Status of Chiral Perturbation Theory for Light Mesons

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Contents

Introduction

mesonic CHPT

- strong and semileptonic weak interactions
- nonleptonic weak interactions
- radiative corrections (dynamical photons/leptons)

review/package/talk for higher-order calculations
J. Bijnens, Prog. Part. Nucl. Phys. 58 (2007) 521
CHIRON, EPJ C75 (2015) 1
leading logs $\longrightarrow$ HSMBI-WG

main issues (esp. for chiral $SU(3)$)

- low-energy constants (LECs)
- "convergence", rescattering $\longrightarrow$ dispersive treatment

Low-energy constants

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_F p^2}(2) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1) + \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{leptons}}_{\text{kin}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{leptons}}_{e^2 p^2}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

recent review Bijnens, GE, Ann. Rev. Nucl. Part. Sci. 64 (2014) 149

restrict discussion to NNLO fit of

NLO LECs $L_i (i = 1, \ldots, 10)$ and NNLO LECs $C_i (i = 1, \ldots, 90)$

update and extension of Bijnens, Jemos 2012

new ingredients

- relations $\bar{l}_j(L_i, C_i)$ $(j = 1, \ldots, 4)$ Gasser, Haefeli, Ivanov, Schmid 2007

  $\longrightarrow$ altogether 17 input data
- penalize bad convergence of meson masses
- intelligent guesses (priors) for 34 (combinations of the) C_i
- renormalization scale $\mu = 0.77$ GeV

C_i	$C_i^r = 0$	JWCW 2015
$10^3 L_1^r$	0.67(06)	0.45(07)
$10^3 L_2^r$	0.17(04)	0.22(04)
$10^3 L_3^r$	−1.76(21)	−1.66(22)
$10^3 L_4^r$	0.73(10)	0.51(12)
$10^3 L_5^r$	0.65(05)	2.61(12)
$10^3 L_6^r$	0.25(09)	0.73(06)
$10^3 L_7^r$	−0.17(06)	−0.54(05)
$10^3 L_8^r$	0.22(08)	1.43(10)
χ^2	26	25
dof	9	9

fitting procedure

- minimization/random walk in restricted C_i-space
- iterate after possible modification of C_i-space
- normal χ^2-fit for L_i for (fixed) "best" values of the C_i

  $\longrightarrow$ "best" values for L_i

	NNLO free fit	NNLO BE14	NLO 2014	GL 1985
$10^3 L_A^r$	0.68(11)	0.24(11)	0.4(2)	
$10^3 L_1^r$	0.64(06)	0.53(06)	1.0(1)	0.7(3)
$10^3 L_2^r$	0.59(04)	0.81(04)	1.6(2)	1.3(7)
$10^3 L_3^r$	−2.80(20)	−3.07(20)	−3.8(3)	−4.4(2.5)
$10^3 L_4^r$	0.76(18)	0.3	0.0(3)	−0.3(5)
$10^3 L_5^r$	0.50(07)	1.01(06)	1.2(1)	1.4(5)
$10^3 L_6^r$	0.49(25)	0.14(05)	0.0(4)	−0.2(3)
$10^3 L_7^r$	−0.19(08)	−0.34(09)	−0.3(2)	−0.4(2)
$10^3 L_8^r$	0.17(11)	0.47(10)	0.5(2)	0.9(3)
F_0 [MeV]	64	71		

- strong sensitivity to (large-N_c) suppressed L_4 $\longrightarrow$ enforce small L_4 (supported by lattice)
- BE14: $10^3 L_4^r = 0.3$; NLO: $-0.3 \leq 10^3 L_4^r \leq 0.3$ fixed $\longrightarrow$ $L_A = 2L_1 - L_2$ and L_6 automatically suppressed
- NNLO fits only make sense with certain set of C_i^r ($\rightarrow$ BE review)
- except for last column: no estimate of higher-order uncertainties

	NNLO free fit	NNLO BE14	NLO 2014	GL 1985
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- reasonable convergence of observables (enforced for masses)
- qualitative evidence for resonance saturation, even for scalars
  e.g.: $L_5 \simeq 2L_8$, also $C_1 + C_3 - C_4, C_{12}$
  η' dominance for some C_i (as for L_7) Kaiser 2007
- last 3 columns: remarkable stability

LECs from τ decay data $\longrightarrow$ Golterman, GB-WG

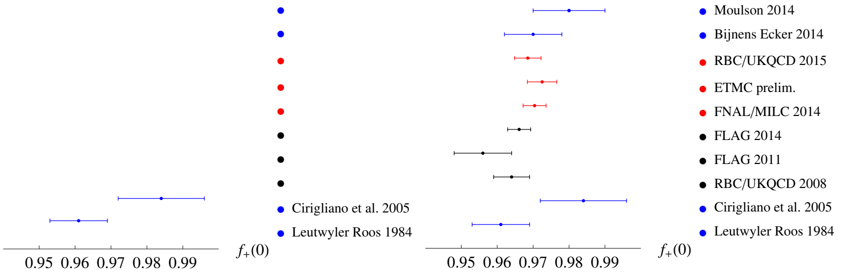
Status of CKM unitarity

main sources for determination of CKM matrix elements (first row)

- V_{ud}: superallowed β decays
  neutron β decay
  (pion β decay)
- V_{us}: K_{l3} decays $\rightarrow |f_+^{K^0\pi^-}(0)V_{us}|$
  $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$, F_K/F_π, V_{ud}
  $\Gamma(\tau \rightarrow X_s \nu)$ vs. $\Gamma(\tau \rightarrow X_{\rm nonstrange}\nu)$, V_{ud}
  (hyperon decays)
- V_{ub}: irrelevant for unitarity test

to determine V_{us} independently of V_{ud}: **need $f_+^{K^0\pi^-}(0)$**

$\longrightarrow$ 30-year-history of estimates/determinations



NNLO CHPT: Bijnens, Talavera 2003 + LECs from BE14

$$f_+^{K^0\pi^-}(0) = 1 - \underbrace{0.02276}_{\text{NLO}} - \underbrace{0.00754}_{\text{NNLO}}$$

$\longrightarrow$ "official" CHPT prediction to NNLO

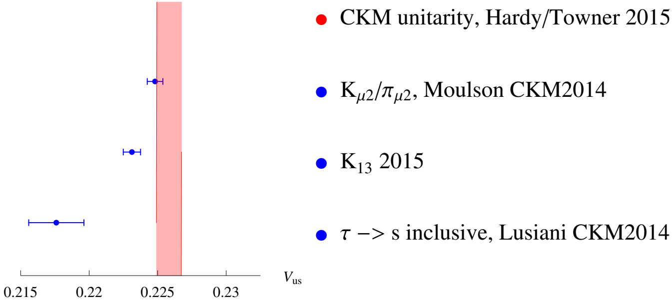
$$f_+^{K^0\pi^-}(0) = 0.970 \pm 0.008$$

compare with average of (three) most recent lattice determinations (red dots), to be used in the following figure:

$$f_+^{K^0\pi^-}(0) = 0.9703 \pm 0.0021$$

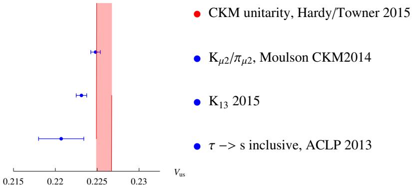
other input: status of CKM2014 (Lusiani, Moulson)

except for update of V_{ud} (Hardy, Towner 2015)



conclusion

worry about input for V_{us} rather than about unitarity



Antonelli, Cirigliano, Lusiani, Passemar

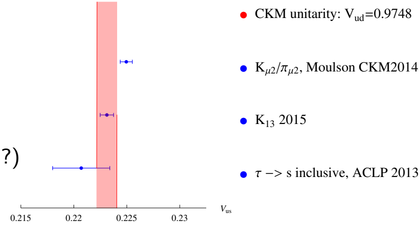
(ACLP 2013): $B(\tau \to X_s \nu)$ too small

$\longrightarrow$ use K decays to calculate 68% of total strange width $\to$ $B(\tau \to K\nu, K\pi\nu)$

possible scenario for CD18

increase V_{ud} by 3 σ

(isospin violation in nuclei under control?)

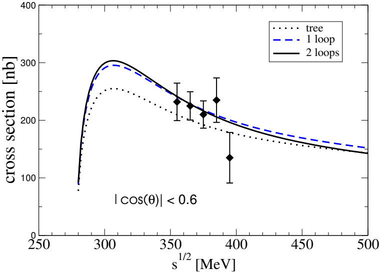


A success story at NNLO: pion polarizabilities

$\gamma\gamma\pi^+\pi^-$ complex: early example of NNLO CHPT

NLO Bijnens, Cornet 1986
NNLO Bürgi 1996
Gasser, Ivanov, Sainio 2006

chiral $SU(2)$ $\longrightarrow$ expect good convergence



$\sigma(\gamma\gamma \to \pi^+\pi^-)$

Gasser, Ivanov, Sainio

electric (α_π) and magnetic (β_π) polarizabilities

threshold expansion for $\gamma\pi^\pm \to \gamma\pi^\pm$

$$z_\pm = 1 \pm \cos\theta_{\rm cm}$$

$$\frac{d\sigma}{d\Omega} - \left(\frac{d\sigma}{d\Omega}\right)_{\rm Born} = -\frac{\alpha M_\pi^3 (s - M_\pi^2)^2}{4s^2(sz_+ + M_\pi^2 z_-)} \times \left(z_-^2(\alpha_\pi - \beta_\pi) + \frac{s}{M_\pi^4} z_+^2 (\alpha_\pi + \beta_\pi)\right) + \ldots$$

3 types of experiment $\longrightarrow$ Friedrich, plenary talk

- $\pi\gamma \to \pi\gamma$ (Primakoff)
- $\gamma\pi \to \gamma\pi$ (pion photoproduction)
- $\gamma\gamma \to \pi\pi$ (via $e^+e^- \to e^+e^-\pi\pi$)

CHPT

polarizabilities in units $10^{-4}\mathrm{fm}^3$

	$O(p^4)$	$O(p^6)$
$\alpha_\pi - \beta_\pi$	6.0	5.7
$\alpha_\pi + \beta_\pi$	0	0.16

numbers from

Gasser, Ivanov, Sainio

NLO

single LEC $2l_5 - l_6 = 2(L_9 + L_{10})$ (well established from $\pi \to e\nu\gamma$)

NNLO

additional NLO LECs l_1, l_2, l_3, l_4 and 3 NNLO LECs

$\alpha_\pi - \beta_\pi$ not very sensitive to NNLO LECs $\longrightarrow$

$\alpha_\pi - \beta_\pi = 5.7 \pm 1.0$ Gasser, Ivanov, Sainio

experimental situation

last century: large uncertainties ($\alpha_\pi - \beta_\pi \sim \; 4 \div 53$)

this century: two more precise experiments

additional dispersive analysis

experiment		$\alpha_\pi - \beta_\pi$
MAMI 2005	$\gamma p \to \gamma \pi^+ n$	$11.6 \pm 1.5_{\mathrm{stat}} \pm 3.0_{\mathrm{syst}} \pm 0.5_{\mathrm{mod}}$
theory		
Fil'kov, Kashevarov 2006	dispersive	$13.0^{+2.6}_{-1.9}$
Gasser, Ivanov, Sainio 2006	CHPT	5.7 ± 1.0

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MAMI 2005	$\gamma p \to \gamma \pi^+ n$	$11.6 \pm 1.5_{\rm stat} \pm 3.0_{\rm syst} \pm 0.5_{\rm mod}$
COMPASS 2015 ($\beta_\pi = -\alpha_\pi$)	$\pi^- \mathrm{Ni} \to \pi^- \gamma\ \mathrm{Ni}$	$4.0 \pm 1.2_{\rm stat} \pm 0.7_{\rm syst}$
theory		
Fil'kov, Kashevarov 2006	dispersive	$13.0^{+2.6}_{-1.9}$
Gasser, Ivanov, Sainio 2006	CHPT	5.7 ± 1.0

perfect agreement between CHPT and most precise experiment

N.B.: disp. treatment criticized by Pasquini, Drechsel, Scherer 2008

$\eta \to 3\pi$

$\eta \to 3\pi$ decays violate isospin

electromagnetic contributions small

Sutherland, . . . , Ditsche, Kubis, Meißner

to a very good approximation

$$A(\eta \to 3\pi) \sim m_d - m_u \sim R^{-1} \sim Q^{-2} \qquad (m_{ud} := (m_u + m_d)/2)$$

$$R = \frac{m_s - m_{ud}}{m_d - m_u}, \quad Q^2 = \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}, \quad Q^2 = \left(1 + \frac{m_s}{m_{ud}}\right) R/2$$

from chiral point-of-view:

best way to determine $m_u - m_d$

problem: convergence of chiral amplitude

		$\Gamma(\eta \to \pi^+\pi^-\pi^0)/\text{eV}$	r
LO	Osborn, Wallace 1970	66^*	1.54
NLO	Gasser, Leutwyler 1985	$168(50)^*$	1.46
NNLO	Bijnens, Ghorbani 2007		1.47
expt.	PDG 2014	300(12)	1.48(5)

* using Dashen's theorem ($Q_{\text{Dashen}} = 24.3$)

$$r = \Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$$

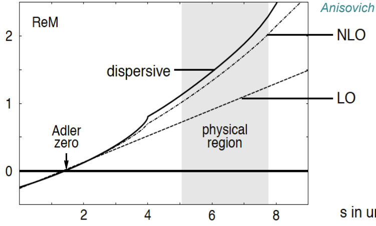
in addition

CHPT amplitudes have problems with measured Dalitz plot distributions (see below)

main deficiency (?)

strong $\pi\pi$ rescattering included only perturbatively

$\longrightarrow$ calls for dispersive treatment



Anisovich & Leutwyler'96

Kambor, Wiesendanger, Wyler 1996

Anisovich, Leutwyler 1996

observations

dispersive effects moderate

Adler zero (along $s = u$) chiral $SU(2)$ prediction

$\longrightarrow$ small higher-order corrections

more recent developments

- all use representation holding up to and including NNLO
  $\longrightarrow$ $\pi\pi$ partial-wave discontinuities for $\ell = 0, 1$ only
  Stern, Sazdjian, Fuchs 1993 ("reconstruction theorem")
  $\rightarrow$ Moussallam, GB-WG
- match to CHPT amplitude to obtain Q from rates

1. Colangelo, Lanz, Leutwyler, Passemar (in progress)

dispersive approach following Anisovich, Leutwyler

- electromagnetic effects to NLO fully taken into account (Ditsche, Kubis, Meißner 2009)
- dispersive amplitudes: Bern $\pi\pi$ phase shifts (effectively) 5 subtraction constants
- absolute magnitude: match to NLO chiral amplitude near center of Dalitz plot (Adler zero satisfied)

2. Schneider, Kubis, Ditsche 2011

2-loop NREFT approach (analogous to $K \to 3\pi$)

- allows investigation of isospin-violating corrections
- relations between charged and neutral Dalitz plots

3. Kampf, Knecht, Novotny, Zdrahal 2011 (and in progress)

- analytic 2-loop representation from general principles
- amplitudes involve 6 parameters (subtraction constants), fitted to Dalitz plot distribution (KLOE 2008: $\eta \to \pi^+\pi^-\pi^0$)
- predict Dalitz plot parameter α (neutral decay mode)
- match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small $\to$ R
- criticized by Bern group: amplitude far away from Adler zero

4. Guo et al. 2015

most recent numerical dispersive analysis (JPAC)

- Madrid/Cracow $\pi\pi$ phase shifts, 3 subtraction constants
- fit experimental Dalitz plot (WASA/COSY 2014: $\eta \to \pi^+\pi^-\pi^0$)
- predict Dalitz plot parameter α
- match to NLO CHPT near Adler zero $\longrightarrow$ Q

definition of Dalitz plot parameters

$$|A_c(s,t,u)|^2 = \Gamma_c(X,Y) = N\left(1 + aY + bY^2 + dX^2 + \dots\right)$$
$$|A_n(s,t,u)|^2 = \Gamma_n(X,Y) = N\left(1 + 2\alpha Z + \dots\right)$$

expansion around center of the Dalitz plot ($X = Y = 0$) $[Q_\eta = M_\eta - \sum_i M_{\pi^i}]$

$$X = \frac{\sqrt{3}}{2M_\eta Q_\eta}(u-t),\quad Y = \frac{3}{2M_\eta Q_\eta}\left((M_\eta - M_{\pi^0})^2 - s\right) - 1,\ Z = (X^2 + Y^2)$$

	$-a$	b	d	α
KLOE 2015	1.095(4)	0.145(6)	0.081(7)	
BESIII 2015	1.128(17)	0.153(17)	0.085(18)	−0.055(15)
WASA/COSY 2014	1.144(18)	0.219(51)	0.086(23)	
NNLO CHPT	1.271(75)	0.394(102)	0.055(57)	0.013(32)
KKNZ				−0.044(4)
NREFT	1.213(14)	0.308(23)	0.050(3)	−0.025(5)
JPAC	1.116(32)	0.188(12)	0.063(4)	−0.022(4)
PDG 2014				−0.0315(15)

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quark mass ratios

$$R = \frac{m_s - m_{ud}}{m_d - m_u}, \quad Q^2 = \left(1 + \frac{m_s}{m_{ud}}\right) R/2$$

	NNLO CHPT	CLLP prel.	KKNZ	JPAC	FLAG 2014 ($N_f = 2+1$)
R	$[36.9]^*$	$[31.9]^*$	37.4(2.2)	$[32.2]^*$	35.8(1.9)(1.8)
Q	22.9	21.3(6)	$[23.1]^*$	21.4(4)	22.6(7)(6)

$[\;\;]^*$ using $\frac{m_s}{m_{ud}}$ from FLAG 2014 ($N_f = 2+1$)

recall: $Q_{\rm Dashen} = 24.3$

$\longrightarrow$ Giovanella, Kolesar, Masjuan, GB-WG

Kaon decays

several studies for $K \to \pi\pi\ell\nu_\ell$ (K_{l4}) decays

main interest

- access to $\pi\pi$ threshold region $\longrightarrow$ $\pi\pi$ scattering lengths
- form factors, LECs, ...

$\longrightarrow$ Knecht, Stoffer, GB-WG

isospin-violating corrections

Cuplov, Nehme 2003: photonic 1-loop corrections

Colangelo, Gasser, Rusetsky 2009:

mass-difference corrections for phases $\longrightarrow$
NA48/2 scattering lengths in agreement with CHPT/Roy equation analysis Colangelo, Gasser, Leutwyler 2000

Stoffer 2014: **complete isospin-violating 1-loop corrections**

- earlier elm. corrections completed, arbitrary cut on photon energy in (semi-)inclusive $K_{\ell 4\gamma}$ decays
- exact matrix element ready for MC simulation (e.g., PHOTOS)
- complete mass effects for form factors, corrections for phases reproduced

Bernard, Descotes-Genon, Knecht 2013:

mass-difference effects on phases beyond 1 loop

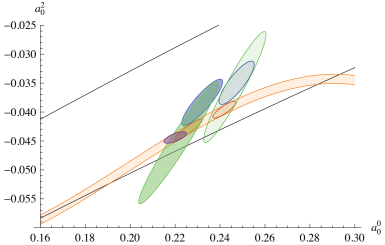
comparison NA48/2 with CHPT/Roy

$$\delta_S(s) - \delta_P(s) = \delta^{S-P}_{\mathrm{Roy}}(s; a_0^0, a_0^2) \quad + \quad \delta^{L=1}_{\mathrm{IB}}(s)$$

Colangelo, Gasser, Rusetsky

$\pi\pi$ rescattering: dependence of $\delta_{\mathrm{IB}}(s)$ on $a_0^0, a_0^2 \longrightarrow$ bias?

explicit dependence $\longrightarrow$ dispersive treatment of $\delta_{\mathrm{IB}}(s; a_0^0, a_0^2)$



small purple ellipse:
Colangelo, Gasser, Leutwyler

other ellipses to the left:
various fits by BDK

ellipses to the right:
without isospin violation

final result (green ellipse)

$a_0^0 = 0.221 \pm 0.018,$ $a_0^2 = -0.0453 \pm 0.0106$ BDK

$a_0^0 = 0.2220(128)_{\rm stat}(50)_{\rm syst}(37)_{\rm th},$ $a_0^2 = -0.0432(86)_{\rm stat}(34)_{\rm syst}(28)_{\rm th}$ NA48/2

isospin breaking in $K^\pm \to \pi^0\pi^0 e^\pm \nu_e$ $\longrightarrow$ Knecht, GB-WG

Colangelo, Passemar, Stoffer 2015:

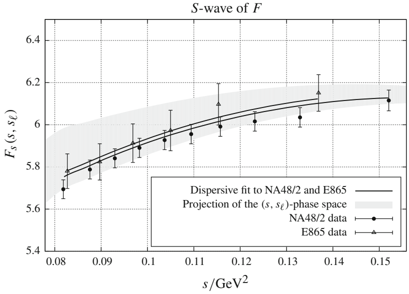
dispersive treatment of $K_{\ell 4}$ decays

NNLO CHPT Amoros, Bijnens, Talavera 2000

standard problem: perturbative treatment of final-state interaction

dispersive treatment

- usual assumption ("reconstruction theorem"): two-particle rescattering with S- and P-waves only
- isospin breaking taken into account
- subtraction constants cannot be determined from data alone
- matching to CHPT at both one- and two-loop levels $\longrightarrow$ values for LECs L_1, L_2, L_3 compatible with fit BE14
- further details $\rightarrow$ Stoffer, GB-WG



unlike NNLO CHPT:
dispersive treatment accounts for measured curvature of
S-wave of form factor *F*

nonleptonic K decays

no recent spectacular developments $\longrightarrow$

Cirigliano et al., Kaon decays in the SM, RMP 84 (2012) 399

essentially up-to-date

related recent work

D'Ambrosio, Greynat, Vulvert 2014, SM and new physics contributions to $K_{L,S} \to 4$ leptons

Crewther, Tunstall 2015, $\Delta I = 1/2$ rule for kaon decays from QCD infrared fixed point $\longrightarrow$ Tunstall, poster session

Buras et al. 2015, $K^+ \to \pi^+ \nu \overline{\nu}$ and $K_L \to \pi^0 \nu \overline{\nu}$ in the SM: status and perspectives

Bai et al. (RBC/UKQCD) 2015, SM prediction for direct CP violation in $K \to \pi\pi$ decay $\longrightarrow$ Garron, GB-WG

Conclusions

main objectives of CHPT

- understand physics of the SM at low energies
- look for evidence of new physics

objectives accomplished?

- we have come quite some way in understanding the SM at low energies, but there is plenty of room for improvement
- on the other hand: we have not found any evidence for new physics
- but neither has the LHC !

Spare slides

Anti-correlation between F_0 and L_4

global fits exhibit strong anticorrelation between F_0 and L_4

$F_0 = \lim_{m_u,m_d,m_s\to 0} F_\pi$ Bijnens,Jemos,GE

$$\mathcal{L}_{p^2}(2) = \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle$$

$$\mathcal{L}_{p^4}(10) = \cdots + L_4\langle D_\mu U D^\mu U^\dagger\rangle\langle \chi U^\dagger + \chi^\dagger U\rangle + \ldots$$

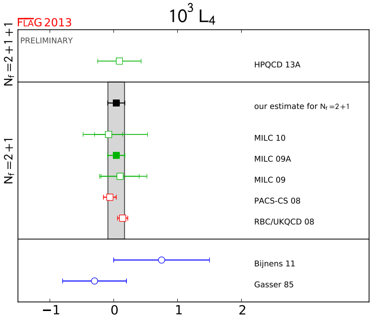
$$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}(10) = \frac{1}{4}\langle D_\mu U D^\mu U^\dagger\rangle\left[F_0^2 + 8L_4\left(2\mathring{M}_K^2 + \mathring{M}_\pi^2\right)\right] + \ldots$$

$\langle \ldots \rangle$ flavour trace, $\chi = 2B_0\mathcal{M}_q$ ($B_0 \sim$ quark condensate)

$\mathring{M}_P$ lowest-order meson mass

$F_\pi^2/(16M_K^2) = 2\times 10^{-3}$ $\sim$ typical size of NLO LEC

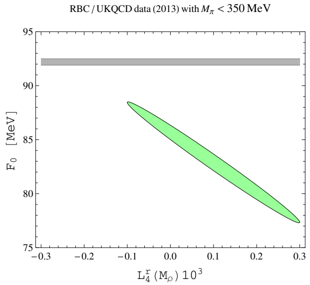
- "convergence" of SU(3) CHPT depends (also) on value of F_0
  chiral expansion: powers of $p^2/(4\pi F_0)^2$
  but F_0 less well known than many higher-order LECs
- rather wide spread in F_0 also from lattice studies
- FLAG 2013/14: published lattice determinations for $L_4^r(M_\rho)$



suggestion — GE, Masjuan, Neufeld 2014

determine F_0, L_4 from $SU(3)$ lattice data for F_π
employ large-N_c motivated approximation for 2-loop calculation
(requires tree- and 1-loop amplitudes only)

advantage — anti-correlation can be softened by tuning quark masses
$\longrightarrow$ smaller errors for F_0, L_4



RBC/UKQCD data (2013)

$F_0 = (82.9 \pm 5.6)$ MeV

$L_4^r(M_\rho) = (0.10 \pm 0.20) \cdot 10^{-3}$

$\mathrm{corr}(F_0, L_4) = -0.991$

persisting anti-correlation between F_0, L_4 despite lattice data

$\longrightarrow$ could be improved by lowering $m_s \sim M_K^2$