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# Octet baryon masses in covariant baryon chiral perturbation theory


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*PRD**84**(2011)074024, JHEP**12**(2012)073, PRD**87**(2013)074001, Eur.Phys.J. C**74**(2014)2754, PRD**91**(2015)051502*

# OUTLINE

- Introduction
- Theoretical Framework
- Numerical Details
- Results and Discussion
- Summary

# Origin of masses

## ◻ Current quark masses – Explained

- Standard Model → Higgs Mechanism
- LHC @ CERN → Higgs particle

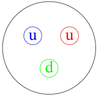
*ATLAS Collaboration, PLB716(2012)1*

*CMS Collaboration, PLB716(2012)30*

**Nobel Prize** 2013


## ◻ Light hadron masses – Complicated

QCD




Proton

$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

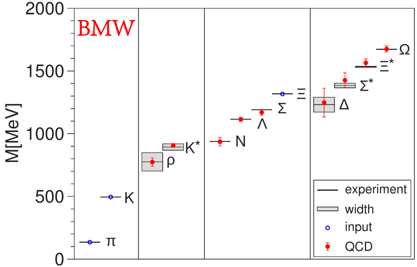
(Strong force)

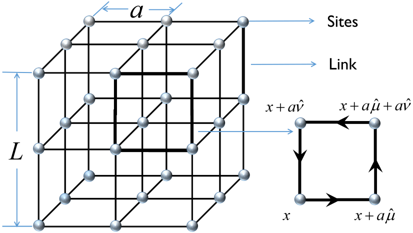
$$M_p\ (938\text{MeV}) \gg m_u + m_u + m_d\ (12\text{MeV})$$

- Current quark masses (1-3%)
- Non-perturbative strong interaction (>95%)
  - Lattice QCD
  - Chiral Perturbation Theory
  - Other Models

# Octet baryon masses in LQCD

- $N_f = 2+1$ lattice simulations
  - BMW, *S. Dürr et al., Science 322 (2008) 1224*
  - PACS-CS, *S. Aoki et al., PRD 79 (2009) 034503*
  - LHPC, *A. Walker-Loud et al., PRD 79 (2009) 054502*
  - HSC, *H.-W. Lin et al., PRD 79 (2009) 034502*
  - UKQCD, *W. Bietenholz et al., PRD 84 (2011) 054509*
  - NPLQCD, *S. Beane et al., PRD 84 (2011) 014507*




☞ Lattice simulations employ **different**:
- **fermion/gauge actions**
- **quark masses**
- **lattice volumes ($V = L^4$)**
- **lattice spacings ($a$)**

☞ In continuum:
**the fundamental theory — QCD**

**It is crucial to test the consistency of different LQCD simulations.**

# LQCD supplemented by BChPT

- Cost of LQCD

$$\text{Cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a}.$$

- Limitation of LQCD

| Input of LQCD | Simulation | Physical World |
|---|---|---|
| Light quark masses $m_{u/d}$ | $\sim 10$ MeV | $3-5$ MeV |
| Lattice box size $L$ | $2-5$ fm | Infinite space time |
| Lattice spacing $a$ | $a \sim 0.1$fm | Continuum |

In order to obtain the physical values

refer to talk- Prof. Claude Bernard

$$m_{u/d}^{\text{Lat.}} \xrightarrow{\textbf{Chiral extrapolation}} m_{u/d}^{\text{Phys.}}$$

$$L \xrightarrow{\textbf{Finite-volume corrections}} \infty$$

$$a \xrightarrow{\textbf{Continuum extrapolation}} 0$$

**Baryon Chiral Perturbation Theory (BChPT)** is a powerful tool to perform **the multi-extrapolation** for LQCD simulations.

# Analysis of LQCD baryon masses in BChPT

- Up to NNLO – **Most** studies
  - Heavy Baryon ChPT
    - ☞ **failed** to describe the lattice data *PACS-CS,PRD(2009), LHPC,PRD(2009)*
  - Extended-on-mass-shell (EOMS) BChPT
    - ☞ **Improved description** of the PACS-CS and LHPC data *J. Martin-Camalich et al., PRD(2010)*
    - ☞ Finite-volume effects in LQCD simulations are very important *L.S. Geng et al., PRD(2011)*
  - Finite-range regularization + HBChPT
    - ☞ nice description of the PACS-CS and LHPC data *R.D. Young et al., PRD(2010)*
- Up to N$^3$LO – **Few** studies
  - Partial summation BChPT
    - ☞ nice description of the BMW, PACS, LHPC and UKQCD data *A. Smeke et al., PRD(2012), M.F.M. Lutz et al., PRD(2013),(2014)*
  - Infrared BChPT
    - ☞ nice description of UKQCD data *P.C. Bruns et al.,PRD(2013)*

**Many low-energy constants (LECs) need to be fixed.**

# In this work

**Calculate the octet baryon masses in the EOMS BChPT up to N$^3$LO to systematically study the LQCD data**

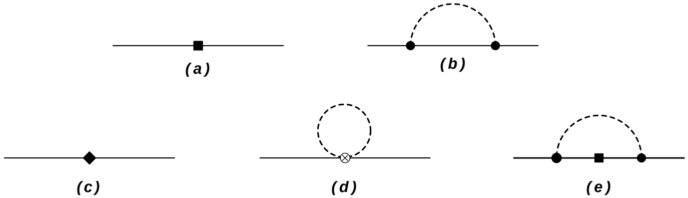
- ☑ Take into account finite volume corrections (FVCs) **self-consistently**
- ☑ Perform **a simultaneous fit** of all the $N_f = 2 + 1$ lattice results
  - ☞ Fix LECs and perform chiral extrapolation
  - ☞ Test the consistency of different LQCD data
- ☑ Perform the continuum extrapolation of LQCD by constructing **Wilson BChPT**
  - ☞ Evaluate the finite lattice spacing discretization effects
- ☑ **Accurately** predict the sigma terms of octet baryon

# Theoretical Framework

## Effective Lagrangians up to N$^3$LO

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}} &= \mathcal{L}_{\phi}^{(2)} + \mathcal{L}_{\phi}^{(4)} + \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} + \mathcal{L}_{\phi B}^{(3)} + \mathcal{L}_{\phi B}^{(4)} \\
&= \frac{F_{\phi}{}^{2}}{4}\langle D_\mu U (D_\mu U)^\dagger\rangle + \frac{F_{\phi}{}^{2}}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle + \sum_{i=4}^{8} L_i \hat{\mathcal{O}}_{\phi}^{(4)} \\
&\quad + \langle \bar{B}(i\not{D} - M_0)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]_\pm\rangle \\
&\quad + b_0\langle\chi_+\rangle\langle B\bar{B}\rangle + b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + \sum_{j=1}^{8} b_j \hat{\mathcal{O}}_{\phi B}^{(2)} + \sum_{k=1}^{7} d_k \hat{\mathcal{O}}_{\phi B}^{(4)}.
\end{aligned}
$$

## Feynman Diagrams up to N$^3$LO




**Fields:** *Solid lines – octet-baryons, Dashed lines – Pseudoscalar mesons*

**Vertex:** *Boxes –* $\mathcal{L}_{\phi B}^{(2)}$, *Diamonds –* $\mathcal{L}_{\phi B}^{(4)}$, *Solid dot –* $\mathcal{L}_{\phi B}^{(1)}$, *circle-cross –* $\mathcal{L}_{\phi B}^{(2)}$

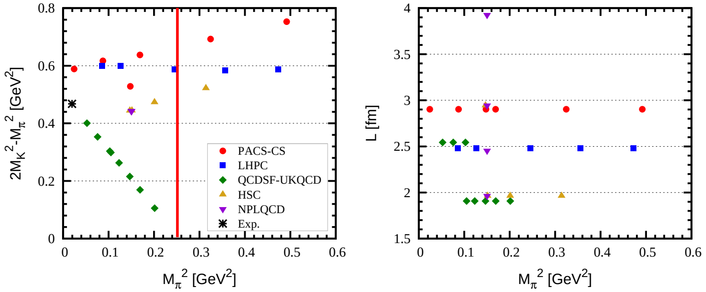
# Octet baryon masses in finite volume box

- Calculate **the baryon self-energy** in covariant BChPT
- Subtract **the PCB terms** with EOMS scheme
- Take into account **the FVCs**

$$
\begin{aligned}
m_B(M_\phi) &= m_0 + m_B^{(2)}(M_\phi) + m_B^{(3)}(M_\phi) + m_B^{(4)}(M_\phi) \\
&= m_0 + \sum_{\phi=\pi,K} \xi^{(a)}_{B,\phi} M_\phi^2 + \sum_{\phi_1,\phi_2=\pi,K,\eta} \xi^{(c)}_{B,\phi_1,\phi_2} M_{\phi_1}^2 M_{\phi_2}^2 \\
&+ \sum_{\phi=\pi,K,\eta} \xi^{(b)}_{B,\phi} \left[ H^{(b)}_{\text{loop}} - H^{(b)}_{\text{pcb}} - \Delta H^{(b)}_{\text{FVC}} \right] \\
&+ \sum_{\phi=\pi,K,\eta} \xi^{(d)}_{B,\phi} \left[ H^{(d)}_{\text{loop}} - \Delta H^{(d)}_{\text{FVC}} \right] \\
&+ \sum_{\phi=\pi,K,\eta} \xi^{(e)}_{B,\phi} \left[ H^{(e)}_{\text{loop}} - H^{(e)}_{\text{pcb}} - \Delta H^{(e)}_{\text{FVC}} \right].
\end{aligned}
$$

# Numerical Details

- Fitting data: **LQCD results (11-sets)** + **Exp. values**
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD
    - Lattice data with $M_\pi < 500$ MeV
      - ☞ reduce the higher order contributions of chiral expansions
    - Lattice data with $M_\phi L > 4$
      - ☞ minimize finite-volume effects of LQCD
  - **Fitting points:** $44(\mathrm{LQCD}) + 4(\mathrm{Exp.}) = 48$



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      - ☞ minimize finite-volume effects of LQCD
  - **Fitting points:** $44(\text{LQCD}) + 4(\text{Exp.}) = 48$
- Fitting methods: **NLO, NNLO, N$^3$LO**
  - **Free parameters**

| | | | |
|---|---|---|---|
| NLO | $m_0 + m_B^{(2)}$ | $m_0, b_0, b_D, b_F$ | **4** |
| NNLO | $m_0 + m_B^{(2)} + m_B^{(3)}$ | $m_0, b_0, b_D, b_F$ | **4** |
| N$^3$LO | $m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}$ | $m_0, b_0, b_D, b_F, b_i, d_j$ | **19** |

  - **Other parameters**
    - $L^r_{4,5,6,7,8}$, *J. Bijnens et al., NPB(2012), with $\mu = 1$ GeV*
    - $F_0 = 0.0871$ GeV, *G. Amoros et al., NPB(2001)*
    - $D = 0.80$, $F = 0.46$

# Best fitting results

| | NLO | NNLO | $N^3LO$ |
|---|---|---|---|
| $m_0$ [MeV] | 900(6) | 767(6) | **880(22)** |
| $b_0$ [GeV$^{-1}$] | $-0.273(6)$ | $-0.886(5)$ | $-0.609(19)$ |
| $b_D$ [GeV$^{-1}$] | 0.0506(17) | 0.0482(17) | 0.225(34) |
| $b_F$ [GeV$^{-1}$] | $-0.179(1)$ | $-0.514(1)$ | $-0.404(27)$ |
| $b_1$ [GeV$^{-1}$] | – | – | 0.550(44) |
| $b_2$ [GeV$^{-1}$] | – | – | $-0.706(99)$ |
| $b_3$ [GeV$^{-1}$] | – | – | $-0.674(115)$ |
| $b_4$ [GeV$^{-1}$] | – | – | $-0.843(81)$ |
| $b_5$ [GeV$^{-2}$] | – | – | $-0.555(144)$ |
| $b_6$ [GeV$^{-2}$] | – | – | 0.160(95) |
| $b_7$ [GeV$^{-2}$] | – | – | 1.98(18) |
| $b_8$ [GeV$^{-2}$] | – | – | 0.473(65) |
| $d_1$ [GeV$^{-3}$] | – | – | 0.0340(143) |
| $d_2$ [GeV$^{-3}$] | – | – | 0.296(53) |
| $d_3$ [GeV$^{-3}$] | – | – | 0.0431(304) |
| $d_4$ [GeV$^{-3}$] | – | – | 0.234(67) |
| $d_5$ [GeV$^{-3}$] | – | – | $-0.328(60)$ |
| $d_7$ [GeV$^{-3}$] | – | – | $-0.0358(269)$ |
| $d_8$ [GeV$^{-3}$] | – | – | $-0.107(32)$ |
| $\chi^2$/d.o.f. | 11.8 | 8.6 | 1.0 |

Table: Values of the LECs.

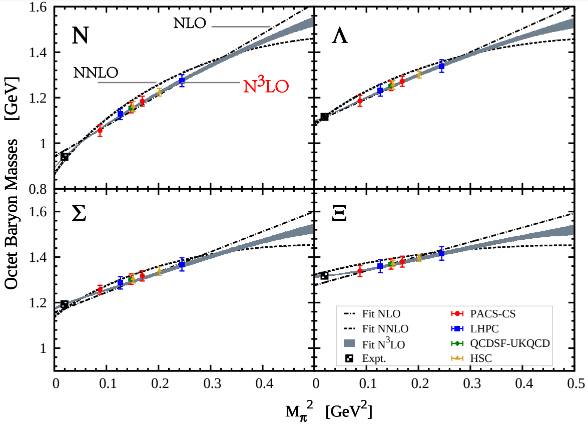
- EOMS-BChPT shows a clear improvement order by order
- Different lattice QCD simulations are consistent with each other
- Values of LECs from EOMS-$N^3LO$ look very natural
- $m_0 = 880$ MeV consistent with the SU(2)-BChPT results.

  *M. Procura et al.,PRD(2003,2006)*

  *L. Alvarez-Ruso et al., PRD(2013)*

- **Neglecting finite-volume corrections would lead to $\chi^2/\mathrm{d.o.f.} = 1.9$.**

# Chiral extrapolation




- **NLO fitting** linear and can not describe the experimental value
- **NNLO fitting** more curved and can not well describe lattice data
- **N$^3$LO fitting** can give a good description of LQCD and Exp. data, confirm the linear dependence of the lattice data on $M_\pi^2$

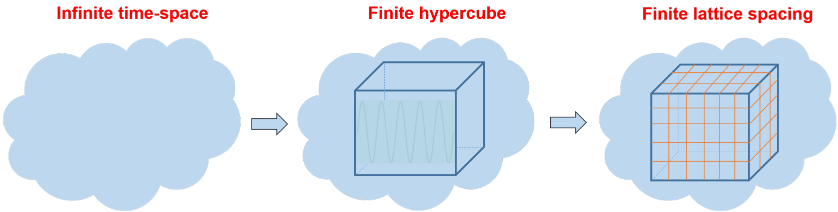
# Up to now...

## Multi-extrapolation of LQCD

$$m_{u/d}^{\text{Lat.}} \xRightarrow{\text{Chiral extrapolation}} m_{u/d}^{\text{Phys.}} \quad ✓$$

$$L \xRightarrow{\text{Finite–volume corrections}} \infty \quad ✓$$

$$a \xRightarrow{\text{Continuum extrapolation}} 0$$



# Discretization effects of LQCD

- **Construct Wilson BChPT** up to $\mathcal{O}(a^2)$

refer to talk-Prof. Claude Bernard

  - Use Symanzik effective action *K.Symanzik, NPB(1983)*

$$S_{\text{eff}} = S_0^{\text{QCD}} + aS_1 + a^2 S_2 + \cdots$$

  - Construct effective Lagrangians up to $\mathcal{O}(p^4)$ $\quad \left(p^2 \sim \frac{m_q}{\Lambda_{\text{QCD}}} \sim a\Lambda_{\text{QCD}}\right)$

$$\mathcal{L}_a^{\text{eff}} = \mathcal{L}^{\mathcal{O}(a)} + \mathcal{L}^{\mathcal{O}(am_q)} + \mathcal{L}^{\mathcal{O}(a^2)}.$$

  - Calculate discretization effects of LQCD with the Wilson fermion

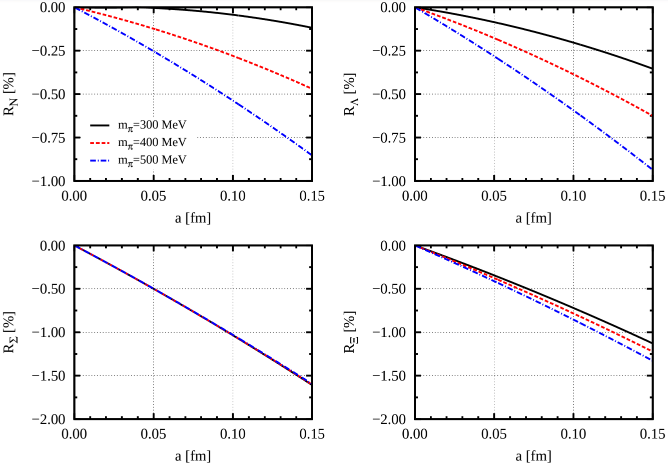
$$m_B^{(a)} = m_B^{\mathcal{O}(a)} + m_B^{\mathcal{O}(am_q)} + m_B^{\mathcal{O}(a^2)}.$$

- **Take into account FVCs and discretization effects** in the octet baryon masses

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + m_B^{(a)}.$$

- Study the LQCD results obtained with $\mathcal{O}(a)$-improved Wilson actions
  - **10 sets**: PACS-CS, QCDSF-UKQCD, HSC, NPLQCD
  - 19 LECs + **4 new** LECs (related to lattice spacing)

# Evolution of discretization effects with $a$ and $m_\pi$




$$R_B = \frac{m_B^{(a)}}{m_B}$$

- Pion mass $m_\pi$ fixed:

$$a \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Lattice spacing $a$ fixed:

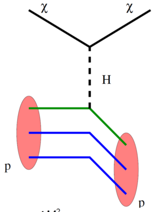
$$m_\pi \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Discretization effects on baryon masses **do not exceed 2% for $a = 0.15$ fm**
- **Consistent** with early LQCD studies *S. Durr et al., Phys. Rev. D79, (2009) 014501.*
- Up to $\mathcal{O}(a^2)$, discretization effects are **small** and can be safely **ignored**

# Pion- and strangeness-octet baryon sigma terms

## Nucleon-sigma term

- Related to chiral quark condensate $\langle \bar{q}q \rangle$
- Understand the **composition of the nucleon**
- Key input for direct **dark matter** searches
- **Strangeness-nucleon sigma term: $0 \sim 300$ MeV**




$\sigma_{SI} = \frac{4M^2}{\pi}[Zf_P + (A-Z)f_N]$

## Feynman-Hellmann Theorem

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle \equiv m_l \frac{\partial m_B}{\partial m_l},$$
$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle \equiv m_s \frac{\partial m_B}{\partial m_s}.$$

## Three key factors to accurately predict baryon sigma terms

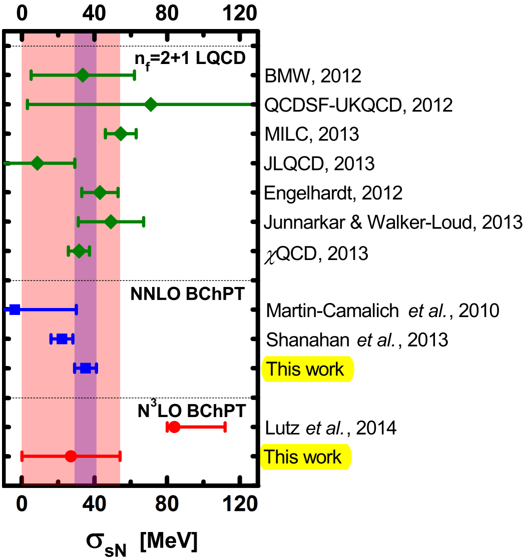
- **Effects of lattice scale setting:** mass independent vs. mass dependent
- **Strong isospin breaking effects:** better constrain the values of LECs
- **Chiral expansion truncations:** systematic uncertainties of sigma terms

# Octet baryon sigma terms from N$^3$LO BChPT

| | MIS | | MDS |
|---|---|---|---|
| | $a$ fixed | $a$ free | |
| $\sigma_{\pi N}$ | 55(1)(4) | 54(1) | 51(2) |
| $\sigma_{\pi\Lambda}$ | 32(1)(2) | 32(1) | 30(2) |
| $\sigma_{\pi\Sigma}$ | 34(1)(3) | 33(1) | 37(2) |
| $\sigma_{\pi\Xi}$ | 16(1)(2) | 18(2) | 15(3) |
| $\sigma_{sN}$ | 27(27)(4) | 23(19) | 26(21) |
| $\sigma_{s\Lambda}$ | 185(24)(17) | 192(15) | 168(14) |
| $\sigma_{s\Sigma}$ | 210(26)(42) | 216(16) | 252(15) |
| $\sigma_{s\Xi}$ | 333(25)(13) | 346(15) | 340(13) |

- Three scale setting methods yield **similar baryon sigma terms**.
- **The scale setting effects** on the sigma terms are very small.
  - MIS: mass independent scale setting
    - a-fixed: LQCD papers
    - a-free: self-consistent determined
  - MDS: mass dependent scale setting
    - $r_0$ for PACS-CS
    - $r_1$ for LHPC
    - $X_\pi$ for QCDSF-UKQCD

# $\sigma_{sN}$: comparison with earlier studies




- Our result **consistent** with the latest LQCD and NNLO BChPT results
- N$^3$LO $\sigma_{sN}$ result has **a larger uncertainty** compared to the NNLO
- Call for more LQCD data of octet baryon masses, especially with **the different strange quark masses**

# Summary

- We have systematically studied the LQCD octet baryon masses with the EOMS BChPT up to N$^3$LO
- Finite-volume and disretizaion effects on the lattice data are taken into account self-consistently
- Through simultaneously fit "all" the current LQCD data:
  - Covariant BChPT shows a clear improvement order by order
  - LQCD results are consistent with each other, though their setups are quite different
  - Up to $\mathcal{O}(a^2)$, the discretization effects on the LQCD baryon masses are shown to be small and can be safely ignored
- An accurate determination of the octet baryon sigma terms via the Feynman-Hellmann theorem.

*Thank you!*

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- **An accurate determination of the octet baryon sigma terms via the Feynman-Hellmann theorem.**

*Thank you!*

# *Back up slides*

# Power-Counting

- Systematic Power-Counting important for **perturbation theory**
  - Due to the calculation with a perturbation theory, we cannot calculate up to infinity order. Therefore, we should perform at a specific order, that's always called **chiral order** in chiral perturbation theory.
  - At this order, the corresponding **chiral effective lagrangians** can be written out with chiral symmetry and other symmetries.
  - Undering these lagrangians, there exist **infinity feynam diagrams**, obviously, we cannot calculate these infinity feynman diagrams.
  - **Then we should have a principle to determine which diagrams are important at the specific order.** Then **this principle is called power-counting**, which can deal with the previous problem to pick out the important feynman diagrams.
  - Using this systematic power-counting, we can **test the convergence of chiral expansion** with increasing the chiral order.

# Power-Counting

- In ChPT, graphs are analyzed in terms of powers of small external momenta over the large scale: $(Q/\Lambda_\chi)^\nu$.
  $Q$ is generic for a momentum (nucleon three momentum or pion four momentum) or a pion mass
  $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale
- **Determining the power $\nu$ has become known as power-counting.**
- **Naïve dimensional analysis**
  Following the Feynman rules of covariant perturbation theory, a nucleon propagator $Q^{-1}$, a pion propagator $Q^{-2}$, each derivative in any interaction is $Q$, and each four-momentum integration $Q^4$.

Chiral order:

$$\nu = 4L - 2N_M - N_B + \sum_k k v_k.$$

# Power-counting breaking

- The nucleon mass does not vanish in the chiral limit.
- The consequence of all this is that **the one-to-one corresponding between the expansion in terms of small external momenta and pion masses (Chiral Expansion) and the expansion in pion loops (Loop Expansion) is destroyed.**

# Power-counting in mesonic and baryonic sector

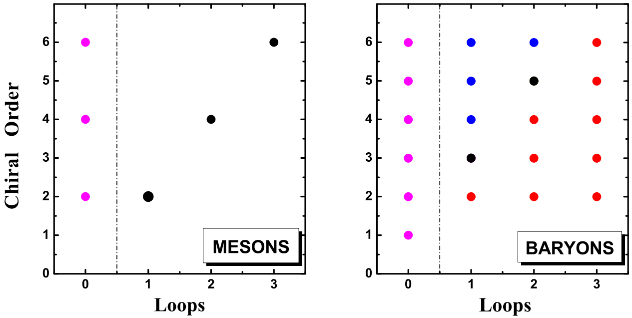
## Mesonic sector

- ChPT has gained great achievements
- Calculation up to $\mathcal{O}(p^6)$ is standard

## Baryonic sector – Baryon ChPT

- A systematic power-counting lost
- Because $m_B \neq 0$ in the chiral limit

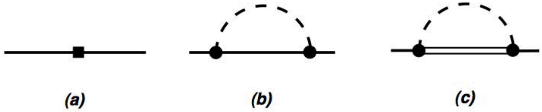
**Chiral Order** $$4L - 2N_M - N_B + \sum_k k v_k.$$




: NPB(1988)

# Power-Counting Breaking Problem

- Take the nucleon mass up to $\mathcal{O}(p^3)$ for example




$$m_N = m_0 + bM_\pi^2 + \text{loops}.$$

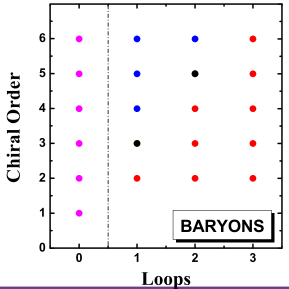
(b or c) Chiral order: 1+1+4-1-2=3.

If the systematic power counting exists:

$$\text{loop} = cM_\pi^3 + \cdots .$$

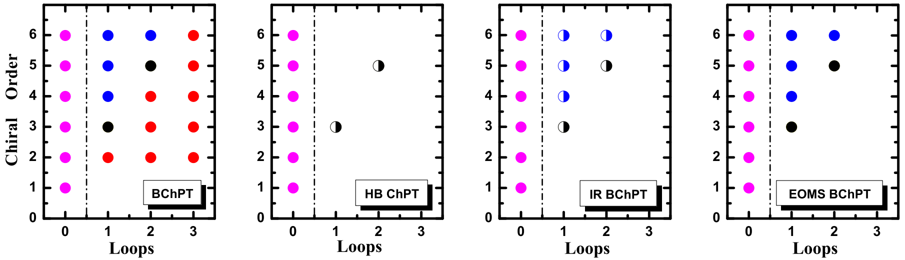
However the truth:

$$\text{loop} = \alpha m_0^3 + \beta m_0 M_\pi^2 + cM_\pi^3 + \cdots .$$



# Solving the PCB problem

| Non-Relativistic | Relativistic | |
|---|---|---|
| Heavy-Baryon ChPT<br>*Jenkins, PLB(1991)* | Infrared BChPT<br>*Becher, EPJC(1999)* | EOMS BChPT<br>*Gegelia, PRD(1999), Fuchs, PRD(2003)* |
| Baryon masses as static sources | $H = I + R$ | the PCB terms subtracted |
| Expansion in powers of $1/m_B$ | $\int_0^1 \cdots = \int_0^\infty \cdots - \int_1^\infty \cdots$ | Redefinition the LECs |
| ☑ strictly power-counting | | ☑ fullfills all symmtery |
| ✗ breaks analyticity | ✗ breaks analyticity | ☑ satisfies analyticity |
| ✗ converges slowly | ✗ converges slowly | ☑ converges relatively fast |



# Extended-On-Mass-Shell (EOMS)

$$m_N = m_0 + \text{tree} + \text{loop}$$
$$= m_0 + bm_\pi^2 + \alpha m_0^3 + \beta m_0 M_\pi^2 + cM_\pi^3 + \cdots .$$

- Directly throw away the power counting breaking terms

$\alpha = 0, \beta = 0$

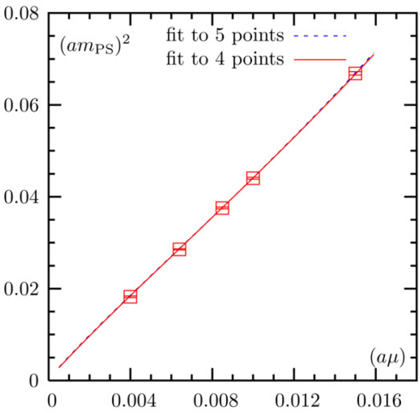
$$m_N = m_0 + bM_\pi^2 + cM_\pi^3 + \cdots \quad (\mathcal{O}(p^3)).$$

- Equivalently, redefinition the corresponding LECs

$m_0^r = m_0(1 + \alpha m_0^2), b^r = b + \beta m_0$

$$m_N = m_0^r + b^r M_\pi^2 + cM_\pi^3 + \cdots \quad (\mathcal{O}(p^3)).$$

# $m_q \propto M_\pi^2$ in LQCD calculation




ETM collaboration, hep-lat/0701012

$$m_\pi^2 \propto m_q$$

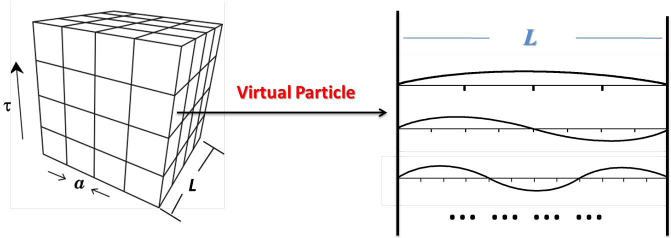
# HB ChPT cannot describe PACS-CS data

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing $D$, $F$, and $\mathcal{C}$ at the phenomenological estimate.

| | LO | NLO Case 1 | NLO Case 2 | Phenomenological |
|---|---|---|---|---|
| $m_B$ | 0.410(14) | 0.391(39) | −0.15(9) | |
| $\alpha_M$ | −2.262(62) | −2.62(62) | −15.3(2.0) | |
| $\beta_M$ | −1.740(58) | −2.6(1.5) | −21.3(3.0) | |
| $\sigma_M$ | −0.53(12) | −0.71(34) | −9.6(1.4) | |
| $D$ | | $0.000(16) \times 10^{-8}$ | 0.80 fixed | 0.80 |
| $F$ | | $0.000(9) \times 10^{-8}$ | 0.47 fixed | 0.47 |
| $\mathcal{C}$ | | 0.36(30) | 1.5 fixed | 1.5 |
| $\chi^2/\text{dof}$ | 1.10(63) | 1.39(77) | 153(82) | |

# Lattice Finite-Volume Corrections

## ☐ Physical picture of FVCs




- Momentum of virtual particle discretized

$$k_i = \frac{2\pi}{L} \cdot n_i, \quad (i = 0, 1, 2, 3) \quad \Longrightarrow \quad \int_{-\infty}^{\infty} dk \sim \sum_{n=-N+1}^{N} \frac{2\pi}{L} \cdot n, \quad N = \frac{L}{2a}.$$

- Definition of FVCs:

$$\Delta H^{(b)}_{\rm FVC} = \int \frac{dk_0}{2\pi} \cdot \left( \frac{1}{L^3} \sum_{\vec{k}} \Box - \int \frac{d\vec{k}}{(2\pi)^3} \Box \right) \qquad \text{with } \; \mathrm{L_{time}} \sim 5\mathrm{L_{space}}.$$

# lattice spacing power-counting

In LQCD simulations, the following hierarchy of energy scales should be satisfied

$$m_q \ll \Lambda_{\rm QCD} \ll \frac{1}{a}. \tag{1}$$

If one assumes that the size of the chiral symmetry breaking due to the light-quark masses and the discretization effects are of comparable size, as done in Refs. [Beane:2003xv,Bar:2003mh,Tiburzi:2005vy], one has the following expansion parameters

$$p^2 \sim \frac{m_q}{\Lambda_{\rm QCD}} \sim a\Lambda_{\rm QCD}, \tag{2}$$

where $p$ denotes a generic small quantity and $\Lambda_{\rm QCD} \approx 300$ MeV denotes the typical low energy scale of QCD.

# $\mathcal{O}(a)$-improved Wilson action

$$S_{\text{quark}} = \sum_{q=u,d,s}\Big[\sum_n \bar{q}_n q_n - \kappa_q c_{\text{SW}} \sum_n \sum_{\mu,\nu} \frac{i}{2} \bar{q}_n \sigma_{\mu\nu} F_{\mu\nu}(n) q_n$$
$$- \kappa_q \sum_n \sum_\mu \{\bar{q}_n (1 - \gamma_\mu) U_{n,\mu} q_{n+\hat{\mu}}$$
$$+ \bar{q}_n (1 + \gamma_\mu) U^\dagger_{n-\hat{\mu},\mu} q_{n-\hat{\mu}}\}\Big], \qquad (2)$$

# Relatively large Strangeness Sigma terms?

## Lattice-scale setting

- PACS-CS data with mass independent scale-setting:
  assume that the lattice scale, at constant bare coupling, is independent of the bare quark mass.

$$\sigma_{sN} = 59 \pm 7 \ (MeV)$$

- PACS-CS data with mass dependent scale-setting:
  The scale for the PACS-CS lattice data was set assuming that the dimensionful Sommer scale $r_0$ is independent of quark mass.

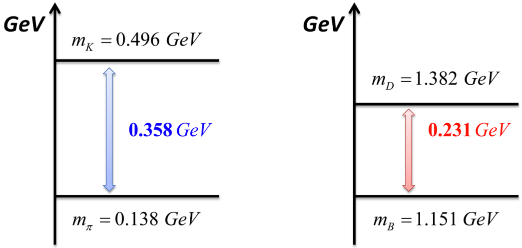
$$\sigma_{sN} = 21 \pm 6 \ (MeV)$$

*P.E. Shanahan, A.W. Thomas and R.D. Young, PRD 87, 074503 (2013)*

## Whether other LQCD data will show the same trend?

# Decuplet resonances in BChPT

- Baryon Spectrum in SU(3)-BChPT




- Perturbative parameters

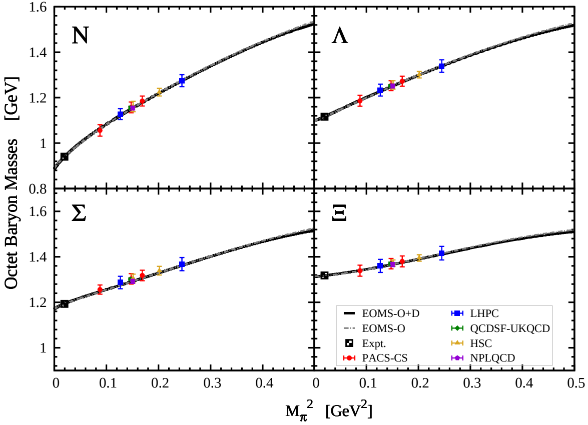
$$\frac{m_K}{\Lambda_{\text{ChPT}}} > \frac{\delta}{\Lambda_{\text{ChPT}}}$$

Effects of virtual decuplet baryons should be carefully studied

# Baryon masses in octet + decuplet EOMS BChPT

- **Virtual decuplet baryons** are explicitly included in BChPT
  - Octet baryon masses: $m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + m_B^{(D)}$
- Fit the **same** lattice data as previous
- There is **no new unknown LECs**
  - Octet-decuplet mass splitting: $\delta = 0.231$ GeV
  - Meson-octet-decuplet coupling constant: $\mathcal{C} = 0.85$ *J. M. Alarcon et al., 1209.2870*
  - Fixed from the experimental decuplet masses *J. Martin-Camalich et al., PRD(2010)*
    - $m_D = m_0 + \delta = m_0 + 0.231$ GeV
    - $t_0 = (m_0 + 0.231 - 1.215)/0.507$ GeV$^{-1}$
    - $t_D = -0.326$ GeV$^{-1}$

# Virtual decuplet effects on the chiral extrapolation

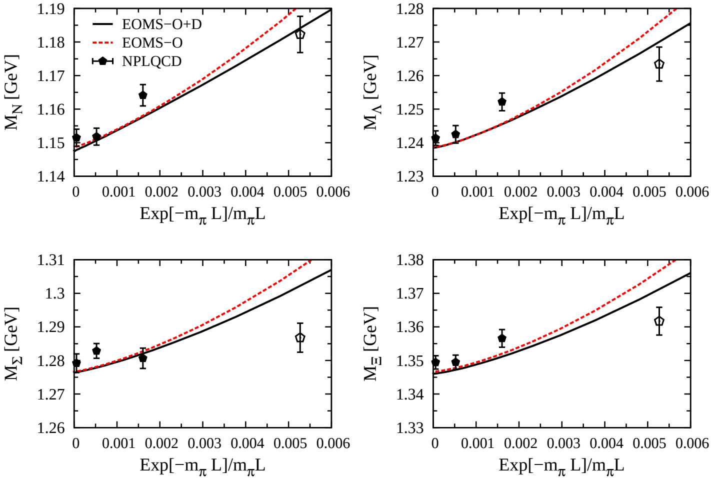



## Fit the 11 LQCD data sets **with and without decuplet**

- **Decuplet effects on the chiral extrapolation are small**
- **Previous assumption is confirmed**: virtual decuplet contributions can be absorbed by 19 LECs of octet only version

# Virtual decuplet effects on finite-volume corrections

- Use the previous best fit results to describe the NPLQCD lattice data
- Virtual decuplet contributions can give a better description of the FVCs at small volume region



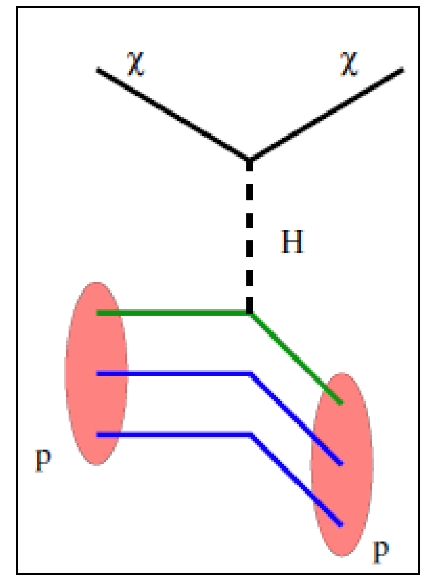
# Searches of Dark Matter

## Dark matter candidates

- Lightest, neutral SUSY particles
- Called Weakly Interactive Massive Particles (WIMP)

## In experiments

- Detect the scattering of a WIMP off a nucleus
- Cross section (spin indep.)




$$\sigma_{\rm SI} = \frac{4m_r^2}{\pi}\left[Z f_p + (A - Z) f_n\right]^2 .$$

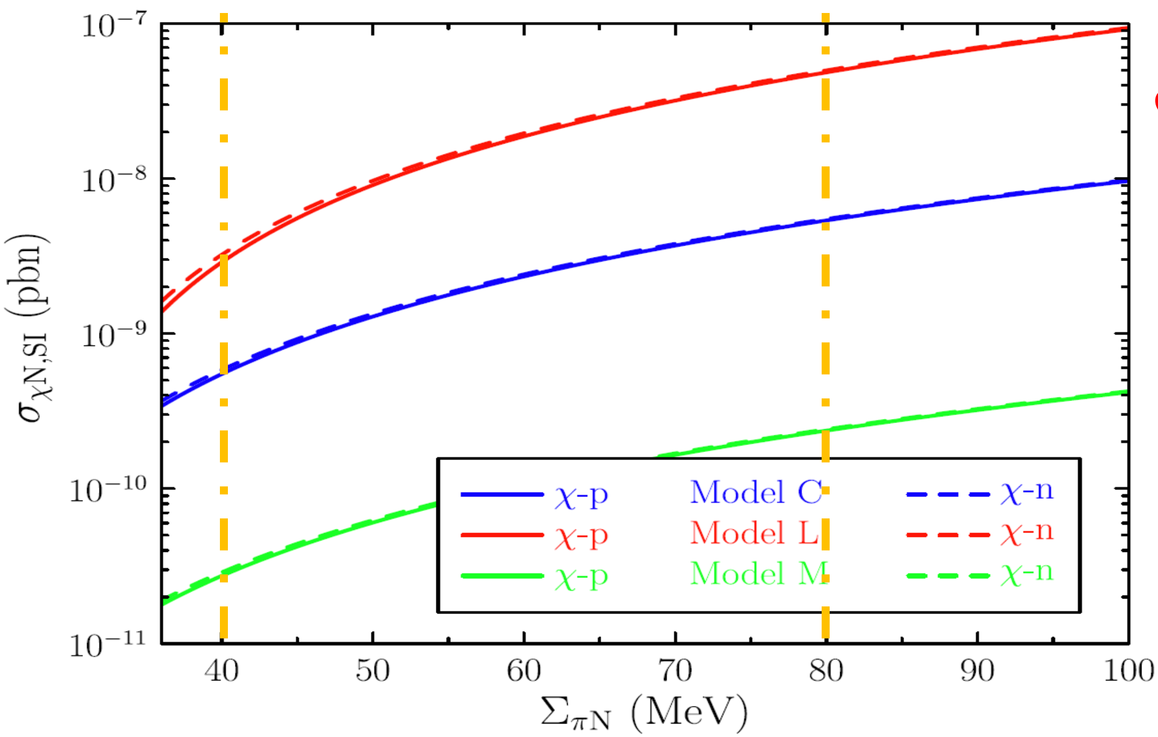
**with**

$$\frac{f_N}{m_N} = \sum f_{T_q}^N \frac{\lambda_q}{m_q} .$$

$$f_{T_{u/d}}^N m_N = \sigma_{\pi N} , \qquad f_{T_s}^N m_N = \sigma_{sN} .$$

# Cross section dependence on the sigma terms

- **J. Ellis, K. A. Olive, C. Savage, PRD 77, 065026 (2008)**
  - constrained minimal supersymmetric extension of the SM (CMSSM)
  - Non-singlet quantity $\sigma_0 = m_{u/d}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$
  - Strangeness sigma term: $\sigma_{sN} = \frac{m_s}{2m_{u/d}}\left(\Sigma_{\pi N} - \sigma_0\right)$.




$\sigma_{sN} \in (0, 400)$ **MeV**, $\sigma_0 = 36 \pm 7$ MeV

$\Downarrow$

40 MeV ~ $\Sigma_{\pi N}$ ~ 80 MeV

$\Downarrow$

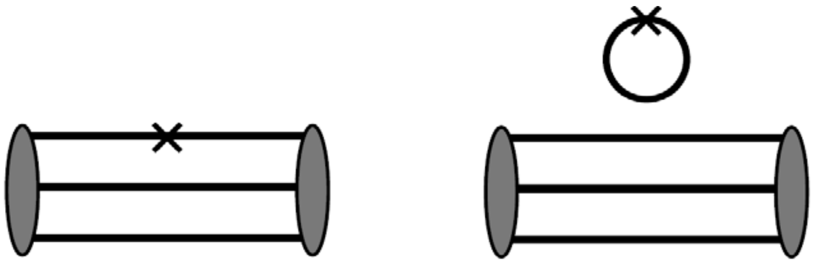
$\sigma_{SI}$ varies more than one order

Decrease the uncertainty of strangeness sigma term is the **KEY.**

# LQCD studies of sigma terms

## Direct method

- Calculate the **3-point connected/disconnected** diagrams



- ✓ JLQCD coll., PRD83,114506 (2011)
- ✓ R. Babich *et al.*, PRD85,054510 (2012)
- ✓ QCDSF coll., PRD85, 054502 (2012)
- ✓ ETM coll., JHEP 1208,037(2012)
- ✓ M. Engelhardt *et al.*, PRD86, 114510 (2012)
- ✓ JLQCD coll., PRD87, 034509 (2013)

## Indirect method

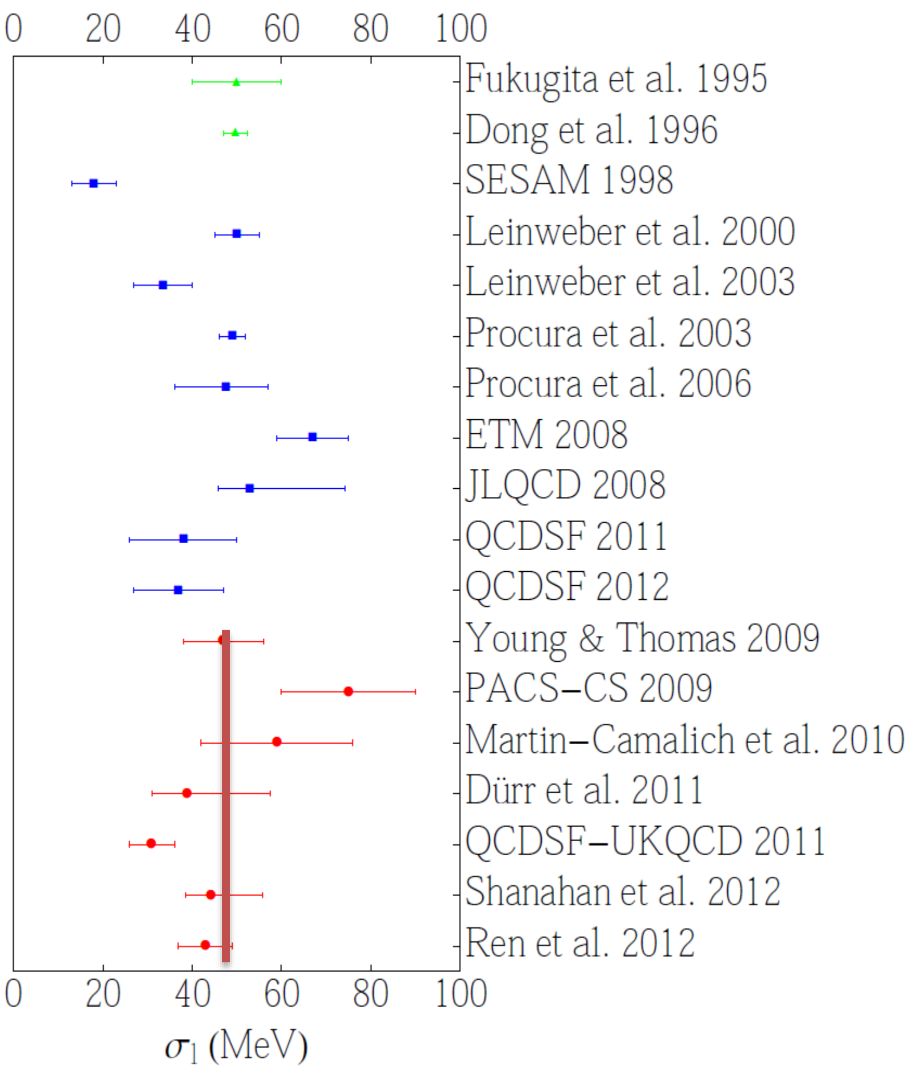
- Analysis the quark mass dependence on the nucleon mass using **chiral perturbation theory**
- **Feynman-Hellmann theorem**

$$\sigma_{\pi B} = m_l \langle B|\bar{u}u + \bar{d}d|B\rangle \equiv m_l \frac{\partial m_B}{\partial m_l},$$

$$\sigma_{sB} = m_s \langle B|\bar{s}s|B\rangle \equiv m_s \frac{\partial m_B}{\partial m_s},$$

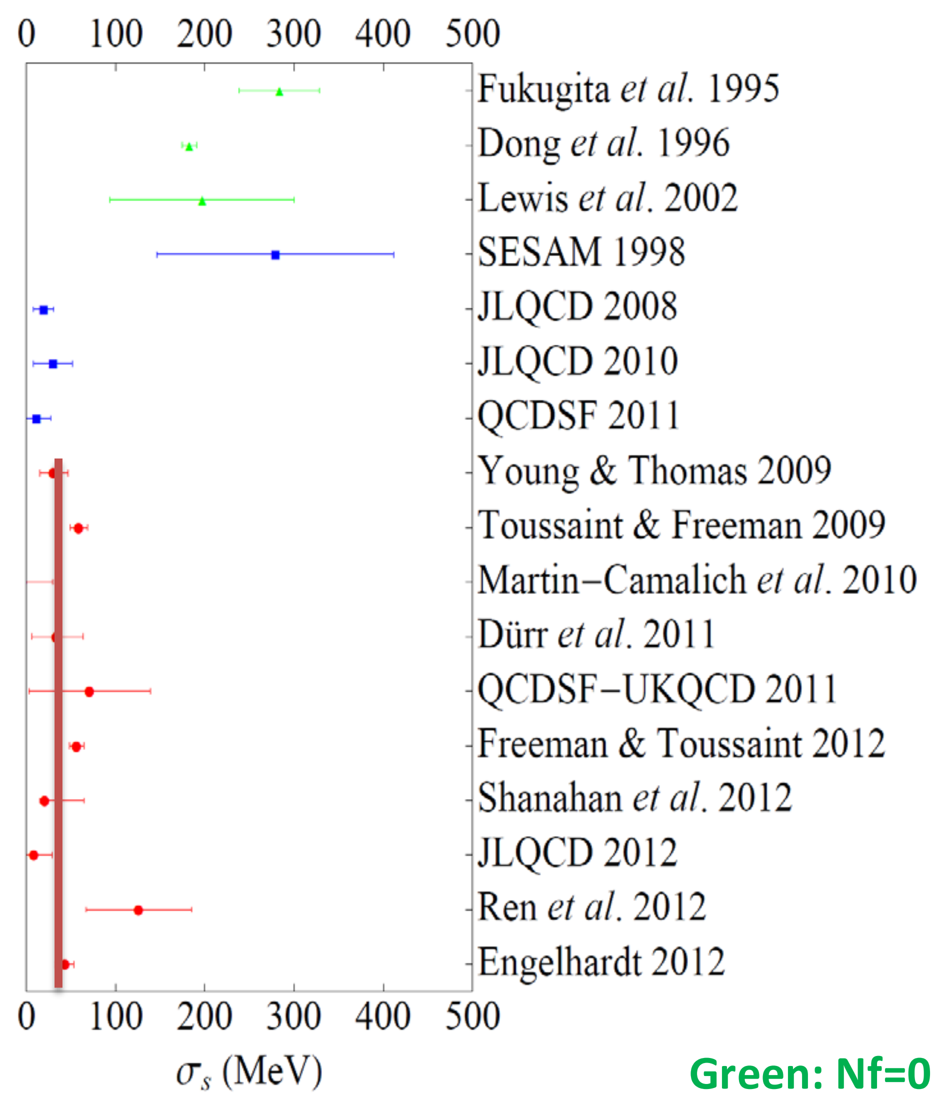
- ✓ R. Young *et al.*, PRD81, 014503 (2010)
- ✓ S. Durr *et al.*, PRD85,014509 (2012)
- ✓ R. Horsley *et al.*, PRD85, 034506 (2012)
- ✓ A. Semke and M. Lutz., PLB717, 242(2012)
- ✓ P. Shanahan *et al.*, PRD87, 074503 (2013)
- ✓ XLR *et al.*, JHEP1212, 073 (2012)
- ✓ P. Junnarkar *et al.*, PRD87, 114510 (2013)

# Summary the present sigma term results




**~ 50 MeV**

**consistent, small uncertainty**




**< 40 MeV**

Green: Nf=0
Bule: Nf=2
Red: Nf>2

From arXiv:1301.1765