Recoil corrections in antikaon-deuteron scattering

V. Baru, E. Epelbaum, M. Mai, A. Rusetsky
**MOTIVATION**

- The $\bar{K}N$ system as a testing ground for low energy $SU(3)$ meson-baryon dynamics
- Two fundamental quantities: $a_{I=0}$ and $a_{I=1}$

⇒ Two experiments

1) $K^-p$: energy shift and width of the 1s level for kaonic hydrogen in SIDDHARTA\(^1\) @ DAΦNE
   - related to $a_{K^-p}$ via modified *Deser-type formula*\(^2\)
2) $K^-d$: X-ray yield of kaonic deuterium derived.
   Important for:
   - planned upgrade to SIDDHARTA-2
   - Kaon implantation experiment @ J-PARC

⇒ GOAL: Explicit relation btw. $a_1$, $a_0$ and $A_{K^-d}$

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\(^1\) Bazzi et al. (2011)
\(^2\) Meißner, Raha, Rusetsky (2004)
1. “Unitarized ChPT“ for meson-baryon scattering
   → adjust free parameters to the scattering data on $\bar{K}N$ system\(^1\)
   ⇒ $\bar{K}d$ not addressed!

2. Three-body Faddeev equation:
   → determine the $\bar{K}NN$ amplitude numerically\(^2\)
   ⇒ knowledge of $NN$ and $\bar{K}N$ potential required!

3. Multiple-scattering series:
   → poor convergence ⇒ resummation\(^3\) in the static limit ($m_N \to \infty$)

   $A_{st} \sim \int d^3r \left( |\Psi(r)|^2 \frac{4r\tilde{a}_0\tilde{a}_1 + r^2(\tilde{a}_0 + 3\tilde{a}_1)}{2r^2 + r(\tilde{a}_0 - \tilde{a}_1) - 2\tilde{a}_1\tilde{a}_0} \right)$ for $\tilde{a} = (1 + \xi)a$, $\xi = \frac{M_K}{m_N}$

   ⇒ What are the nucleon recoil corrections?
   ⇒ in principle they start with $\sqrt{\xi} \approx 0.7$
   ⇒ numerical solutions of Faddeev eqn. suggest $\sim 15\%$ effect\(^4\)

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\(^1\) Ikeda, Hyodo, Weise (2012) MM, Meißner (2012)...
\(^2\) Shevchenko (2012) ...
\(^3\) Kamalov, Oset, Ramos (2001)
\(^4\) Gal (2008)
Recoil corrections - idea

- $\bar{K}N$ scale is large ($\sim M_\rho$) → effective range expansion
- NN potential is characterized by a soft scale ($\sim M_\pi$) → taken explicitly

$\Rightarrow$ Three types of interactions:

- In the static limit only (b) contributes
  → rewrite the $\bar{K}NN$ propagator: $g = (g - g_{st}) + g_{st} := \Delta g + g_{st}$

\[
A = \tilde{a} + \tilde{a}^2g + \tilde{a}^3g^2 + \cdots \\
= \{\tilde{a} + \tilde{a}^2g_{st} + \cdots\} + \{\tilde{a} + \tilde{a}^2g_{st} + \cdots\} (\Delta g) \{\tilde{a} + \tilde{a}^2g_{st} + \cdots\} + \cdots \\
= A_{st} + A^{(1)} + A^{(2)} + ...
\]

→ include interactions of type (a) and (c)
Recoil corrections - idea

- **Full set of Feynman diagrams:**
  - No recoil insertions $A_{st} \Rightarrow$ one diagram
  - One recoil insertion $A^{(1)} \Rightarrow$ three diagrams
  - Two recoil insertions $A^{(2)} \Rightarrow$ six diagrams
  - ...  

  $\Rightarrow$ computational challenge: rising number of integrals
  $\Rightarrow$ one insertion is done, two is in preparation
IS THIS A GOOD APPROACH?

IF “YES“, WHAT DOES IT PREDICT?
Test of the framework - results

- Convergence of $A^{(n)}$ series in powers of $\sqrt{\xi}$
  - $\rightarrow$ sign for a good counting scheme of $A = A_{st} + A^{(1)} + A^{(2)} + ...$
  - $\rightarrow$ uniform expansion\(^5\) of $A^{(n)}$
    - $\rightarrow$ independent of the regularization procedure
    - $\rightarrow$ applicable to any Feynman diagram
  - $\rightarrow$ isospin decomposition ($NN$ interm. state) reveals additional cancellation pattern:
    1) $I=1$ ($S=1$, $L=1$): cancels exactly at $\mathcal{O}(\sqrt{\xi})$
    2) $I=0$ ($S=1$, $L=0$): cancels exactly at $\mathcal{O}(\sqrt{\xi})$, if $[\text{module}]$ is constant
    - $\rightarrow$ orthogonality of bound state and continuum w.f.
  - $\rightarrow$ assume for now Hulthén potential ($\beta = 1.4 \text{fm}^{-1}$) for the $NN$
    interaction:
    \[
    V_{NN}(p, q) = \lambda g(p)g(q), \quad g(p) = \frac{1}{\beta^2 + p^2}, \quad \lambda = 32\pi m_N \beta (\beta + \gamma)^2
    \]

Test of the framework - results

- Expansion in $\tilde{\xi} := \xi/(1 + \xi/2)$ yields:

$$A_1 = \frac{8\pi}{(1 + \xi/2)^2} \left( c_{1,1} \tilde{\xi} + c_{1,1} \tilde{\xi}^2 + \ldots + b_{1,1} \tilde{\xi}^{3/2} + b_{1,2} \tilde{\xi}^{5/2} + \ldots \right)$$

$$A_0 = \frac{8\pi}{(1 + \xi/2)^2} \left( c_{0,1} \tilde{\xi} + c_{0,1} \tilde{\xi}^2 + \ldots + b_{0,0} \tilde{\xi}^{1/2} + b_{0,1} \tilde{\xi}^{3/2} + \ldots \right)$$

$$A_c = \frac{8\pi}{(1 + \xi/2)^2} \left( C_1 \tilde{\xi}^2 + C_2 \tilde{\xi}^2 + \ldots + B_1 \tilde{\xi}^{1/2} + B_2 \tilde{\xi}^{3/2} + \ldots \right)$$

\[ \Rightarrow \text{convergence after a few orders in} \ \tilde{\xi}^{1/2} \]

\[ \Rightarrow \text{LO sizable cancellations:} \quad b_{0,0} = -0.047 + i0.154 \leftrightarrow B_1 = +0.047 - i0.132 \]
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⇒ convergence after a few orders in $\tilde{\xi}^{1/2}$

⇒ LO sizable cancellations: $b_{0,0} = -0.047 + i0.154 \leftrightarrow B_1 = +0.047 - i0.132$
IS THIS A GOOD APPROACH? ✓

WHAT DOES IT PREDICT?
## I. Choice of the $NN$ potential

- $NN$: Hulthén and PEST\(^6\) potential (short range physics)
- $\bar{K}N$: $a_1 = -1.62 + i0.78$ fm, $a_0 = +0.18 + i0.68$ fm

<table>
<thead>
<tr>
<th></th>
<th>Hulthén</th>
<th>PEST</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{st}$</td>
<td>$-1.492 + i1.187$</td>
<td>$-1.549 + i1.245$</td>
</tr>
<tr>
<td>$A^{(1)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>$-0.004 - i0.045$</td>
<td>$+0.002 - i0.039$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$-0.380 + i1.192$</td>
<td>$-0.401 + i1.309$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$+0.352 - i1.058$</td>
<td>$+0.356 - i1.193$</td>
</tr>
<tr>
<td>Sum:</td>
<td>$-0.031 + i0.090$</td>
<td>$-0.043 + i0.076$</td>
</tr>
</tbody>
</table>

$A_{st} + A^{(1)}$ | $-1.523 + i1.277$ | $-1.593 + i1.322$

⇒ one insertion corrections are moderate for both NN potentials

\(^6\)Zankel et al. (1983)
II. Higher orders? *(preliminary)*

- Do we need the two, three, ... recoil insertions corrections?
- Formally they start at $(\xi^{1/2})^2$, $(\xi^{1/2})^3$, ...
- First estimation of two insertion corrections (for Hulthén):

<table>
<thead>
<tr>
<th>$A_{st}$ [fm]</th>
<th>$A^{(1)}$ [fm]</th>
<th>$A^{(2)}$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>1</td>
<td>$-0.00 - i0.04$</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>$-0.03 + i0.13$</td>
<td>00</td>
</tr>
<tr>
<td></td>
<td>$-0.03 + i0.09$</td>
<td>10</td>
</tr>
<tr>
<td>$\sum$</td>
<td>$-1.49 + i1.19$</td>
<td></td>
</tr>
<tr>
<td>$\sum$</td>
<td>$-1.49 + i1.19$</td>
<td></td>
</tr>
</tbody>
</table>

$\Rightarrow$ Estimate two recoil corrections:

$\xi^{1/2} Im(A^{(1)}_1) = -0.03$ fm, $\xi^{1/2} Im(A^{(1)}_0) = 0.09$ fm

$\Rightarrow$ Estimate three recoil corrections: $\xi Im(A^{(1)}_0) = 0.07$ fm $\approx 6\% A_{st}$

$\Rightarrow$ Further cancellations might reduce the size of recoil corrections
III. Syntetic data

- $NN$: PEST potential
- $\bar{K}N$: syntetic data around literature values, restricted by SIDDHARTA

$\Rightarrow (A_{st} + A^{(1)})$ depends strongly on the choice of $\bar{K}N$ s.l.

$\Rightarrow$ precise exp. data on $\bar{K}d$ system can restrict $a_0$ and $a_1$ significantly!
Conclusion

✓ Analytic formulas for multiple insertion corrections
✓ Expansion of $A^{(1)}$ in powers of $\xi$ converges
✓ Large cancellations at LO in one insertion corr.
✓ One insertion corr.: $7 - 8\%$ of the static result $\Rightarrow$ Good news!
✓ $A$ is sensitive to $a_0$ and $a_1$ $\Rightarrow$ Good news for future experiment on kaonic deuterium

! Finite range/relativistic corrections
! Investigation of results for two insertion correction

... in progress