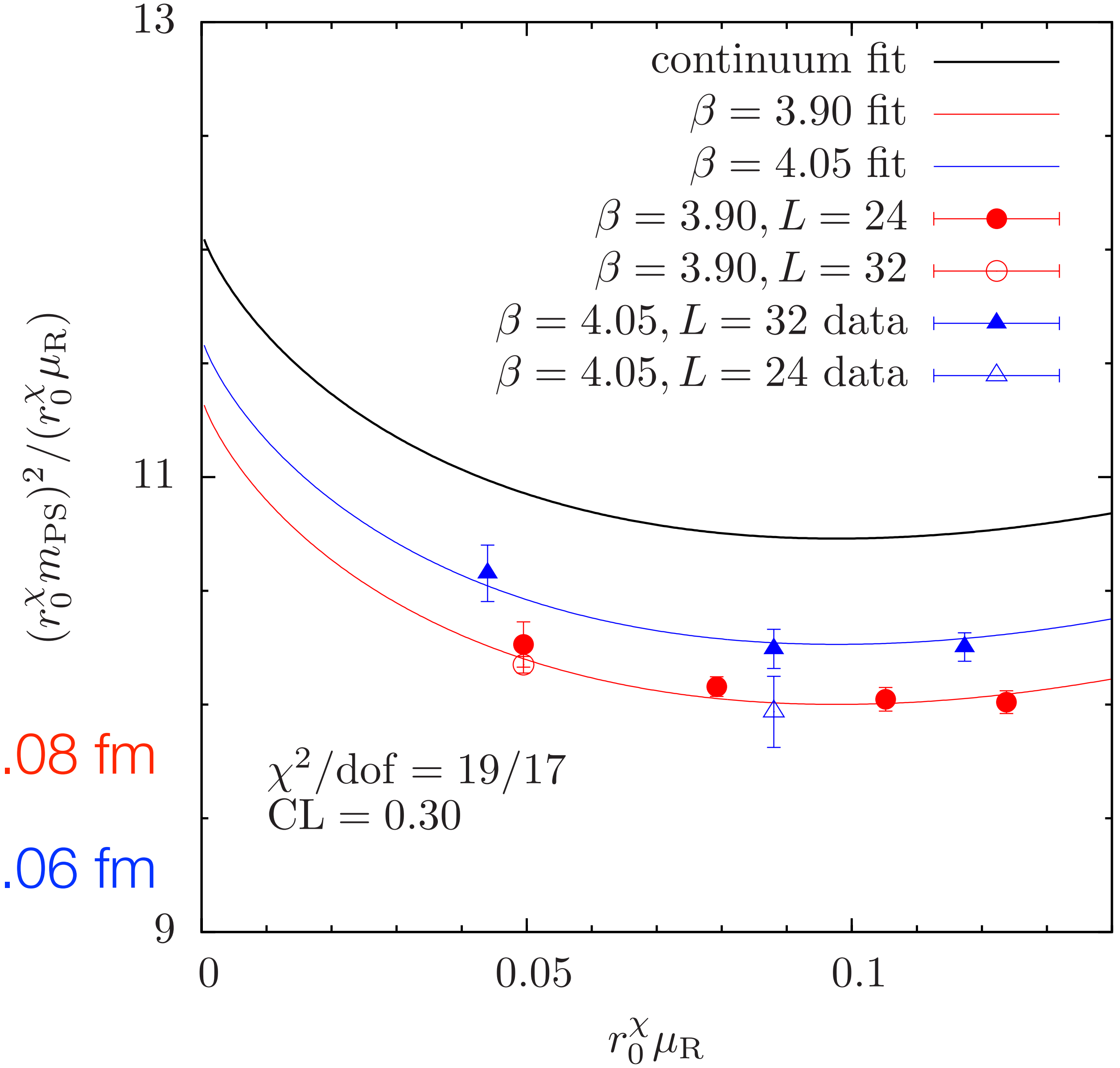

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- Here, simple analytic terms, const.× a^2, do the trick.
- In some other cases (very precise lattice data, many degrees of freedom, larger discretization errors...) this approach may not be adequate.

Use of EFTs in Lattice QCD

✦ Key insight: ChPT can be modified to include lattice discretization errors. [Sharpe & Singleton, 1998]

- Relates a-dependence to mass dependence, so better controlled extrapolations.
- Non-analytic terms in a arise from loops.
- Method uses another EFT: Symanzik Effective Theory (SET) [Symanzik, 1983].
    - For SET, the lattice QCD theory at fixed lattice spacing a is taken as "fundamental."
    - SET is the EFT that describes the lattice theory at energy scales small compared to the cutoff: $p \ll 1/a$.
    - Leading order Lagrangian is just the continuum QCD Lagrangian.
    - Since $ap \ll 1$, need to keep only low powers of a as corrections:
        - add on local operators with dimension > 4, multiplied by appropriate powers of a.
    - Needed local operators $\rightarrow$ determined by the underlying lattice symmetries.

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    - new LECs c_1 and c_2 encode leading discretization effects in ChPT.
    - Sharpe & Singleton showed from this ChPT that a new lattice-artifact phase ("Aoki phase") was possible at fixed a for very small m.

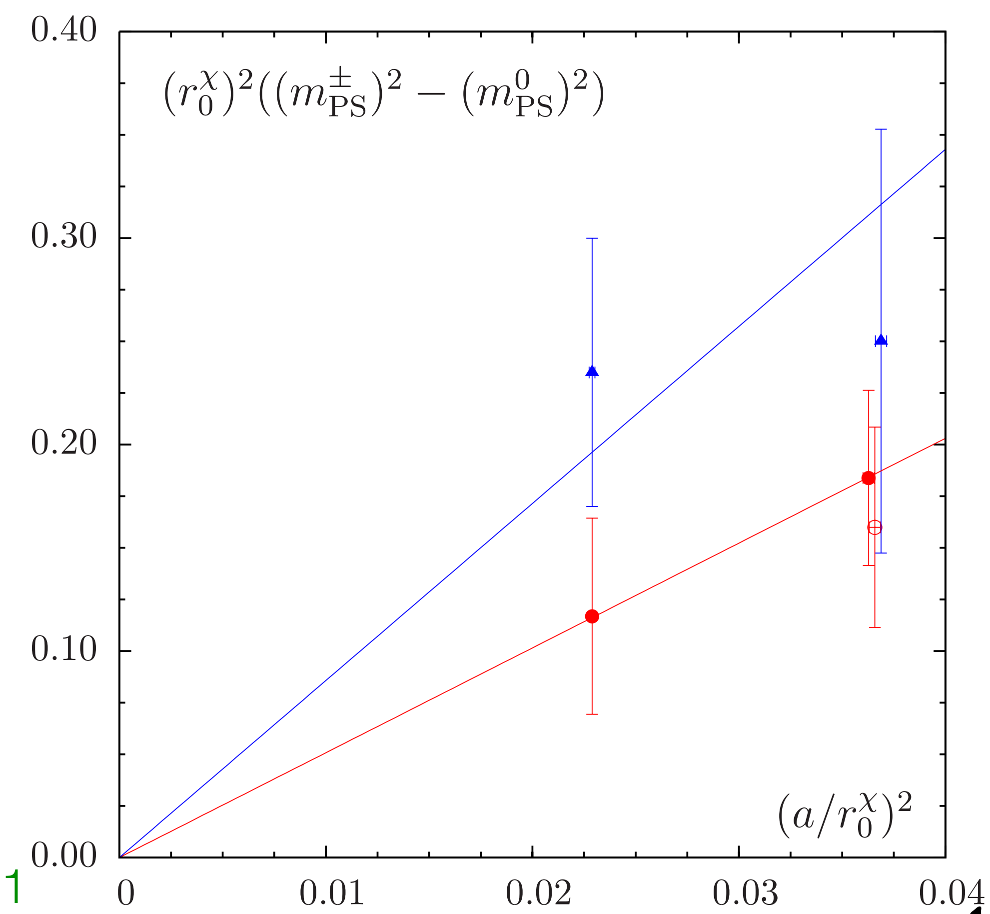
Lattice QCD: twisted-mass quarks

- Start with a doublet of Wilson quarks.
- Add a *twisted mass* [Frezzotti, Grassi, Sint & Weisz, 2001]:

$$\bar{q}\,(\not{D} + m)q \quad \rightarrow \quad \bar{q}\,(\not{D} + m + i\mu\gamma_5\tau_3)q$$

- In continuum, μ term can be rotated away by non-singlet SU(2) chiral rotation.
- But on lattice, since Wilson term (to remove doublers) is in "*m* direction", twist is nontrivial:
  - Avoid "exceptional configurations" in which statistical fluctuations from Wilson term bring mass to zero.
  - If m tuned to 0, physical quantities have errors starting at $O(a^2)$, not $O(a)$ [Frezzotti and Rossi, 2004].
  - Price is violation of isospin symmetry at nonzero *a*.
    - twisted mass ChPT [Munster, Schmidt & Scholz, 2004; Sharpe & Wu, 2004]

      $\Rightarrow$ $O(a^2)$ splitting of π^0 from π^+.



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    - where Σ is now a $4n \times 4n$ matrix.

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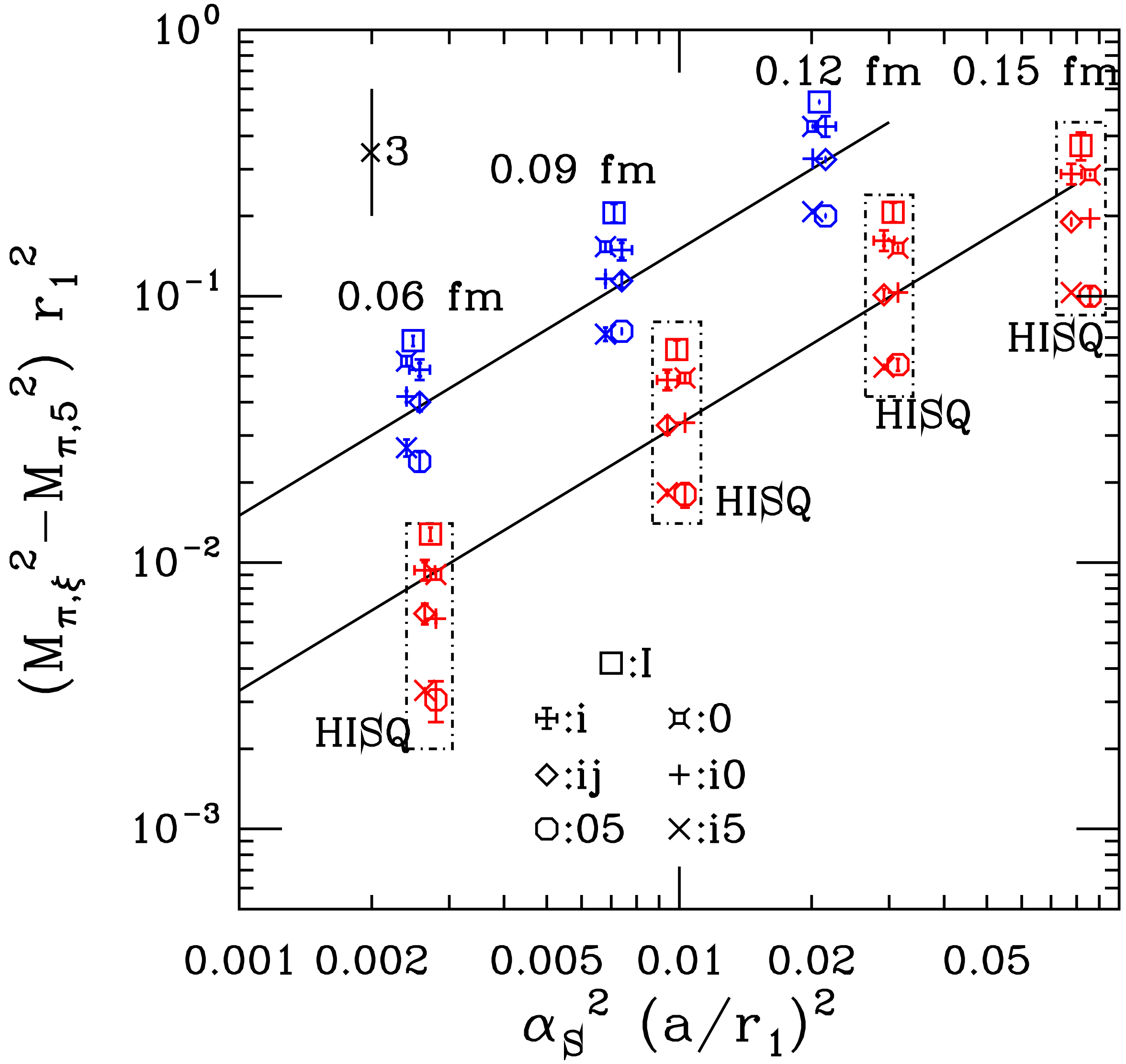

Pion taste splittings vs $(\alpha_s a)^2$ for two versions of staggered quarks: “asqtad” and newer, more highly improved version, “HISQ”.

MILC [A. Bazavov, et al.], PRD 87, 054505 (2013) [arXiv:1212.4768]

ChPT and extrapolations to continuum

- ✦ So lattice-spacing-dependent ChPT can explain/control lattice artifacts:
  - Aoki phase (Wilson)
  - pion isospin-violations (twisted mass)
  - pion taste-splittings (staggered)
- ✦ Another key use to is guide continuum extrapolations:
  - Fit quark-mass dependence and lattice-spacing dependence together, using expressions from the appropriate chpt.
  - Can significantly reduce systematic errors.
  - Such fits often done in *partially quenched* context: choose valence quarks to have different masses than sea quarks.
    - Useful because valence quarks are cheap compared to sea quarks: extract as much as possible for a given configuration (generated with sea quark back-effects).
    - "Partially quenched" because valence quarks are quenched: forbidden from appearing in virtual loops, but sea quarks are not quenched.
    - Add corresponding ghost (bosonic!) quarks, with same mass matrix as the valence quarks, to cancel the virtual loops (determinant) of the valence quarks [Morel, 1987].

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  - graded group has some Grassman generators, because some transformations take fermions into bosons, and vice-versa, as in supersymmetry.

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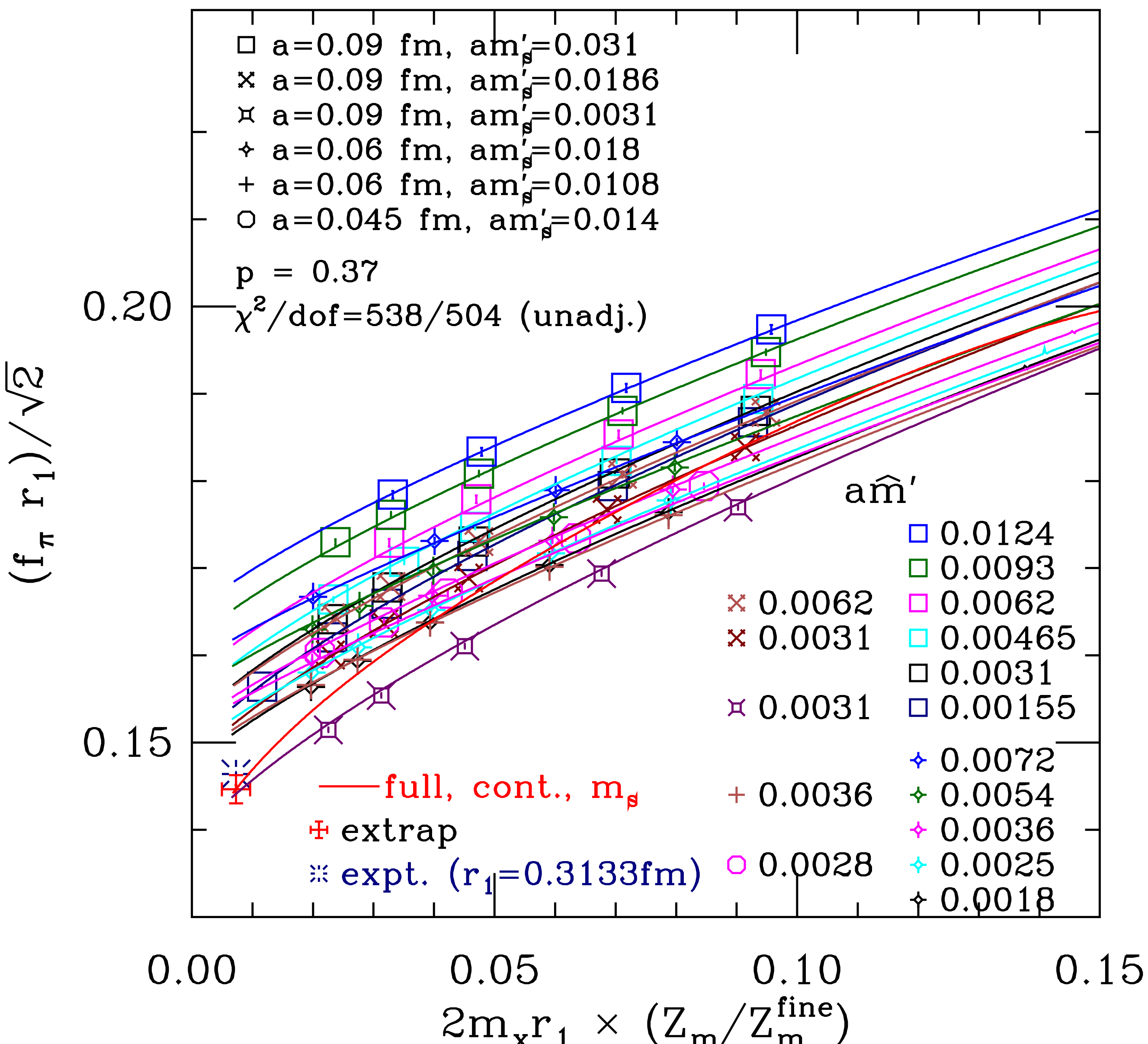

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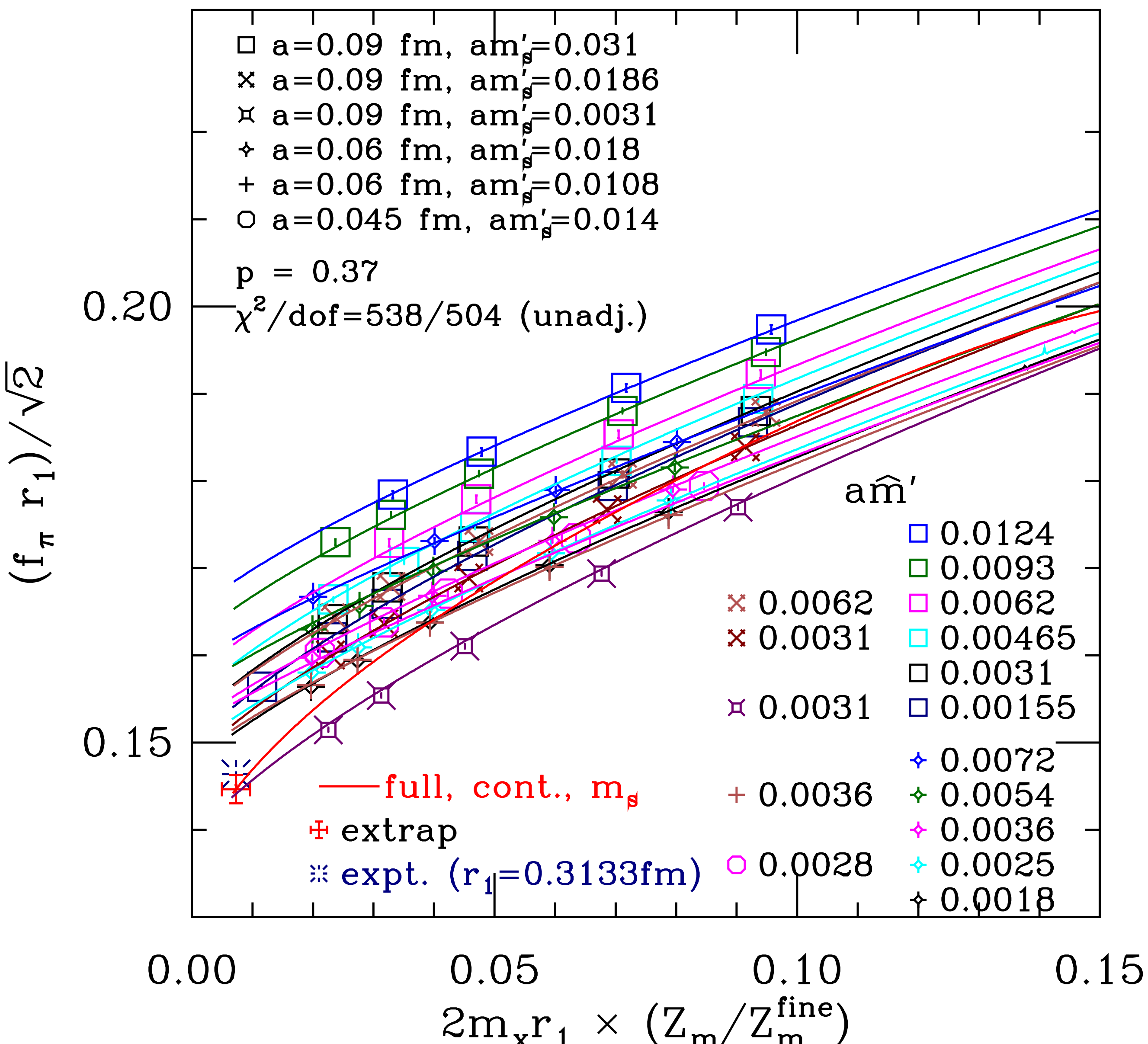

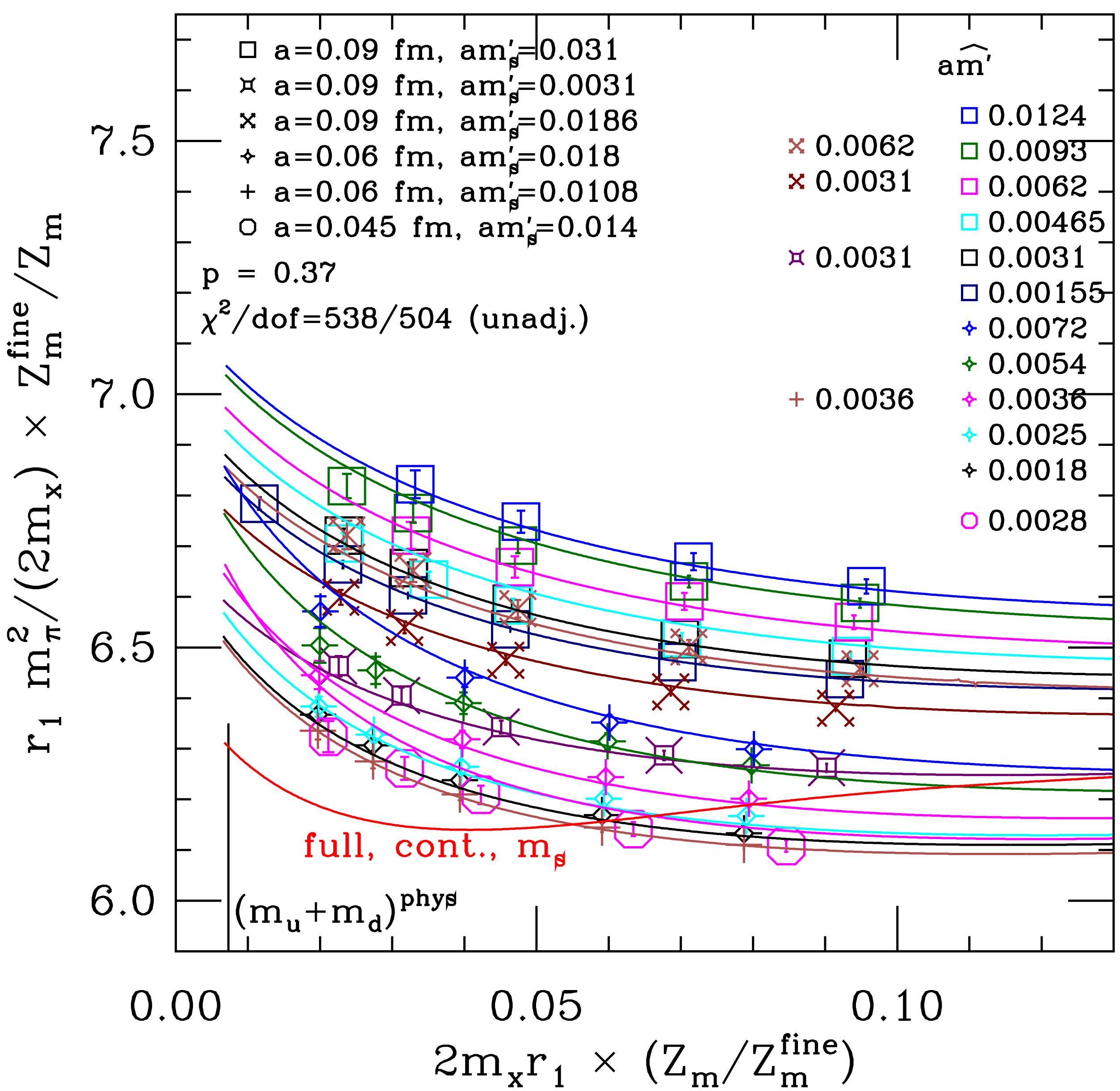

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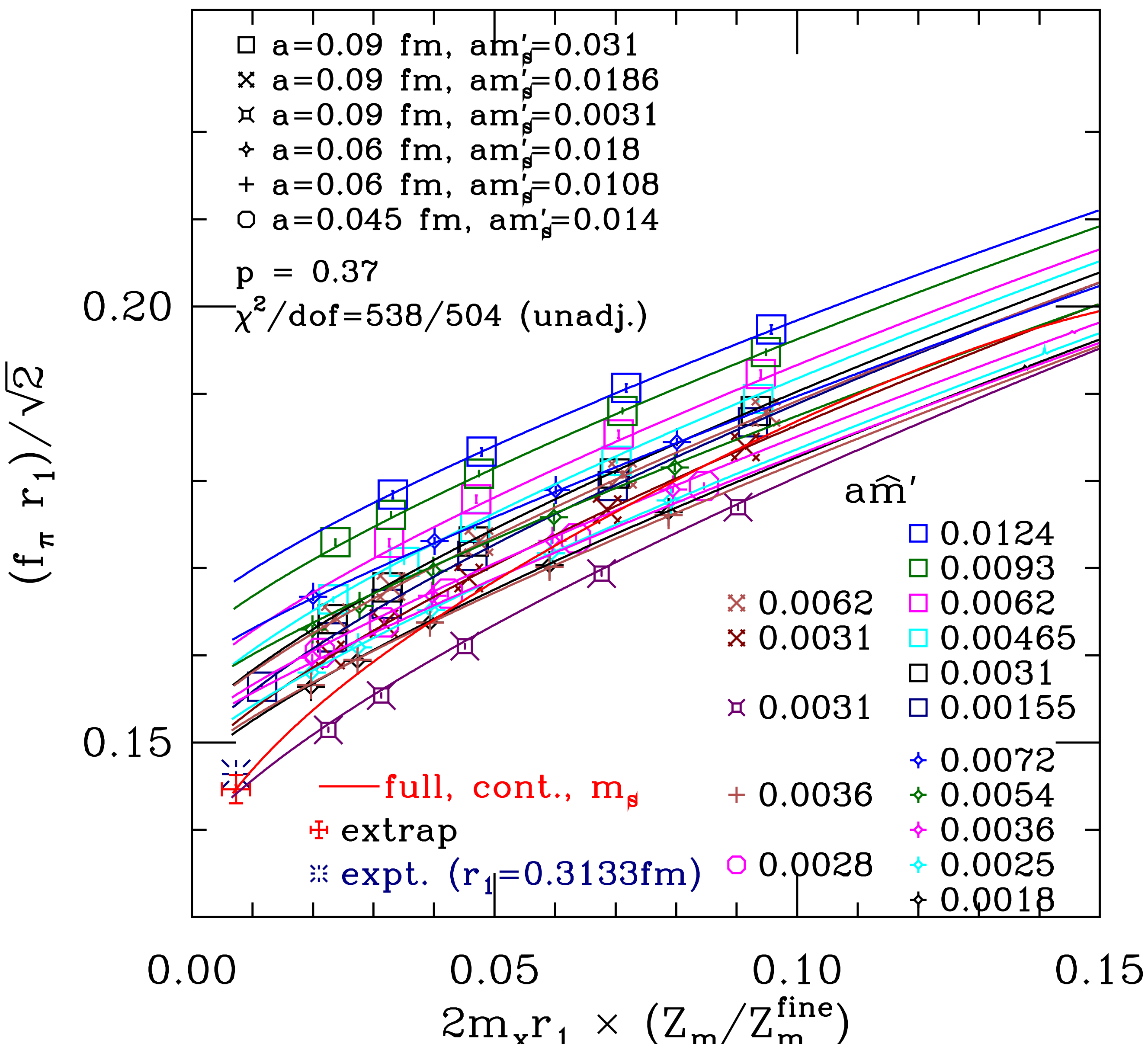

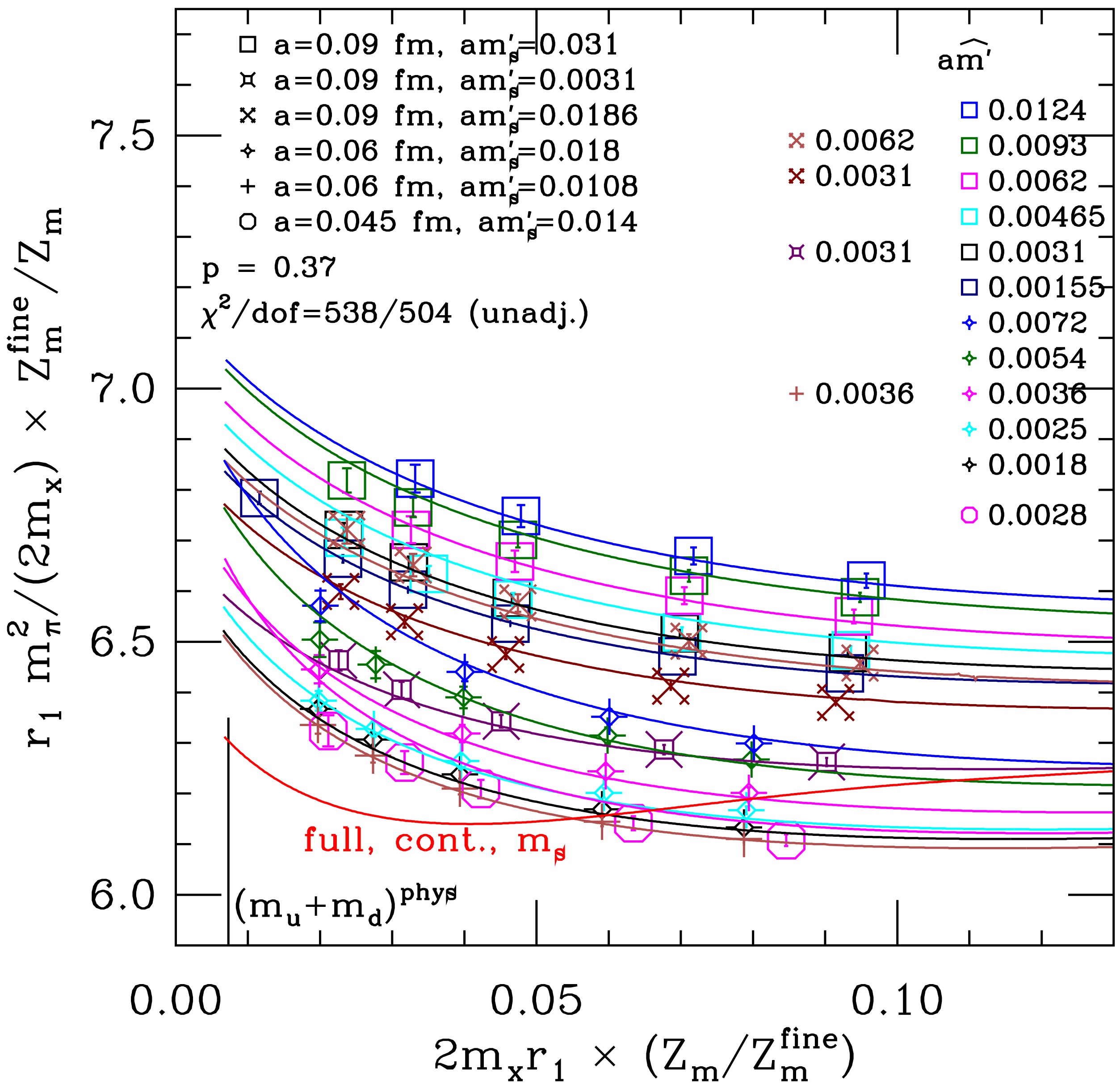

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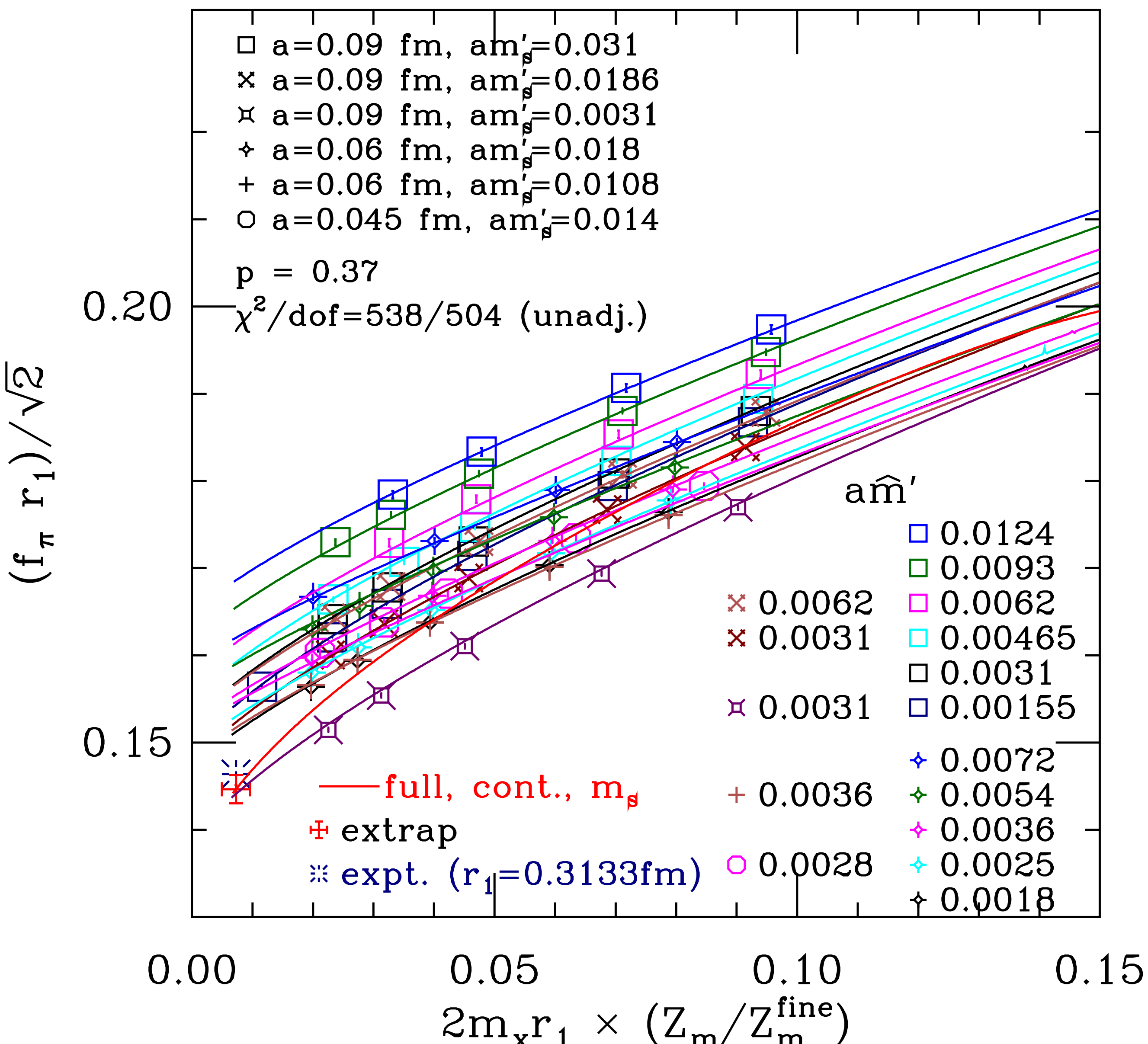

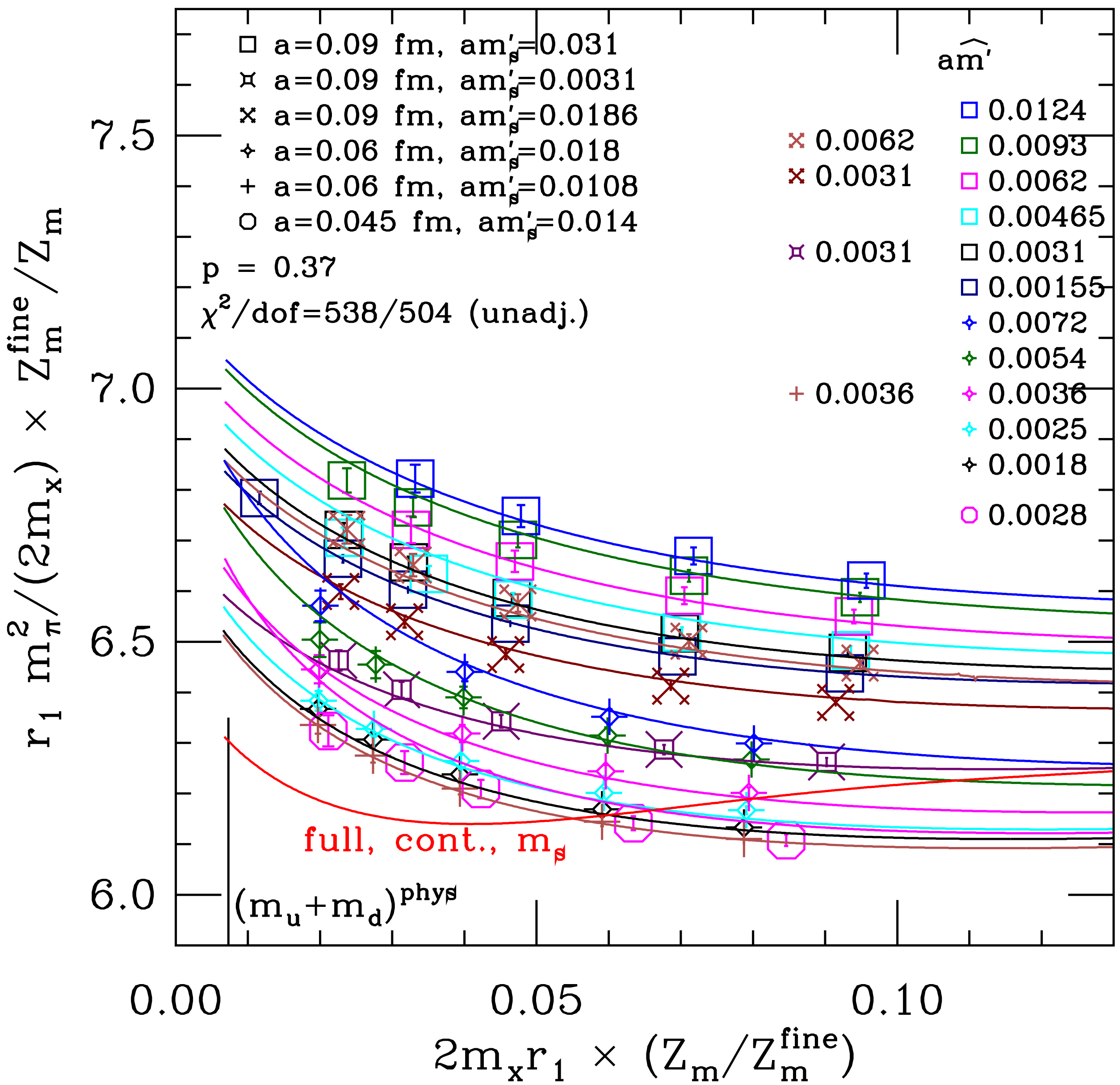

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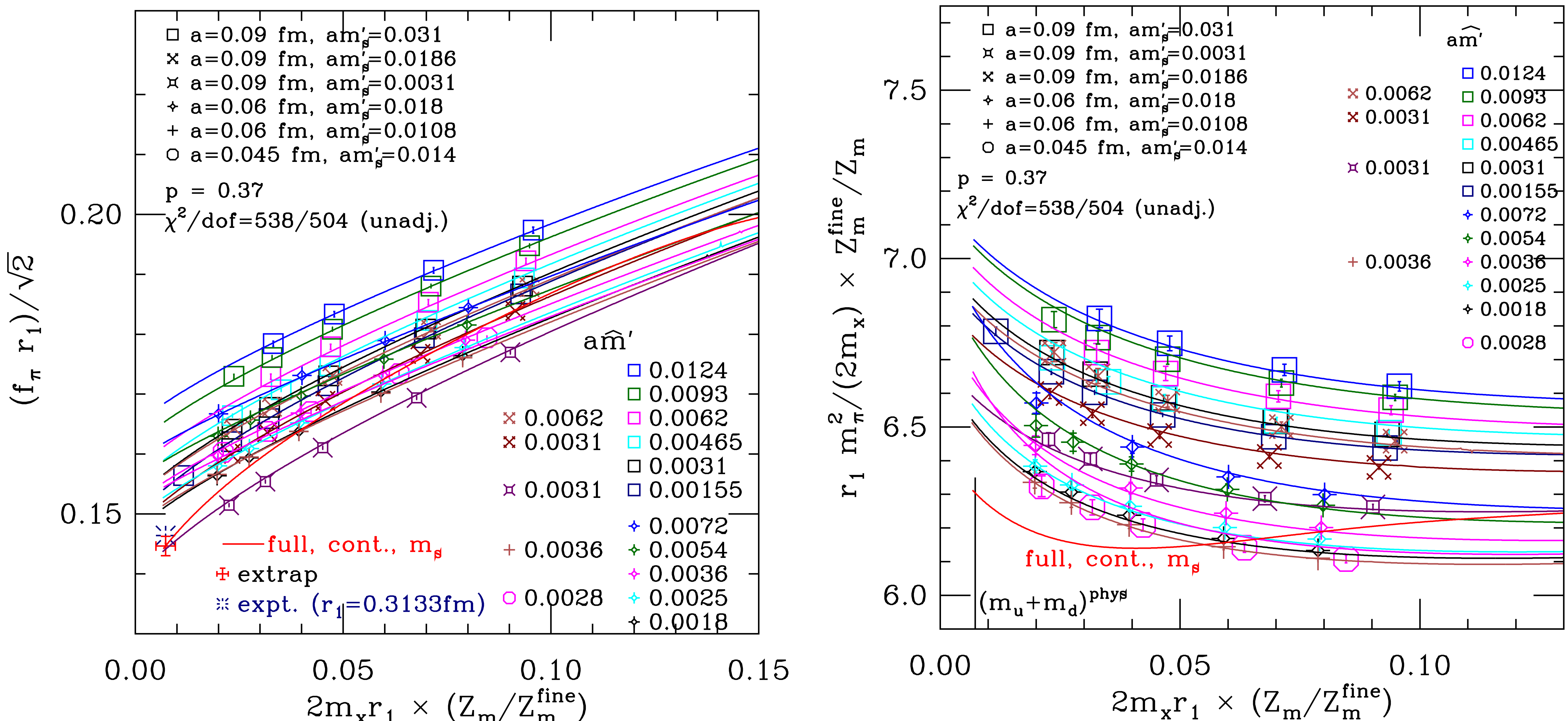

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  - Even though bulk of lattice data have ~10% discretization or mass corrections.

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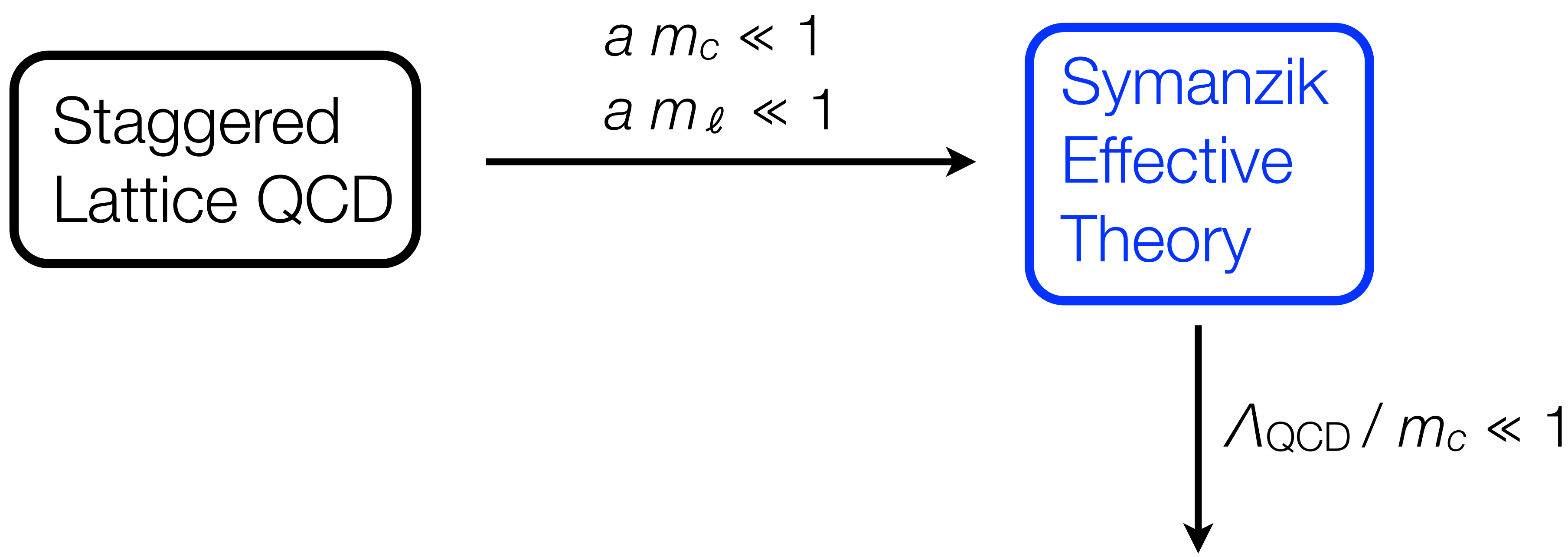

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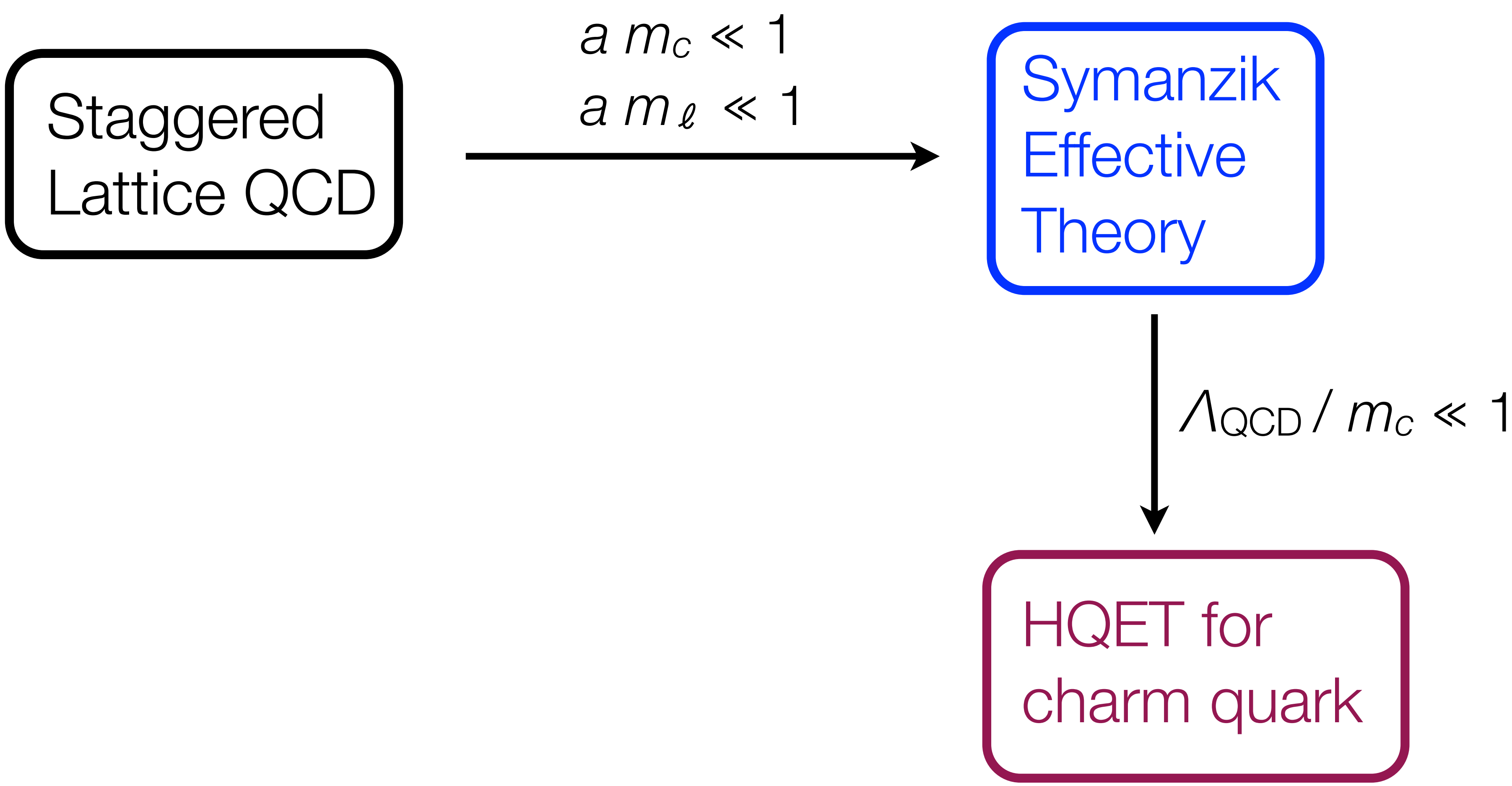

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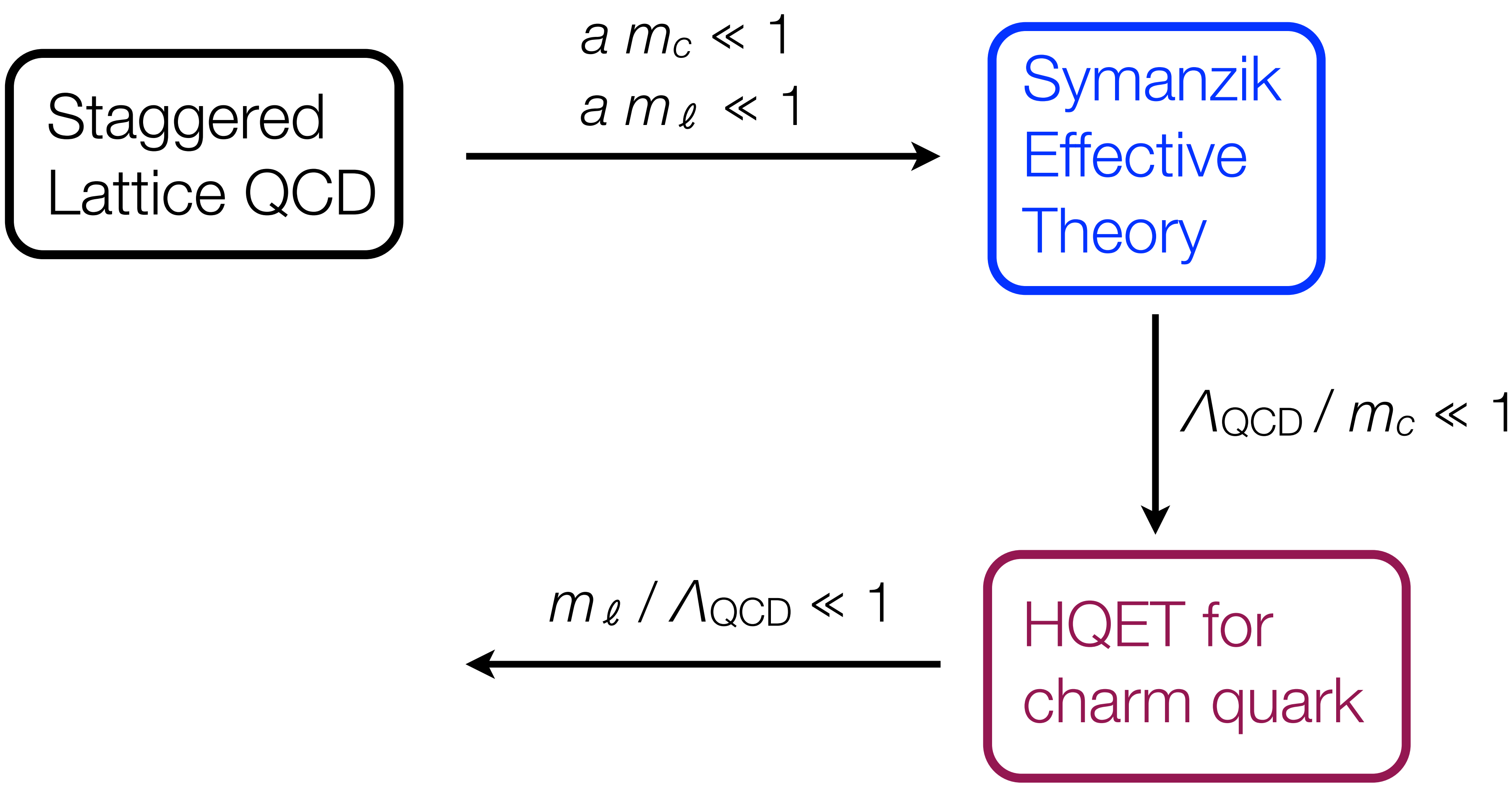

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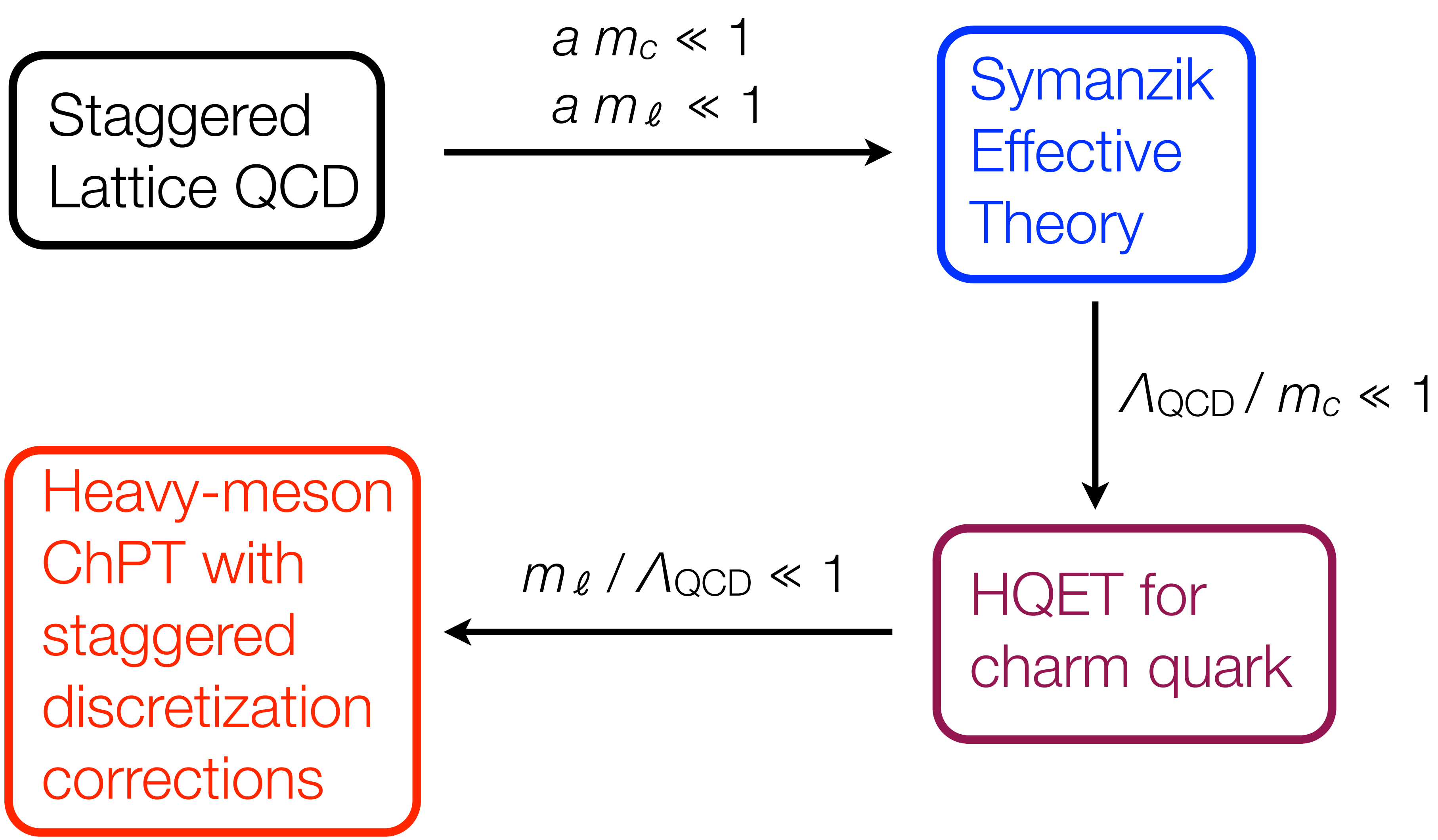
  - Highly improved action so, effectively, $a\, m_c \ll 1$, even though may have only $a\, m_c \lesssim 1$.



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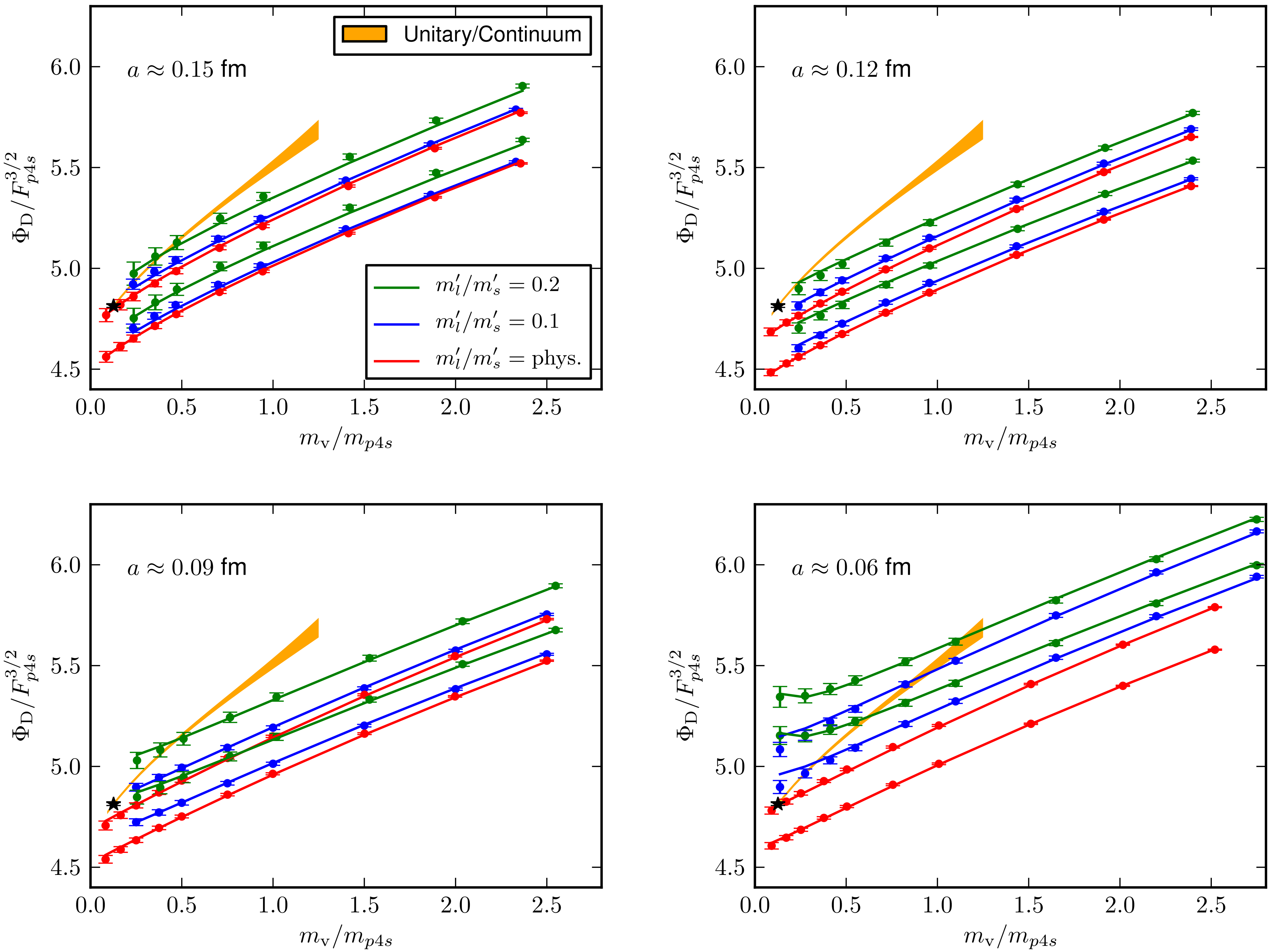

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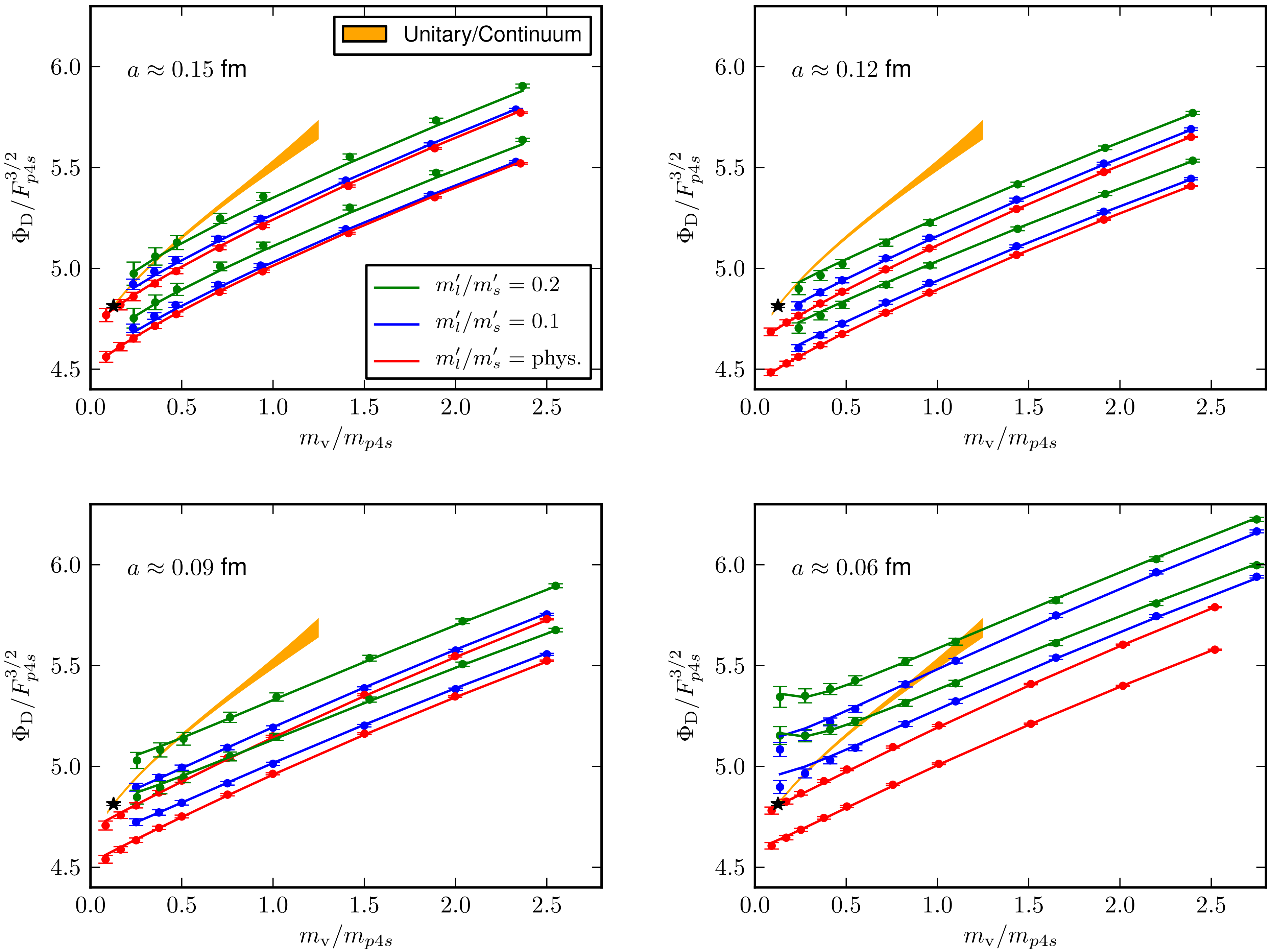

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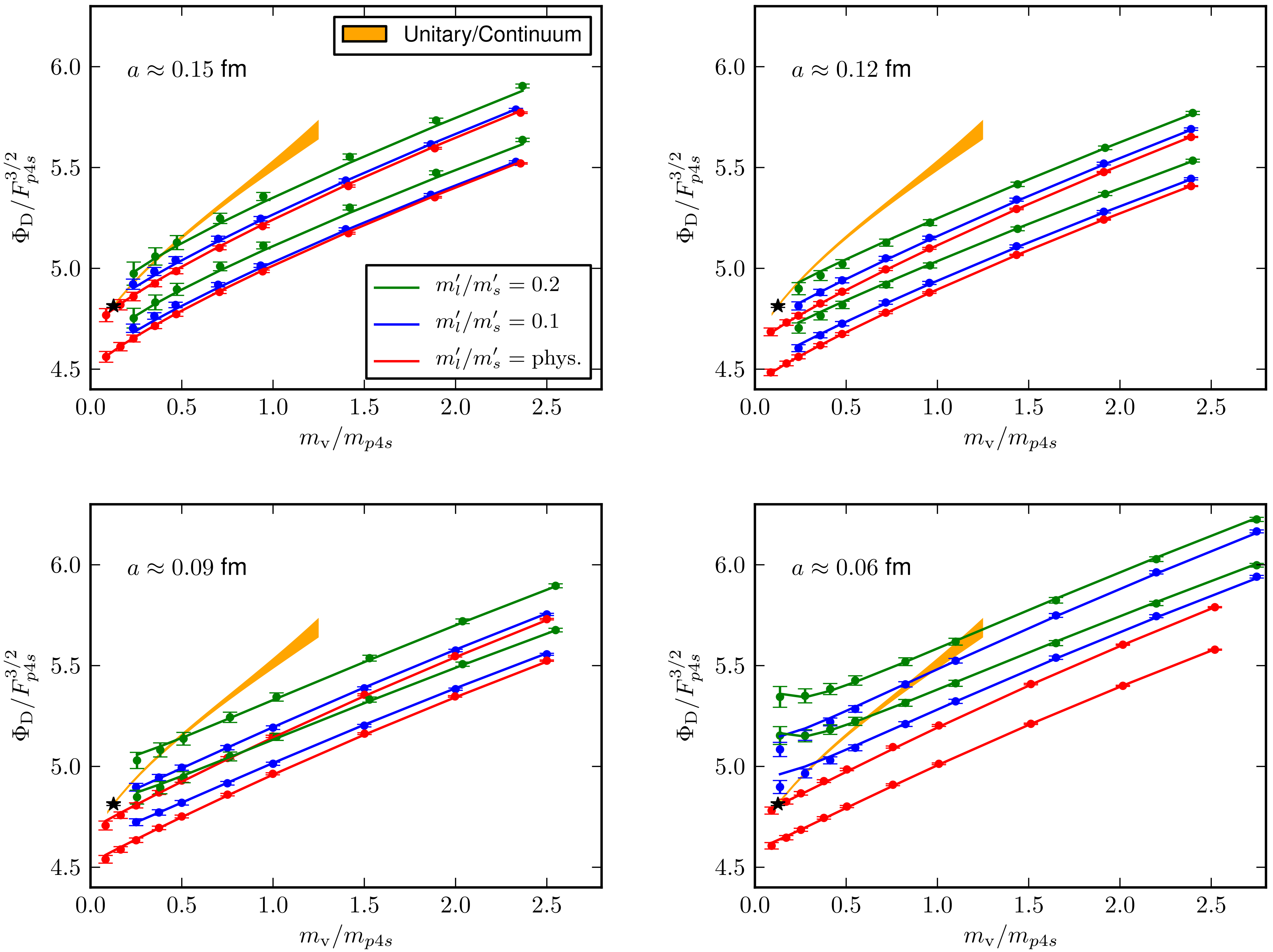

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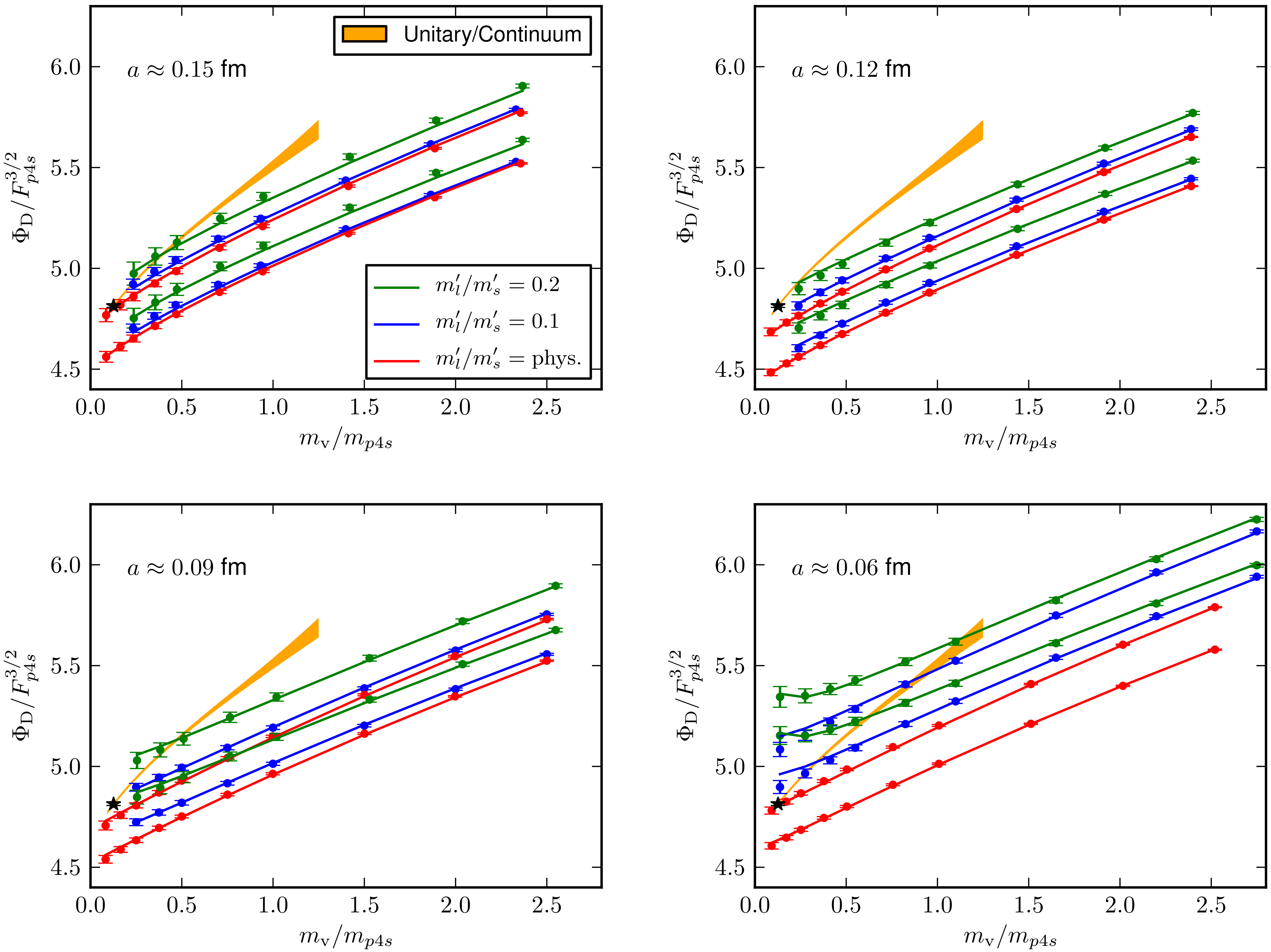

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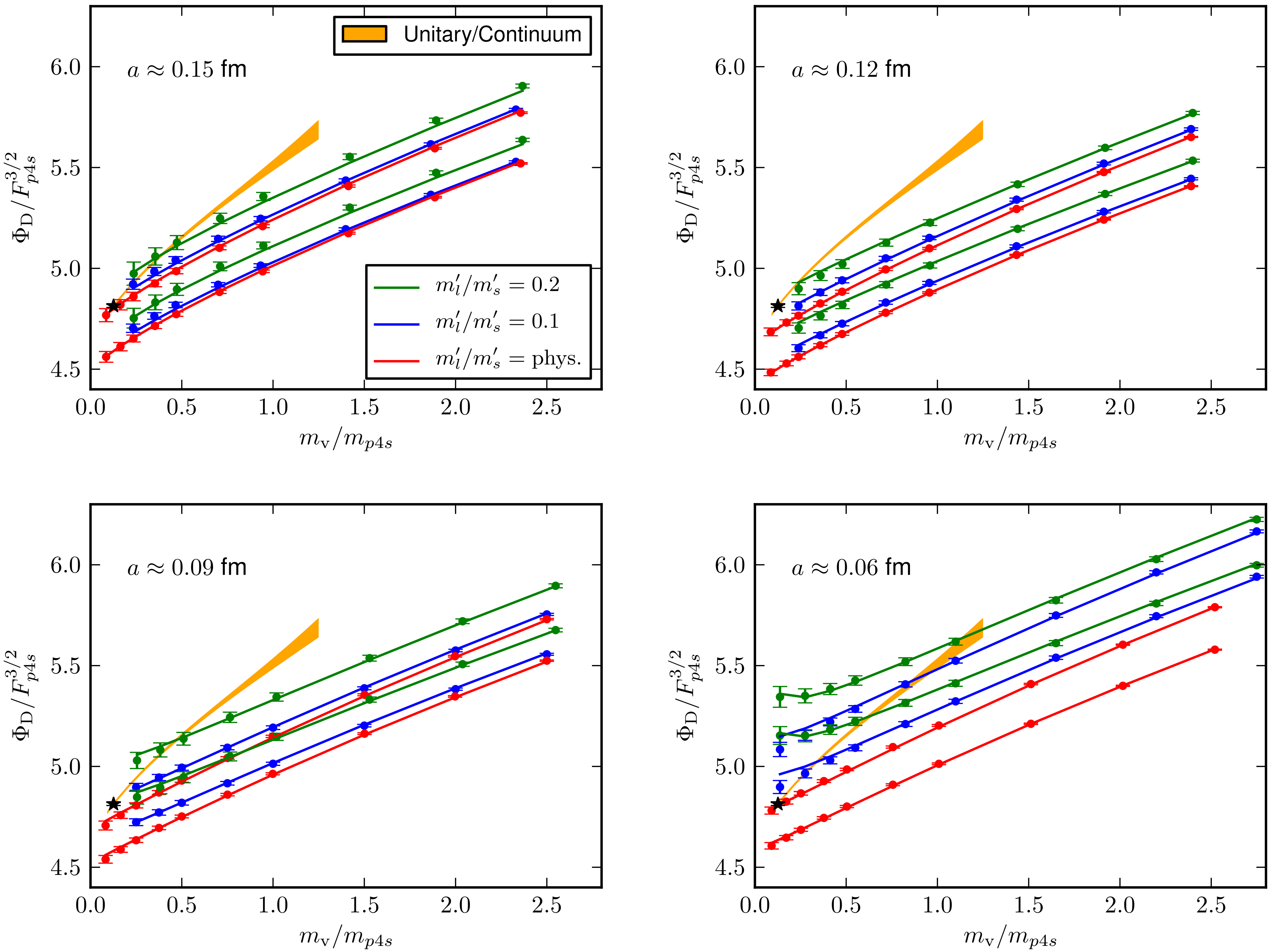

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- Get:

$$f_{D^+} = 212.6(0.4)\left(^{+1.0}_{-1.2}\right)\ \text{MeV}$$

$$f_{D_s} = 249.0(0.3)\left(^{+1.1}_{-1.5}\right)\ \text{MeV}$$

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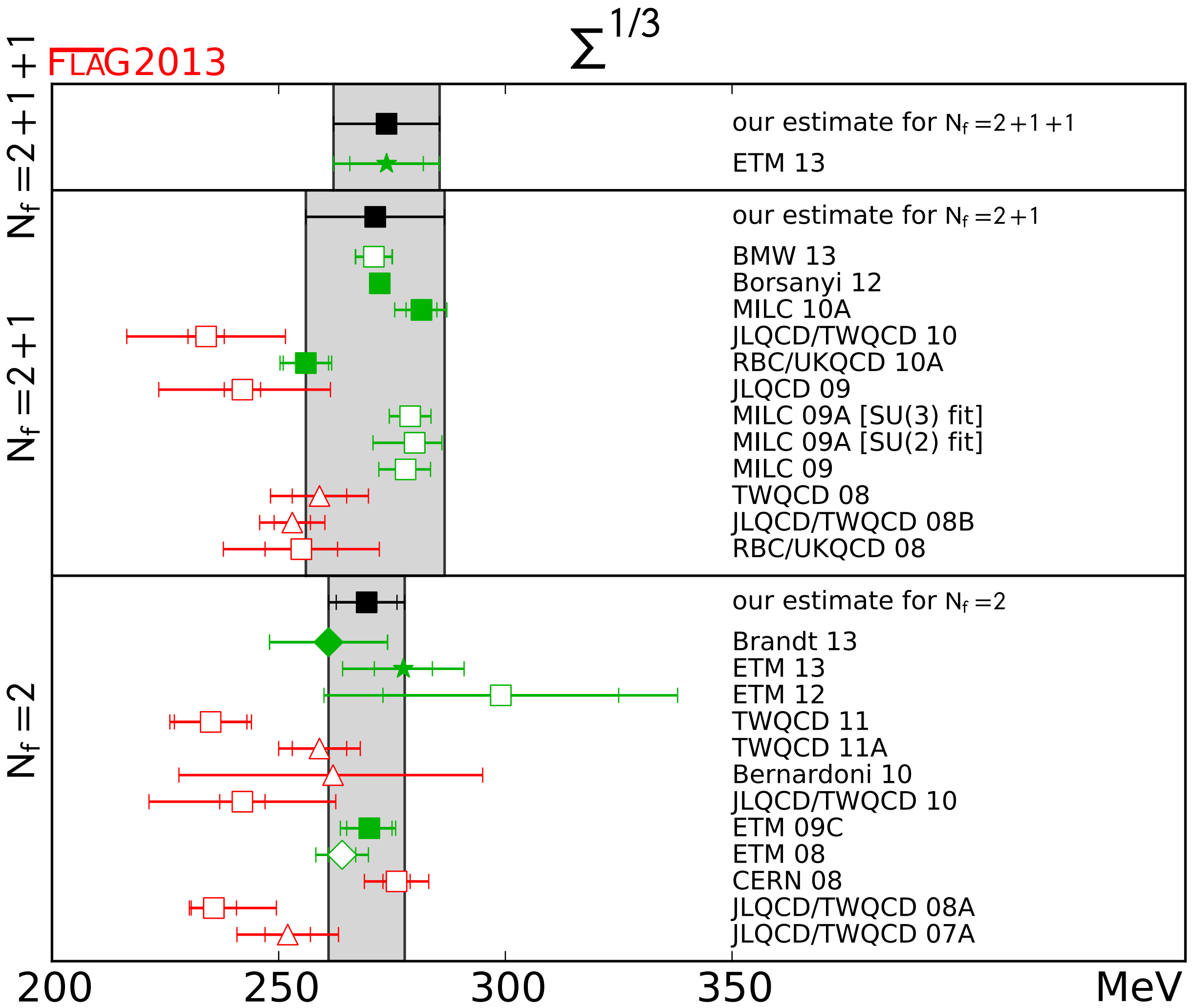
infinite volume: odd integral

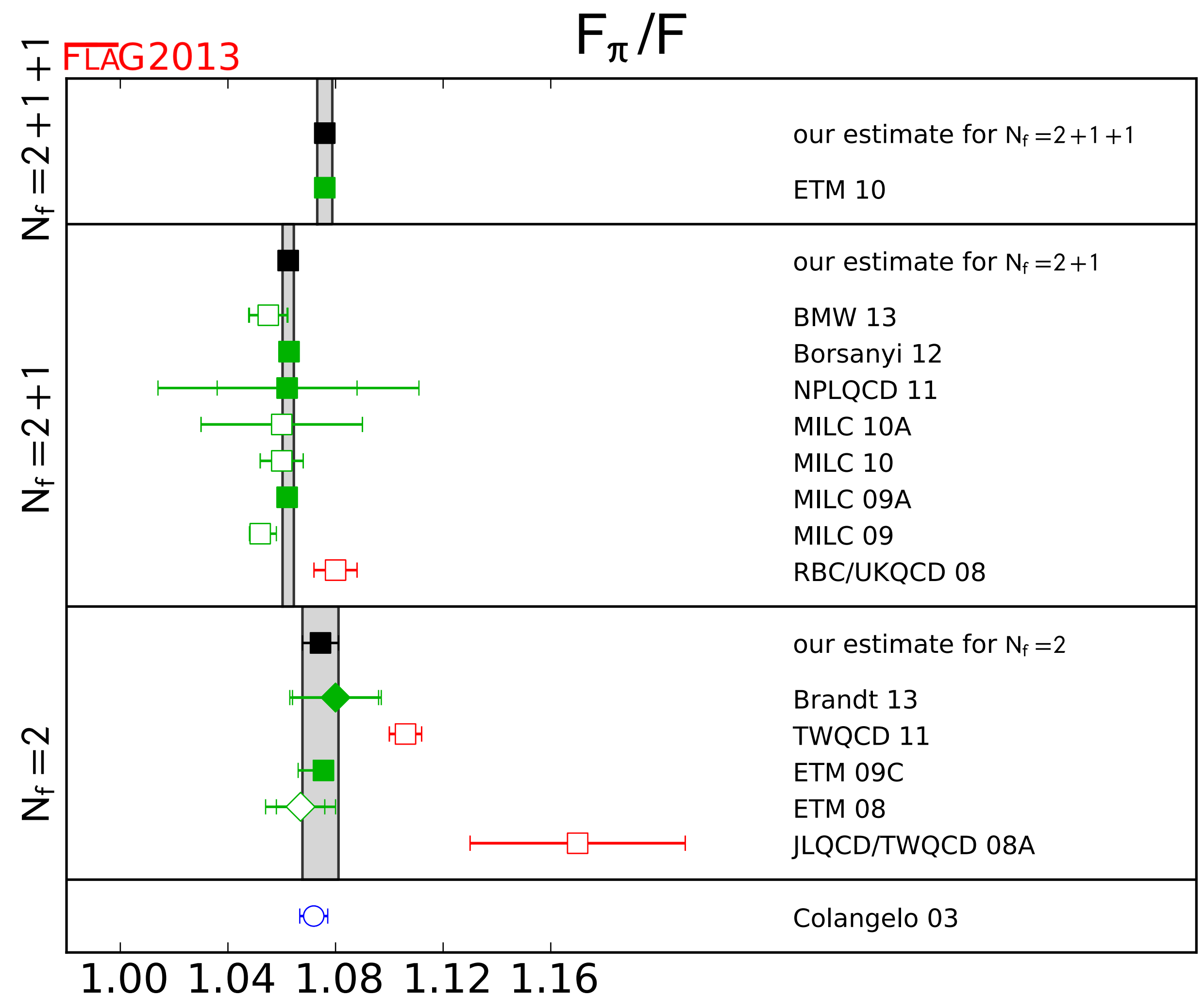
finite volume:
shift spoils k → -k symmetry

ChPT Results from the Lattice

- ✦ Lattice allows first-principles computation of LECs of the effective theory from fundamental QCD.
  - In practice, is easiest for LECs affecting pseudoscalar meson masses and leptonic decay constants.
    - Can be calculated from quark-mass dependence of 2-point Euclidean Green's functions.
    - Nice complement to experiments, which give little constraint on quark-mass dependence since quark masses fixed in Nature.
    - LECs affecting momentum dependence of scattering amplitudes are just the opposite:
      - Difficult on the lattice: n-point functions; must pull out (indirectly) Minkowski-space amplitudes from Euclidean space calculations. [Maiani & Testa, 1990; Lüscher, 1991].

SU(2) LECs





$$\Sigma \equiv |\langle \bar{u}u \rangle| = (271(15)\ \mathrm{MeV})^3 \qquad \frac{F_\pi}{F} = 1.0624(21)$$

- in the two flavor chiral limit: $m_u, m_d \to 0$.
- values quoted are for $N_f = 2+1$ theory.
- For F_π/F, looks like systematic errors of one or more calculations may be underestimated.

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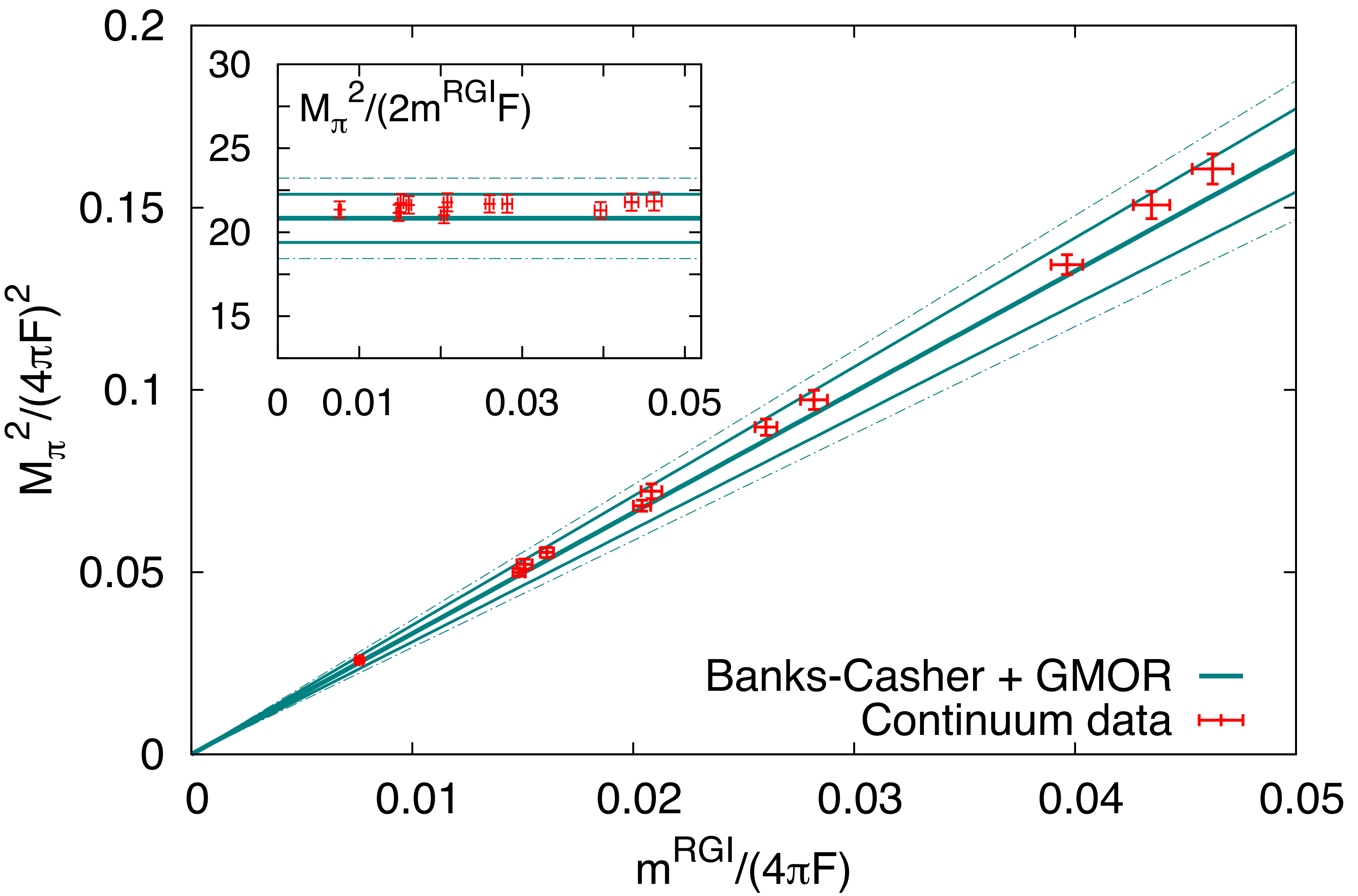

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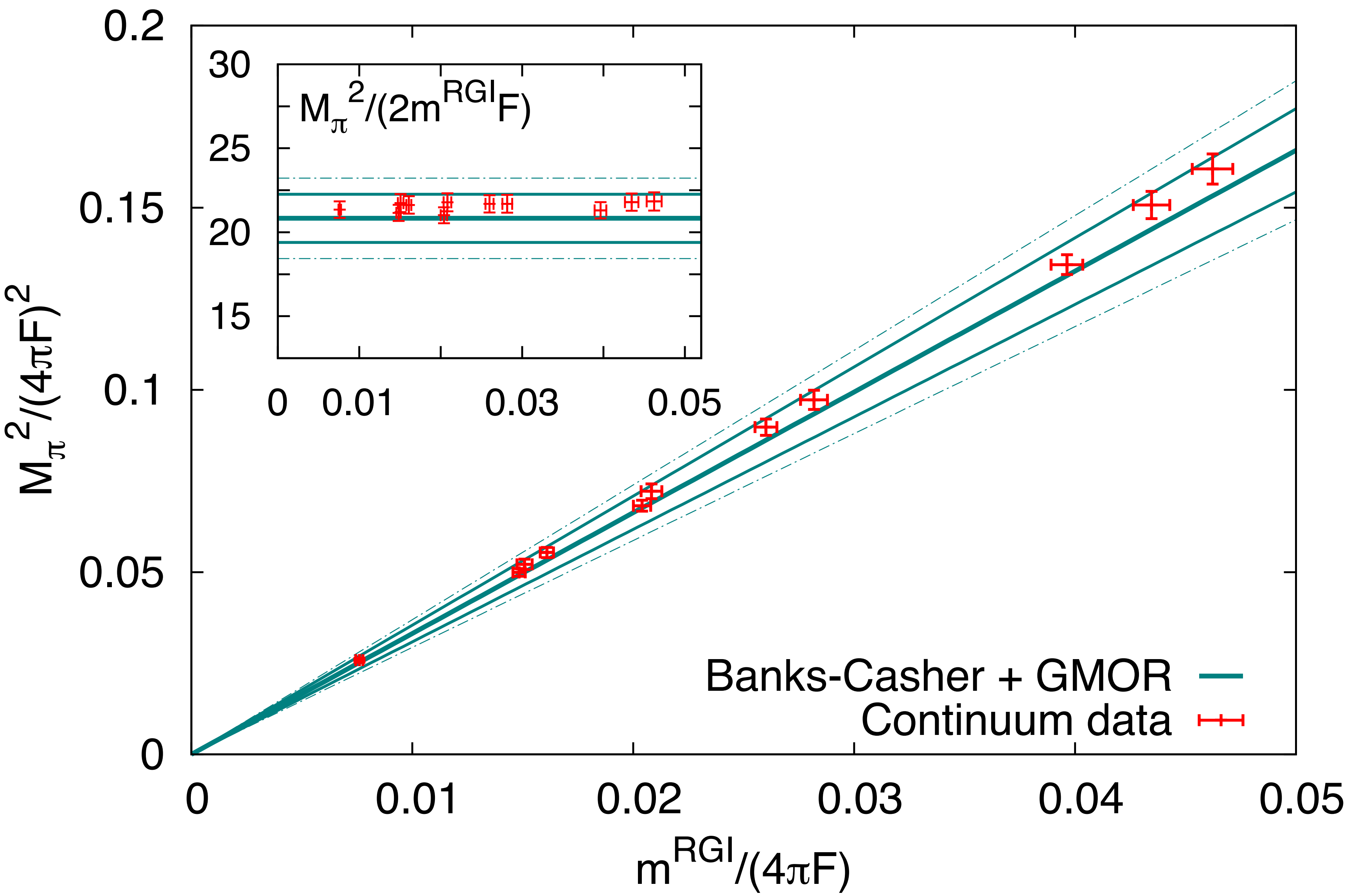

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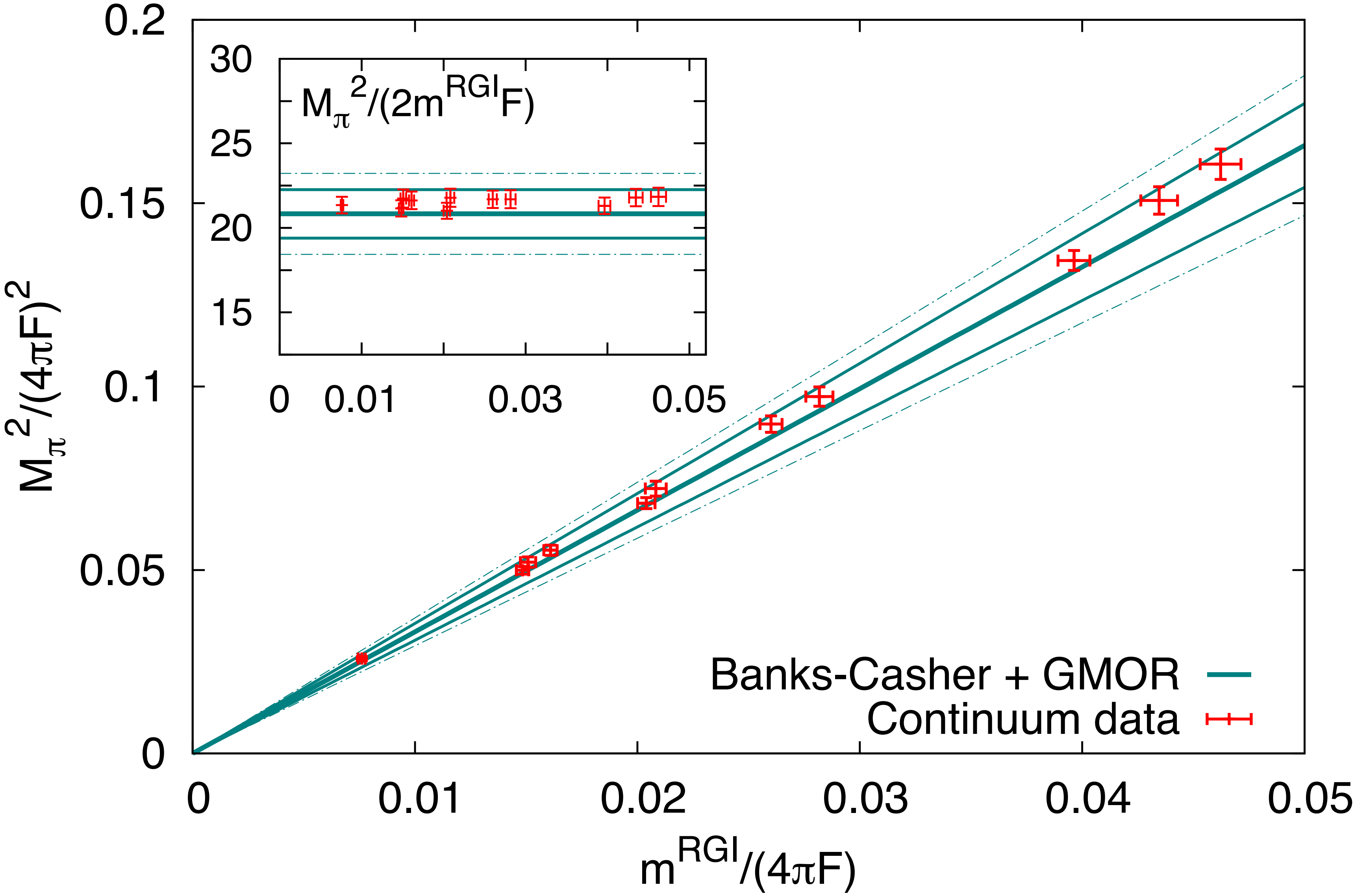

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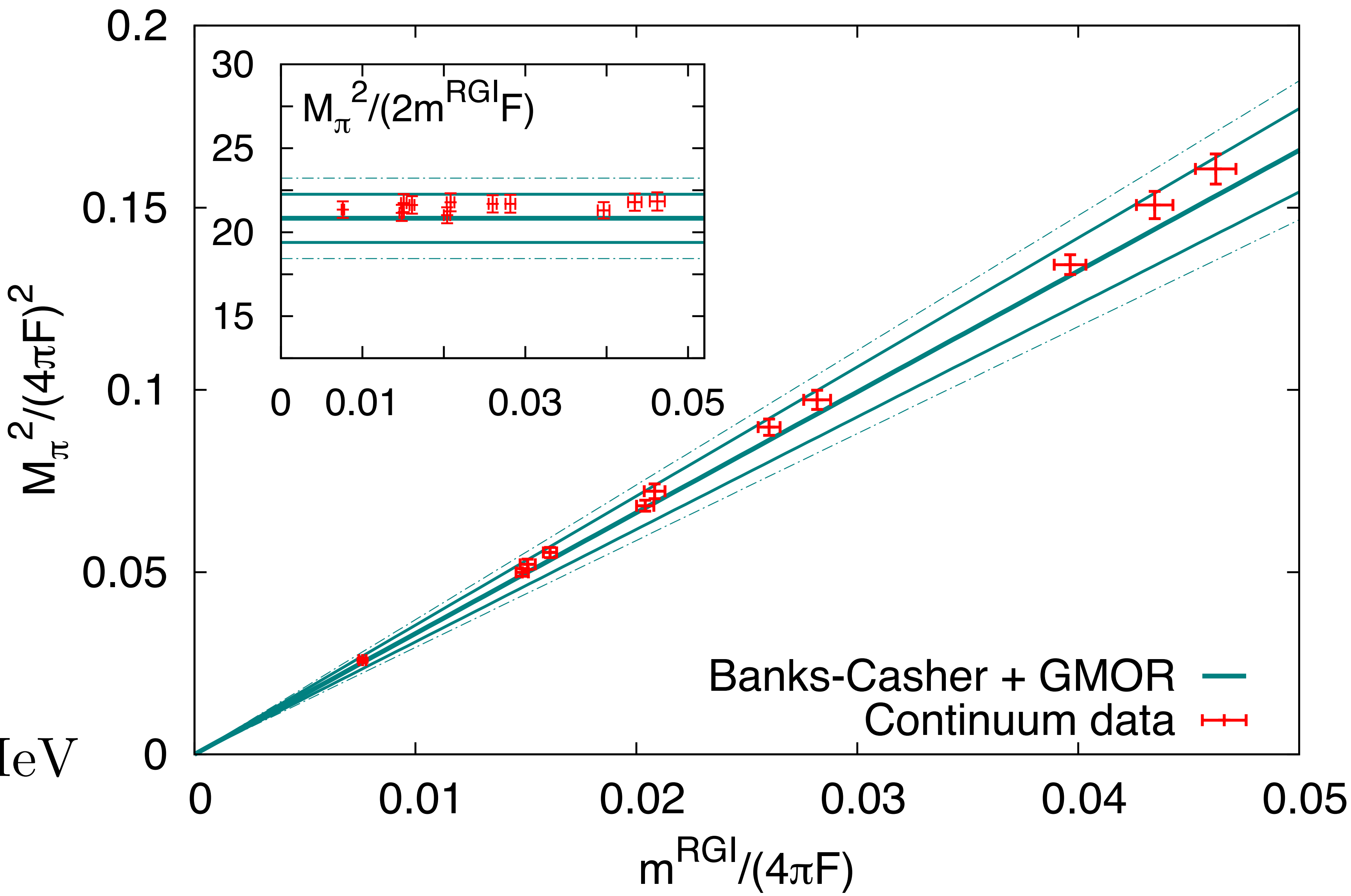

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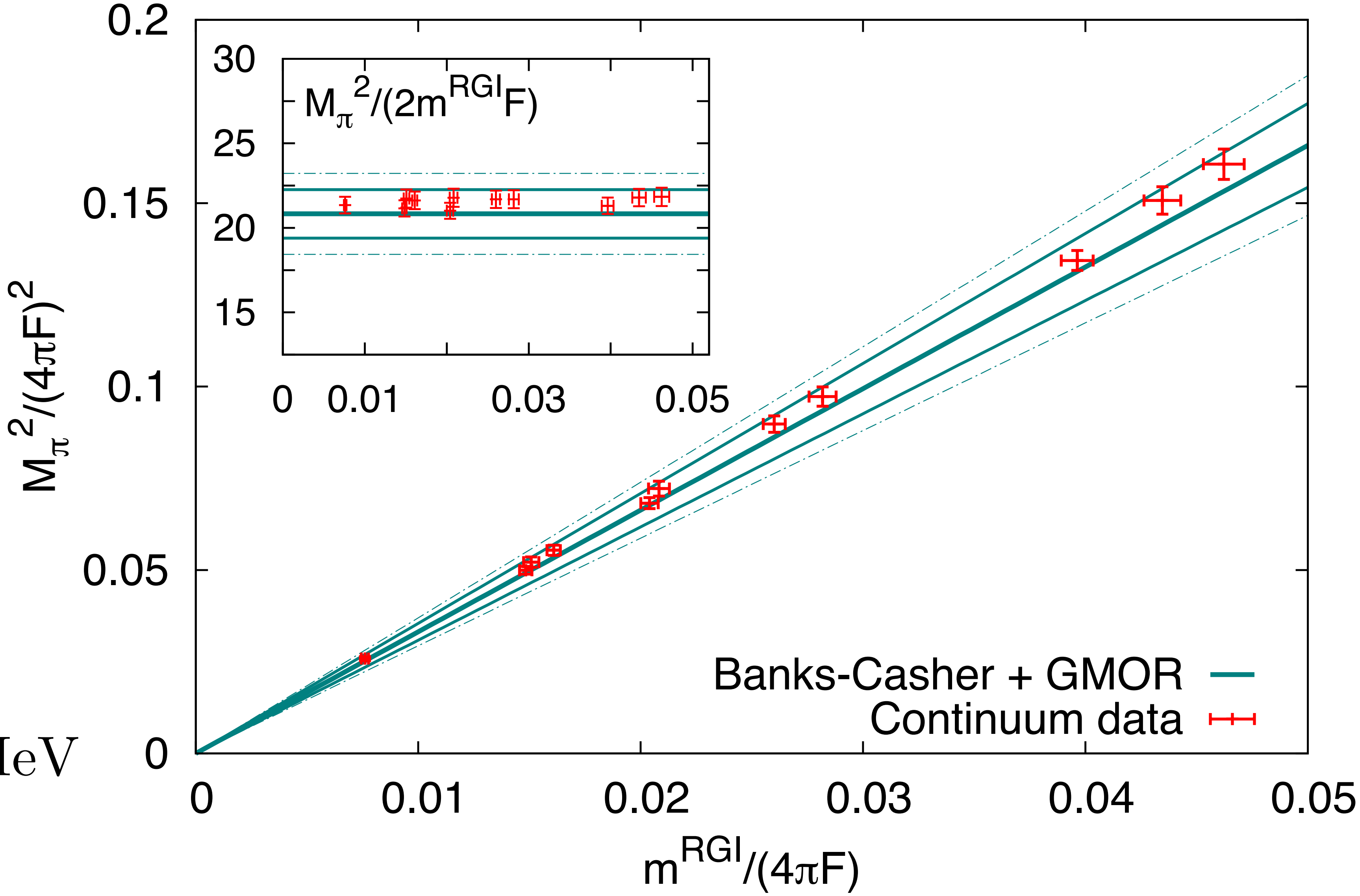
- FLAG, 2013 average is
$$|\langle \bar{u}u \rangle|^{1/3} = 269(8)\ \mathrm{MeV}$$
(N_f=2; μ = 2 GeV).



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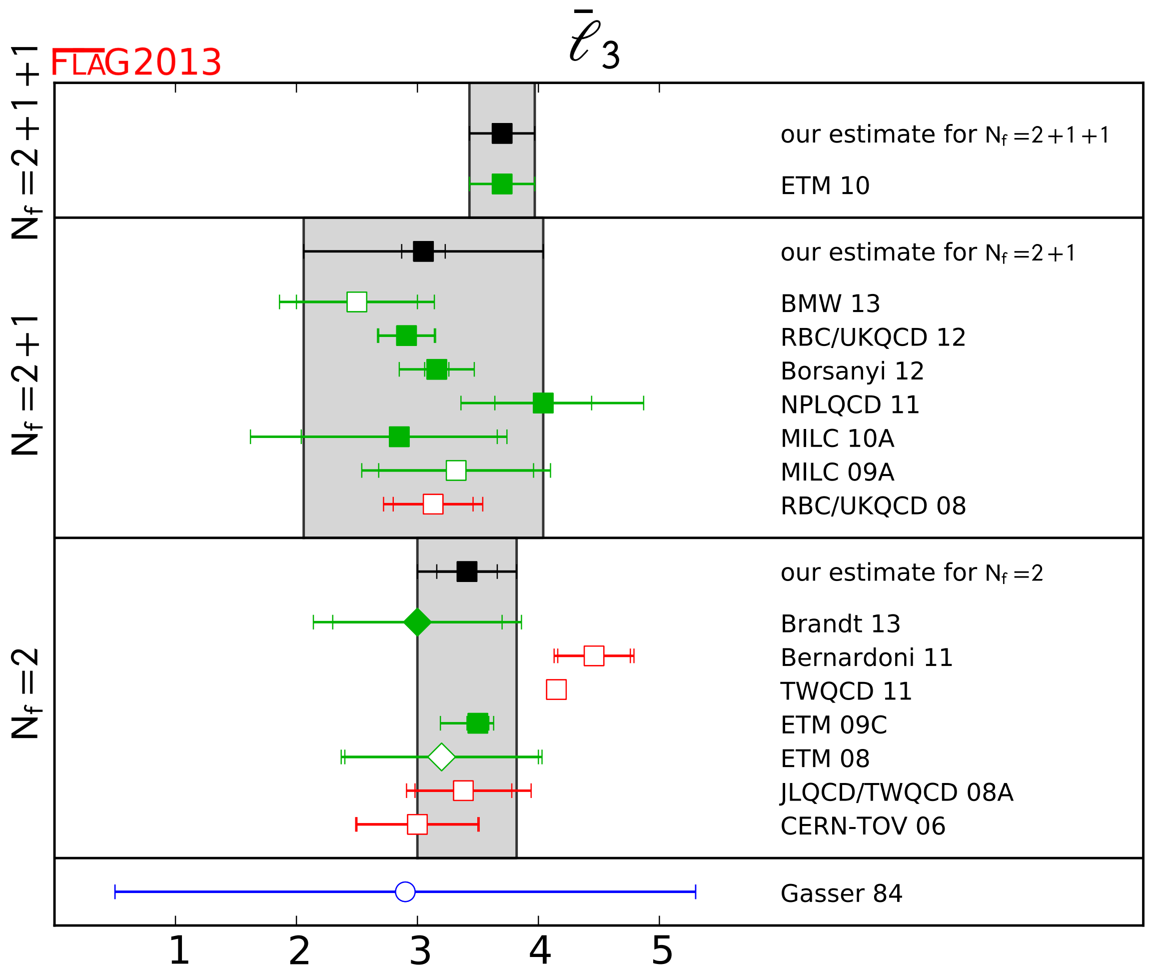
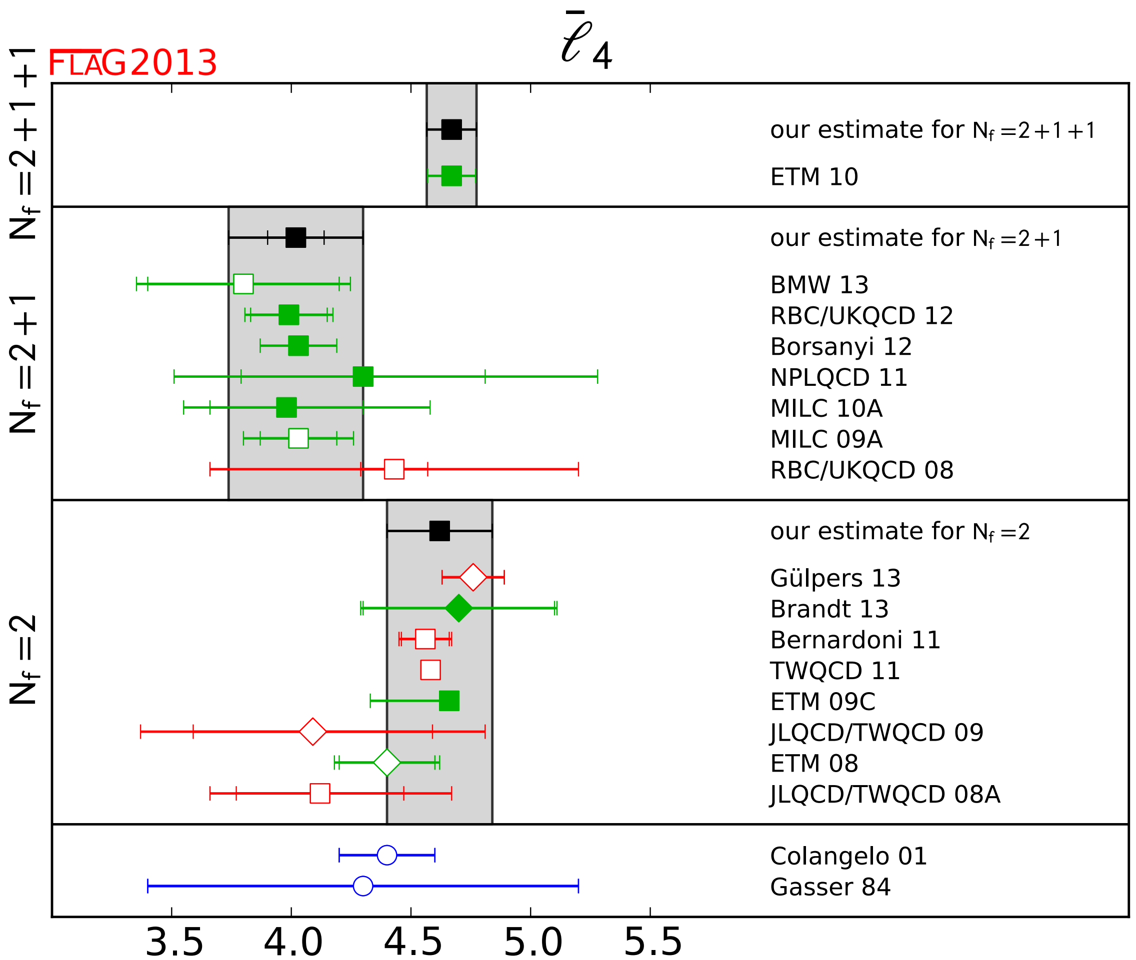


FLAG2013
$\bar{\ell}_3$
$N_f=2+1+1$
$N_f=2+1$
$N_f=2$
our estimate for $N_f=2+1+1$
ETM 10
our estimate for $N_f=2+1$
BMW 13
RBC/UKQCD 12
Borsanyi 12
NPLQCD 11
MILC 10A
MILC 09A
RBC/UKQCD 08
our estimate for $N_f=2$
Brandt 13
Bernardoni 11
TWQCD 11
ETM 09C
ETM 08
JLQCD/TWQCD 08A
CERN-TOV 06
Gasser 84
1
2
3
4
5



FLAG2013
$\bar{\ell}_4$
$N_f=2+1+1$
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our estimate for $N_f=2+1+1$
ETM 10
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our estimate for $N_f=2$
Gülpers 13
Brandt 13
Bernardoni 11
TWQCD 11
ETM 09C
JLQCD/TWQCD 09
ETM 08
JLQCD/TWQCD 08A
Colangelo 01
Gasser 84
3.5
4.0
4.5
5.0
5.5

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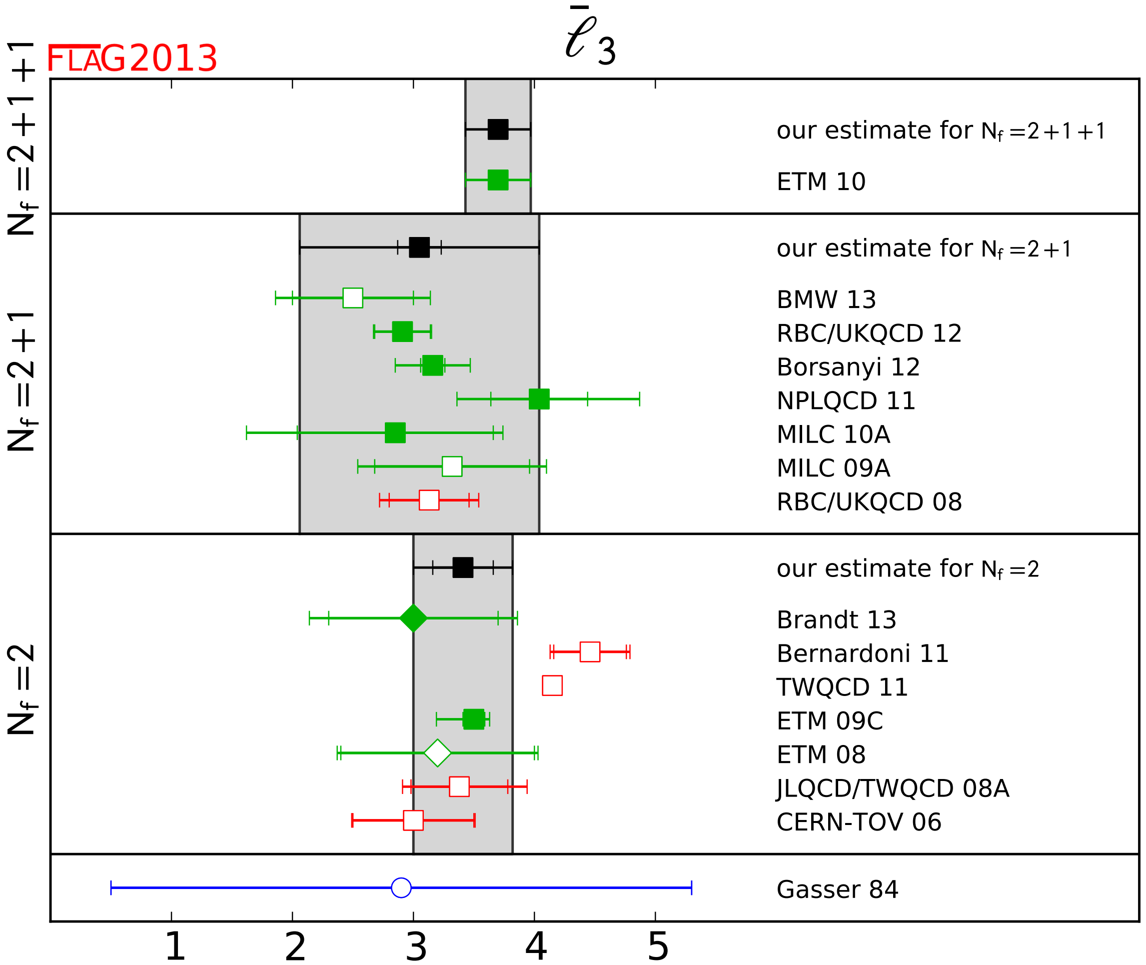

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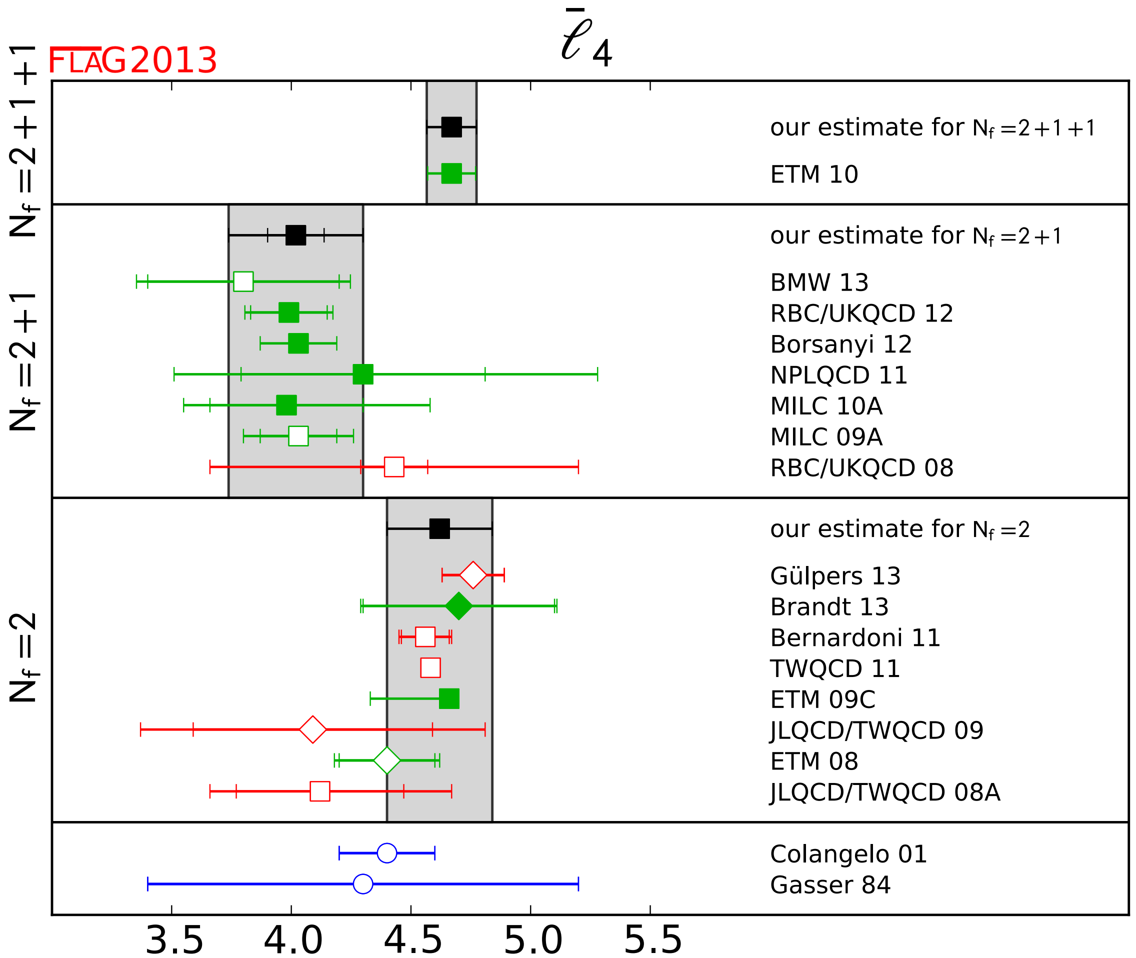

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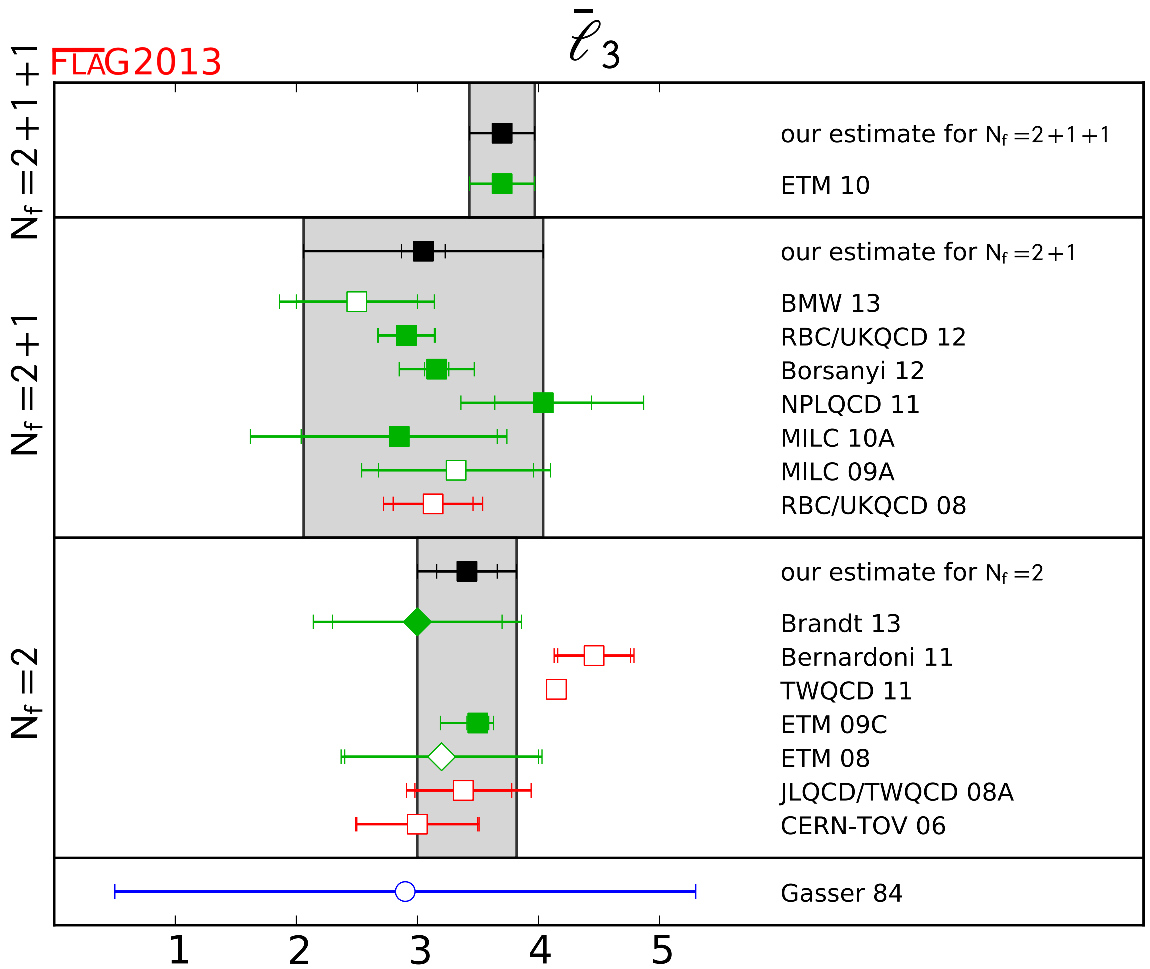

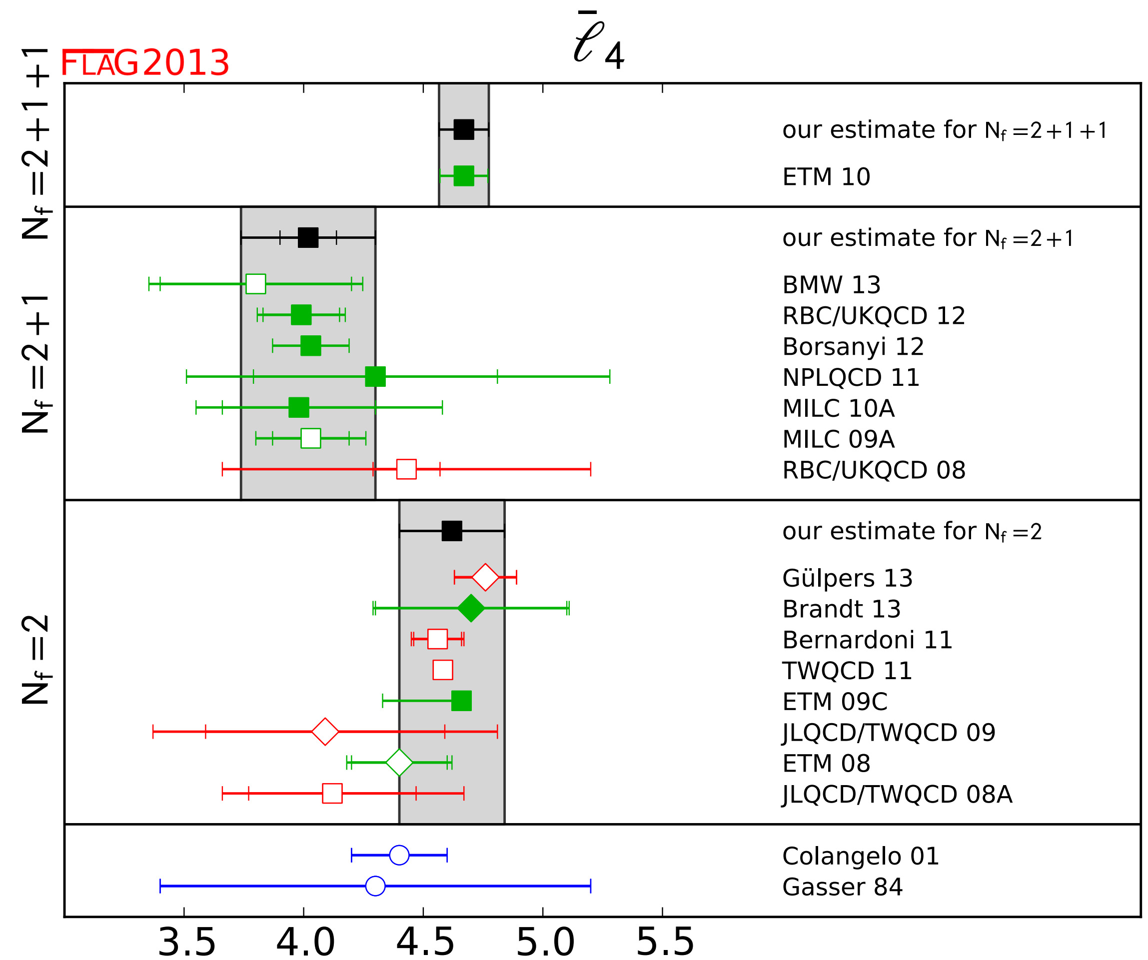

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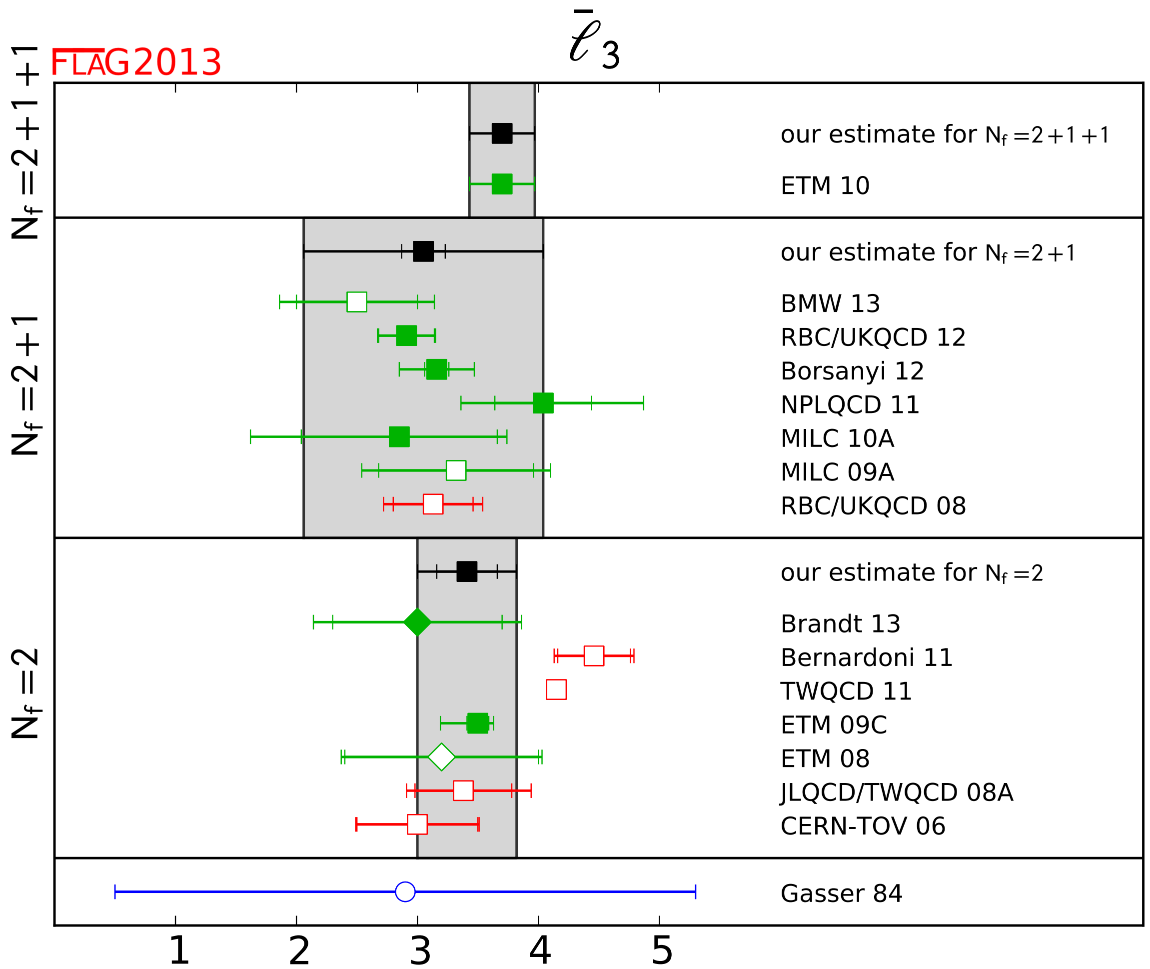

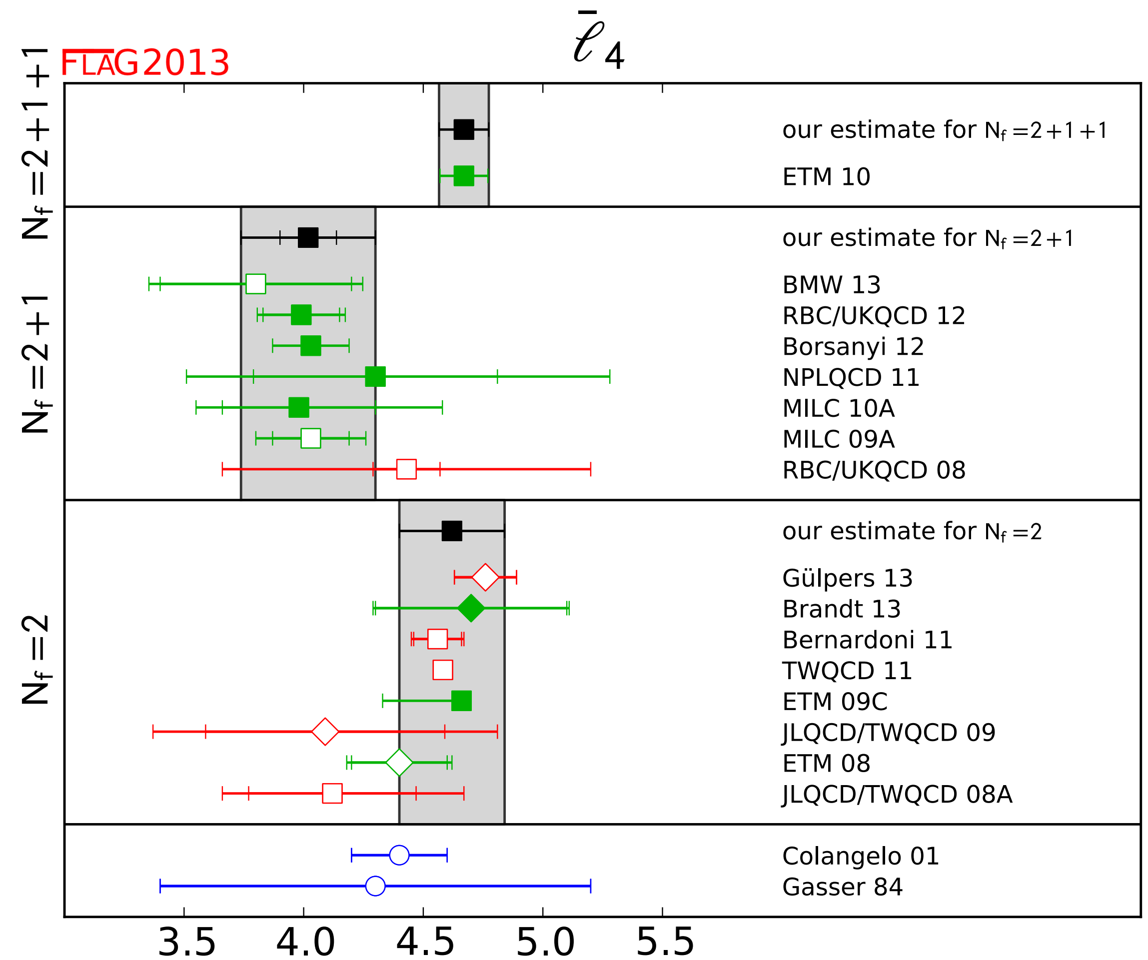

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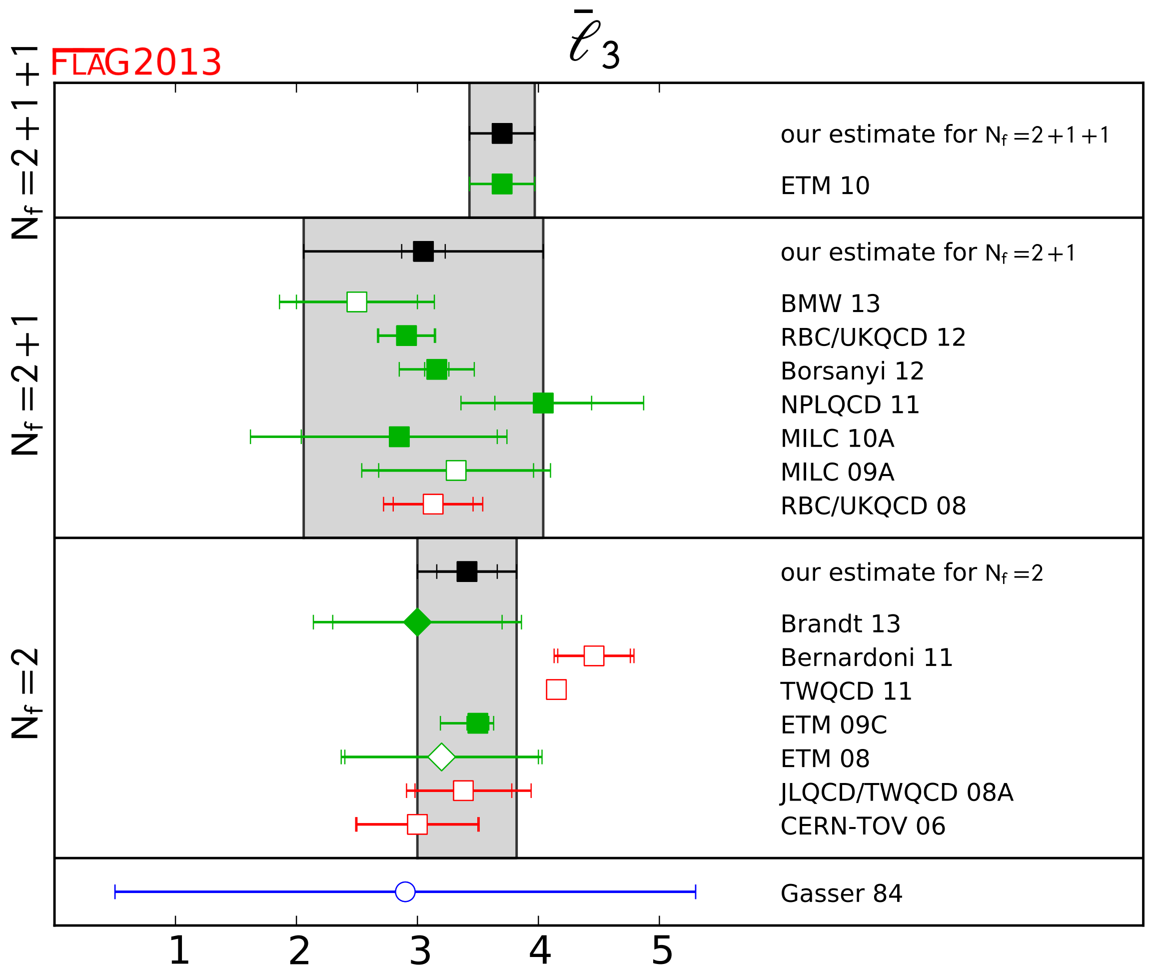

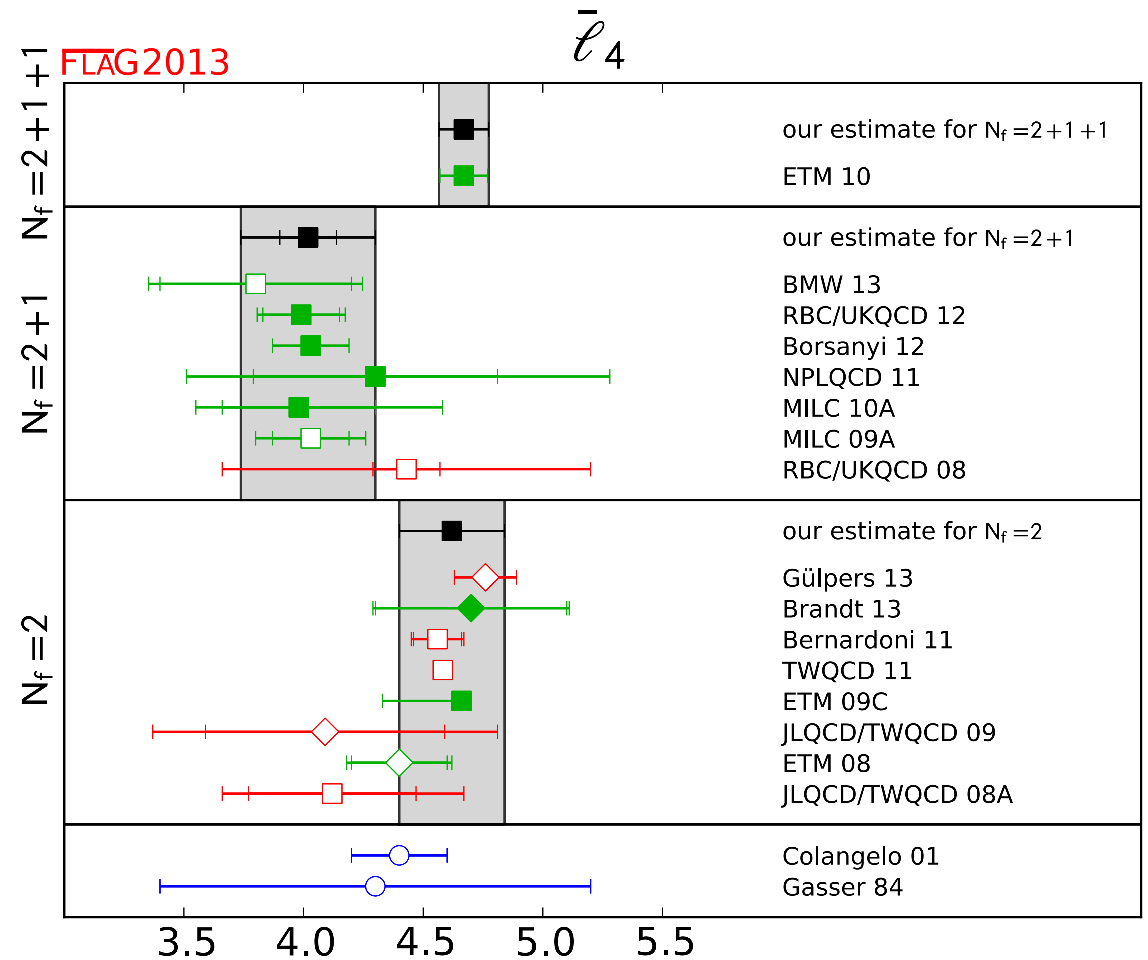

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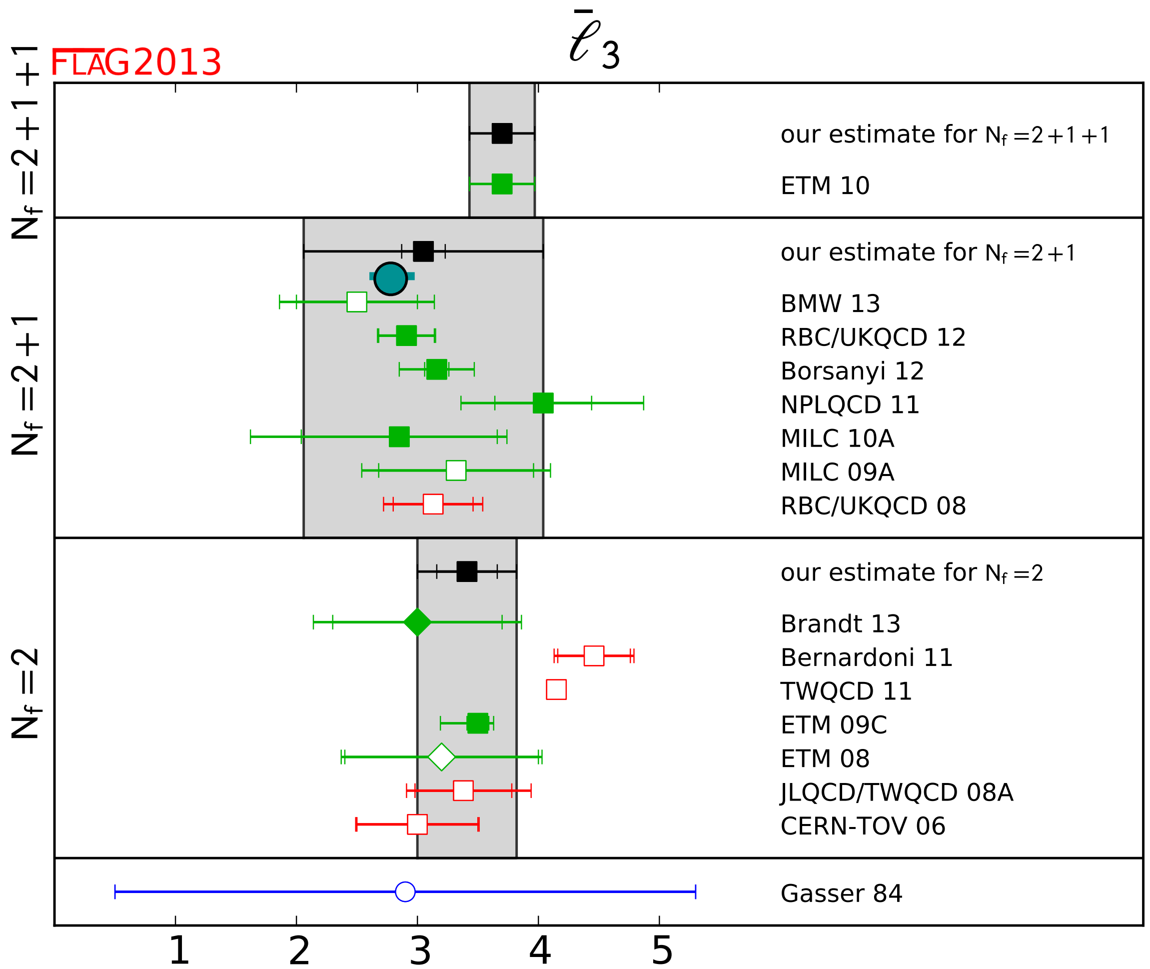

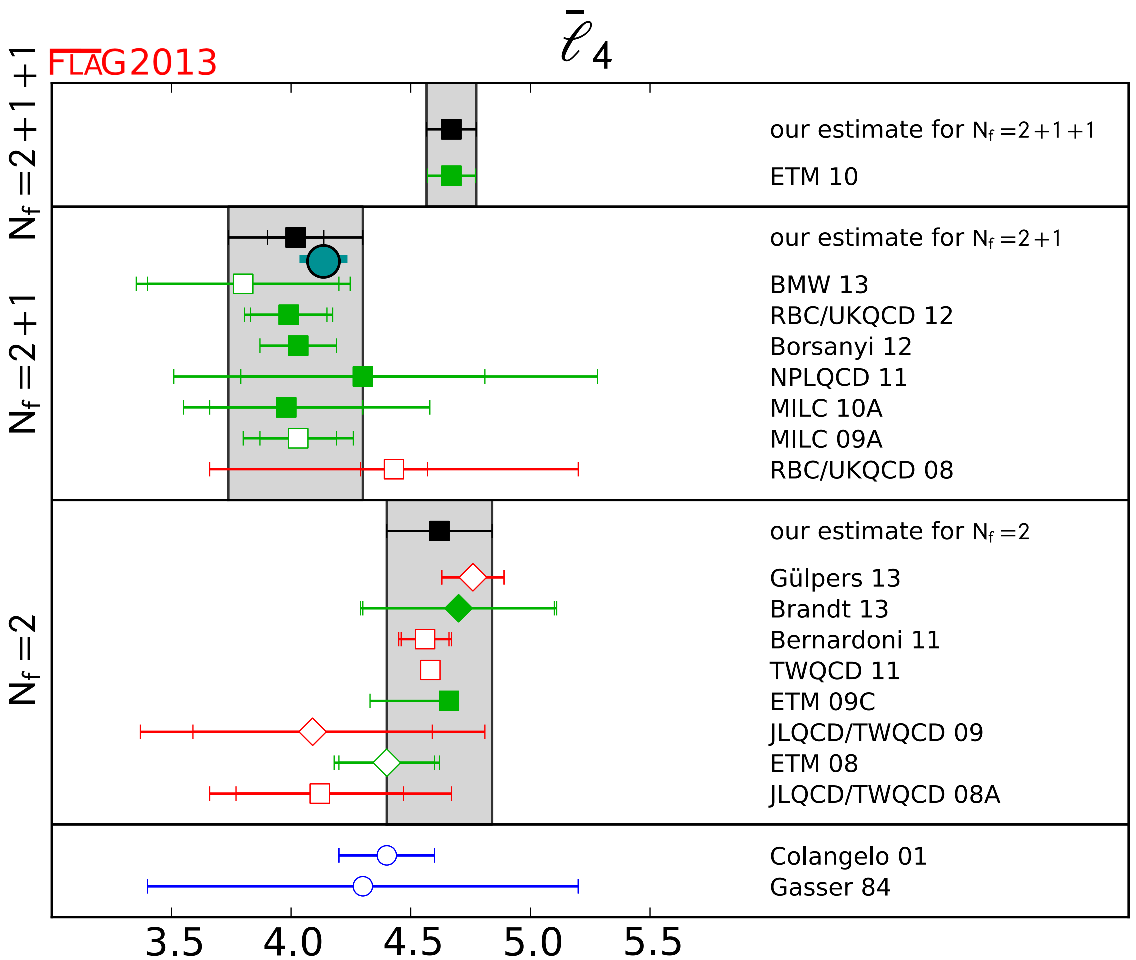

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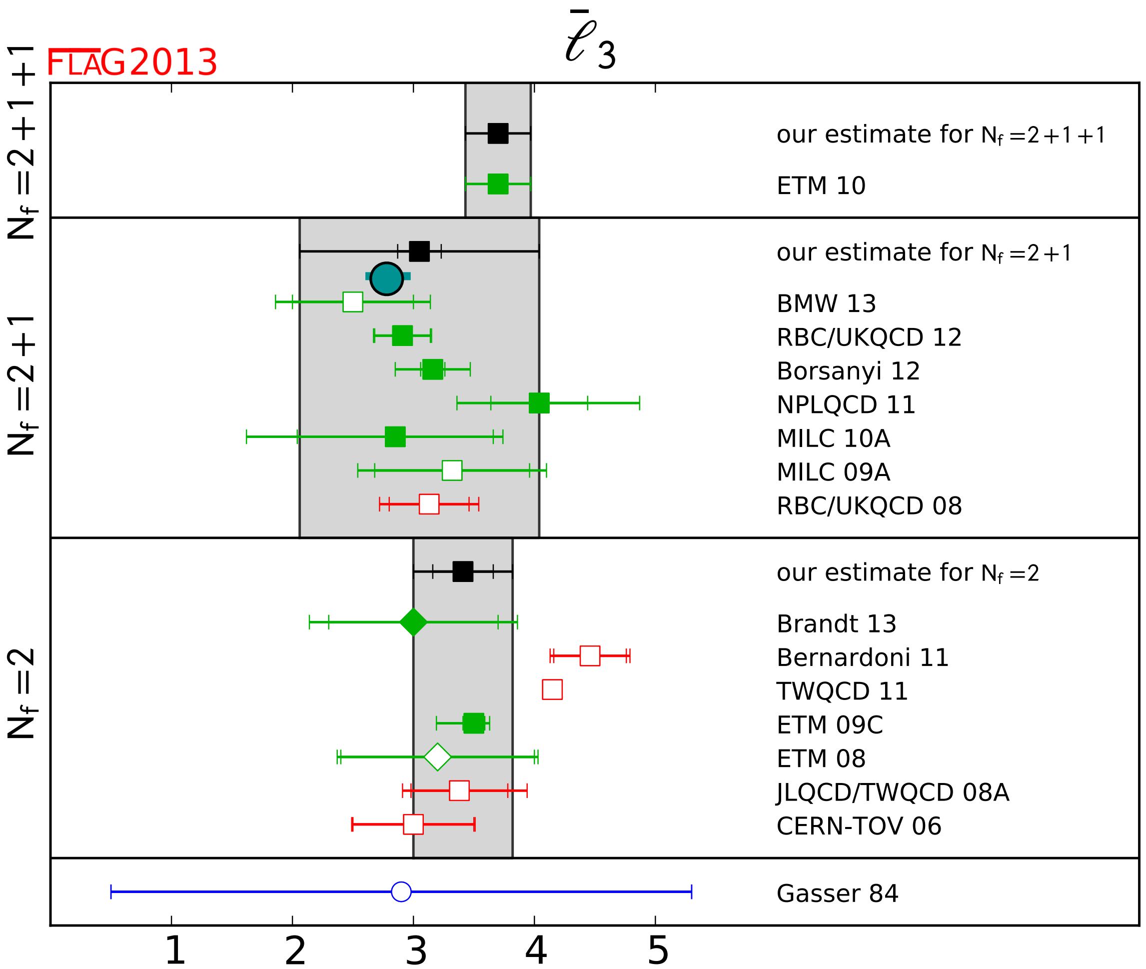

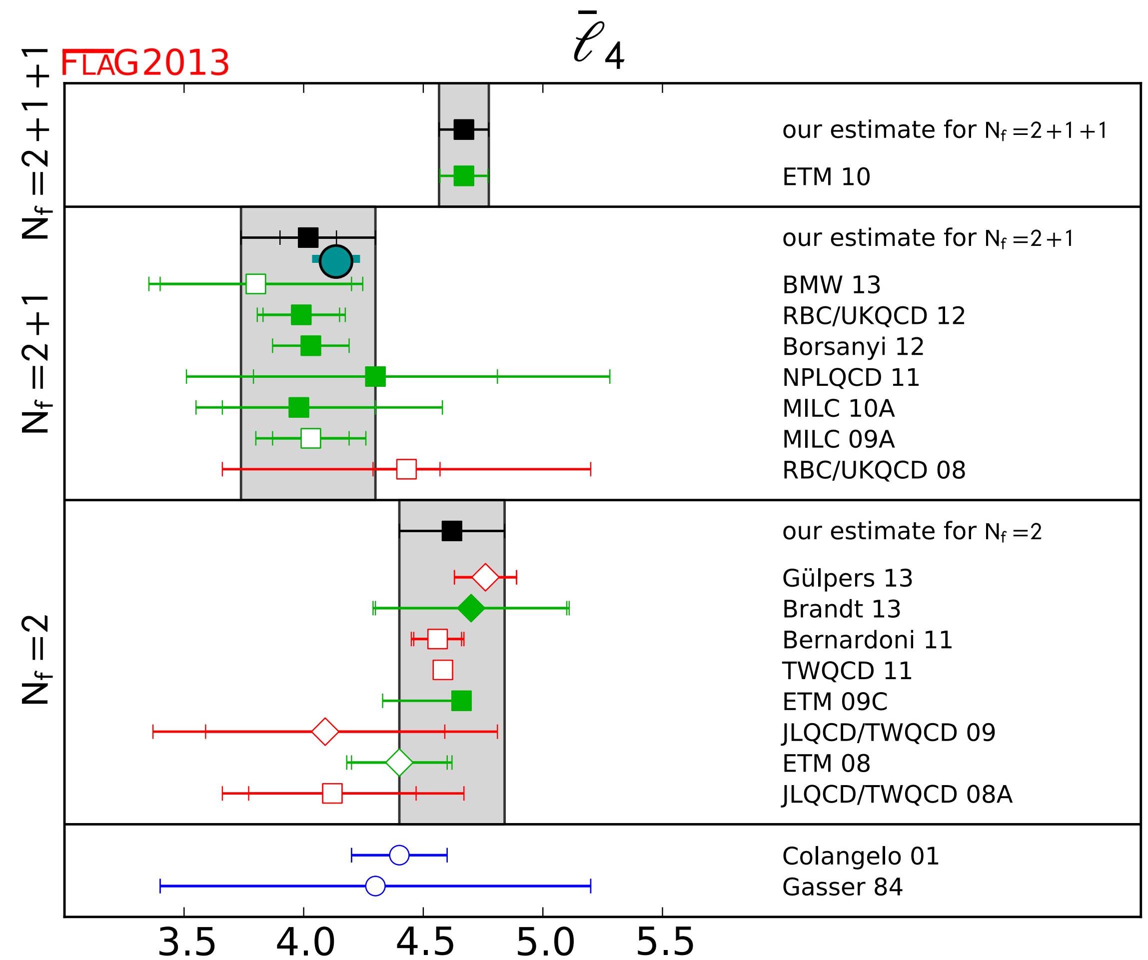

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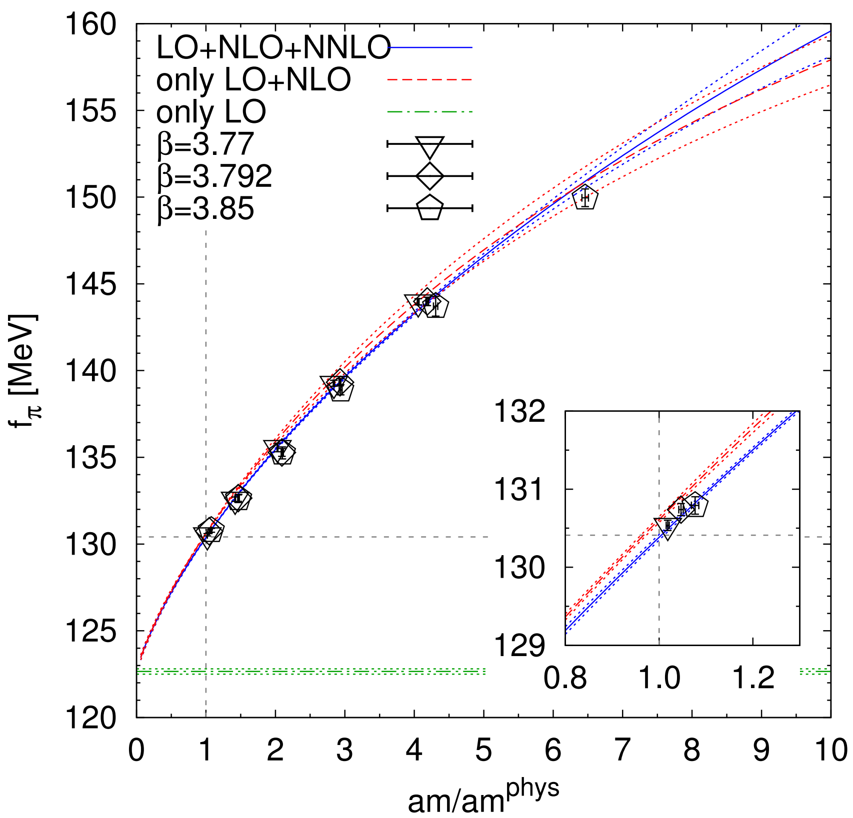

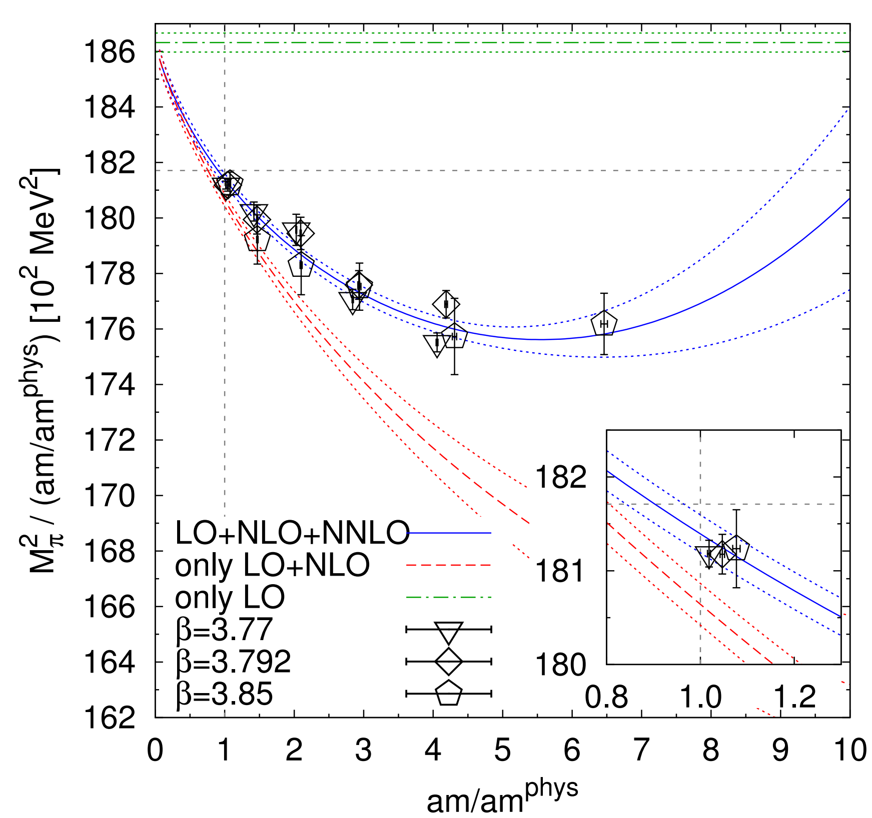

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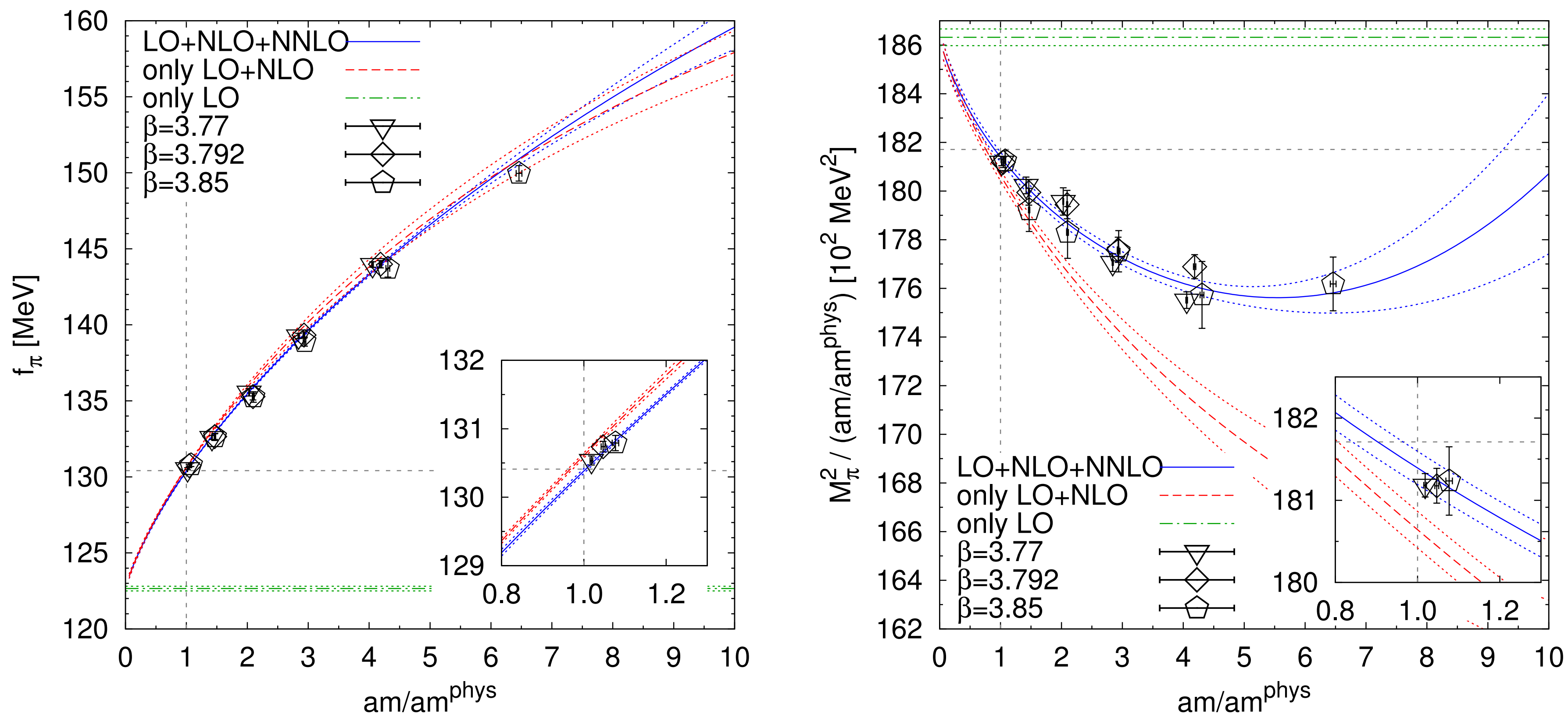

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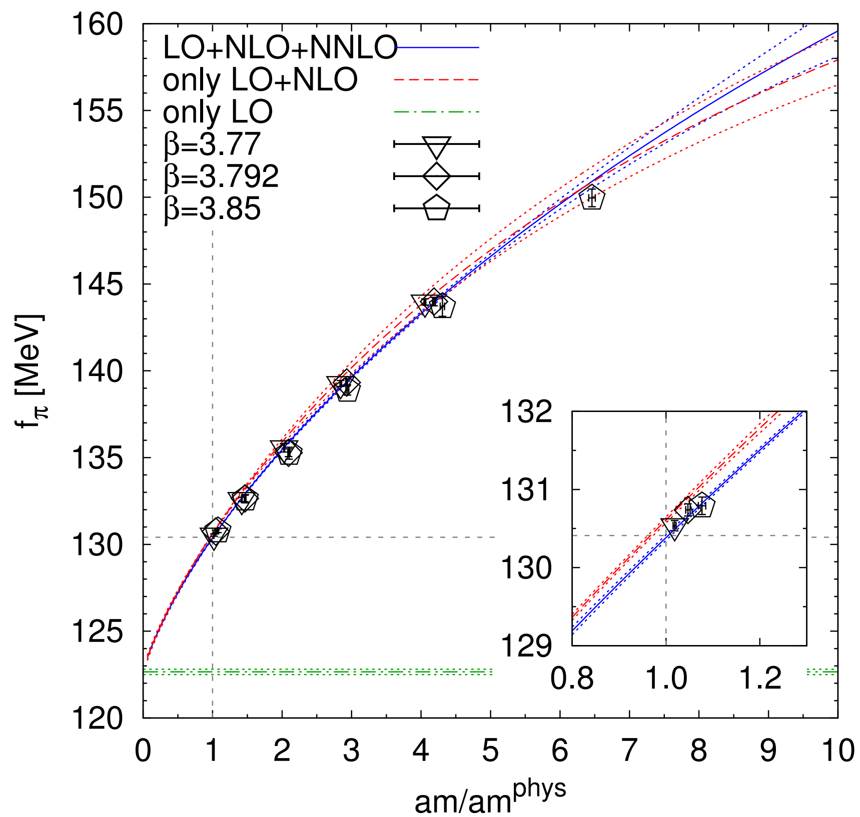

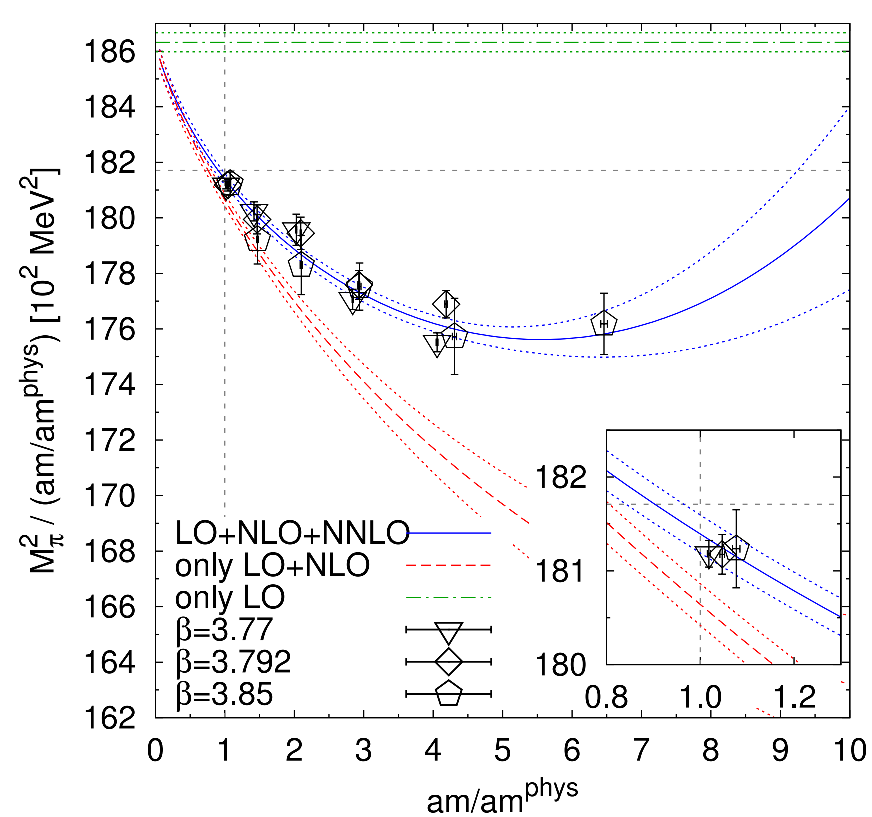

- Convergence good for f_π and reasonable for $M_\pi^2/\hat{m}$.
  - up to limit of lattice data (~7 or 8 times physical $\hat{m}$).

Convergence of ChPT: SU(2)

S. Borsanyi et al., Phys. Rev. D88 (2013) 014513, [arXiv: 1205.0788].

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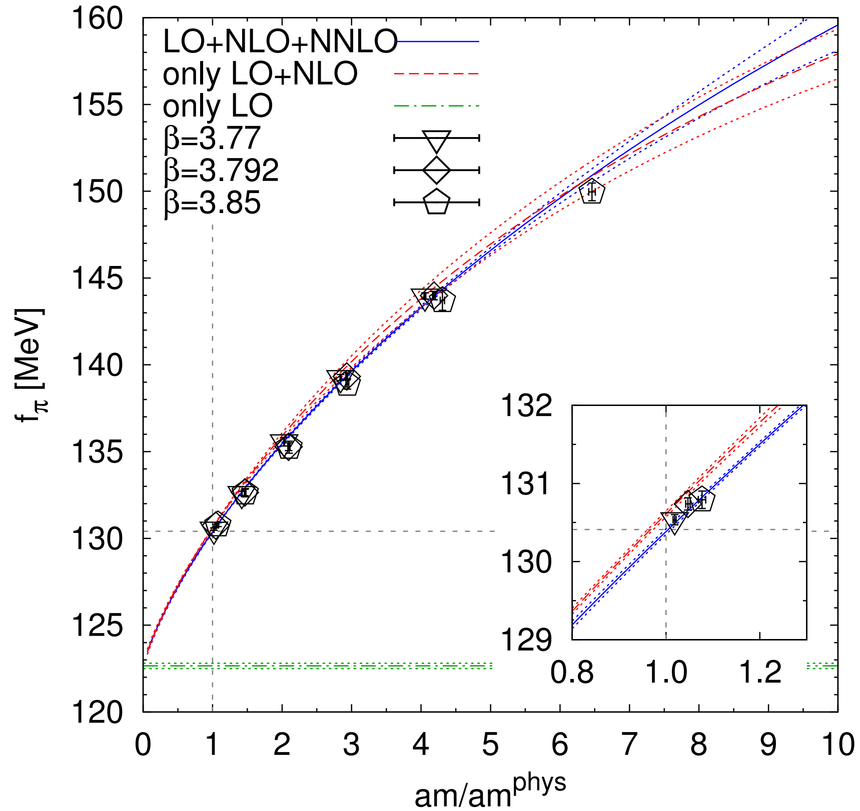
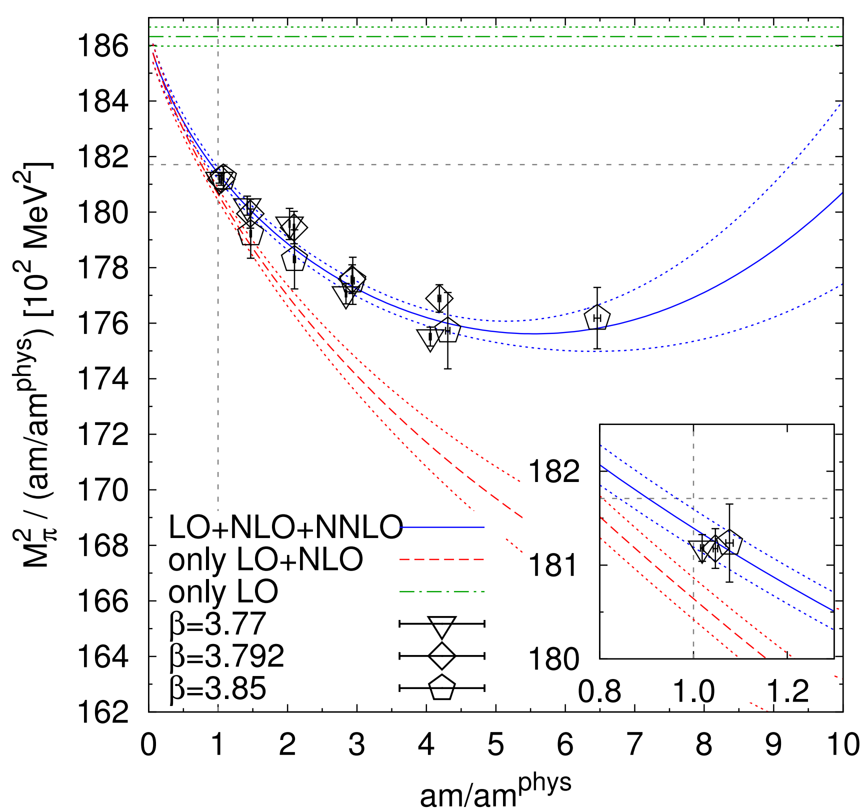
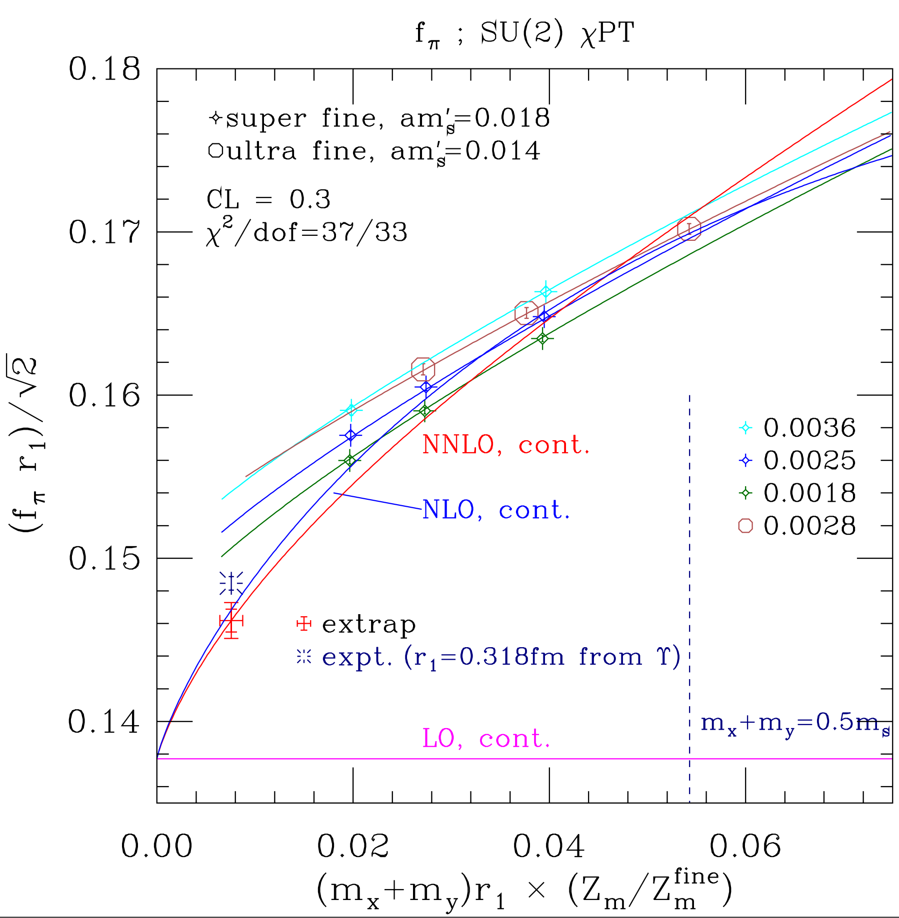
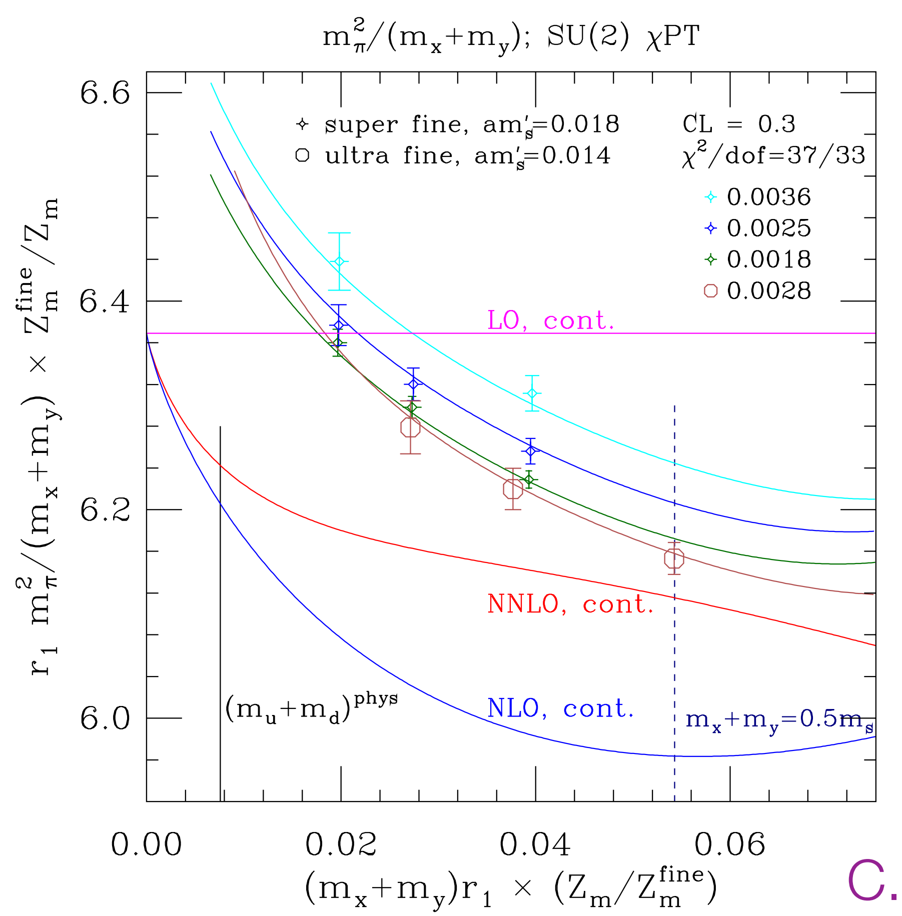




MILC [A. Bazavov et al.], PoS(LAT09), 077, [arXiv: 0911.0472]

2+1; a~0.06 to 0.045 fm





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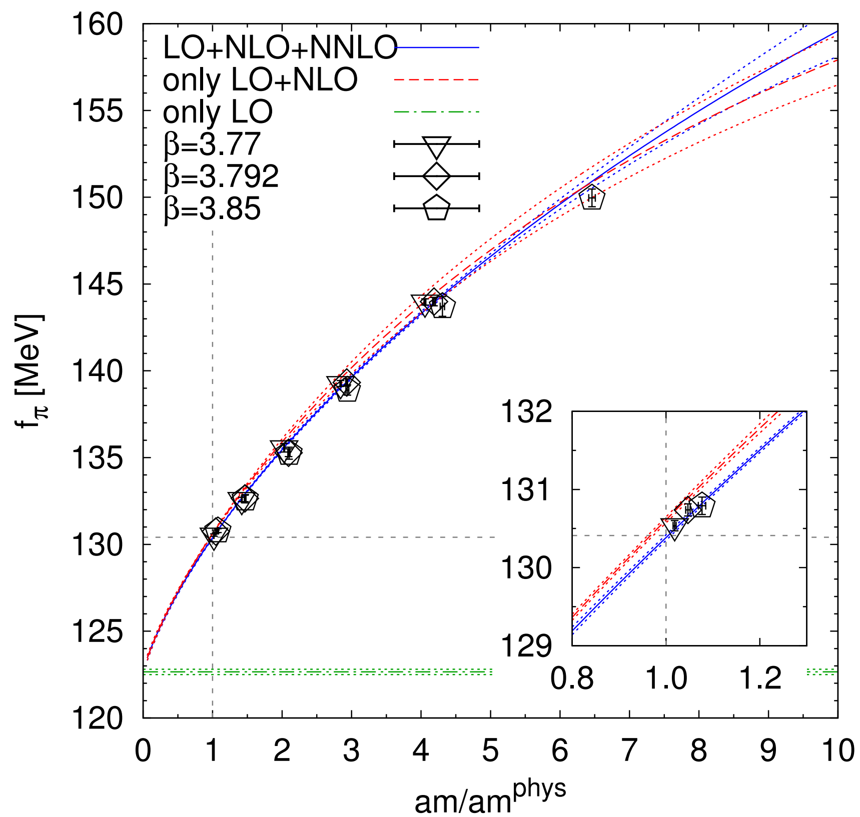

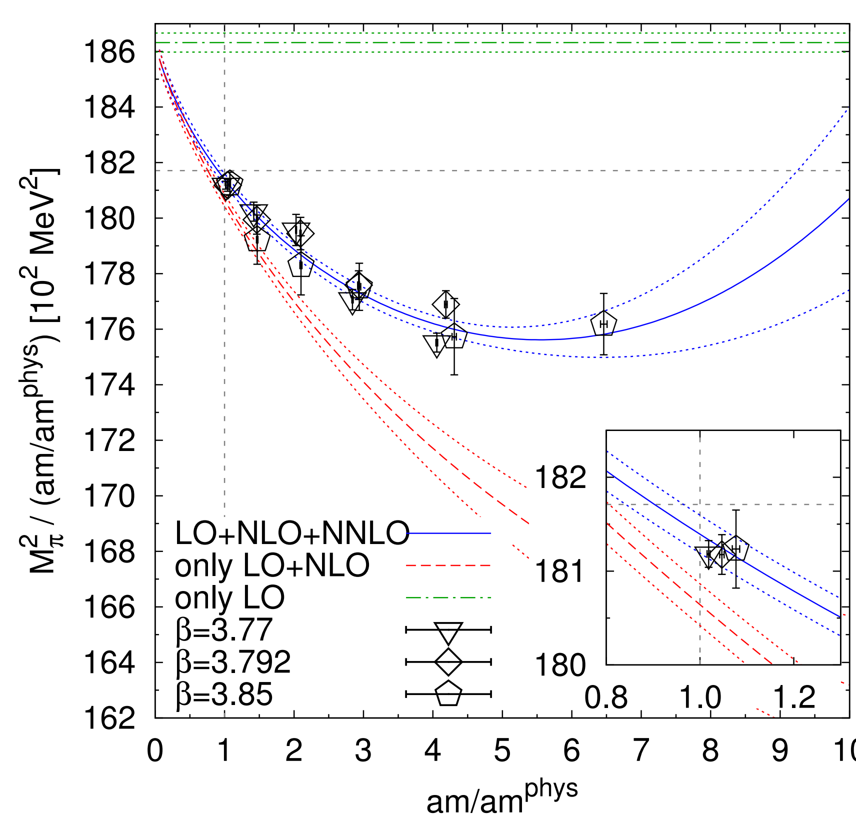

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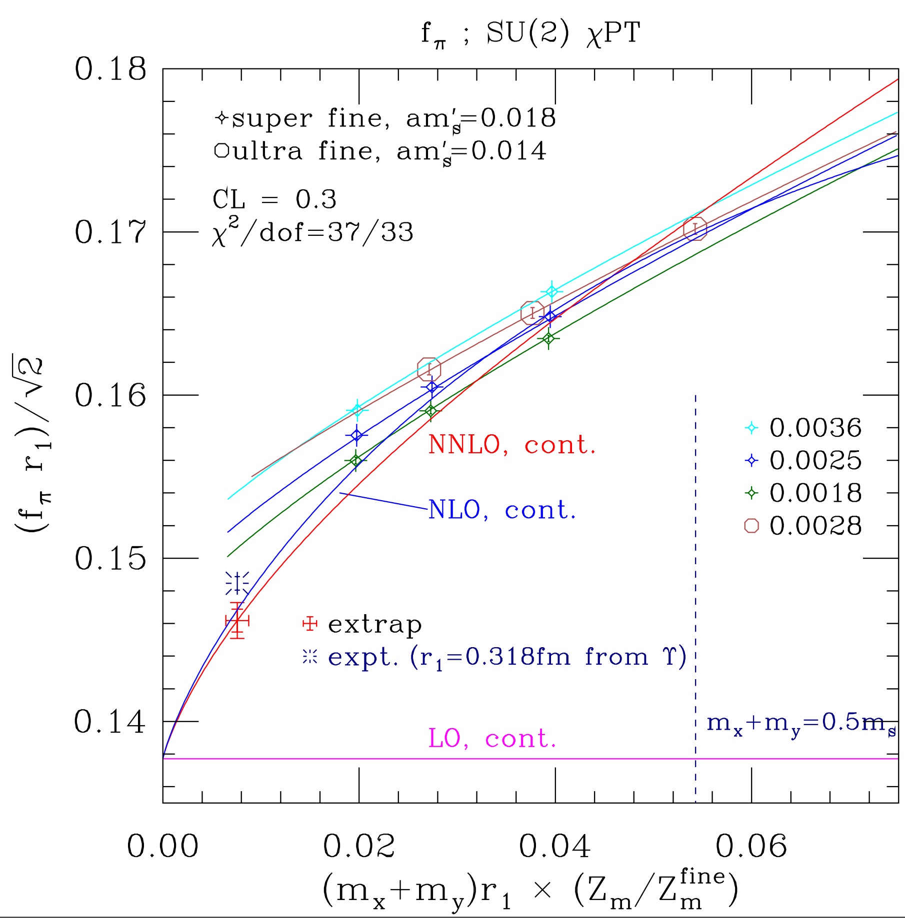

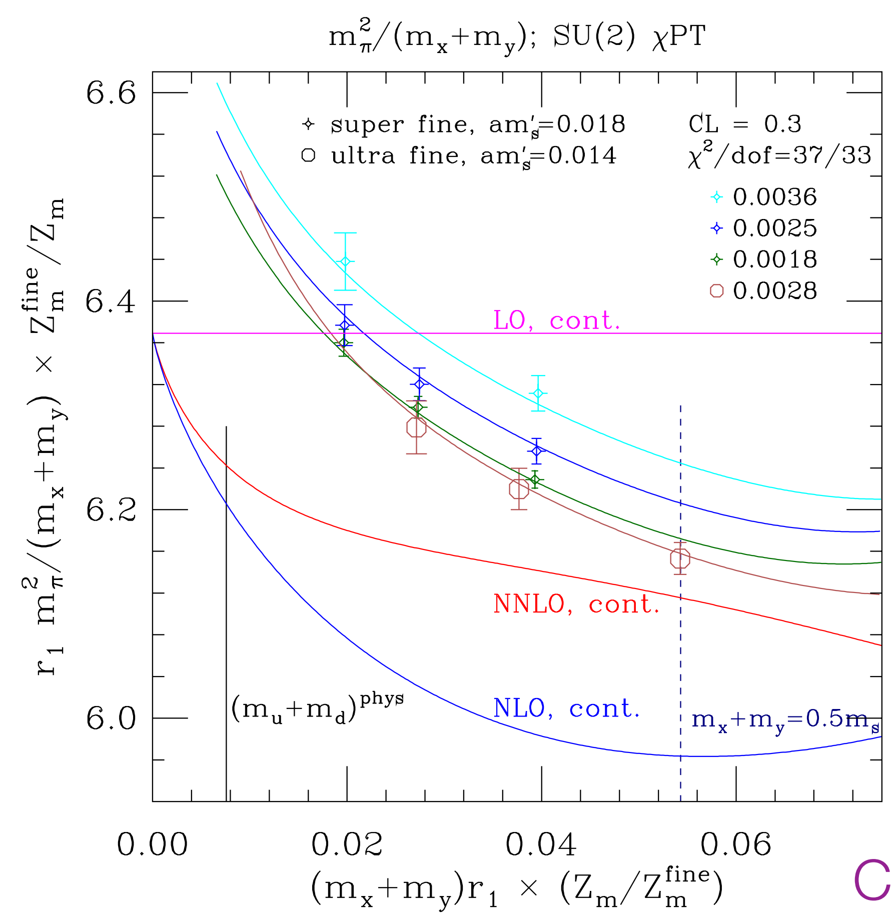

- Convergence good for f_π and reasonable for $M_\pi^2/\hat{m}$.
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- Reasonable agreement between computations.
  - MILC lattice data is partially quenched.

Convergence of ChPT: SU(2)

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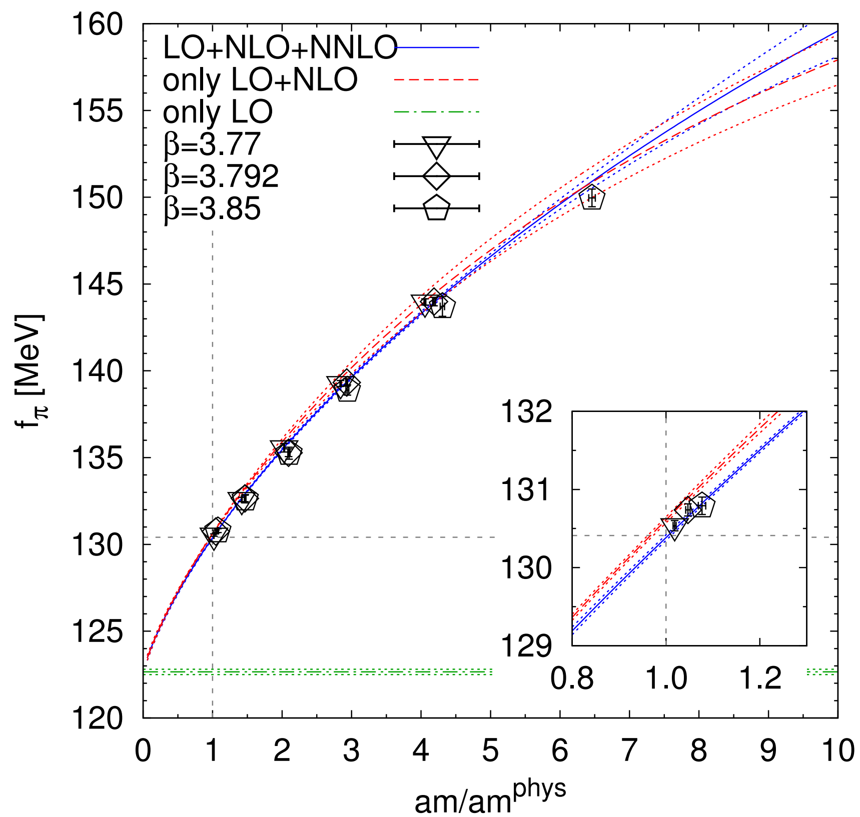

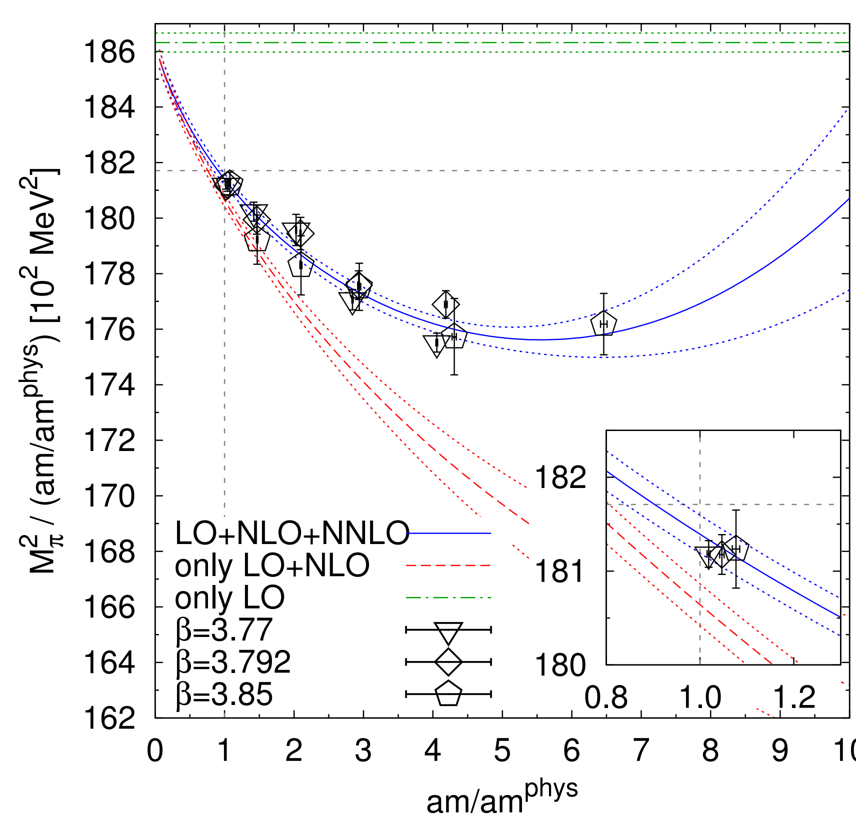

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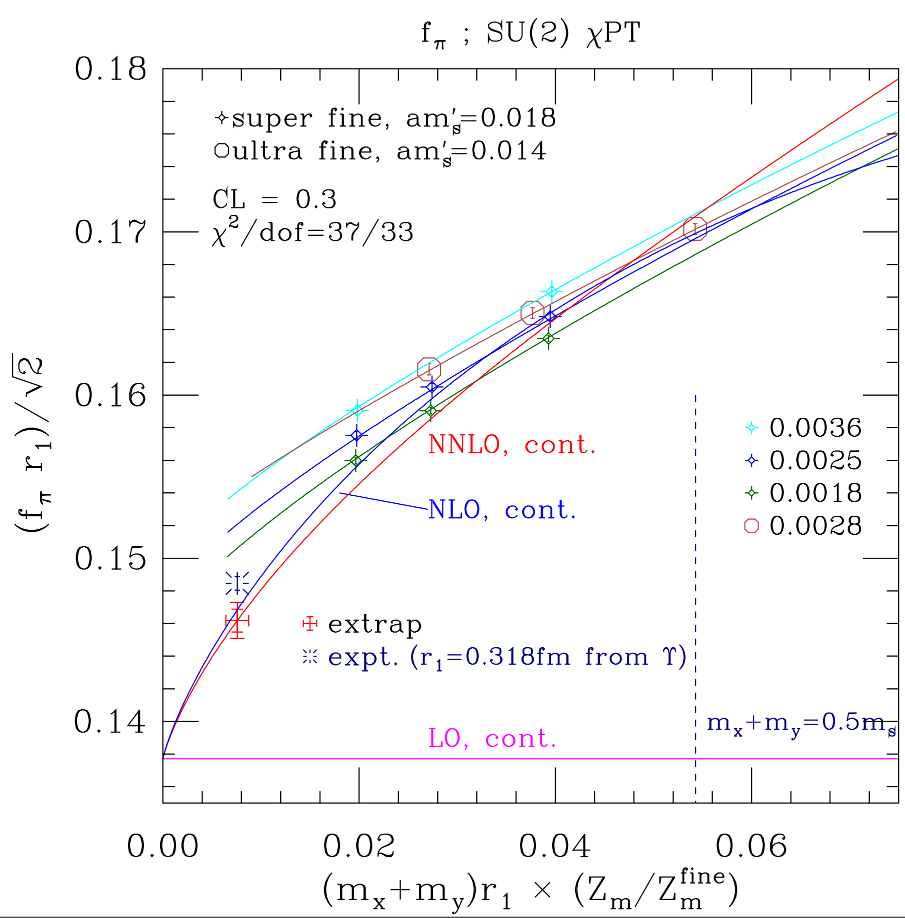

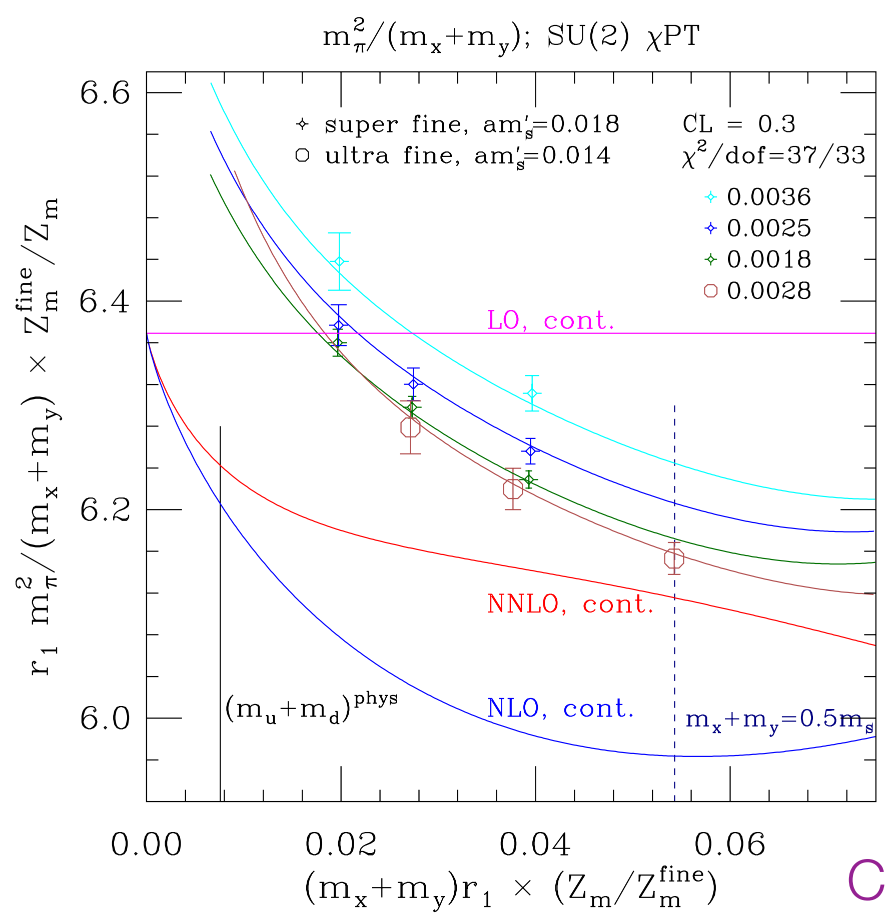

- Convergence good for f_π and reasonable for $M_\pi^2/\hat{m}$.
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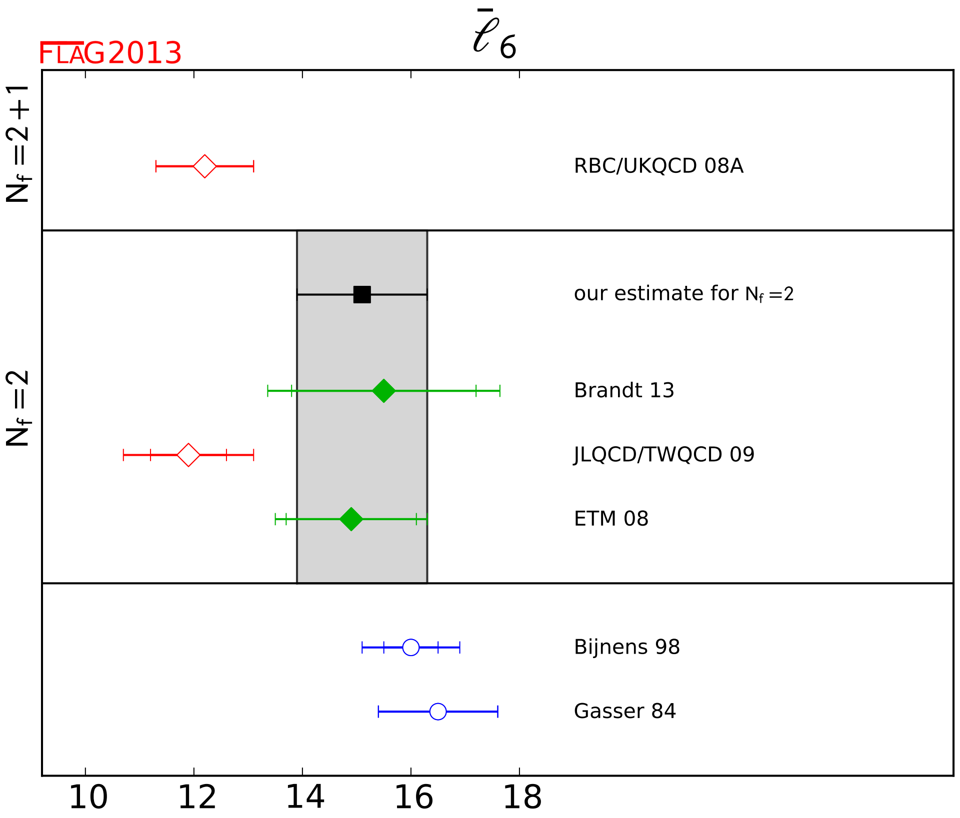
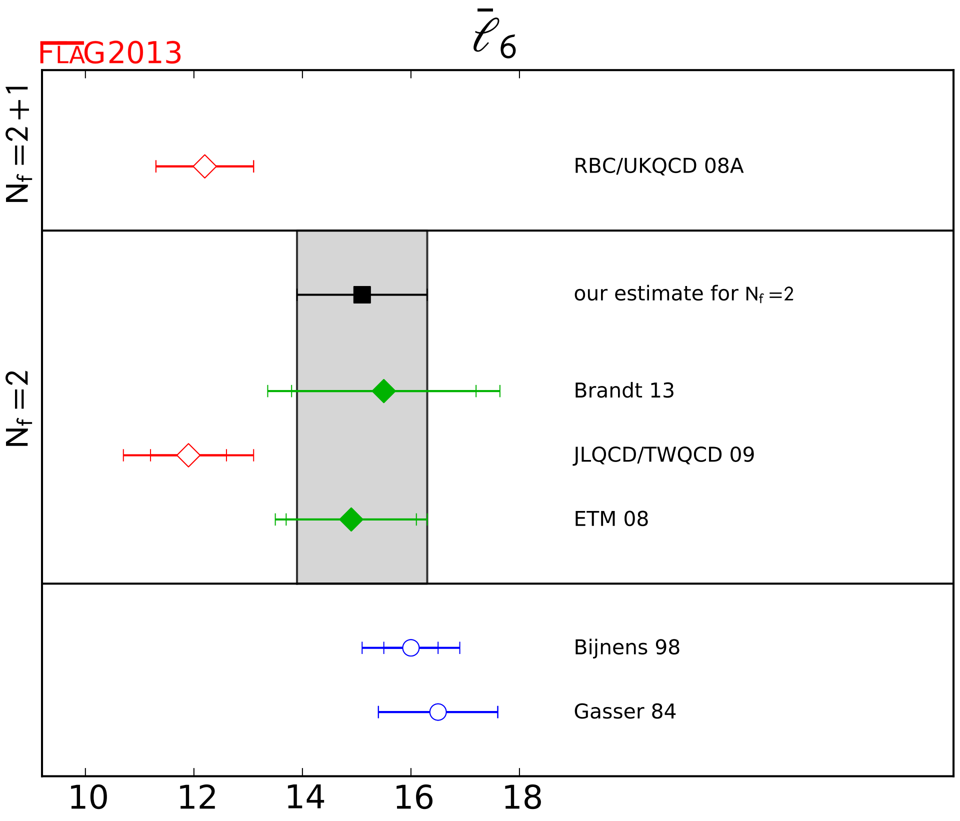
- Reasonable agreement between computations.
  - MILC lattice data is partially quenched.
    - corrected for in lattice ChPT.

SU(2) LECs



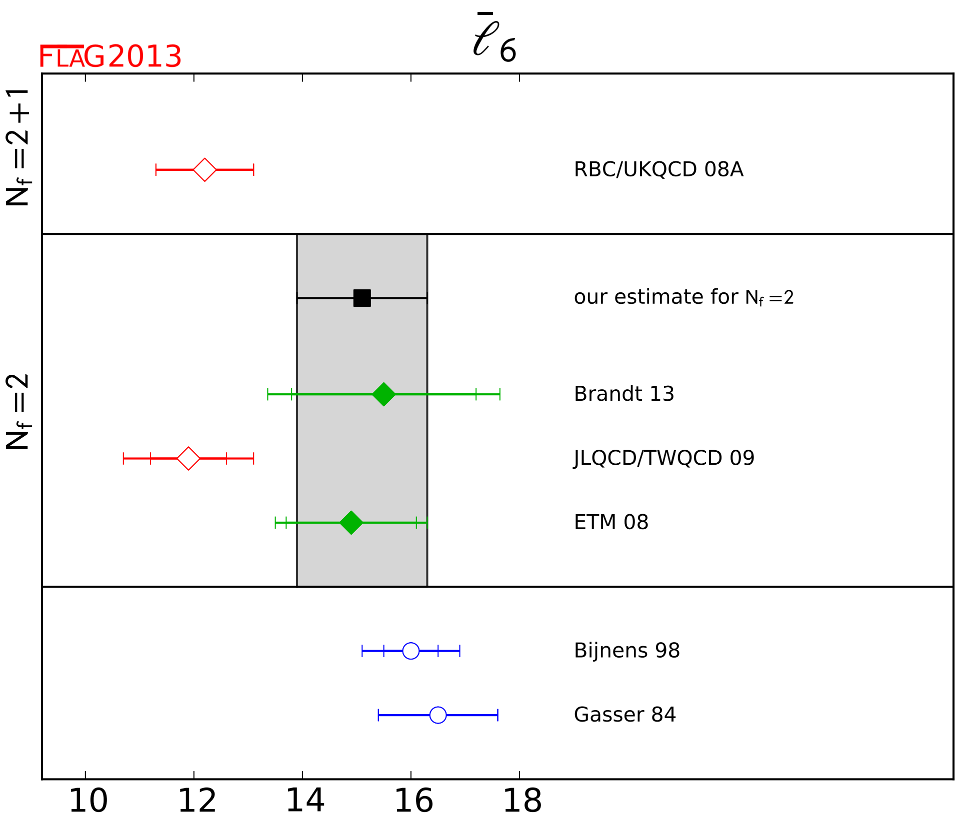
FLAG2013
$\bar{\ell}_6$
$N_f=2+1$
RBC/UKQCD 08A
$N_f=2$
our estimate for $N_f=2$
Brandt 13
JLQCD/TWQCD 09
ETM 08
Bijnens 98
Gasser 84
10
12
14
16
18

SU(2) LECs



$$\bar{\ell}_6 = 15.1(1.2)$$

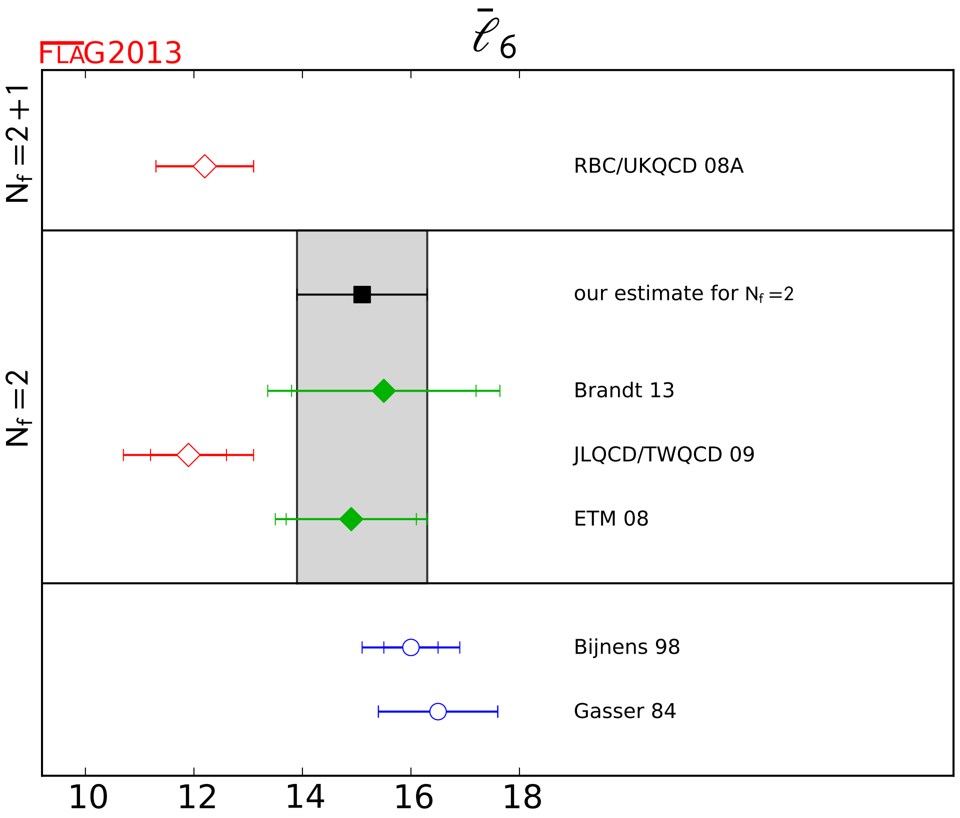
SU(2) LECs



$$\bar{\ell}_6 = 15.1(1.2)$$

- from pion vector form factor.

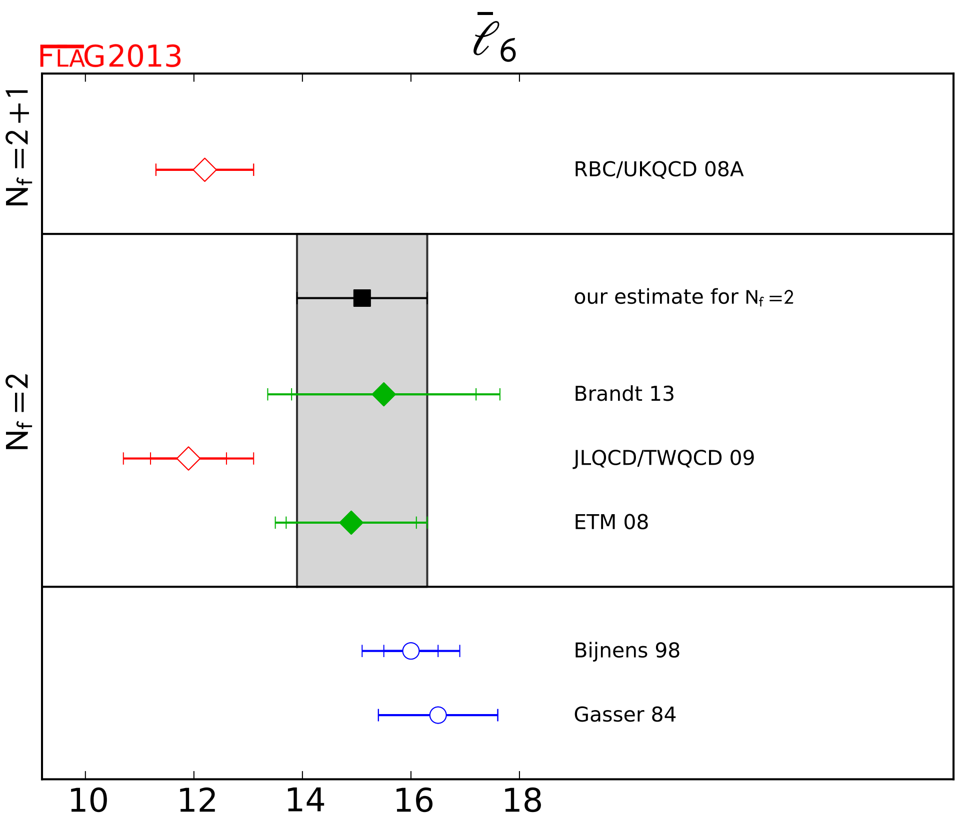
SU(2) LECs



$$\bar{\ell}_6 = 15.1(1.2)$$

- from pion vector form factor.
- result from $N_f = 2$.

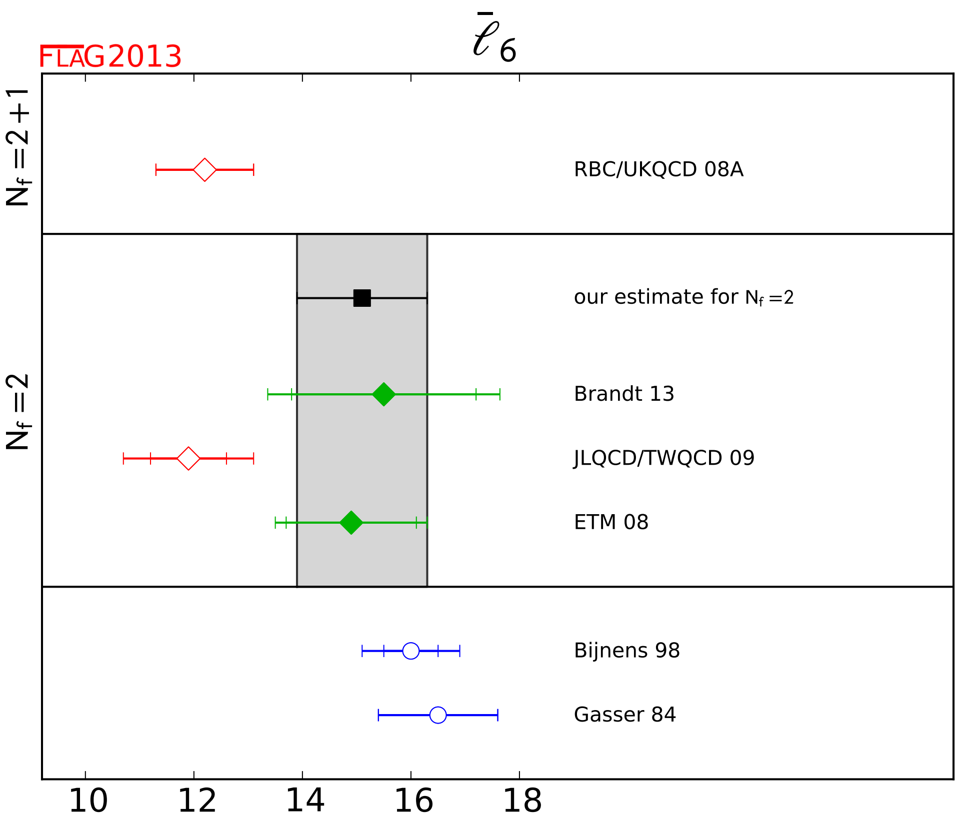
SU(2) LECs



$$\bar{\ell}_6 = 15.1(1.2)$$

- from pion vector form factor.
- result from $N_f = 2$.
- also determines the radius $\langle r^2 \rangle^\pi_{\rm V}$ and the curvature $c_{\rm V}$.

SU(2) LECs



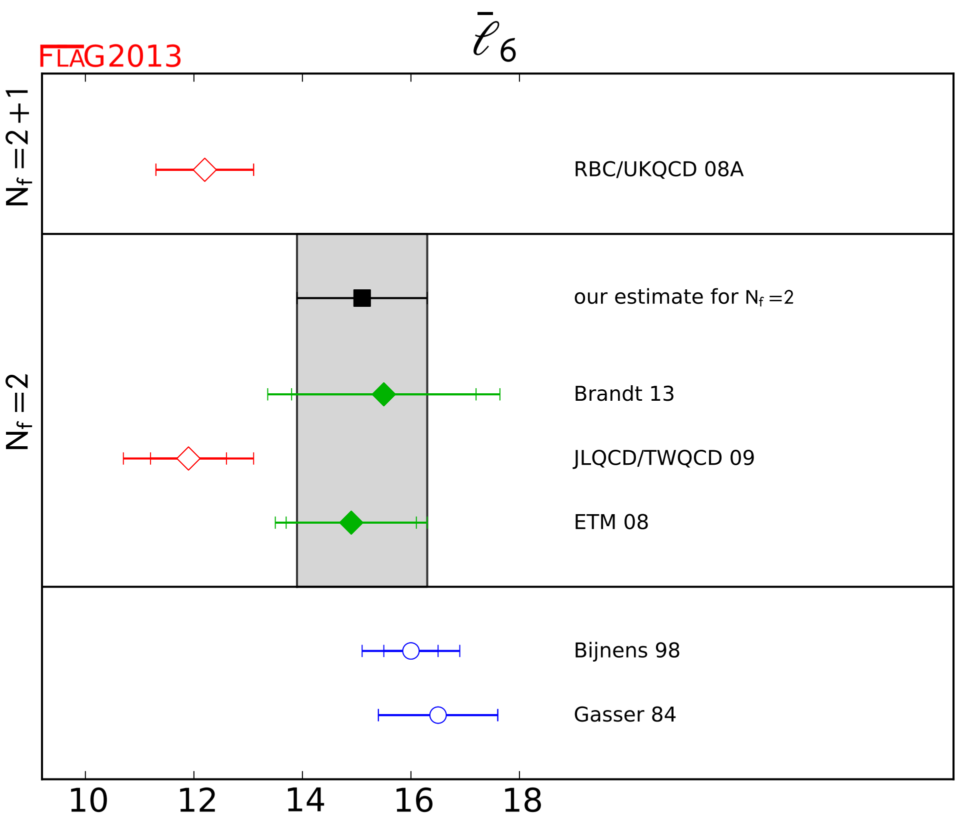
$$\bar{\ell}_6 = 15.1(1.2)$$

- from pion vector form factor.
- result from $N_f = 2$.
- also determines the radius $\langle r^2 \rangle_V^\pi$ and the curvature c_V.

Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	$\langle r^2 \rangle_V^\pi$	c_V	$\bar{\ell}_6$
RBC/UKQCD 08A	[252]	2+1	A	○	■	★	0.418(31)		12.2(9)
LHP 04	[274]	2+1	A	○	■	○	0.310(46)		
Brandt 13	[256]	2	A	○	★	★	0.481(33)(13)		15.5(1.7)(1.3)
JLQCD/TWQCD 09	[270]	2	A	○	■	■	0.409(23)(37)	3.22(17)(36)	11.9(0.7)(1.0)
ETM 08	[237]	2	A	○	○	○	0.456(30)(24)	3.37(31)(27)	14.9(1.2)(0.7)
QCDSF/UKQCD 06A	[275]	2	A	○	★	○	0.441(19)(56)(29)		
Bijnens 98	[236]						0.437(16)	3.85(60)	16.0(0.5)(0.7)
NA7 86	[276]						0.439(8)		
Gasser 84	[58]								16.5(1.1)

From, FLAG [S. Aoki et al.], Eur. Phys. J. C 74, 2890 (2014).

SU(2) LECs



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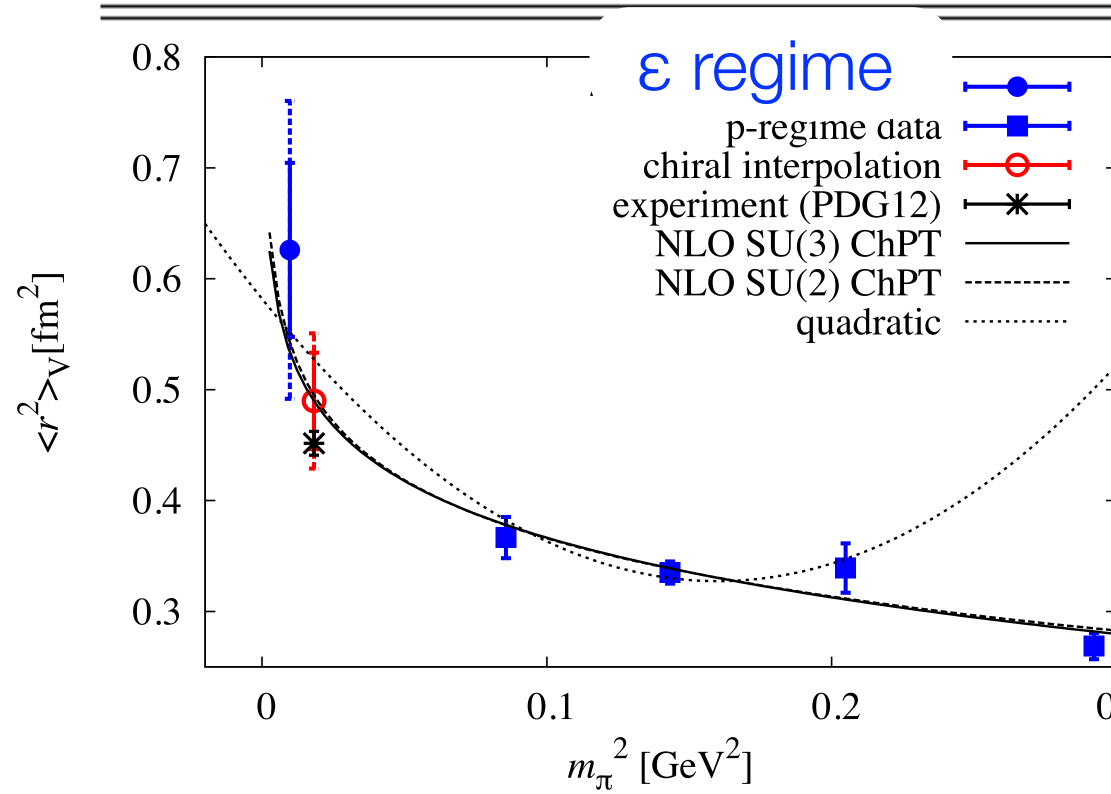
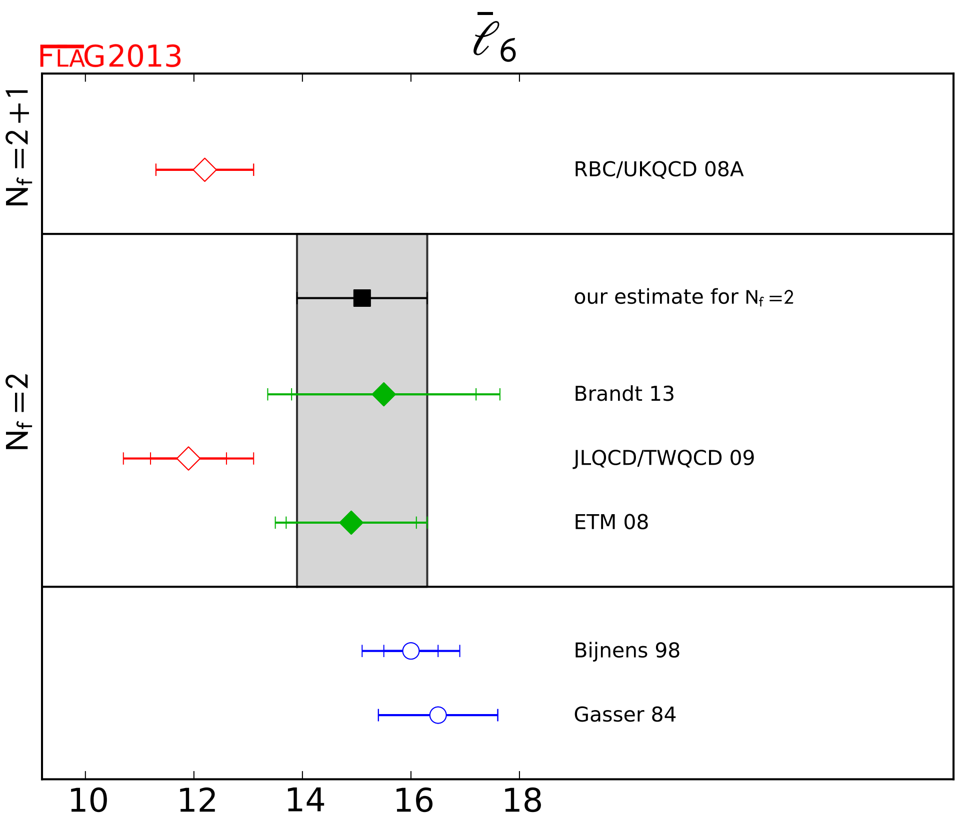


H. Fukaya, et al., PRD 90, 034506 (2014); $N_f = 2+1$

SU(2) LECs



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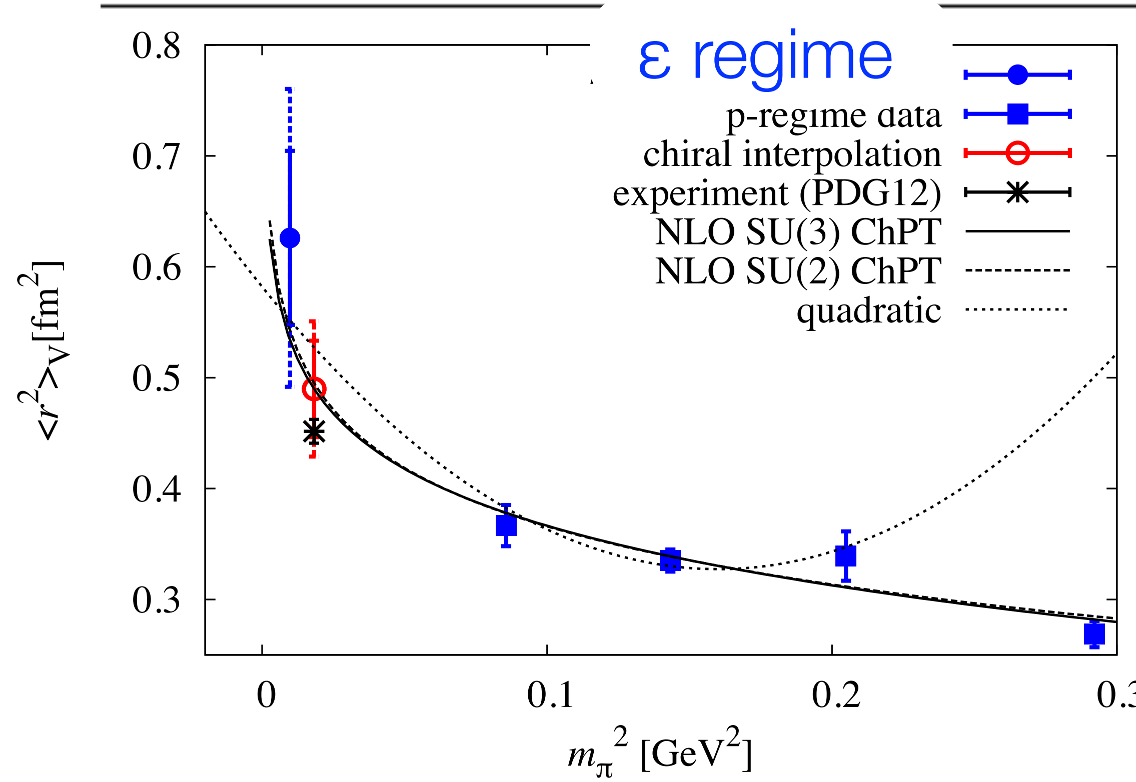
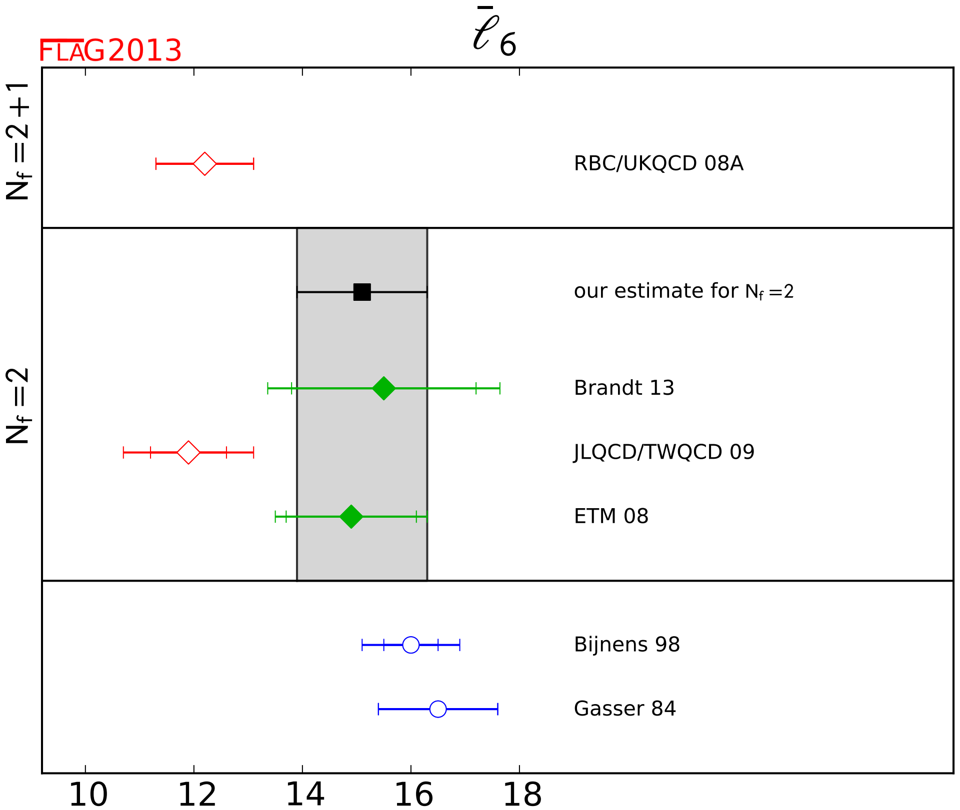


$$\langle r^2\rangle_V = 0.49(4)(4)\ \mathrm{fm}^2$$
$$\bar{\ell}_6 = 7.5(1.3)(1.5)$$
$$F_\pi/F = 1.6(2)(3)$$

H. Fukaya, et al., PRD 90, 034506 (2014); $N_f = 2+1$

SU(2) LECs



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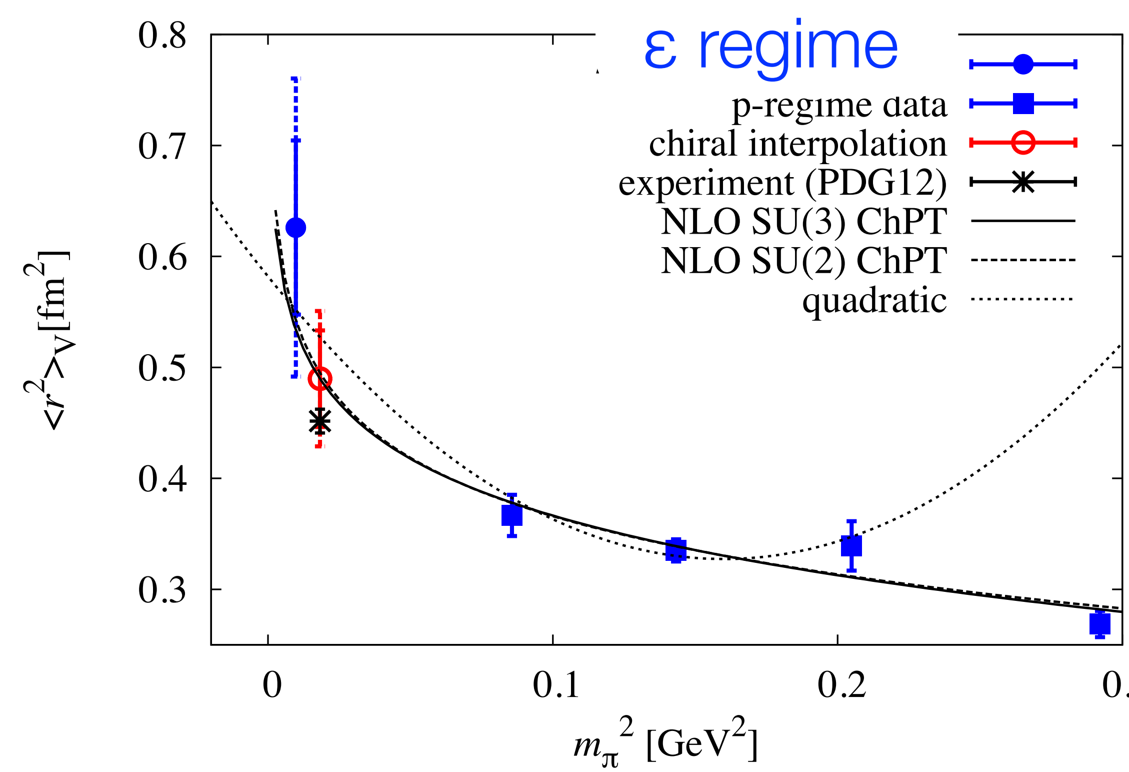


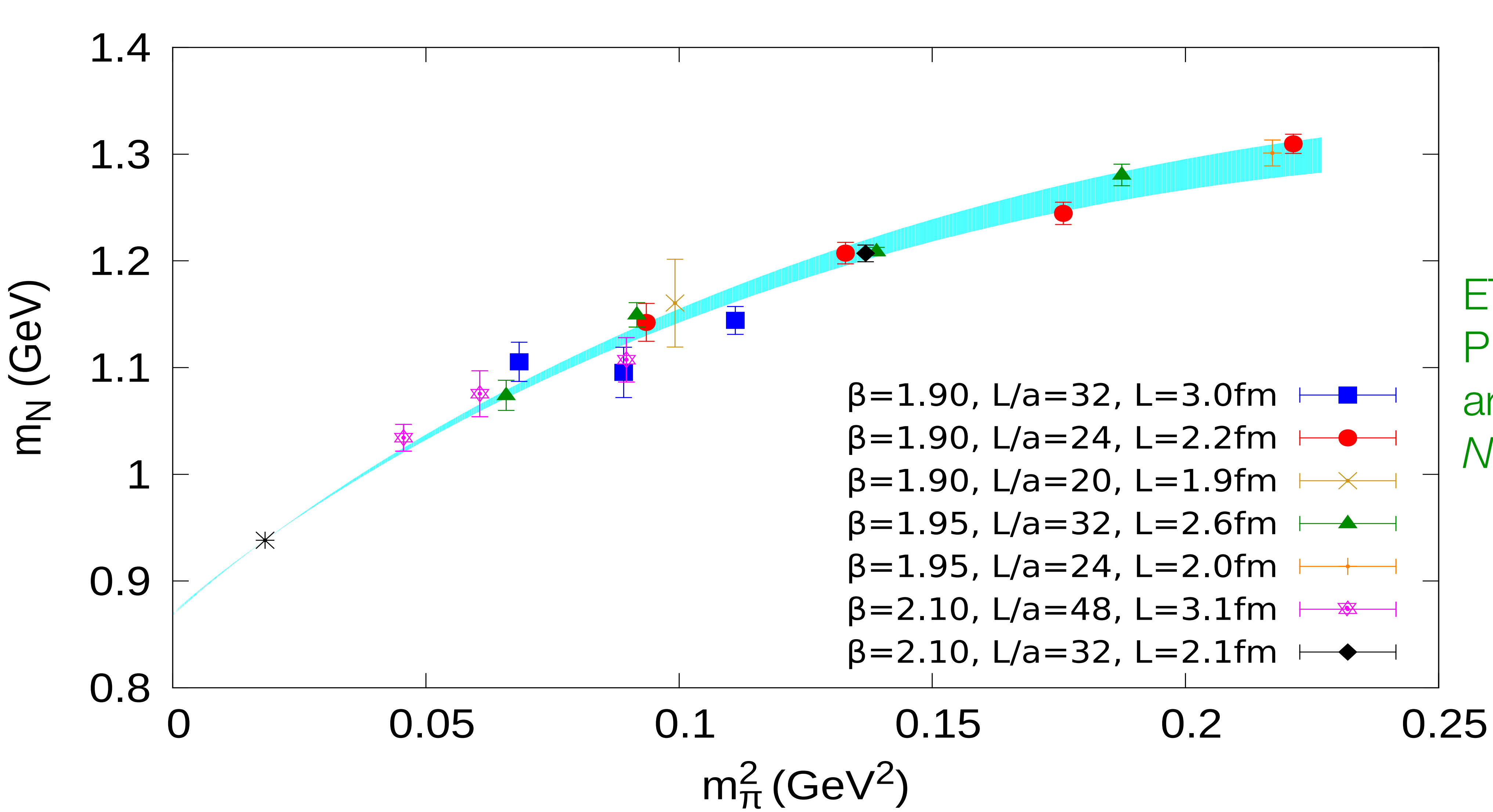
$$\langle r^2\rangle_V = 0.49(4)(4)\ \mathrm{fm}^2$$
$$\bar{\ell}_6 = 7.5(1.3)(1.5)$$
$$F_\pi/F = 1.6(2)(3)$$

radius ok but F and $\bar{\ell}_6$ low: problems with finite volume effects (fixed topology) and/or NNLO terms?

H. Fukaya, et al., PRD 90, 034506 (2014); $N_f = 2+1$

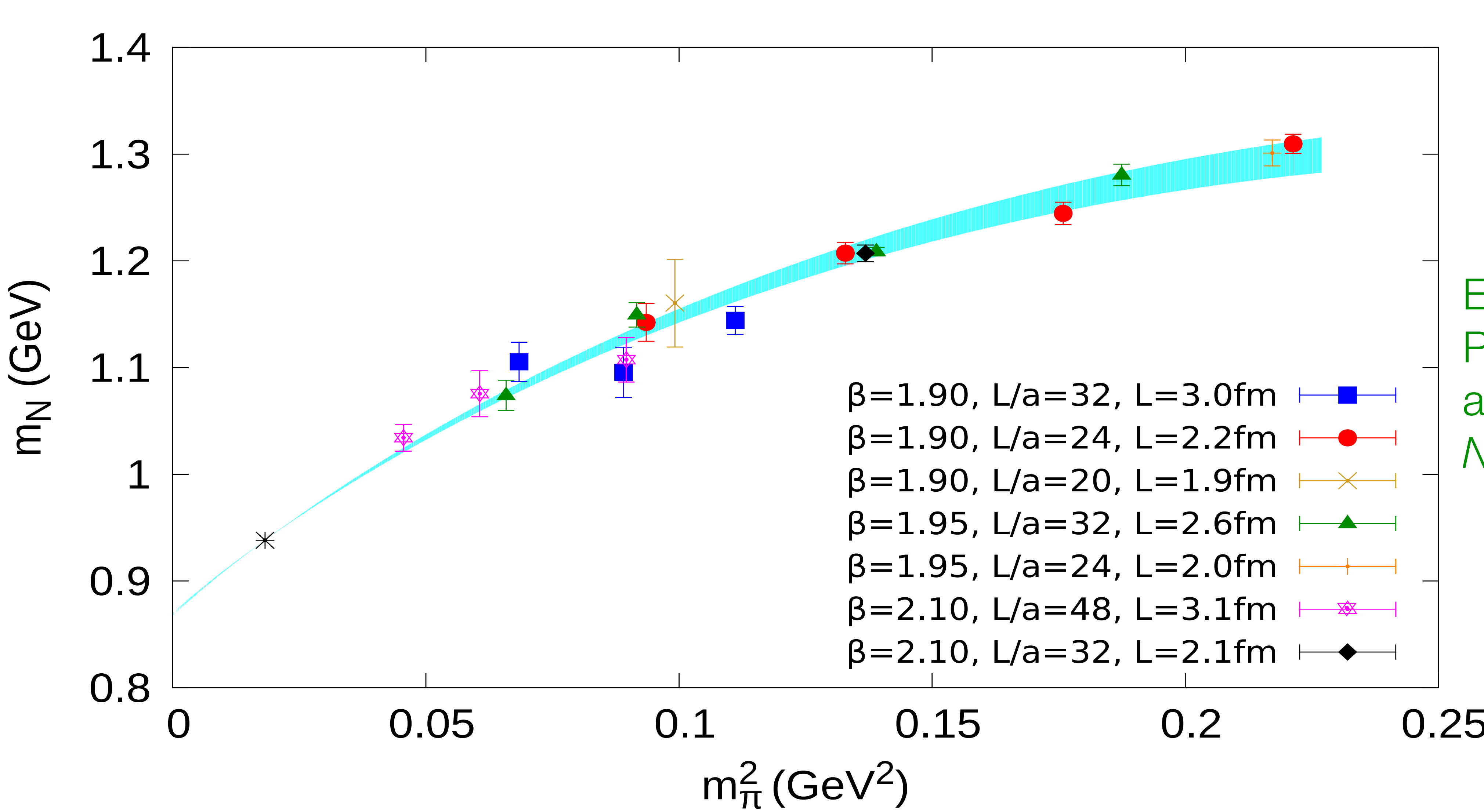
Nucleon Chiral Extrapolation

Nucleon Chiral Extrapolation



ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310; N_f = 2+1+1

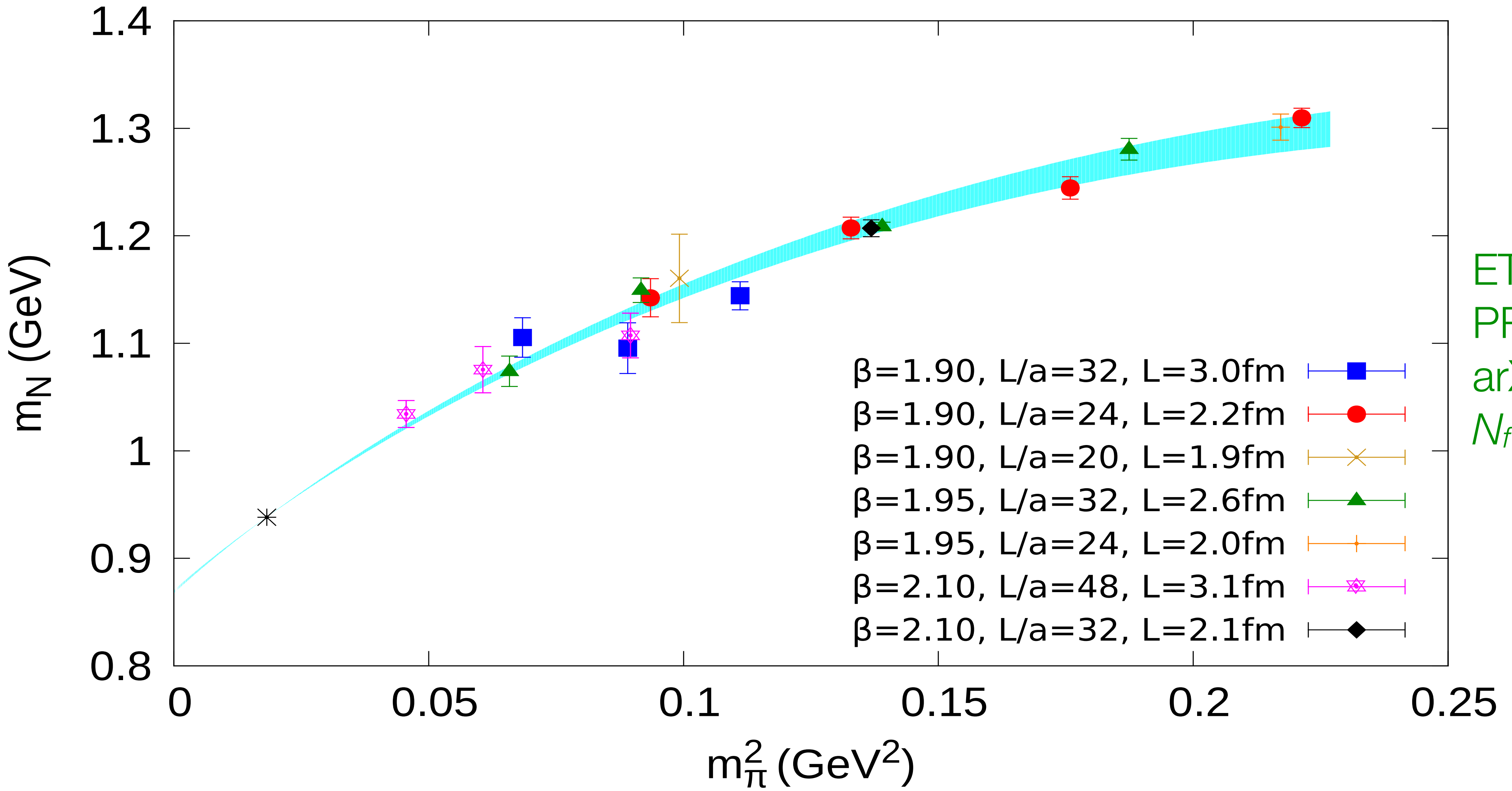
Nucleon Chiral Extrapolation



ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310; N_f = 2+1+1

- Fit is to NLO SU(2) HBChPT: $m_N = m_N^{(0)} - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi F_\pi^2} m_\pi^3$

Nucleon Chiral Extrapolation



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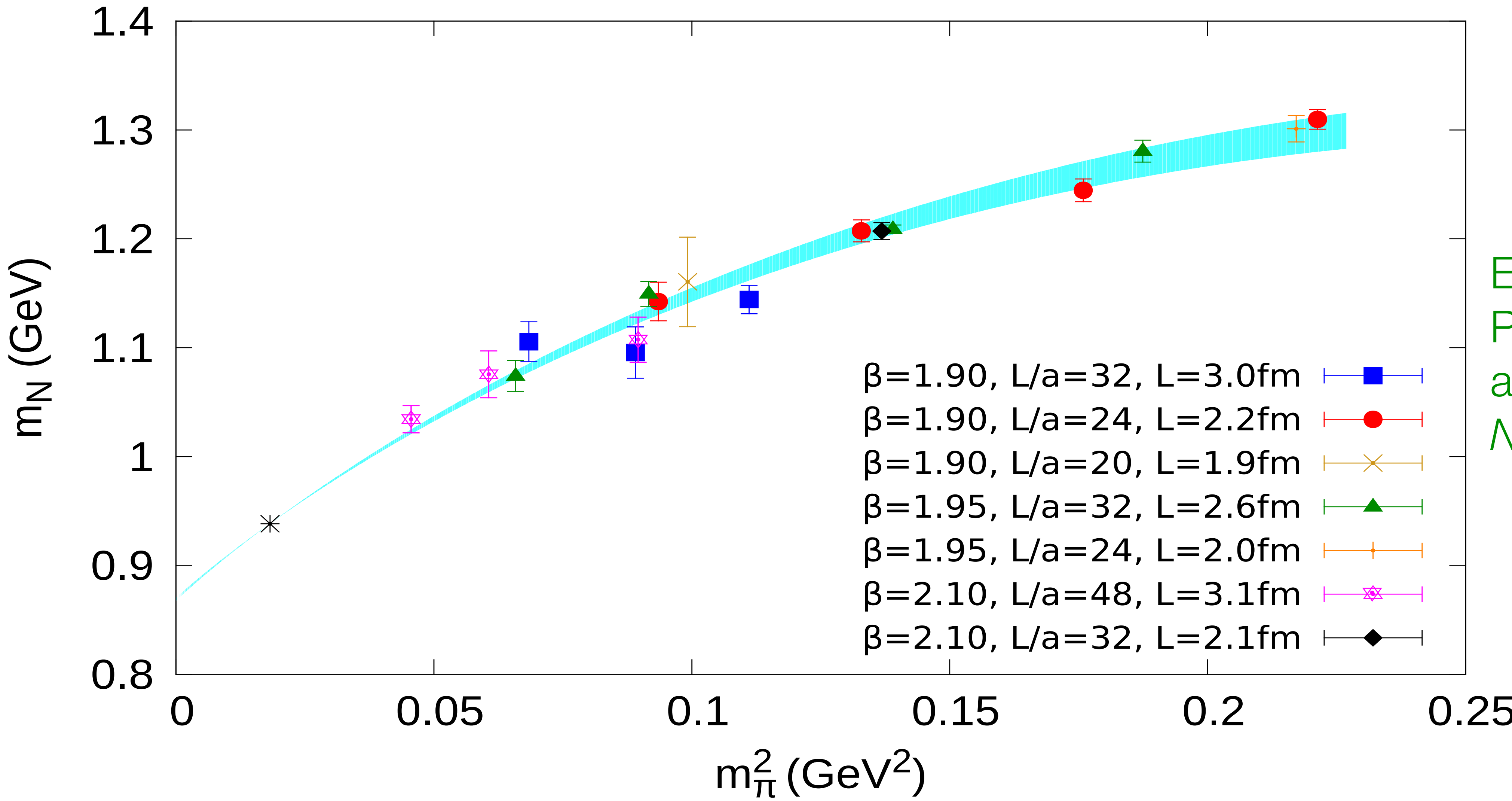
  - constrained to go through physical point.

Nucleon Chiral Extrapolation



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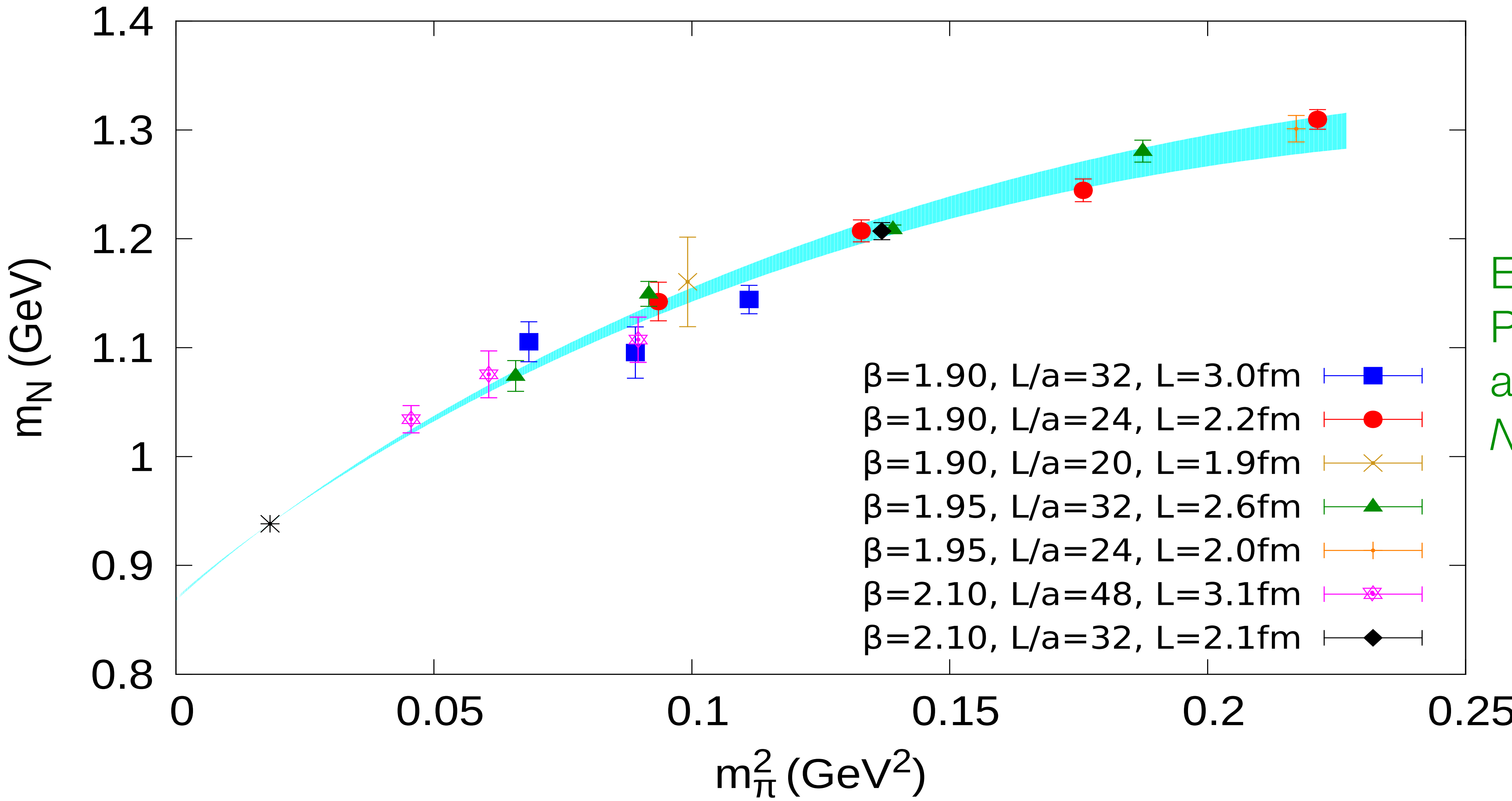
  - constrained to go through physical point.
- c_1 ~ -1.1 is large but not crazy: m_N changes by ~40% from physical point to $m_\pi = m_K$.

Nucleon Chiral Extrapolation



ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310;
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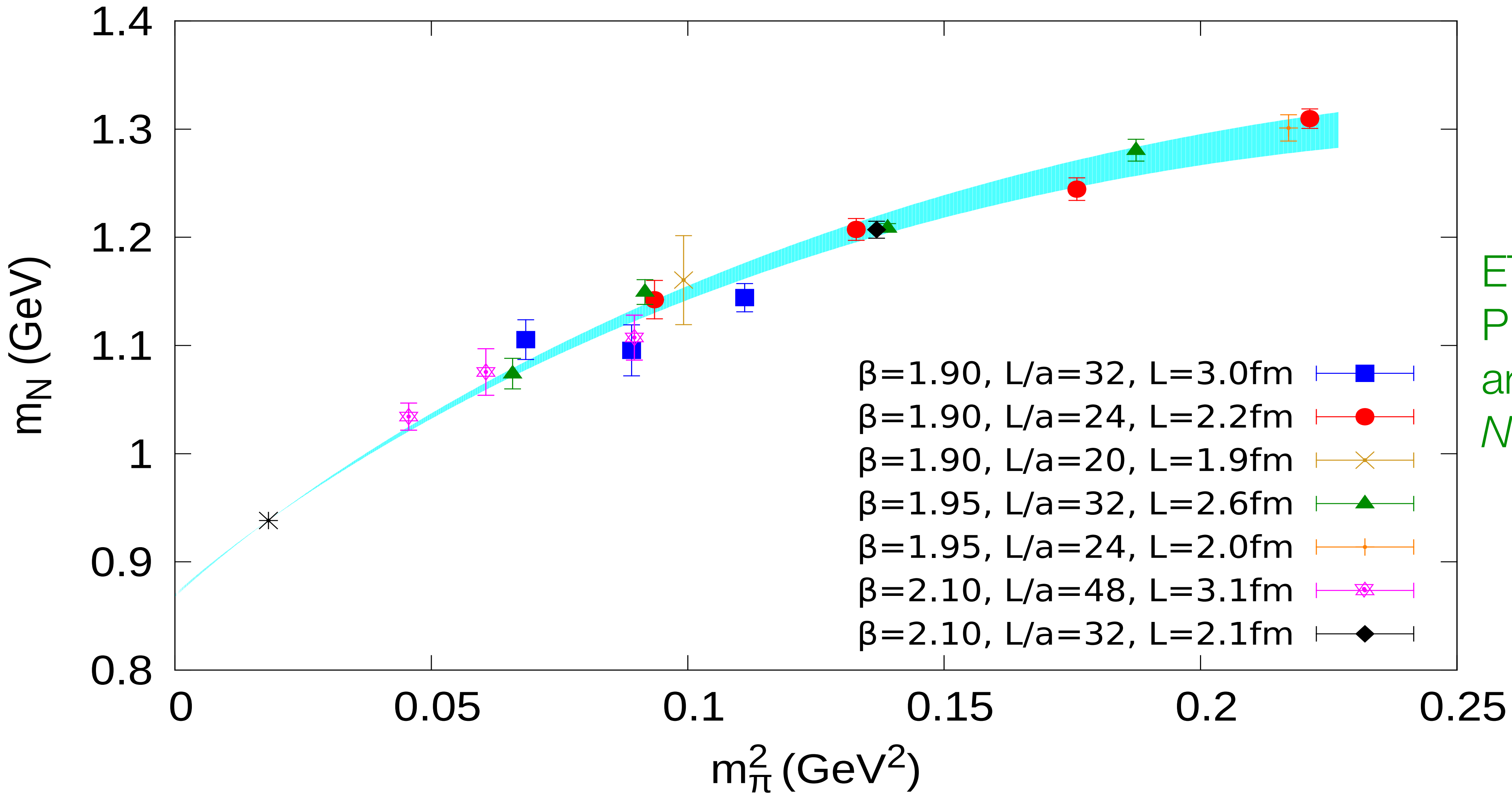
  - about twice as much as $f_\pi \rightarrow f_K$.

Nucleon Chiral Extrapolation



ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310; N_f = 2+1+1

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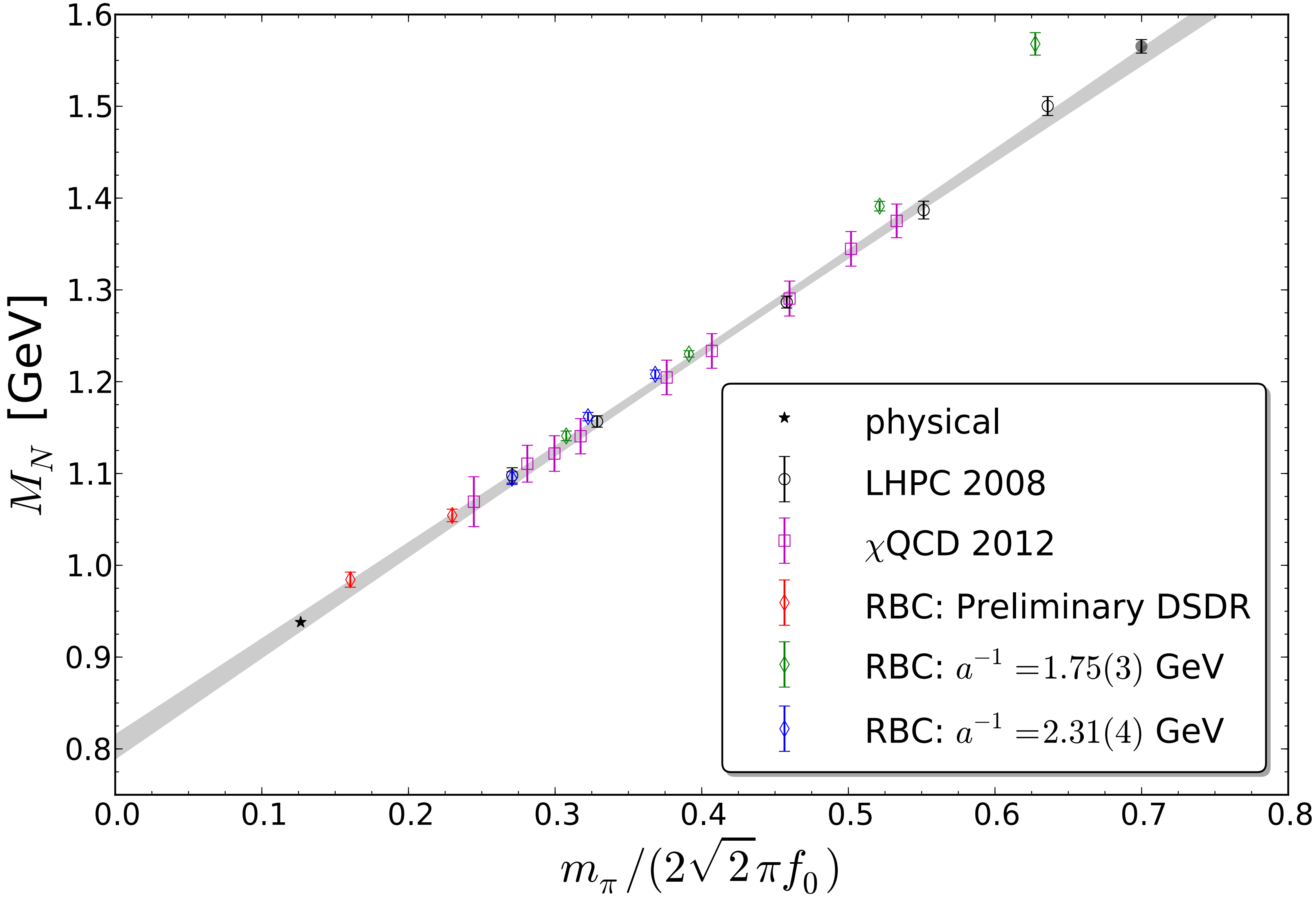
  - constrained to go through physical point.
- c_1 ~ -1.1 is large but not crazy: m_N changes by ~40% from physical point to $m_\pi = m_K$.
  - about twice as much as $f_\pi \rightarrow f_K$.
- Fit including Δ is very similar.

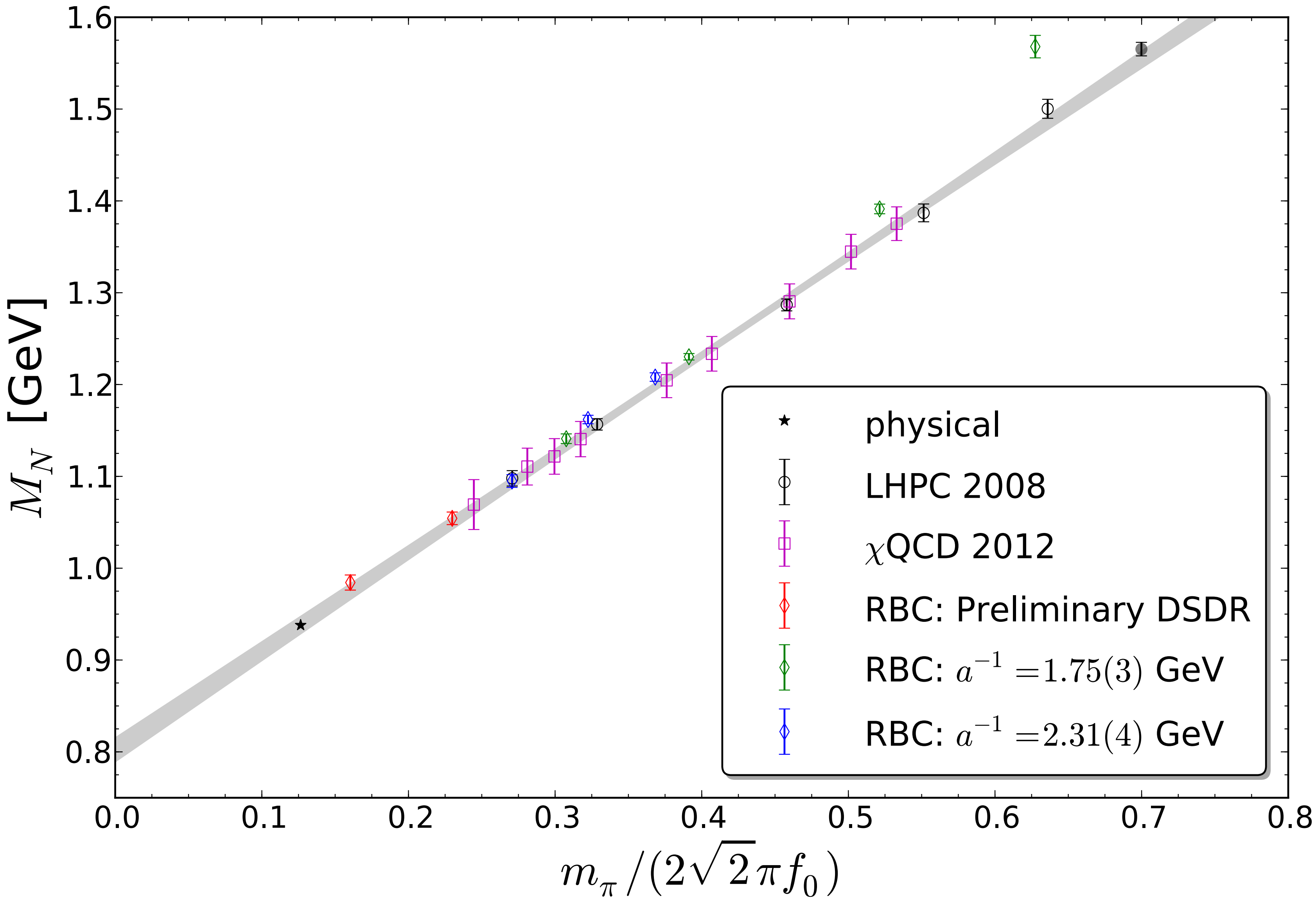
Nucleon Chiral Extrapolation

Nucleon Chiral Extrapolation



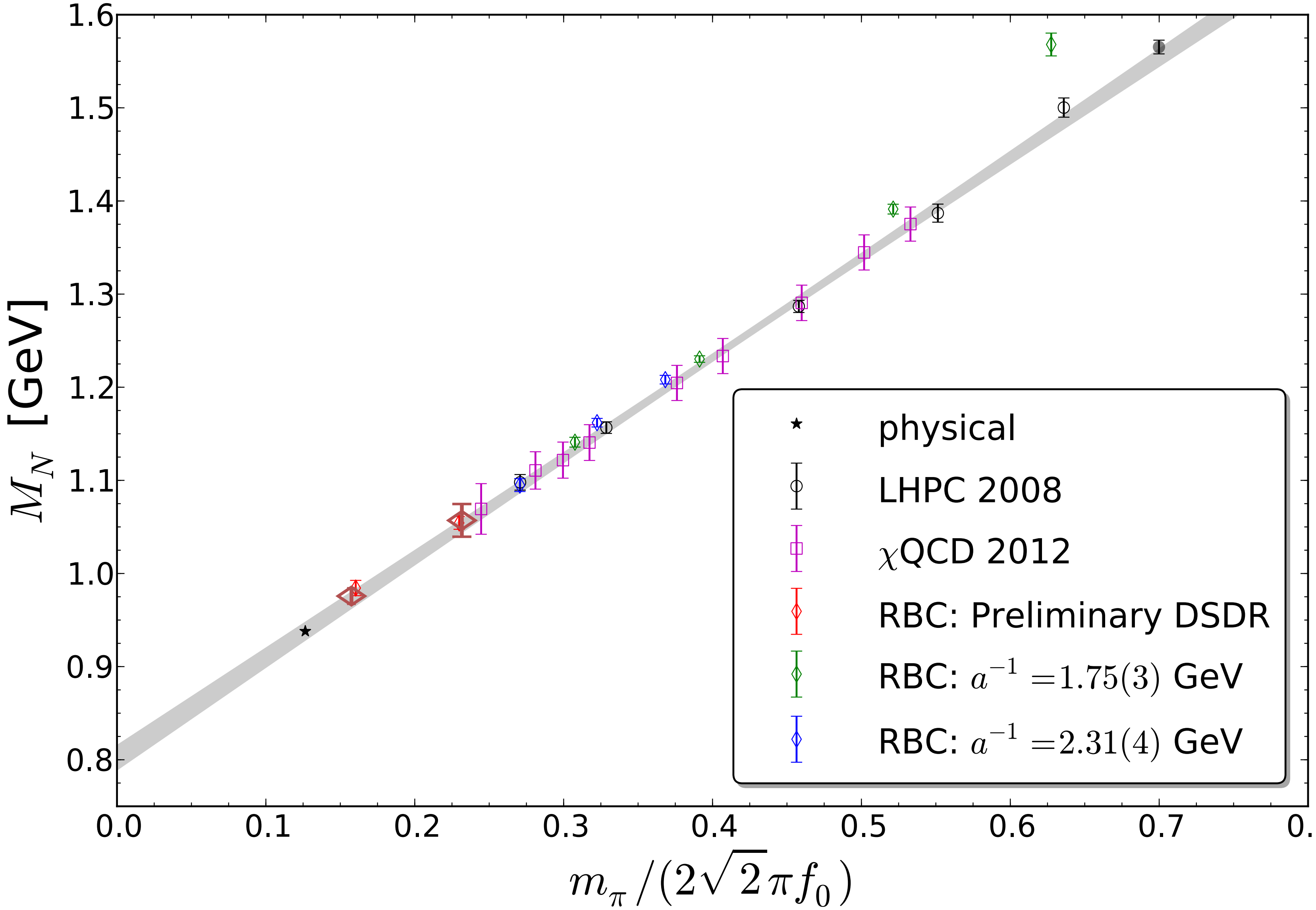
- Walker-Loud, Lattice '13, arXiv:1401.8259.

Nucleon Chiral Extrapolation



- Walker-Loud, Lattice '13, arXiv:1401.8259.
- "Ruler plot" [named by B. Tiburzi].

Nucleon Chiral Extrapolation



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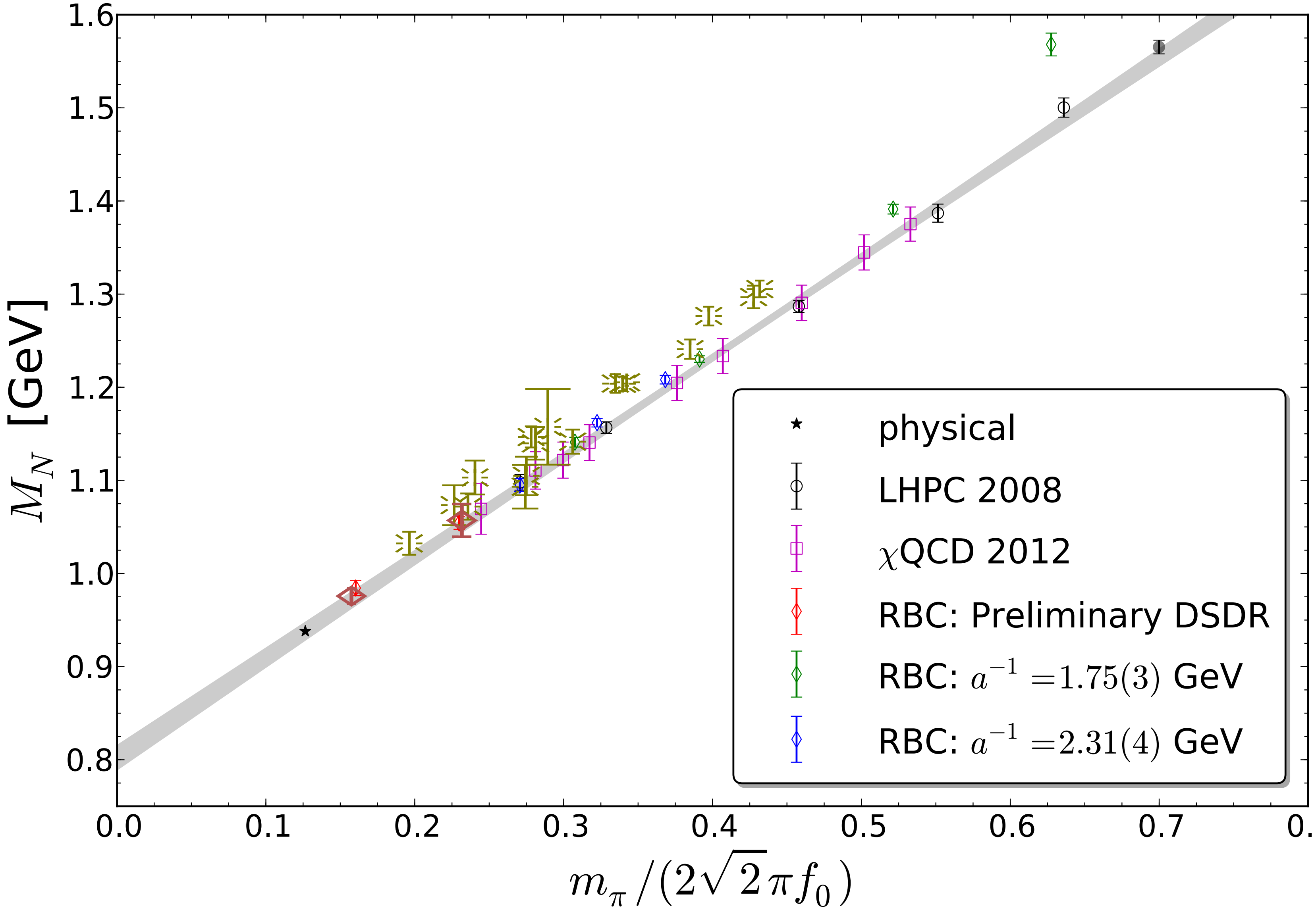
- Update of RBC points.*

*thanks to T. Blum for unpublished data.

Nucleon Chiral Extrapolation



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- ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310.

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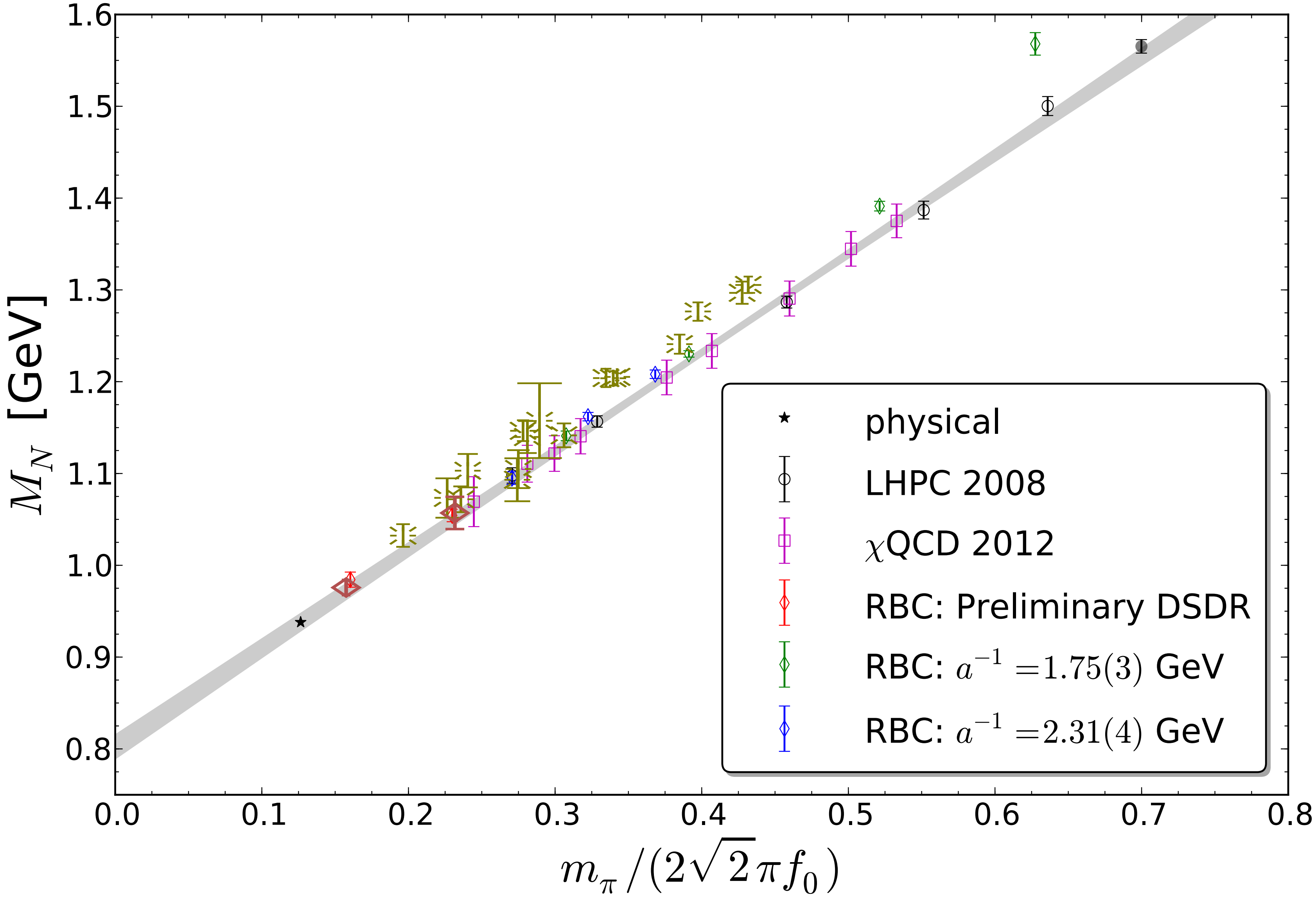
Nucleon Chiral Extrapolation



- Walker-Loud, Lattice ’13, arXiv:1401.8259.
- “Ruler plot” [named by B. Tiburzi].
- Update of RBC points.*
- ETM [C. Alexandrou et al.], PRD 90, 074501 (2014), arXiv:1406.4310.
- Seems to be a curious accident; doesn’t contradict expected chiral behavior.

*thanks to T. Blum for unpublished data.

SU(3) LECs

✦ L_{10} from lattice+continuum: P. Boyle et al., PRD 89, 094510 (2014).

SU(3) LECs

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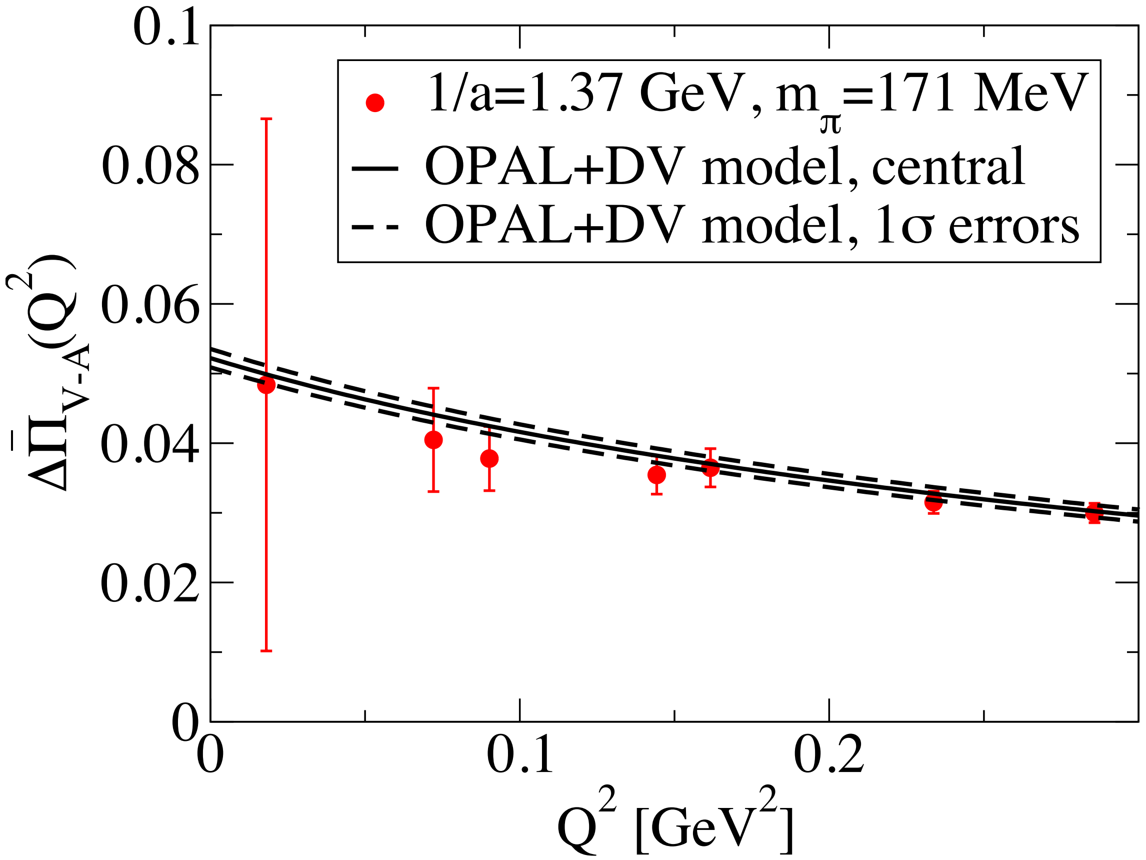

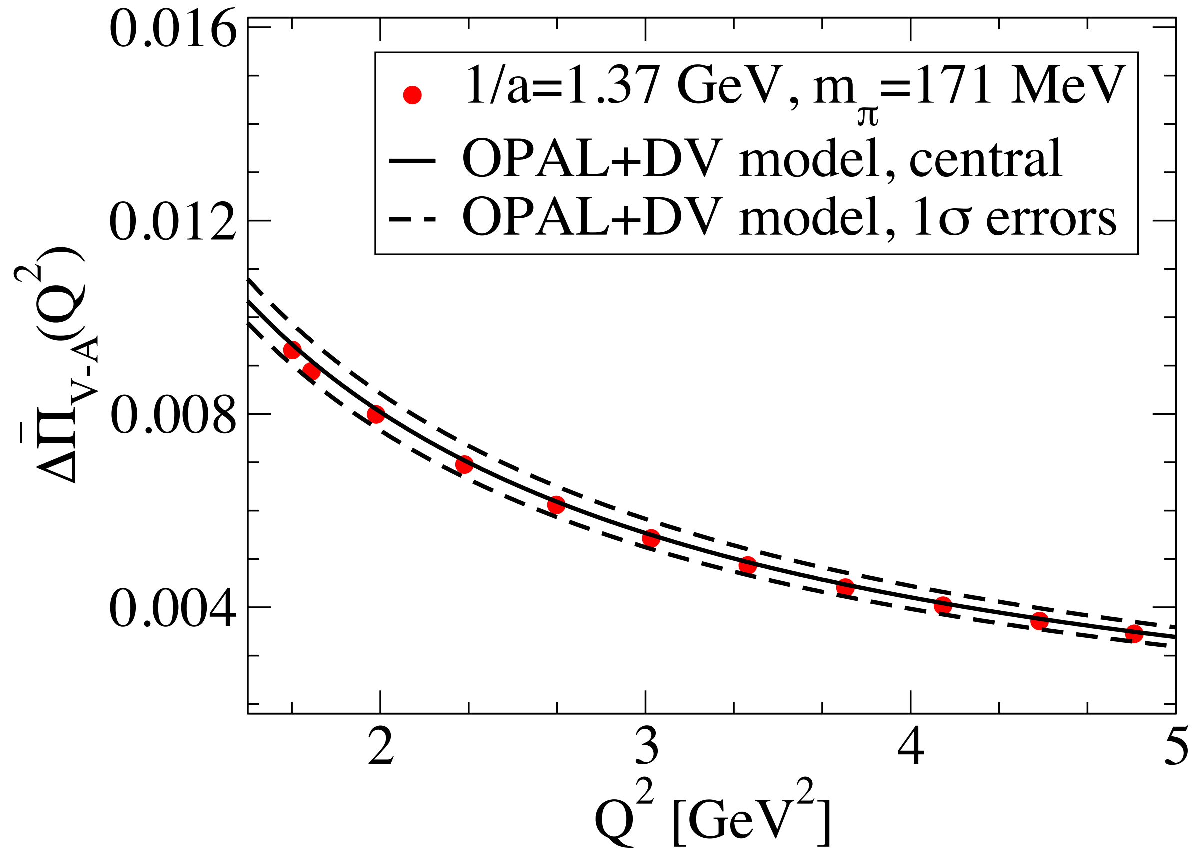

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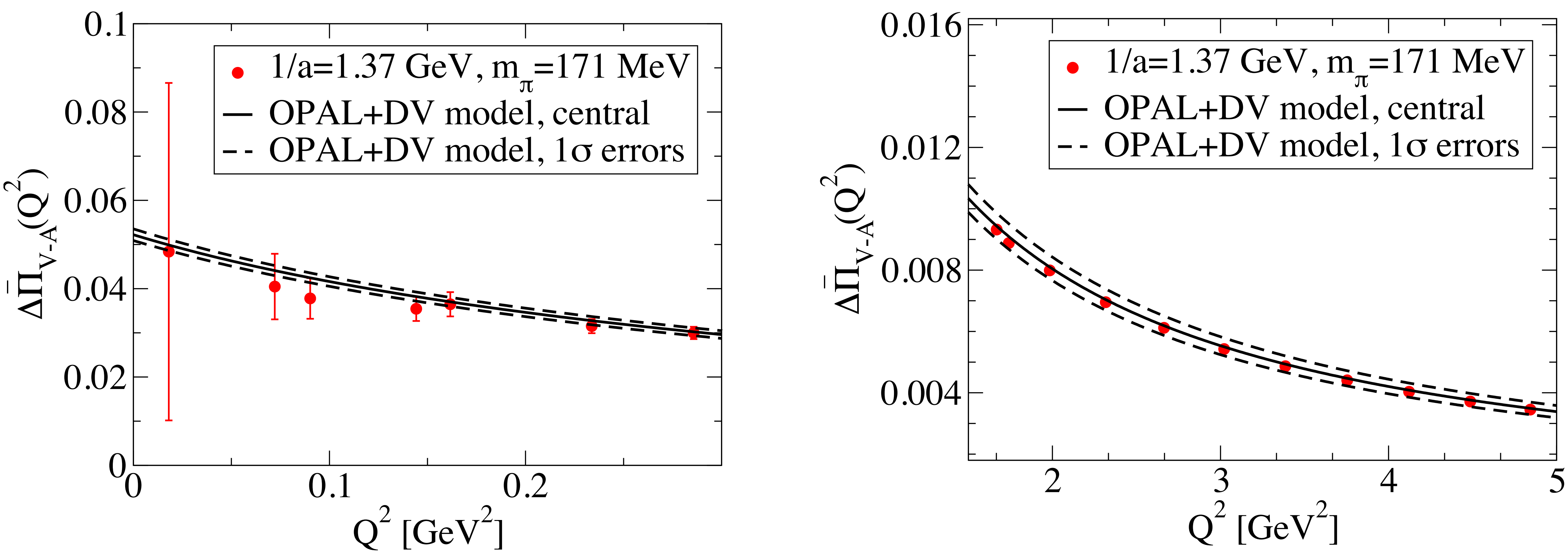

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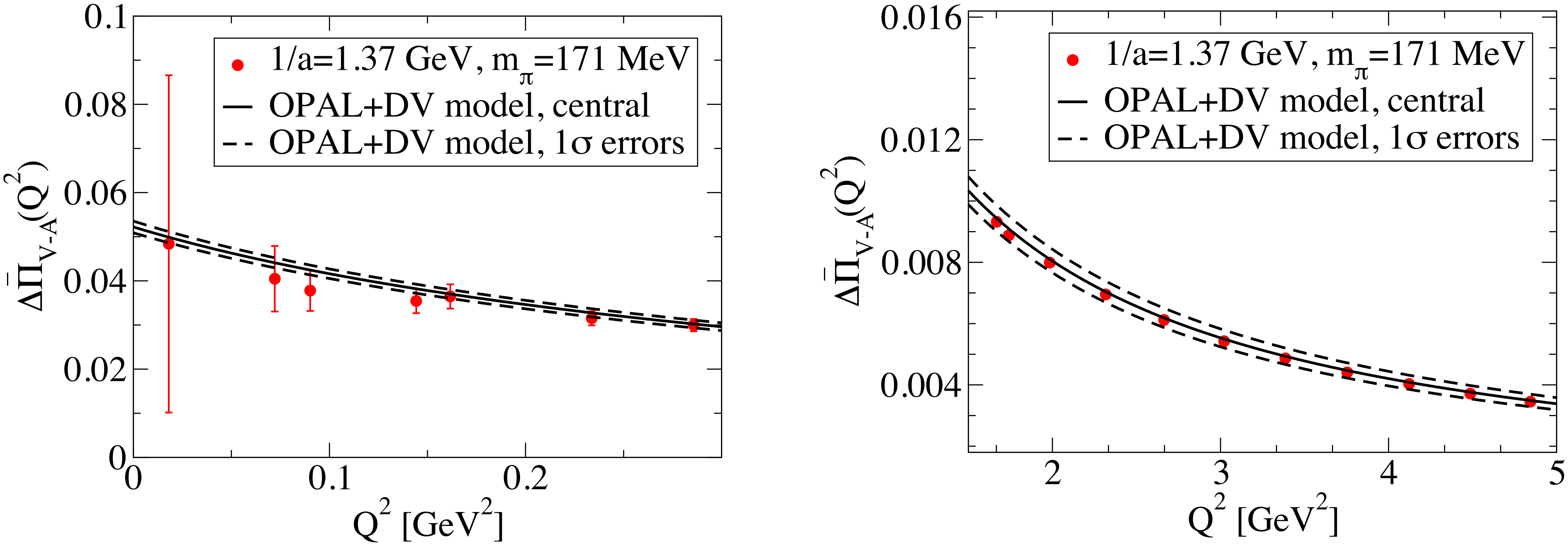

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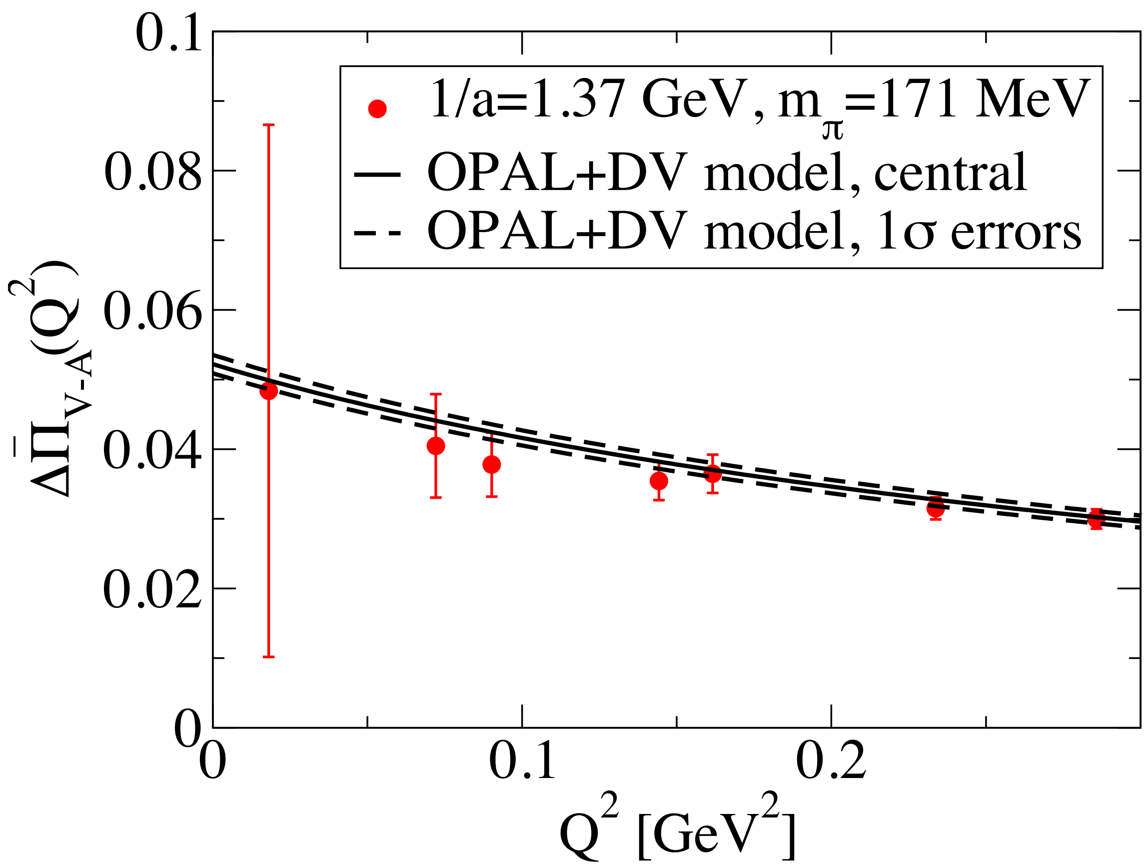

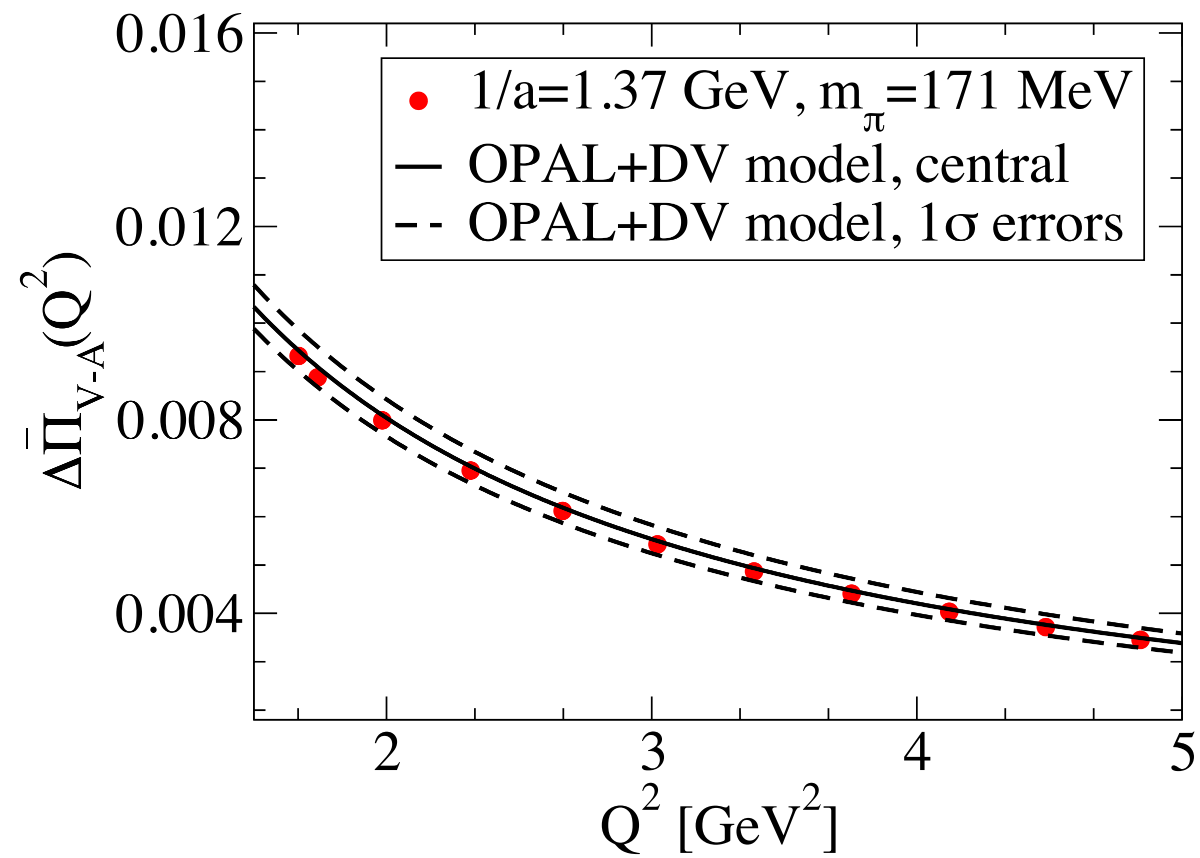

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- Reliable control of the SU(3) ChPT seems only possible for the simulated strange-quark mass, m_s', chosen less than its physical value, m_s.

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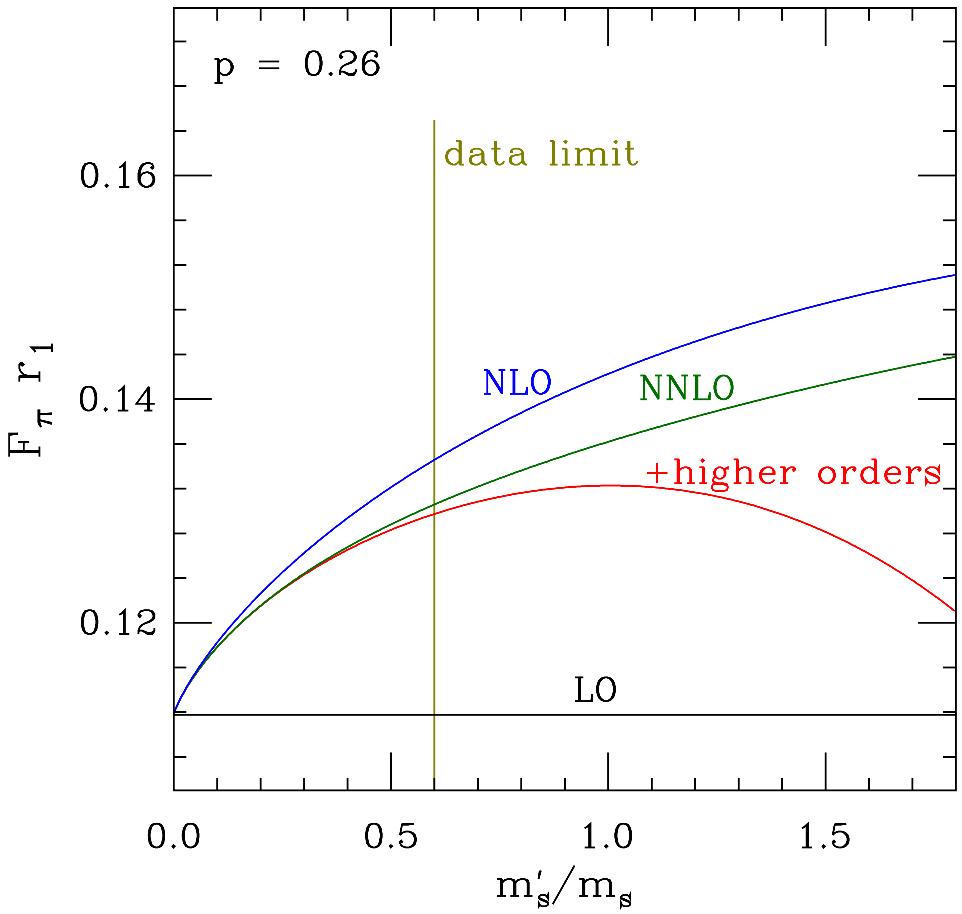

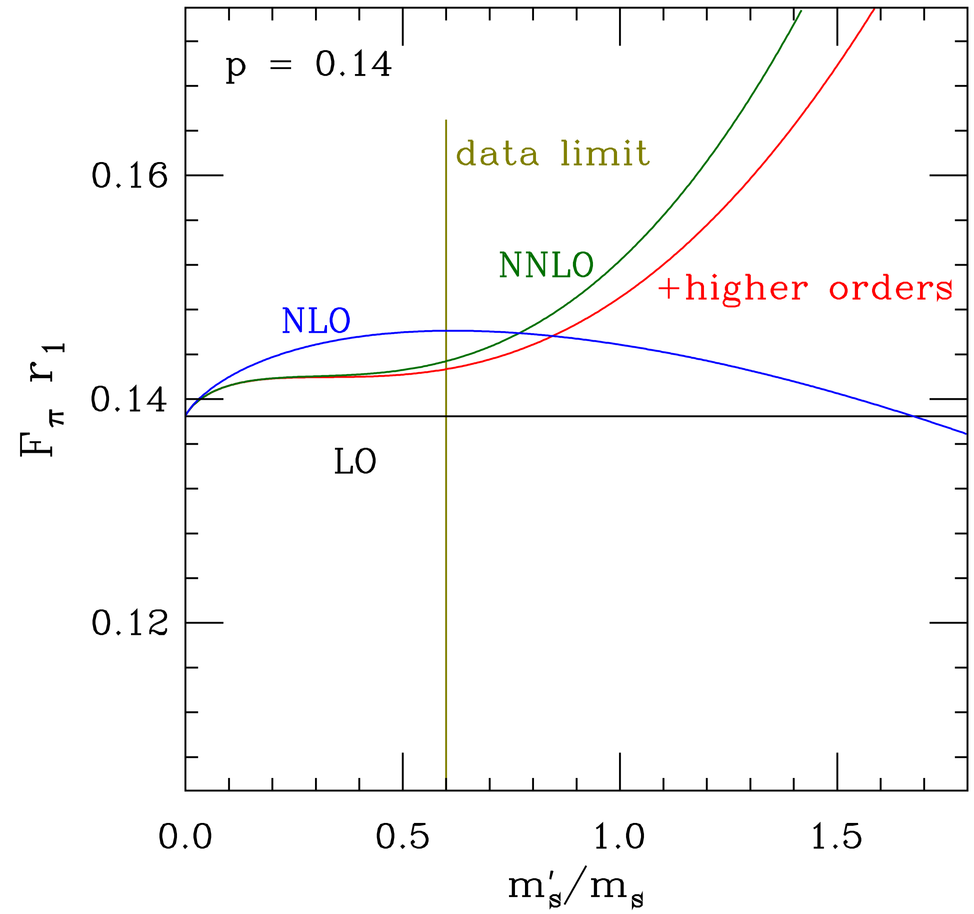

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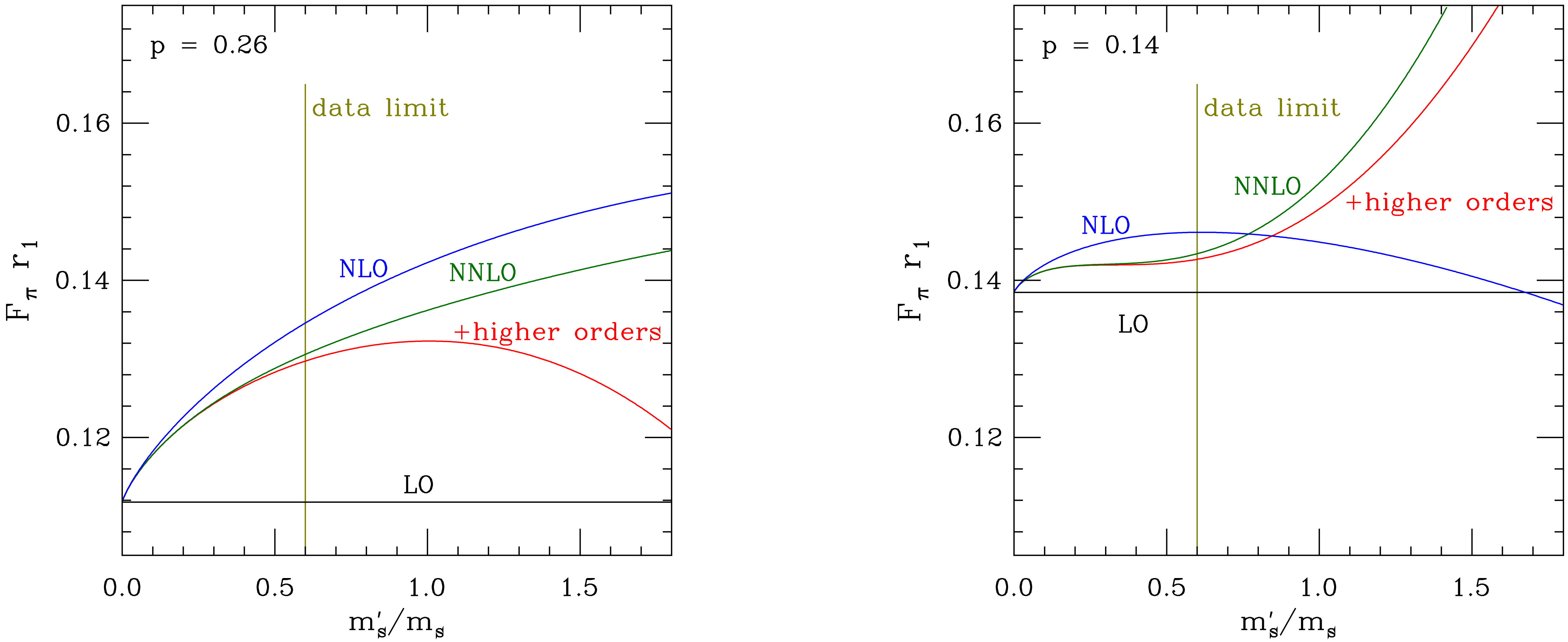

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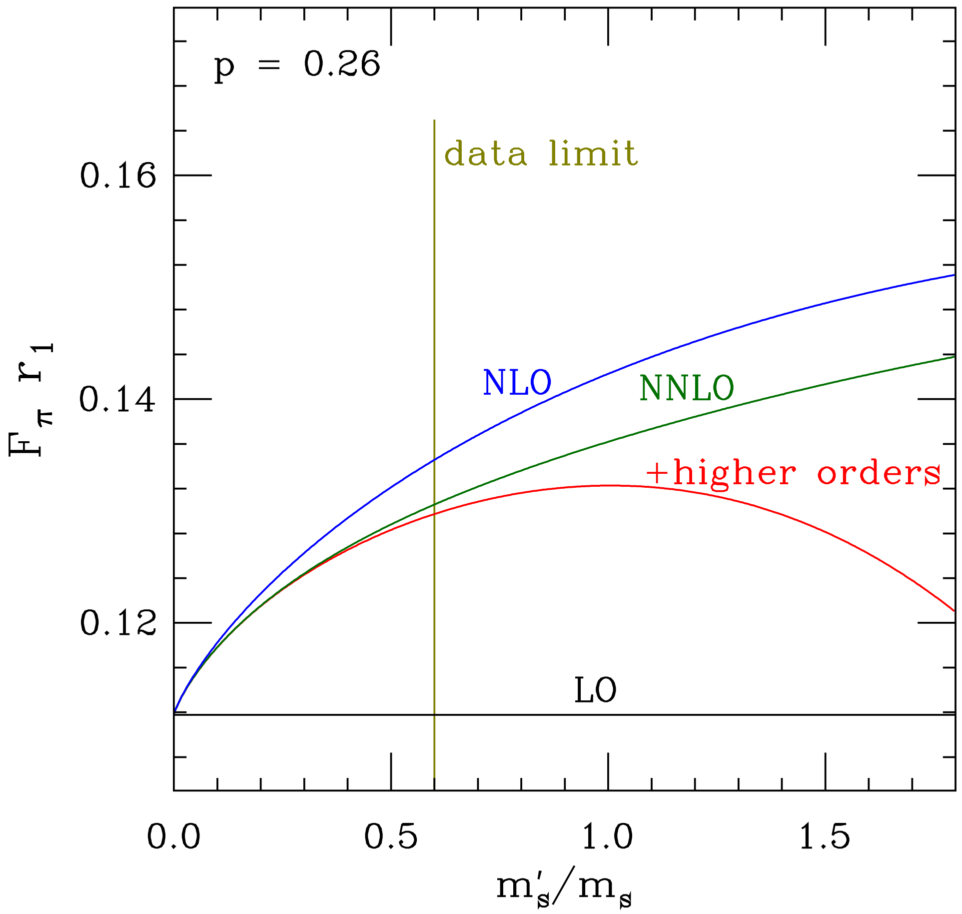

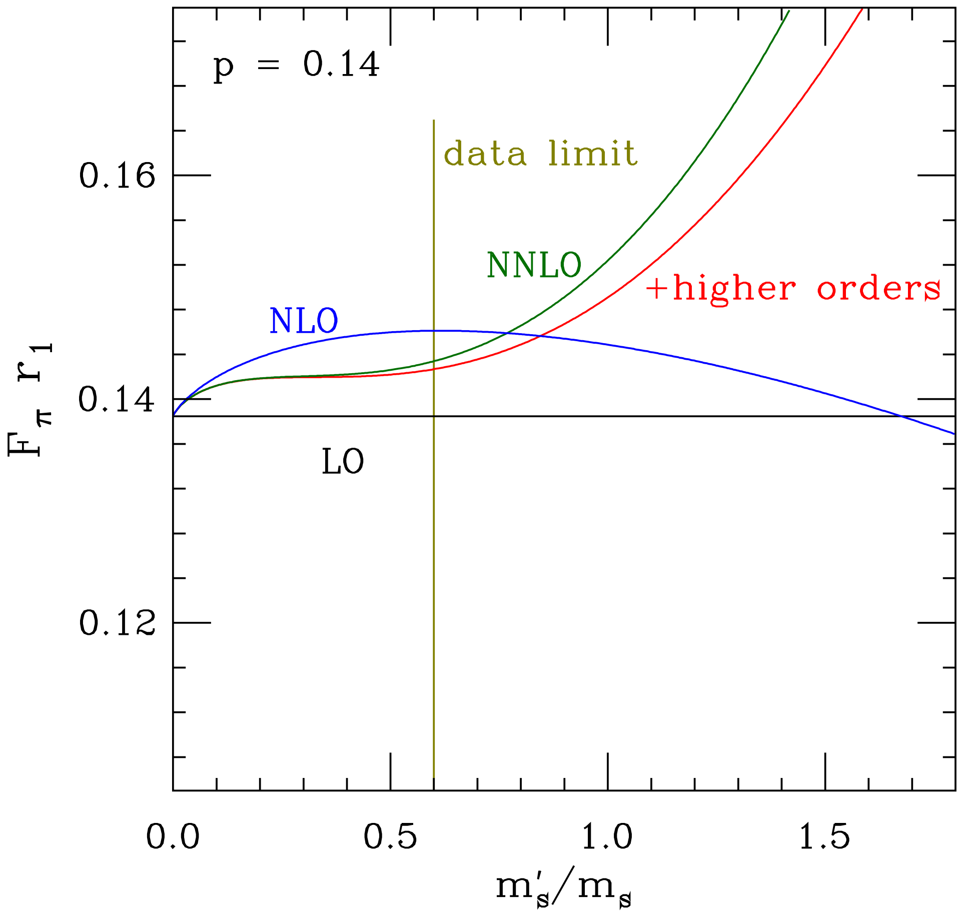

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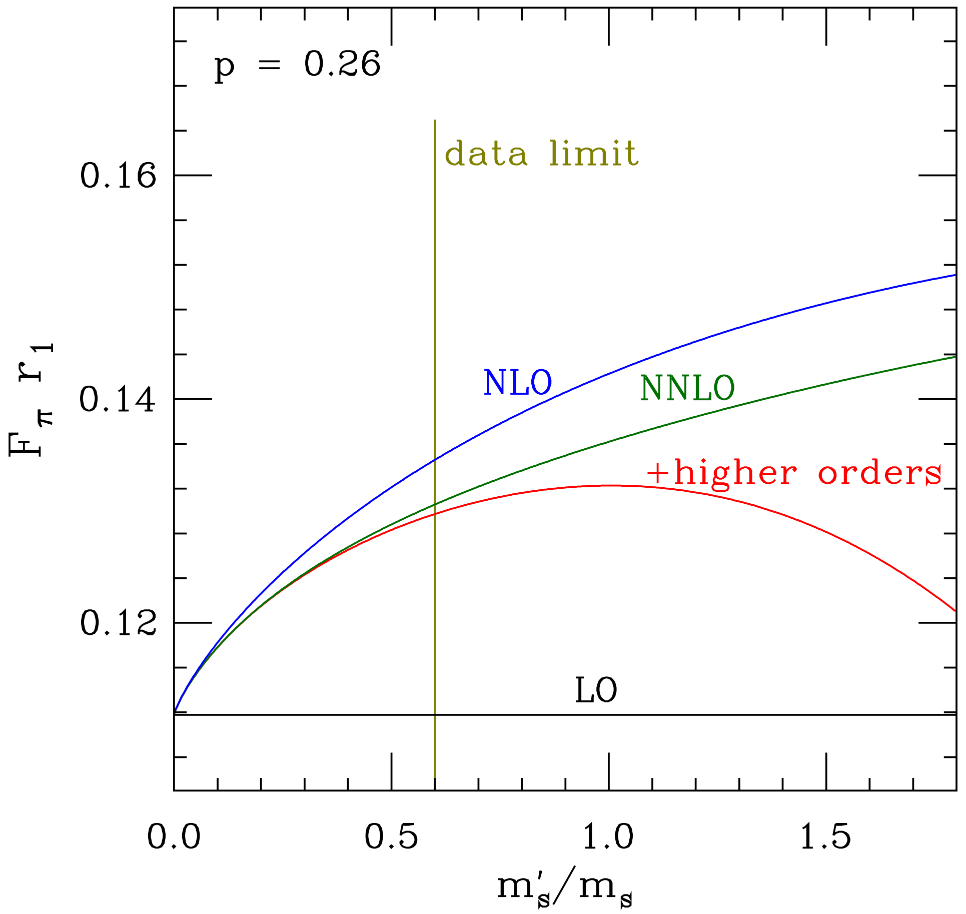

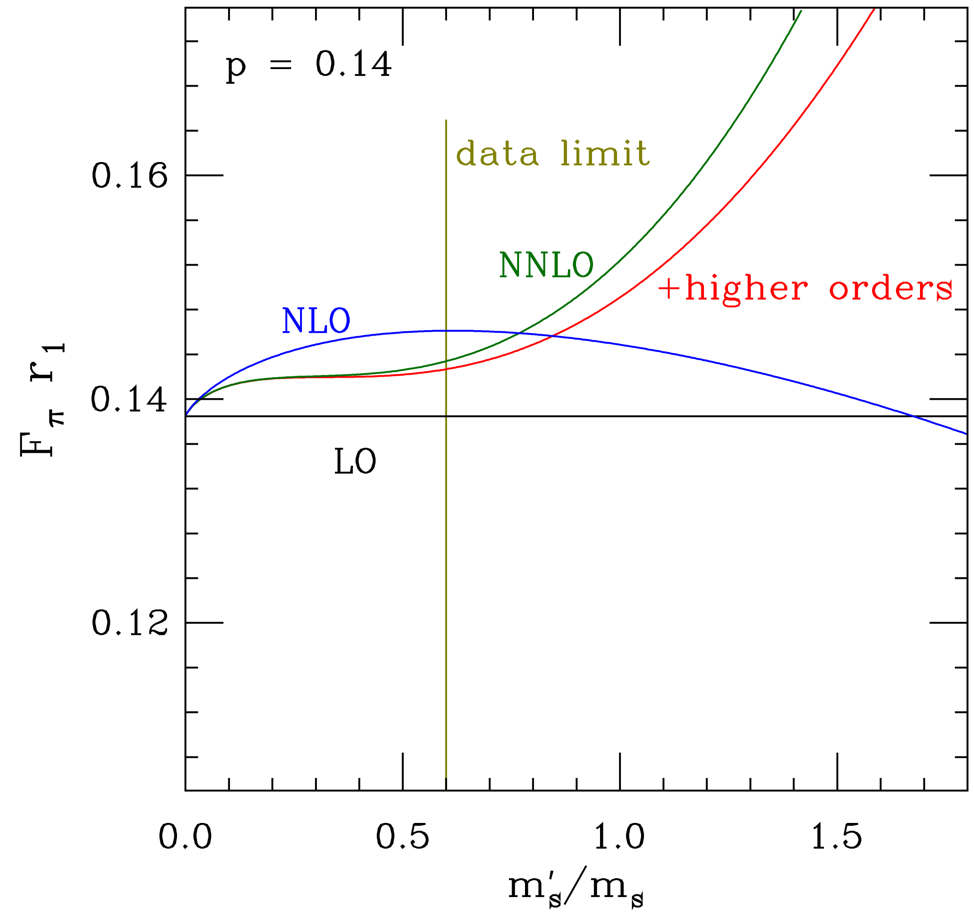

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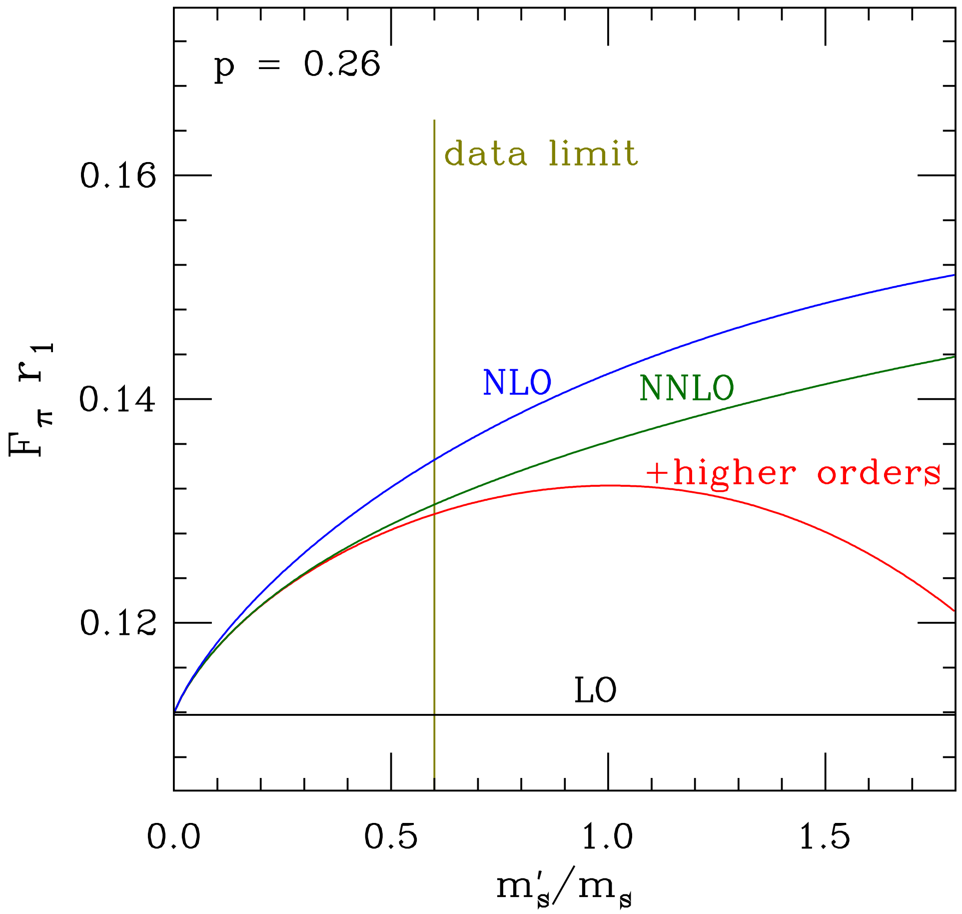

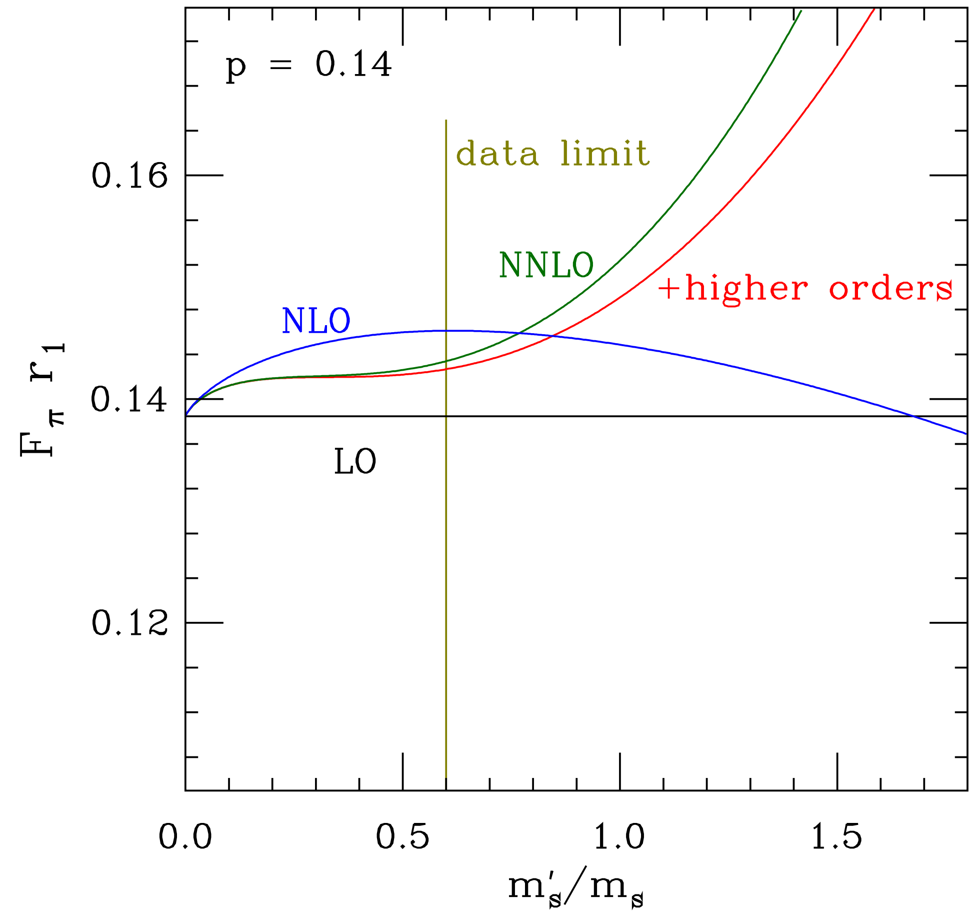

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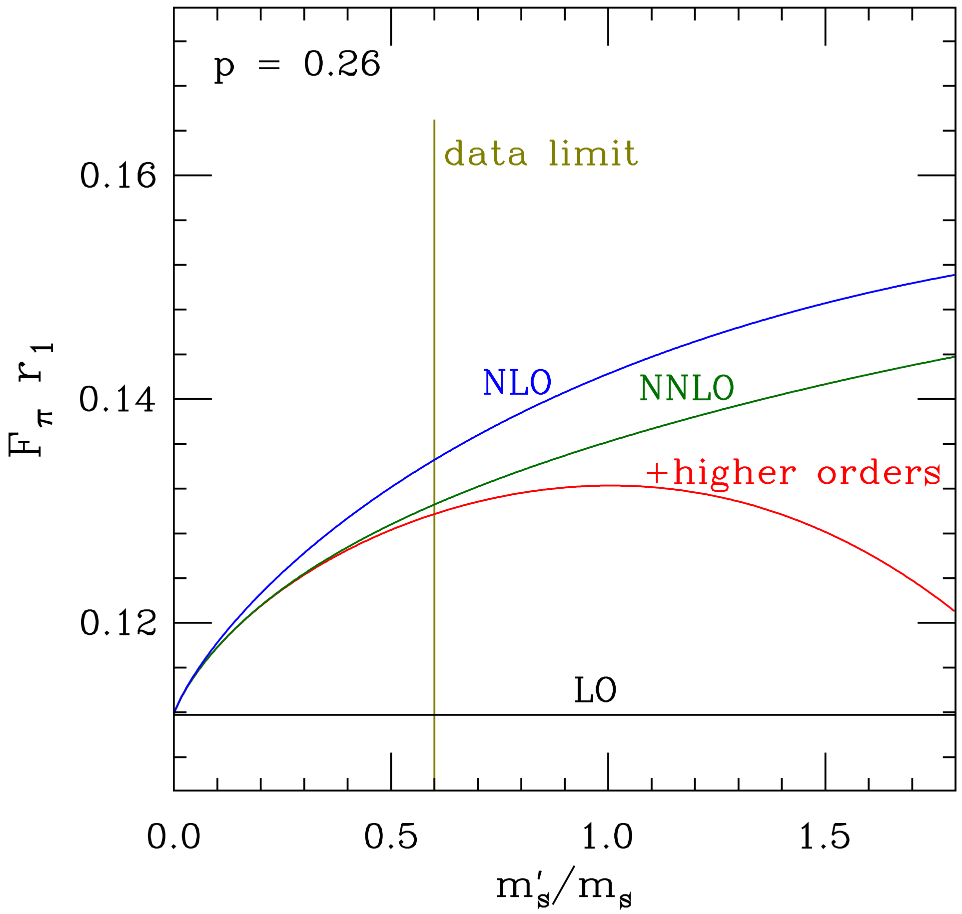

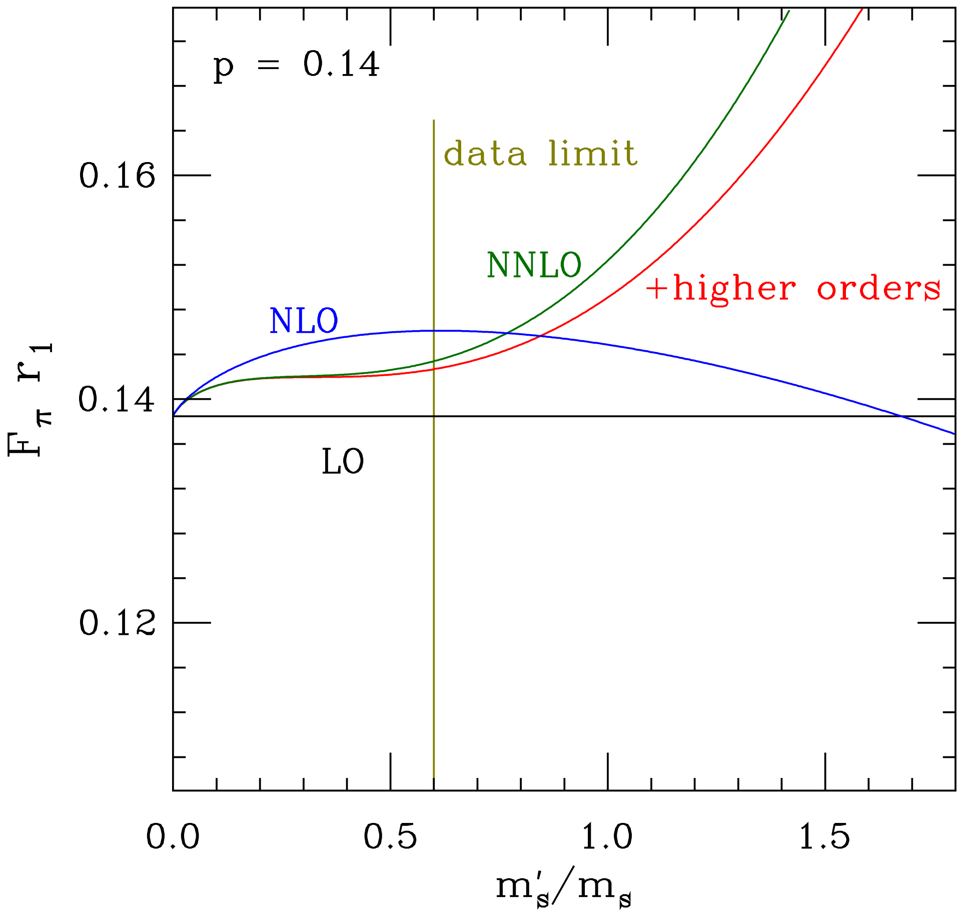

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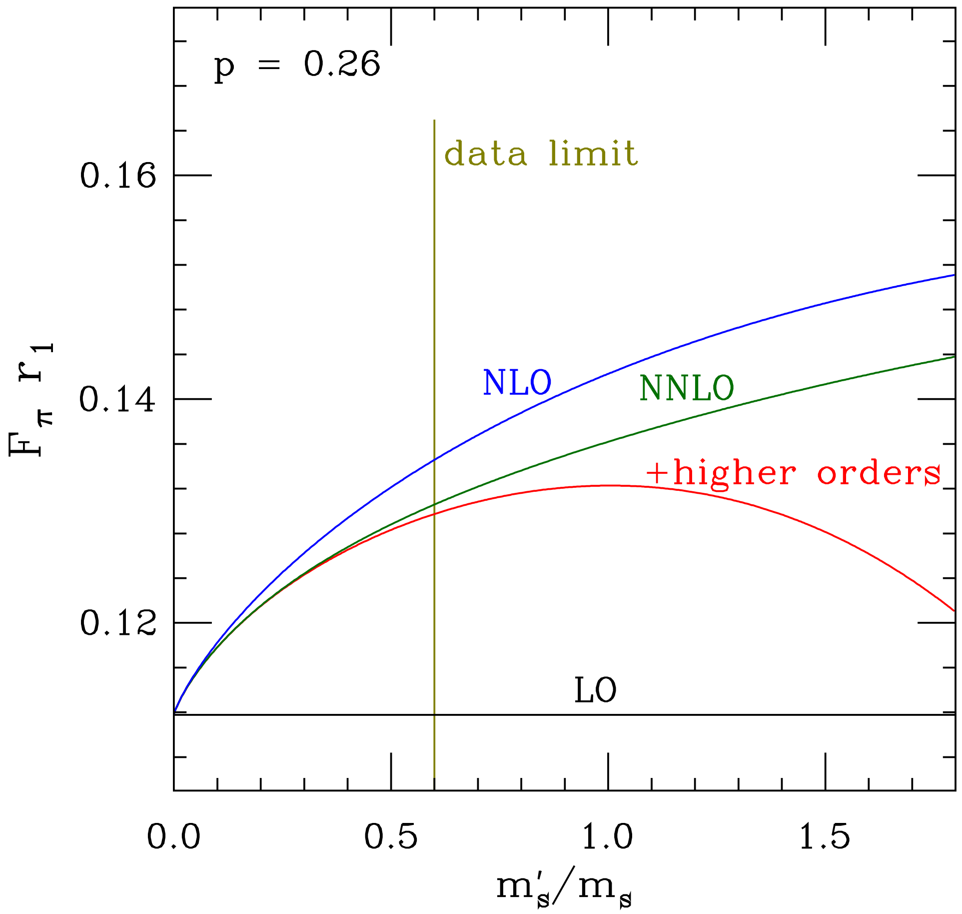

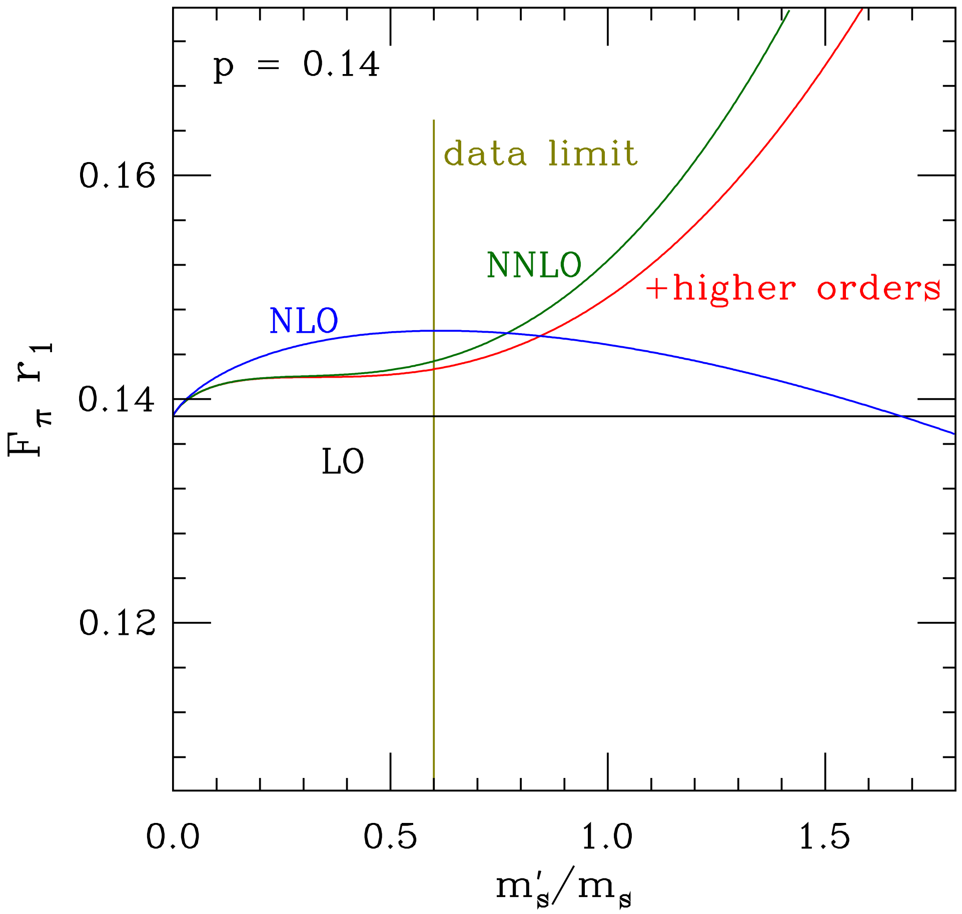

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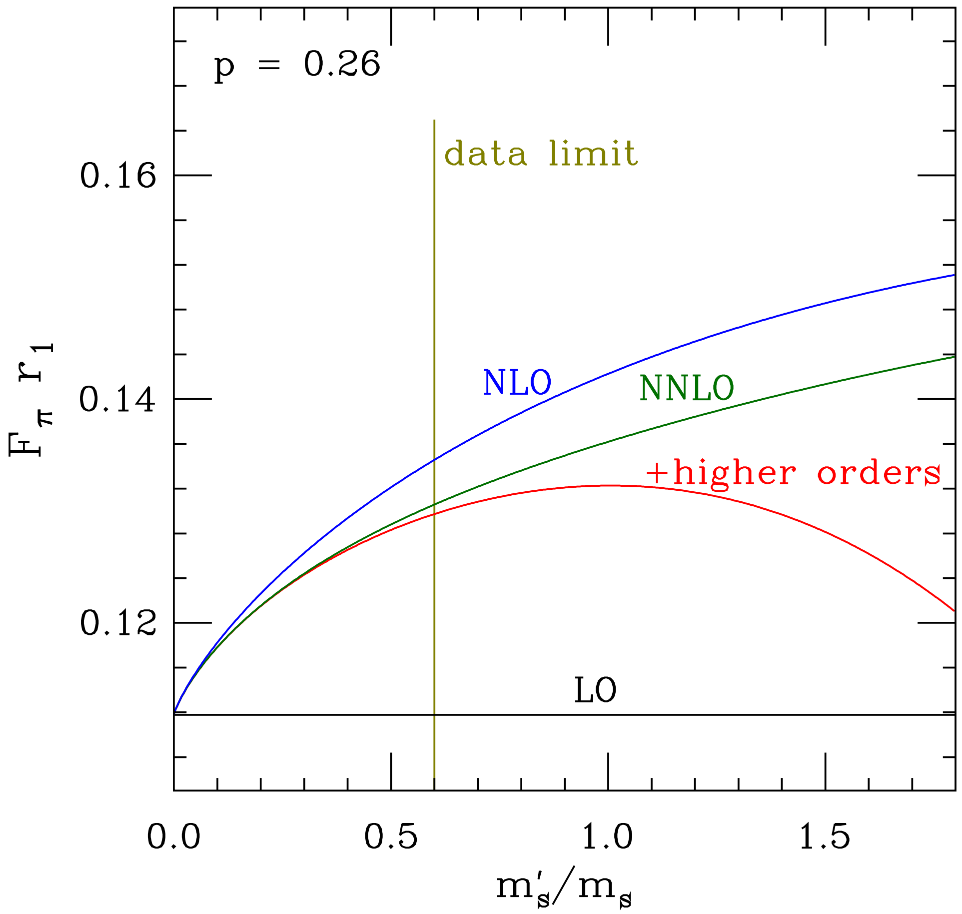

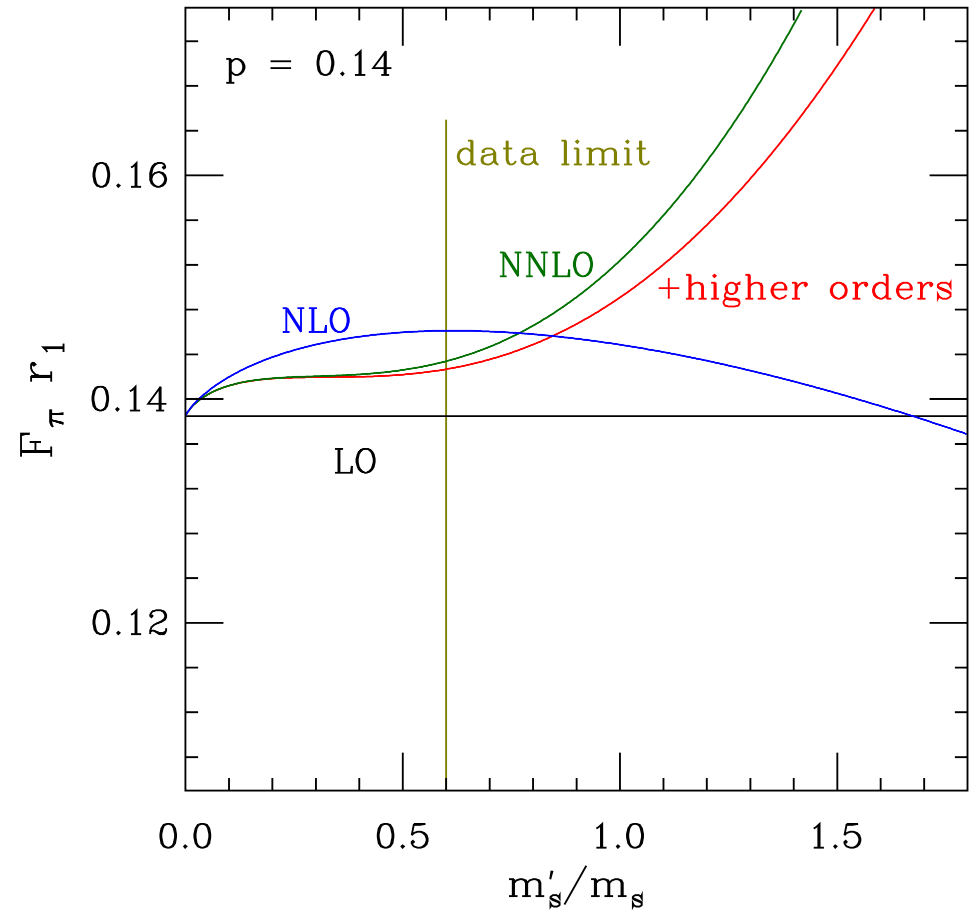

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"Paramagnetic effect:" Descotes, Girlanda & Stern, 1999

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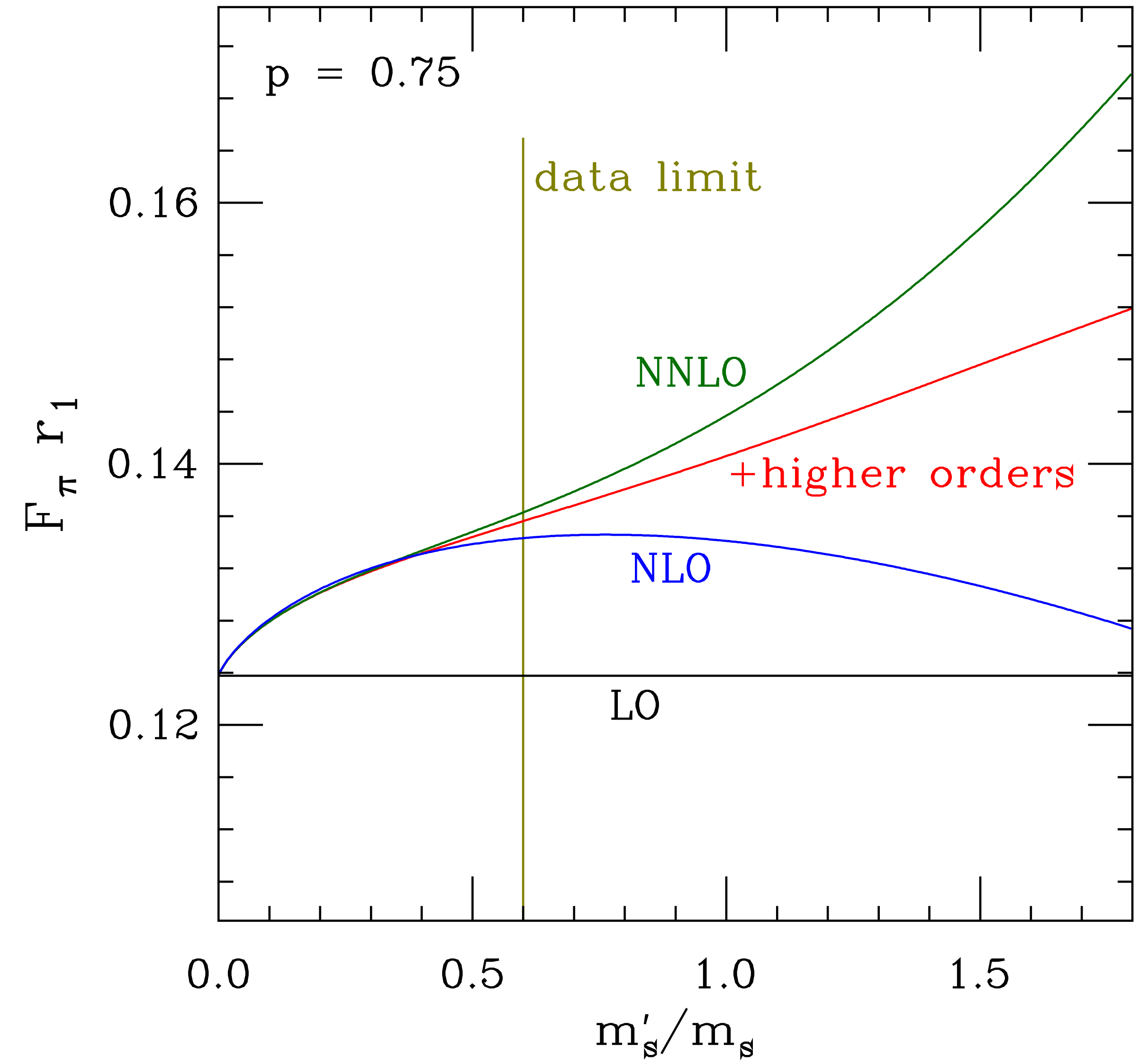

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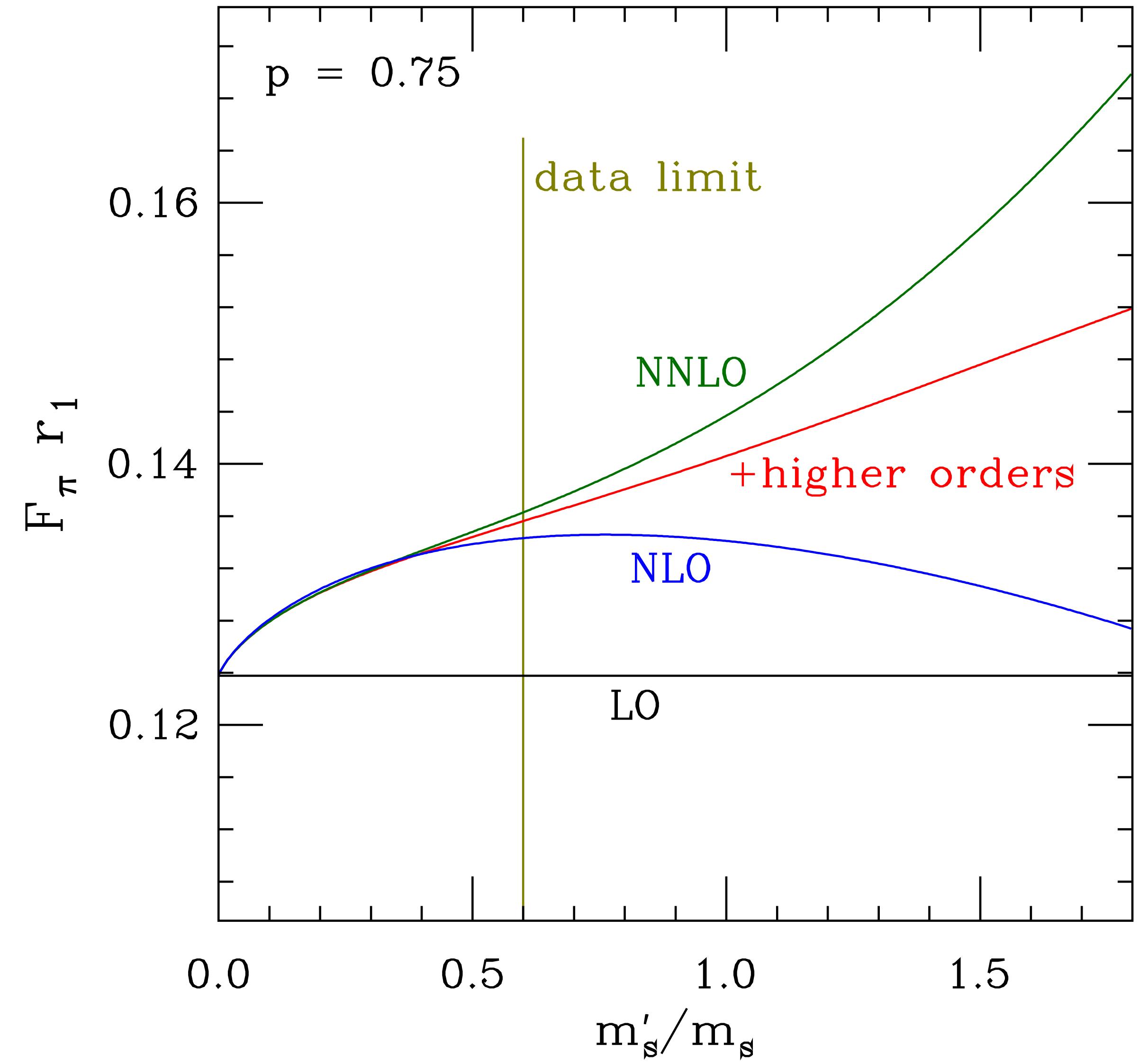

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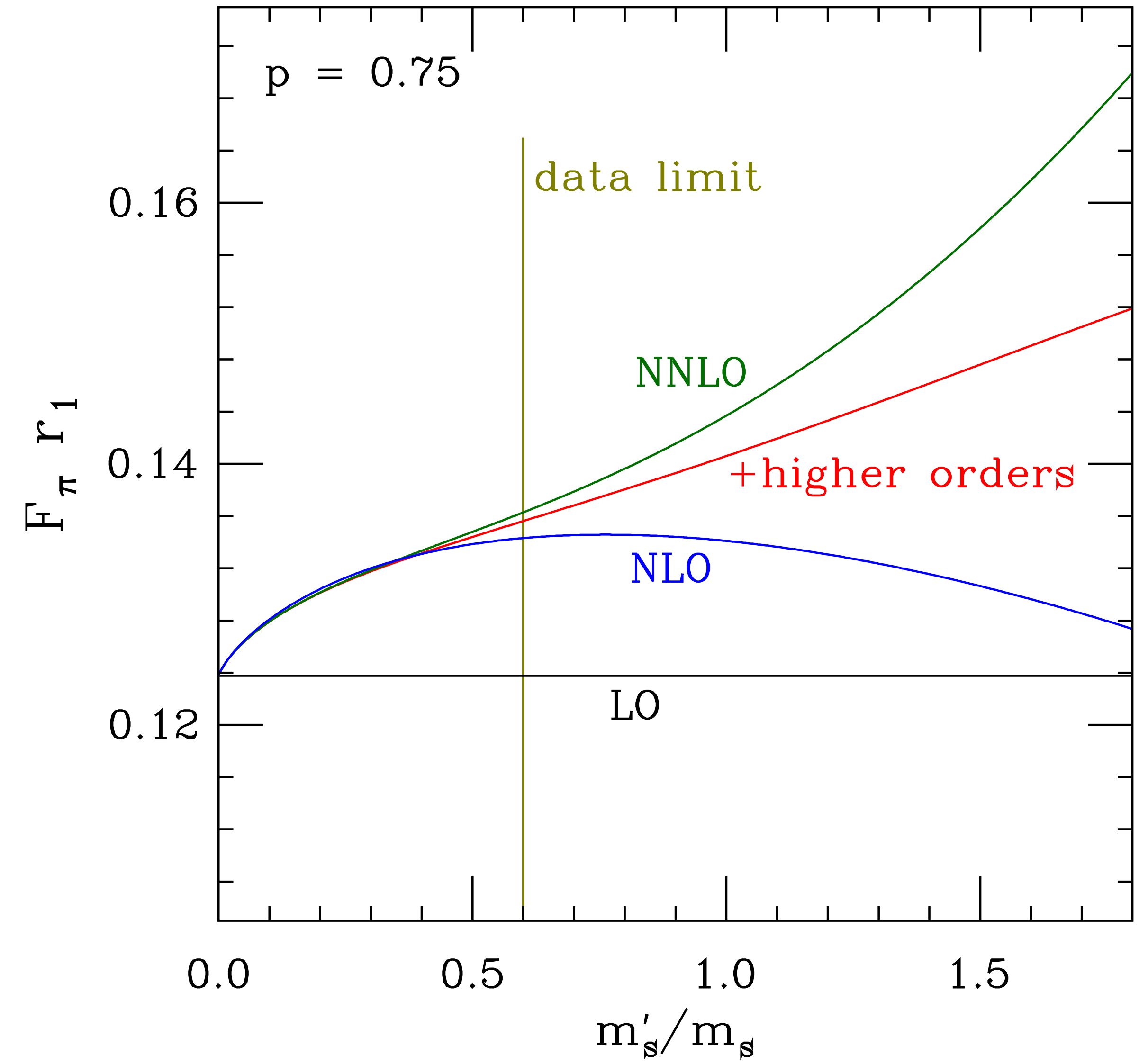

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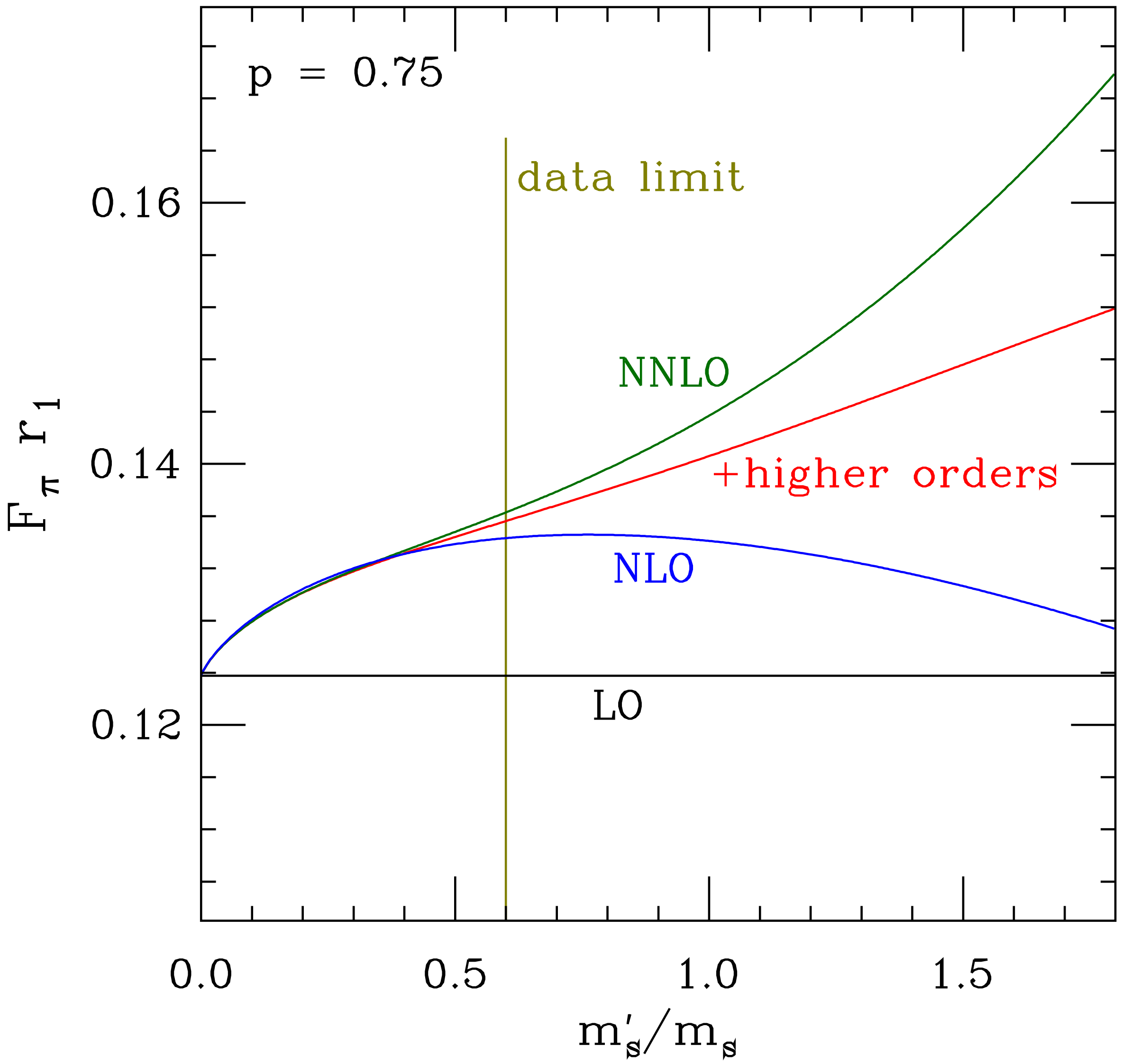

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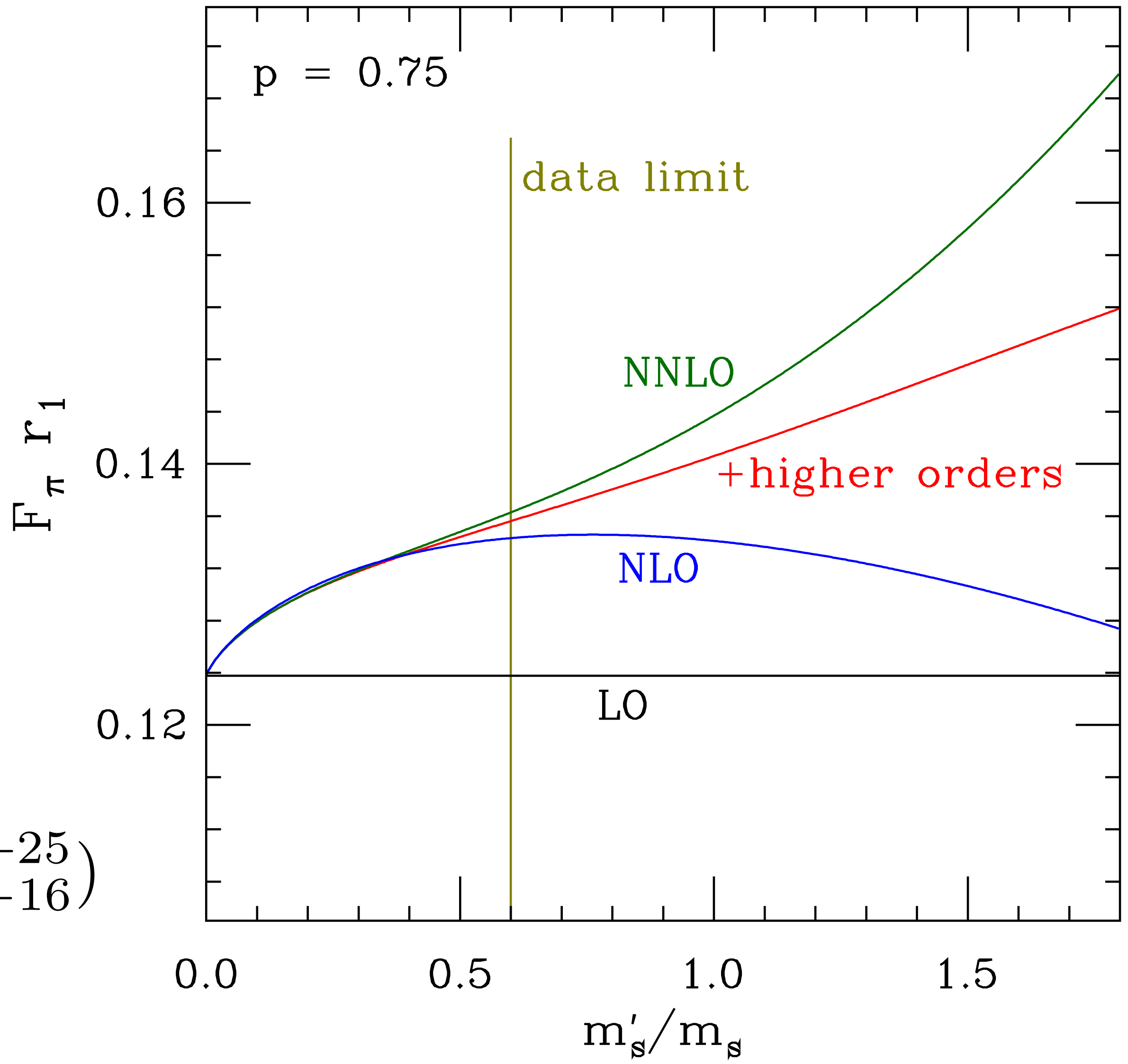

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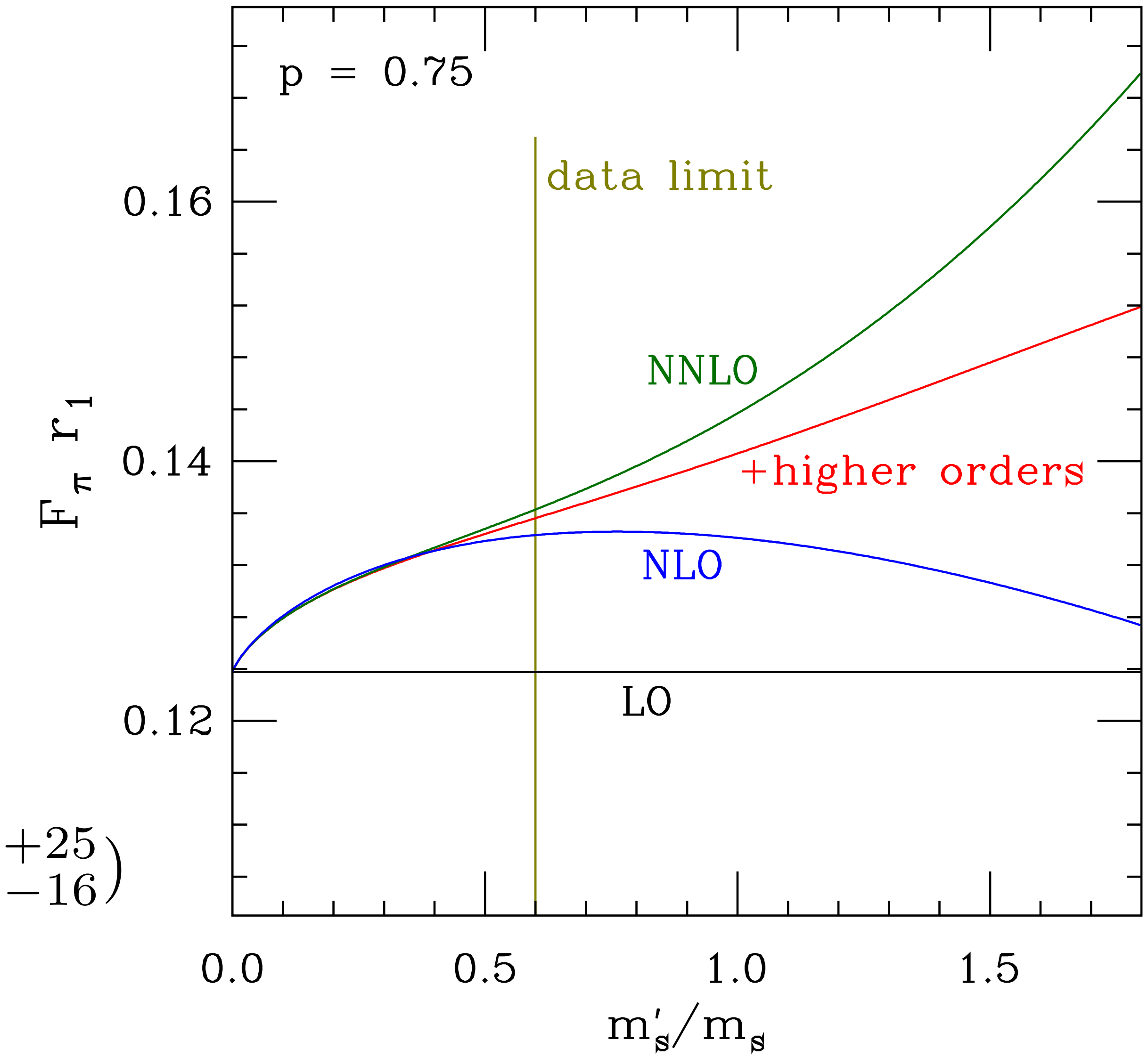

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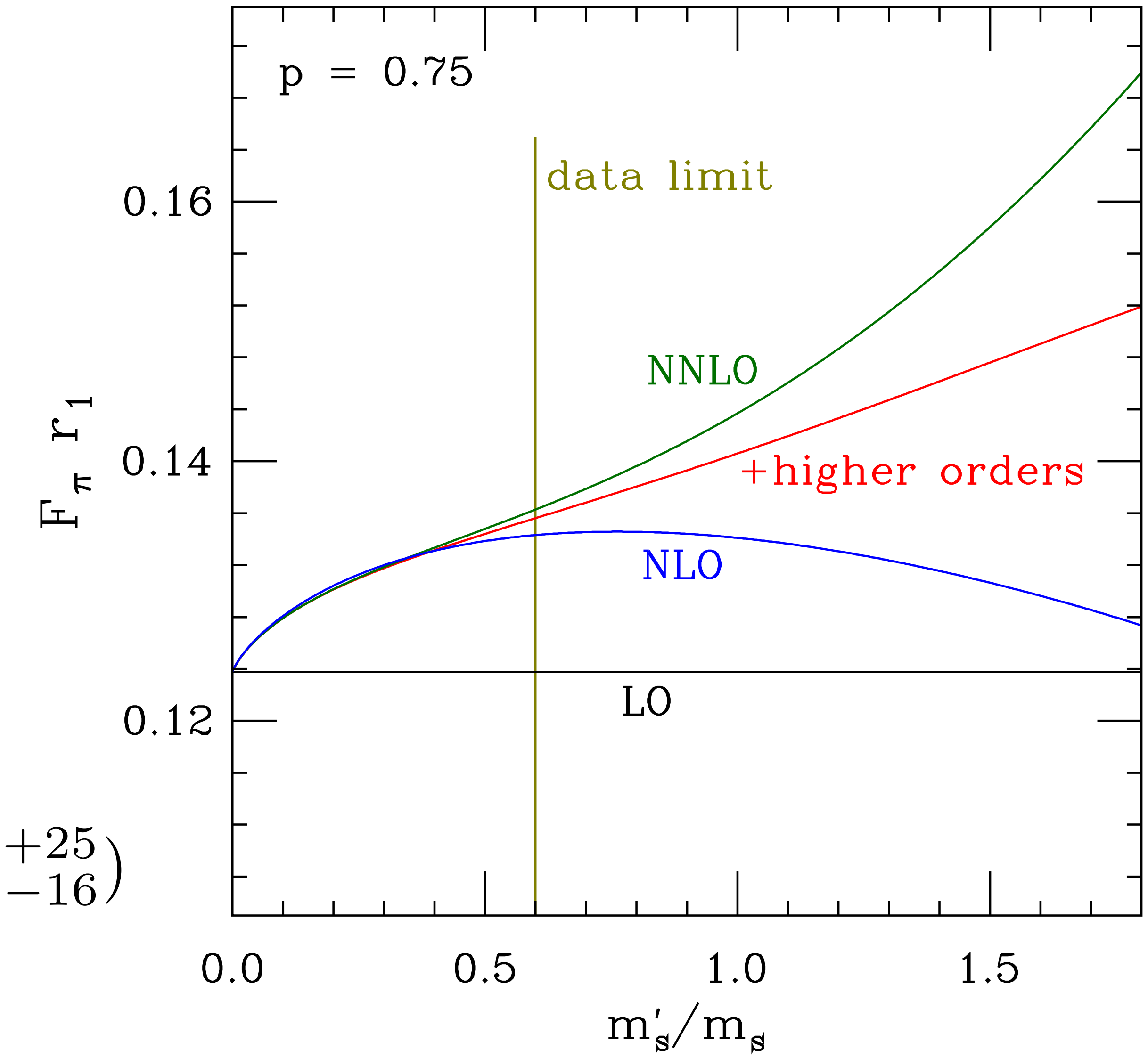

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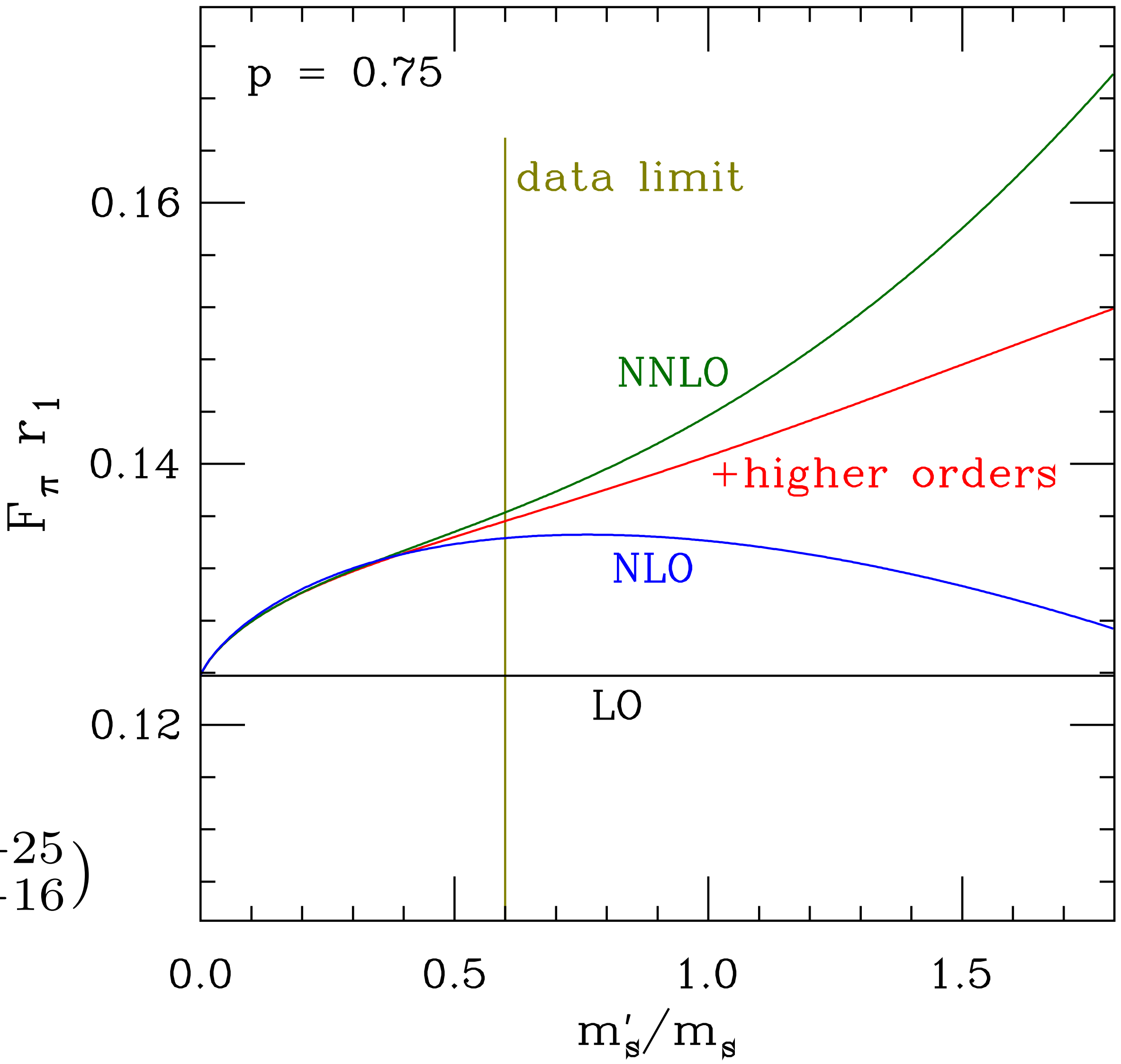

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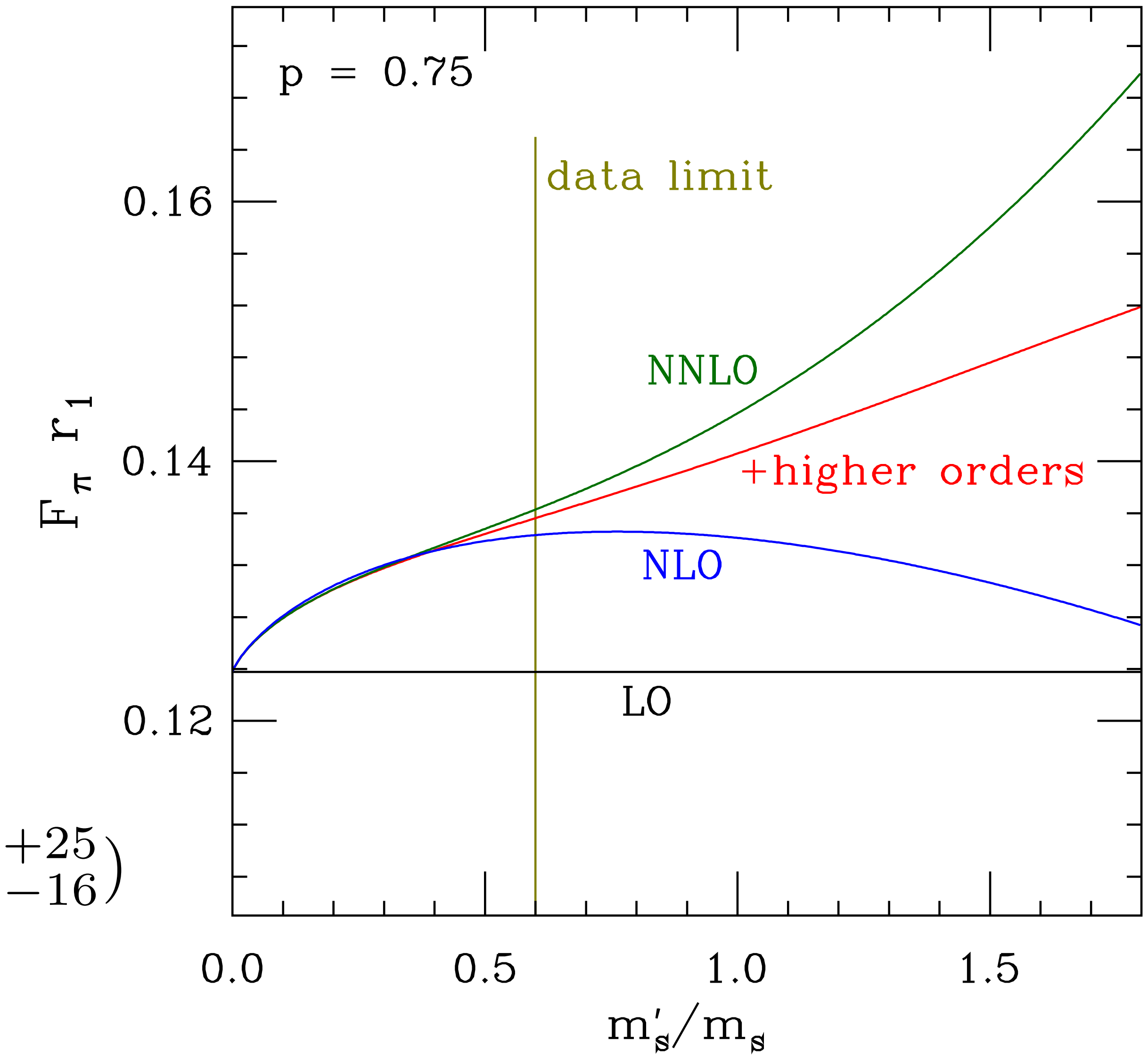

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- Need data with smaller discretization errors and smaller m_s' (in progress). [Higher order staggered ChPT would also help.]

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      - If want interesting, but not experimentally accessible, quantities like f_0, (decay constant in 3-flavor chiral limit), staggered ChPT will still be needed for foreseeable future.

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        - can reduce already small sea-quark effect on scale even further by fitting to their formula.

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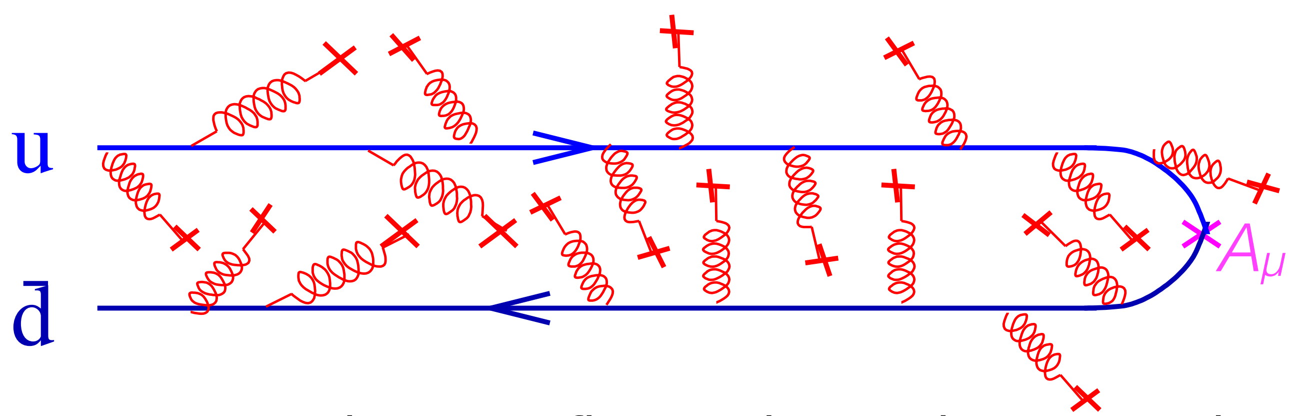
  - Many paybacks to come!

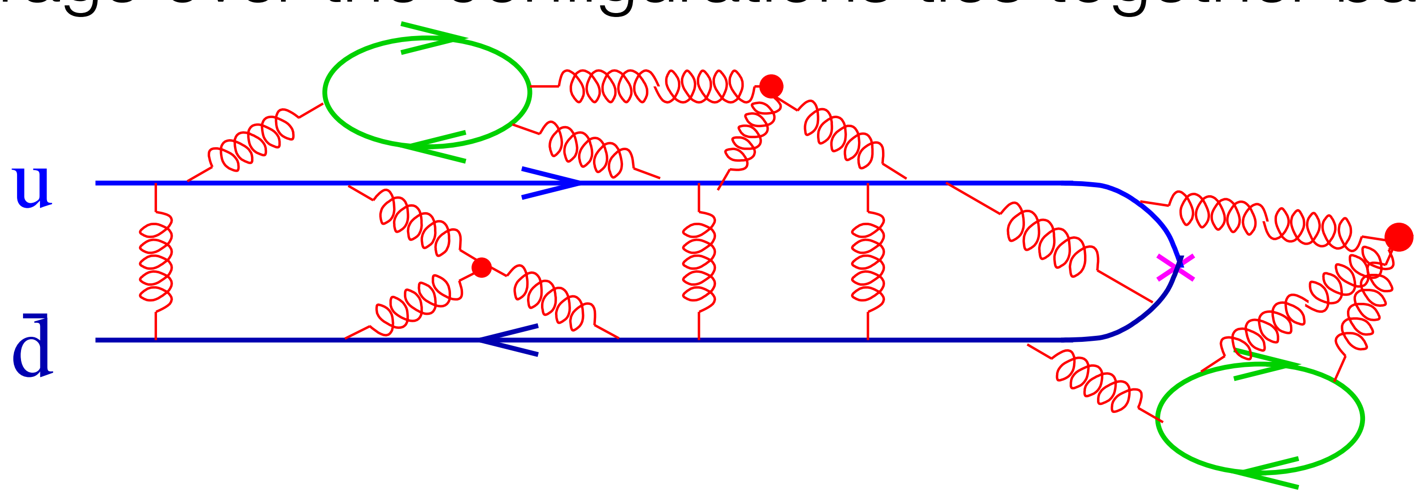
Backup Slides

QCD Simulations

- ✦ Generate an ensemble of gluon fields according to a probability distribution given by the QCD gluon action and back-effect of sea quarks (virtual quark loops).
  - Expensive! (mainly because of sea quarks, whose effect is encoded in a determinant).
- ✦ In each gluon-field background (a "configuration"), calculate propagation of valence quarks.
  - Relatively cheap (for each quark, need one column of a matrix inverse).
  - E.g., for $\langle 0|\, A_\mu\, |\pi(p)\rangle = i\, f_\pi\, p_\mu$, in a given background:



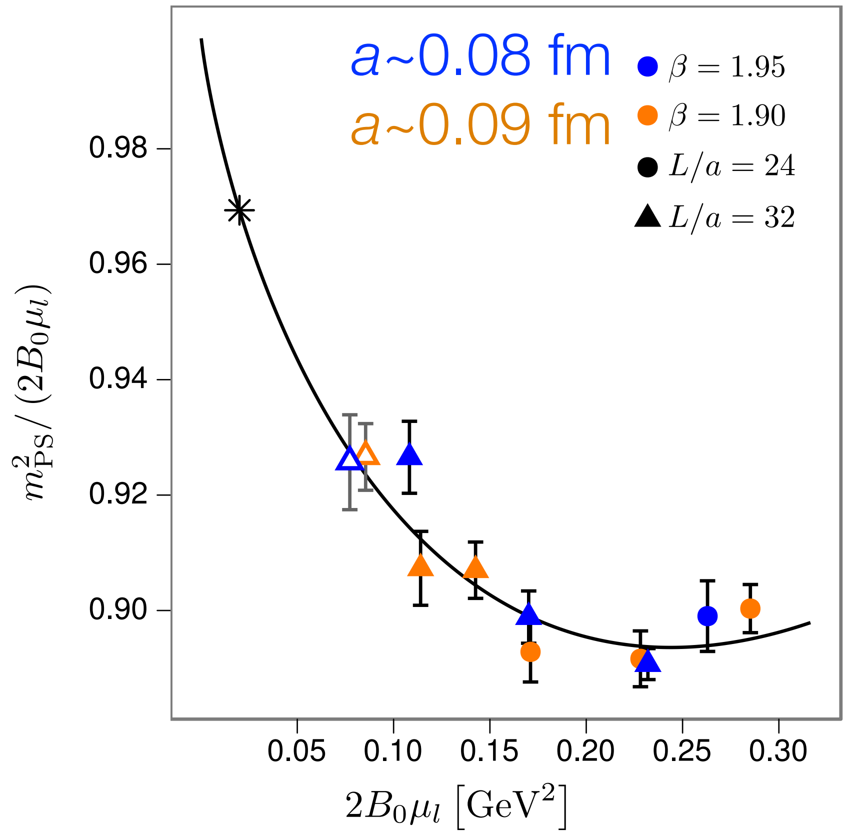
- ✦ Then average over the configurations ties together background gluon fields to make:

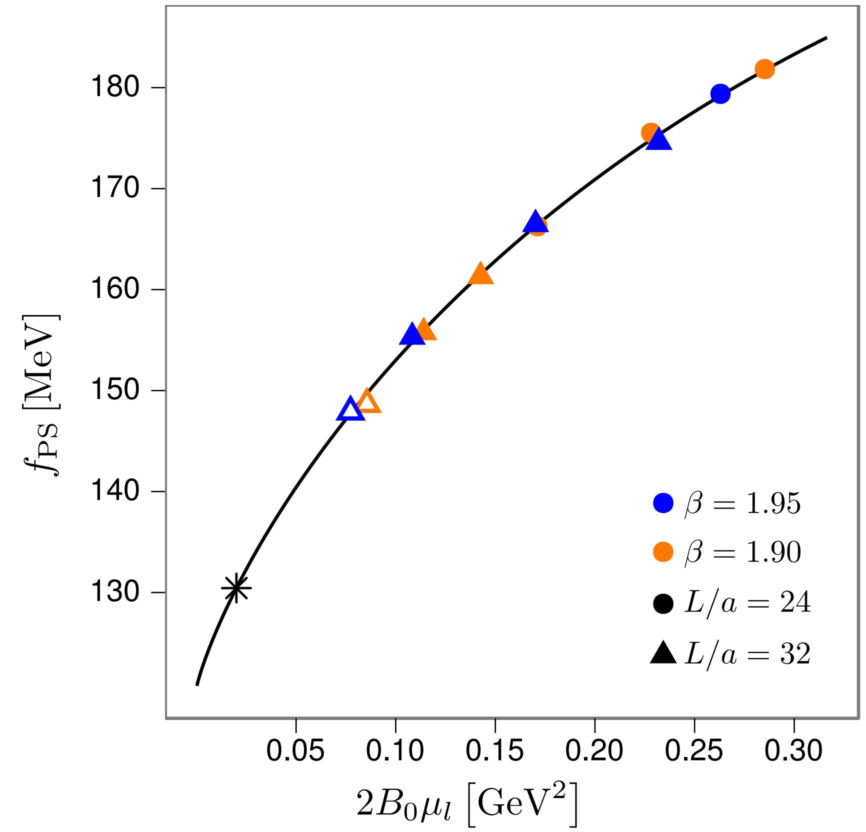


Some SU(2) Fits

ETM, [R. Baron et al.], JHEP 1006 (2010) 111, [arXiv: 1004.5284].

2+1+1



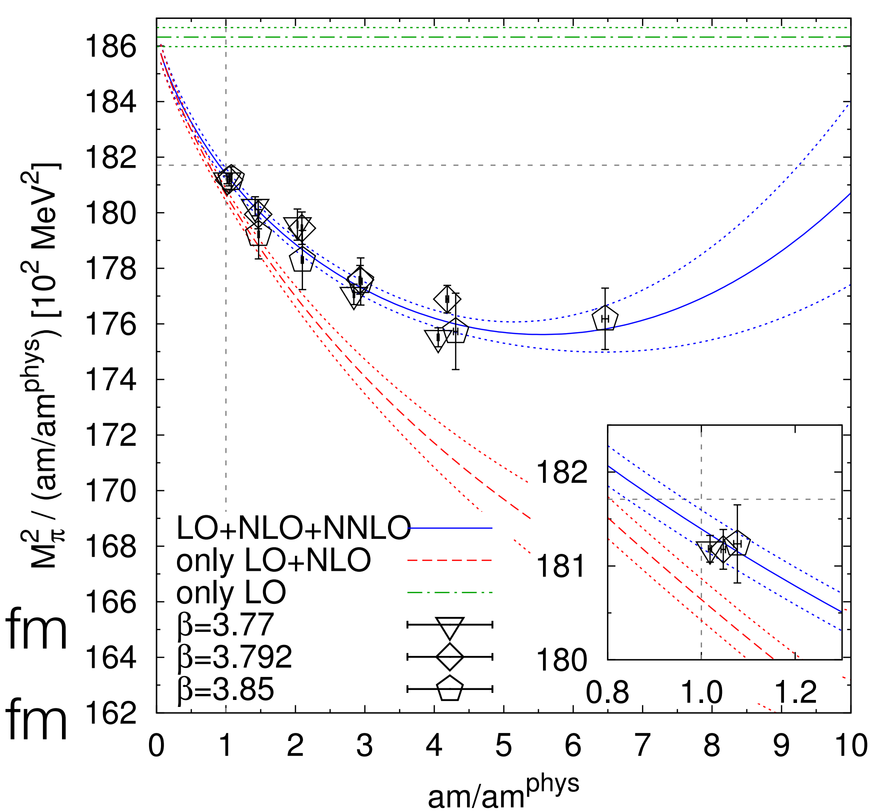


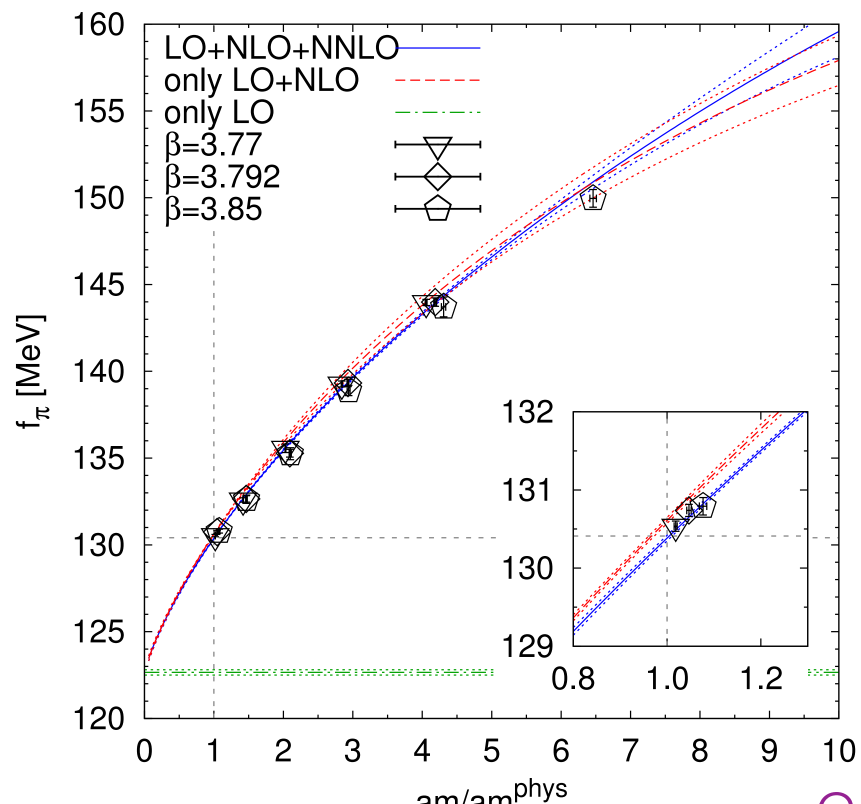
S. Borsanyi et al., Phys. Rev. D88 (2013) 014513, [arXiv: 1205.0788].

2+1

a~0.12 fm

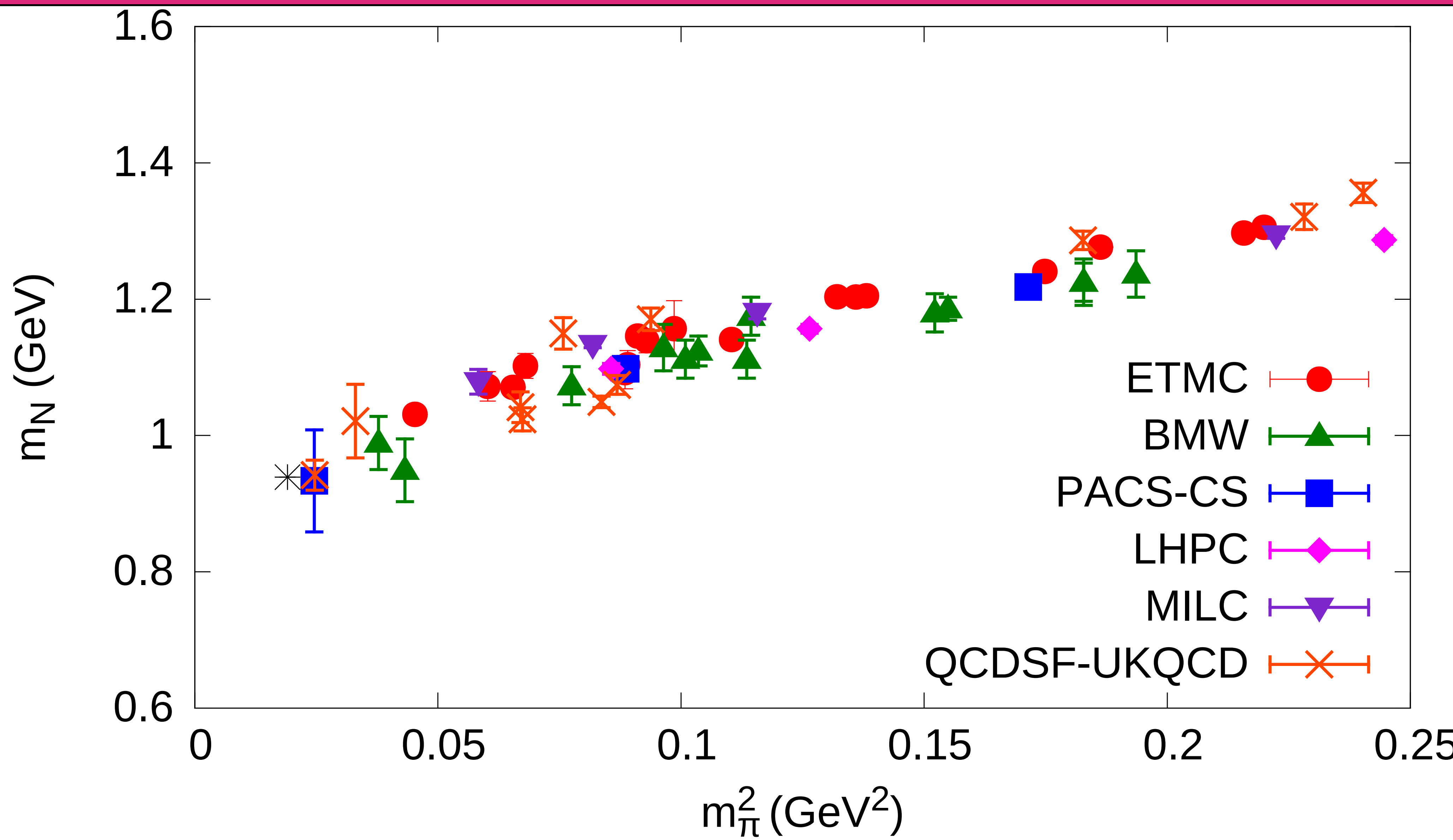
a~0.10 fm





- Note: Borsanyi et al paper includes physical quark masses.
- Discretization errors small in both cases.

Nucleon Chiral Extrapolation



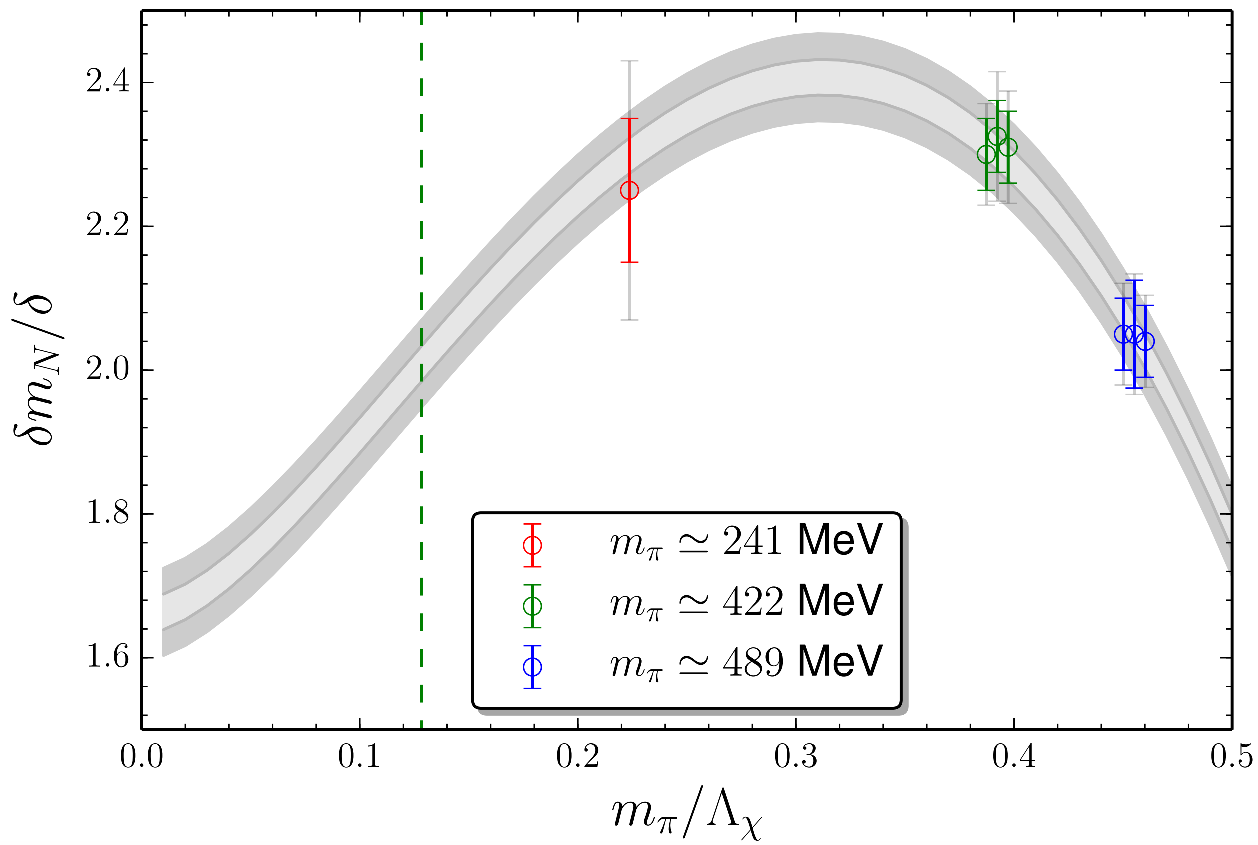
- Plot from: ETM [C. Alexandrou et al.], PoS LATTICE 2014, 100 [arXiv:1412.0925].
- Good agreement among groups.
- Note QCDSF point close to physical, with relatively small errors.
  - G. Bali et al., NPB 866 (2013) 1 [arXiv:1206.7034].

Nucleon isospin violation

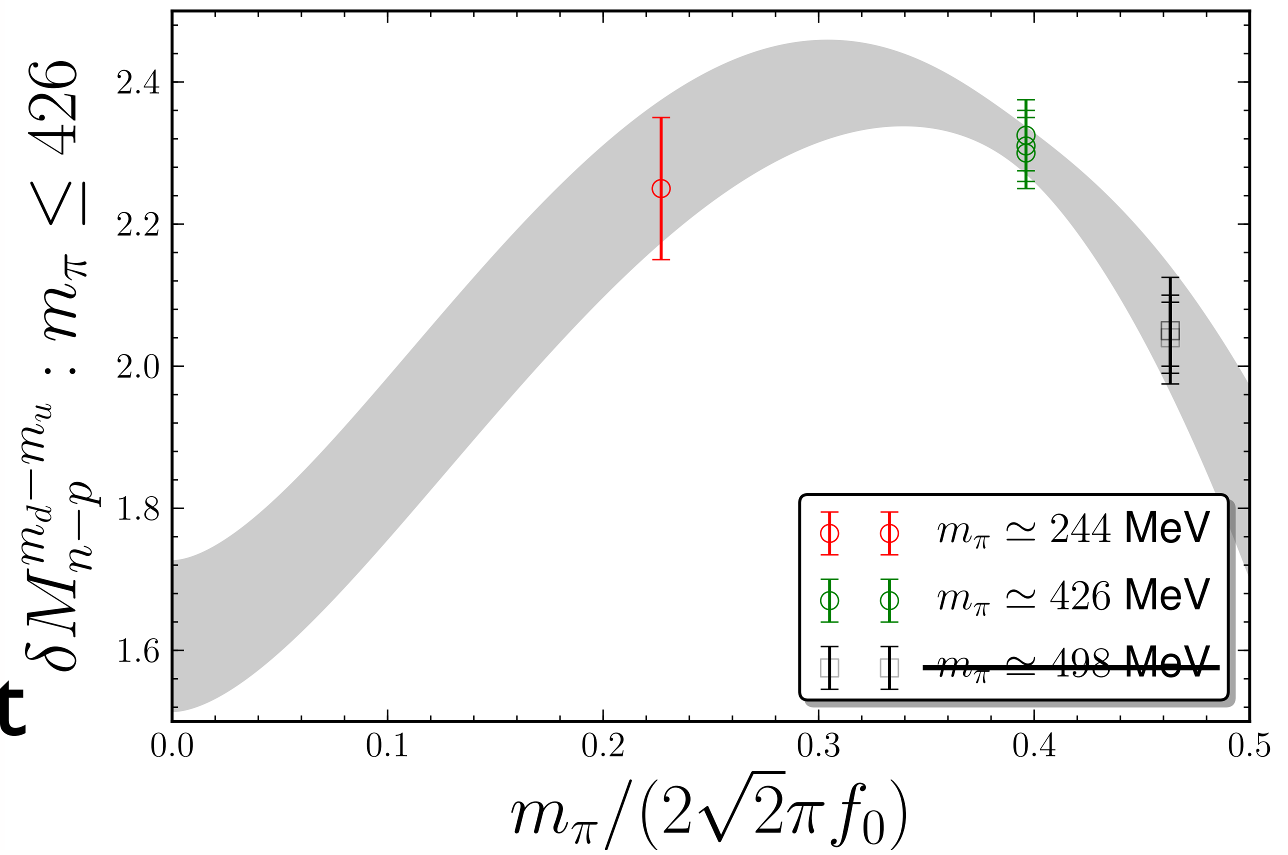
(Slide from A. Walker-Loud)

PRELIMINARY

C. Aubin, W. Detmold, E. Mereghetti, K. Orginos, S. Syritsyn, B. Tiburzi, A. Walker-Loud



Text



NNLO χPT

$$\delta M_{n-p}^{m_d-m_u} = \delta\left\{\alpha\left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

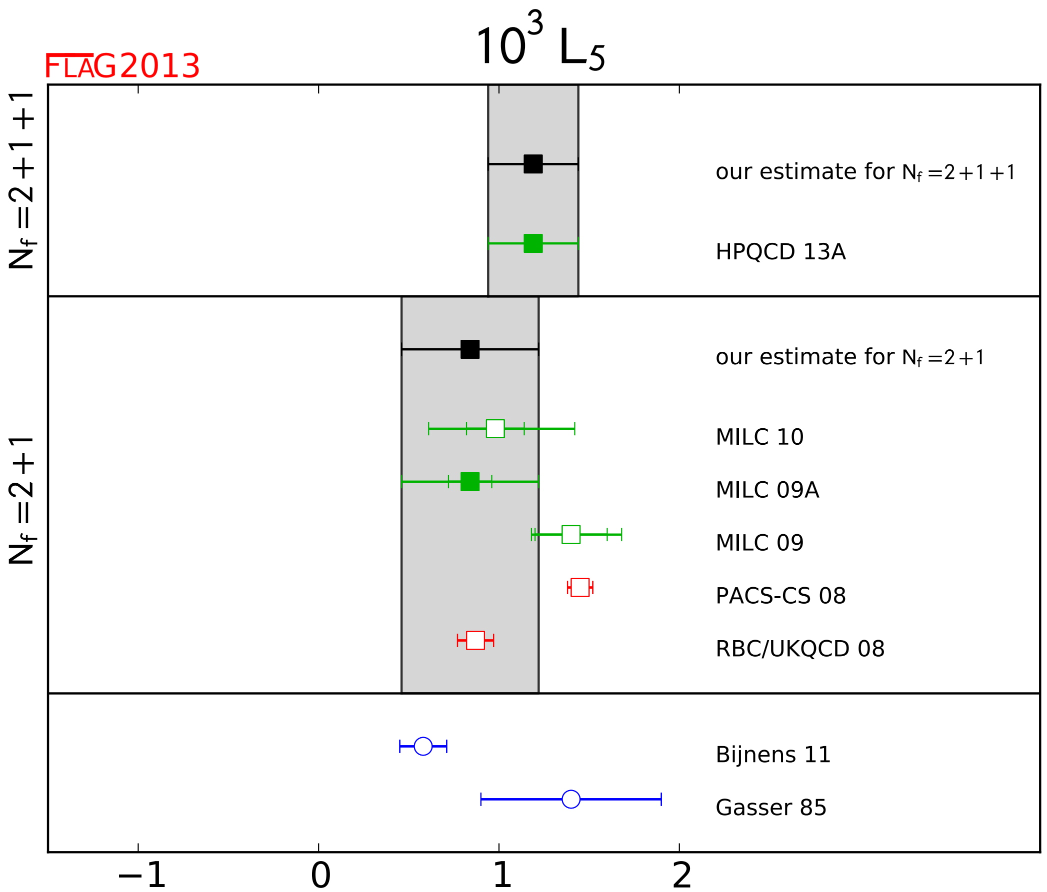
$(g_A = 1.27, f_\pi = 130\text{ MeV})$

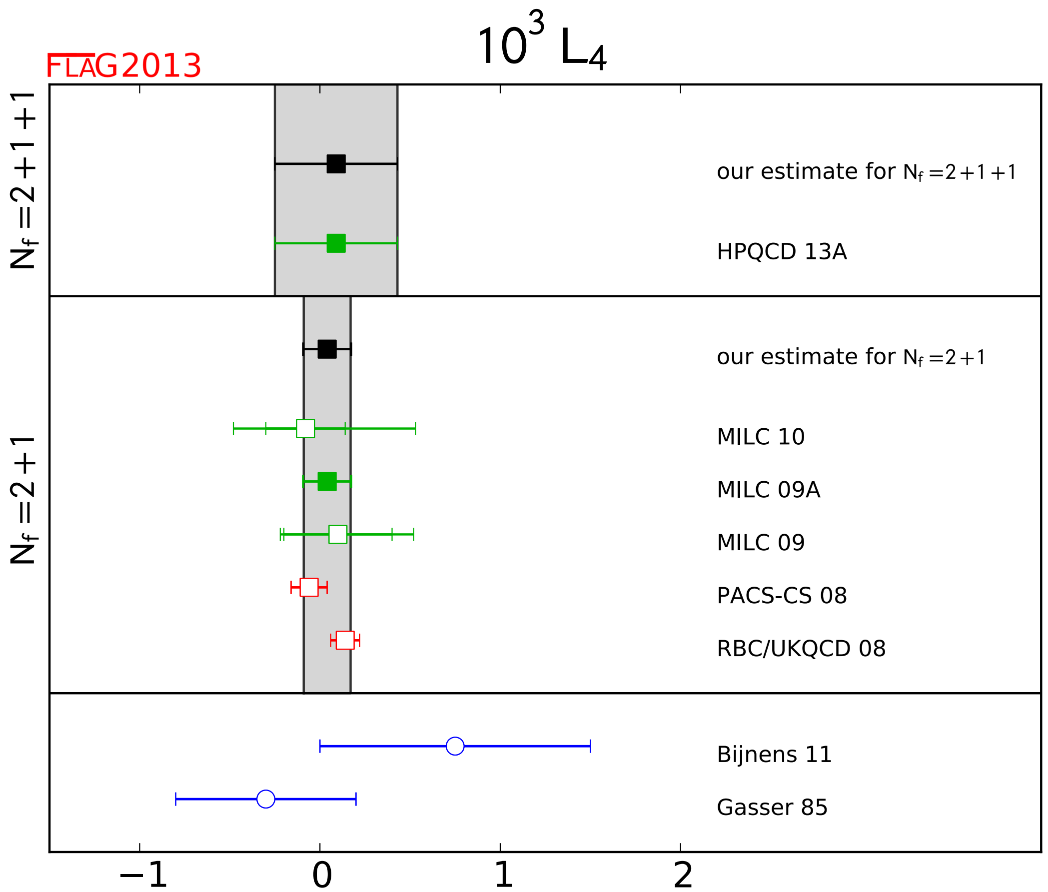
$$\chi^2/dof = 1.66/5 = 0.33$$

exclude heavy mass point

this is striking evidence of a chiral logarithm

SU(3) LECs





$$L_5 = 0.84(38) \times 10^{-3} \qquad L_4 = 0.04(14) \times 10^{-3}$$

- Results quoted are for $N_f = 2+1$.
- From decay constant.
- L_5 controls valence mass dependence; L_4 controls sea mass dependence.

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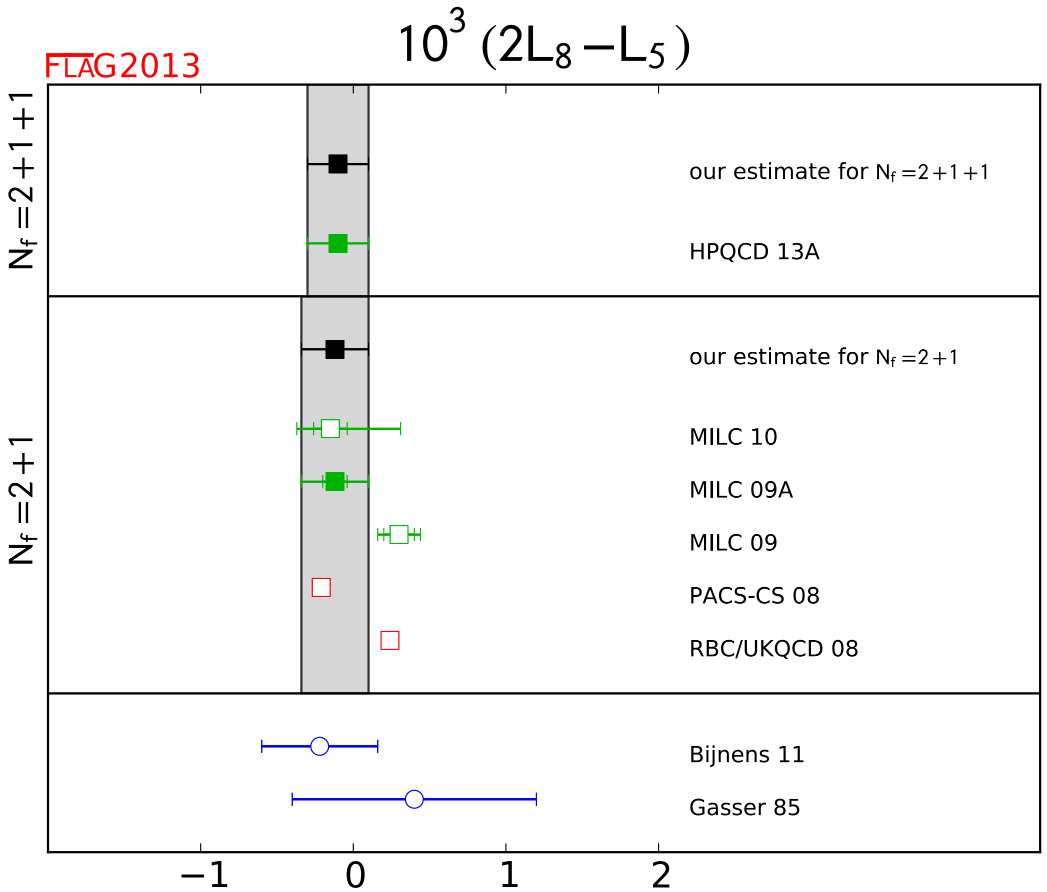
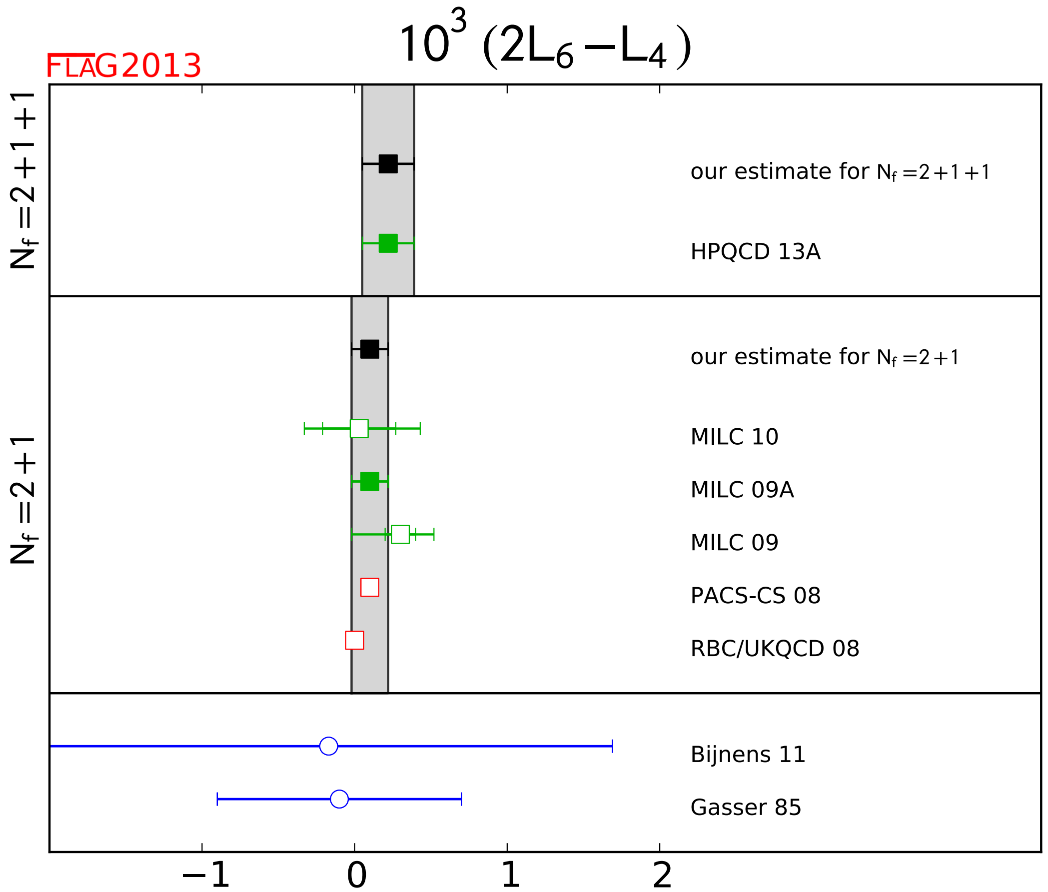


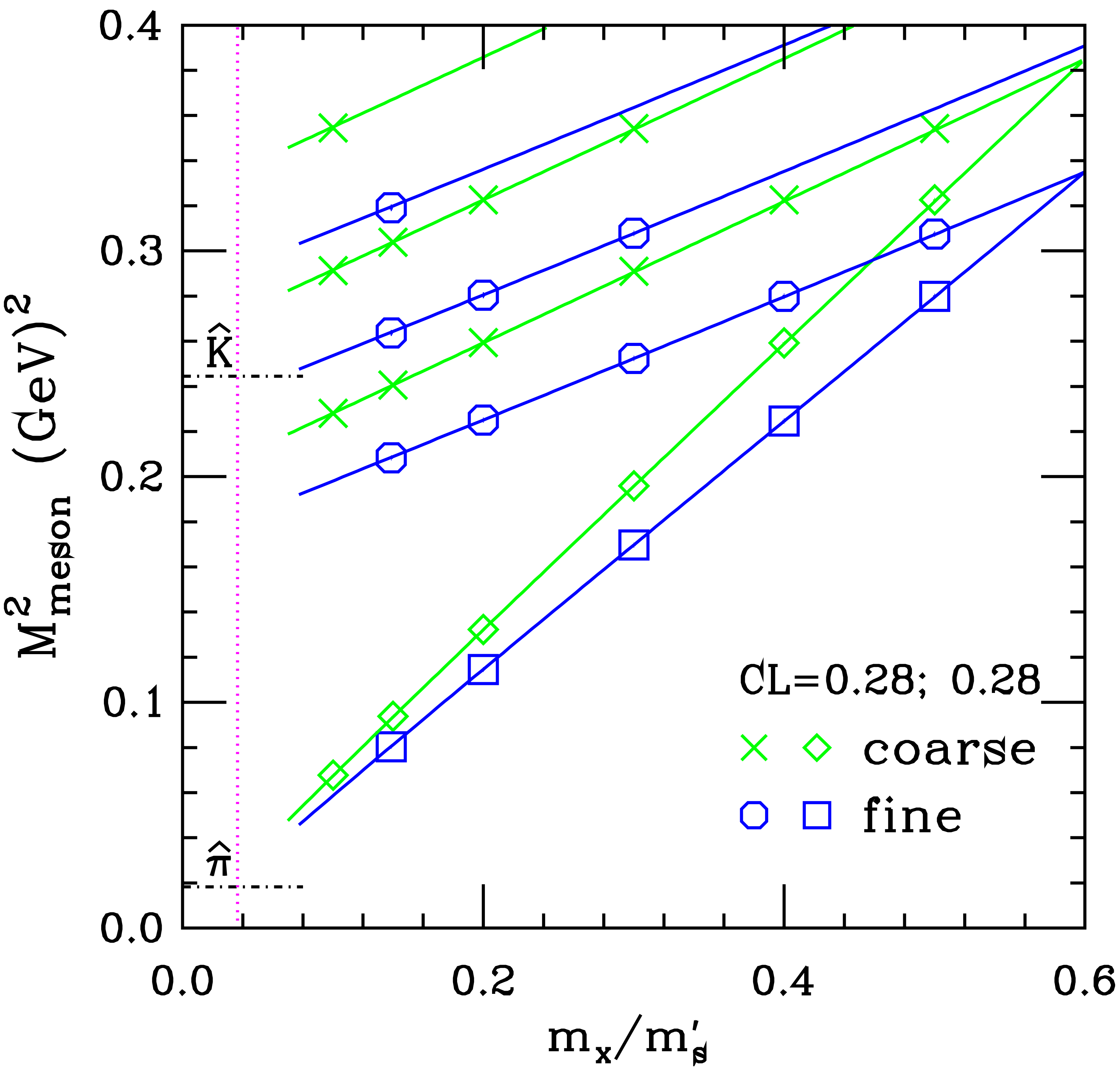


$$2L_8 - L_5 = -0.12(22) \times 10^{-3}$$

$$2L_6 - L_4 = 0.10(12) \times 10^{-3}$$

- Results quoted are for $N_f = 2+1$.
- From meson mass.
- $2L_8$-L_5 controls valence mass dependence; $2L_6$-L_4 controls sea mass dependence.
- Small because m_π^2 is nearly linear in quark mass (small NLO corrections).

Meson (mass)^2



- m_π^2 and m_K^2 vs. $\hat{m}$
- shows how linear the (mass)^2 is.
- old: from MILC, 2004!
- usually people divide by $\hat{m}$ to show non-linearity.