χEFT Studies of Light Nuclei: a Status Report

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Overview

- Nuclear χEFT: from field-theory amplitudes to nuclear potentials and electroweak currents
- Constraining the currents
- Recent applications to:
  - EM structure of few-nucleon ground states and M1 transitions in $A > 4$
  - WK transitions in $A = 2$ and 3 systems
  - HWI and the PV potential: constraints on the LEC's from PV few-nucleon observables
- Summary and outlook
- A minimally non-local r-space potential with Δ intermediate states

Collaborators:

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The χEFT approach

- χEFT is a low-energy approximation of QCD
- Lagrangians describing the interactions of π, N, $\dots$ are expanded in powers of Q/Λ_χ ($\Lambda_\chi \sim 1$ GeV)
- Their construction has been codified in a number of papers [1]

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \dots \\ &\quad + \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \dots\end{aligned}$$

- Initial impetus to the development of χEFT for nuclei in the early nineties [2]
- First χEFT studies of nuclear electroweak currents soon followed [3]

[1] Gasser and Leutwyler (1984); Gasser, Sainio, and Švarc (1988); Bernard *et al.* (1992); Fettes *et al.* (2000)

[2] Weinberg (1990)–(1992); [3] Park, Min, and Rho (1993) & (1996)

General considerations

- Time-ordered perturbation theory (TOPT):

$$\langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid i \rangle$$

- Momentum scaling of contribution

$$\underbrace{\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

- Each of the N_K energy denominators involving only nucleons is of order Q^{-2}
- Each of the other $N - N_K - 1$ energy denominators involving also pion energies is expanded as

$$\frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[1 + \frac{E_i - E_I}{\omega_\pi} + \frac{(E_i - E_I)^2}{\omega_\pi^2} + \dots \right]$$

- Power counting:

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \dots, \quad \text{and} \quad T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

From amplitudes to potentials

- Construct v such that when inserted in LS equation

$$v + v\, G_0\, v + v\, G_0\, v\, G_0\, v + \dots \qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

leads to T-matrix order by order in the power counting

- Assume

$$v = v^{(0)} + v^{(1)} + v^{(2)} + \dots \qquad v^{(n)} \sim Q^n$$

- Determine $v^{(n)}$ from

$$\begin{aligned}
v^{(0)} &= T^{(0)} \\
v^{(1)} &= T^{(1)} - \left[v^{(0)}\, G_0\, v^{(0)}\right] \\
v^{(2)} &= T^{(2)} - \left[v^{(0)}\, G_0\, v^{(0)}\, G_0\, v^{(0)}\right] - \left[v^{(1)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v^{(1)}\right]
\end{aligned}$$

where

$$v^{(m)}\, G_0\, v^{(n)} \sim Q^{m+n+1}$$

Pastore, Goity, and Schiavilla (2008); Pastore *et al.* (2009); Pastore *et al.* (2011)

Including EM interactions

- Power counting of EM interactions (treated in first order)

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \dots \qquad T_\gamma^{(n)} \sim e\, Q^n$$

- For $v_\gamma = A^0\,\rho - \mathbf{A}\cdot\mathbf{j}$ to match T_γ order by order

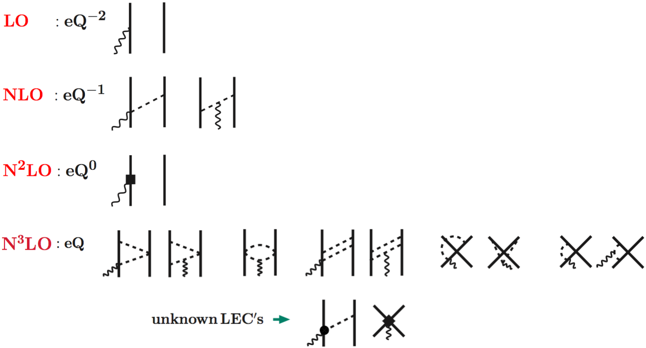
$$\begin{aligned}
v_\gamma^{(-3)} &= T_\gamma^{(-3)} \\
v_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[v_\gamma^{(-3)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v_\gamma^{(-3)}\right] \\
v_\gamma^{(-1)} &= T_\gamma^{(-1)} - \left[v_\gamma^{(-3)}\, G_0\, v^{(0)}\, G_0\, v^{(0)} + \text{permutations}\right] \\
&\quad - \left[v_\gamma^{(-2)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v_\gamma^{(-2)}\right]
\end{aligned}$$

and so on up to $n = 1$ ($\sim e\, Q$)

- These relations determine $\rho^{(n)}$ (LO has $n = -3$) and $\mathbf{j}^{(n)}$ (LO has $n = -2$)

EM operators up to one loop

- Contributions to **j** up to order $e\,Q$:



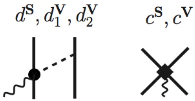
- Five unknown LEC's in **j** at $e\,Q$
- LO for ρ at $e\,Q^{-3}$ and no OPE corrections at $e\,Q^{-1}$
- No unknown LEC's in ρ up to $e\,Q$

Pastore *et al.* (2009) & (2011); Kölling *et al.* (2009) & (2011); Piarulli *et al.* (2013)

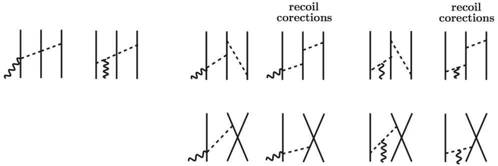
The LEC's characterizing j at N^{3}LO

Nuclear χEFT

- d's could be determined by (γ, π) data on the nucleon



- d_1^V, d_2^V fixed by assuming Δ dominance, d_1^S, c^S, and c^V fixed by fitting $A = 2$ and 3 EM observables
- Three-body currents at N^{3}LO vanish:



Determining the unknown LEC's in j

Nuclear χEFT

- After (perturbative) renormalization, resulting operators still need to be regularized:

$$C_\Lambda(k) = \mathrm{e}^{-(k/\Lambda)^n}$$

  and $n \geq 4$ not to generate spurious contributions
- For the given Λ ($\sim c^{S,V}/\Lambda^4$ and $\sim d^S/\Lambda^2$):
  - Isovector (d_1^V, d_2^V) from Δ-resonance saturation; isoscalar (d^S, c^S) by reproducing μ_d and μ^S
  - Isovector c^V fixed by reproducing either σ_{np} or μ^V

Λ	c^S	$d^S \times 10$	$c^V(\sigma_{np})$	$c^V(\mu^V)$
500	4.1	2.2	–13	–8.0
600	11	3.2	–22	–12

Girlanda *et al.* (2010); Piarulli *et al.* (2013)

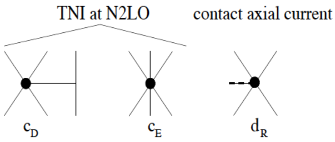
Applications to EM Structure

EM structure of light nuclei

- A sample of available results:
  - EM properties of light nuclei with $A \leq 10$ (charge radii, magnetic moments, . . .)
  - Elastic f.f.'s of few-nucleon systems
- See Pastore's talk for further results

Unless otherwise noted, all calculations are based on chiral NN and NNN potentials at N3LO [1] and N2LO [2,3]

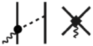
c_D and c_E in N2LO NNN potential fixed by reproducing GT m.e. in ^{3}H β-decay and ^{3}H/^{3}He binding energies [4]

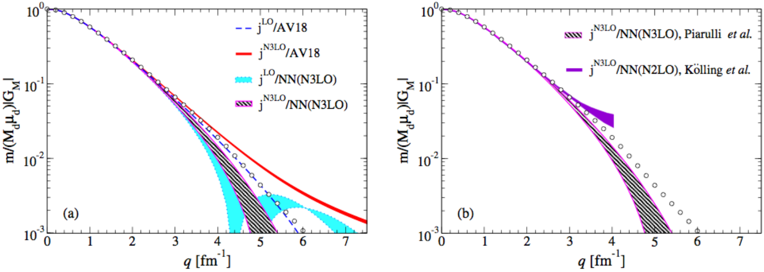


[1] Machleidt and Entem (2011); [2] van Kolck (1994); [3] Epelbaum *et al.* (2002); [4] Gardestig and Phillips (2006)

Isoscalar magnetic structure: deuteron

- OPE (NLO) and TPE (N3LO) vanish (isovector); largest corrections beyond LO from (isoscalar) N3LO terms

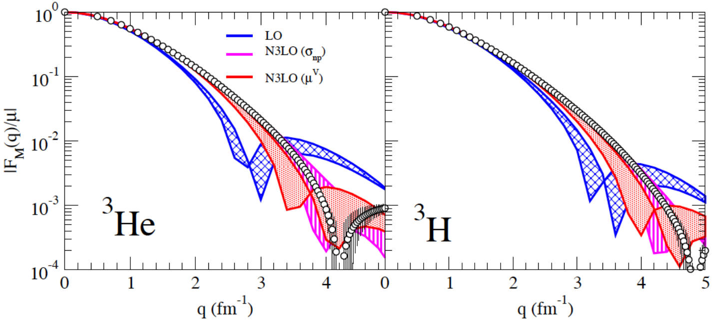




Courtesy of Bacca and Pastore, J. Phys. G: Nucl. Part. Phys. **41**, 123002 (2014)

^{3}He/^{3}H magnetic f.f.'s

- Two-body (isovector) contributions play crucial role

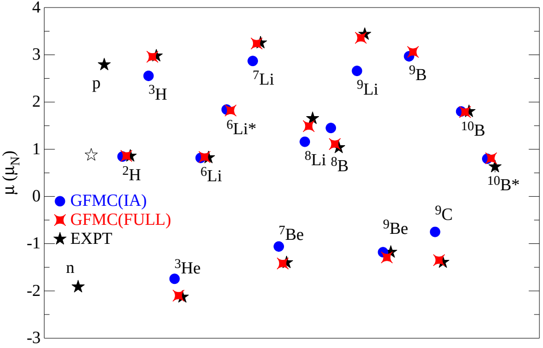


- Magenta shaded area: LEC c^V fixed by σ^γ_{np}
- Red shaded area: LEC c^V fixed by μ^V(^{3}He/^{3}H)

Piarulli *et al.* (2013)

Magnetic moments in $A \leq 10$ nuclei

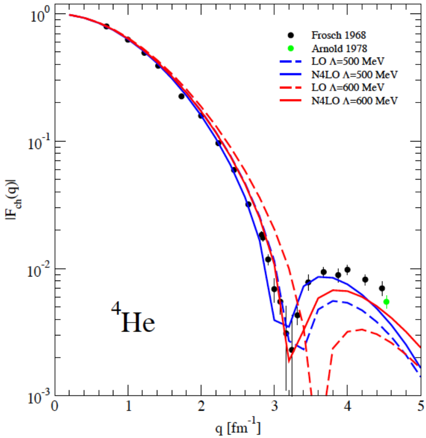
- GFMC calculations use the AV18/IL7 (rather than chiral) potentials in combination with the χEFT currents
- Predictions for $A > 3$; 40% of $\mu(^9\mathrm{C})$ due to corrections beyond LO



Pastore *et al.* (2013)

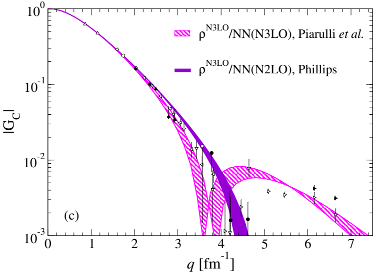
^{4}He charge f.f.

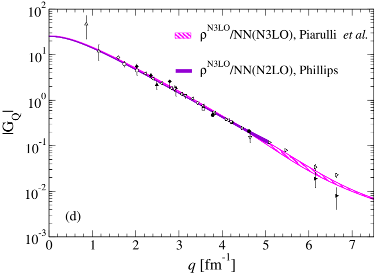
- Isoscalar corrections beyond N3LO ($\sim Q^0$) suppressed by Q^2: loop corrections at N4LO ($\sim Q^1$) are isovector



Marcucci *et al.* (2015)

G_C and G_Q f.f.'s in ^{2}H; $A = 2$–4 static properties





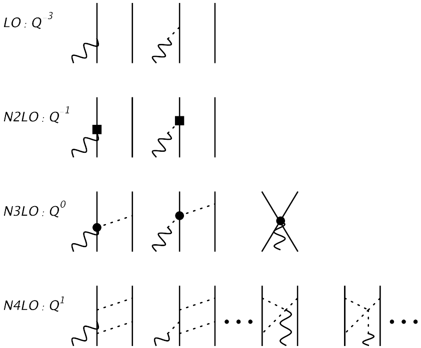
	$r_c(^2\mathrm{H})$(fm)	$Q_d{}^\dagger$(fm^2)	$r_c(^3\mathrm{He})$(fm)	$r_c(^4\mathrm{He})$(fm)
χEFT	2.126(4)	0.2836(16)	1.962(4)	1.663(11)
EXP	2.130(10)	0.2859(6)	1.973(14)	1.681(4)

†See Gross' talk for a calculation of Q_d in a relativistic OBE approach based on covariant spectator theory

Axial Currents and Weak Transitions

Weak axial operators up to one loop

- Axial current (and charge) operators derived up to one loop in TOPT (Baroni's talk)



- $\mathbf{j}_{5a}$ and ρ_{5a} differ from those of an earlier derivation[1]

[1] Park, Min, and Rho (1993); Park *et al.* (2003)

μ^- capture on ^{2}H and ^{3}He

- χEFT predictions with conservative error estimates [1]:

$$\Gamma(^2\mathrm{H}) = (399 \pm 3)\,\mathrm{sec}^{-1} \qquad \Gamma(^3\mathrm{He}) = (1494 \pm 21)\,\mathrm{sec}^{-1}$$

- Errors due primarily to:
  - experimental error on GT$^{\mathrm{EXP}}$ (0.5%)
  - uncertainties in EW radiative corrections [2] (0.4%)
  - cutoff dependence
- Using $\Gamma^{\mathrm{EXP}}(^3\mathrm{He}) = (1496 \pm 4)$ sec^{-1}, one extracts

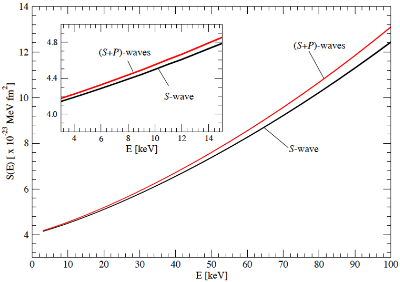
$$G_{PS}(q_0^2 = -0.95\, m_\mu^2) = 8.2 \pm 0.7$$

versus $G_{PS}^{\mathrm{EXP}}(q_0^2 = -0.88\, m_\mu^2) = 8.06 \pm 0.55$ [3] and a χPT prediction of 7.99 ± 0.20 [4]
- Upcoming measurement of $\Gamma(^2\mathrm{H})$ by the MuSun collaboration at PSI with a projected 1% error ...

[1] Marcucci *et al.* (2012); [2] These corrections increase rate by 3%, see Czarnecki *et al.* (2007); [3] From a measurement of $\Gamma^{\mathrm{EXP}}(^1\mathrm{H})$, Andreev *et al.* (2013); [4] Bernard *et al.* (1994), Kaiser (2003)

pp weak capture

- Process ignites the pp cycle in the sun
- N^{3}LO chiral potential, including full treatment of EM interactions
- $S(0) = (4.030 \pm 0.006) \times 10^{-23}$ MeV-fm^2
- Decrease ($\lesssim 1\%$) due to higher-order EM effects offset by increase due to P-wave capture



Marcucci, Schiavilla, and Viviani (2013)

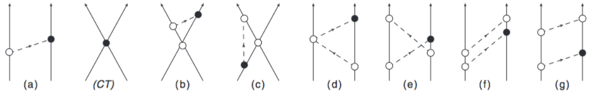
Hadronic Weak Interactions and PV Observables

Hadronic weak interactions: the PV potential

- Chiral Lagrangian up to order Q derived in the early nineties [1]

$$\mathcal{L}_{\rm PV} = \overbrace{\frac{h^1_\pi}{2\sqrt{2}}\, f_\pi\, \overline{\psi}\, X^3_-\, \psi}^{\mathcal{L}^{(0)}_{\rm PV}} + \mathcal{L}^{(1)}_{\rm PV}$$

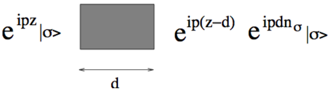
- Complete analysis of $\Delta I = 0, 1, 2$ terms contributing to $\mathcal{L}^{(2)}_{\rm PV}$ carried out recently [2]
- In addition, there are 5 independent $4\,N$ contact terms with one gradient at lowest order [3]
- PV potential up to order Q (N2LO) depends [2,4] on h^1_π and the LEC's $C_{i=1,\ldots,5}$



[1] Kaplan and Savage (1993); [2] Viviani *et al.* (2014); [3] Girlanda (2008); [4] Zhu *et al.* (2005)

Constraining the PV potential in $A = 2$–5 nuclei

- A number of PV observables have been, are being, or could be measured in few-nucleon systems[1]:
  - A_z in $\vec{p}p$ and $\vec{p}\alpha$, and $\vec{n}\,{}^3\mathrm{He} \rightarrow p\,{}^3\mathrm{H}$ scattering[2]
  - Neutron spin rotation in $\vec{n}p$, $\vec{n}d$, and $\vec{n}\alpha$ scattering
  - A^γ in $\vec{n}p$ capture[3] and P^γ in $d(\vec{\gamma}, n)p$
- Beautiful but difficult experiments: asymmetries are expected to be of order $\lesssim G_F\, f_\pi^2 \sim 10^{-7}$



$$n_\sigma = 1 + \frac{2\pi\,\rho}{p^2} M_\sigma(\theta = 0)$$

$$\frac{d\phi}{dd} = -\frac{2\pi\,\rho}{p} Re\,[M_+(\theta = 0) - M_-(\theta = 0)]$$

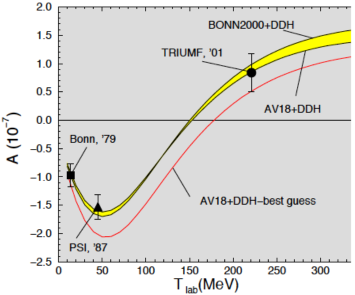
[1] Schindler's talk; [2] Gericke's talk; [3] Barròn-Palos' talk

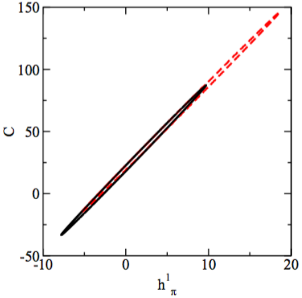
A_z in $\vec{p}p$ scattering

- Pion contribution enters via TPE

$$A_z^{pp}(E,\Lambda) = a_0^{(pp)}(E,\Lambda)\, h_\pi^1 + a_1^{(pp)}(E,\Lambda)\, C$$

with $C = C_1 + C_2 + 2\,(C_4 + C_5)$





de Vries *et al.* (2013); Viviani *et al.* (2014)

A_z in $\vec{n}\,^3\mathrm{He}$ charge-exchange reaction

- Experiment in progress at the Oak Ridge SNS facility
- PV observables $a_z \cos\theta$

$$a_z = a_0\, h_\pi^1 + a_1\, C_1 + a_2\, C_2 + a_3\, C_3 + a_4\, C_4 + a_5\, C_5$$

Λ(MeV)	a_0	a_1	a_2	a_3	a_4	a_5
500	-0.1444	0.0061	0.0226	-0.0199	-0.0174	-0.0005
600	-0.1293	0.0081	0.0320	-0.0161	-0.0156	-0.0001

- a_0 large relative to a_i, but in particular (isoscalar) C_2 is expected to be large

$$C_1^{(\mathrm{DDH})} \approx 1 \quad C_2^{(\mathrm{DDH})} \approx 30 \quad C_3^{(\mathrm{DDH})} \approx -2$$
$$C_4^{(\mathrm{DDH})} \approx 0 \quad C_5^{(\mathrm{DDH})} \approx 7$$

by matching the C_i to DDH vector-meson couplings

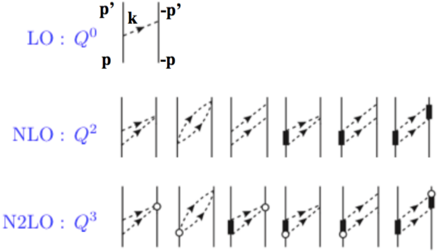
Viviani *et al.* (2010); Viviani *et al.* (2014)

Summary and outlook

- Electroweak currents up to one loop have been derived in TOPT
- Quantitatively accurate description of EW properties: static properties of $A \leq 10$, $A = 2$–4 f.f.'s, weak captures
- In μ-capture nuclear χEFT leads to predictions with $\lesssim 1$% accuracy (extract nucleon G_{PS} from ^{3}He capture)
- Theory results (in search of experimental data!) obtained for a variety of few-nucleon PV observables
- Nuclear χEFT for $A > 4$ and quantum Monte Carlo methods: need for r-space formulation

Minimally non-local r-space potential with Δ's

- Two-nucleon potential: $v = v^{\mathrm{EM}} + v^{\mathrm{LR}} + v^{\mathrm{SR}}$
- EM component v^{EM} including corrections up to α^2
- Chiral OPE and TPE component v^{LR} with Δ intermediate states up to order Q^3 (local in r-space)

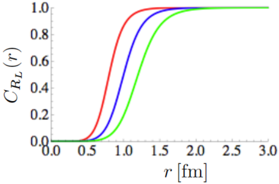


- Short-range contact component v^{SR} up to order Q^4 with Fierz rearrangement of terms to yield at most $\mathbf{p}^2$

Piarulli *et al.* (2015)

Fitting procedure

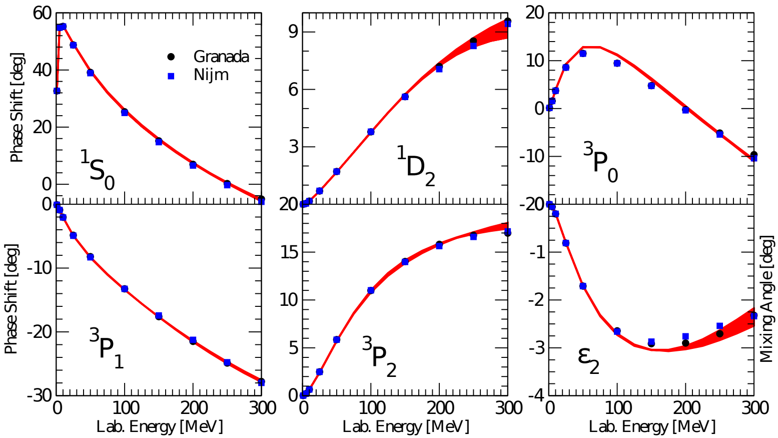
- Short-range cutoff $C_{R_S}(r) \propto \mathrm{e}^{-(r/R_S)^2}$; long-range cutoff $C_{R_L}(r)$



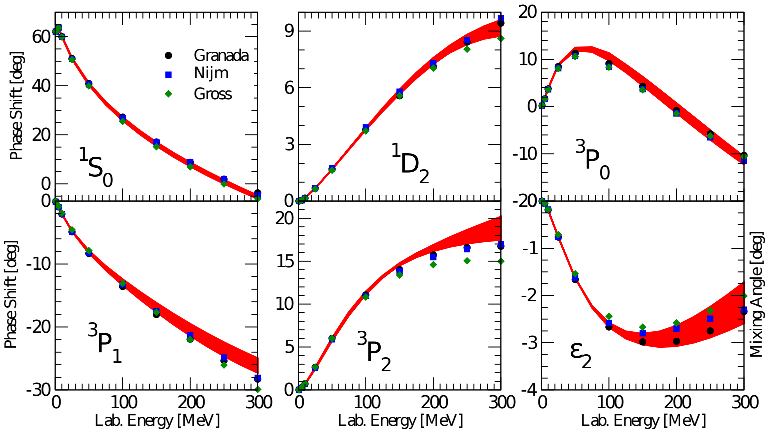
- Total 34 LEC's in v^{SR} fitted to 2161 pp and 2764 np data up to $E_{\mathrm{lab}} = 300$ MeV in the Granada database
- For $(R_L, R_S) = \underbrace{(1.2, 0.8)}_{a}, \underbrace{(1.0, 0.7)}_{b}, \underbrace{(0.8, 0.6)}_{c}$ fm

	a	b	c
$\chi^2(pp)$/datum	1.48	1.48	1.52
$\chi^2(np)$/datum	1.20	1.19	1.23

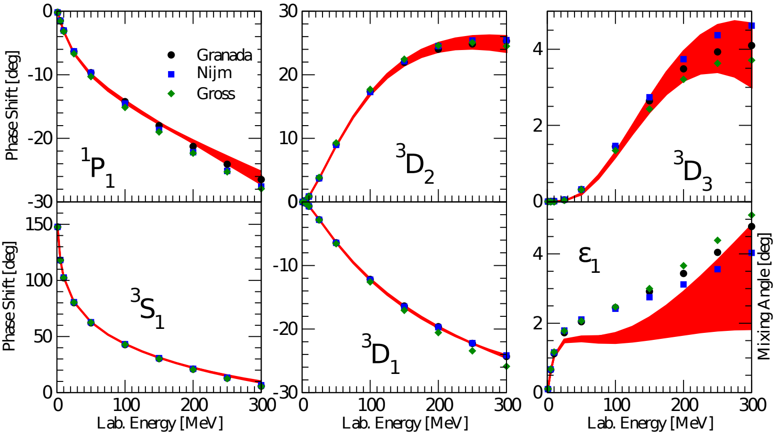
pp phase shifts



$np\ T = 1$ phase shifts



$np\ T = 0$ phase shifts



Deuteron properties

	Experiment	v_{12}^{a}	v_{12}^{b}	v_{12}^{c}
E_d [MeV]	2.224575(9)	2.224575	2.224574	2.224575
η	0.0256(4)	0.0245	0.0248	0.0246
r_d [fm]	1.97535(85)	1.948	1.975	1.989
μ_d [μ_0]	0.857406(1)	0.852	0.850	0.848
Q_d [fm^2]	0.2859(3)	0.257	0.268	0.269
P_d [%]		4.94	5.29	5.55