Recoil corrections in antikaon-deuteron scattering

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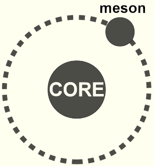
MOTIVATION

- The $\bar{K}N$ system as a testing ground for low energy $SU(3)$ meson-baryon dynamics
- Two fundamental quantities: $a_{I=0}$ and $a_{I=1}$

$\Rightarrow$ **Two experiments**

1) K^-p: energy shift and width of the $1s$ level for kaonic hydrogen in SIDDHARTA[1] @ DAΦNE
   - related to a_{K^-p} via modified *Deser-type formula*[2]
2) K^-d: X-ray yield of kaonic deuterium derived. Important for:
   - *planned* upgrade to SIDDHARTA-2
   - Kaon implantation experiment @ J-PARC



$\Rightarrow$ **GOAL: Explicit relation btw. a_1, a_0 and A_{K^-d}**

[1] **Bazzi et al.(2011)**
[2] **Meißner, Raha, Rusetsky(2004)**

MOTIVATION

1. **“Unitarized ChPT“ for meson-baryon scattering**

   → adjust free parameters to the scattering data on $\bar{K}N$ system[1]

   ⇒$\bar{K}d$ not addressed!

2. **Three-body Faddeev equation:**

   → determine the $\bar{K}NN$ amplitude numerically[2]

   ⇒knowledge of NN and $\bar{K}N$ potential required!

3. **Multiple-scattering series:**

   → poor convergence ⇒ resummation[3] in the *static limit* $(m_N \to \infty)$

$$A_{st} \sim \int d^3r \left(|\Psi(r)|^2 \frac{4r\tilde{a}_0\tilde{a}_1 + r^2(\tilde{a}_0 + 3\tilde{a}_1)}{2r^2 + r(\tilde{a}_0 - \tilde{a}_1) - 2\tilde{a}_1\tilde{a}_0} \right) \quad \text{for } \tilde{a} = (1+\xi)a\,, \ \xi = \frac{M_K}{m_N}$$

   → **What are the nucleon recoil corrections?**

   ⇒in principle they start with $\sqrt{\xi} \approx 0.7$

   ⇒numerical solutions of Faddeev eqn. suggest $\sim 15\%$ effect[4]

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[1] Ikeda, Hyodo, Weise(2012) MM, Meißner(2012)...

[2] Shevchenko (2012) ...

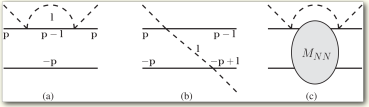
[3] Kamalov, Oset, Ramos (2001)

[4] Gal (2008)

Recoil corrections - idea

- $\bar{K}N$ scale is large ($\sim M_\rho$) $\rightarrow$ effective range expansion
- NN potential is characterized by a soft scale ($\sim M_\pi$) $\rightarrow$ taken explicitly

$\Rightarrow$**Three types of interactions**:



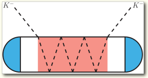
- In the static limit only **(b)** contributes
  $\rightarrow$ rewrite the $\bar{K}NN$ propagator: $g = (g - g_{st}) + g_{st} := \Delta g + g_{st}$

$$\begin{aligned} A =& \; \tilde{a} + \tilde{a}^2 g + \tilde{a}^3 g^2 + \cdots \\ =& \; \{\tilde{a} + \tilde{a}^2 g_{\mathrm{st}} + \cdots\} + \{\tilde{a} + \tilde{a}^2 g_{\mathrm{st}} + \cdots\} \, (\Delta g) \, \{\tilde{a} + \tilde{a}^2 g_{\mathrm{st}} + \cdots\} + \cdots \\ =& \; A_{st} + A^{(1)} + A^{(2)} + \ldots \end{aligned}$$

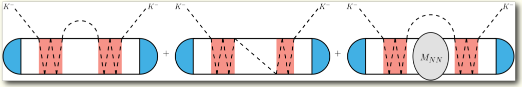
$\rightarrow$ include interactions of type **(a)** and **(c)**

Recoil corrections - idea

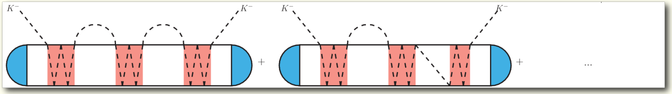
- **Full set of Feynman diagrams:**
  - → No recoil insertions A_{st} ⇒ one diagram



  - → One recoil insertion $A^{(1)}$ ⇒ three diagrams



  - → Two recoil insertions $A^{(2)}$ ⇒ six diagrams



  - → ...

⇒ computational challenge: rising number of integrals

⇒ ***one*** *insertion is done,* ***two*** *is in preparation*

IS THIS A GOOD APPROACH?

IF “YES“, WHAT DOES IT PREDICT?

Test of the framework - results

- Convergence of $A^{(n)}$ series in powers of $\sqrt{\xi}$
  - $\rightarrow$ sign for a good counting scheme of $A = A_{\mathsf{st}} + A^{(1)} + A^{(2)} + \ldots$
  - $\rightarrow$ uniform expansion[5] of $A^{(n)}$

    $\rightarrow$ *independent of the regularization procedure*

    $\rightarrow$ *applicable to any Feynman diagram*
  - $\rightarrow$ isospin decomposition (NN interm. state) reveals additional cancellation pattern:
    1) I=1 (S=1, L=1): cancels exactly at $\mathcal{O}(\sqrt{\xi})$
    2) I=0 (S=1, L=0): cancels exactly at $\mathcal{O}(\sqrt{\xi})$, if  is constant

       $\rightarrow$ *orthogonality of bound state and continuum w.f.*
  - $\rightarrow$ assume *for now* Hulthén potential ($\beta = 1.4 fm^{-1}$) for the NN interaction:

$$V_{NN}(p,q) = \lambda g(p) g(q)\,, \quad g(p) = \frac{1}{\beta^2 + p^2}\,, \quad \lambda = 32\pi m_N \beta\,(\beta+\gamma)^2$$

[5] **Beneke, Smirnov (1998) Baru, Epelbaum, Rusetsky (2009)**

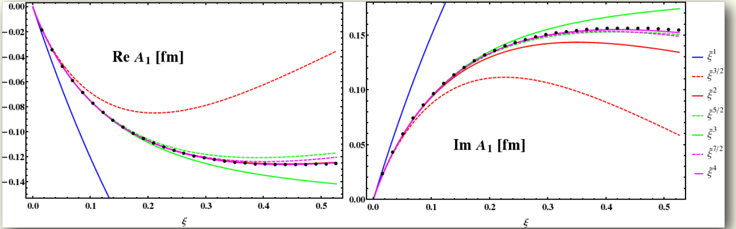
Test of the framework - results

- Expansion in $\tilde{\xi} := \xi/(1+\xi/2)$ yields:

$$A_1 = \frac{8\pi}{(1+\xi/2)^2}\left(c_{1,1}\tilde{\xi} + c_{1,1}\tilde{\xi}^2 + \ldots + b_{1,1}\tilde{\xi}^{3/2} + b_{1,2}\tilde{\xi}^{5/2} + \ldots\right)$$

$$A_0 = \frac{8\pi}{(1+\xi/2)^2}\left(c_{0,1}\tilde{\xi} + c_{0,1}\tilde{\xi}^2 + \ldots + b_{0,0}\tilde{\xi}^{1/2} + b_{0,1}\tilde{\xi}^{3/2} + \ldots\right)$$

$$A_c = \frac{8\pi}{(1+\xi/2)^2}\left(C_1\tilde{\xi} + C_2\tilde{\xi}^2 + \ldots. + B_1\tilde{\xi}^{1/2} + B_2\tilde{\xi}^{3/2} + \ldots\right)$$



$\Rightarrow$ convergence after a few orders in $\tilde{\xi}^{1/2}$

$\Rightarrow$ LO sizable cancellations: $b_{0,0} = -0.047 + i0.154 \leftrightarrow B_1 = +0.047 - i0.132$

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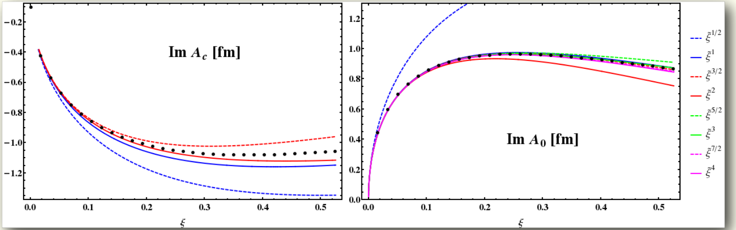

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IS THIS A GOOD APPROACH? ✓

WHAT DOES IT PREDICT?

I. Choice of the NN potential

- NN: Hulthén and PEST[6] potential (short range physics)
- $\bar{K}N$: $a_1 = -1.62 + i0.78$ fm, $a_0 = +0.18 \ + i0.78$ fm

Hulthén		
A_{st}		$-1.492 + i1.187$
$A^{(1)}$	A_1	$-0.004 - i0.045$
	A_0	$-0.380 + i1.192$
	A_c	$+0.352 - i1.058$
	Sum:	$-0.031 + i0.090$
$A_{st} + A^{(1)}$		**-1.523+i1.277**

PEST		
A_{st}		$-1.549 + i1.245$
$A^{(1)}$	A_1	$+0.002 - i0.039$
	A_0	$-0.401 + i1.309$
	A_c	$+0.356 - i1.193$
	Sum:	$-0.043 + i0.076$
$A_{st} + A^{(1)}$		**-1.593+i1.322**

⇒ one insertion corrections are moderate for both NN potentials

[6] **Zankel et al. (1983)**

II. Higher orders? *(preliminary)*

- Do we need the two, three, ... recoil insertions corrections?
- Formally they start at $(\xi^{1/2})^2$, $(\xi^{1/2})^3$, ...
- First estimation of two insertion corrections (for Hulthén):

A_{st} [fm]		$A^{(1)}$ [fm]		$A^{(2)}$ [fm]	
		I		I	
		1	$-0.00 - i0.04$	11	$+0.01 - i0.01$
		0	$-0.03 + i0.13$	00	$+0.04 + i0.09$
				10	$+0.01 - i0.00$
$\sum$	$-1.49 + i1.19$	$\sum$	$-0.03 + i0.09$	$\sum$	$+0.06 + 0.07$

→ Estimate two recoil corrections:

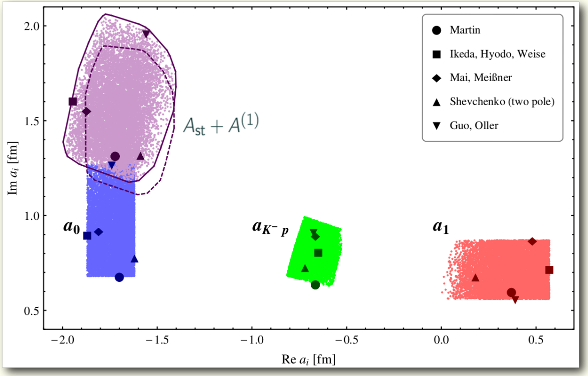
$$\xi^{1/2} Im(A_1^{(1)}) = -0.03 \text{ fm}, \; \xi^{1/2} Im(A_0^{(1)}) = 0.09 \text{ fm}$$

→ Estimate three recoil corrections: $\underline{\xi Im(A_0^{(1)}) = 0.07 \text{ fm} \approx 6\% A_{\text{st}}}$

→ Further cancellations might reduce the size of recoil corrections

III. Syntetic data

- NN: PEST potential
- $\bar{K}N$: syntetic data around literature values, restricted by SIDDHARTA



→ $(A_{st} + A^{(1)})$ depends strongly on the choice of $\bar{K}N$ s.l.

⇒precise exp. data on $\bar{K}d$ system can restrict a_0 and a_1 significantly!

Conclusion

- ✓ Analytic formulas for multiple insertion corrections
- ✓ Expansion of $A^{(1)}$ in powers of ξ converges
- ✓ Large cancellations at LO in one insertion corr.
- ✓ One insertion corr.: $7-8\%$ of the static result $\Rightarrow$ Good news!
- ✓ A is sensitive to a_0 and a_1 $\Rightarrow$ Good news for future experiment on kaonic deuterium

- ! Finite range/relativistic corrections
- ! Investigation of results for two insertion correction

... *in progress*