# Multiparton corelations in underlying event .

B.Blok (Technion, Haifa)

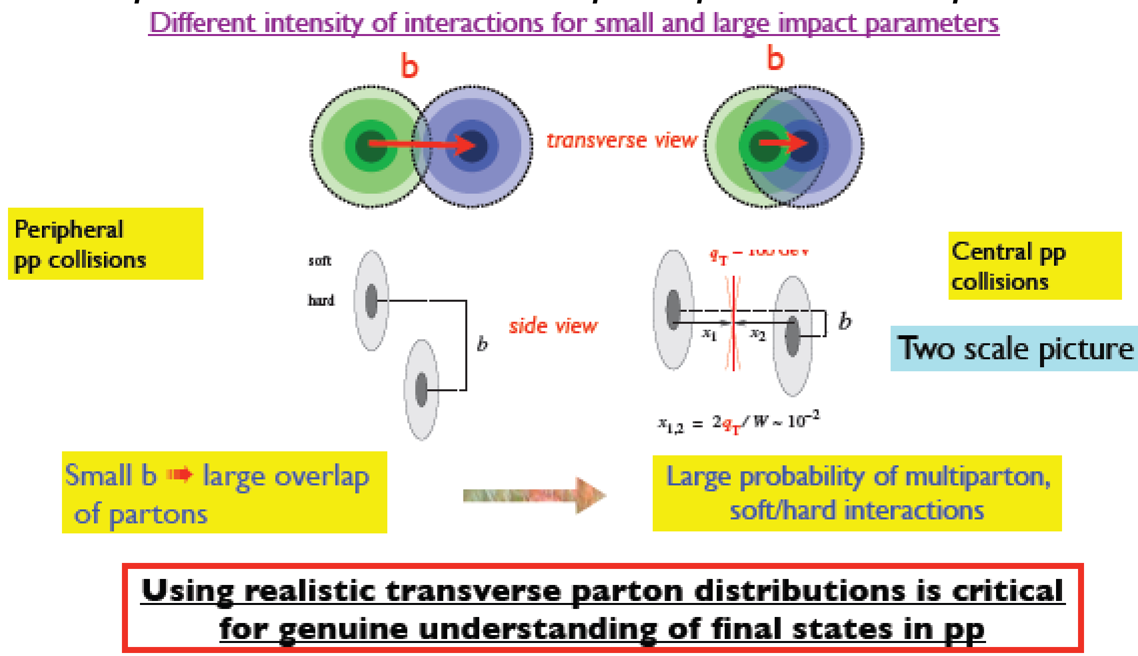
*ISMD 2014 Bologna*

*1. Underlying event-more or less everything exept hard trigger (say soft+ softer components of jets).*

*Aim of present talk – we argue that standard geometric picture is not enough and one needs to understand different types of correlations, both soft and hard to have full understanding of underlying event physics.*

## *2. Geometrical picture of underlying event.*

*Consider proton-proton collisions in impact parameter space*



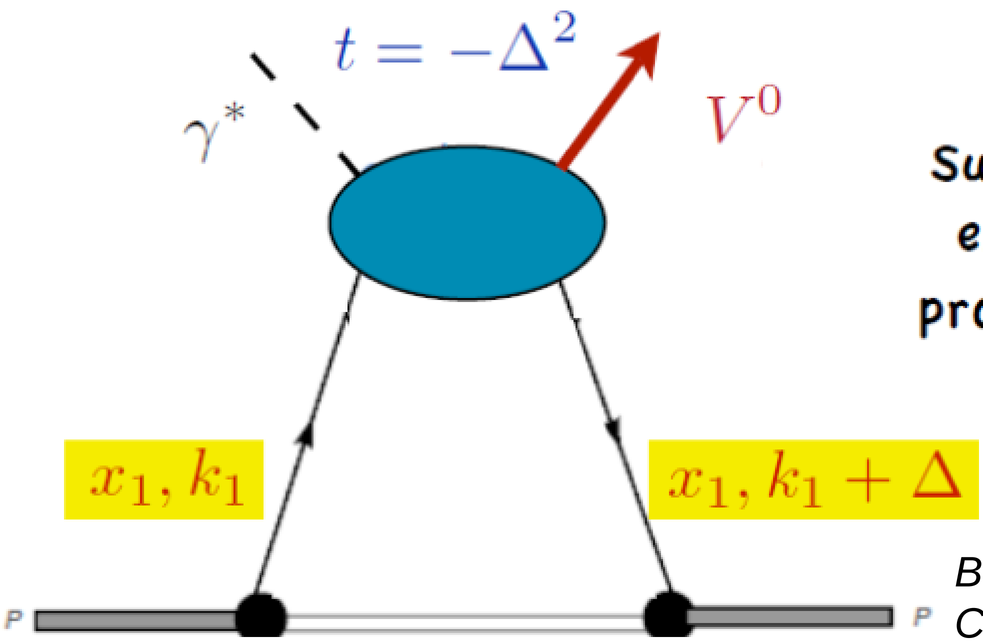
*Standard approach go to impact parameter space and use optical theorem*

$$\sigma_{tot}(s) = 2\int d^2\mathbf{b}\,\mathrm{Re}\,\Gamma(s,b),$$
$$\sigma_{el}(s) = \int d^2\mathbf{b}\,|\Gamma(s,b)|^2,$$
$$\sigma_{inel}(s) = \int d^2\mathbf{b}\left(2\,\mathrm{Re}\,\Gamma(s,b) - |\Gamma(s,b)|^2\right);$$

$$\Gamma(s,b) = \frac{1}{2is(2\pi)^2}\int d^2\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{b}}A(s,t).$$

*A-scattering amplitude for 2 to 2 process. Formula is rather general-pp, photon+p (DIS),…-usually parametrised by experimental data.*

*using impact parameter space picture it is straightforward to develope a mean field geometric approach to pp collisions. The parameters of such model can be fixed by HERA data (dipole approach), and one can proceed to calculation of multijet probabilities.*




**G P D**

**Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !**

*Based on QCD factorisation theorem Collins,Frankfurt,Strikman*

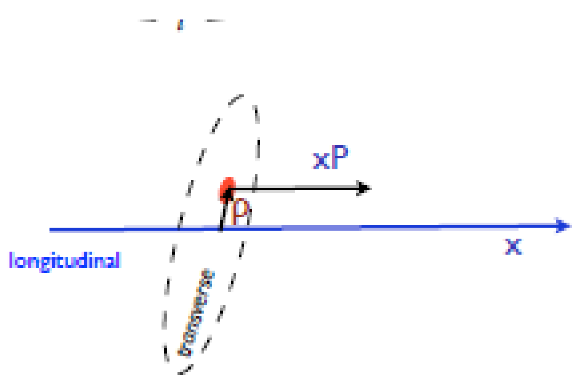
*The impact parameter dependence of profile functions at HERA is determined by the same GPD that is determined through analysis of exclusive production of vector mesons. So if we know the total cross section in HERA, we can directly connect impact factor behaviour of profile functions at HERA with that of GPD, thus expressing the cross sections in terms of parton densities.*
*This gives explicit connection between cross sections and transverse parton distributions in nucleon.*

*1-parton GPD is HERA is well described in  factorised form, as product of usual pdf and two gluon formfactor*

$$x f_g(x,t,\mu^2) = x f_g(x,\mu^2)\, F_g(x,t,\mu)$$

$$F_g(x,t,\mu) = \frac{1}{\left(1 - \frac{t}{m_g^2(x,\mu)}\right)^2}$$

$m_g^2$ *is of order  GeV and decreases with x*




*transverse spatial distribution of partons*

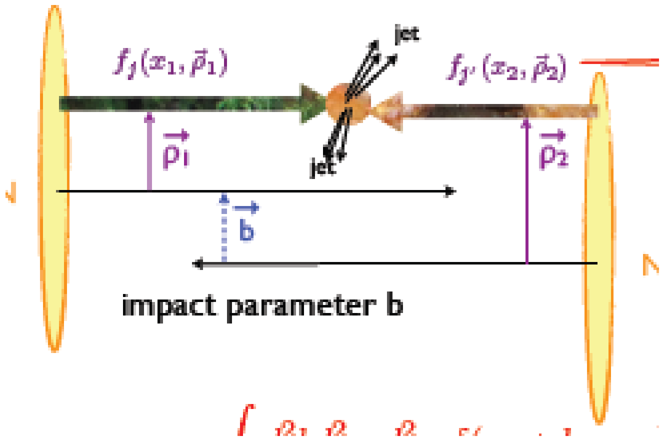
$$f(x,\rho) \equiv \int d^2\vec{\Delta} e^{i\vec{\Delta}_\perp \rho} f(x,x,t), \; -t = \Delta^2$$

ρ - transverse distance from the c.m. of proton

$$\rho_{c.m.} = \sum_i \rho_i x_i$$

$$\mathcal{F}_g(x,\rho,\mu) = \frac{m_g^3(x,\mu)\rho}{4\pi} K_1(m_g(x,\mu)\rho).$$

*Can use this simple transverse picture of nucleon to describe pp collisions.Example-multijet events (Frankfurt,Strikman,Weiss, 2003)*

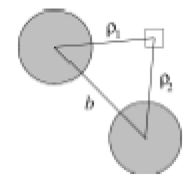



Gluon GPDs and difference between transverse s b distributions for four jet, dijet triggers and minimal bias collisions

The distribution of interactions over b for events with inclusive dijet trigger (Higgs production,...) is given by

$$P_2(b) = \int d^2\rho_1 \int d^2\rho_2 \delta^{(2)}(\vec{b} - \vec{\rho}_1 + \vec{\rho}_2) F_g(x_1,\rho_1) F_g(x_2,\rho_2),$$

differential probability for dijet production to occur at given b




for $F_g(x,t) = 1/(1 - t/m_g(x)^2)$ $\quad F_g(x,\rho) = \frac{m_g^2}{2\pi}\left(\frac{m_g\rho}{2}\right) K_1(m_g\rho)$

$$P_2(b) = \frac{m_g^2}{12\pi}\left(\frac{m_g b}{2}\right)^3 K_3(m_g b)$$

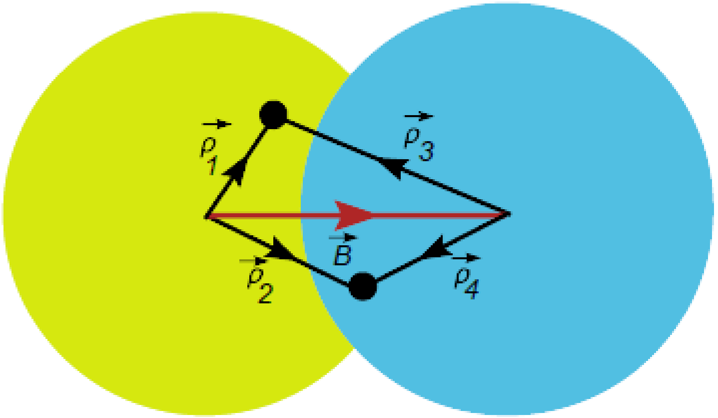
$$P_4(b) = \frac{P_2(b)^2}{\int d^2b P_2(b)^2}$$

*This gives distribution of dijet probability, similar –n dijet. In the same way-dijet profile function:*

$$\Gamma^{inel}_{jets}(s,b) = 1 - \exp\left[-\sigma^{inc}_{2jet} P_2(b,\bar{x},p_t^c)\right] .$$

*Rogers,Strikman,Stasto 2008*

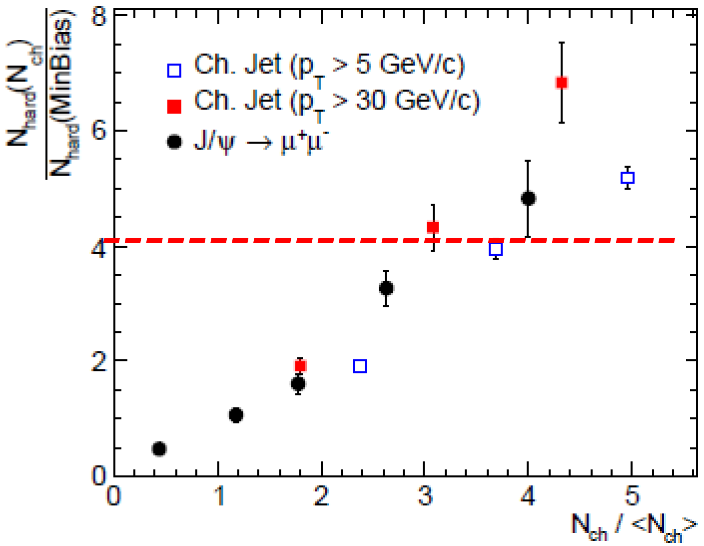
*Very similar picture can be applied for mean field calculation of MPI*




*Geometry of double parton collision in impact parameter picture*

*How good is geometric approach (especially if one starts to use experimental parametrisations of profile functions?*
*Two examples: minijet multiplicities and MPI*

*Test of geometrical picture in dijet event ( Azarkin,Dremin,Strikman 2014)*




Relative yield of hard momentum processes as a function of $N_{\rm ch}$,

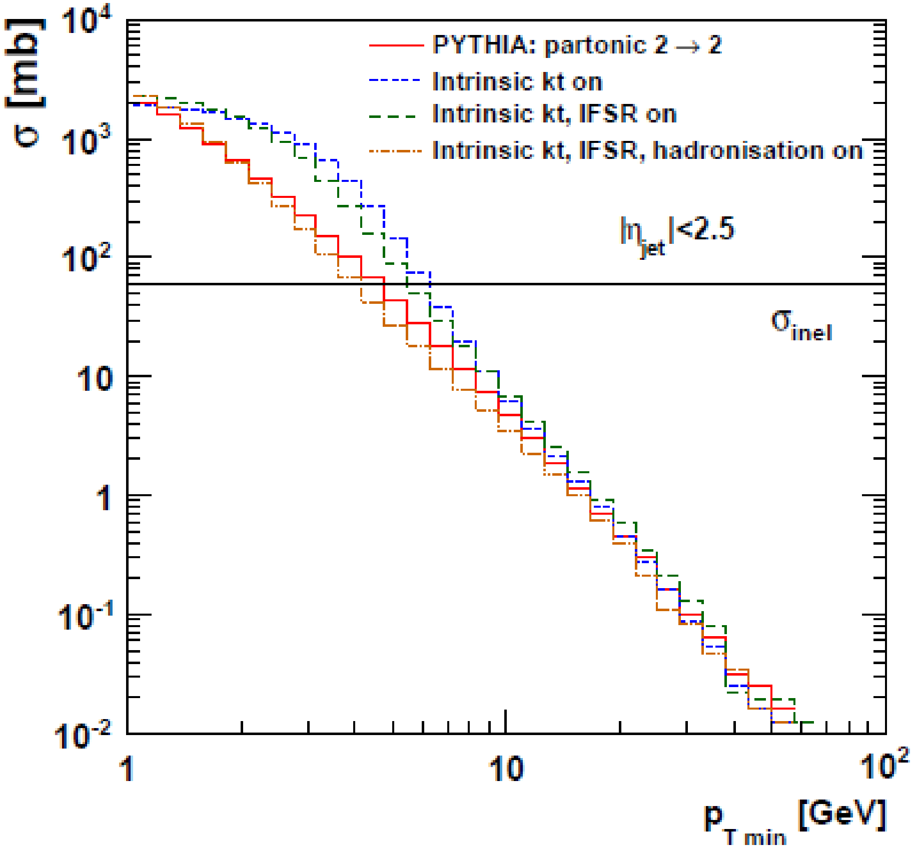
$$R = \frac{M(trigger)}{M(minimal\, bias)} \qquad R(b) = P_2(b)\sigma_{inel}.$$

*Maximum-at b=0, and we get 4 using*

$$P_2(b,s,p_t^c) = \frac{m_g^2(\bar{x},p_t^c)}{12\pi}\left(\frac{m_g(\bar{x},p_t^c)b}{2}\right)^3 K_3(m_g(\bar{x},p_t^c)b).$$

*Need correlations/hot spots to explain very high multiplicities*

# Another example-MPI




Jet multiplicity=σ(inclusive jet)/σ(inelastic nondiffractive)(pp)

At least 10 dijet events per nondiffractive event even at $p_t$ >3 GeV

Grebenyuk,Hautmann,Jung,Katsas,Knutsson 2012

Blok,Dokshitzer,Frankfurt,Strikman 2010

*Mean field calculations of MPI-done first in eighties in coordinate space-* Treleani Paver (1985), Mekhfi (1985)

*The four jet calculations can be done directly in the momentum space*

$$\sigma_4(x_1,x_2,x_3,x_4) = \int \frac{d^2\overrightarrow{\Delta}}{(2\pi)^2} D_a(x_1,x_2,p_1^2,p_2^2,\overrightarrow{\Delta}) \times D_b(x_3,x_4,p_1^2,p_2^2,-\overrightarrow{\Delta}) \times \frac{d\sigma^{13}}{d\hat{t}_1}\frac{d\sigma^{24}}{d\hat{t}_2} d\hat{t}_1 d\hat{t}_2.$$

$D_\alpha(x_1,x_2,p_1^2,p_2^2,\overrightarrow{\Delta})$ are the new ${}_2$GPDs

$$D\ (x_1,x_2,p_1^2,p_2^2,\overrightarrow{\Delta}) = \sum_{n=3}^{\infty}\int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\theta(p_1^2-k_1^2)$$

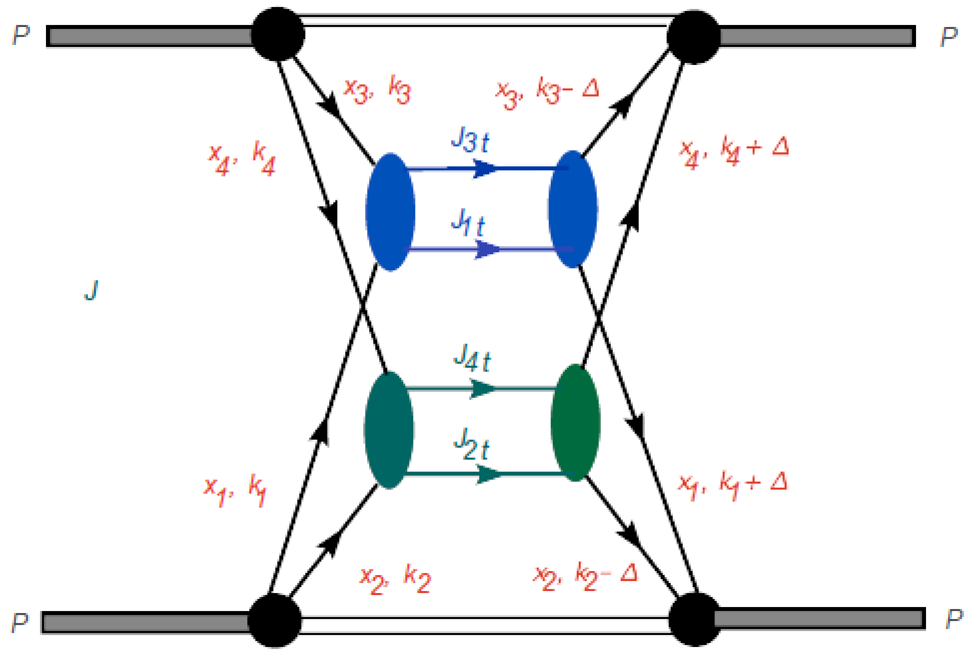
$$\times\ \theta(p_2^2-k_2^2)\int\prod_{i\neq 1,2}\frac{d^2k_i}{(2\pi)^2}\int_0^1\prod_{i\neq 1,2}dx_i$$

*Expressed through nucleon light cone wave function.*

$$\times\ \psi_n(x_1,\vec{k}_1,x_2,\vec{k}_2,..,\vec{k}_i,x_i..)$$

$$\times\ \psi_n^+(x_1,\overrightarrow{k_1}+\overrightarrow{\Delta},x_2,\overrightarrow{k_2}-\overrightarrow{\Delta},x_3,\vec{k}_3,...)$$

$$\times\ (2\pi)^3\delta(\sum_{i=1}^{i=n}x_i-1)\delta(\sum_{i=1}^{i=n}\vec{k}_i).$$




Kinematics of double hard collision

$$\frac{1}{\pi R_{\mathrm{int}}^2} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} \frac{D(x_1, x_2, -\vec{\Delta}) D(x_3, x_4, \vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)},$$

## The approximation of independent particles.

*Suppose the multiparton wave faction factorise, i.e. we neglect possible interparton correlations and recoil effects. Then it's straightforward to see that the two particle GPDs factorise and acquire a form:*

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta}) G(x_2, p_2^2, \vec{\Delta}),$$

*The one-particle GPD-s G are conventionally written in the dipole form:*

$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2) F_{2g}(\Delta)$$

G - the usual 1-parton distribution (determining DIS structure functions)

F - the two-gluon form factor of the nucleon the dipole fit : $F_{2g}(\Delta) \simeq \frac{1}{\left(1+\Delta^2/m_g^2\right)^2}$ $m_g^2(x \sim 0.03, Q^2 \sim 3\mathrm{GeV}^2)$ $\simeq 1.1\mathrm{GeV}^2$

$$\frac{1}{\pi R_{\mathrm{int}}^2} = \int \frac{d^2\Delta}{(2\pi)^2} F_{2g}^4(\Delta) = \frac{m_g^2}{28\pi}.$$

$$R_{\mathrm{int}}^2 = 7/2 r_g^2, \qquad r_g^2/4 = dF_{2g}(t)/dt_{t=0}.$$

*The experimental result is 15 mb, while the use of the electromagnetic radius of the nucleon leads to this area being 60 mb while we obtain in independent particle approximation 34 mb*

*Conclusion: new insights beyond geometric/mean field approach are needed to fully understand UE.*

# Example:multiparton correlations in MPI

*Blok,Doksitzer,Frankfurt,Strikman,2010,2011,2012,2013;*
*Ryskin and Snigirev,2011, Gaunt and Stirling, 2011, Gaunt 2013,Gaunt,Maciula,Szczurec 2014,*
*Diehl,Osterwalder and Schafer. 2012*

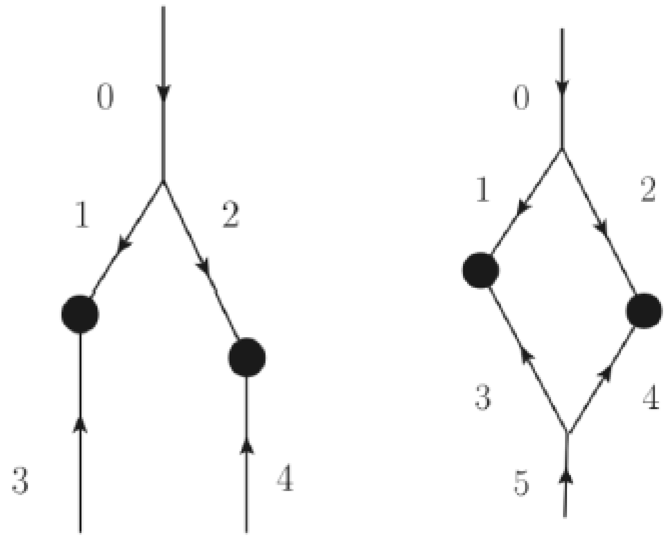
Two basic ideas (relative to conventional one dijet processes-2 to2 in our notations):

1. Double collinear enhancement in total cross sections-i.e. double pole enhancement in differential two dijet cross sections.

2. new topologies-in addition to conventional pQCD bremsstralung-parton/ladder splitting .

*a) Four to four     b) three to four   ------------ but no two to four*

*Blok,Dokshitzer,Frankfurt,Strikman*
*2010,2011*
*Gaunt Stirling 2011*




*Parton model structure of new topologies*

*There are two types of three to four contributions. The first –when there is radiation after the split, the second –when there is not. This leads to different types of singularities:*

$$\frac{\alpha_s^2}{\delta'^2\,\delta^2}$$

*Short split singularity*

$$\delta' = \vec{\delta}_{13} + \vec{\delta}_{24}$$

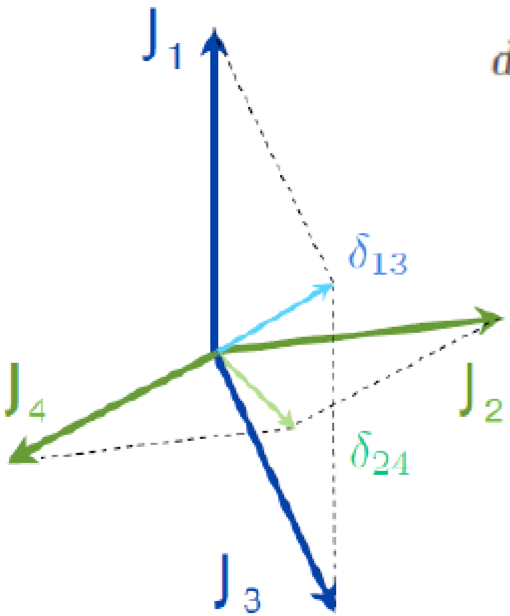
$$\frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}$$

*Long split singularity*

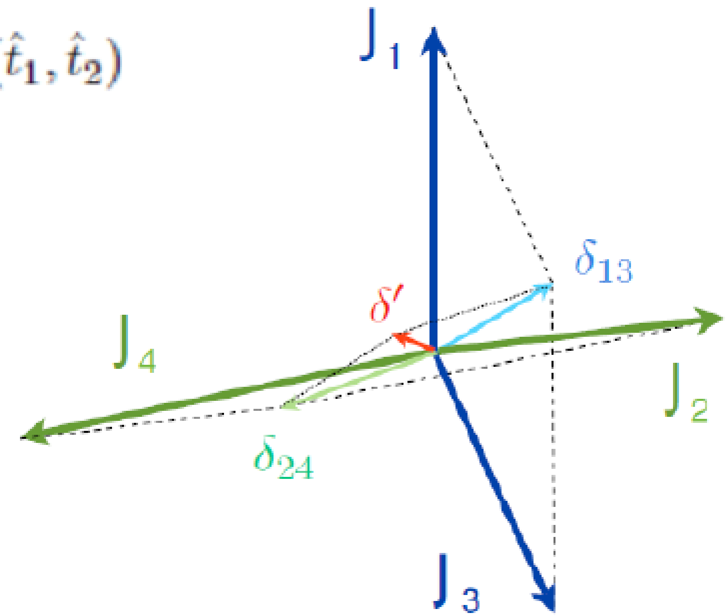
## back-to-back kinematics

$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$

$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$




$d\Sigma(\hat{t}_1, \hat{t}_2)$




$$d\sigma^{(4\to 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$d\sigma^{(3\to 4)} \propto \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$\pi^2 \frac{d\sigma_1^{(3\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \Big\{ {}_{[1]}D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \cdot {}_{[2]}D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2)$$
$$\times\ S_1\left(Q^2, \delta_{13}^2\right) S_3\left(Q^2, \delta_{13}^2\right) \cdot S_2\left(Q^2, \delta_{24}^2\right) S_4\left(Q^2, \delta_{24}^2\right) \Big\}.$$

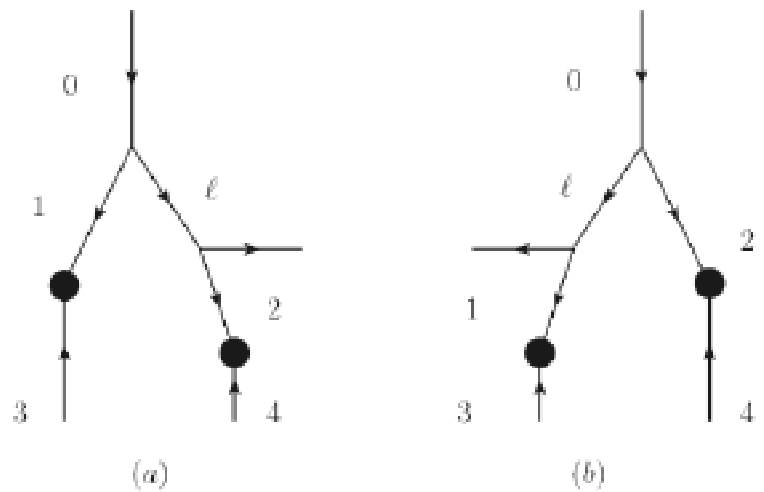
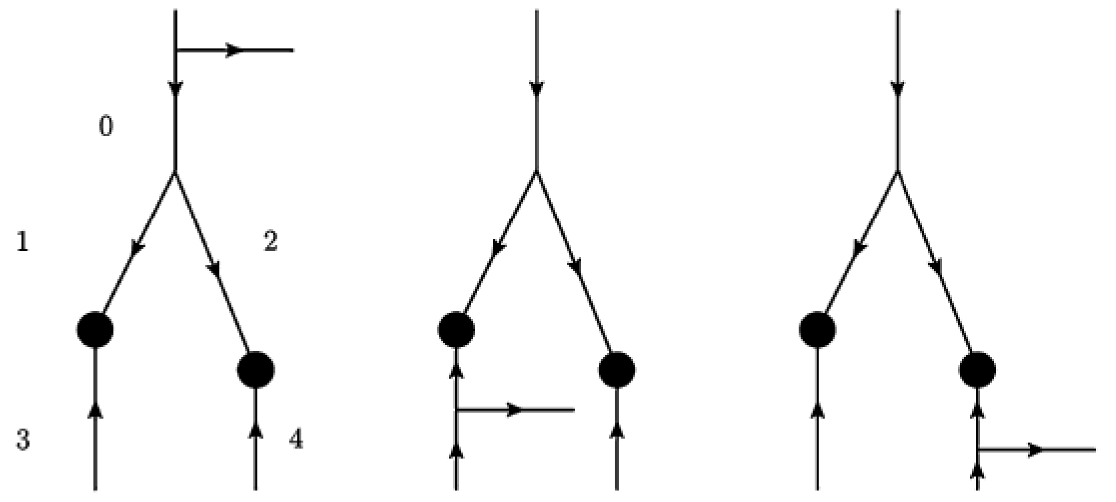



FIG. 3: Three-to-four amplitudes with extra emission from inside the splitting fork

*Short split*

$$\frac{d\sigma}{d^2\delta_{13}\, d^2\delta_{24}} \propto \frac{\alpha_s}{\delta^2}\delta(\vec{\delta}_{13}+\vec{\delta}_{24}), \qquad \delta^2 \equiv \delta_{13}^2 = \delta_{24}^2.$$

$$\frac{d\sigma}{d^2\delta_{13}\, d^2\delta_{24}} \propto \frac{\alpha_s^2}{\delta^2\,\delta'^2}, \qquad \delta'^2 \ll \delta^2 \equiv \delta_{13}^2 \simeq \delta_{24}^2.$$




(a) (b) (c)

$$\frac{\pi^2 d\sigma_2^{(3\to 4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\alpha_s(\delta^2)}{2\pi\,\delta^2} \sum_c P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right)$$

$$\times S_1(Q^2,\delta^2)\, S_2(Q^2,\delta^2)$$

$$\times \frac{\partial}{\partial\delta'^2}\left\{ S_c(\delta^2,\delta'^2)\frac{G_a^c(x_1+x_2;\delta'^2,Q_0^2)}{x_1+x_2}\right.$$

$$\times S_3(Q^2,\delta'^2) S_4(Q^2,\delta'^2)$$

## *The total double dijet cross section*

$$\frac{d\sigma(x_1,x_2,x_3,x_4)}{d\hat{t}_1\, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1}\frac{d\sigma^{24}}{d\hat{t}_2} \times \left\{\frac{1}{S_4}+\frac{1}{S_3}\right\}.$$

$$S_4^{-1}(x_1,x_2,x_3,x_4;Q^2) = \int \frac{d^2\Delta}{(2\pi)^2}\, {}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta}) \times {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}),$$

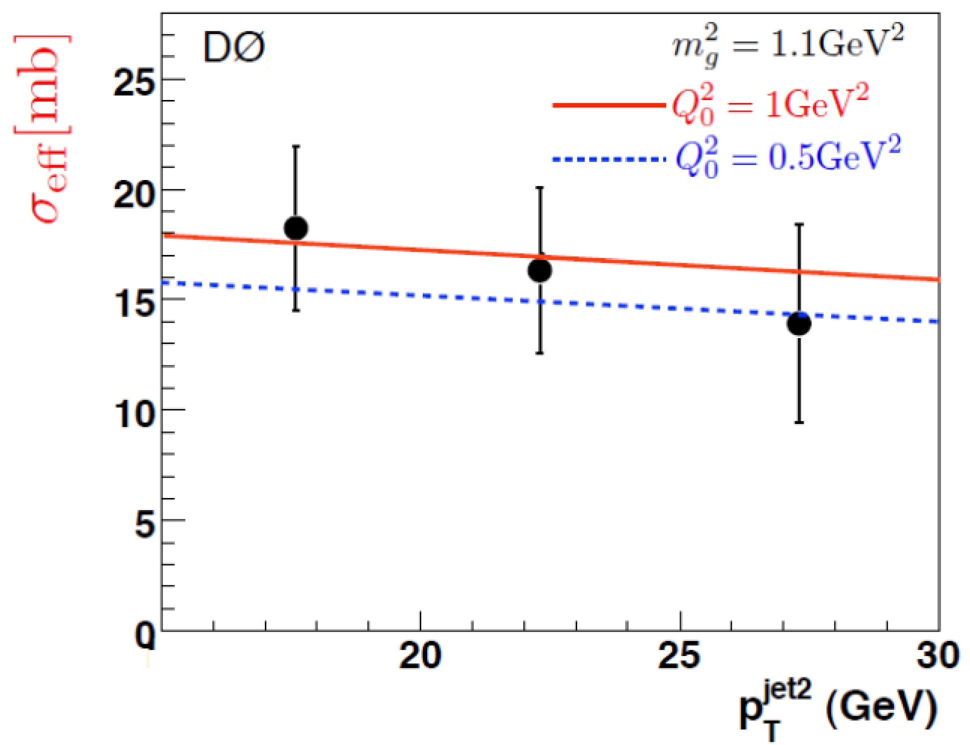
$$S_3^{-1}(x_1,x_2,x_3,x_4;Q^2) = \int \frac{d^2\Delta}{(2\pi)^2}\, {}_{[1]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta}) \times {}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta})$$

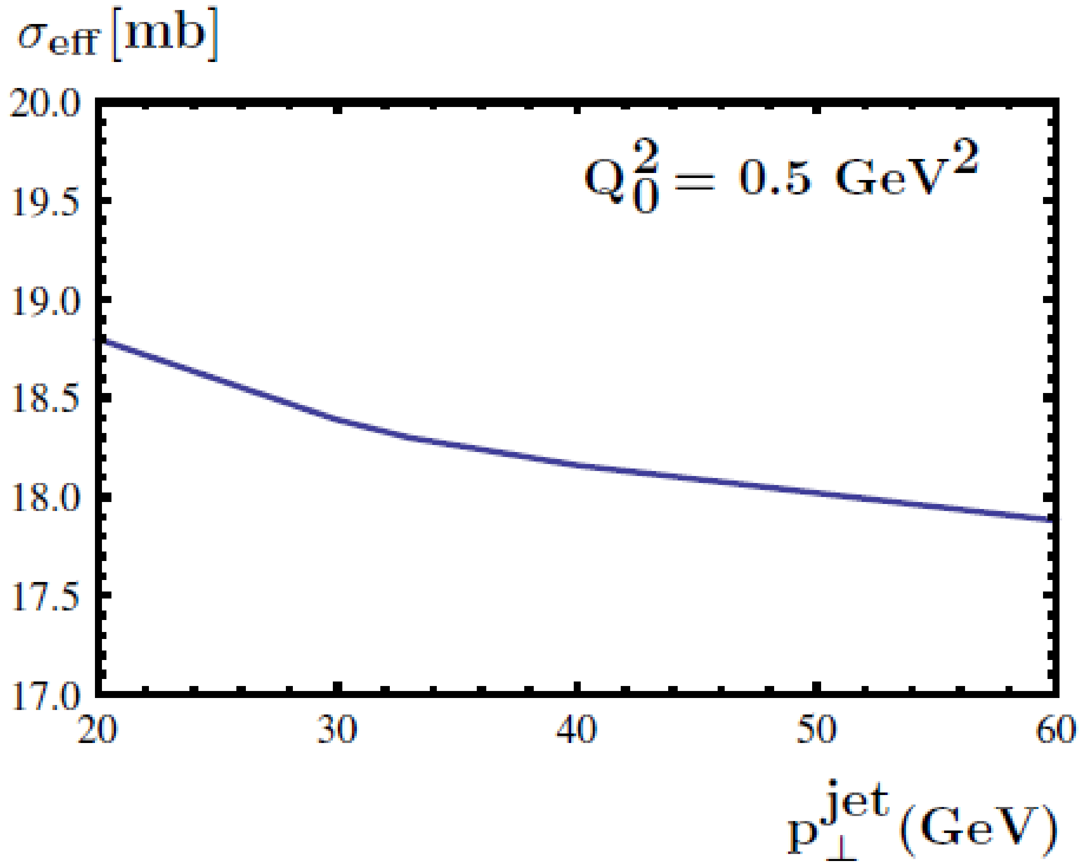
*Nonlinear QCD evolution equations:*

$$\begin{aligned} {}_{[2]}D_a^{b,c}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}) &= S_b(q_1^2,Q_{\min}^2)S_c(q_2^2,Q_{\min}^2)\, {}_{[2]}D_a^{b,c}(x_1,x_2;Q_0^2,Q_0^2;\vec{\Delta}) \\ &+ \sum_{b'}\int_{Q_{\min}^2}^{q_1^2}\frac{dk^2}{k^2}\frac{\alpha_s(k^2)}{2\pi}S_b(q_1^2,k^2) \times \int\frac{dz}{z}P_{b'}^b(z)\, {}_{[2]}D_a^{b',c}\left(\frac{x_1}{z},x_2;k^2,q_2^2;\vec{\Delta}\right) \\ &+ \sum_{c'}\int_{Q_{\min}^2}^{q_2^2}\frac{dk^2}{k^2}\frac{\alpha_s(k^2)}{2\pi}S_c(q_2^2,k^2) \times \int\frac{dz}{z}P_{c'}^c(z)\, {}_{[2]}D_a^{b,c'}\left(x_1,\frac{x_2}{z};q_1^2,k^2;\vec{\Delta}\right). \end{aligned}$$

$$\begin{aligned} {}_{[1]}D_h^{(a,b)}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta}) &= \sum_{a',b',c'}\int^{\min(q_1^2,q_2^2)}\frac{dk^2}{k^2}\frac{\alpha_s(k^2)}{2\pi}\int\frac{dy}{y^2}D_h^c(y,k^2) \\ &\times \int\frac{dz}{z(1-z)}P_c^{(a',b')}(z)\,\mathcal{D}_{a'}^a\left(\frac{x_1}{zy},q_1^2;k^2\right) \times \mathcal{D}_{b'}^b\left(\frac{x_2}{(1-z)y},q_2^2;k^2\right). \end{aligned}$$

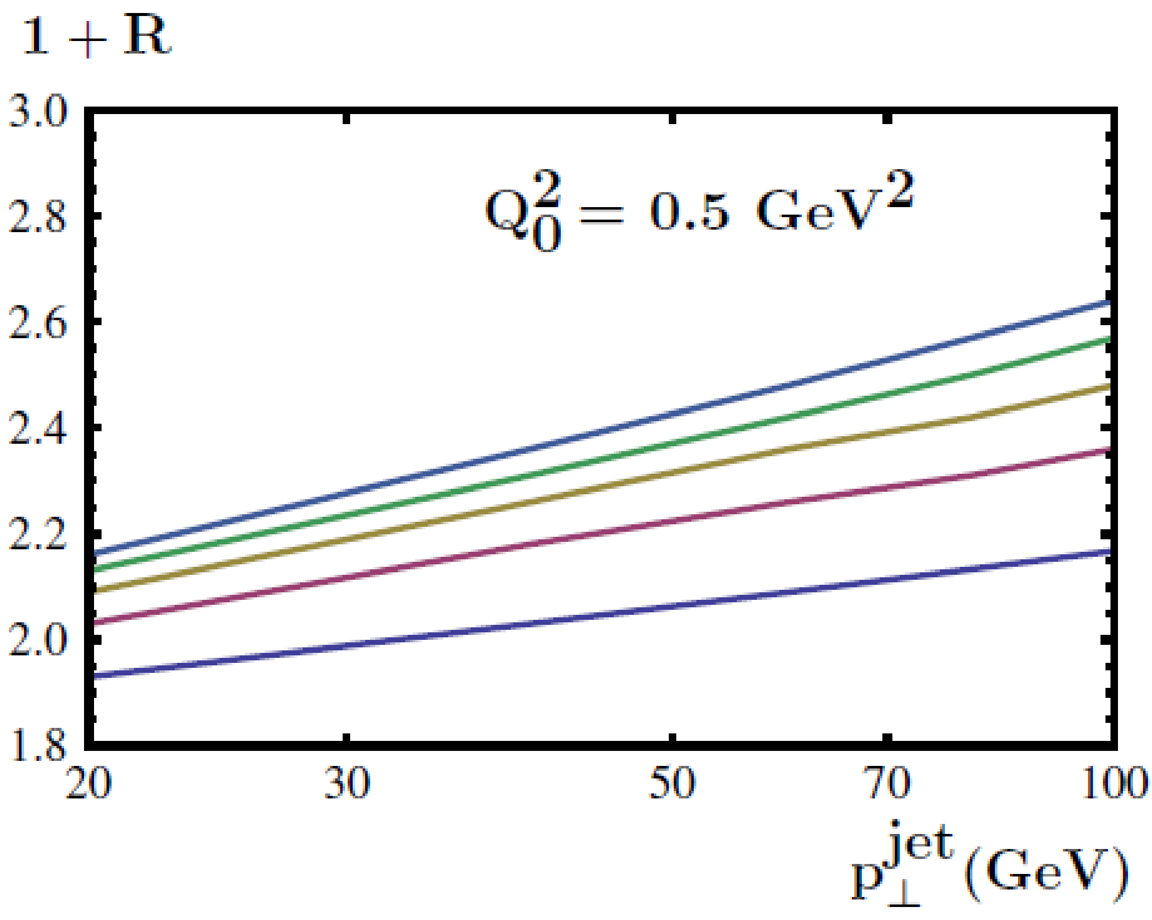
*D0 physics (slightly larger energies )*






$\sigma_{\rm eff}$ for the $Wjj$ process at LHC energy

*4 gluonic jets at LHC.*



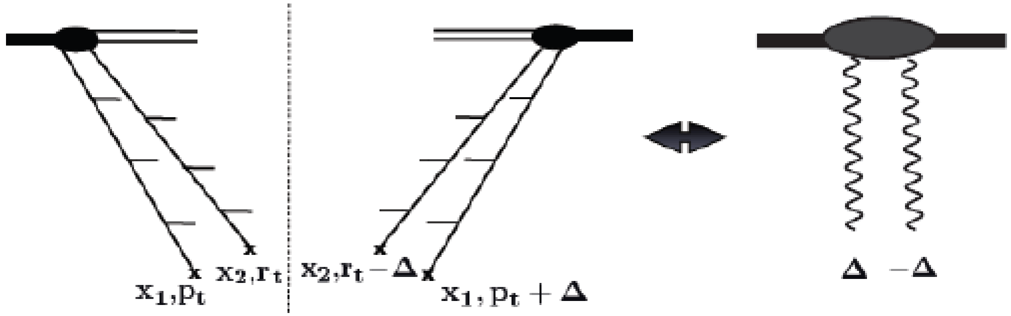

*Similar calculations for Wjj,ZZ,WW at LHC, CDF –see BDFS 2013*

*Recently confirmed Gaunt,Maciula,Szczurec 2014*

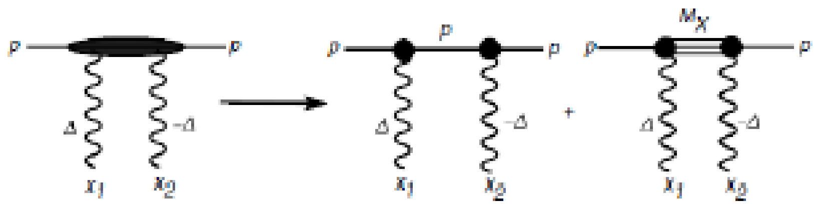
*Multiparton correlations decrease the $\sigma_{eff}$ by the order of 2, and consequently Increase dijets numbers in UE. The $\sigma_{eff}$ is now dynamical and depends on hard scale.*

## *Soft correlations in MPI (BDFS arXiv 1206.5594)*

**At such x the Gribov–Regge high energy hadron interactions picture based on soft Pomeron exchange becomes applicable. In this picture the two soft partons originate from two independent "multiperipheral ladders" represented by cut Pomerons**




Pomeron amplitude is practically pure imaginary. As a result, this amplitude equals that for the diffractive cut of the two-Pomeron diagram




Hence $R(x_1, x_2, -\Delta^2) = \dfrac{{}_{[2]}D(x_1, x_2; Q_0^2, Q_0^2; \Delta)}{G(x_1, Q_0^2)\, G(x_2, Q_0^2)}$

$$= R_{\text{el}}(x_1, x_2, t) + R_{\text{diff}}(x_1, x_2, t) \qquad t = \Delta^2$$

$$R_{\text{el}}(x_1, x_2, t) = g_1(x_1, t) g_2(x_2, t) \qquad (g_i = \text{single GPD})$$

$$\frac{R_{\text{diff}}(x_1, x_2, t)}{R_{\text{el}}(x_1, x_2, t)}$$

from ratio of elastic and diffractive production of vector mesons at x at HERA

$$\left.\frac{R_{\text{diff}}(x_1, x_2, t)}{R_{\text{el}}(x_1, x_2, t)}\right|_{t=0} \equiv \omega_g = 0.2 \pm 0.05$$

sensitive to cutoff in $M_Y$

measure of the longitudinal correlation of partons

$B_{inel}/B_{el} \sim 0.28$

$$\eta \equiv \frac{(1/S)_{\text{corr}}}{(1/S)_{\text{uncorr}}} = 1 + 2\omega_g \frac{2B_{\text{el}}}{B_{\text{el}} + B_{\text{inel}}} + \omega_g^2 \frac{B_{\text{el}}}{B_{\text{inel}}}.$$

$\omega_g = 0.2 \pm 0.05$ *yields* $\eta = 1.8 \pm .2$

This mechanism grows with decrease of x - very small at x > $10^{-2}$ but drops at fixed x with increase of $Q^2$. Works to compensate the increase of gluon radius with decrease of x.

*Conclusions:*

*1. There are important additions to a simple mean field/geometrical picture of underlying event.*

*2. Such corrections may include multiparton correlations, both soft and perturbative, That explain MPI and for small pt will give significant contributions to multiplicities.*

*3. Other types of correlations are currently under study by many groups, In particular large MPI at zero impact paramter (3 to 4) must lead to large color Correlations. Such correlations are in initial state of study (see e.g. HERWIG people talks+ recent work by Argyropoulos and Sjostrand*

Effects of color reconnection on $t\bar{t}$ final states at the LHC
Spyros Argyropoulos, Torbjörn Sjöstrand. Jul 24, 2014. 21 pp.
LU-TP-14-23, DESY-14-134, MCNET-14-15
e-Print: **arXiv:1407.6653**