Evidence that the $\Lambda(1405)$ is a molecular anti-kaon–nucleon bound state

Jonathan Hall, Waseem Kamleh, Derek Leinweber, Ben Menadue, Ben Owen, Tony Thomas, Ross Young

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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The $\Lambda(1405)$

- The $\Lambda(1405)$ is the lowest-lying odd-parity state of the Λ baryon.
- Even though it contains a heavy strange quark and has odd parity its mass is lower than any other excited spin-1/2 baryon.

$\Lambda(1405)$ 1/2$^-$ $I(J^P) = 0(\frac{1}{2}^-)$

Mass $m = 1405.1^{+1.3}_{-1.0}$ MeV
Full width $\Gamma = 50.5 \pm 2.0$ MeV
Below $\overline{K}N$ threshold

$\Lambda(1405)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\Sigma\pi$	100 %	155

- Its mass is lower than the lowest odd-parity nucleon state $N(1535)$, even though it has a valence strange quark.

The $\Lambda(1405)$

- Before the existence of quarks was confirmed, Dalitz and co-workers speculated that it might be a molecular state of an anti-kaon bound to a nucleon.
- The structure of the $\Lambda(1405)$ resonance has been the subject of extensive debate.
- We present a lattice QCD simulation that
  - Shows the $\Lambda(1405)$ strange magnetic form factor vanishes,
  - Together with a Hamiltonian effective field theory analysis of the lattice QCD energy levels,

  reveals that the structure is dominated by a bound anti-kaon–nucleon component.

Why focus on the strange magnetic form factor?

- It provides direct insight into the possible dominance of a molecular $\overline{K}N$ bound state.
- In forming such a molecular state, the $\Lambda(u, d, s)$ valence quark configuration is complemented by
  - A $u, \overline{u}$ pair making a $K^-(s, \overline{u})$ - proton (u, u, d) bound state, or
  - A $d, \overline{d}$ pair making a $\overline{K}^0(s, \overline{d})$ - neutron (d, d, u) bound state.
- In both cases the strange quark is confined within a spin-0 kaon and has no preferred spin orientation.
- To conserve parity, the kaon has zero orbital angular momentum.
- Thus, in a $\overline{K}N$ molecule the strange quark does not contribute to the magnetic form factor of the $\Lambda(1405)$.

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- Thus, in a $\overline{K}N$ molecule the strange quark does not contribute to the magnetic form factor of the $\Lambda(1405)$.

Lattice QCD Evidence that the $\Lambda(1405)$ Resonance is an Antikaon-Nucleon Molecule

Jonathan M. M. Hall,[1] Waseem Kamleh,[1] Derek B. Leinweber,[1,*] Benjamin J. Menadue,[1,2] Benjamin J. Owen,[1] Anthony W. Thomas,[1,3] and Ross D. Young[1,3]

[1]*Special Research Centre for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide, South Australia 5005, Australia*

[2]*National Computational Infrastructure (NCI), Australian National University, Canberra, Australian Capital Territory 0200, Australia*

[3]*ARC Centre of Excellence for Particle Physics at the Terascale (CoEPP), Department of Physics, University of Adelaide, Adelaide, South Australia 5005, Australia*

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For almost 50 years the structure of the $\Lambda(1405)$ resonance has been a mystery. Even though it contains a heavy strange quark and has odd parity, its mass is lower than any other excited spin-$1/2$ baryon. Dalitz and co-workers speculated that it might be a molecular state of an antikaon bound to a nucleon. However, a standard quark-model structure is also admissible. Although the intervening years have seen considerable effort, there has been no convincing resolution. Here we present a new lattice QCD simulation showing that the strange magnetic form factor of the $\Lambda(1405)$ vanishes, signaling the formation of an antikaon-nucleon molecule. Together with a Hamiltonian effective-field-theory model analysis of the lattice QCD energy levels, this strongly suggests that the structure is dominated by a bound antikaon-nucleon component. This result clarifies that not all states occurring in nature can be described within a simple quark model framework and points to the existence of exotic molecular meson-nucleon bound states.

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Outline

Variational Analysis

By using multiple operators, we can isolate and analyse individual energy eigenstates:

- Construct the correlation matrix

$$G_{ij}(\vec{p}; t) = \sum_{\vec{x}} e^{-i\,\vec{p}\cdot\vec{x}}\, \mathrm{tr}\left(\Gamma\ \langle \Omega |\, \chi_i(x)\, \overline{\chi}_j(0)\, | \Omega \rangle \right),$$

  for some set $\{\, \chi_i \,\}$ operators
- We seek the linear combinations of the operators $\{\, \chi_i \,\}$ that perfectly isolate individual energy eigenstates, α, at momentum $\vec{p}$:

$$\phi^\alpha = v_i^\alpha(\vec{p})\, \chi_i\,, \qquad \overline{\phi}^\alpha = u_i^\alpha(\vec{p})\, \overline{\chi}_i\,.$$

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$$\phi^\alpha = v_i^\alpha(\vec{p})\,\chi_i\,, \qquad \overline{\phi}^\alpha = u_i^\alpha(\vec{p})\,\overline{\chi}_i\,.$$

Variational Analysis

- Using a basis of n operators $\{\, \chi_i \,\}$ that couple to the states of interest, we can isolate and analyse individual energy eigenstates.
- Construct an $n \times n$ correlation matrix,

$$G_{ij}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | \chi_i(x) \bar{\chi}_j(0) | \Omega \rangle .$$

- We seek to find the optimised linear combination of operators that isolate an individual energy eigenstate α at momentum $\vec{p}$:

$$\bar{\phi}^\alpha = \sum_{i=1}^{N} u_i^\alpha \, \bar{\chi}_i , \qquad \phi^\alpha = \sum_{i=1}^{N} v_i^\alpha \, \chi_i$$

Variational Analysis

- In the case where a single state α participates in the optimised correlation function, one can solve the generalised eigenproblems

$$[G^{-1}(\vec{p};t)G(\vec{p};t+\delta t)]\,\mathbf{u}^{\alpha}(\vec{p}) = e^{-E_{\alpha}(\vec{p})\,\delta t}\,\mathbf{u}^{\alpha}(\vec{p})$$
$$\mathbf{v}^{\alpha\mathsf{T}}(\vec{p})\,[G(\vec{p};t+\delta t)G^{-1}(\vec{p};t)] = e^{-E_{\alpha}(\vec{p})\,\delta t}\,\mathbf{v}^{\alpha\mathsf{T}}(\vec{p})$$

  such that the the left and right eigenvectors diagonalise the correlation matrix at times t and $t+\delta t$,

$$\mathbf{v}^{\alpha\mathsf{T}}(\vec{p})G(\vec{p},t)\mathbf{u}^{\beta}(\vec{p}) = \delta^{\alpha\beta}z^{\alpha}\bar{z}^{\beta}e^{-E_{\alpha}t}.$$

Eigenstate-Projected Correlators

- The left and right vectors are used to define the eigenstate-projected correlators

$$G^{\alpha}(\vec{p};t) = \sum_{\vec{x}} \mathrm{e}^{-\mathrm{i}\,\vec{p}\cdot\vec{x}} \, \langle \Omega | \phi^{\alpha}(x) \, \overline{\phi}^{\alpha}(0) | \Omega \rangle$$
$$= \mathbf{v}^{\alpha\mathsf{T}}(\vec{p}) \, G(\vec{p};t) \, \mathbf{u}^{\alpha}(\vec{p}).$$

- Effective masses of different states are then analysed from the eigenstate-projected correlators in the usual way.
- Careful χ^2 analysis to fit single-state ansatz ensures a robust extraction of eigenstate energies,

$$G^{\alpha}(\vec{p},t) = z_{\alpha}\bar{z}_{\alpha} e^{-E_{\alpha}(\vec{p})t}.$$

Simulation Details

We are using the PACS-CS $(2+1)$-flavour ensembles, available through the ILDG.

S. Aoki *et al* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009)

- Lattice size of $32^3 \times 64$ with $\beta = 1.90$. $L \simeq 3$ fm.
- 5 pion masses, ranging from 640 MeV down to 156 MeV.
- Single strange quark mass, with $\kappa_s = 0.13640$.
  - We use $\kappa_s = 0.13665$ for the valence strange quarks to reproduce the physical kaon mass.
- The strange quark κ_s is held fixed as the light quark masses vary.
  - Changes in the strange quark contributions are environmental effects.

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The Λ(1405) and Lattice QCD

Our variational analysis successfully isolated three low-lying odd-parity spin-1/2 states.

B. Menadue, W. Kamleh, D. B. Leinweber, M. S. Mahbub, Phys. Rev. Lett. **108**, 112001 (2012)

- An extrapolation of the trend of the lowest state reproduces the mass of the Λ(1405).
- Subsequent studies have confirmed these results.

G. P. Engel, C. B. Lang, A. Schäfer, Phys. Rev. D **87**, 034502 (2013)

Operators Used in Λ(1405) Analysis

We consider local three-quark operators with the correct quantum numbers for the Λ channel, including

- Flavour-octet operators

$$\chi_1^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C\gamma_5 d^b)s^c + (u^a C\gamma_5 s^b)d^c - (d^a C\gamma_5 s^b)u^c\right)$$
$$\chi_2^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C d^b)\gamma_5 s^c + (u^a C s^b)\gamma_5 d^c - (d^a C s^b)\gamma_5 u^c\right)$$

- Flavour-singlet operator

$$\chi^1 = 2\varepsilon^{abc}\left((u^a C\gamma_5 d^b)s^c - (u^a C\gamma_5 s^b)d^c + (d^a C\gamma_5 s^b)u^c\right)$$

Operators Used in $\Lambda(1405)$ Analysis

We also use gauge-invariant Gaussian smearing to increase our operator basis.

- These results use 16 and 100 sweeps.
  - Gives a 6×6 matrix.
- Also considered 35 and 100 sweeps.
  - Results are consistent with larger statistical uncertainties.

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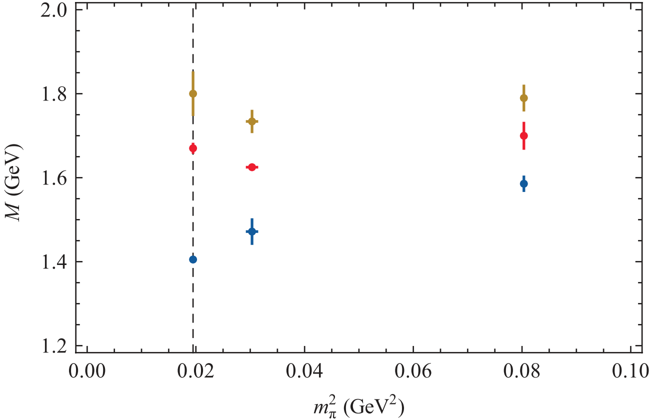
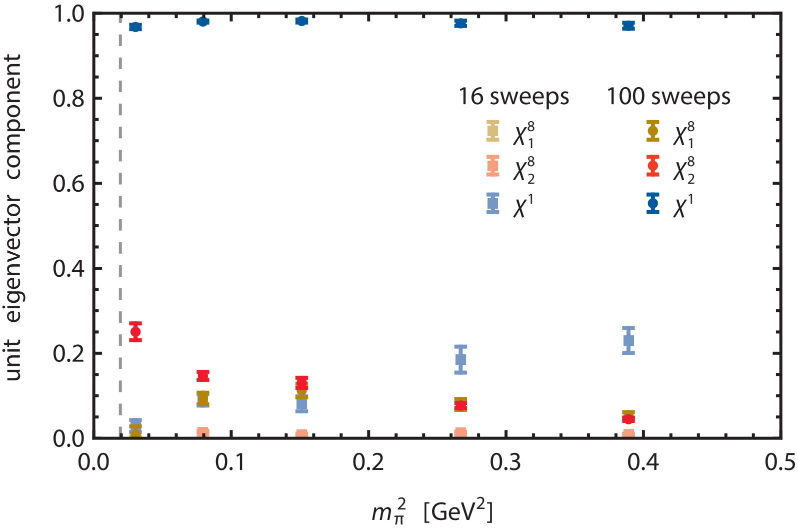
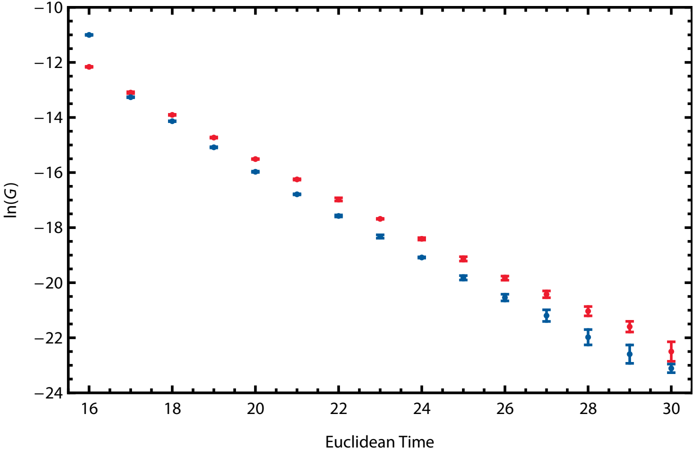
Λ(1405) and Baryon Octet dominated states



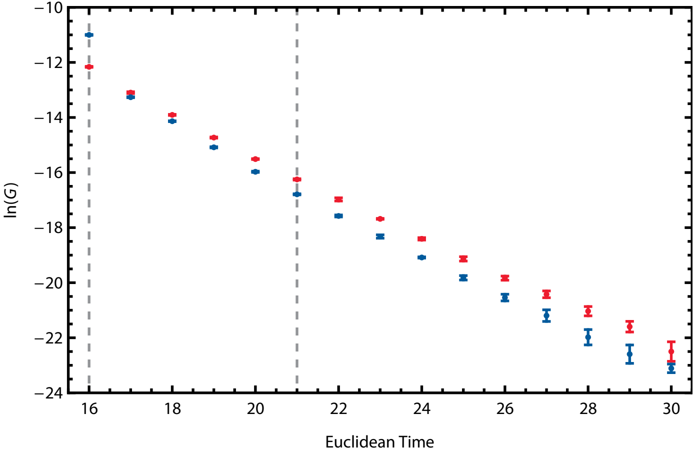
Flavour structure of the Λ(1405)



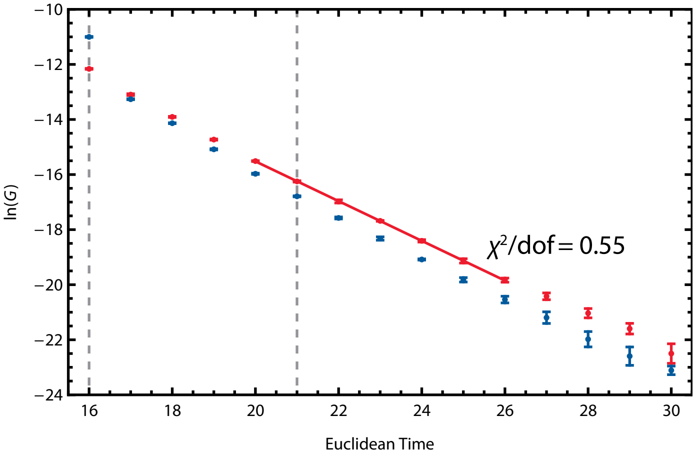
The importance of eigenstate isolation (red)



Probing with the electromagnetic current



Only the projected correlator has acceptable χ^2/dof



Extracting Form Factors from Lattice QCD

- To extract the form factors for a state α, we need to calculate the three-point correlation function

$$G_\alpha^\mu(\vec{p}', \vec{p}; t_2, t_1) = \sum_{\vec{x}_1, \vec{x}_2} \mathrm{e}^{-\mathrm{i}\,\vec{p}'\cdot\vec{x}_2} \mathrm{e}^{\mathrm{i}(\vec{p}'-\vec{p})\cdot\vec{x}_1} \langle\Omega|\phi^\alpha(x_2)\, j^\mu(x_1)\,\overline{\phi}^\alpha(0)|\Omega\rangle$$

- This takes the form

$$\mathrm{e}^{-E_\alpha(\vec{p}')(t_2-t_1)}\mathrm{e}^{-E_\alpha(\vec{p})t_1} \sum_{s,s'} \langle\Omega|\phi^\alpha|p', s'\rangle \langle p', s'|j^\mu|p, s\rangle \langle p, s|\overline{\phi}^\alpha|\Omega\rangle$$

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Current Matrix Elements for Spin-$1/2$ Baryons

The current matrix element for spin-$1/2$ baryons has the form

$$\langle p', s' | j^\mu | p, s \rangle = \left(\frac{m_\alpha^2}{E_\alpha(\vec{p}) E_\alpha(\vec{p}')} \right)^{1/2} \times \\ \times \, \overline{u}(\vec{p}') \left(F_1(q^2)\, \gamma^\mu + \mathrm{i}\, F_2(q^2)\, \sigma^{\mu\nu} \frac{q^\nu}{2 m_\alpha} \right) u(\vec{p})$$

- The Dirac and Pauli form factors are related to the Sachs form factors through

$$\mathcal{G}_\mathrm{E}(q^2) = F_1(q^2) - \frac{q^2}{(2m^\alpha)^2} F_2(q^2)$$
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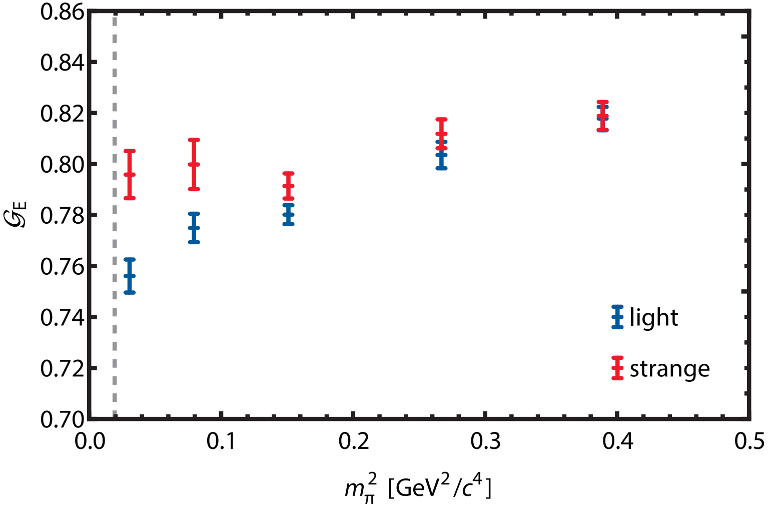
Current Matrix Elements for Spin-$1/2$ Baryons

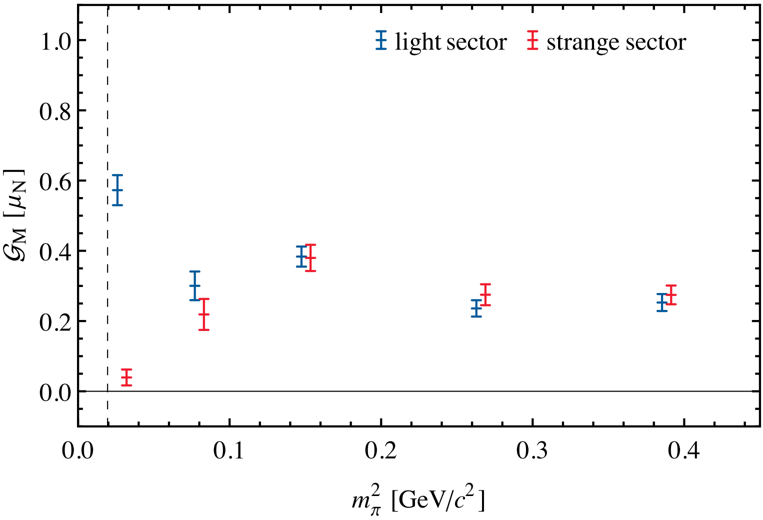
The light- and strange-quark sector contributions can be isolated.

- Eg. The strange sector is isolated by setting $q_u = q_d = 0$.
- q_s is set to unity such that we report results for single quarks of unit charge.
- Symmetry in the *u*-*d* sector provides $\mathcal{G}^u(Q^2) = \mathcal{G}^d(Q^2) \equiv \mathcal{G}^\ell(Q^2)$ for $q_u = q_d = 1$.

$\mathcal{G}_E$ for the $\Lambda(1405)$ at $Q^2 \sim 0.16\,\mathrm{GeV}^2$



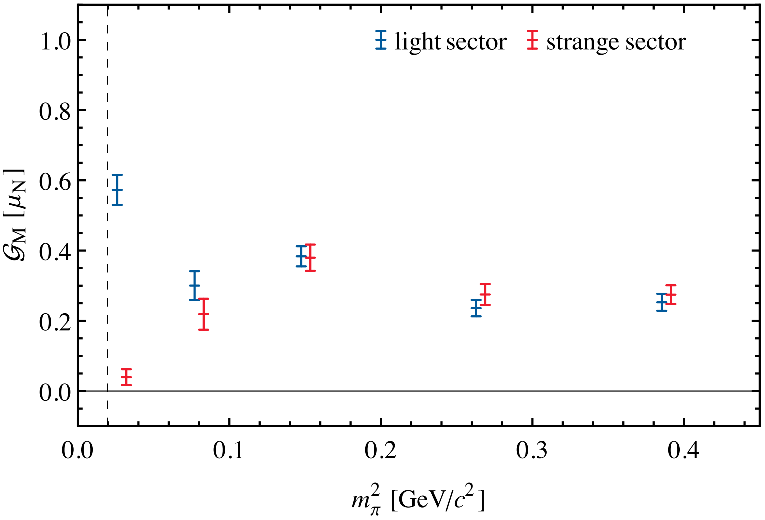
$\mathcal{G}_M$ for the $\Lambda(1405)$ at $Q^2 \sim 0.16\,\mathrm{GeV}^2$



$\Lambda(1405)$ magnetic form factor observations

- $SU(3)$-flavour symmetry is manifest for $m_\ell \sim m_s$. All three quark flavours play a similar role.
- $\mathcal{G}_M^\ell \equiv \mathcal{G}_M^u \equiv \mathcal{G}_M^d \simeq \mathcal{G}_M^s$ for the heaviest three masses.

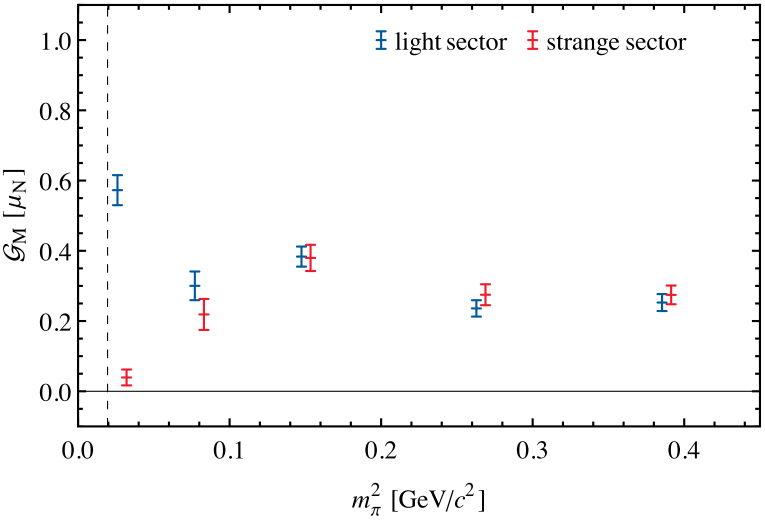
Λ(1405) magnetic form factor observations



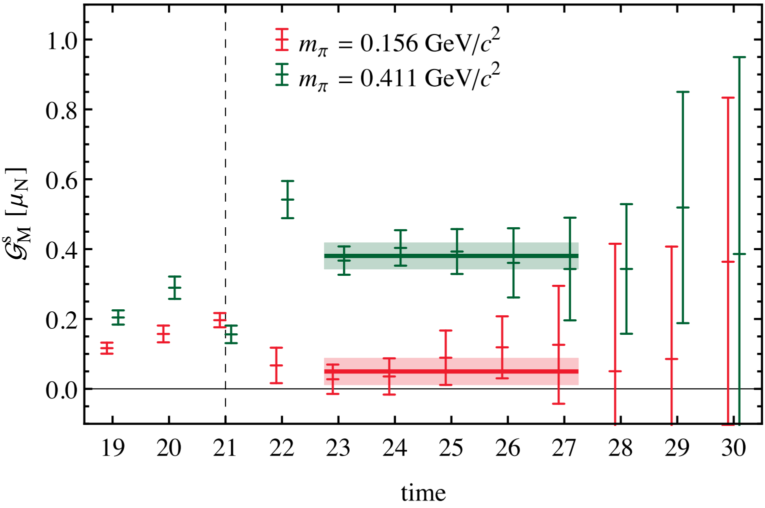
$\Lambda(1405)$ magnetic form factor observations

- The internal structure of the $\Lambda(1405)$ reorganises at the lightest quark mass.
- The strange quark contribution to the magnetic form factor of the $\Lambda(1405)$ drops by an order of magnitude and approaches zero.

Λ(1405) magnetic form factor observations



Correlation function ratio providing $\mathcal{G}_M^s(Q^2)$



$\Lambda(1405)$ magnetic form factor observations

- As the simulation parameters describing the strange quark are held fixed, this is a remarkable environmental effect of unprecedented strength.
- We observe an important rearrangement of the quark structure within the $\Lambda(1405)$ consistent with the dominance of a molecular $\overline{K}N$ bound state.

Hamiltonian Effective Field Theory Model

- Can use matrix Hamiltonian model to study resonance structure in a finite-volume.

J. M. M. Hall, A. C.-P. Hsu, D. B. Leinweber, A. W. Thomas, R. D. Young., Phys. Rev. D **87**, 094510 (2013)

- Details of matrix Hamiltonian analysis for $\Lambda(1405)$

J. M. M. Hall, WK, D. B. Leinweber, B. J. Menadue, et. al., Proc. Sci., LATTICE2014 (2014) 094

  - The four octet meson-baryon interaction channels of the $\Lambda(1405)$ are included: $\pi\Sigma$, $\overline{K}N$, $K\Xi$ and $\eta\Lambda$.
  - It also includes a single-particle state with bare mass, $m_0 + \alpha_0\, m_\pi^2$.

- In a finite periodic volume, momentum is quantised to $n\,(2\pi/L)$.
  - Working on a cubic volume of extent L on each side, it is convenient to define the momentum magnitudes

$$k_n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,\frac{2\pi}{L}\,,$$

with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.

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Hamiltonian model, H_0

Denoting each meson-baryon energy by $\omega_{MB}(k_n) = \omega_M(k_n) + \omega_B(k_n)$, with $\omega_A(k_n) \equiv \sqrt{k_n^2 + m_A^2}$, the non-interacting Hamiltonian takes the form

$$H_0 = \begin{pmatrix} m_0 + \alpha_0\, m_\pi^2 & 0 & 0 & \cdots \\ 0 & \begin{matrix} \omega_{\pi\Sigma}(k_0) & \\ & \ddots \\ & & \omega_{\eta\Lambda}(k_0) \end{matrix} & 0 & \cdots \\ 0 & 0 & \begin{matrix} \omega_{\pi\Sigma}(k_1) & \\ & \ddots \\ & & \omega_{\eta\Lambda}(k_1) \end{matrix} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Hamiltonian model, H_I

- Interaction entries describe the coupling of the single-particle state to the two-particle meson-baryon states.
- Each entry represents the S-wave interaction energy of the $\Lambda(1405)$ with one of the four channels at a certain value for k_n.

$$H_I = \begin{pmatrix} 0 & g_{\pi\Sigma}(k_0) & \cdots & g_{\eta\Lambda}(k_0) & g_{\pi\Sigma}(k_1) & \cdots & g_{\eta\Lambda}(k_1)\cdots \\ g_{\pi\Sigma}(k_0) & 0 & \cdots & & & & \\ \vdots & \vdots & 0 & & & & \\ & & & \ddots & & & \\ g_{\eta\Lambda}(k_0) & & & & & & \\ g_{\pi\Sigma}(k_1) & & & & & & \\ \vdots & & & & & & \\ g_{\eta\Lambda}(k_1) & & & & & & \\ \vdots & & & & & & \end{pmatrix} .$$

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Eigenvalue Equation Form

- The eigenvalue equation corresponding to our Hamiltonian model is

$$\lambda = m_0 + \alpha_0\, m_\pi^2 - \sum_{M,B} \sum_{n=0}^{\infty} \frac{g_{MB}^2(k_n)}{\omega_{MB}(k_n) - \lambda}\,.$$

  with λ denoting the energy eigenvalue.
- The bare mass $m_0 + \alpha_0\, m_\pi^2$ encounters self-energy corrections that lead to avoided level-crossings in the finite-volume energy eigenstates.

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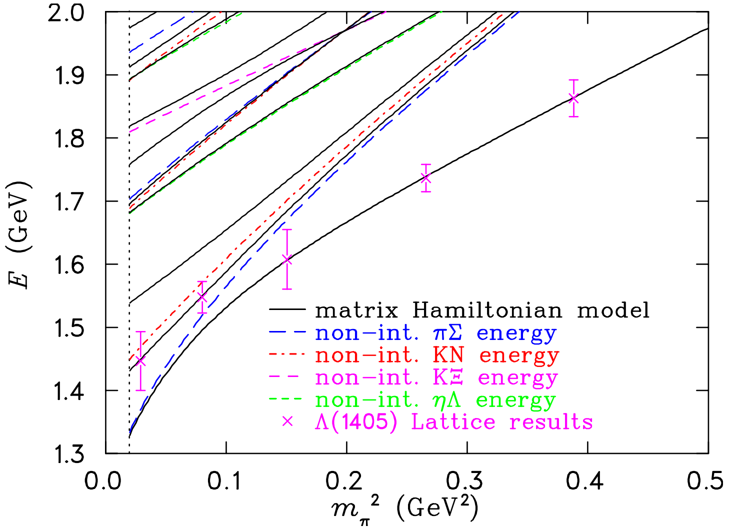
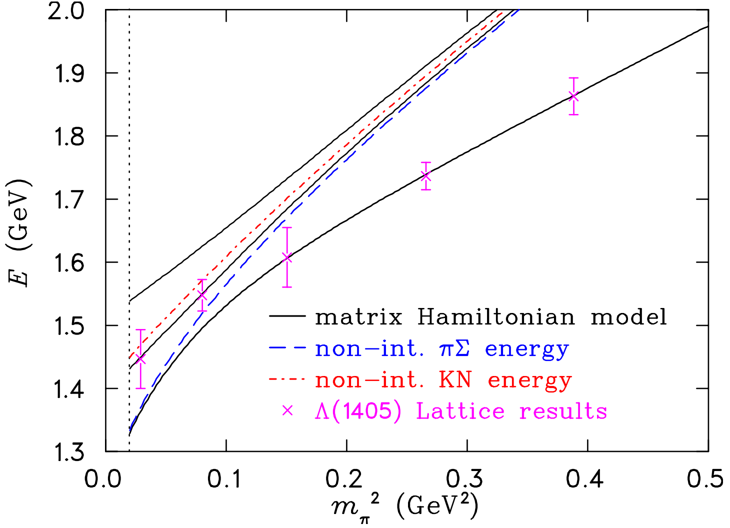
Hamiltonian model solution and fit

- The LAPACK software library routine `dgeev` is used to obtain the eigenvalues and eigenvectors of $H = H_0 + H_I$.
- The bare mass parameters m_0 and α_0 are determined by a fit to the lattice QCD results.
- Reference to chiral effective field theory provides the form of $g_{MB}(k_n)$.

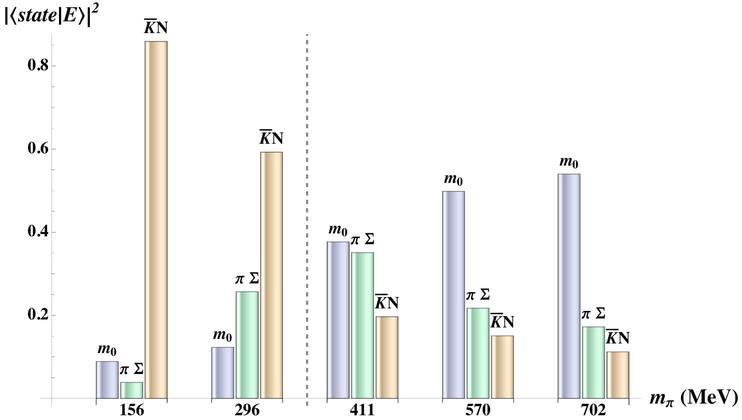
Hamiltonian model fit



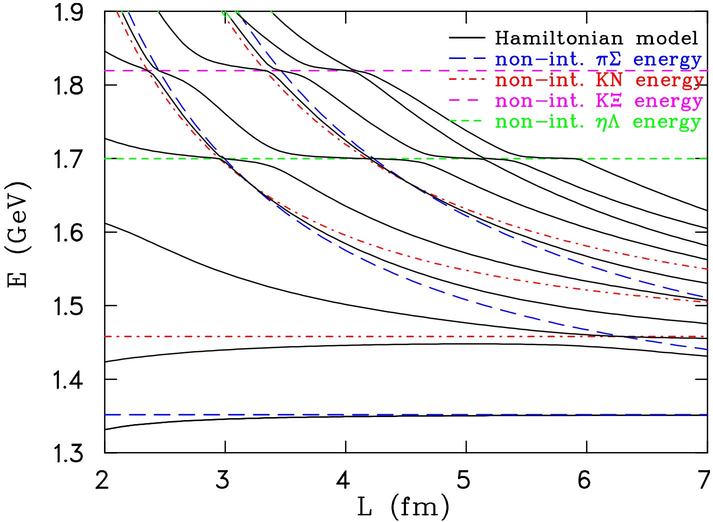
Avoided Level Crossing



Energy eigenstate, $|E\rangle$, basis $|state\rangle$ composition

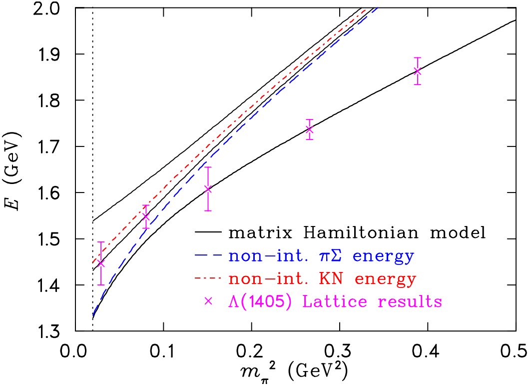


Volume dependence of the odd-parity Λ spectrum

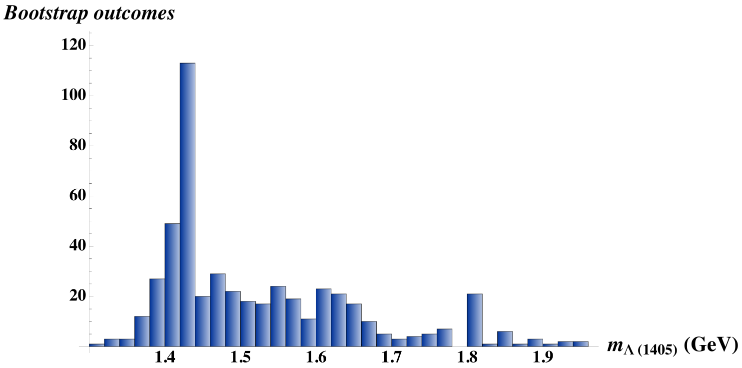


Infinite-volume reconstruction of the Λ(1405) energy

- Bootstraps are calculated by altering the value of each lattice data point by a Gaussian-distributed random number, weighted by the uncertainty.



Infinite-volume Λ(1405) mass distribution at m_π^{phys}



Conclusions

- The $\Lambda(1405)$ has been identified on the lattice through a study of its quark mass dependence and its relation to avoided level crossings in effective field theory.
- The structure of the $\Lambda(1405)$ is dominated by a molecular bound state of an anti-kaon and a nucleon.
- This structure is signified by:
  - The vanishing of the strange quark contribution to the magnetic moment of the $\Lambda(1405)$, and
  - The dominance of the $\overline{K}N$ component found in the finite-volume effective field theory Hamiltonian treatment.

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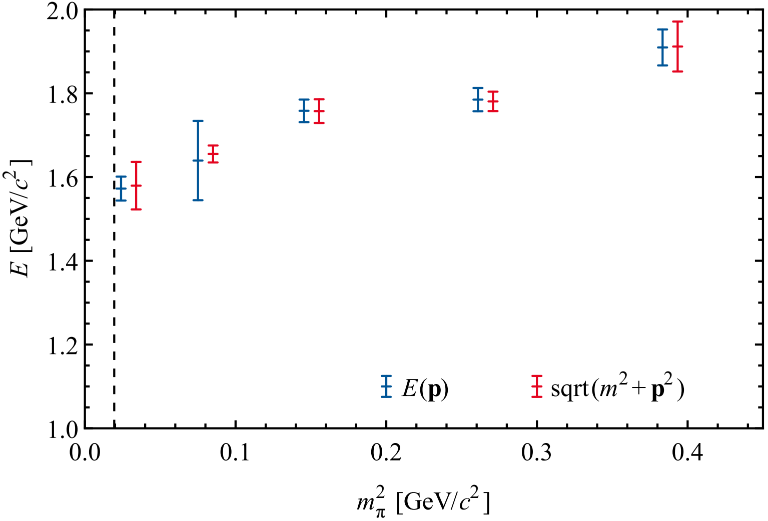
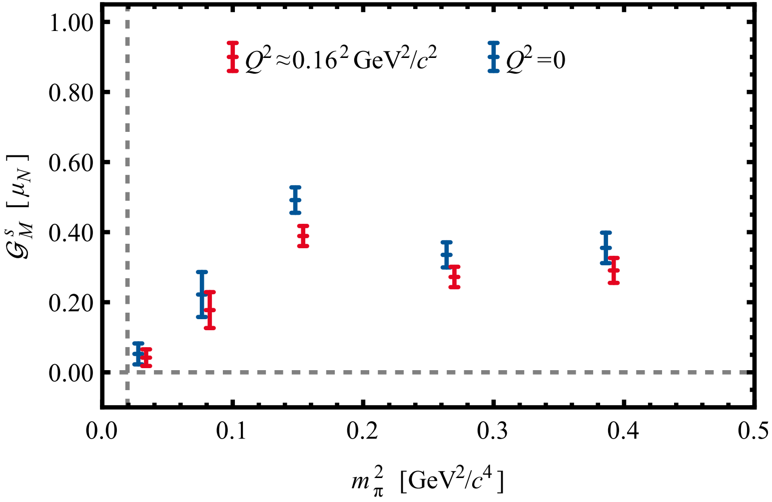
Supplementary Information

The following slides provide additional information which may be of interest.

Dispersion Relation Test for the $\Lambda(1405)$



$\mathcal{G}_{\mathrm{M}}^{s}(q^2)$ scaled to $\mathcal{G}_{\mathrm{M}}^{s}(0)$ via $\mathcal{G}_{\mathrm{M}}^{s}(q^2)/\mathcal{G}_{\mathrm{E}}^{s}(q^2)$



$\mathcal{G}_{\mathrm{E}}$ for the $\Lambda(1405)$

When compared to the ground state, the results for $\mathcal{G}_{\mathrm{E}}$ are consistent with the development of a non-trivial $\overline{\mathrm{K}}$N component at light quark masses.

- Noting that the centre of mass of the $\overline{K}(s,\bar{\ell})\ N(\ell,u,d)$ is nearer the heavier N,
  - The anti–light-quark contribution, $\bar{\ell}$, is distributed further out by the $\overline{\mathrm{K}}$ and leaves an enhanced light-quark form factor.
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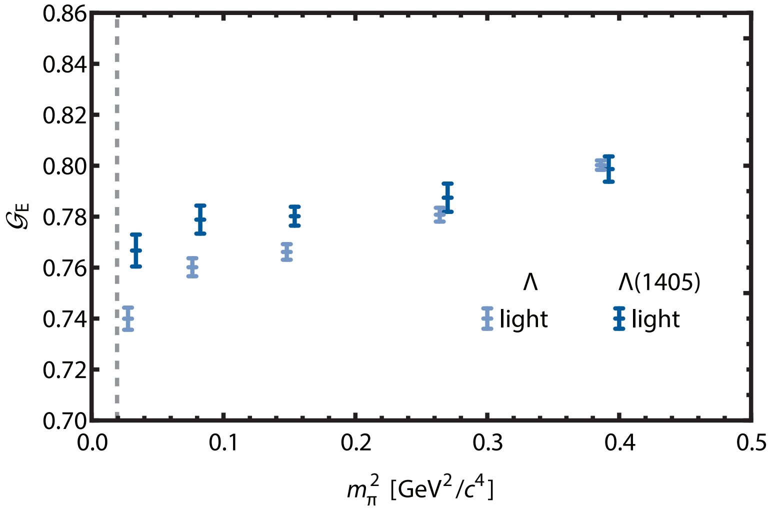
$\mathcal{G}_E$ for the Λ(1405)



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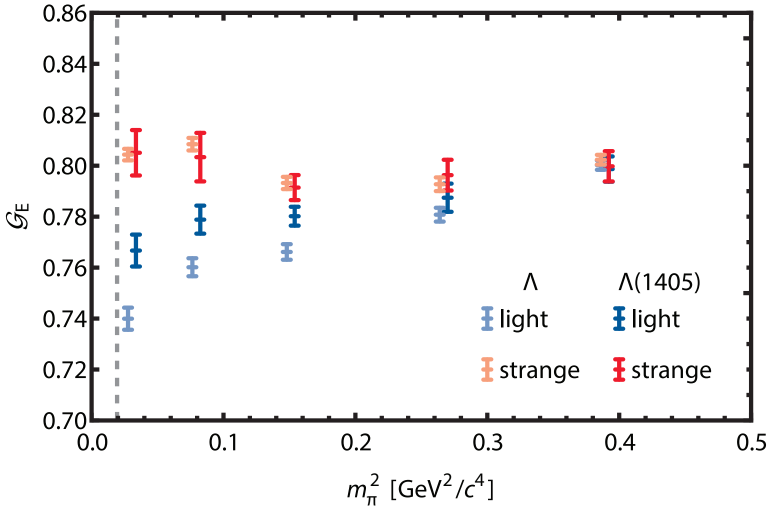


Hamiltonian model, H_I

- The form of the interaction is derived from chiral effective field theory.

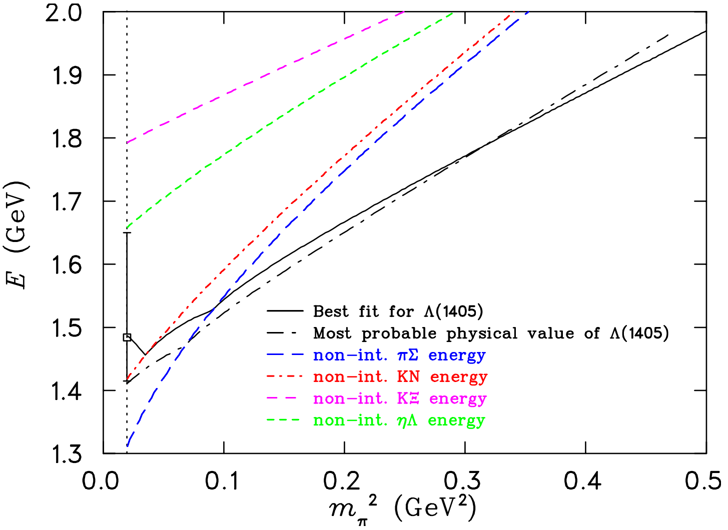
$$g_{MB}(k_n) = \left(\frac{\kappa_{MB}}{16\pi^2 f_\pi^2} \frac{C_3(n)}{4\pi} \left(\frac{2\pi}{L}\right)^3 \omega_M(k_n)\, u^2(k_n) \right)^{1/2} .$$

- κ_{MB} denotes the $SU(3)$-flavour singlet couplings

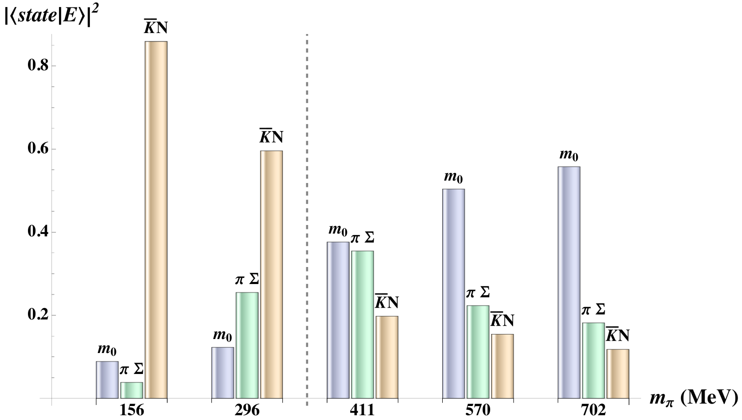
$$\kappa_{\pi\Sigma} = 3\xi_0 , \qquad \kappa_{\bar{K}N} = 2\xi_0 , \qquad \kappa_{K\Xi} = 2\xi_0 , \qquad \kappa_{\eta\Lambda} = \xi_0 ,$$

  with $\xi_0 = 0.75$ reproducing the physical $\Lambda(1405) \to \pi\Sigma$ width.
- $C_3(n)$ is a combinatorial factor equal to the number of unique permutations of the momenta indices $\pm n_x$, $\pm n_y$ and $\pm n_z$.
- $u(k_n)$ is a dipole regulator, with regularization scale $\Lambda = 0.8$ GeV.

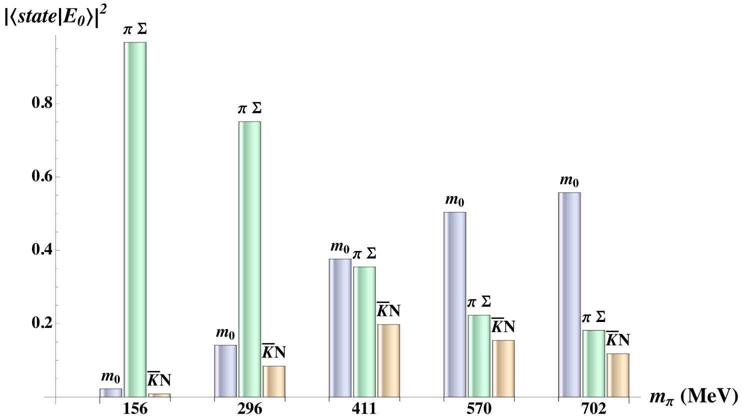
Infinite-volume reconstruction of the Λ(1405) energy



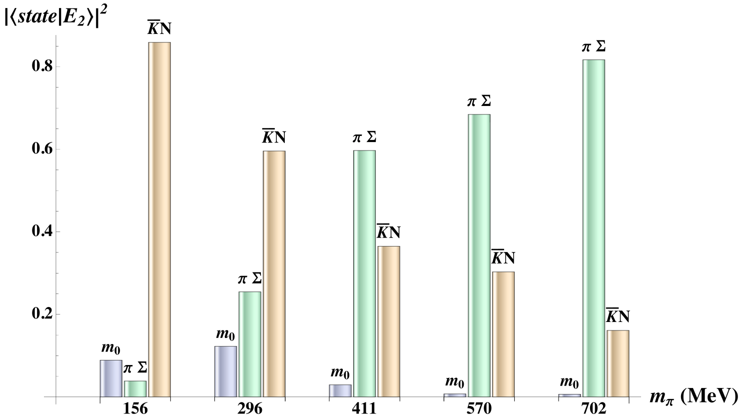
Energy eigenstate, $|E\rangle$, basis $|state\rangle$ composition



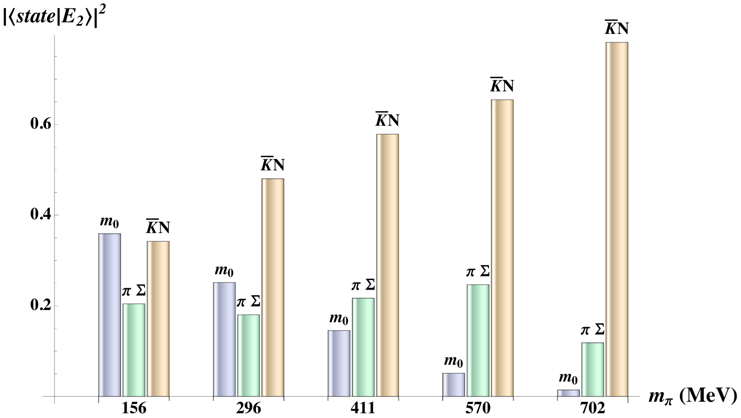
Energy eigenstate, $|E_1\rangle$, basis $|state\rangle$ composition



Energy eigenstate, $|E_2\rangle$, basis $|state\rangle$ composition



Energy eigenstate, $|E_3\rangle$, basis $|state\rangle$ composition



N- spectrum with 5-quark operators