Extracting (pion) form factors from finite volume lattices

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Hidenori Fukaya
and Takashi Suzuki (Osaka Univ.)
for JLQCD collaboration

What I would like to talk

E-mail from organizers :

"We would like to invite you to give a talk in a plenary session of the conference about the recent lattice investigations of the pion and nucleon form factors"

pion form factor [review + our work]

$$\langle \pi(p') | J^\mu_{em}(q) | \pi(p) \rangle = F_\pi(q^2)(p+p')^\mu$$

nuculeon form factor [review]

$$\langle N(p',s') | V^\mu_{\rm em}(0) | N(p,s) \rangle =$$

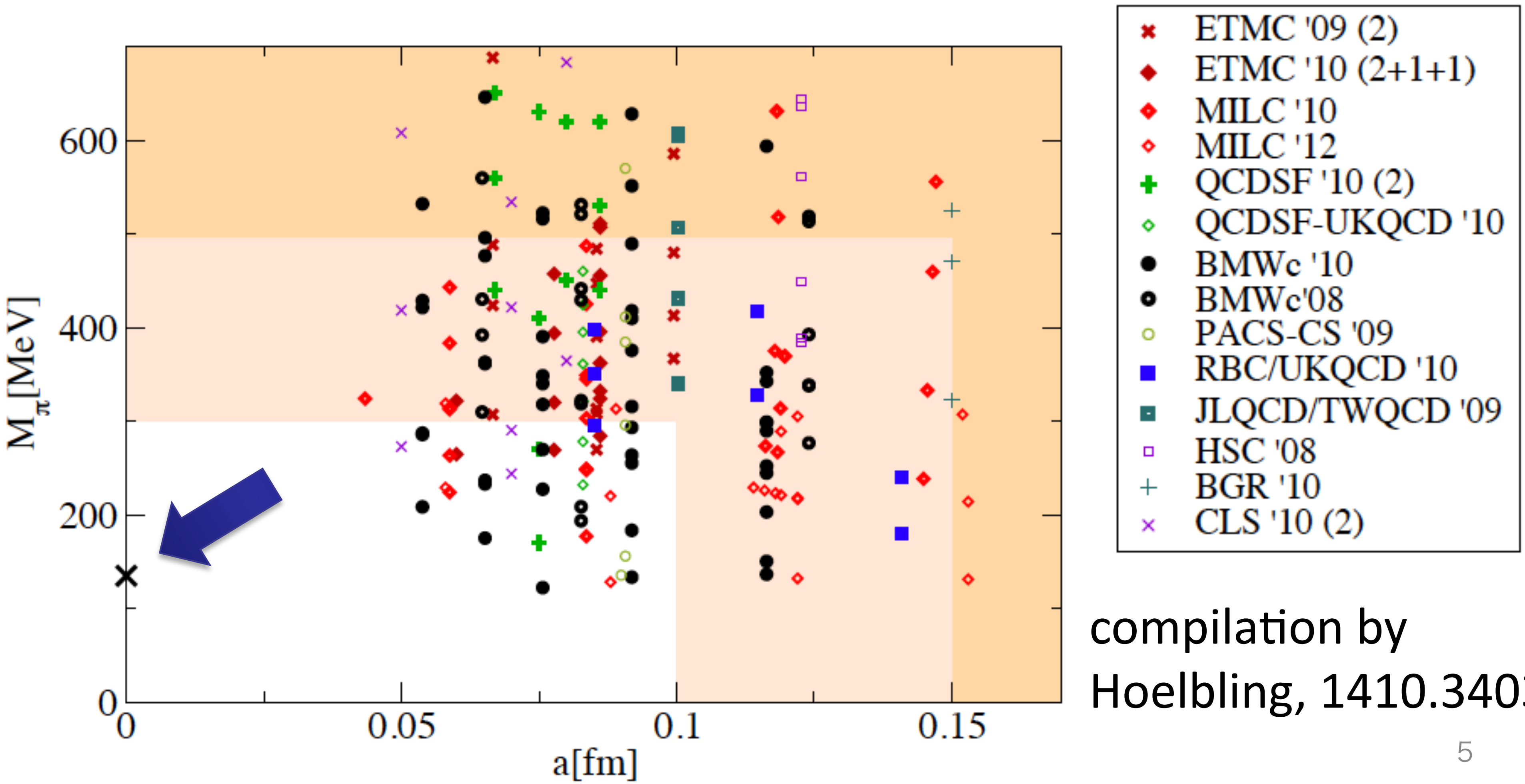
$$\bar{u}(p',s') \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_{\rm N}} F_2(q^2) \right] u(p,s)$$

CONTENTS

1. Recent lattice results (review)

Current status of lattice QCD

Reaching to physical & continuum limits



compilation by
Hoelbling, 1410.3403

New results after CD2012

Pion form factors

Lattice

Mainz [Brandt et al. 1301.3513]
HPQCD [Koponen et al. 1311.3513]
JLQCD [1405.4077] -> this talk
X. Feng et al. 1412.6319
CSSM [Owen et al. 1501.02561]

ChPT (on lattice systematics)

Bijnens & Relefors 1402.1385
Tiburzi 1407.4059
F & Suzuki 1409.0327 -> this talk

What's new ?

(partially) twisted boundary method becomes standard.

More studies on finite V effects (including twisted boundary)

Partially twisted boundary

Periodic boundary

p is discrete :

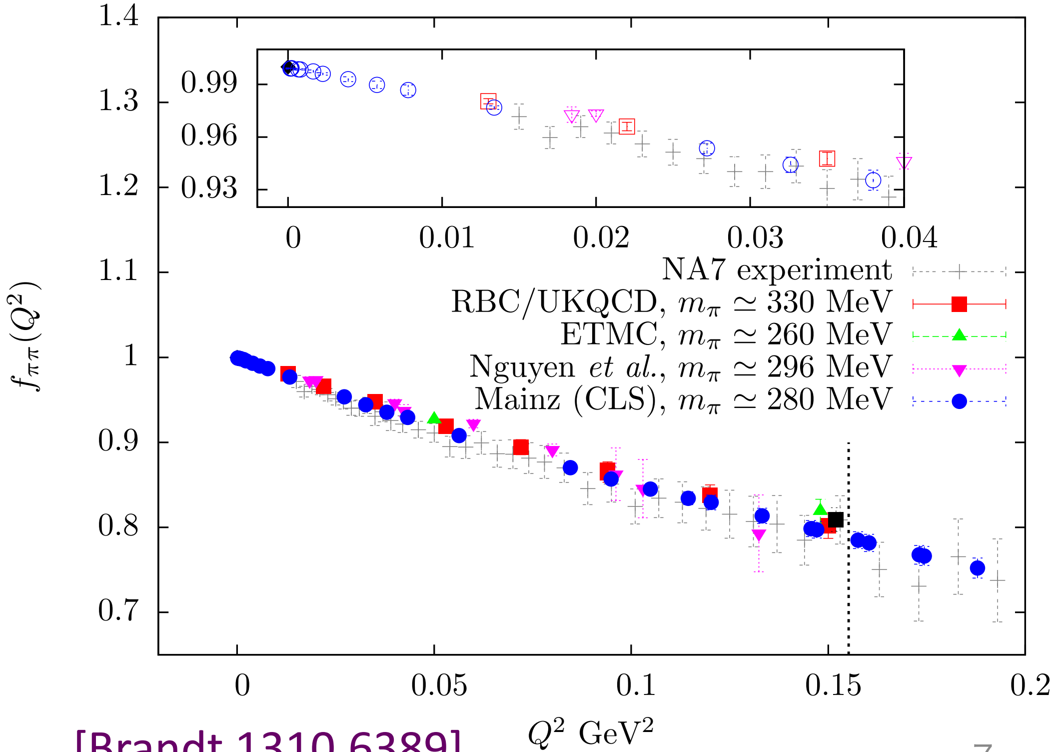
$$p = \frac{2\pi n}{L}$$

$\rightarrow$ small momentum transfer is difficult.

Twisted boundary

$$q_v(x + L_\mu) = e^{i\theta_\mu / L} q_v(x)$$

$$\rightarrow \quad p_{\min} = \theta_\mu / L$$



[Brandt 1310.6389]

Any artifact on twisting ?

Bijnens and Relefors 1402.1385

[talk by Relefors Monday]

Twisted boundary breaks (cubic part of) Lorentz inv. and reflection symmetry.

→ new terms having unusual p dep.

→ EM form factor can have sizable effect (as a part of finite V effect).

New results after CD2012

Nucleon form factors

Lattice: ETMC 1303.5979

PNDME 1306.5435, 1506.04196, 1506.06411

RBC/UKQCD 1401.1476

CSSM/QCDSF/UKQCD

1401.5862,1403.1965

Engelhardt et al. 1404.4029

Capitani et al. 1504.04628

What's new ?

Removing effects of excited states is important.

Effective theory:

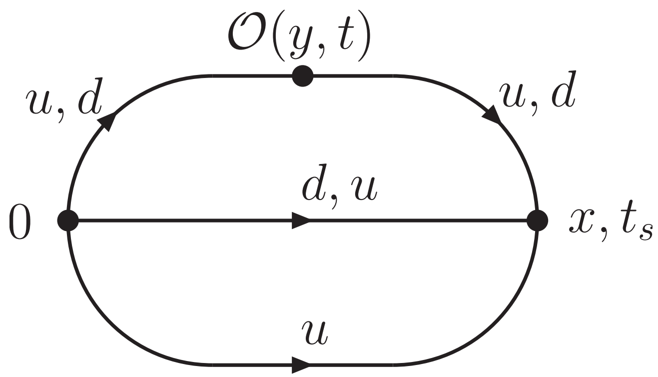
-> talk by Tiburzi

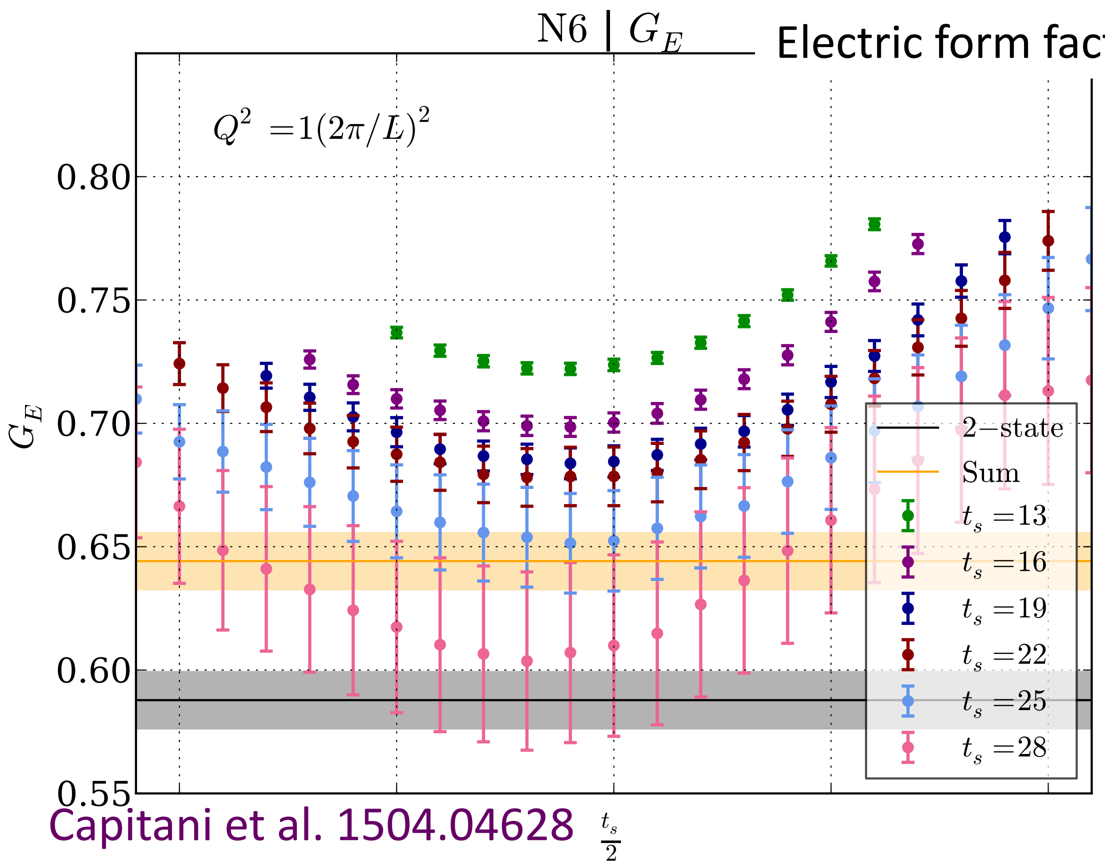
Tuesday,Hadron structure & Meson-Baryon interactions

Looking for a plateau

Long separation -> noisy.

Short separation -> no good plateau.





Capitani et al. 1504.04628

Summation method
~ linear extrapolation
to $t_s = \infty$

Summary of recent results (except for JLQCD)

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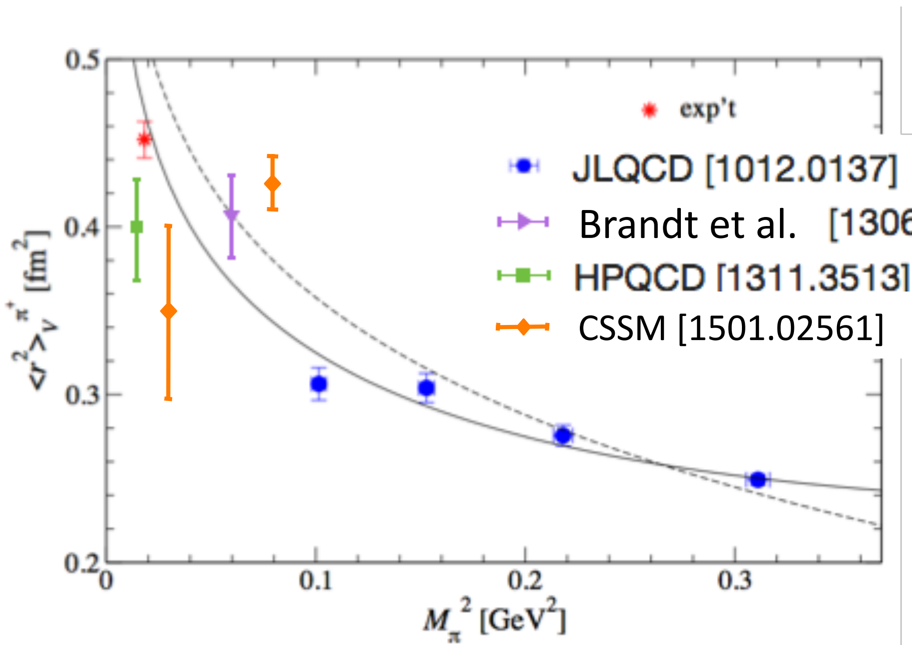
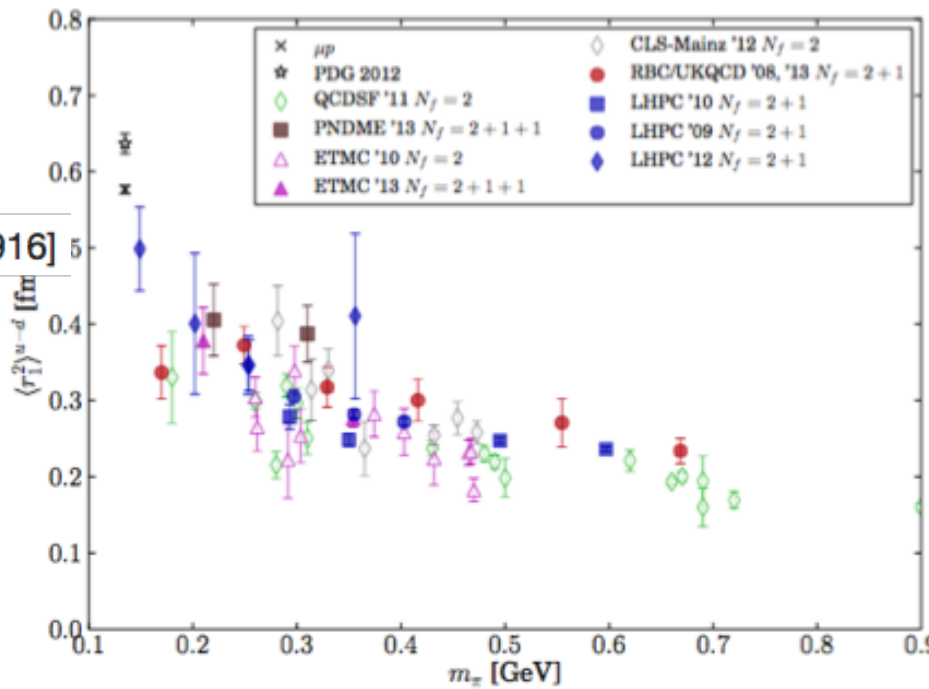
Pion's charge radius



Nucleon's isovector radius



S. Syritsyn[arXiv: 1403.4686]

Closer to the physical point, but charge radii still look under-estimated.

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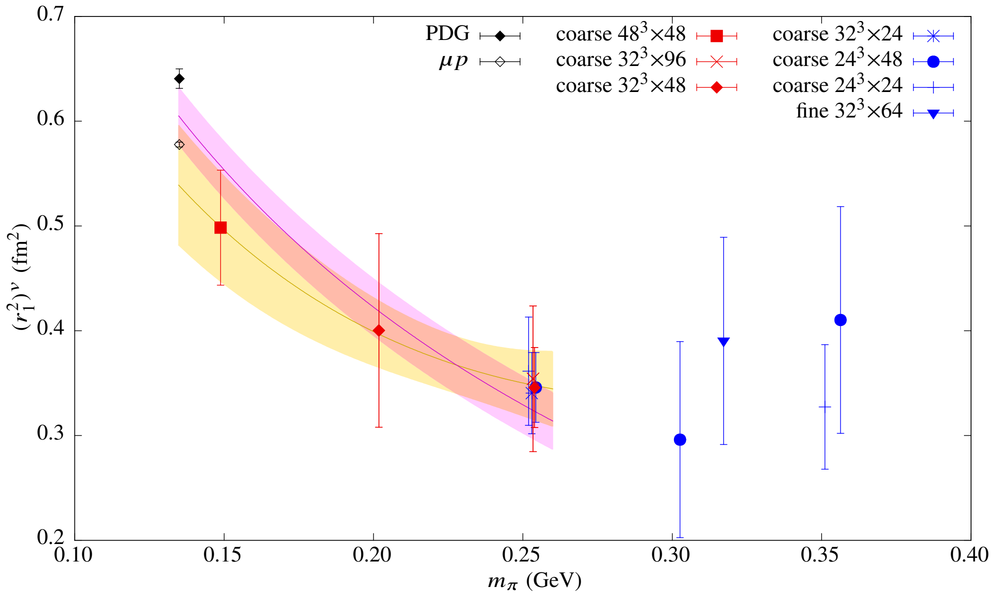
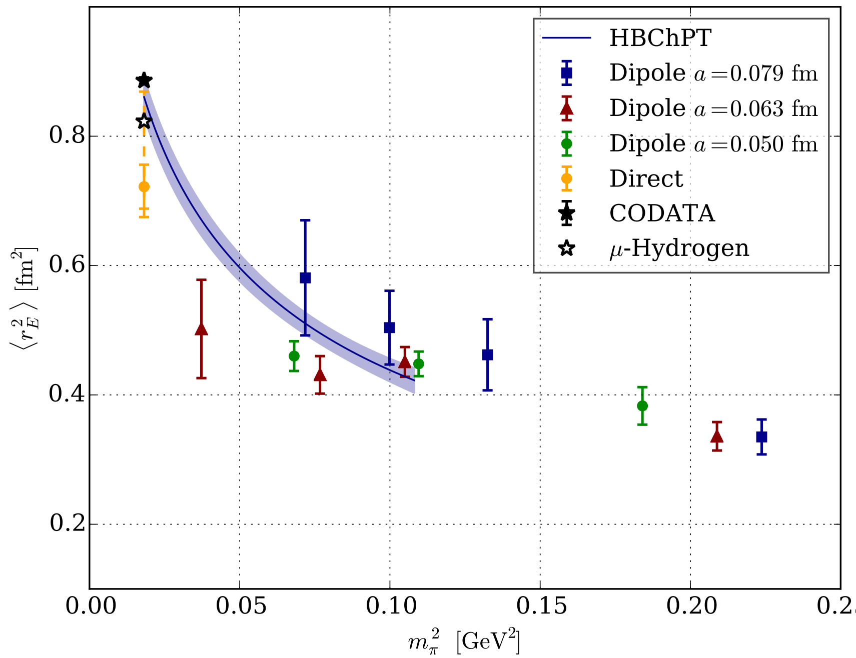
Summary of recent results (except for JLQCD)

Nucleon's isovector radius



Engelhardt et al. 1404.4029

Nucleon's electric radius



Capitani et al. 1504.04628

Closer to the physical point, but charge radii still look under-estimated.

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2. Why hadron radii under-estimated?

Many reasons…

Any unphysical scale distorts

$$\ln M_\pi^2 \to \ln(M_\pi^2 + \delta m^2)$$

Possible sources :

Heavier simulated quarkmass $\delta m^2 = \Delta M_\pi^2$

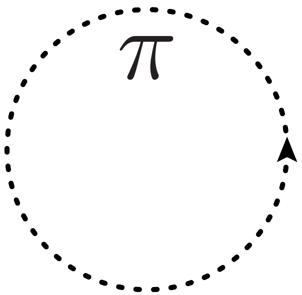
cut-off effect (Wilson term etc.) $\delta m^2 = \Lambda_{\rm QCD}^3 a$

finite V effect $\delta m^2 = 1/L^2$

Note : it is unlikely to have negative δm^2

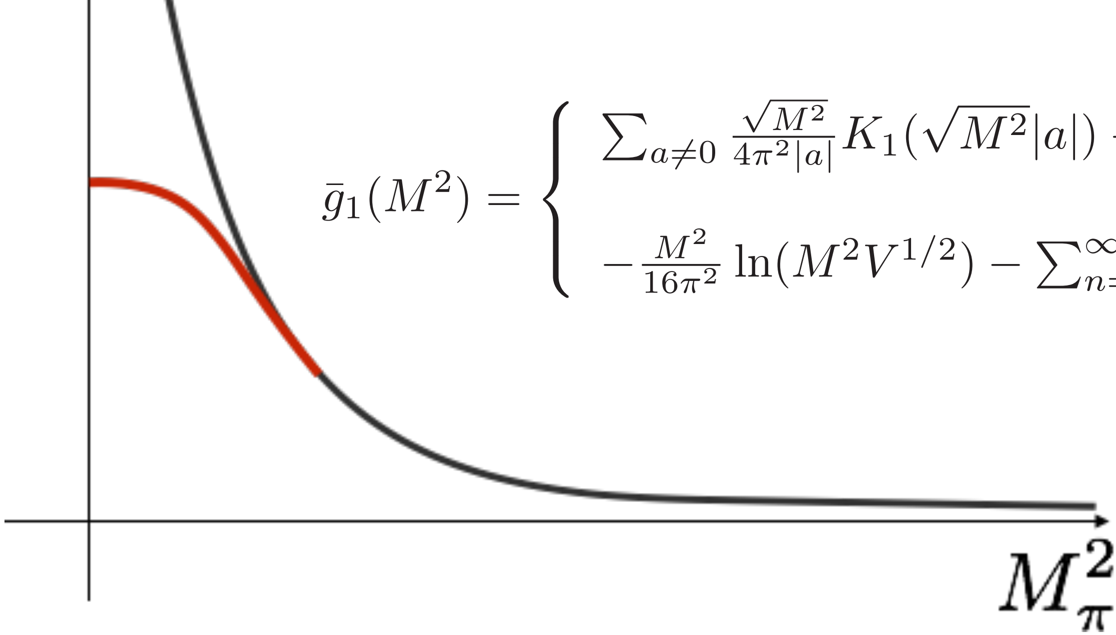
Beautiful example : finite V

ε expansion by Gasser & Leutwyler 1987



$$= \frac{M_\pi^2}{16\pi^2} \ln \frac{M_\pi^2}{\mu_{sub}^2} + \bar{g}_1(M_\pi^2)$$

$$\bar{g}_1(M^2) = \begin{cases} \sum_{a\neq 0} \frac{\sqrt{M^2}}{4\pi^2|a|} K_1(\sqrt{M^2}|a|) - \frac{1}{M^2 V} & (|M|L > 1) \\ -\frac{M^2}{16\pi^2}\ln(M^2 V^{1/2}) - \sum_{n=1}^{\infty} \frac{\beta_n}{(n-1)!} M^{2(n-1)} V^{(n-2)/2} & (|M|L \leq 1) \end{cases}$$



What we need

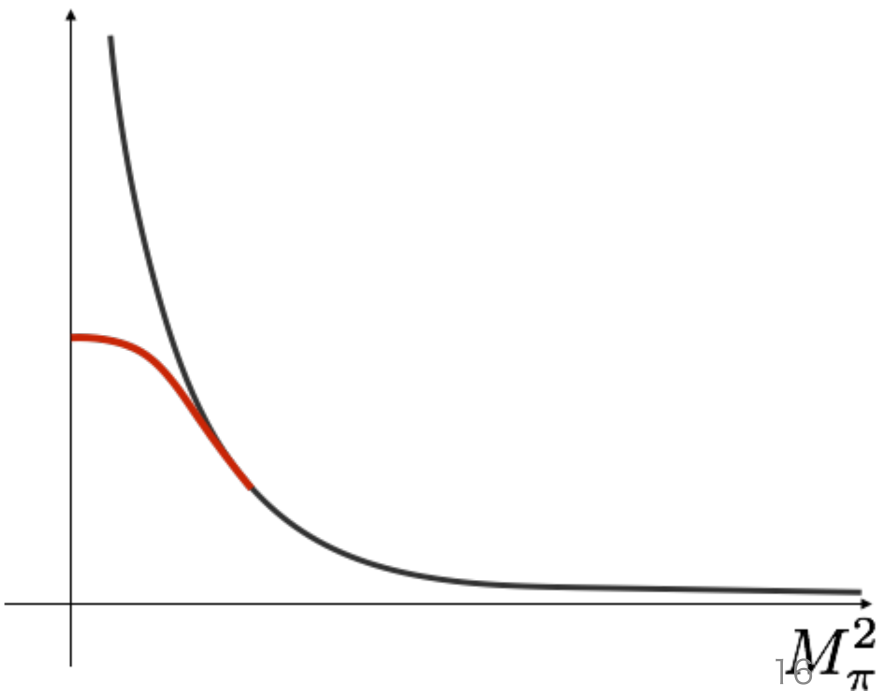
To avoid $\ln M_\pi^2 \to \ln(M_\pi^2 + \delta m^2)$

in the form factor computations,

we need to control

1. Chiral symmetry
2. Cut-off effect
3. Finite V effect

at the same time.



Our strategy

JLQCD's chiral fermion project

2006-2012 : dynamical overlap quarks

L~1.8fm, 1/a~1.8GeV, m_π~100MeV (ε regime)

2012-present : Mobius domain-wall quarks

L~2.4-4fm, 1/a~2.4-4GeV, m_π~220MeV

😀 Good chiral symmetry

😀 O(a) error automatically removed.

😥 Small V → our main concern.

3. How to remove finite volume effect

Finite volume = pion physics

Correlation length (1/M) of QCD particles

Pion(∼140MeV) ∼ 1.4fm

Kaon(∼500MeV)∼0.4fm

rho (∼800MeV)∼0.26fm

proton (∼1GeV) ∼0.2fm

$$e^{-M_\pi L} = 0.03\text{–}0.25$$

$$e^{-m_K L} \sim 0.007$$

$$e^{-M_\rho L} < 0.0005$$

$$(\text{for} \;\; 2\text{fm} < L < 4\text{fm})$$

Finite volume correction in

QCD = pion theory (chiral perturbation theory)

weakly coupled = analytically predictable.

How large should our lattice be ?

FLAG (flavor lattice averaging group)'s criterion [FLAG 2013]

- Finite-volume effects:
  - ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 volumes
  - ○ $M_{\pi,\mathrm{min}}L > 3$ and at least 2 volumes
  - ■ otherwise

Green star = $e^{-m_\pi L} \sim 0.018.$

For physical pion mass, L > 5.8 fm.

Zero momentum mode dominates.

For non-zero momentum pion mode,

$$e^{-\sqrt{m_\pi^2+\mathbf{p}^2}L} \leq e^{-\sqrt{m_\pi^2+(2\pi/L)^2}L} \leq e^{-2\pi} = 0.0019$$

(* This is true even in the chiral limit.)

$$\ll e^{-m_\pi L} \quad !$$

IF we could remove the effect of zero-mode, L could be smaller.

Can we remove pion's zero-mode ?

Zero-mode has no space-time dependence.

$$f(x) = A + Bg(x)$$

Two mathematical tools to remove them :

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Two mathematical tools to remove them :

1. Subtraction :

$$f(x) - f(x_{\rm ref}) = B(g(x) - g(x_{\rm ref}))$$

Can we remove pion's zero-mode ?

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Zero-mode has no space-time dependence.

$$f(x) = A + Bg(x)$$

Two mathematical tools to remove them :

1. Subtraction :

$$f(x) - f(x_{\mathrm{ref}}) = B(g(x) - g(x_{\mathrm{ref}}))$$

2. Division :

$$\frac{f(x) - f(x_0)}{f(x_1) - f(x_0)} = \frac{g(x) - g(x_0)}{g(x_1) - g(x_0)}$$

ε expansion of ChPT

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What is the ε expansion ? [Gasser & Leutwyler 1987]

p expansion : every mode is perturbative

$$U(x) = 1 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right), \quad \in SU(N_f)$$

epsilon expansion :

$$U(x) = U_0 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right),$$

U_0 non-perturbatively treated.

(good for expansion when $M_\pi L < 1$)

ε expansion of ChPT

Pion theory at finite V

$$\mathcal{L} = -\frac{\Sigma}{2}\mathrm{Tr}\left[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right] + \frac{1}{2}\mathrm{Tr}(\partial_\mu \xi)^2 + \frac{\Sigma}{2F^2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 \xi^2 + \xi^2 U_0^\dagger \mathcal{M}] + \cdots,$$

Zero-mode = SU(N) (or U(N)) matrix model

Non-zero-mode = massless bosons

(perturbative) interactions

= hybrid system of
matrix model and bosonic fields

Let us compute 2pt function.

$$\langle P(x)P(0)\rangle = A + B\sum_{p'\neq 0}\frac{1}{V}\frac{e^{ip'x}}{p'^2} + C\sum_{p'\neq 0}\frac{e^{ip'x}}{p'^4} + \cdots$$

$A, B, C \cdots$: zero-mode's effect
(Bessel functions of $m\Sigma V$)

$\sum_{p'\neq 0}(\cdots)$: non-zero mode's

Subtraction

Constant A is eliminated by

$$\langle P(x)P(0)\rangle - \langle P(x_0)P(0)\rangle = B \sum_{p' \neq 0} \frac{1}{V} \frac{e^{ip'x} - e^{ip'x_0}}{p'^2} + \cdots$$

Another subtraction

Fourier transform ~ subtraction(s)

After d³x integral with $\mathbf{p} \neq 0$

$$\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle = \frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x} A + B\sum_{p'_0}\frac{1}{V}\frac{e^{-ip'_0 t}}{p'^2_0 + \mathbf{p}^2} + \cdots$$

$$= 0 + \frac{B}{2E(\mathbf{p})L^3\sinh(E(\mathbf{p})T/2)}\cosh(E(\mathbf{p})(t - T/2)) + \cdots ,$$

where $E(\mathbf{p}) = |\mathbf{p}|$.

the same form as the p-expansion

But zero-mode's effects still contained in B .

Division

Ratio of 2pt functions at different p's

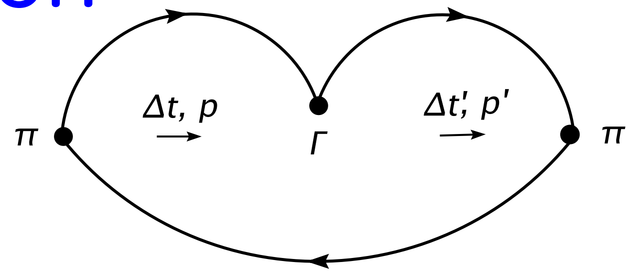
$$\frac{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle}{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}'x}\langle P(x)P(0)\rangle} = \frac{\cosh(E(\mathbf{p})(t-T/2))}{\cosh(E(\mathbf{p}')(t-T/2))} + \cdots,$$

Dominant finite V effect is eliminated !

NLO $\sim \dfrac{1}{4\pi F^2 L^2}$

3pt functions at finite V

PVP correlator in ε expansion



$$C^{\mathrm{3pt}}_{PV_0P}(\Delta t, \Delta t'; \mathbf{p}_i, \mathbf{p}_f)$$
$$= B^{\mathrm{3pt}}(m\Sigma V)\left[E(\mathbf{p}_i) + E(\mathbf{p}_f)\right] F_V(q^2)$$
$$\times \cosh(E(\mathbf{p}_i)(\Delta t - T/2)) \cosh(E(\mathbf{p}_f)(\Delta t' - T/2))^{(conn)} + \cdots$$

$$R_V(t, t'; |\mathbf{p}_i|, |\mathbf{p}_f|, q^2) \equiv \frac{\frac{1}{N^{\mathrm{3pt}}_{|\mathbf{p}_i|,|\mathbf{p}_f|}} \sum_{\mathrm{fixed}|\mathbf{p}_i|,|\mathbf{p}_f|,q^2} \frac{C^{\mathrm{3pt}}_{PVP}(t, t'; \mathbf{p}_i, \mathbf{p}_f)}{E(\mathbf{p}_i) + E(\mathbf{p}_f)}}{\left(\frac{1}{N^{\mathrm{2pt}}_{|\mathbf{p}_i|}} \sum_{\mathrm{fixed}|\mathbf{p}_i|} C^{\mathrm{2pt}}_{PP}(t, \mathbf{p}_i)\right)\left(\frac{1}{N^{\mathrm{2pt}}_{|\mathbf{p}_f|}} \sum_{\mathrm{fixed}|\mathbf{p}_f|} C^{\mathrm{2pt}}_{PP}(t', \mathbf{p}_f)\right)}$$

$$= B^{\mathrm{3pt/2pt}}(m\Sigma V) F_V(q^2) + \cdots$$

Subtraction

Correlator with zero momentum

The constant term can be cancelled by

$$\Delta_{t'} C^{\rm 3pt}_{PVP}(t,t';\mathbf{p}_i,0) \equiv C^{\rm 3pt}_{PVP}(t,t';\mathbf{p}_i,0) - C^{\rm 3pt}_{PVP}(t,t_{\rm ref};\mathbf{p}_i,0)$$

$$\begin{aligned}\Delta_t \Delta_{t'} C^{\rm 3pt}_{PVP}(t,t';0,0) \equiv\ & C^{\rm 3pt}_{PVP}(t,t';0,0) - C^{\rm 3pt}_{PVP}(t,t_{\rm ref};0,0) \\ & - C^{\rm 3pt}_{PVP}(t_{\rm ref},t';0,0) + C^{\rm 3pt}_{PVP}(t_{\rm ref},t_{\rm ref};0,0)\end{aligned}$$

Division 1

Correlator with zero momentum

The both ratios

$$R_V^1(t,t';|\mathbf{p}_i|,0,q^2) \equiv \frac{\frac{1}{N^{3\mathrm{pt}}_{|\mathbf{p}_i|}}\sum_{\mathrm{fixed}|\mathbf{p}_i|,q^2}\left(C^{3\mathrm{pt}}_{PVP}(t,t';\mathbf{p}_i,0) - C^{3\mathrm{pt}}_{PVP}(t,t_{\mathrm{ref}};\mathbf{p}_i,0)\right)}{\frac{1}{N^{2\mathrm{pt}}_{|\mathbf{p}_i|}}\sum_{\mathrm{fixed}|\mathbf{p}_i|}C^{2\mathrm{pt}}_{PP}(t,\mathbf{p}_i)\left[-\Delta_{t'}\partial C^{2\mathrm{pt}}_{PP}(t',0) + E(\mathbf{p}_i)\Delta_{t'}C^{2\mathrm{pt}}_{PP}(t',0)\right]},$$

$$R_V^2(t,t';0,0,0) \equiv \frac{\Delta_t\Delta_{t'}C^{3\mathrm{pt}}_{PVP}(t,t';0,0)}{-\Delta_t C^{2\mathrm{pt}}_{PP}(t,0)\Delta_{t'}\partial C^{2\mathrm{pt}}_{PP}(t',0) - \Delta_t\partial C^{2\mathrm{pt}}_{PP}(t,0)\Delta_{t'}C^{2\mathrm{pt}}_{PP}(t',0)},$$

$$= B^{3\mathrm{pt}/2\mathrm{pt}}(m\Sigma V)F_V(q^2) + \cdots$$

Division 2

Ratio of 3pt functions

$$F_V(t,t',q^2) \equiv \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V^2(t,t';0,0,0)} \left(\text{or} \quad \frac{R_V^1(t,t';|\mathbf{p}_i|,0,q^2)}{R_V^2(t,t';0,0,0)} \right)$$

$$= F_V(q^2) + \mathcal{O}\left(\frac{1}{4\pi F^2 L^2}\right)$$

LO finite V effect is eliminated !

(

Cf. in the p regime, this ratio method is conventionally used
for canceling the smearing effect, renormalization and so on.

$$F_V(t,t';q^2) = \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V(t,t';0,0,0)}$$

$$= F_V(q^2) + \mathcal{O}\left(e^{-m_\pi L}\right)$$

)

[Hashimoto et al. 2000]

4. Results by JLQCD collaboration

Our simulations

JLQCD (& TWQCD) project [2006-2014]

= QCD with overlap quarks (exact chiral sym.).

1/a ~ 1.8 GeV, L ~ 1.8 fm

p regime lattices : m_π = 290-780 MeV

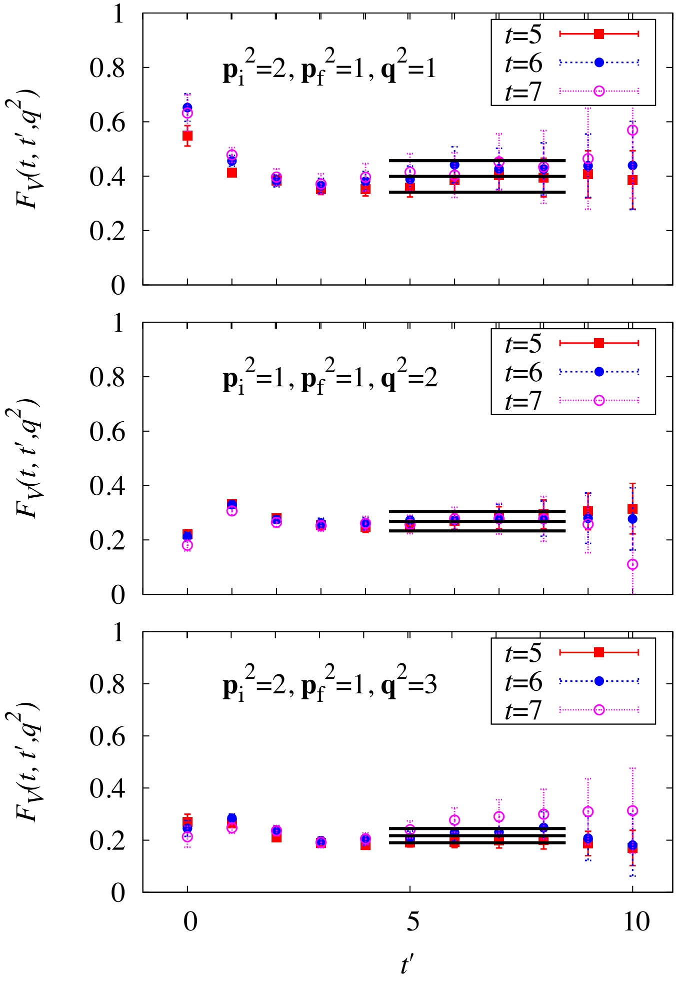
ε regime lattice : m_π ~ 100 MeV

$m_\pi L \sim 0.9$ → Large finite V effects

2pt/3pt correlators use
all-to-all propagator
+ low-mode averaging
-> noise reduction.

Data with subtaction/division trick
[JLQCD 1405.4077]

$$F_V(t,t',q^2) \equiv \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V^2(t,t';0,0,0)}$$



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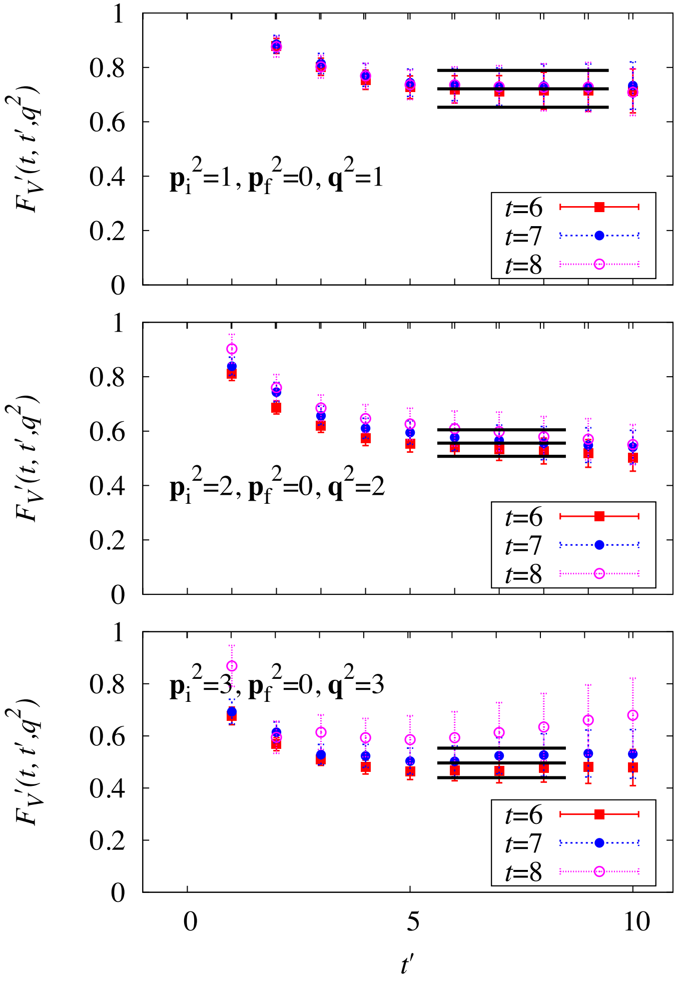
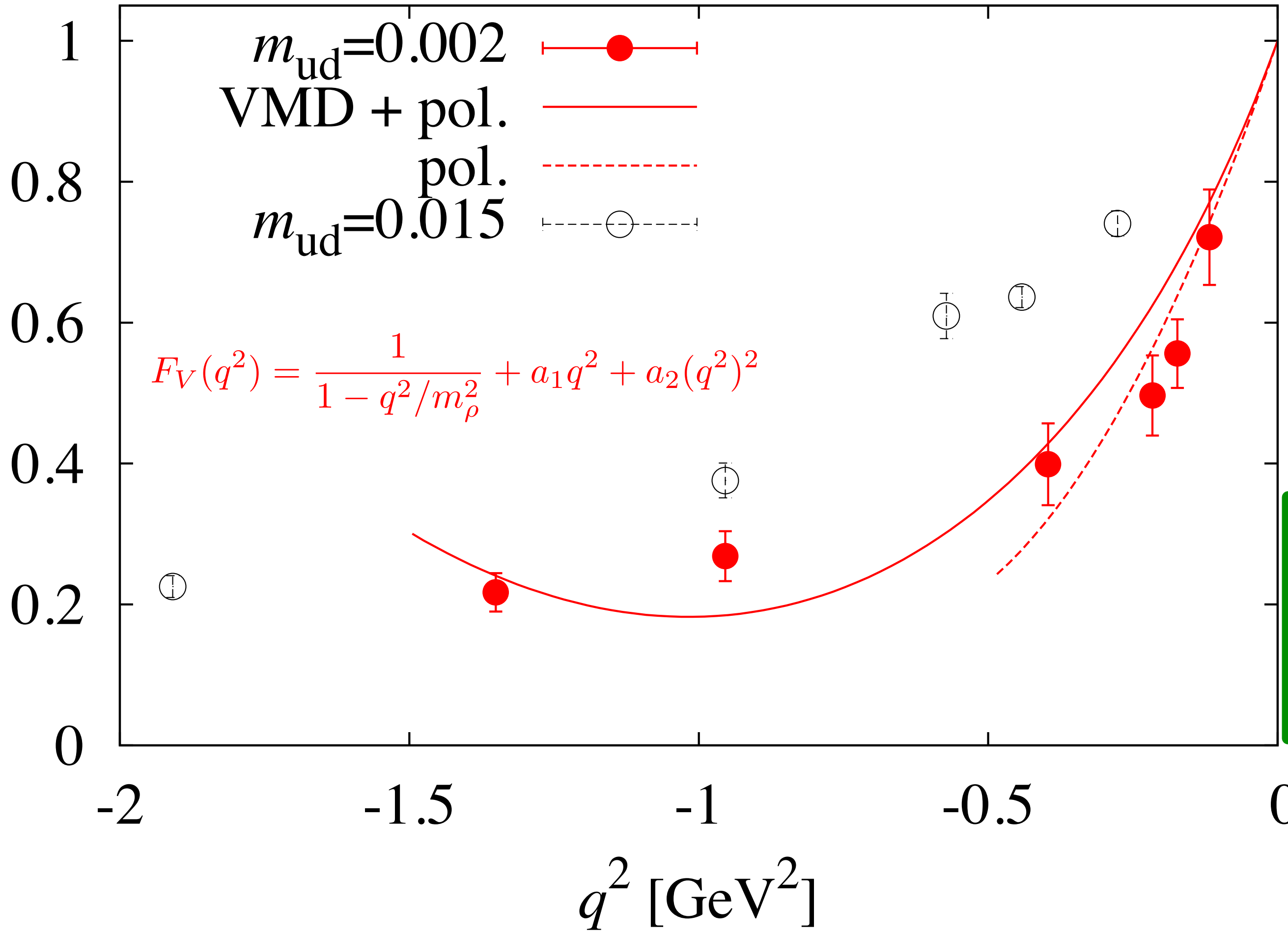


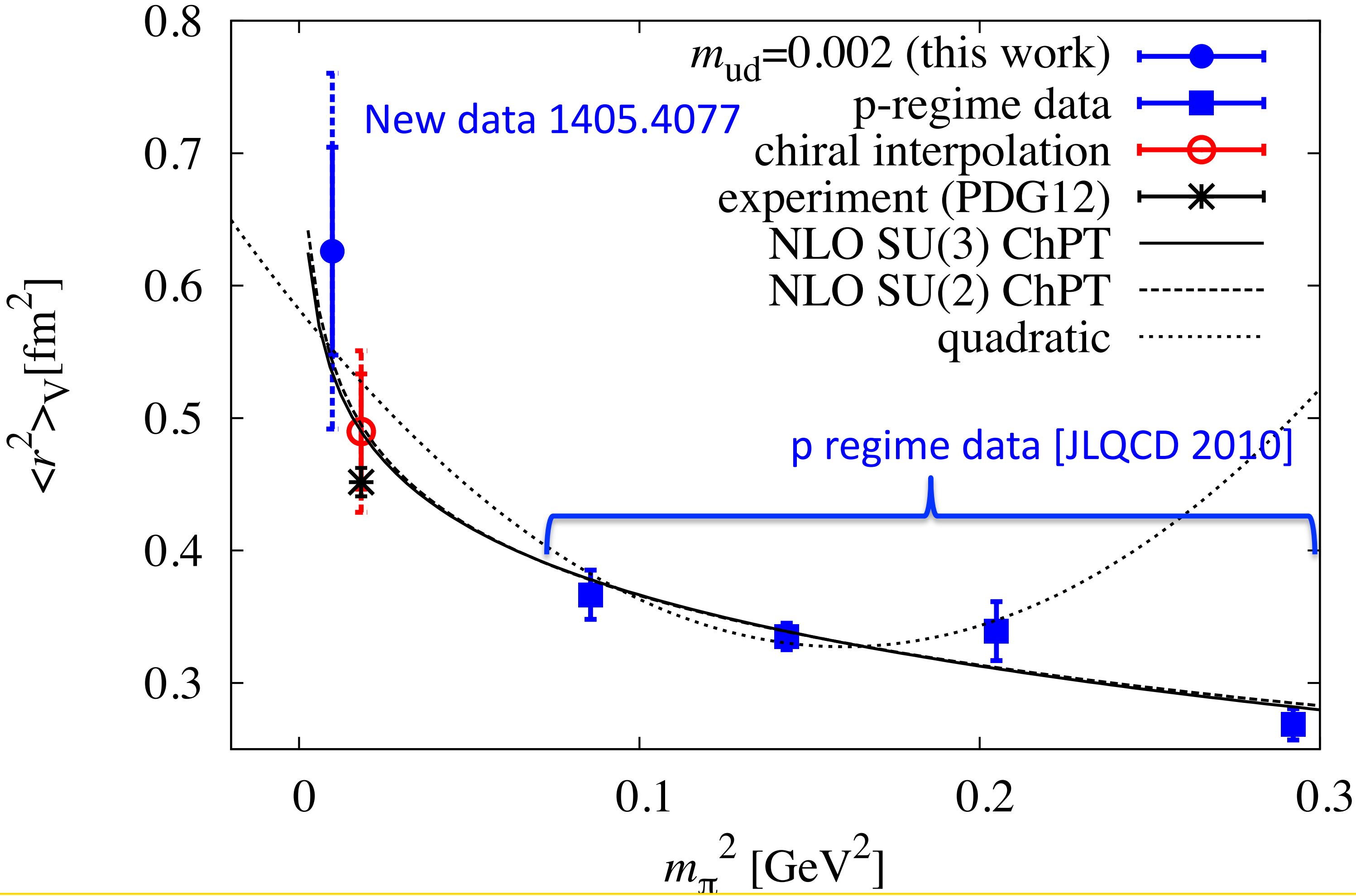
Vector (EM) form factor
[JLQCD 1405.4077]

$F_V(q^2)$



$$\langle r^2 \rangle_V = 6 \frac{\partial F_V(q^2)}{\partial q^2}\bigg|_{q^2=0} = 0.53(4)\ \text{fm}^2.$$

Charge radius

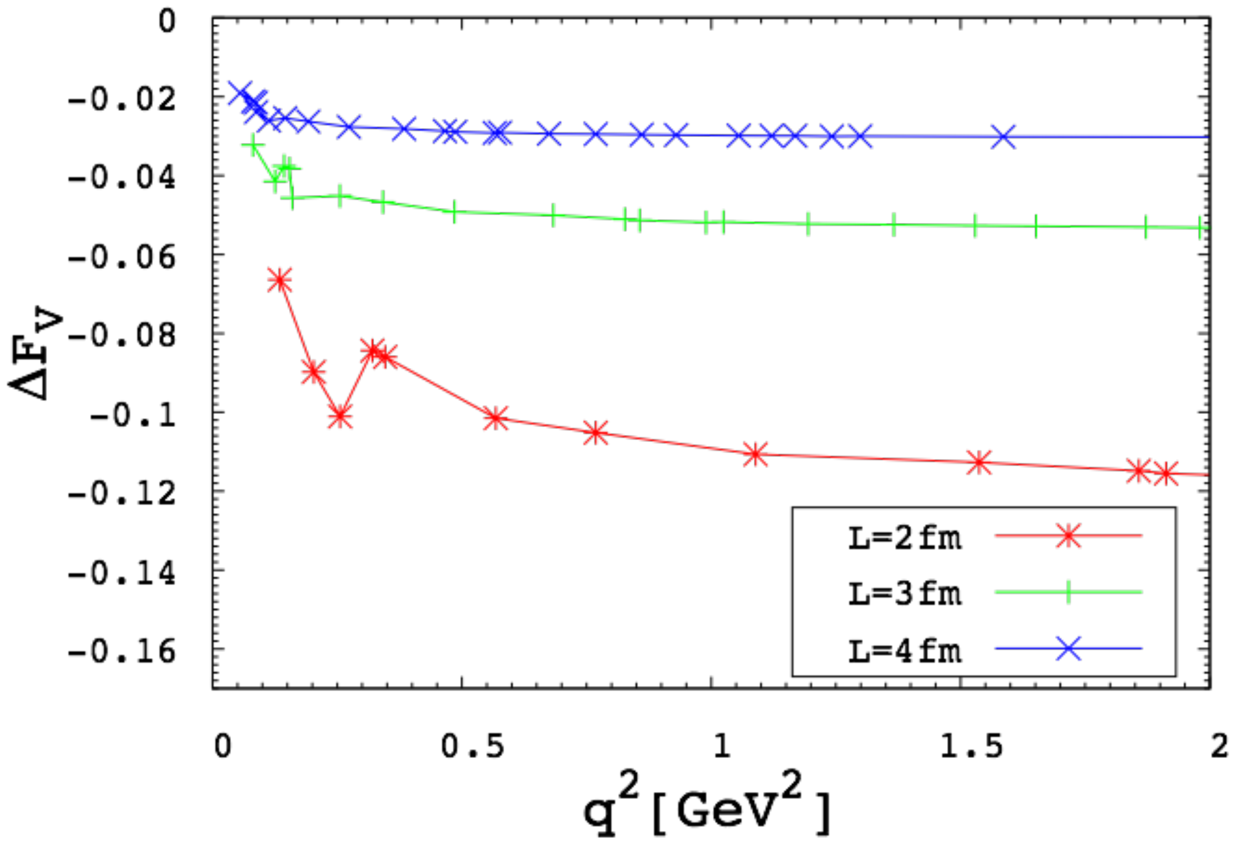


Our data at lightest m_{ud} is greater than the PDG value !

NLO corrections

NLO corrections in ChPT [HF & T.Suzuki 1409.0327]

On our L~2fm lattices, corrections $\sim \frac{1}{4\pi F^2 V^{1/2}} \sim 7\% - 12\%$



$$\Delta F_V = F_V^{\text{finite}} - F_V^{\infty}$$

Cf. for L~5.8 fm,

$$e^{-m_\pi L} \sim 0.018.$$

5. Summary and discussion

Summary

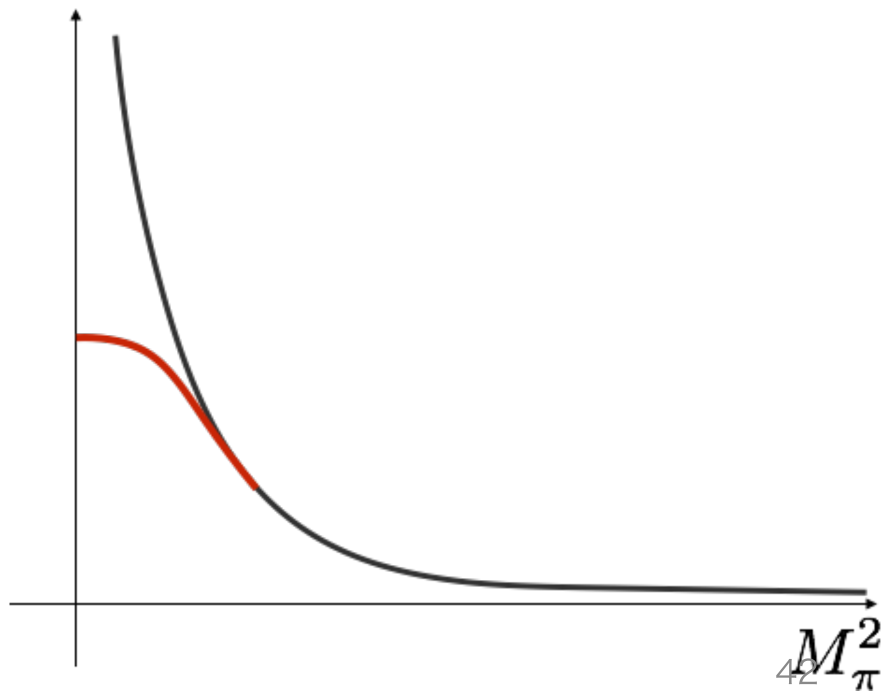
To avoid $\ln M_\pi^2 \to \ln(M_\pi^2 + \delta m^2)$

in the form factor computations,

we need to control

1. Chiral symmetry
2. Cut-off effect
3. Finite V effect

at the same time.



Summary

JLQCD's project

1. Chiral symmetry-> overlap(DW) quarks
2. Cut-off effect -> automatic O(a) improvement
3. Finite V effect-> find insensitive combinations using ChPT:
 cancellation of zero-mode.

$$e^{-\sqrt{m_\pi^2+(2\pi/L)^2}L} \ll 0.01 \ll e^{-m_\pi L}$$

JLQCD's new project

Simulations on bigger & finer lattices started.

Computers @KEK: SR11000 (2 TFLOPS) + BG/L (57 TFLOPS)

→ SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)

Lattice cut-off : 1.8 GeV → 2.4, 3.6, 4.2 GeV

Lattice size : 16^3 x48 → 32^3 x64, 48^3x96, 64^3x128

(Physical size : 1.8 fm → 2.6 fm ~ 4 fm)

Fermion action : overlap fermion→ (Mobius) DomainWall

Pion mass : 200-400 MeV

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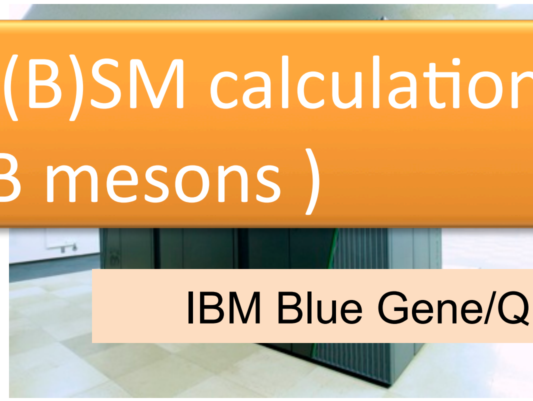
(Physical size : 1.8 fm → 2.6 fm ~ 4 fm)

Fermion action : overlap fermion→ (Mobius) DomainWall

Pion mass : 200-400 MeV

Our goal = 1 % precision of (B)SM calculations
(in particular, D & B mesons)

Hitachi SR16000

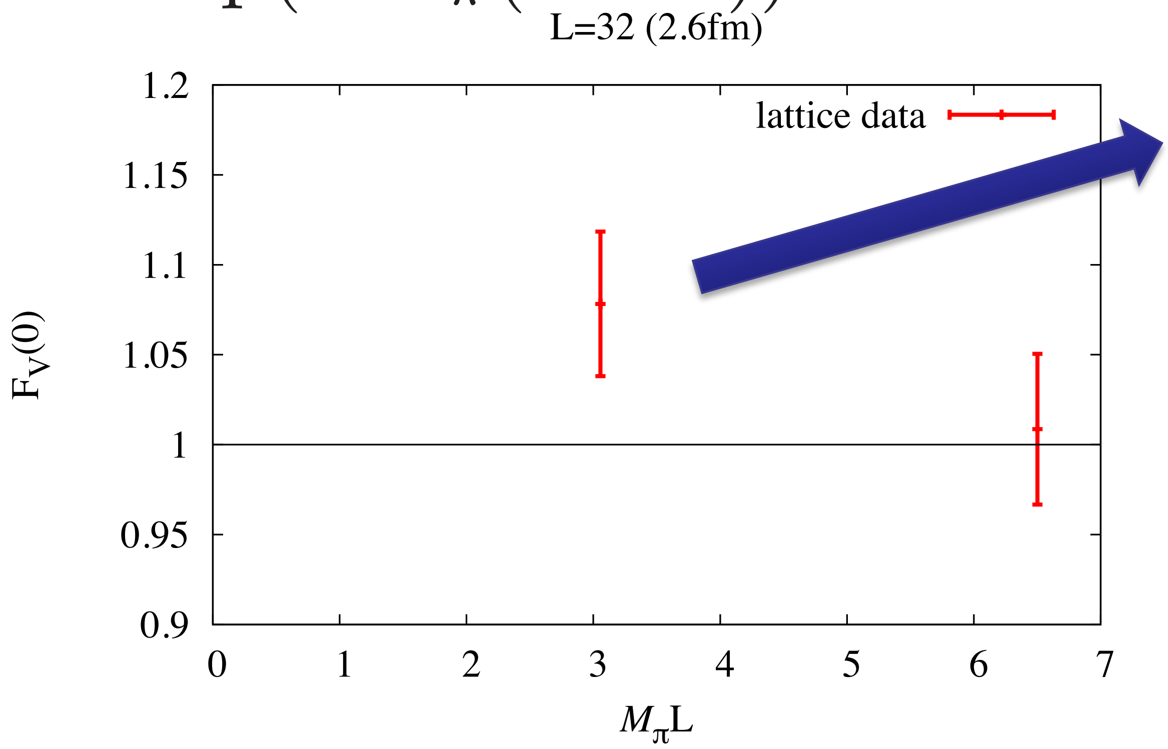


IBM Blue Gene/Q

New data (preliminary)

pion form factor without sub./div. trick

$$\frac{\langle P(x)V^0(y)P(z)\rangle}{\exp(-m_\pi(x-z))} \times 4m_\pi Z_V \to F_V(0) = 1 \ \ ?$$



Overshooting !
consistent with
$\exp(-m_\pi L) \sim 0.05$

Even in the p regime, the zero-mode effect is sizable.

Let's remove zero-mode !

We tend to use zero-momentum mode's correlator, since it has a good signal.
However, the zero-mode is most sensitive to the finite V effects.

→ Let's use subtraction/division trick to cancel the zero-mode of pions.

$$e^{-\sqrt{m_\pi^2+(2\pi/L)^2}L} \ll 0.01 \ll e^{-m_\pi L}$$