Hadronic Light-by-Light Scattering and the Muon $g-2$

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in collaboration with G. Colangelo, M. Hoferichter and M. Procura

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JHEP **09** (2014) 091 [arXiv:1402.7081 [hep-ph]]

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Outline

Magnetic moment

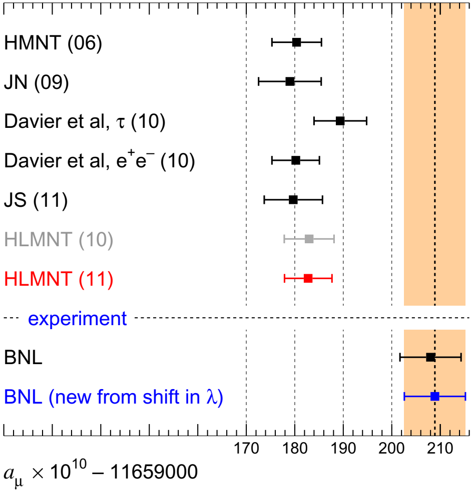
- relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

- Dirac's prediction: $g_e = 2$
- anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- helped to establish QED and QFT as the framework for elementary particle physics
- today: probing not only QED but entire SM

a_μ: comparison of theory and experiment

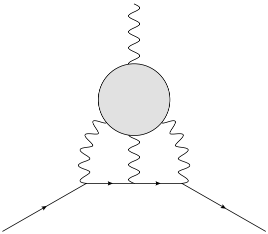


→ Hagiwara et al. 2012

a_μ: theory vs. experiment

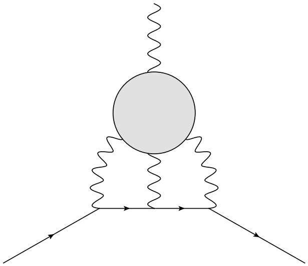
- discrepancy between SM and experiment $\sim 3\sigma$
- hint to new physics?
- new experiments (FNAL, J-PARC) aim at reducing the experimental error by a factor of 4
- theory error completely dominated by hadronic effects
- hadronic vacuum polarisation responsible for largest uncertainty, but will be systematically improved with better data input

Hadronic light-by-light (HLbL) scattering



- up to now only model calculations
- lattice QCD not yet competitive
- uncertainty estimate based rather on consensus than on a systematic method
- will dominate theory error in a few years

How to improve HLbL calculation?



- make use of unitarity, analyticity, gauge invariance and crossing symmetry
- relate HLbL to experimentally accessible quantities

The HLbL tensor

- object in question: $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$
- a priori 138 Lorentz structures
- gauge invariance: 95 linear relations
  $\Rightarrow$ (off-shell) basis: 43 independent structures
- in 4 space-time dimensions: 2 more linear relations
  $\Rightarrow$ 41 helicity amplitudes
- six dynamical variables, e.g. two Mandelstam variables
  $$s = (q_1 + q_2)^2, \quad t = (q_1 + q_3)^2$$
  and the photon virtualities q_1^2, q_2^2, q_3^2, q_4^2

HLbL tensor: Lorentz decomposition

Problem: find a decomposition

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_i T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

with the following properties:

- Lorentz structures $T_i^{\mu\nu\lambda\sigma}$ manifestly gauge invariant
- scalar functions Π_i free of kinematic singularities and zeros

HLbL tensor: Lorentz decomposition

Recipe by Bardeen, Tung (1968) and Tarrach (1975):

- apply gauge projectors to the 138 initial structures:

$$I_{12}^{\mu\nu} = g^{\mu\nu} - \frac{q_2^\mu q_1^\nu}{q_1 \cdot q_2}, \quad I_{34}^{\lambda\sigma} = g^{\lambda\sigma} - \frac{q_4^\lambda q_3^\sigma}{q_3 \cdot q_4}$$

- remove poles taking appropriate linear combinations
- Tarrach: no kinematic-free basis of 43 elements exists
- extend basis by additional structures taking care of remaining kinematic singularities

HLbL tensor: Lorentz decomposition

Solution for the Lorentz decomposition:

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i(s, t, u; q_j^2)$$

- Lorentz structures manifestly gauge invariant
- crossing symmetry manifest
- scalar functions Π_i free of kinematics
  $\Rightarrow$ ideal quantities for a dispersive treatment

Master formula: contribution to $(g-2)_\mu$

$$a_\mu^{\mathrm{HLbL}} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p + q_1)^2 - m_\mu^2][(p - q_2)^2 - m_\mu^2]}$$

- $\hat{T}_i$: known integration kernel functions
- $\hat{\Pi}_i$: linear combinations of the scalar functions Π_i
- five loop integrals can be performed with Gegenbauer polynomial techniques
- Wick rotation possible even in the presence of anomalous thresholds

Analytic properties of scalar functions

- right- and left-hand cuts in each Mandelstam variable
- double-spectral regions (box topologies)
- anomalous thresholds for large photon virtualities

Mandelstam representation

- we limit ourselves to intermediate states of at most two pions
- writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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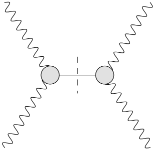
one-pion intermediate state:



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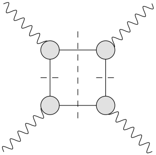
two-pion intermediate state in both channels:



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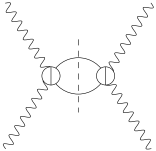
two-pion intermediate state in first channel:



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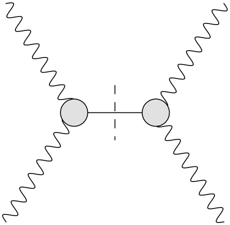
neglected: higher intermediate states

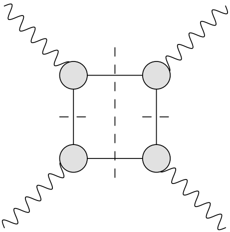
Pion pole



- input: doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- dispersive analysis of transition form factor:

  → Hoferichter et al., EPJC **74** (2014) 3180

Box contributions

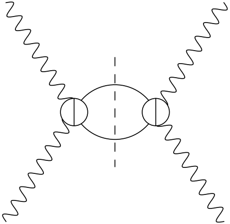


- simultaneous two-pion cuts in two channels
- analytic properties correspond to sQED loop
- Mandelstam representation explicitly constructed

$$\Pi_i = \frac{1}{\pi^2} \int ds' dt' \frac{\rho_i^{st}(s',t')}{(s'-s)(t'-t)} + (t \leftrightarrow u) + (s \leftrightarrow u)$$

- q^2-dependence given by multiplication with pion vector form factor $F_\pi^V(q^2)$ for each off-shell photon

Rescattering contribution

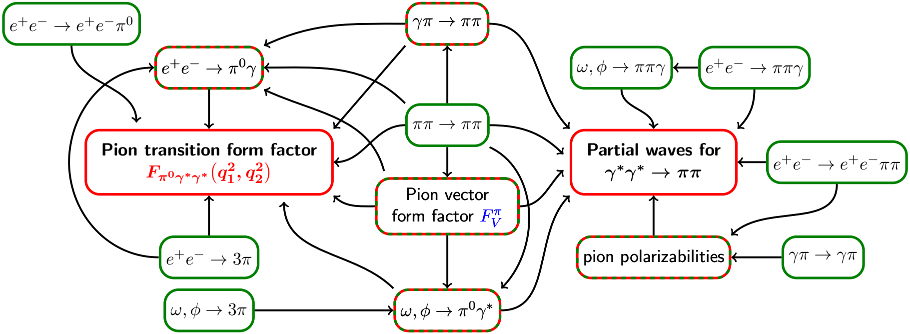


- neglect left-hand cut due to multi-particle intermediate states in crossed channel
- two-pion cut in only one channel
- expansion into partial waves
- unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$

Summary

- our dispersive approach to HLbL scattering is based on fundamental principles:
    - gauge invariance, crossing symmetry
    - unitarity, analyticity
- we take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- a step towards a model-independent calculation of a_μ
- numerical evaluation is work in progress

A roadmap for HLbL

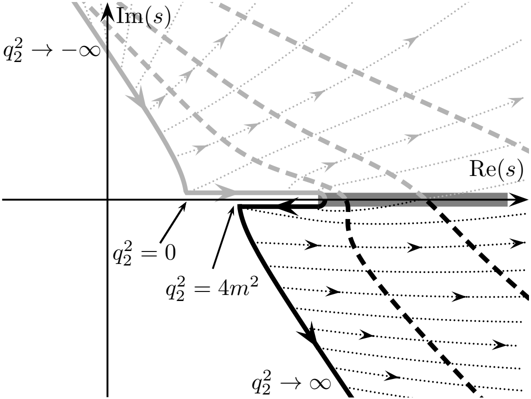


→ Flowchart by M. Hoferichter

Backup

Wick rotation

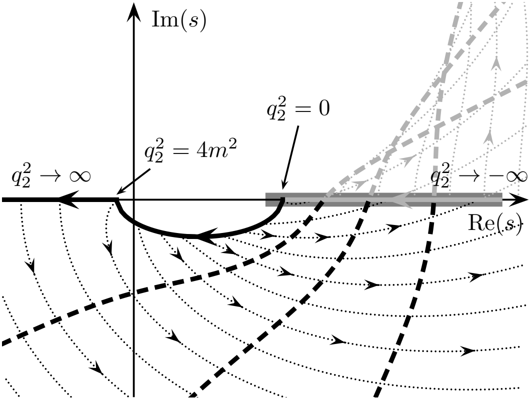
Trajectory of triangle anomalous threshold:



$$0 < q_1^2 < 4m^2$$

Wick rotation

Trajectory of triangle anomalous threshold:



$$4m^2 < q_1^2$$

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$	
BNL E821	$116\,592\,091$	63	→ PDG 2013
QED total	$116\,584\,718.95$	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
LO HVP	$6\,949$	43	→ Hagiwara et al. 2011
NLO HVP	-98	1	→ Hagiwara et al. 2011
NNLO HVP	12.4	0.1	→ Kurz et al. 2014
LO HLbL	116	40	→ Jegerlehner, Nyffeler 2009
NLO HLbL	3	2	→ Colangelo et al. 2014
Hadronic total	6982	59	
Theory total	$116\,591\,855$	59	

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$	
BNL E821	116 592 091	63	→ PDG 2013
QED $\mathcal{O}(\alpha)$	116 140 973.32	0.08	
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01	
QED $\mathcal{O}(\alpha^3)$	30 141.90	0.00	
QED $\mathcal{O}(\alpha^4)$	381.01	0.02	
QED $\mathcal{O}(\alpha^5)$	5.09	0.01	
QED total	116 584 718.95	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
Hadronic total	6982	59	
Theory total	116 591 855	59	

Model calculations of HLbL

Table 13
Summary of the most recent results for the various contributions to $a_\mu^{\mathrm{LbL;had}} \times 10^{11}$. The last column is our estimate based on our new evaluation for the pseudoscalars and some of the other results.

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

→ Jegerlehner, Nyffeler 2009

- pseudoscalar pole contribution most important
- pion-loop second most important
- differences between models, large uncertainties