

System size dependence of the log-periodic oscillations of transverse momentum spectra

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Statistical description

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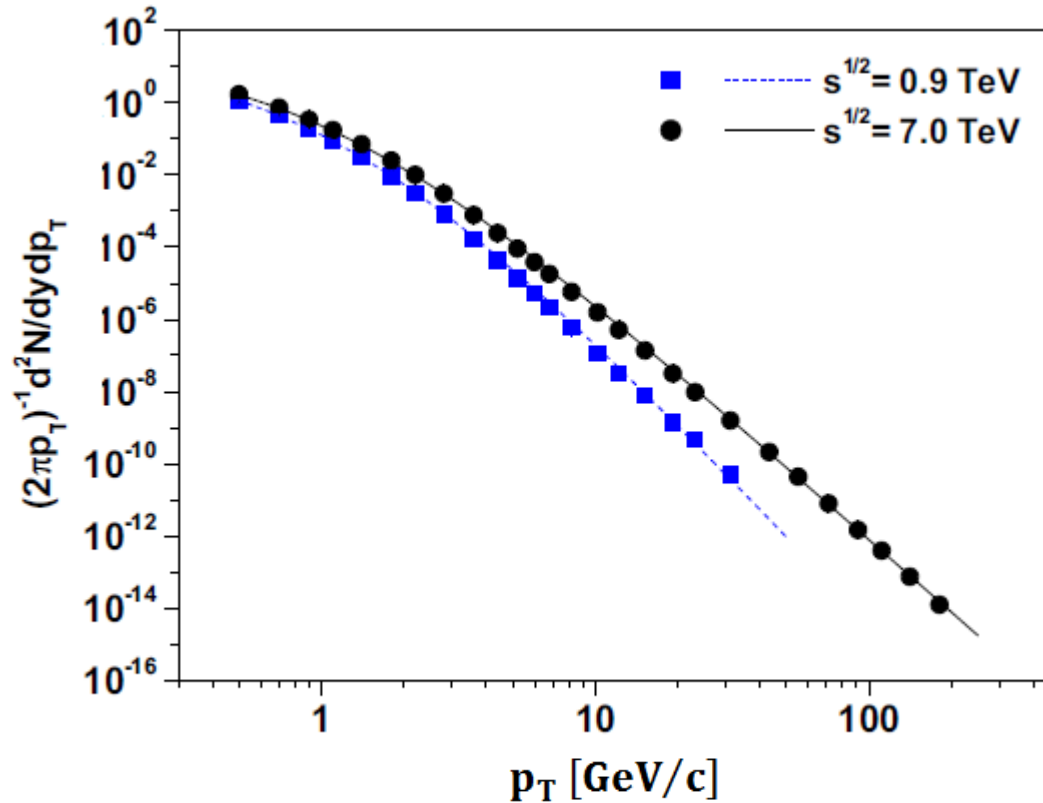
$$f(p_T) = C \cdot \left[1 - (1 - q) \frac{p_T}{T} \right]^{\frac{1}{1-q}} \quad (1)$$

or, in the so called „**QCD inspired**” **Hagedorn form** (with parameters: m and T)

$$h(p_T) = C \cdot \left(1 + \frac{p_T}{mT} \right)^{-m} \quad (2)$$

$$m = \frac{1}{q - 1}$$

p_T distributions in p+p interactions



\sqrt{s} [TeV]	T	m
0.9	0.135	8
7.0	0.145	6.7

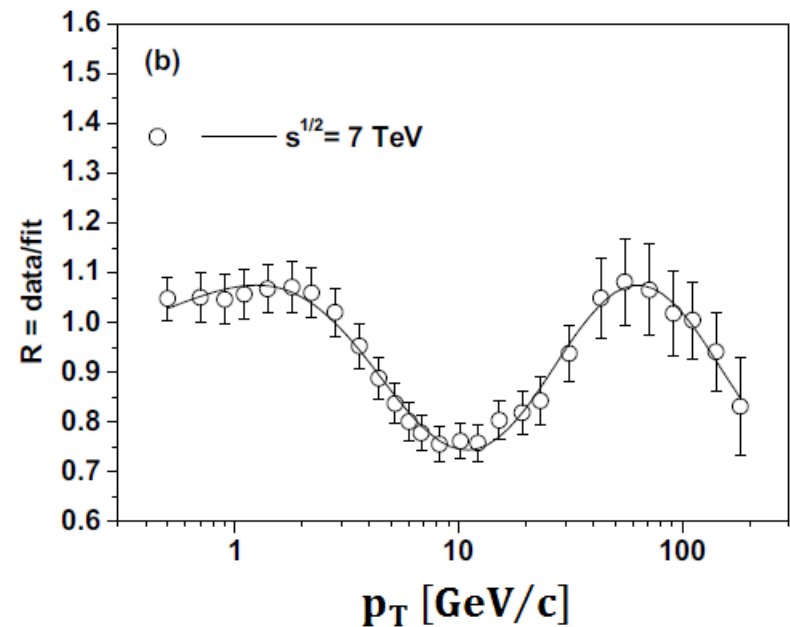
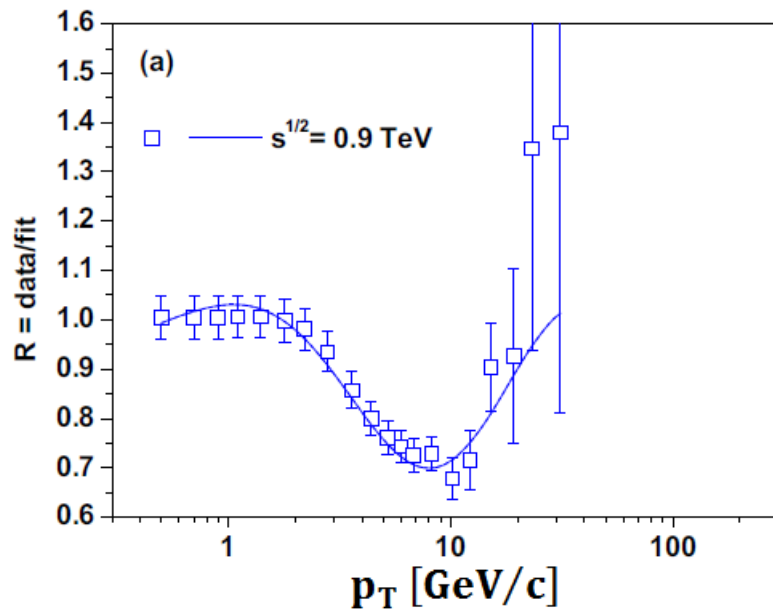
Data from:

V. Khachatryan et al. (CMS Collaboration), JHEP 02 (2010) 041 and JHEP 08 (2011) 086;

Phys. Rev. Lett. 105 (2010) 022002

p_T distributions in p+p interactions

$$R(p_T) = a + b \cos[c \cdot \ln(p_T + d) + f] \quad (3)$$



data/fit ratio is not flat but shows some kind of clearly visible oscillations

Details in:

G. Wilk and Z. Włodarczyk, arXiv:1403.3508 [hep-ph]

Scale invariance – digression (I)

If some function $O(x)$ is scale invariant, i.e.:

$$O(\lambda x) = \mu O(x)$$

then it must have a power law behavior:

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The evolution of the differential $df(p_T)/dp_T$ of a Tsallis distribution $f(p_T)$ with power index n performed for finite differences $\delta p_T = \alpha(nT + p_T)$ results in the following scale invariant relation:

$$g[(1 + \alpha)x] = (1 - \alpha n)g(x) \quad x = 1 + \frac{p_T}{nT}$$

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In general, one can write $h(p_T) = C \cdot \left(1 + \frac{p_T}{mT}\right)^{-m}$ in the form:

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$$g(p_T) \cong \left(1 + \frac{p_T}{nT}\right)^{-m_0} \left\{ w_0 + w_1 \cos \left[\frac{2\pi}{\ln(1 + \alpha)} \ln \left(1 + \frac{p_T}{nT}\right) \right] \right\}$$

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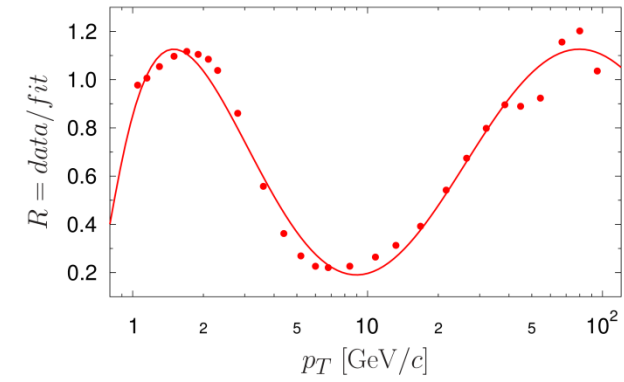
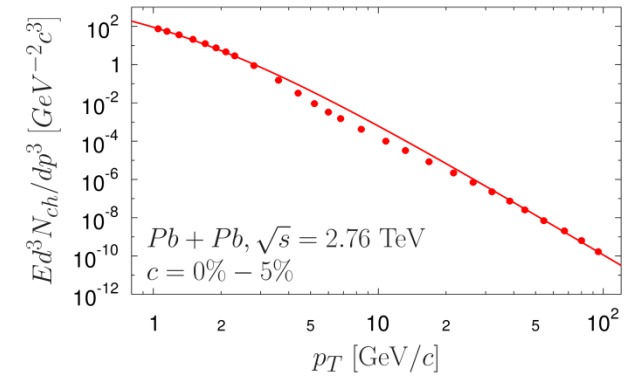
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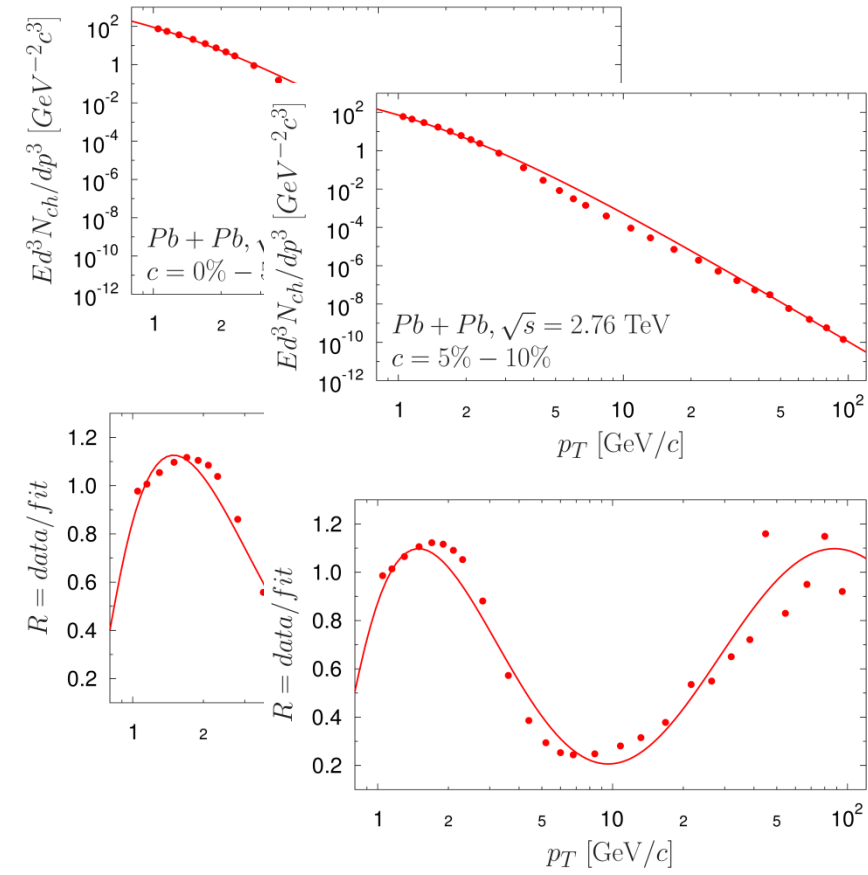
The parameters in general modulating factor $R(p_T) = a + b \cos[c \cdot \ln(p_T + d) + f]$ can be indentified as follows:

$$\begin{aligned} a &= w_0 & c &= \frac{2\pi}{\ln(1 + \alpha)} & d &= nT \\ b &= w_1 & f &= -c \cdot \ln(nT) \end{aligned}$$

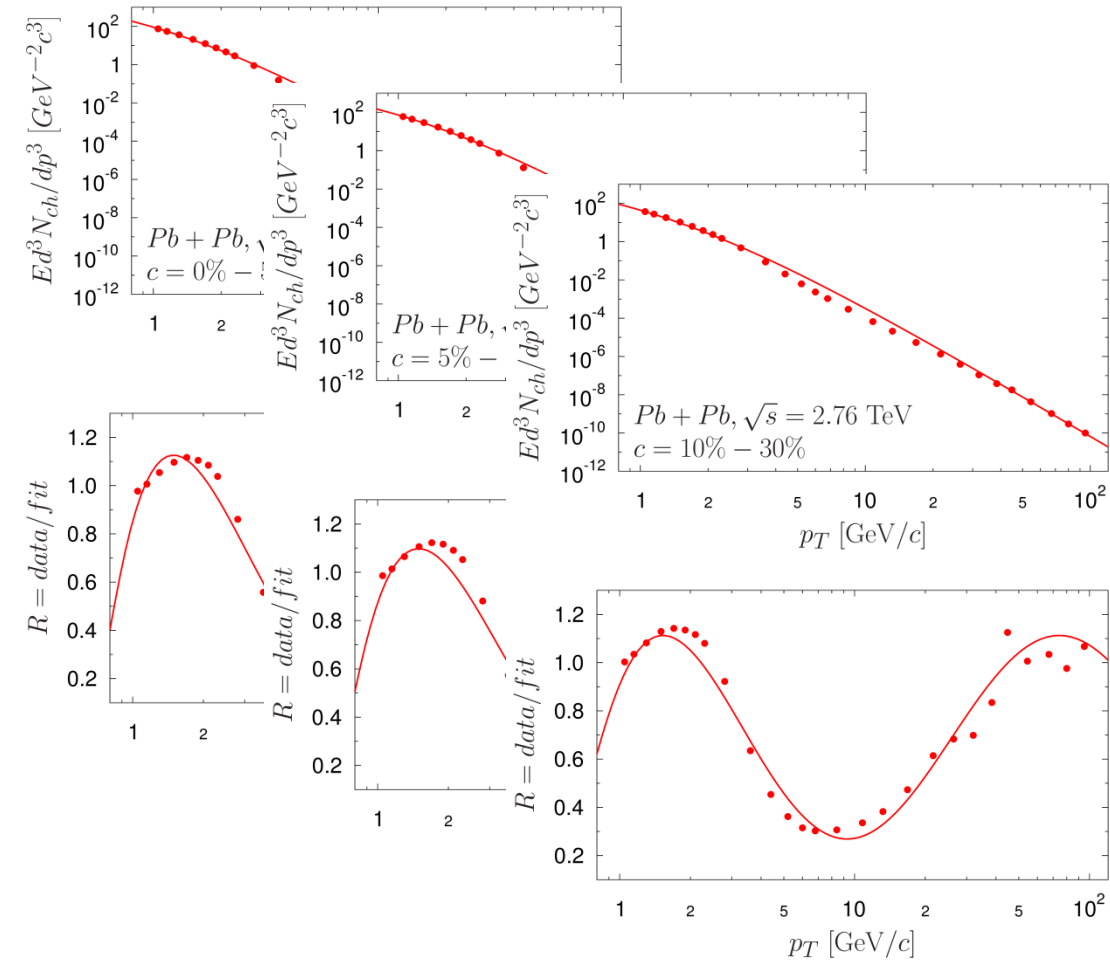
p_T distributions in Pb+Pb collisions



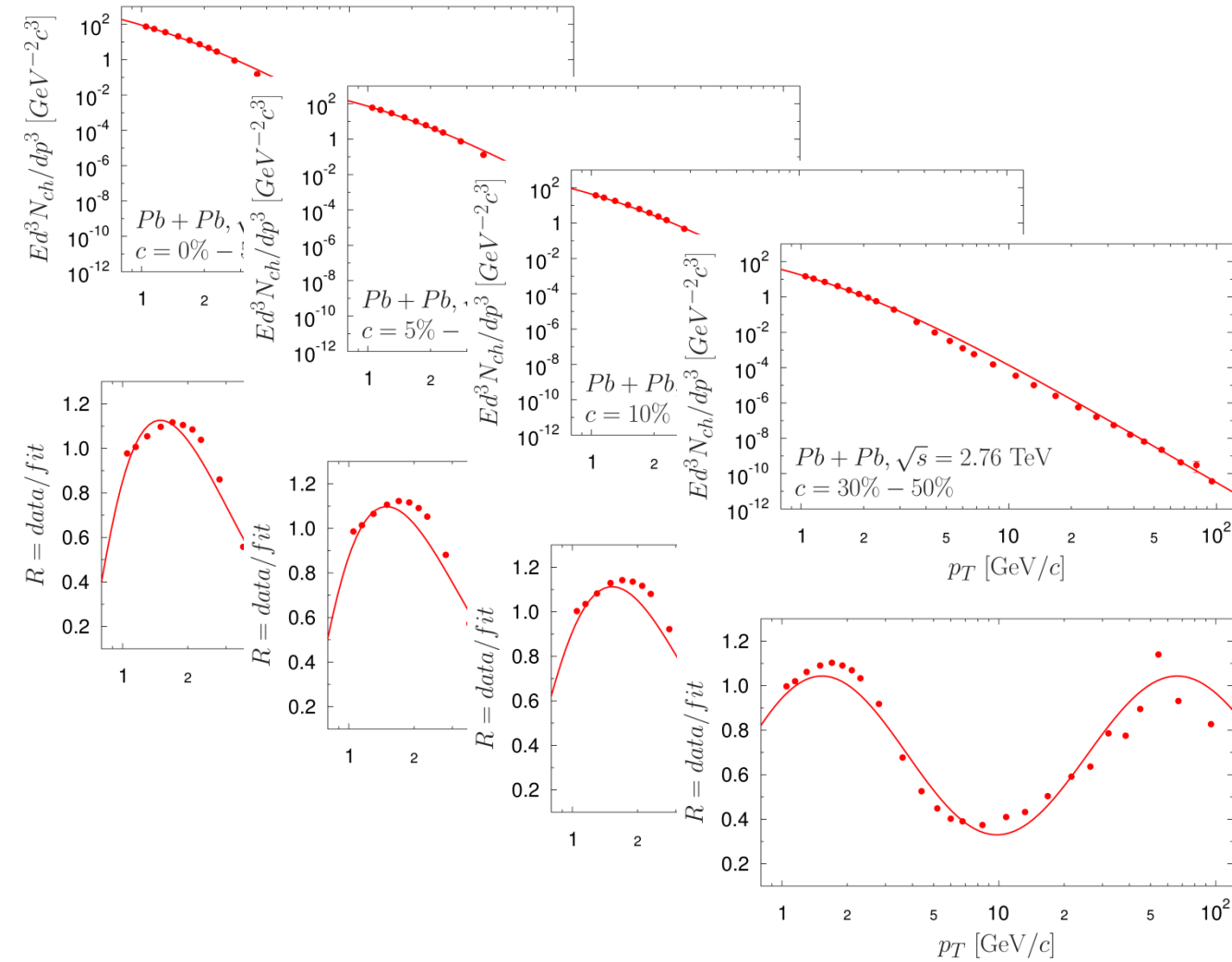
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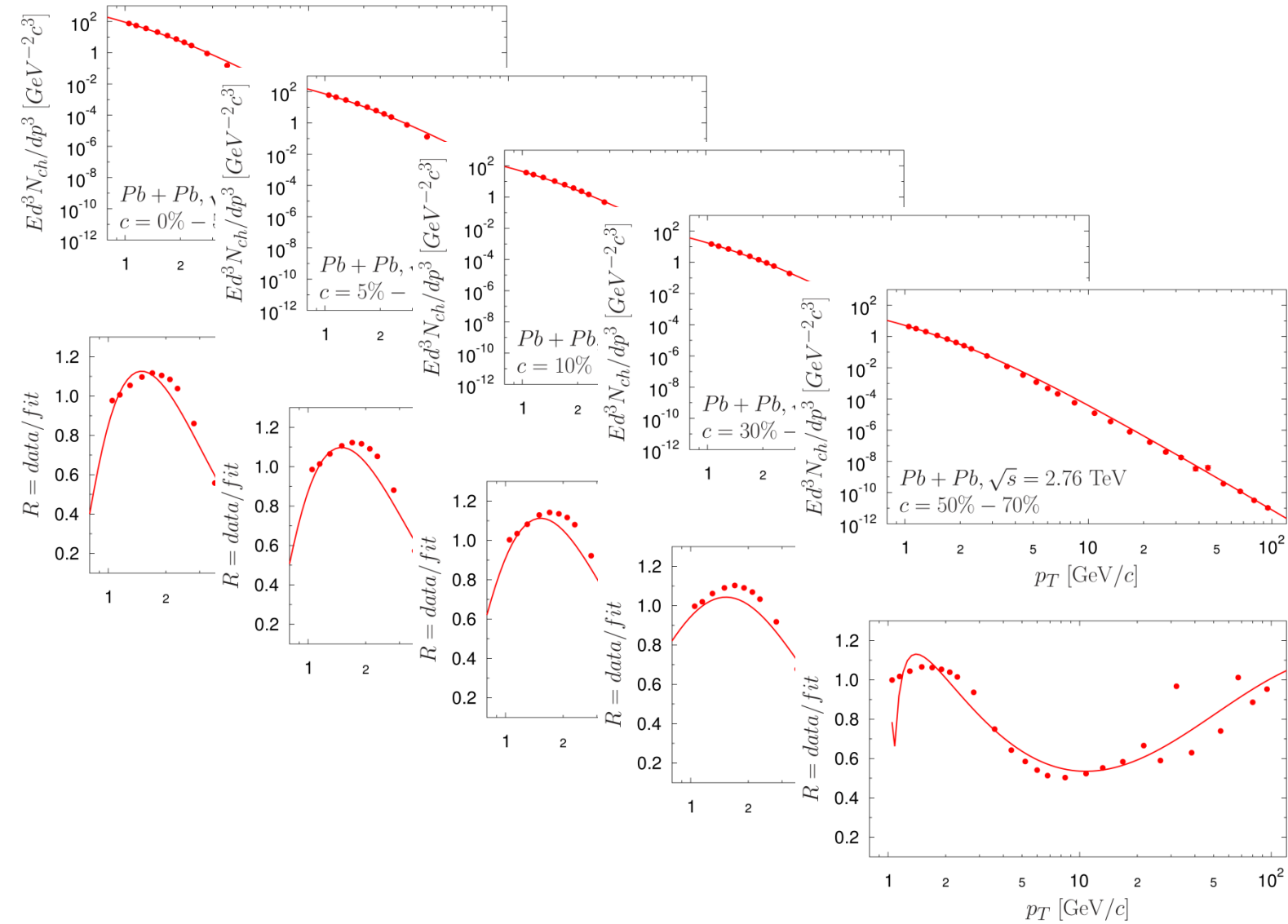
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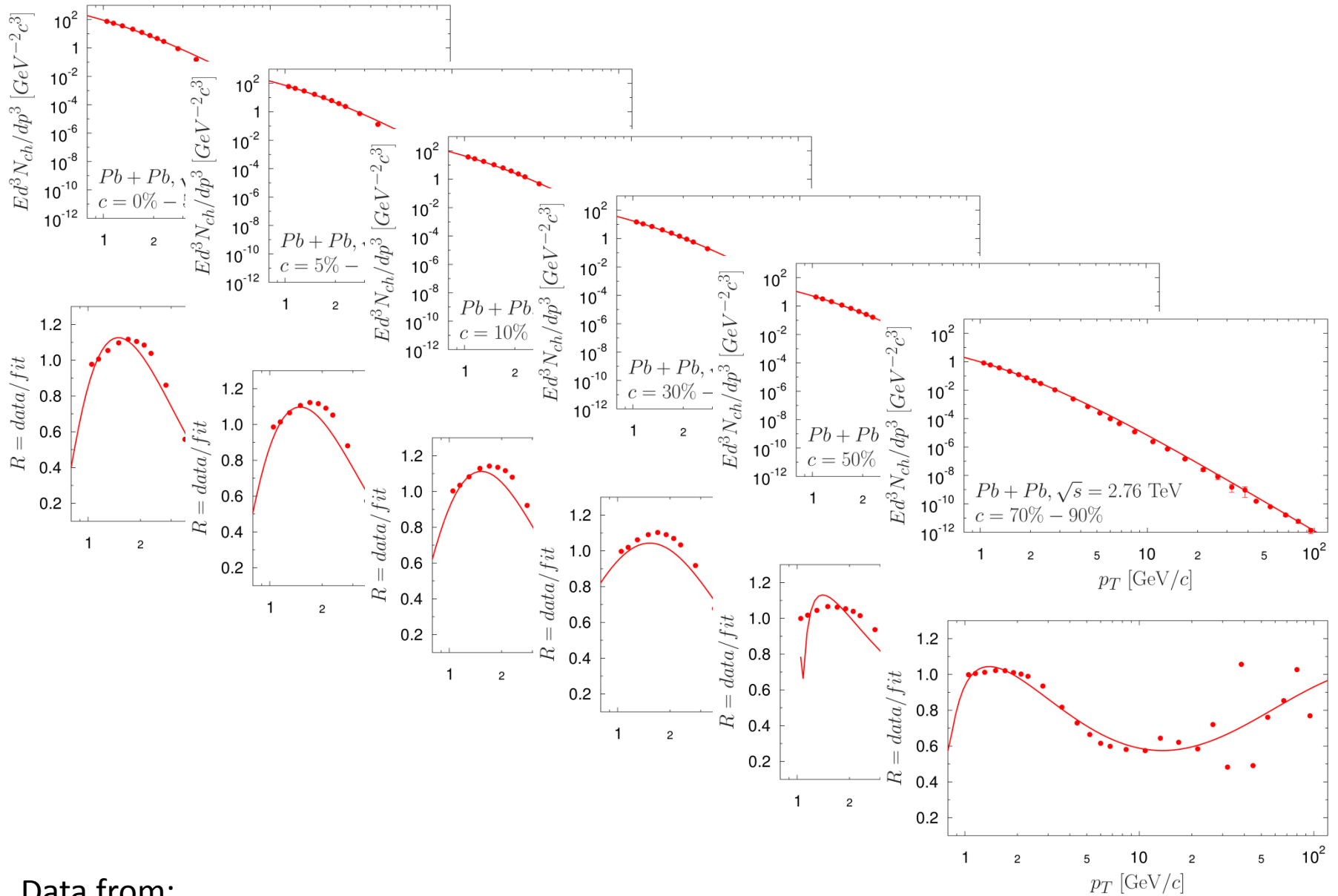
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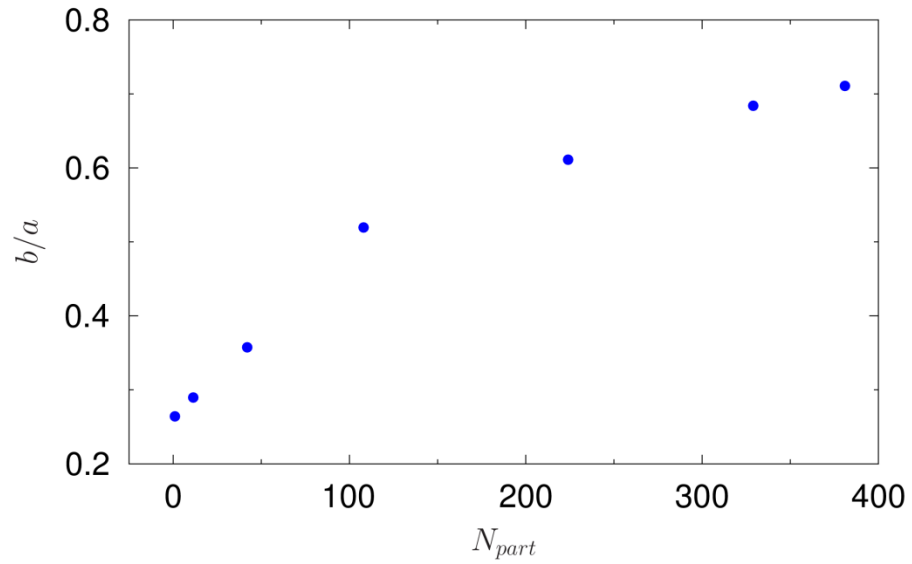


Data from:

S. Chatrchyan et al. (CMS Collaboration), EPJ C72 (2012) 1945

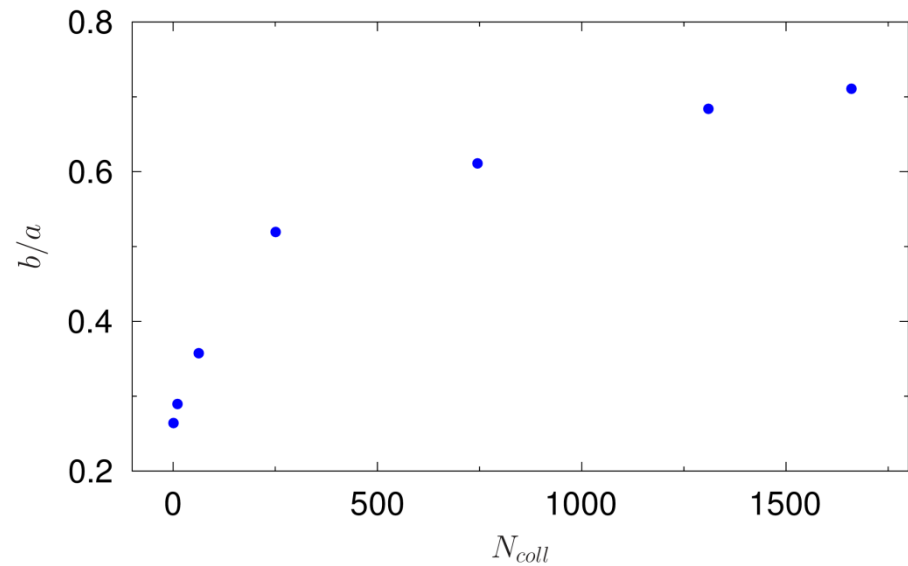
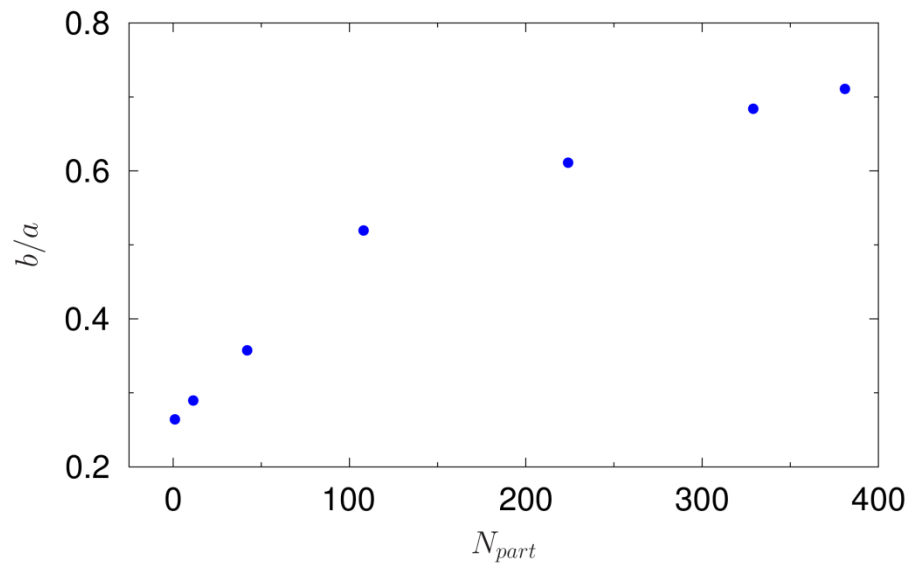
Parameters – centrality dependence

$$R(p_T) = a + b \cos[c \cdot \ln(p_T + d) + f]$$



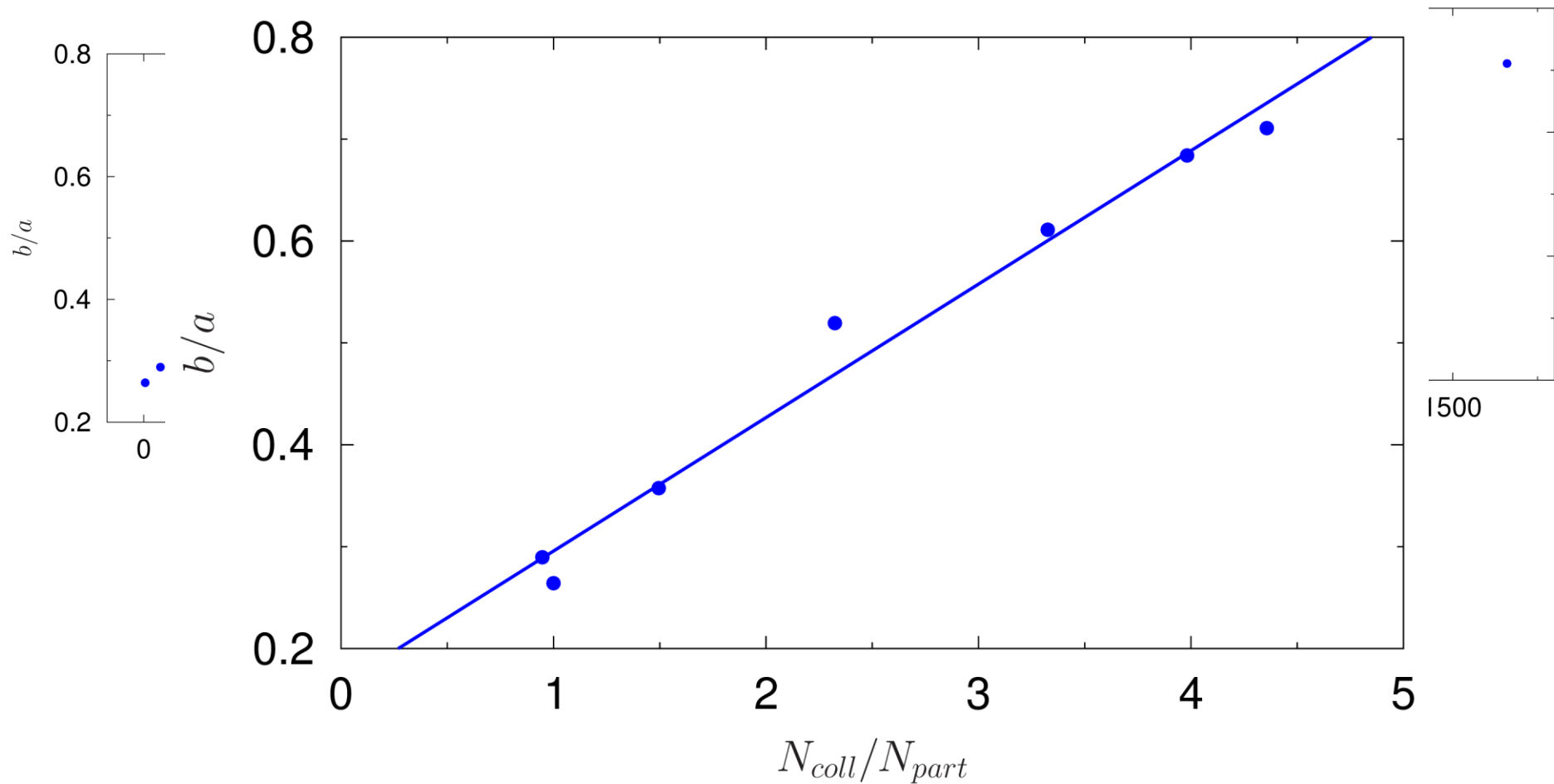
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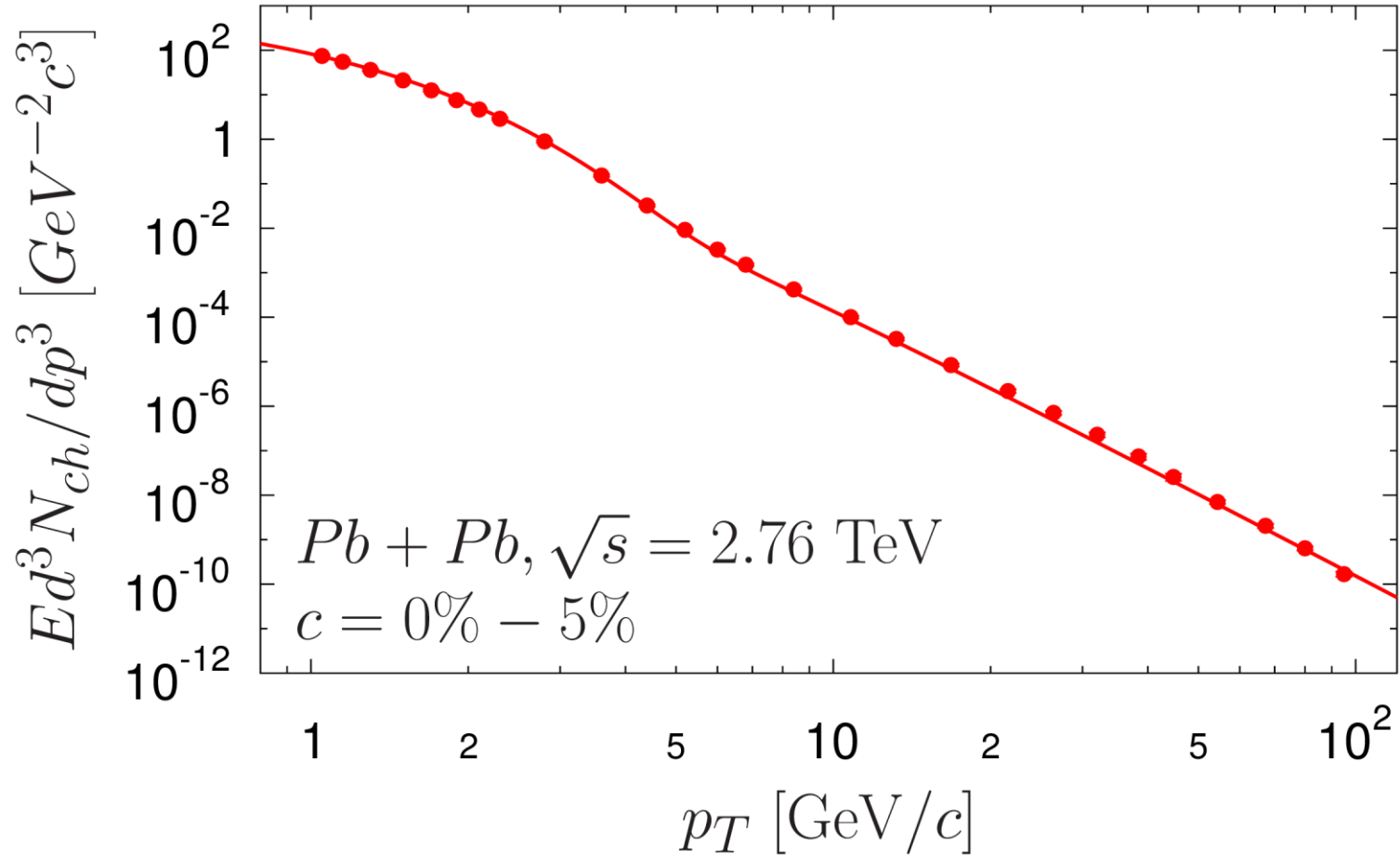
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Two component fit

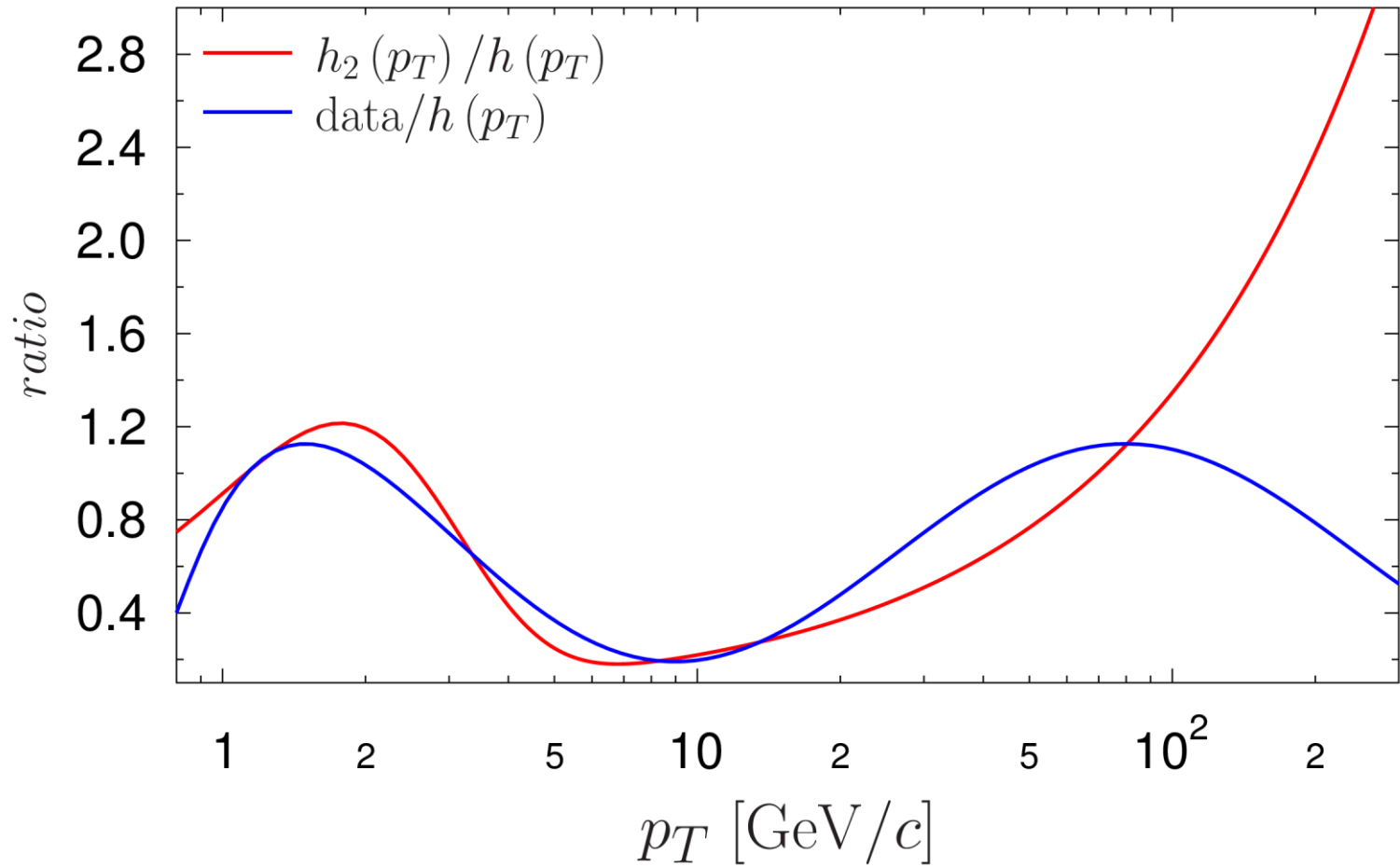
$$h_2(p_T) = C_1 \cdot \left(1 + \frac{p_T}{m_1 T_1}\right)^{-m_1} + C_2 \cdot \left(1 + \frac{p_T}{m_2 T_2}\right)^{-m_2}$$



Details in:

T.S. Biro, G.G. Barnafoldi, P. Van and K. Urmossy, arXiv:1404.1256 [hep-ph]

Ratios



Data from:

S. Chatrchyan et al. (CMS Collaboration), EPJ C72 (2012) 1945

Summary

Recently the inclusive transverse momentum distributions of primary charged particles are measured for different centralities in Pb+Pb collisions at $\sqrt{s}=2.76$ TeV/nucleon.

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In the case of Pb+Pb collisions the amplitude of this oscillations increases linearly as a function of $N_{\text{coll}} / N_{\text{par}}$, and for the most central collisions we observe a spectacular oscillations.

Additional slides