Evgeny Epelbaum, Ruhr-University Bochum

The 8th International Workshop on Chiral Dynamics 2015
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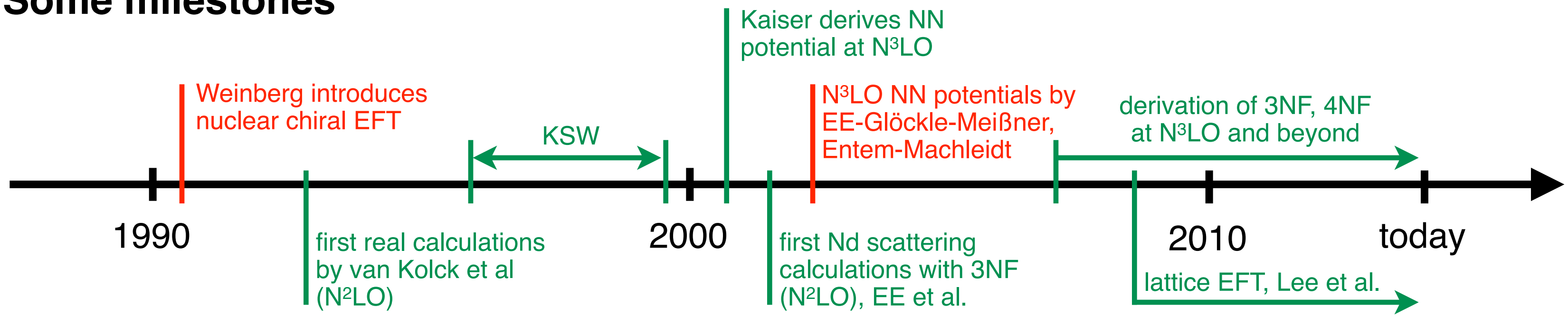
Nuclear chiral EFT
in the precision era

- Introduction
- A new generation of NN potentials up to N^{4}LO
- Quantification of theoretical uncertainties
- Applications to few-N systems
- Summary

The 25 years of nuclear chiral EFT

Some milestones



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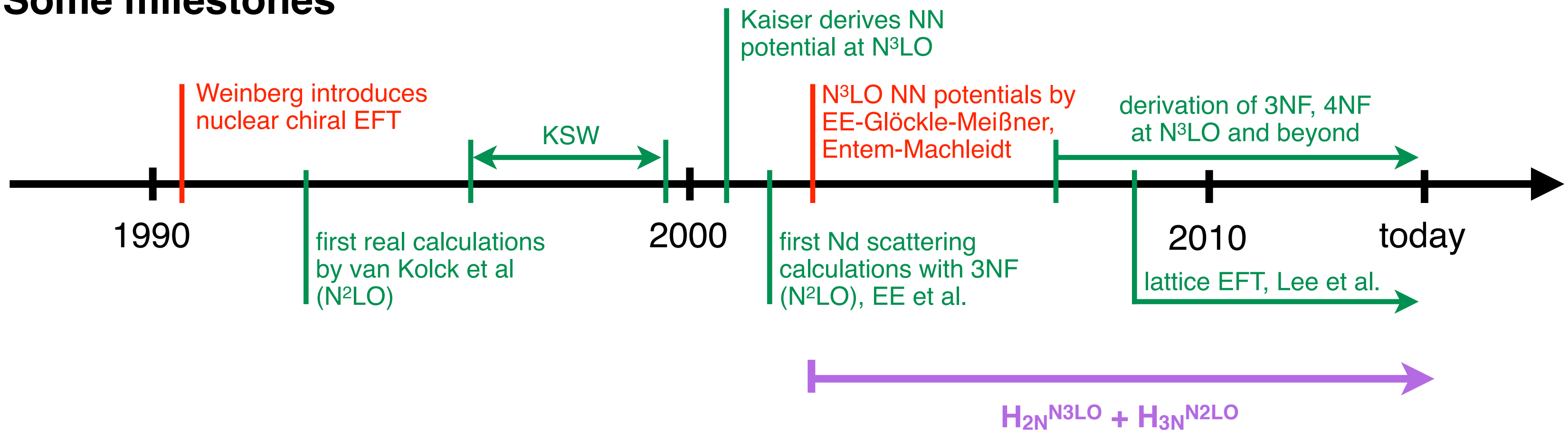

Many new insights including

- explanation of the observed hierarchy of many-body forces
- promising results using 2NF at N³LO combined with 3NF at N²LO
- electroweak structure of light nuclei, pion-deuteron scattering, Compton, …

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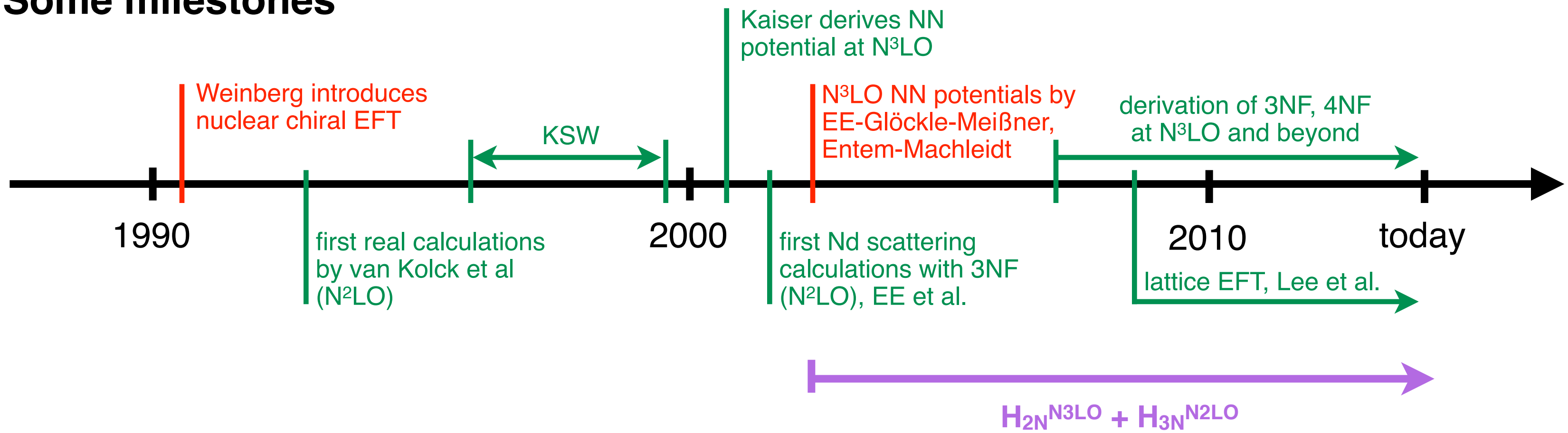

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Today: precision frontier…

1. Address **unsolved problems** (especially **3NF**)

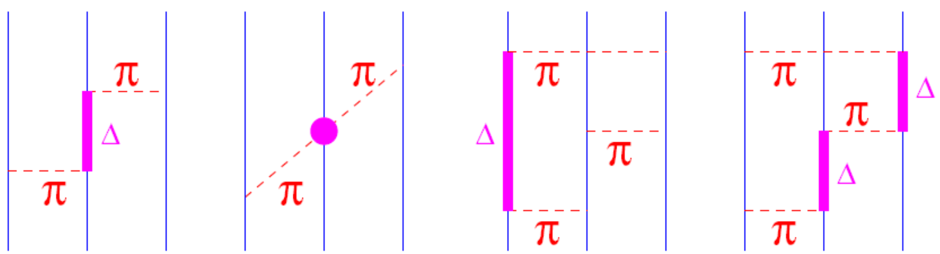
The 25 years of nuclear chiral EFT

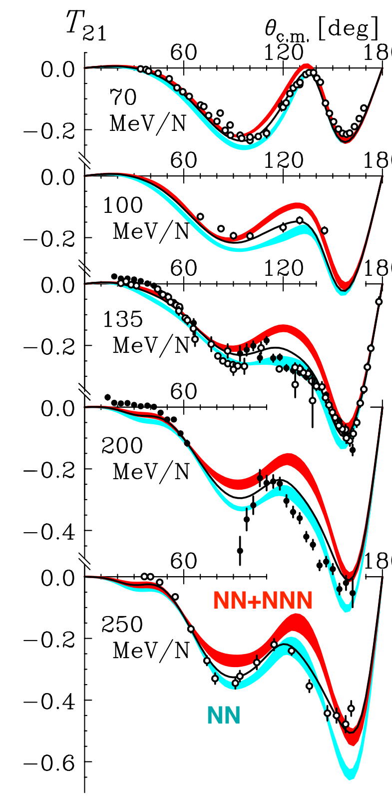
In spite of decades of effort, the spin structure of the 3NF is NOT properly described by 3NF models…

Kalantar-Nayestanaki et al., Rept. Prog. Phys. 75 (2012) 016301

Phenomenological 3NF models

Fujita-Miyazawa, Brasil, Tucson-Melbourne, Urbana, Illinois,…





In Nd scattering, large discrepancies between theory and data are observed at higher energies especially for spin observables

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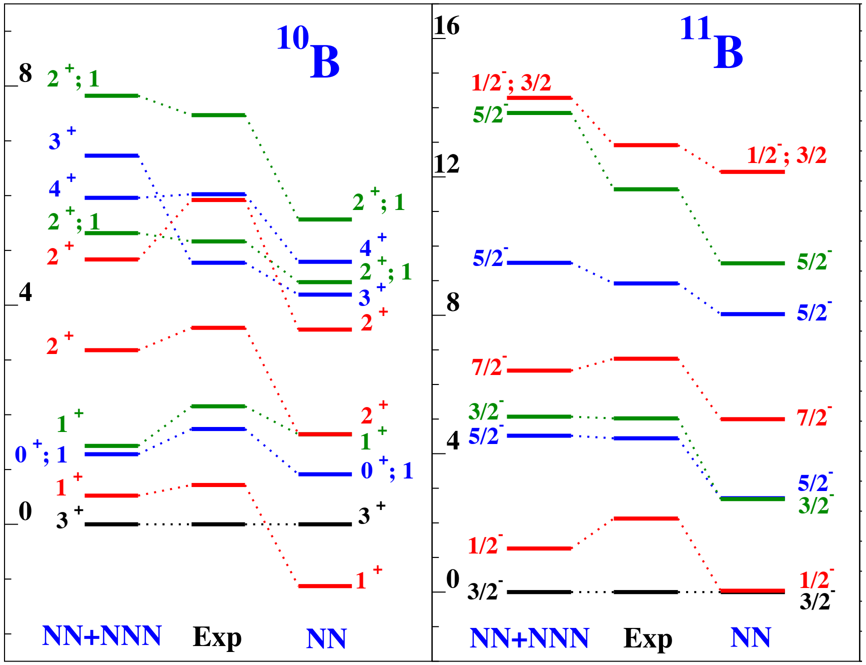
The 25 years of nuclear chiral EFT

NCSM calculation based on chiral 2NF$_{@N3LO}$ + 3NF$_{@N2LO}$

Navratil et al. ’07



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2. Any predictive theory has to come with **uncertainty quantification**

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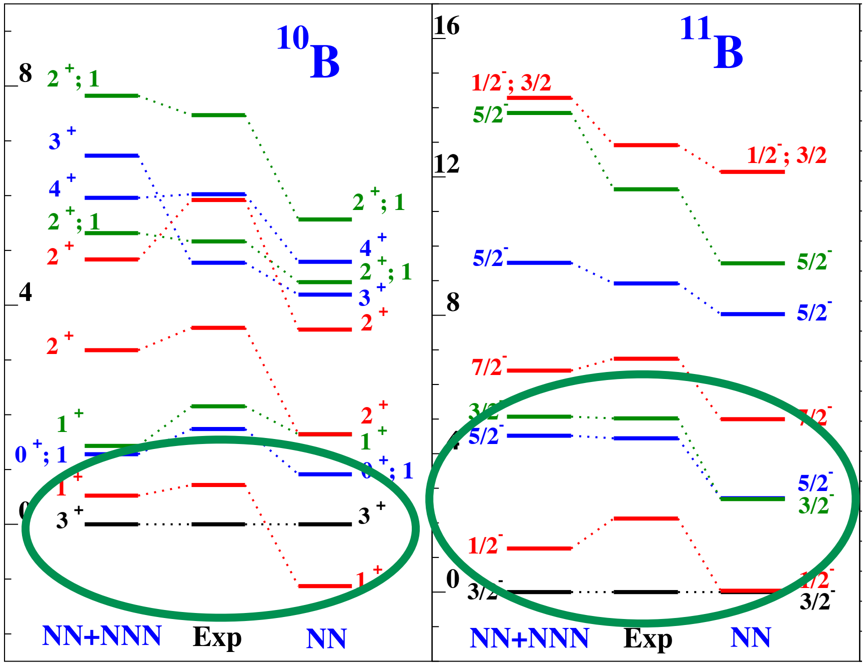

Success

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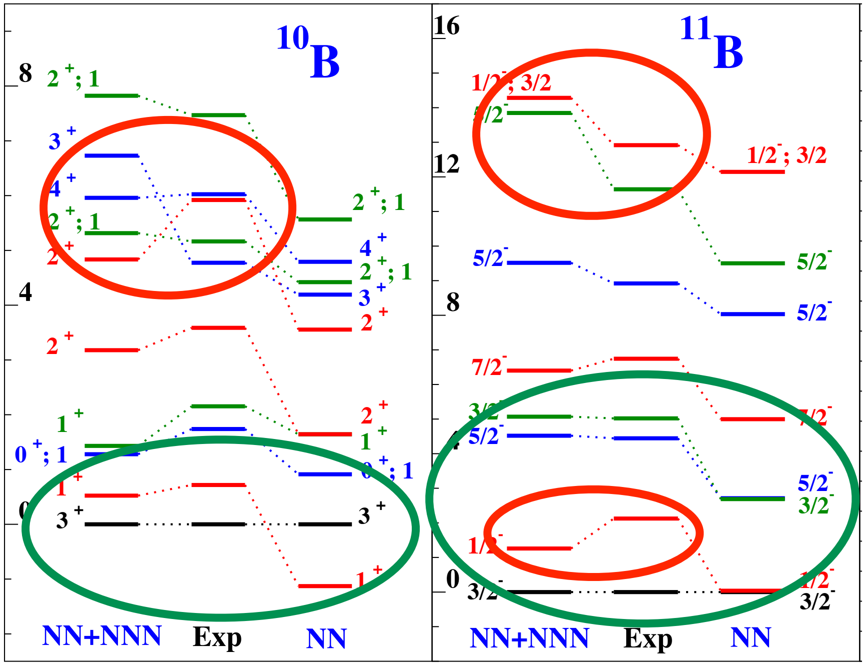

Success **or failure?**

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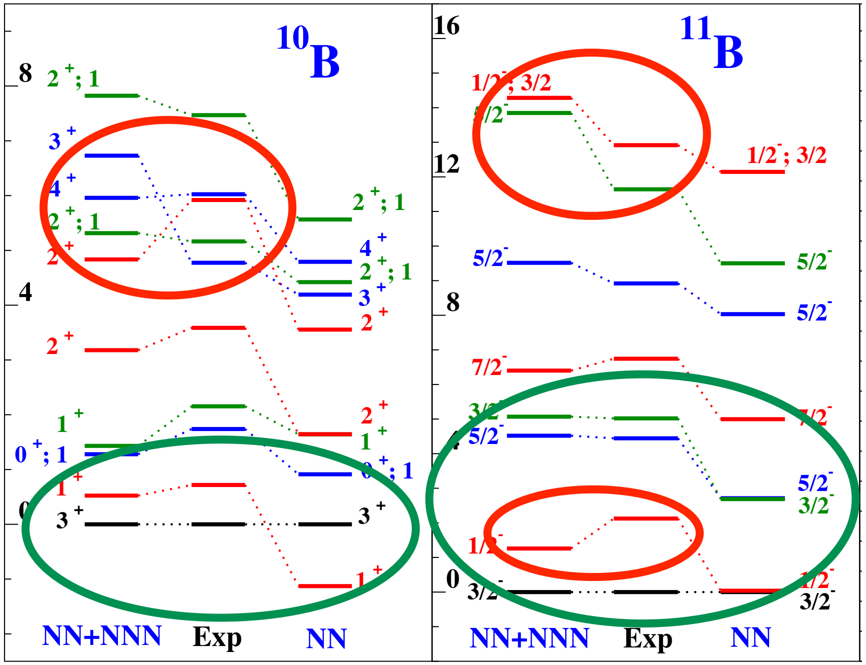


Success **or failure?**

Today: precision frontier…

1. Address **unsolved problems** (especially **3NF**)
2. Any predictive theory has to come with **uncertainty quantification**
3. (High-order) chiral EFT for nuclear forces + *ab-initio* few-N methods + error analysis
   ➛ **testing chiral dynamics in nuclei**…

Chiral NN potentials: some recent developments

- Local NN potentials up to N^{2}LO: well suited for QMC calculations
  Gezerlis, Tews, EE, Freunek, Gandolfi, Hebeler, Nogga, Schwenk '13,'14; Lynn et al.'14

- Minimally nonlocal NN potentials up to N^{3}LO (including N^{2}LO Δ contributions)
  Piarulli, Girlanda, Schiavilla, Navarro Perez, Amaro, Ruiz Arriola '15

- Optimized N^{2}LO NN potential (πN LECs are tuned to NN peripheral scattering); quantification of the statistical uncertainty of the LECs
  Ekström et al.'13,'15

- N^{2}LO potential: a simultaneous fit to πN, NN and few-N data
  Carlsson, Ekström, Forssen, Fahlin Strömberg, Lilja, Lindby, Mattsson, Wendt '15

- Chiral 2π and 3π exchange up to N^{4}LO and (partially) up to N^{5}LO in NN peripheral scattering ⟵ **talk by Ruprecht Machleidt**
  Entem, Kaiser, Machleidt, Nosyk '14,'15

- New generation of chiral NN potentials up to N^{4}LO: improved choice of the regulator, no SFR, no fine tuning of πN LECs, simple algorithm to quantify the systematic uncertainty due to truncation of the chiral expansion
  EE, Krebs, Meißner '14,'15

Chiral Effective Field Theory

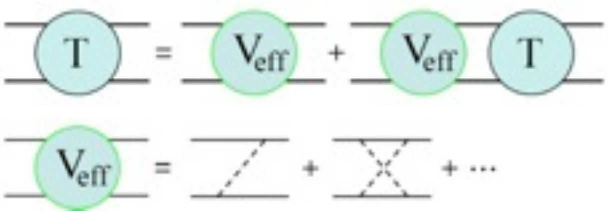
Chiral Perturbation Theory: expansion of the scattering amplitude in powers of Q,
Weinberg, Gasser, Leutwyler, Meißner, ...

$$\boxed{Q = \frac{\text{momenta of external particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}}$$

Write down $L_{eff}[\pi, N, \ldots]$,
identify relevant diagrams at a given order,
do Feynman calculus,
fit LECs to exp data,
make predictions…

Chiral EFT for nuclear systems: expansion for nuclear forces + resummation (Schrödinger eq.)
Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$



- systematically improvable
- access to heavier systems e.g. via a discretized formulation [lattice]
  see talks in the Few-Body WG by Ulf Meißner, Jose Alarcon, Serdar Elhatisari, Alexander Rokash
- unified approach for $\pi\pi$, πN, NN [e.g. LECs in the nuclear force from πN scattering]
- consistent many-body forces and currents
- error estimations

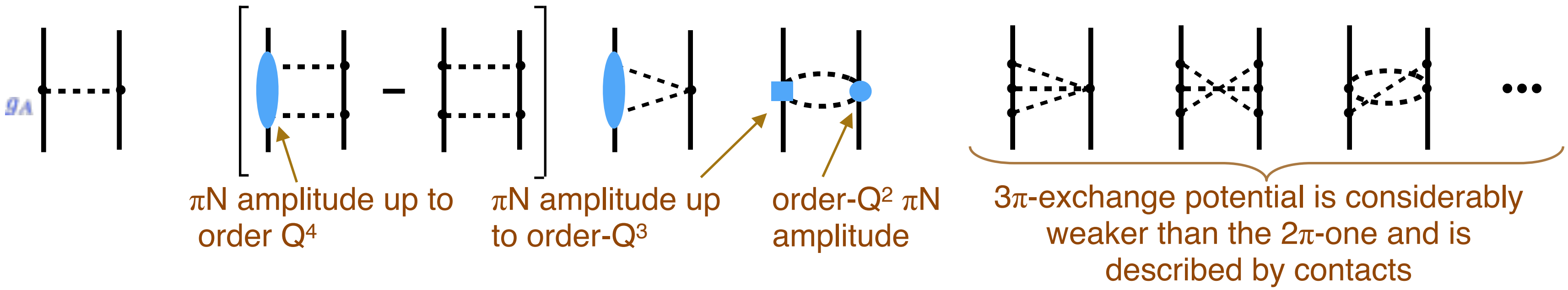
Notice: nonperturbative treatment of chiral nuclear forces in the Schrödinger eq. requires the introduction of a finite cutoff **[Alternatively, use semi-relativistic approach, EE, Gegelia, et al. '12…'15]**

Nucleon-nucleon force up to N^{4}LO

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

The long-range part

Ordonez et al.; Kaiser; EE, Krebs, Meißner; Entem, Machleidt; …



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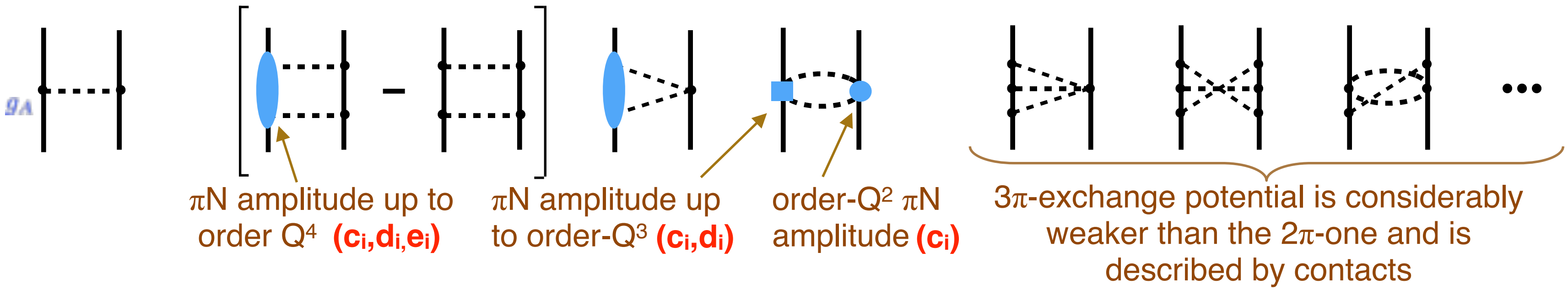

LECs extracted from πN scattering

Krebs, Gasparyan, EE '12

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q^4 fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

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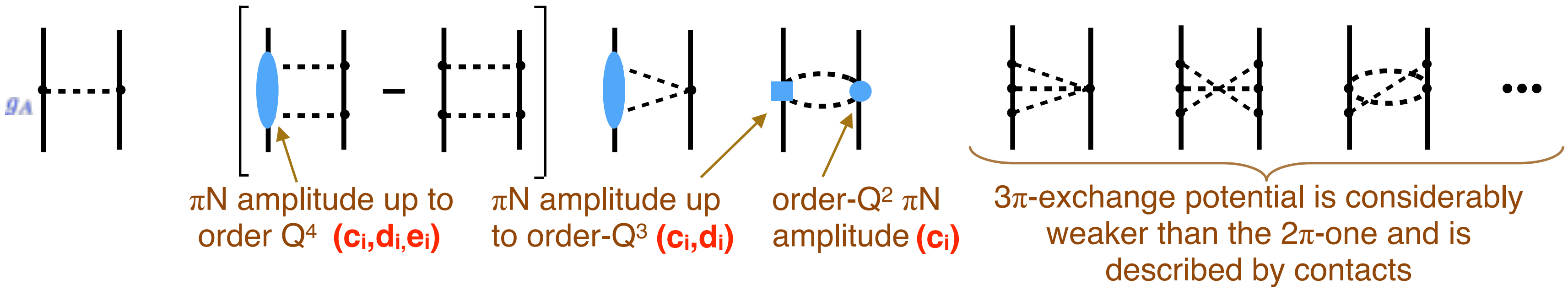

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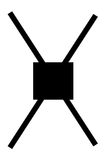
The short-range part (contact terms)



Here the organizational principle for contact terms is assumed to be according to NDA (Weinberg counting)

LO [Q^0]: 2 operators (S-waves)

NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)

N^{2}LO [Q^3]: no new isospin-conserving operators

N^{3}LO [Q^4]: + 15 operators (S-, P-, D-waves and ε_1, ε_2)

N^{4}LO [Q^5]: no new isospin-conserving operators

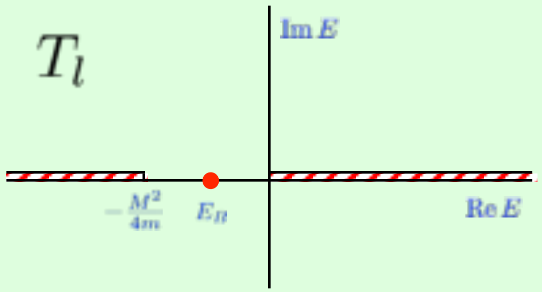
Regularization

Employ regularization which maintains the analytic structure of the amplitude

Old EM/EGM potentials

$$V^{\rm reg}_{\rm long-range}(\vec{p}\,', \vec{p}) = V_{\rm long-range}(\vec{p}\,', \vec{p}) \; \exp\left[- \frac{p^n + p'^n}{\Lambda^n} \right]$$

affects the discontinuity across the left-hand cut
(i.e. some distortions of the long-range potential)



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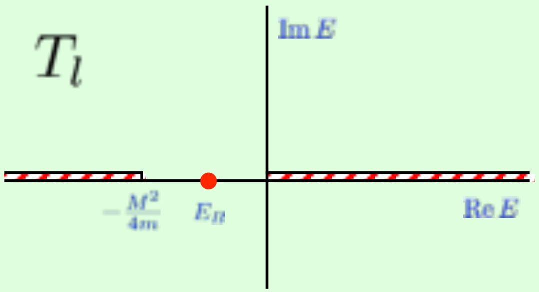

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New potentials

$$\left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

$$V^{\text{reg}}_{\text{long-range}}(\vec{r}) = V_{\text{long-range}}(\vec{r})\; f_{\text{reg}}\left(\frac{r}{R}\right)$$

$$\text{FourierTransform}[f(r/R) - 1]$$

$$V^{\text{reg}}_{\text{long-range}}(\vec{q}) = V_{\text{long-range}}(\vec{q}) - \underbrace{\int d^3l\; V_{\text{long-range}}(\vec{q} - \vec{l})\; \tilde{f}_{\text{reg}}(\vec{l})}_{\text{manifestly short-range}}$$

Advantages:

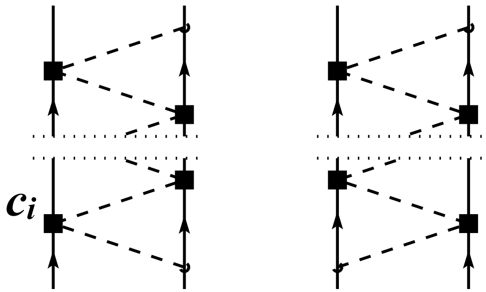
No distortion of the long-range potential $\longrightarrow$ better performance (at high energy)

No need for an additional spectral function regularization in the TPEP

For contact interactions, a Gaussian nonlocal regulator is employed

The cutoff R should be taken above/of the order of the breakdown distance R_b of the chiral expansion of π-exchanges [Lepage '97; EE, Gegelia '09]. What is R_b?

Choice of the cutoff



Certain classes of multiple-pion exchange diagrams (MSS) can be calculated analytically to an infinite order and resummed

Baru, EE, Hanhart, Hoferichter, Kudryavtsev, Phillips, EPJA 48 (12) 69

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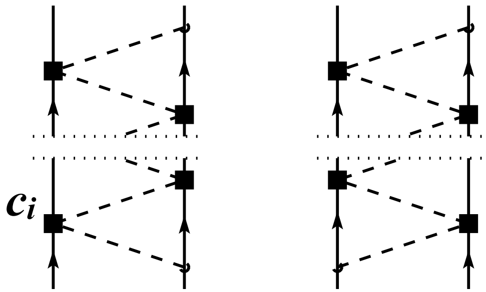

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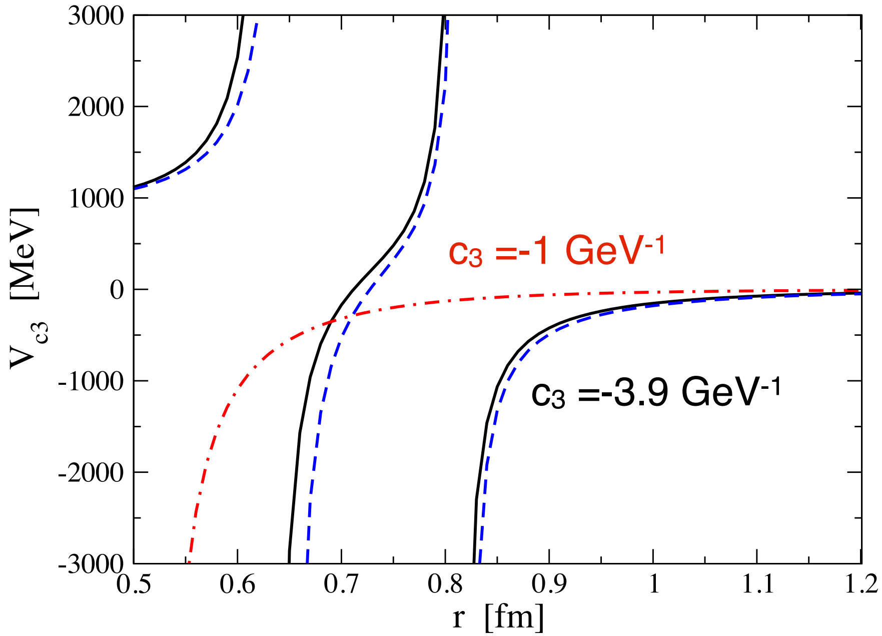
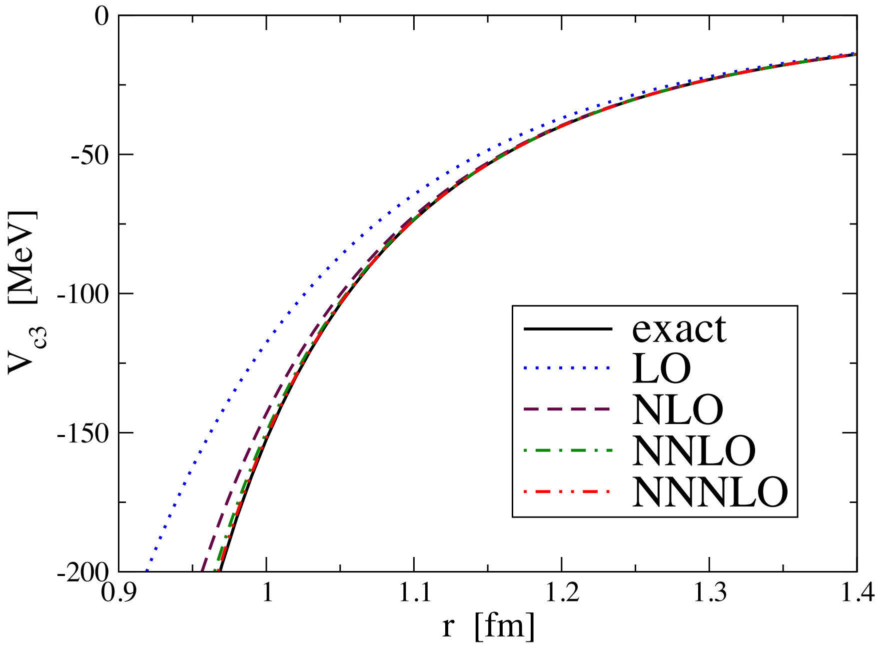
Resummed central potential generated by multi-pion exchange (c_3-part)



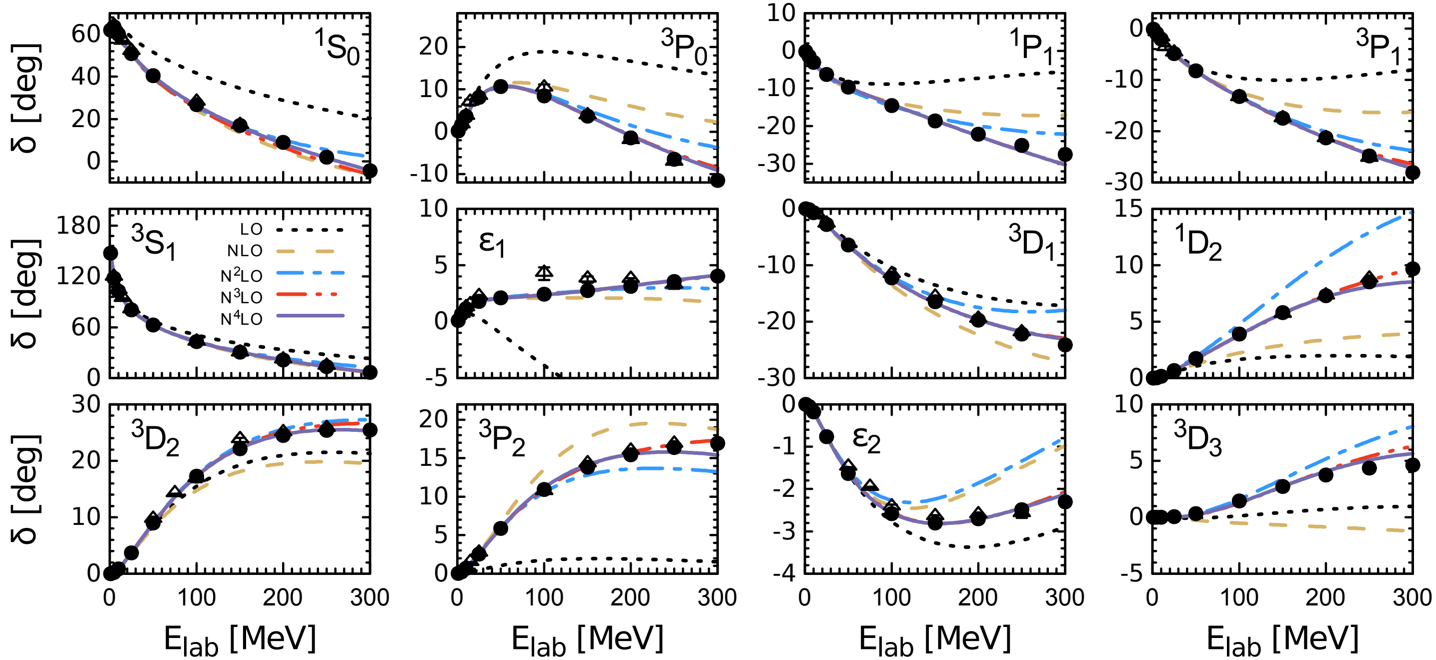


pole(!) ar r ~ 0.8 fm (breakdown distance) ➙ we use R = 0.8…1.2 fm

NN phase shifts order by order

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]



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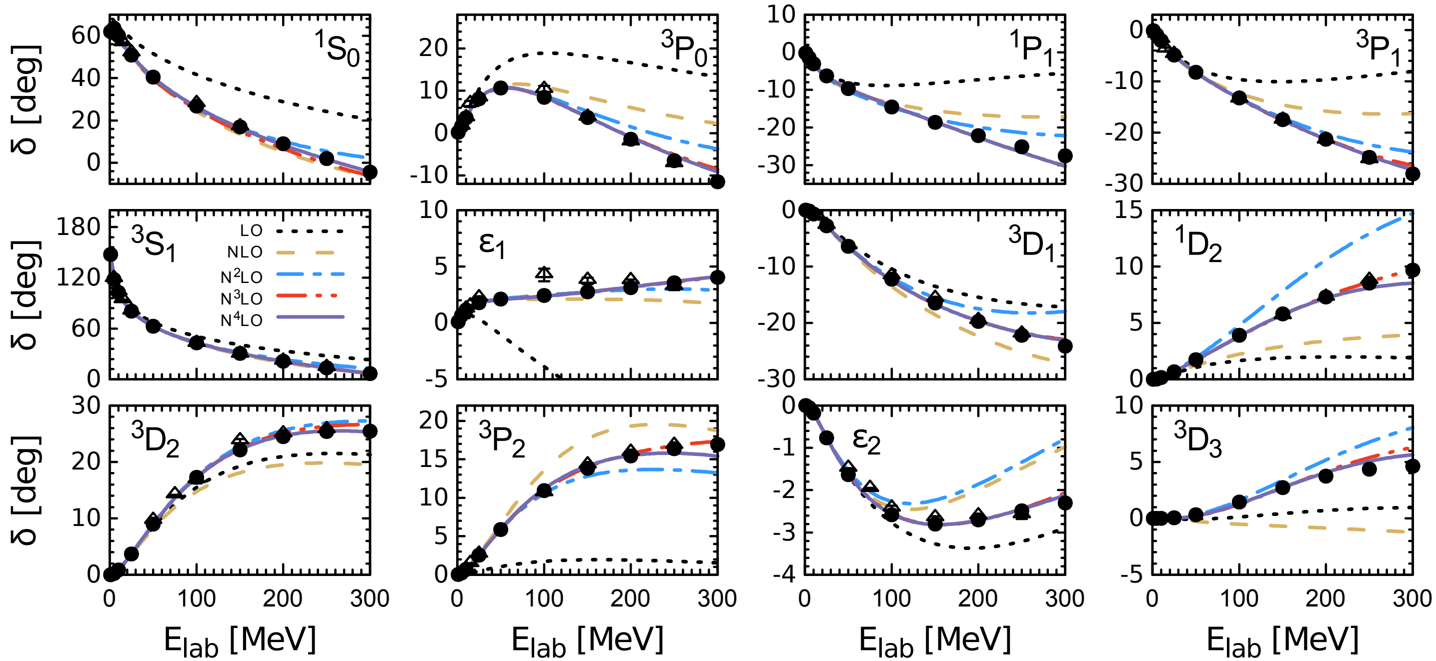

$\tilde{\chi}^2_{datum}$ for the reproduction of the Nijmegen phase shifts [using R = 0.9 fm]

E_{lab} bin	LO [Q^0]	NLO [Q^2]	N^{2}LO [Q^3]	N^{3}LO [Q^4]	N^{4}LO [Q^5]
neutron-proton phase shifts					
0–100	360	31	4.5	0.7	0.3
0–200	480	63	21	0.7	0.3
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0–100	5750	102	15	0.8	0.3
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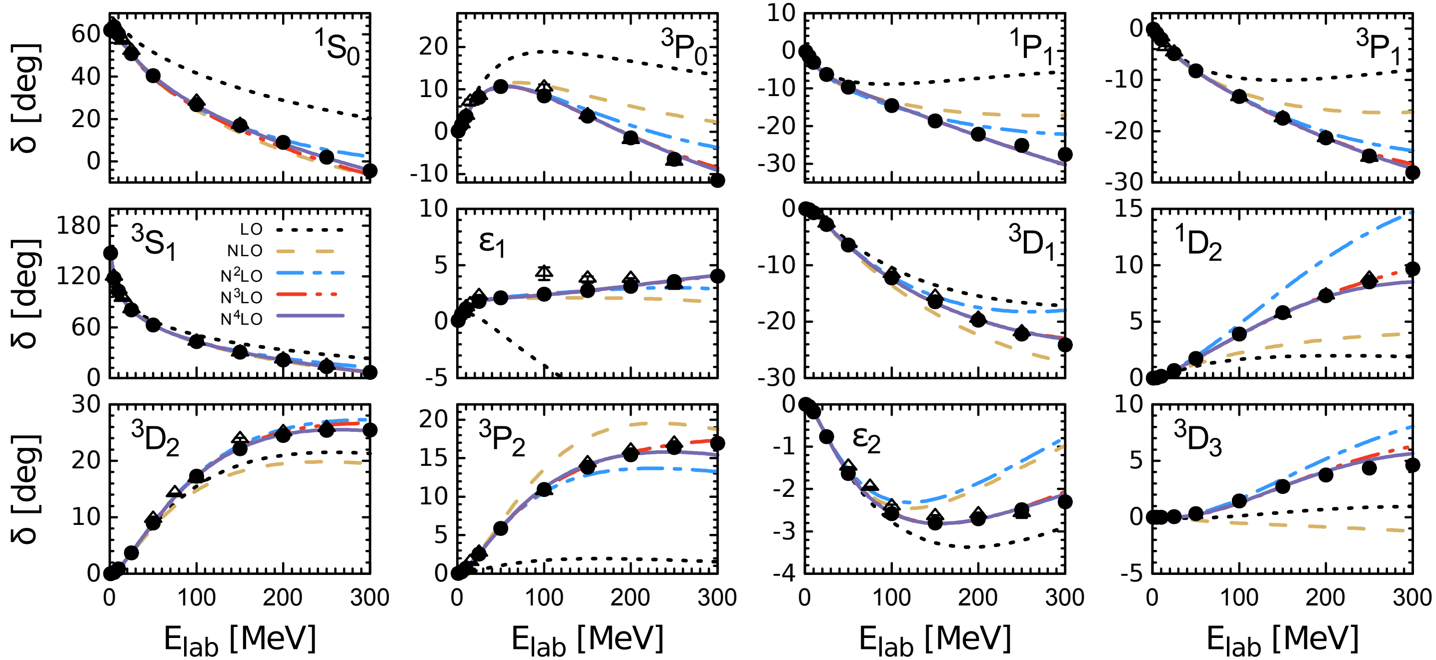

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	2 LECs	**+ 7 LECs + 2 IB LECs**		**+ 15 LECs**	**+ 1 IB LEC**

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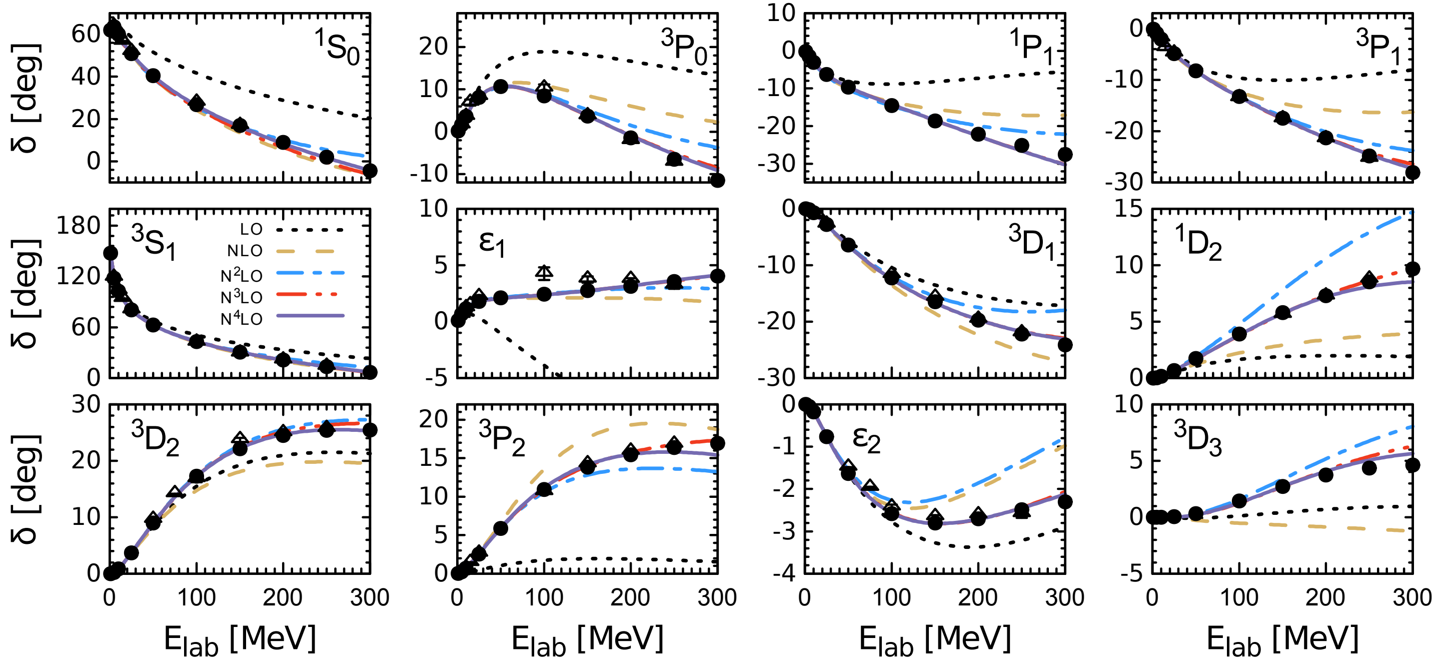

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no new LECs

1 LEC ($^1S_0$)

no new LECs

2 LECs | + 7 LECs + 2 IB LECs | + 15 LECs | + 1 IB LEC

Cutoff dependence of phase shifts

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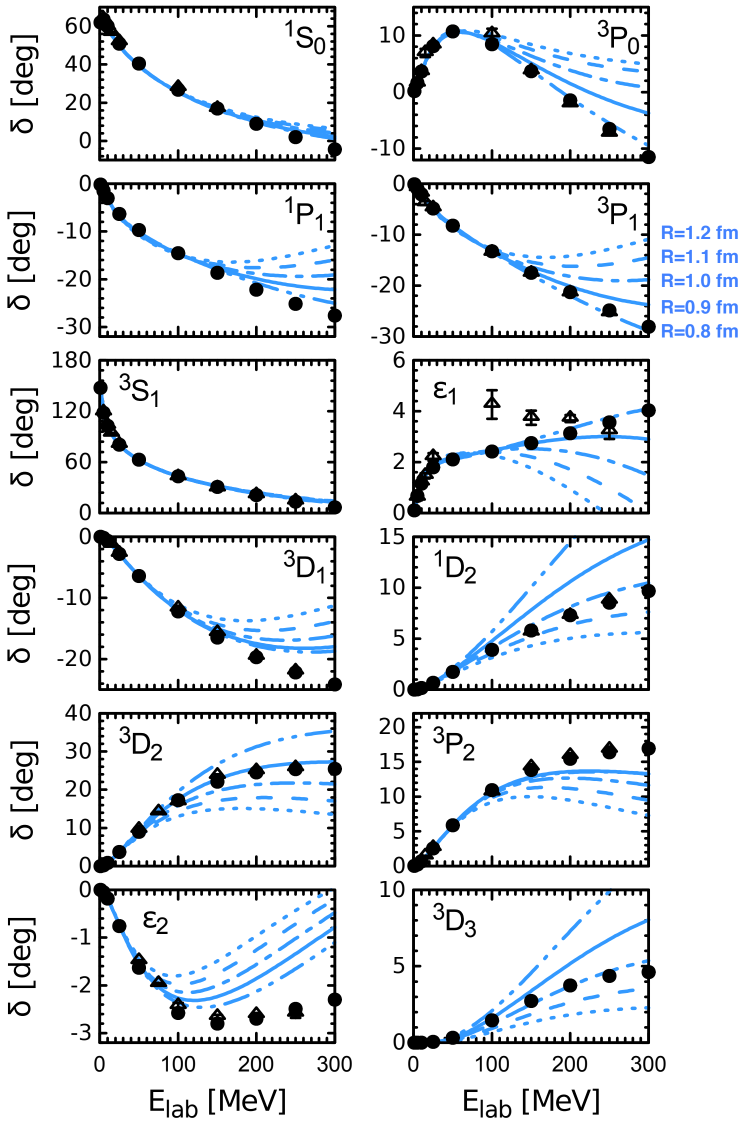
Neutron-proton phase shifts at N²LO



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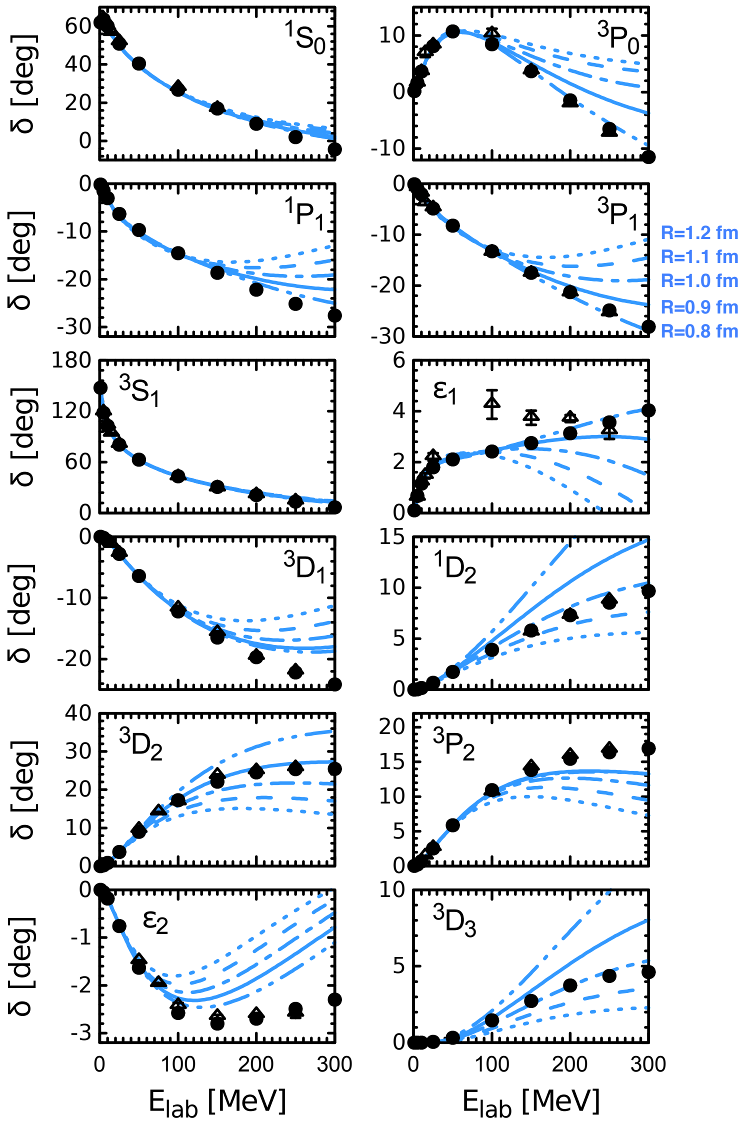

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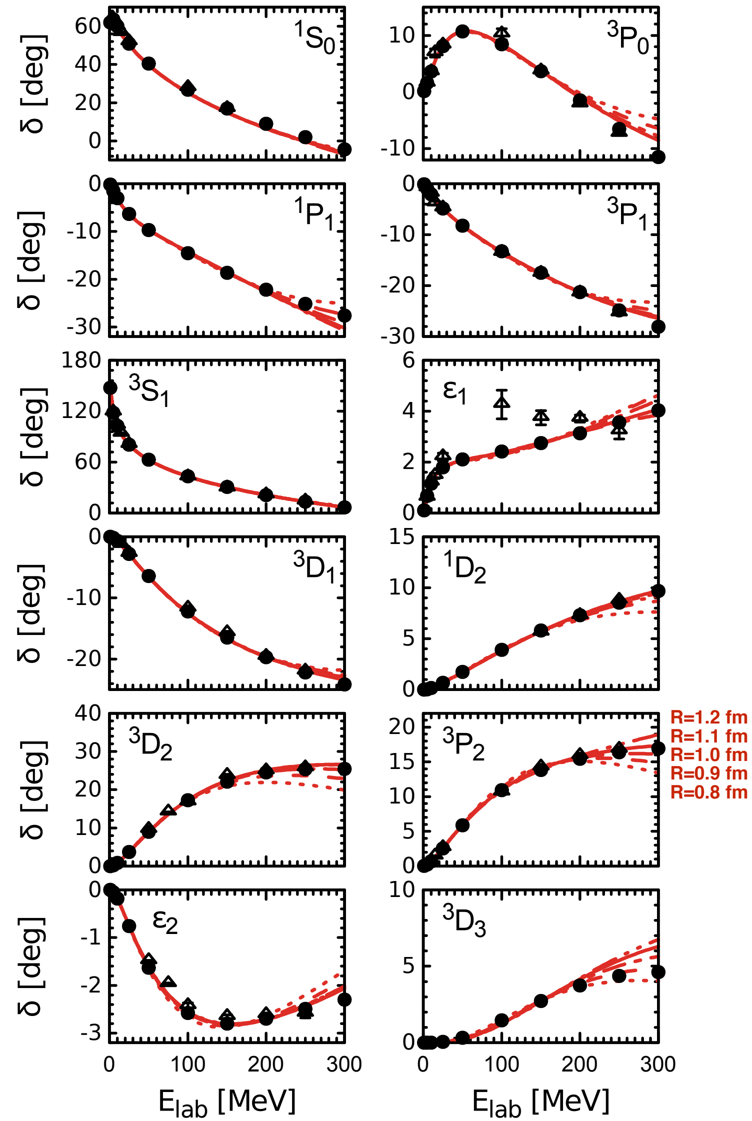

Quantification of

Theoretical Uncertainties

Theoretical uncertainties

Sources of uncertainty:

- Uncertainty in NN PWA used as input to fix contact interactions (probably small)
- Uncertainty in the values of πN LECs (might be significant, Carlsson et al.'15)
- Statistical uncertainty for NN LECs (small, Ekström et al.'14)
- **Uncertainty due to truncation of the chiral expansion at a given order**

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Often estimated by means of a **cutoff variation**. However…

- expected to underestimate the uncertainty at NLO and N^3LO

$$\left.\begin{array}{r} \text{LO: neglected order-}Q^2\text{ contact terms [NLO]} \\ \text{NLO, N}^2\text{LO: neglected order-}Q^4\text{ contact terms [N}^3\text{LO]} \\ \text{N}^3\text{LO, N}^4\text{LO: neglected order-}Q^6\text{ contact terms [N}^5\text{LO]} \end{array}\right\} \begin{array}{c} \text{cutoff dependence} \\ \text{should decrease from} \\ \text{LO to NLO(N}^2\text{LO)} \\ \text{to N}^3\text{LO(N}^4\text{LO)} \end{array}$$

- depends on the chosen range of cutoffs which in practice is rather restricted
- softer cutoffs Λ tend to overestimate the true uncertainty

$\longrightarrow$ **does not provide a reliable way to estimate theoretical uncertainty**

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LO: neglected order-Q^2 contact terms [NLO]
NLO, N^{2}LO: neglected order-Q^4 contact terms [N^{3}LO]
N^{3}LO, N^{4}LO: neglected order-Q^6 contact terms [N^{5}LO]

cutoff dependence should decrease from LO to NLO(N^{2}LO) to N^{3}LO(N^{4}LO)

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Theoretical uncertainty by estimating the size of higher-order contributions

Expansion parameter: $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

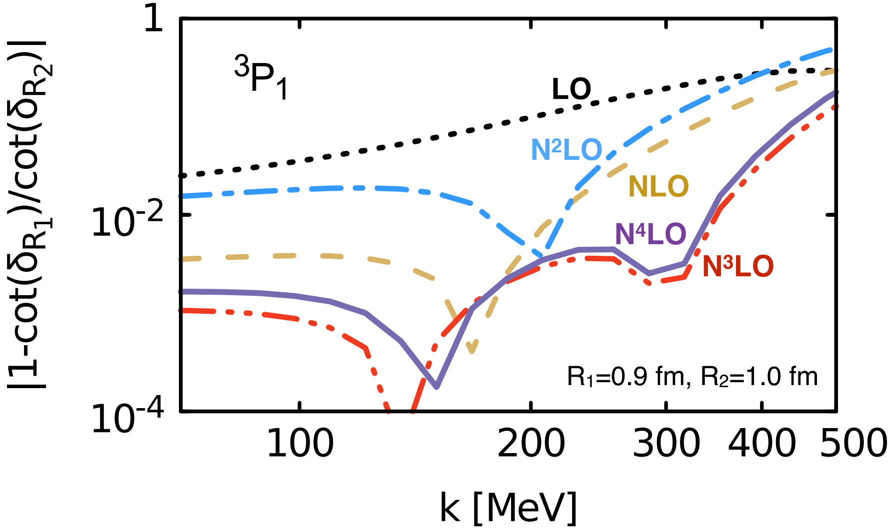
Theoretical uncertainties

Reading off the breakdown scale Λ_b from error plots

As a measure of the residual cutoff dependence of phase shifts, one can look at the error plots:

$$\left| 1 - \frac{\cot \delta_{R_i}(k)}{\cot \delta_{R_j}(k)} \right|$$

Grießhammer '04,'13; see also Lepage '97



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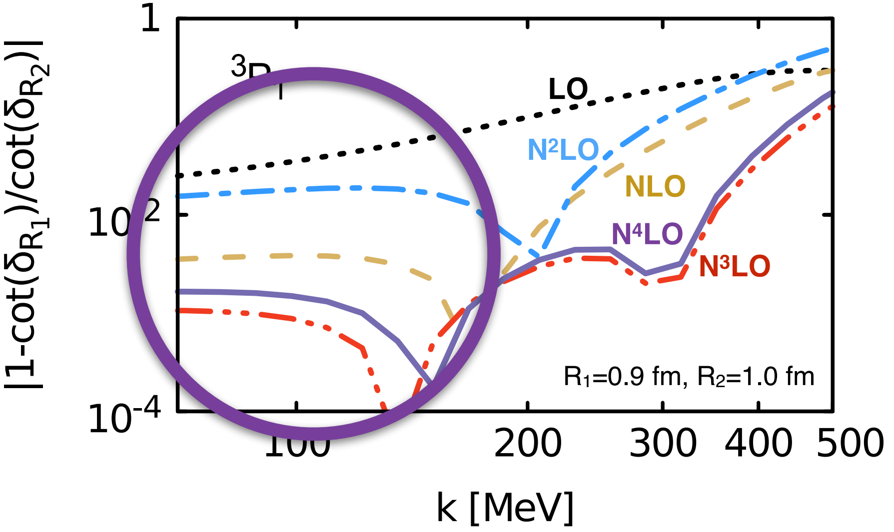
Theoretical uncertainties

Reading off the breakdown scale Λ_b from error plots

As a measure of the residual cutoff dependence of phase shifts, one can look at the error plots:

$$\left| 1 - \frac{\cot \delta_{R_i}(k)}{\cot \delta_{R_j}(k)} \right|$$

Grießhammer '04,'13; see also Lepage '97



Interpretation (simplified):

- error governed by powers of M_π/Λ_b

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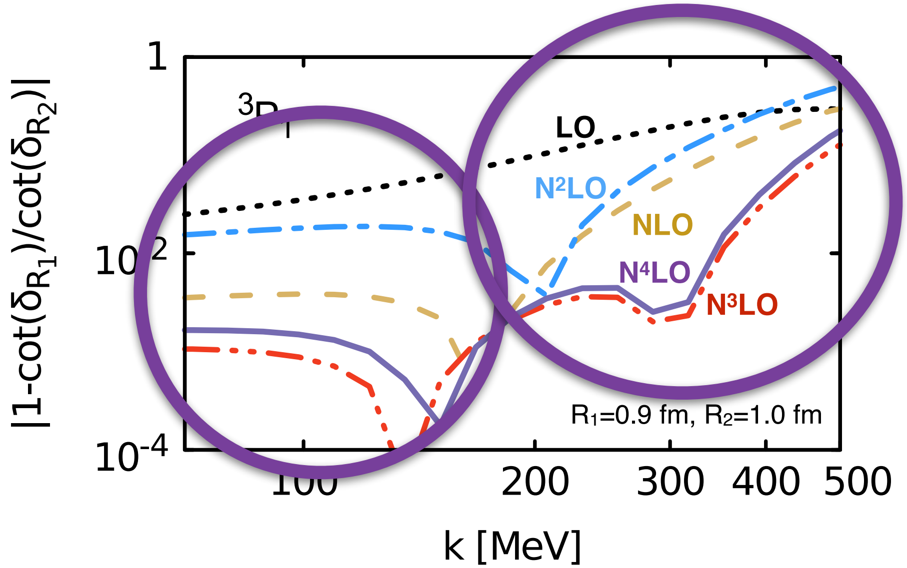

Interpretation (simplified):

- error governed by powers of M_π/Λ_b
- error governed by powers of k/Λ_b

Theoretical uncertainty by estimating the size of higher-order contributions

Expansion parameter: $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

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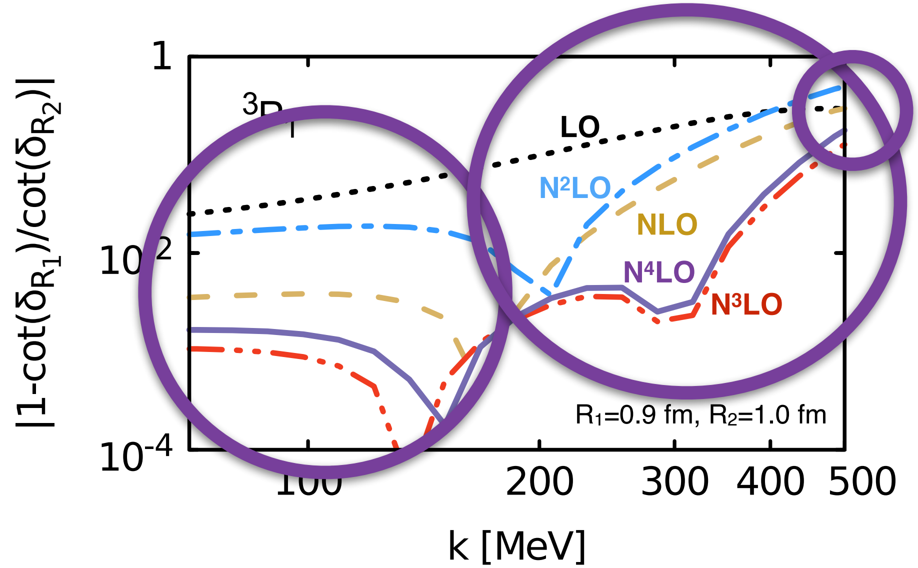

Interpretation (simplified):

- error governed by powers of M_π/Λ_b
- error governed by powers of k/Λ_b
- the breakdown scale corresponds to momenta at which the lines cross each other
  At N³LO, $\Lambda_b \sim 600$ MeV (for R = 0.8…1.0 fm)

Theoretical uncertainty by estimating the size of higher-order contributions

Expansion parameter: $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

Convergence of the chiral expansion

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

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Example: neutron-proton total cross section $_{R=0.9\ \mathrm{fm}}$

E_{lab} = 96 MeV [p = 212 MeV]: $$\sigma_{tot} = \overbrace{84.8}^{Q^0} - \overbrace{9.7}^{Q^2} + \overbrace{3.2}^{Q^3} - \overbrace{0.8}^{Q^4} + \overbrace{0.5}^{Q^5} = 78.0 \text{ mb}$$

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$$\sigma_{tot} = \overbrace{84.8}^{Q^0} - \underbrace{\overbrace{9.7}^{Q^2}}_{\sim 11} + \underbrace{\overbrace{3.2}^{Q^3}}_{\sim 4} - \underbrace{\overbrace{0.8}^{Q^4}}_{\sim 1.3} + \underbrace{\overbrace{0.5}^{Q^5}}_{\sim 0.5} = 78.0 \text{ mb}$$

Q = 212 / 600 ~ 0.35 ⟶ expect:

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Q = 212 / 600 ~ 0.35 ⟶ expect: ~ 11, ~ 4, ~ 1.3, ~ 0.5

E_{lab} = 200 MeV [p = 307 MeV]: $\sigma_{tot} = 34.9 + \underbrace{1.0}_{\sim 9} + \underbrace{6.7}_{\sim 5} + \underbrace{0.6}_{\sim 2.4} - \underbrace{0.5}_{\sim 1.2} = 42.7$ mb

Q = 307 / 600 ~ 0.5 ⟶ expect: ~ 9, ~ 5, ~ 2.4, ~ 1.2

⟶ good convergence of the chiral expansion

Uncertainty quantification

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

Let $X(p)$ be some observable with p denoting the corresponding momentum scale and $X^{(n)}(p), \quad n = 0, 2, 3, 4, \ldots$ a prediction at order Q^n in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(n)}$$

calculated in the chiral expansion

For the order-n contribution one expects $\Delta X^{(n)} \sim \mathcal{O}\big(Q^n \, X^{(0)}\big)$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \; \frac{p}{\Lambda_b}\right)$

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Theoretical uncertainty $\delta X^{(n)}$ estimated via the size of neglected higher-order contributions*:

$$\begin{aligned}
\delta X^{(0)} &= Q^2 \, |X^{(0)}| \, , \\
\delta X^{(2)} &= \max\Big(Q^3 \, |X^{(0)}|, \;\; Q \, |\Delta X^{(2)}|\Big) \, , \\
\delta X^{(3)} &= \max\Big(Q^4 \, |X^{(0)}|, \;\; Q^2 \, |\Delta X^{(2)}|, \;\; Q \, |\Delta X^{(3)}|\Big) \, , \\
&\ldots
\end{aligned}$$

(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)

Uncertainty quantification

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

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& \ldots
\end{aligned}$$

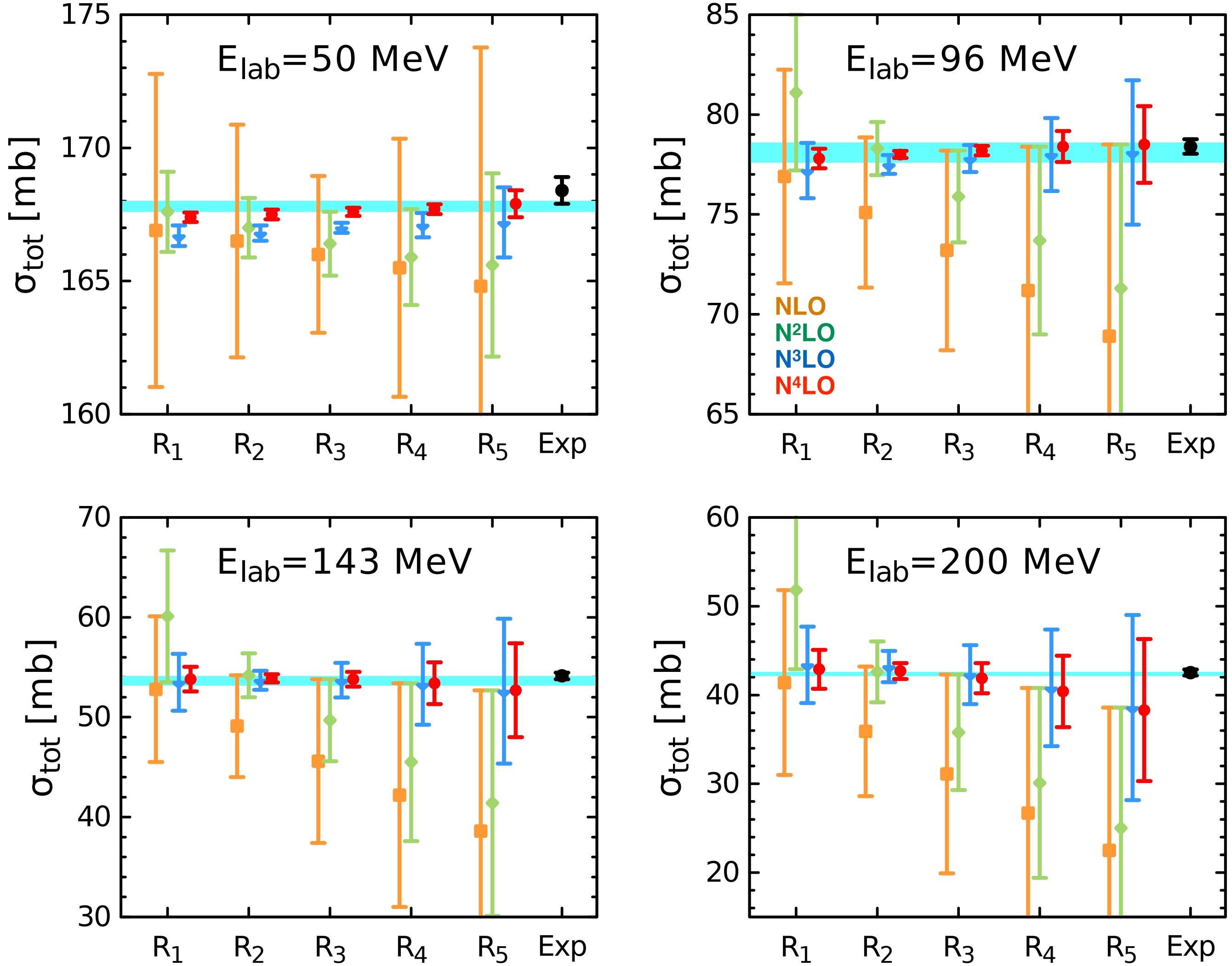
(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)

$\longrightarrow$ **a simple approach applicable for any observable and any choice of the regulator; no reliance on experimental data**

Uncertainty quantification

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

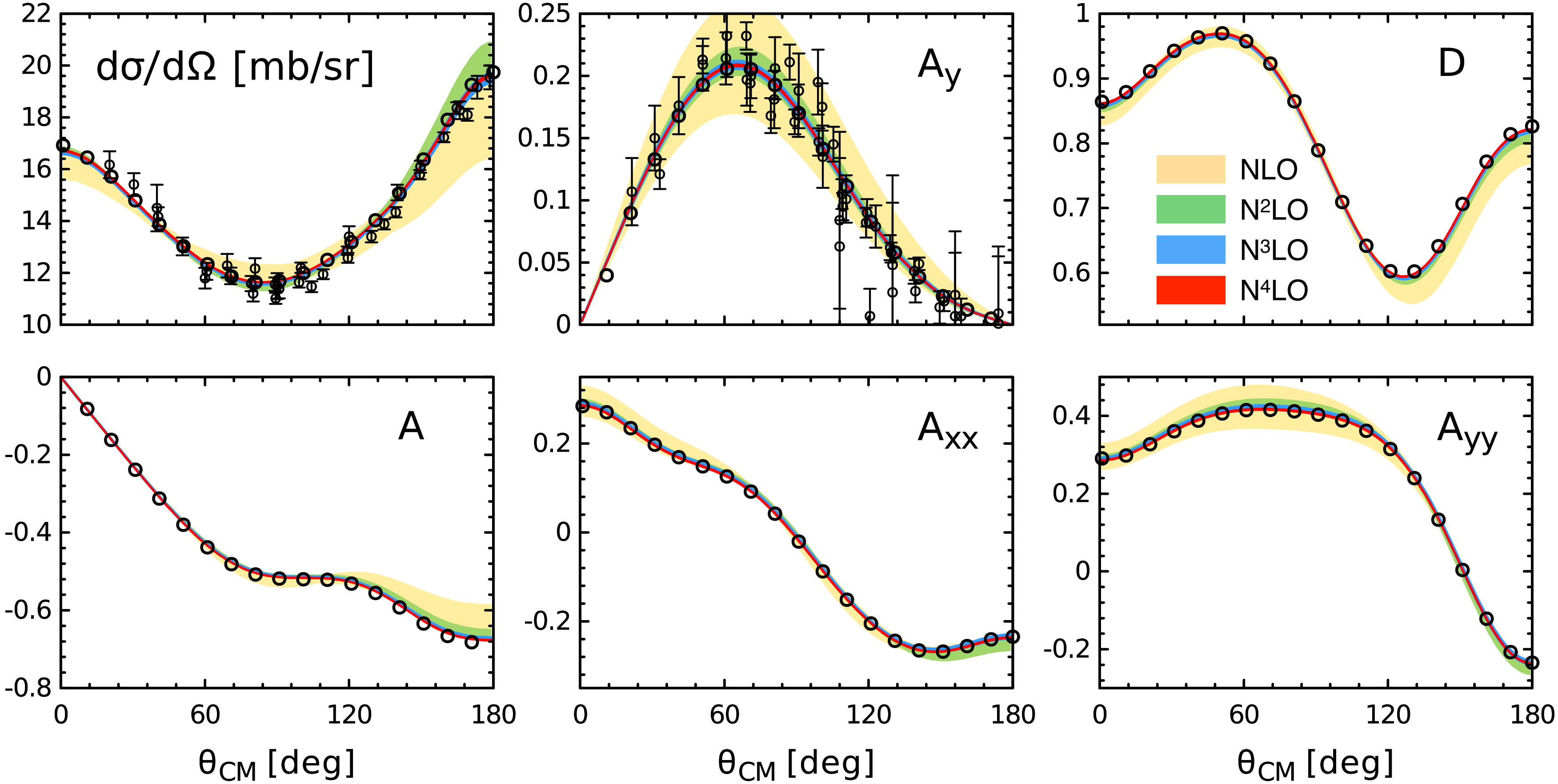
np total cross section for $R_{1,\dots,5} = \{0.8, 0.9, 1.0, 1.1, 1.2\}$ fm



Neutron-proton scattering

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

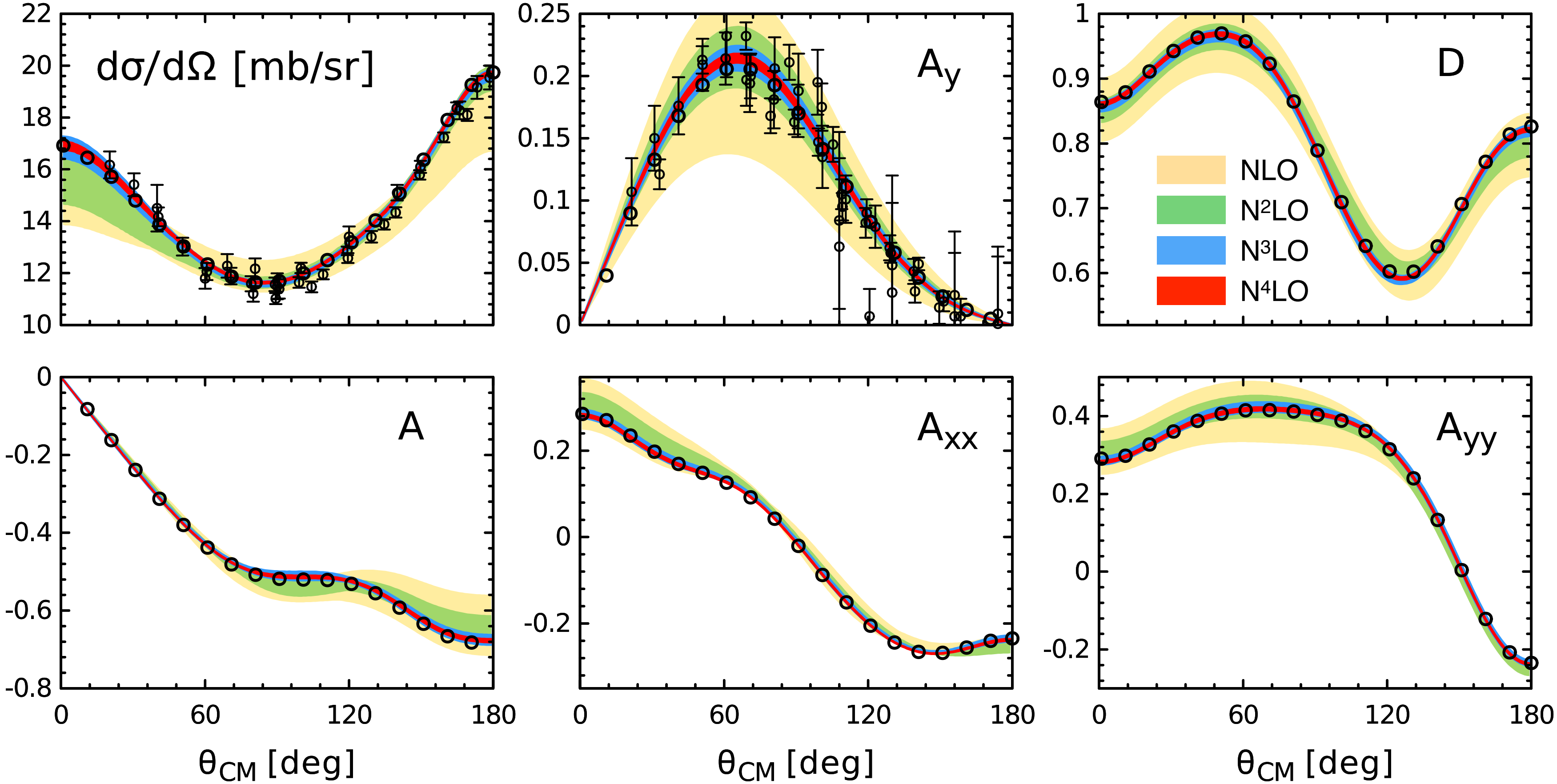
Selected neutron-proton scattering observables at 50 MeV R=0.9fm



Neutron-proton scattering

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

Selected neutron-proton scattering observables at 50 MeV $_{R=1.2fm}$

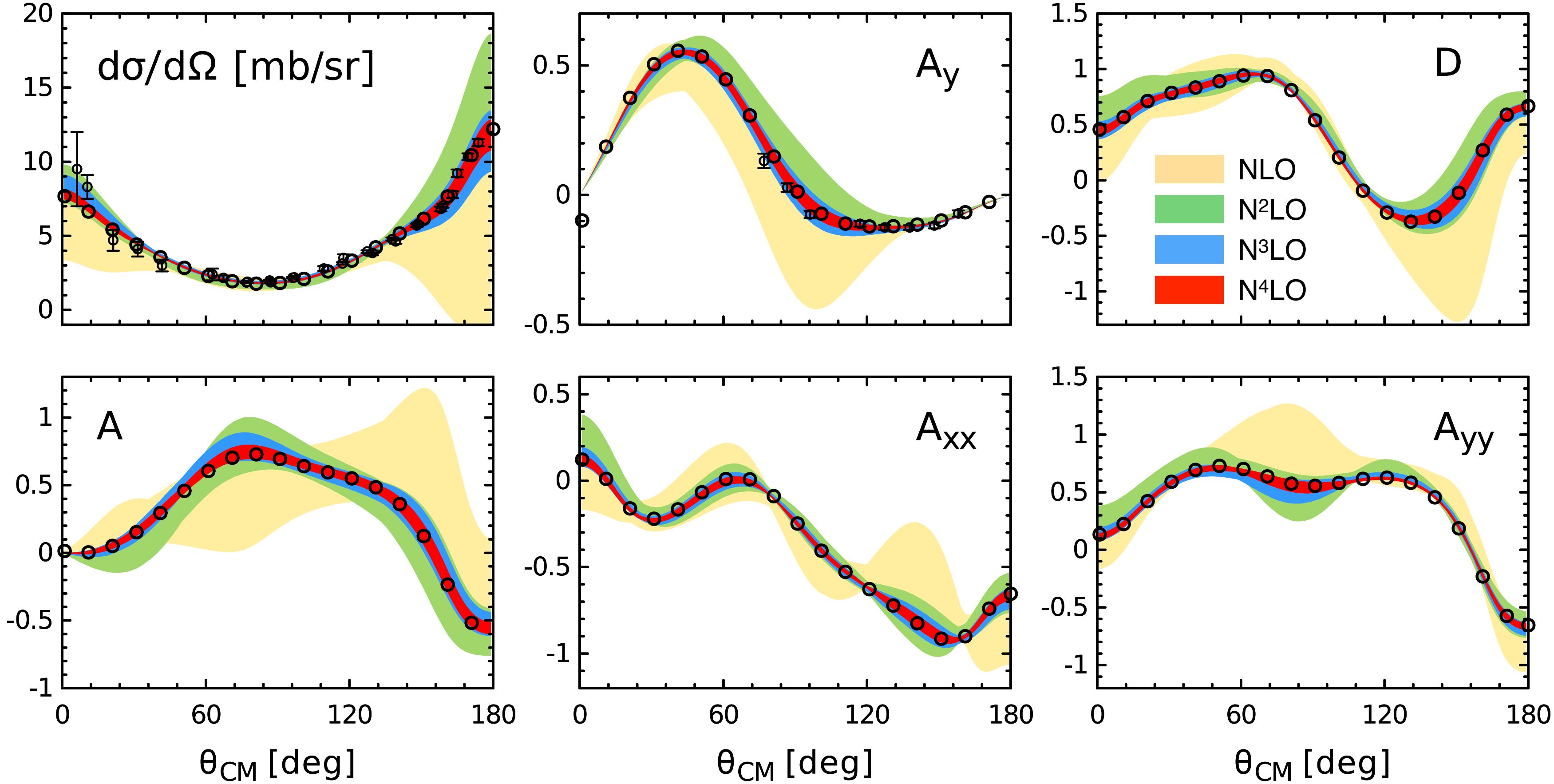


- Theoretical predictions for different cutoff choices are consistent with each other
- Softer cutoffs lead to larger theoretical uncertainties

Neutron-proton scattering

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

Selected neutron-proton scattering observables at 200 MeV $_{R=0.9\text{fm}}$



- Accurate results even at the energy of E_{lab} = 200 MeV (for R = 0.9 fm)

Deuteron properties R=0.9 fm

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; arXiv:1412.4623 [nucl-th]

	LO	NLO	N	N	N	empirical
B	2.0235	2.1987	2.2311	2.2246*	2.2246*	2.224575(9)
A	0.8333	0.8772	0.8865	0.8845	0.8844	0.8846(9)
η	0.0212	0.0256	0.0256	0.0255	0.0255	0.0256(4)
r_d	1.990	1.968	1.966	1.972	1.972	1.97535(85)
Q [fm	0.230	0.273	0.270	0.271	0.271	0.2859(3)
P_D	2.54	4.73	4.50	4.19	4.29	

* has been used in the fit

— fast convergence of the chiral expansion (P_D is not observable)

— error estimation (assuming $Q=M_\pi/\Lambda_b$)

A_S: LO: 0.83(5) → NLO: 0.878(13) → N²LO: 0.887(3) → N³LO: 0.8845(8) → N⁴LO: 0.8844(2)

η: LO: 0.021(5) → NLO: 0.026(1) → N²LO: 0.0256(3) → N³LO: 0.0255(1) → N⁴LO: 0.0255

→ theoretical results for A_S,η at N⁴LO are more accurate than empirical numbers

— results for r_d and Q do not take into account MECs and relativistic corrections:

r_d: $|\Delta r_d| \simeq 0.004$ fm [Kohno '83] → predictions in agreement with the data

Q: rel. corrections + 1π-exchange MEC: $\Delta Q \simeq +0.008 \text{ fm}^2$ [Phillips '07] → $Q \simeq 0.279 \text{ fm}^2$

the remaining deviation of 0.007 fm² agrees with the expected size of  [Phillips '07]

Regulator (in)dependence

How do our results depend on the specific form of the regulator $f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^{n}$

and/or additional spectral function regularization $V_C(q) = \frac{2}{\pi}\int_{2M_\pi}^{\Lambda_{\rm SFR}} d\mu\, \mu\, \frac{\rho_C(\mu)}{\mu^2 + q^2}$

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Selected phase shifts (in deg.) for different values of $\Lambda_{\rm SFR}$ and n at $\mathrm{N^3LO}_{[R\,=\,0.9\ \mathrm{fm}]}$

Lab. energy	NPWA	our result
proton-proton $^1S_0$ phase shift		
10 MeV	55.23	55.22 ± 0.08
100 MeV	24.99	24.98 ± 0.60
200 MeV	6.55	6.56 ± 2.2
neutron-proton $^3S_1$ phase shift		
10 MeV	102.61	102.61 ± 0.07
100 MeV	43.23	43.22 ± 0.30
200 MeV	21.22	21.2 ± 1.4
proton-proton $^3P_0$ phase shift		
10 MeV	3.73	3.75 ± 0.04
100 MeV	9.45	9.17 ± 0.30
200 MeV	-0.37	-0.1 ± 2.3
proton-proton $^3P_1$ phase shift		
10 MeV	-2.06	-2.04 ± 0.01
100 MeV	-13.26	-13.42 ± 0.17
200 MeV	-21.25	-21.2 ± 1.6
proton-proton $^3P_2$ phase shift		
10 MeV	0.65	0.65 ± 0.01
100 MeV	11.01	11.03 ± 0.50
200 MeV	15.63	15.6 ± 1.9

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Selected phase shifts (in deg.) for different values of $\Lambda_{\rm SFR}$ and n at $\mathrm{N^3LO}_{[R = 0.9\ \mathrm{fm}]}$

Lab. energy	NPWA	our result	DR, $n=5$	DR, $n=7$
proton-proton $^1S_0$ phase shift				
10 MeV	55.23	55.22 ± 0.08	55.22	55.22
100 MeV	24.99	24.98 ± 0.60	24.98	24.98
200 MeV	6.55	6.56 ± 2.2	6.55	6.56
neutron-proton $^3S_1$ phase shift				
10 MeV	102.61	102.61 ± 0.07	102.61	102.61
100 MeV	43.23	43.22 ± 0.30	43.28	43.20
200 MeV	21.22	21.2 ± 1.4	21.2	21.2
proton-proton $^3P_0$ phase shift				
10 MeV	3.73	3.75 ± 0.04	3.75	3.75
100 MeV	9.45	9.17 ± 0.30	9.15	9.18
200 MeV	-0.37	-0.1 ± 2.3	-0.1	-0.1
proton-proton $^3P_1$ phase shift				
10 MeV	-2.06	-2.04 ± 0.01	-2.04	-2.04
100 MeV	-13.26	-13.42 ± 0.17	-13.43	-13.41
200 MeV	-21.25	-21.2 ± 1.6	-21.2	-21.2
proton-proton $^3P_2$ phase shift				
10 MeV	0.65	0.65 ± 0.01	0.66	0.65
100 MeV	11.01	11.03 ± 0.50	10.97	11.06
200 MeV	15.63	15.6 ± 1.9	15.6	15.5

Regulator (in)dependence

How do our results depend on the specific form of the regulator $f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$

and/or additional spectral function regularization $V_C(q) = \frac{2}{\pi} \int_{2M_\pi}^{\Lambda_{\rm SFR}} d\mu\, \mu\, \frac{\rho_C(\mu)}{\mu^2 + q^2}$

Selected phase shifts (in deg.) for different values of $\Lambda_{\rm SFR}$ and n at N^{3}LO$_{[R = 0.9\ {\rm fm}]}$

Lab. energy	NPWA	our result	DR, $n = 5$	DR, $n = 7$	SFR, 1.0 GeV	SFR, 1.5 GeV	SFR, 2.0 GeV
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10 MeV	55.23	55.22 ± 0.08	55.22	55.22	55.22	55.22	55.22
100 MeV	24.99	24.98 ± 0.60	24.98	24.98	24.98	24.98	24.98
200 MeV	6.55	6.56 ± 2.2	6.55	6.56	6.56	6.56	6.57
neutron-proton $^3S_1$ phase shift							
10 MeV	102.61	102.61 ± 0.07	102.61	102.61	102.61	102.61	102.61
100 MeV	43.23	43.22 ± 0.30	43.28	43.20	43.17	43.21	43.22
200 MeV	21.22	21.2 ± 1.4	21.2	21.2	21.2	21.2	21.2
proton-proton $^3P_0$ phase shift							
10 MeV	3.73	3.75 ± 0.04	3.75	3.75	3.75	3.75	3.75
100 MeV	9.45	9.17 ± 0.30	9.15	9.18	9.18	9.17	9.17
200 MeV	−0.37	-0.1 ± 2.3	−0.1	−0.1	−0.1	−0.1	−0.1
proton-proton $^3P_1$ phase shift							
10 MeV	−2.06	-2.04 ± 0.01	−2.04	−2.04	−2.04	−2.04	−2.04
100 MeV	−13.26	-13.42 ± 0.17	−13.43	−13.41	−13.41	−13.42	−13.42
200 MeV	−21.25	-21.2 ± 1.6	−21.2	−21.2	−21.2	−21.2	−21.2
proton-proton $^3P_2$ phase shift							
10 MeV	0.65	0.65 ± 0.01	0.66	0.65	0.65	0.65	0.65
100 MeV	11.01	11.03 ± 0.50	10.97	11.06	11.07	11.05	11.04
200 MeV	15.63	15.6 ± 1.9	15.6	15.5	15.5	15.5	15.6

Regulator (in)dependence

How do our results depend on the specific form of the regulator $f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$

and/or additional spectral function regularization $V_C(q) = \frac{2}{\pi} \int_{2M_\pi}^{\Lambda_{\rm SFR}} d\mu\, \mu \, \frac{\rho_C(\mu)}{\mu^2 + q^2}$

Selected phase shifts (in deg.) for different values of $\Lambda_{\rm SFR}$ and n at $N^3LO_{[R = 0.9\ {\rm fm}]}$

Lab. energy	NPWA	our result	DR, $n = 5$	DR, $n = 7$	SFR, 1.0 GeV	SFR, 1.5 GeV	SFR, 2.0 GeV
proton-proton $^1S_0$ phase shift							
10 MeV	55.23	55.22 ± 0.08	55.22	55.22	55.22	55.22	55.22
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200 MeV	6.55	6.56 ± 2.2	6.55	6.56	6.56	6.56	6.57
neutron-proton $^3S_1$ phase shift							
10 MeV	102.61	102.61 ± 0.07	102.61	102.61	102.61	102.61	102.61
100 MeV	43.23	43.22 ± 0.30	43.28	43.20	43.17	43.21	43.22
200 MeV	21.22	21.2 ± 1.4	21.2	21.2	21.2	21.2	21.2
proton-proton $^3P_0$ phase shift							
10 MeV	3.73	3.75 ± 0.04	3.75	3.75	3.75	3.75	3.75
100 MeV	9.45	9.17 ± 0.30	9.15	9.18	9.18	9.17	9.17
200 MeV	−0.37	-0.1 ± 2.3	−0.1	−0.1	−0.1	−0.1	−0.1
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100 MeV	−13.26	-13.42 ± 0.17	−13.43	−13.41	−13.41	−13.42	−13.42
200 MeV	−21.25	-21.2 ± 1.6	−21.2	−21.2	−21.2	−21.2	−21.2
proton-proton $^3P_2$ phase shift							
10 MeV	0.65	0.65 ± 0.01	0.66	0.65	0.65	0.65	0.65
100 MeV	11.01	11.03 ± 0.50	10.97	11.06	11.07	11.05	11.04
200 MeV	15.63	15.6 ± 1.9	15.6	15.5	15.5	15.5	15.6

→ negligible regulator dependence (compared to the estimated theor. accuracy)

Few-nucleon systems with the new chiral NN forces

Low-Energy Nuclear Physics International Collaboration (LENPIC), arXiv:1505.07218 [nucl-th]

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www.lenpic.org

Nd total cross sections

LENPIC, arXiv:1505.07218[nucl-th]

While no complete calculations based on the new 2N+3N forces are available yet, we performed **incomplete calculations based on 2N forces only** in order to:

— identify observables/kinematics best suitable for searches of 3NF effects

— estimate the achievable accuracy of of chiral EFT

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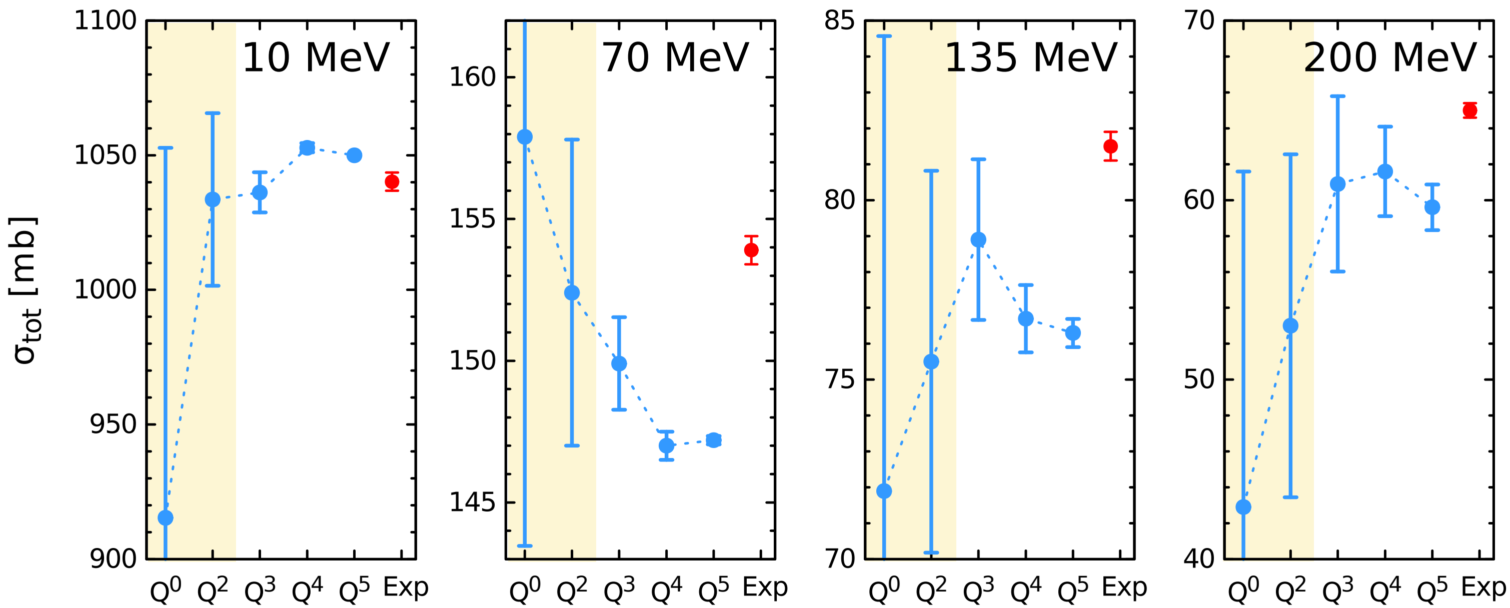
- estimate the achievable accuracy of of chiral EFT

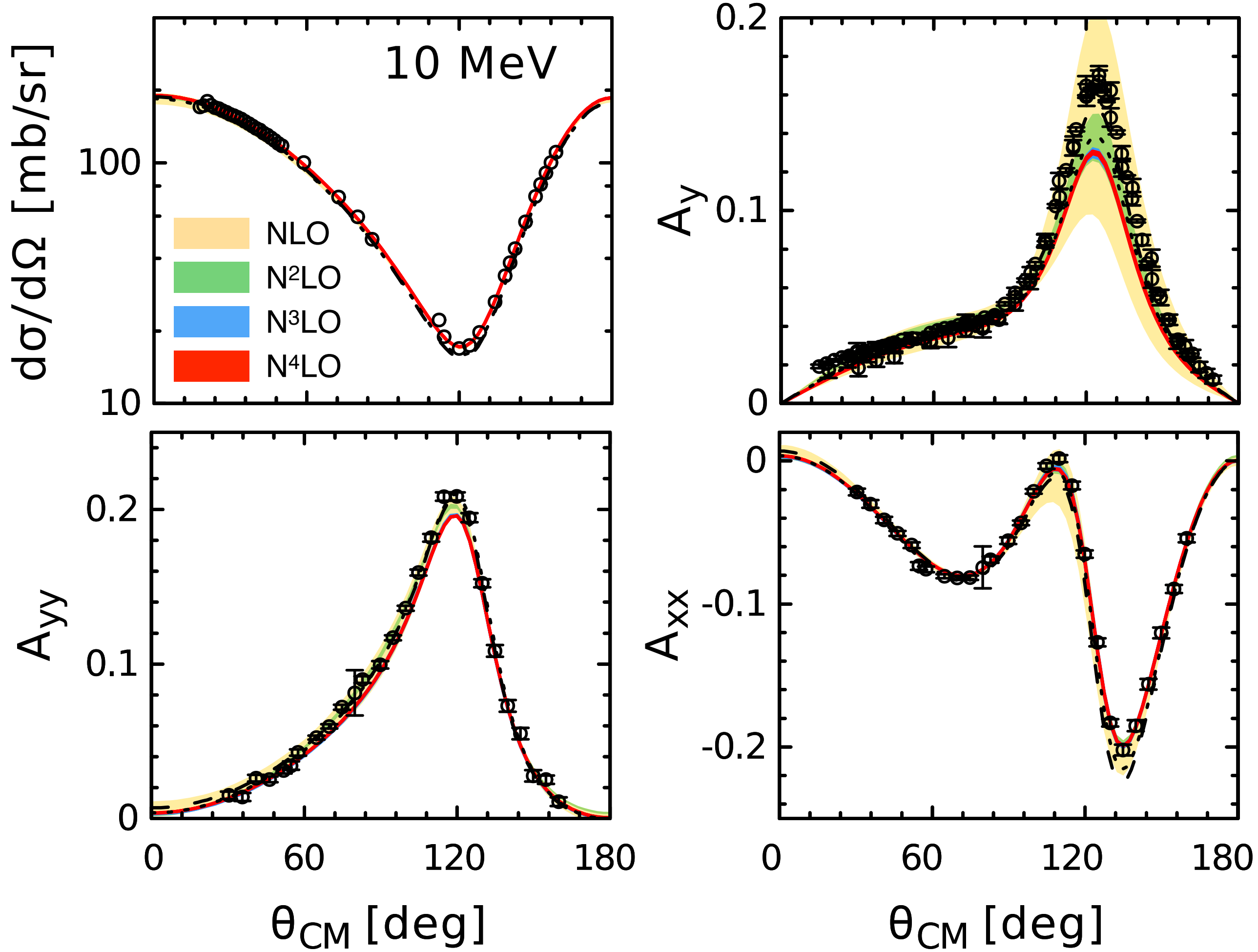
Neutron-deuteron total cross section based on NN forces only R=0.9fm



- Unambiguous evidence for missing three-nucleon forces (within our scheme)
- The size of the missing 3NF contribution agrees well with power counting (N^2LO)

Nd elastic scattering at 10 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th]

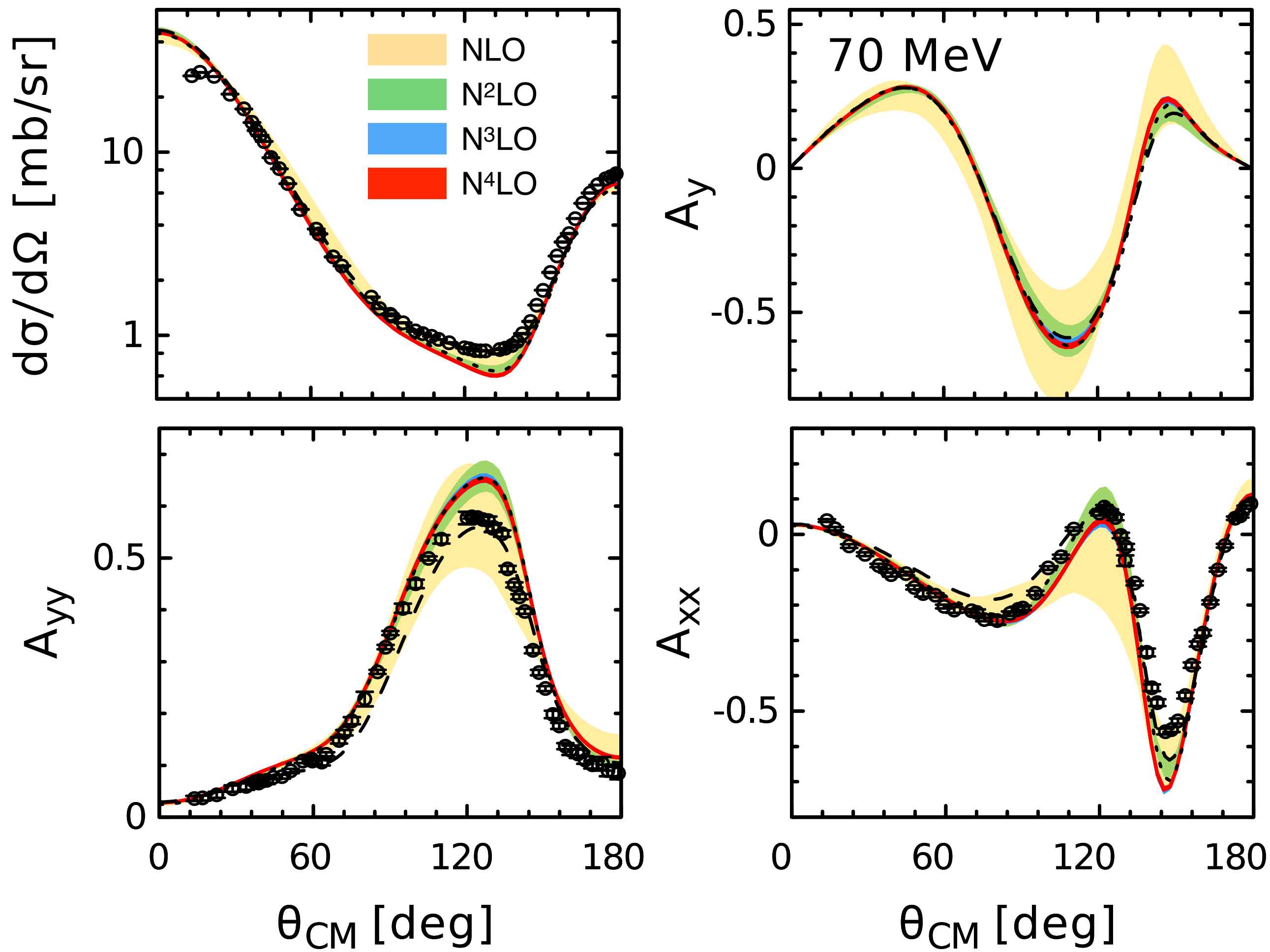


— very accurate results are expected

— not much room for missing 3NF effects except for the A_y (A_y-puzzle)

Nd elastic scattering at 70 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th]

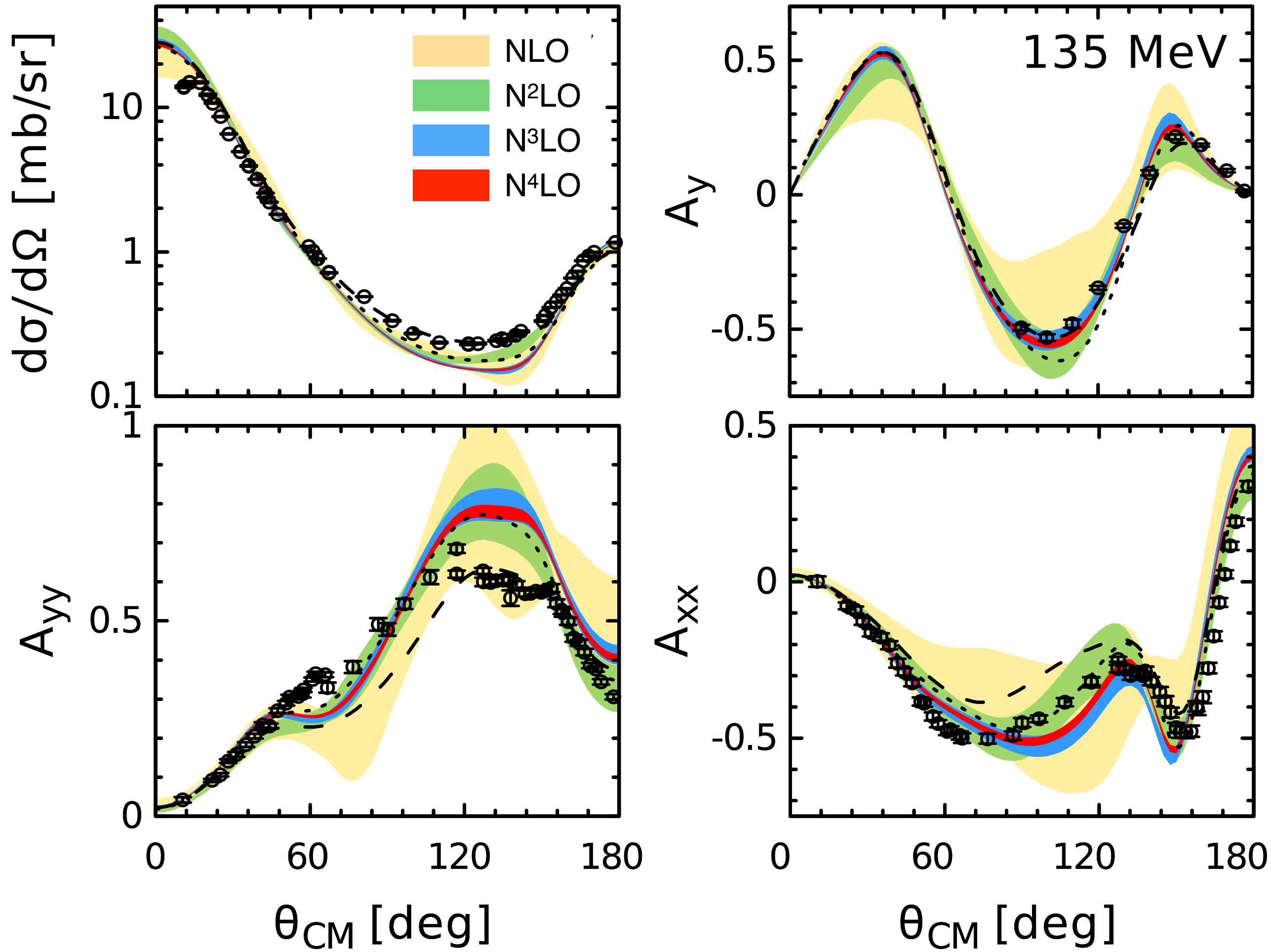


— very accurate results starting from N³LO are expected

— clear room for 3NF contributions to the cross section and tensor analyzing powers

Nd elastic scattering at 135 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th]

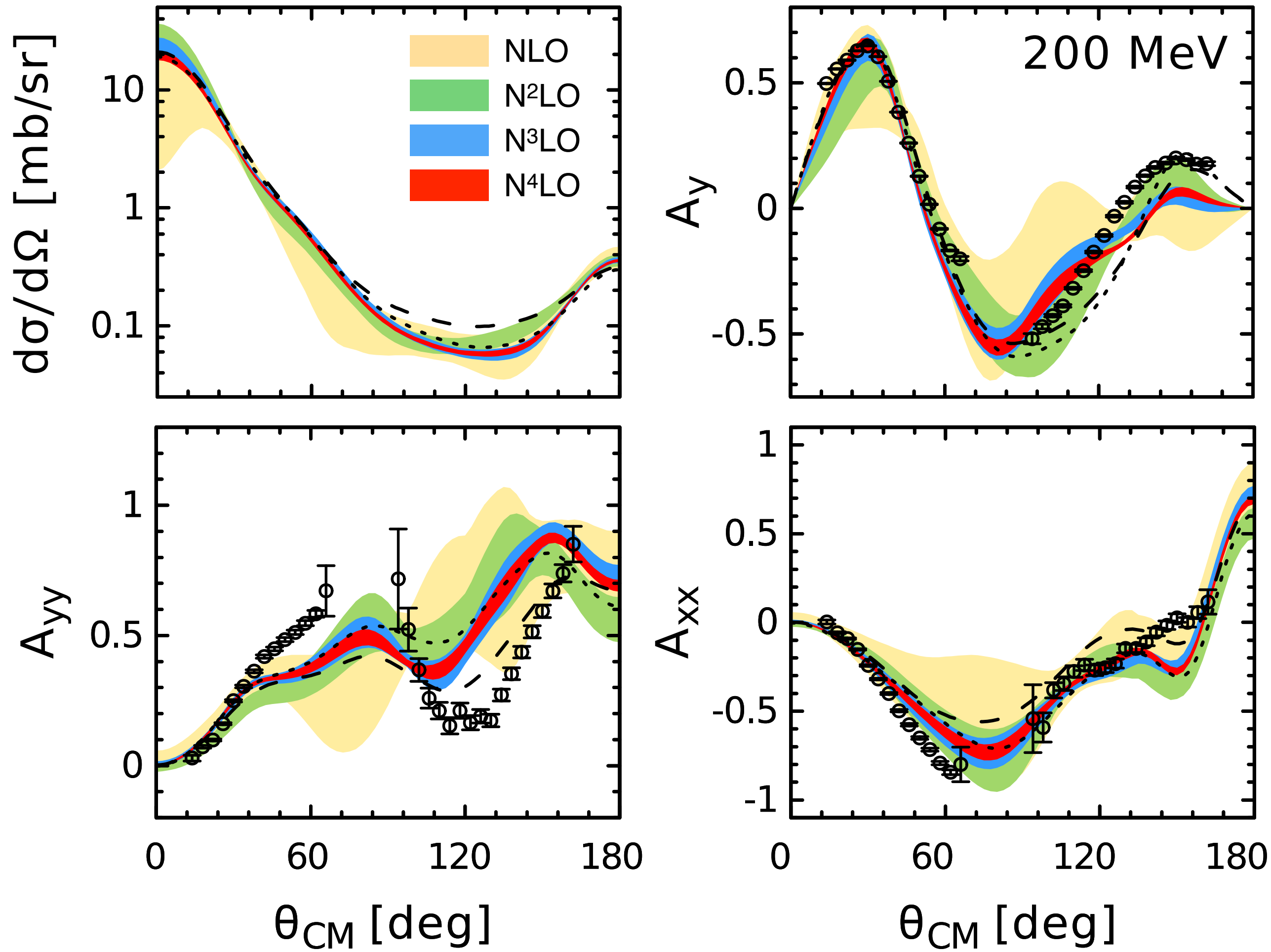


— accurate results at N^4LO are expected

— clear room for 3NF contributions to the cross section and tensor analyzing powers

Nd elastic scattering at 200 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th]



— fairly accurate results at N^{4}LO are expected

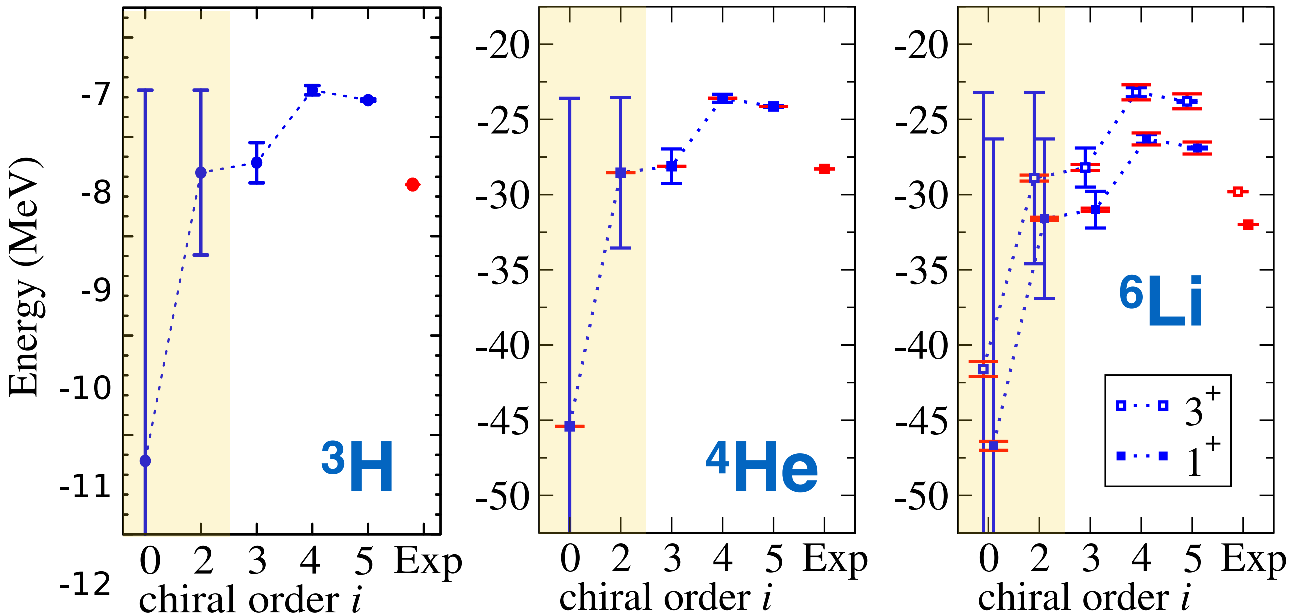
— clear room for 3NF contributions

more results for Nd scattering in the talk by Henryk Witala

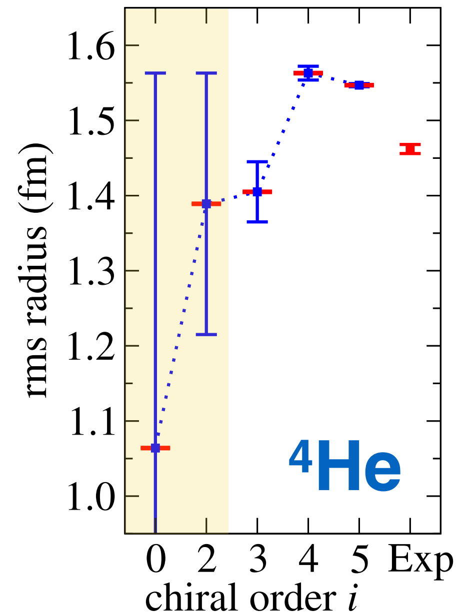
Light nuclei with chiral NN forces [r = 1.0 fm]

LENPIC, arXiv:1505.07218[nucl-th]

Energies of ^3H, ^4He, ^6Li



^4He rms radius



- Faddeev-Yakubovsky for ^3H, ^4He; NCSM with SRG-evolved 2NF (+ $3NF_{induced}$) for ^6Li
- For NCSM, the numerical uncertainty dominates starting from N^3LO
- Theoretical uncertainties at N^4LO for the energies estimated to be below 0.5%
- Required size of the 3NF consistent with the estimated size of N^2LO contributions

Summary

A new generation of chiral NN potentials up to N^{4}LO is being developed

— excellent description of NN data

— good convergence of the chiral expansion

A simple approach to estimate theoretical uncertainty at a given order

— applicable to any observable and for a particular choice of the regulator

— results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties)

Application to few-N systems:

— clear evidence for missing 3NF effects within our scheme

— expect accurate results for Nd scattering up to $E_{lab} \sim 200$ MeV (at N^{4}LO)

— Nd scattering at intermediate ($E_{lab} \sim 50 \ldots 200$ MeV): a golden window to test/probe the 3NF in chiral EFT

Next step: explicit inclusion of the 3NF ⟵ **talks by Hermann Krebs, Luca Girlanda**

Goal: reliable *ab initio* few- and many-body calculations based on chiral EFT with quantified theoretical uncertainties!