Double parton correlations in Light-Front constituent quark models

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In collaboration with:

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Outline

- Introduction: double parton correlations (DPCs) in double parton scattering (DPS)

- Double parton distribution functions (dPDFs) from DPS

- dPDFs in non relativistic (NR) constituent quark models:
  (M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013))

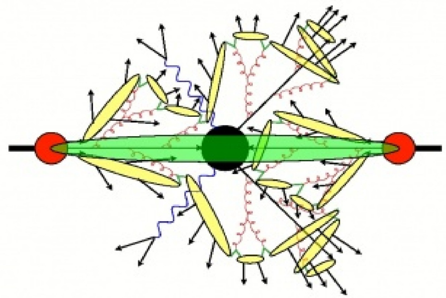
- dPDFs in constituent quark models: a Light-Front approach
  (M. R., S. Scopetta, M. Traini and V.Vento, arXiv:1409.1500v1 [hep-ph])

- pQCD evolution of the model calculation of dPDFs

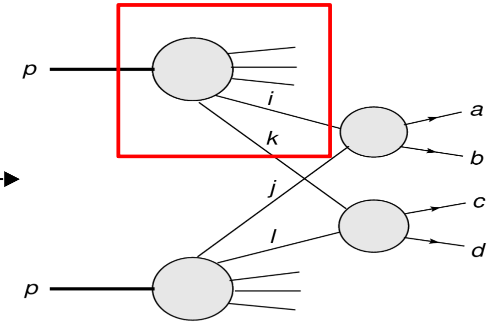
- Conclusions

DPS and dPDFs from multi parton interactions

- Multi parton interaction (MPI) can contribute to the, *pp* and *pA,* cross section @ the LHC:



DPS →



- The cross section for a DPS event can be written in the following way:
(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))

$$d\sigma = \frac{1}{S}\sum_{i,j,k,l}\int d\vec{z}_\perp F_{ik}(x_1,x_2,\vec{z}_\perp,\mu)F_{jl}(x_3,x_4,\vec{z}_\perp,\mu)$$

$$\times\hat{\sigma}_{ij}(x_1,x_2\sqrt{s},\mu)\hat{\sigma}_{kl}(x_3,x_4\sqrt{s},\mu)$$

dPDF

Momentum scale

Transverse distance between the two partons

Momentum fraction carried by the parton inside the hadron

DPS processes can be important!

- background for fundamental studies, *e.g:*

$$pp \to W\ \ H \to l\nu b\bar{b}$$

- new information on the parton structure of protons from dPDF

Parton correlations and dPDFs

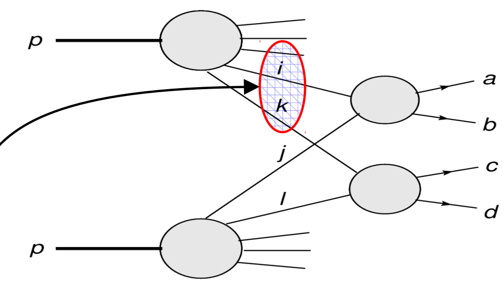
@ LHC kinematics it is often used a factorized form of the dPDFs: $(x_1, x_2) - z_\perp$ factorization:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

and $x_1,\ x_2$ factorization:

$$F_{ij}(x_1, x_2, \mu) = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF}}\, q_j(x_2, \mu)}\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

NO CORRELATION ANSATZ



In this scenario, parton correlations inside the proton are neglected!

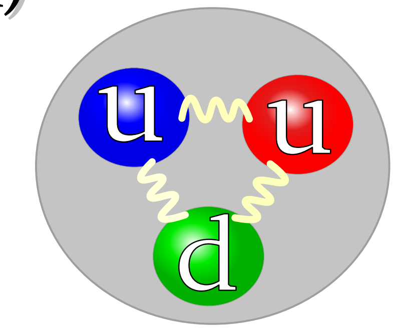
* Is it a good approximation? important for LHC fundamental studies

*Can we have new information on the proton structure observing MPI?

Double Parton Correlations (DPCs)

- In principle, they are present!
- Several authors addressing this issue:
  Many published papers: Korotkikh and Snigirev (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012), Schweitzer, Strikman and Weiss (2013)...
- dPDFs are non-perturbative quantities ⟺ DPCs not calculated directly from QCD
- Our contributions (quark models as possible tools):
  - → calculations of dPDFs with:
    (M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013))
  - → Calculations of dPDFs within a Light-Front approach
    (M. R., S. Scopetta, M. Traini and V.Vento, arXiv:1409.1500v1 [hep-ph])
- A first important investigation, which motivated our work, has been presented in a modified version of the MIT bag model
  (H.-M. Chang, A.V. Manohar, and W.J. Waalewijn, PRD 87, 034009 (2013))
  (In its simplest version, the MIT bag model is an independent particle model and no correlations are found)

DPCs in constituent quark models (CQM)



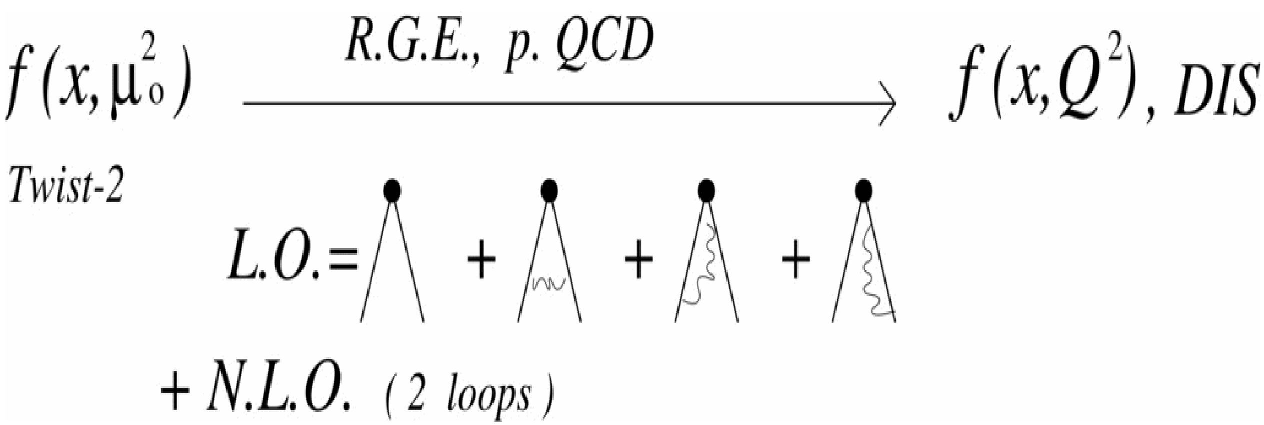
- Main features:
  - potential model
  - effective particles
  - particles are strongly bound and correlated
- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region, while LHC data are available at small x
- At very low x , due to the large population of partons, the role of correlations may be less relevant BUT there is no quantitative theoretical estimate available
- CQM calculations are able to reproduce the gross-feature of experimental PDF in the valence region
- Results can be quite general. In DIS Physics, CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

Model calculations of PDFs in the valence region

- In order to consistently compare data of twist-2 PDFs with the predictions of a CQM, one has to follow a 2-steps procedure: **(firstly suggested by R.L. Jaffe and G.G. Ross, PLB 398 (1980) 313)**

  1. evaluate in the model the twist-2 part of the corresponding observable, which has to be related to a low momentum scale, μ_0^2
  2. perform a perturbative QCD evolution to the DIS experimental scale, Q^2



Caveat: in the simplest CQM picture, ALL the gluons and sea quarks are perturbatively generated

Model calculations of quark-quark dPDFs

- Unpolarized quark-quark dPDFs are defined through Light-Cone quantized states and fields (see, *e.g.*, **M. Diehl, D. Ostermeier, A. Schäfer JHEP 03 (2012) 089**)

- A Non Relativistic (NR) reduction allows one to calculate it, in momentum space, in terms of intrinsic wave functions (WFs):

$$F_{12}(x_1, x_2, \vec{k}_\perp) = 3 \int d\vec{k}_1 d\vec{k}_2$$

A.V. Manhoar and W.J. Waalewijn, PRD 85 114009 (2012))

$$\times \Psi^* \left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2} \right) \hat{P}_{q_1}(1) \hat{P}_{q_2}(2) \Psi \left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2} \right)$$

$$\times \delta \left(x_1 - \frac{k_1^+}{P^+} \right) \delta \left(x_2 - \frac{k_2^+}{P^+} \right)$$

We need to choose the WF corresponding to a suitable CQM to perform the calculation

$$a^\pm = a_0 \pm a_3$$

Flavor projector:

$$\hat{P}_{u(d)} = \frac{1 \pm \tau_3(i)}{2}$$

Conjugated variable to $z_\perp$

dPDF fulfills sum rules, when, *e.g.* q_1= u, q_2=u:

$$\int dx_1 dx_2 uu(x_1, x_2, 0) = 2$$

J.R. Gaunt and W.J. Stirling, JHEP 03 (2010) 005

Our first choice: the Isgur and Karl (IK) model

IK is based on a One Gluon Exchange (OGE) correction to the harmonic oscillator (H.O.), generating a hyperfine interaction which breaks SU(6). Nucleon state (up to the 2nd energy shell):

$$|N\rangle = a|^2S_{1/2}\rangle_S + b|^2S'_{1/2}\rangle_S + c|^2S_{1/2}\rangle_M + d|^4D_{1/2}\rangle_M$$

$$|^{2S+1}X\rangle_t, \quad t = A, M, S = \text{ symmetry type}$$

$$\begin{cases} a = 0.931, \; b = 0.274, \; c = 0.233, \; d = 0.067 & \text{From spectroscopy} \\ a = 1, \; b = c = d = 0 & \text{H.O. is recovered} \end{cases}$$

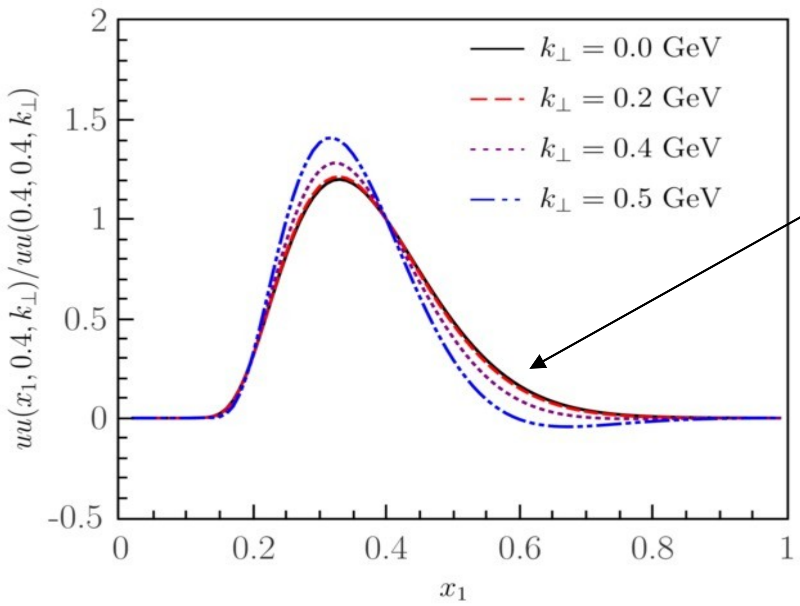
IK is a suitable framework for a first CQM calculation of DPCs:

- IK is the prototype of any other CQM, low energy properties of the nucleon (spectrum and electromagnetic form factors at small momentum transfer) are basically reproduced
- Gross features of the standard PDFs are reproduced

The model results correspond to a low momentum scale (hadronic scale, μ_0). There are only valence quarks: the scale has to be very low ($\mu_0 \simeq 0.300$ GeV according to NLO pQCD). Data are taken at a high momentum scale t. QCD evolution needed!

RESULTS: $(x_1, x_2) - k_\perp$-factorization

(M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013))



The ratio: $\dfrac{uu(x_1, 0.4, k_\perp)}{uu(0.4, 0.4, k_\perp)}$

for four values of $k_\perp$

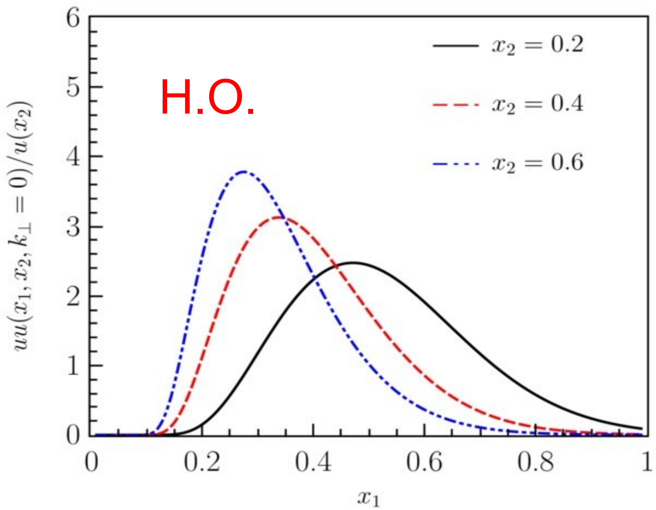
If the factorization worked, the shown ratio should not depend on $k_\perp$

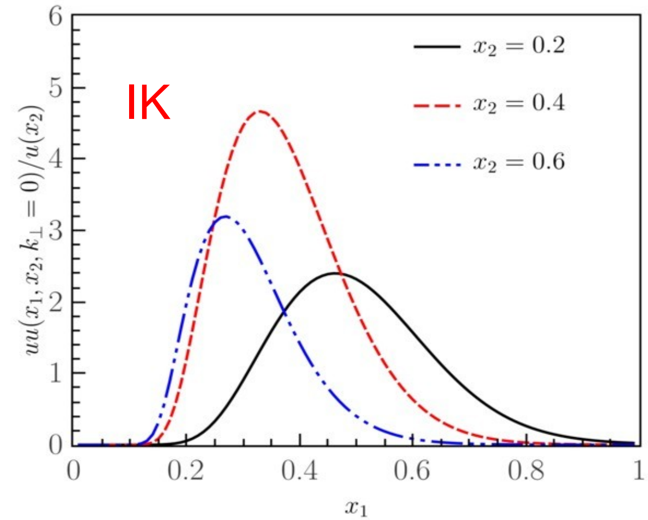
valid in H.O. model

violated in the IK model
(and in the modified bag model calculated by **H.-M. Chang, A.V. Manohar, and W.J. Waalewijn, PRD 87, 034009 (2013)**)

RESULTS: $x_1 - x_2$ factorization

(M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013))





Here it is shown the ratio: $\dfrac{uu(x_1, x_2, k_\perp = 0)}{u(x_2)}$ for three values of x_2

No x_2 dependence would be found if the $x_1 - x_2$-factorization were valid

- This factorization is badly violated
- Result already found in the Bag calculation

→ Model independent feature

Are some improvements necessary? Yes!

To pQCD evolve

Overcome the so called "bad support problem ":
the dPDFs should be zero when: $x_1 + x_2 > 1$
This condition is not realized in the described CQM!

Implement the Relativity

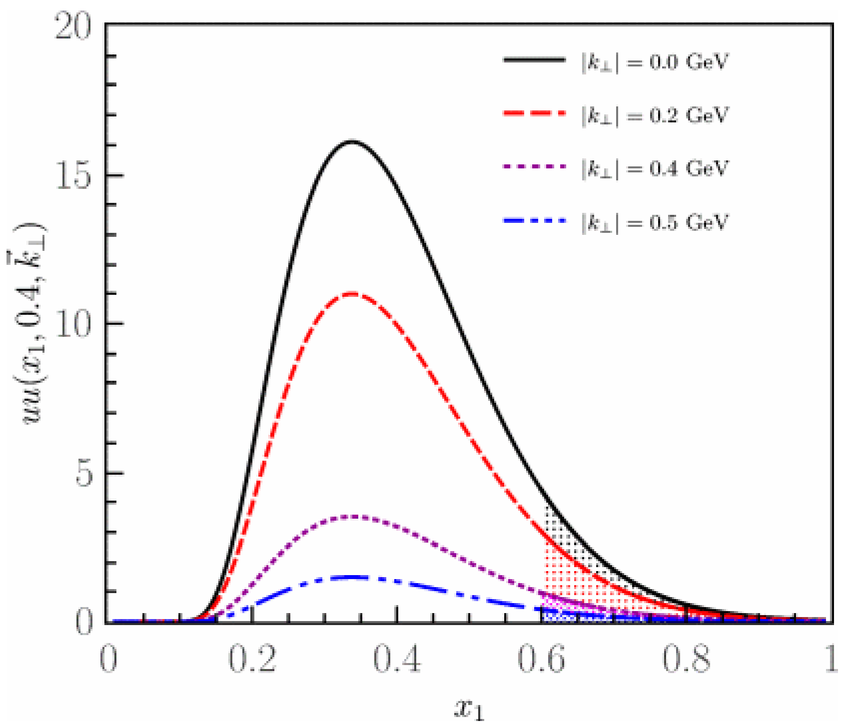
In this approach:

$$\delta\left(x_i - \frac{k_i^+}{P^+}\right) \Longrightarrow \delta\left(x_i - \frac{k_i^+}{M}\right)$$

Proton Mass

so that one finds:

$$\sum_i x_i > 1$$



we are not describing properly the off-shell *"i-"* parton inside the hadron

we need a new approach: the Light-Front

Solution: the Light-Front approach

Relativity can be implemented by using a Light-Front (LF) approach yielding, among other good features, the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

- Full Poincaré covariance
- fixed number of on-mass-shell particles

Among the 3 possibles forms of RHD we have chosen the LF one since there are several advantages. The most relevant are the following:

- ✓ 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) P^+, $P_\perp$, iii) Rotation around z.
- ✓ The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- ✓ in a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- ✓ The IMF (Infinite Momentum Frame) description of DIS is easily included.

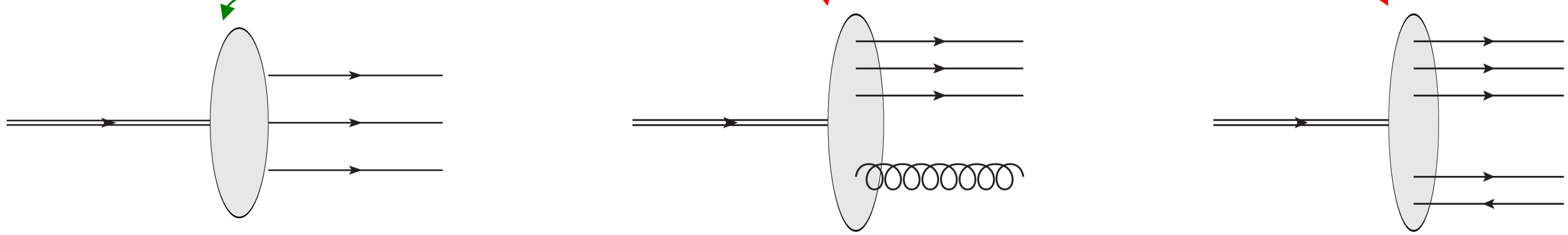
The LF approach is extensively used for hadronic studies (e.m.form factors, PDFs, GPDs, TMDs........)

A Light-Front wave function representation

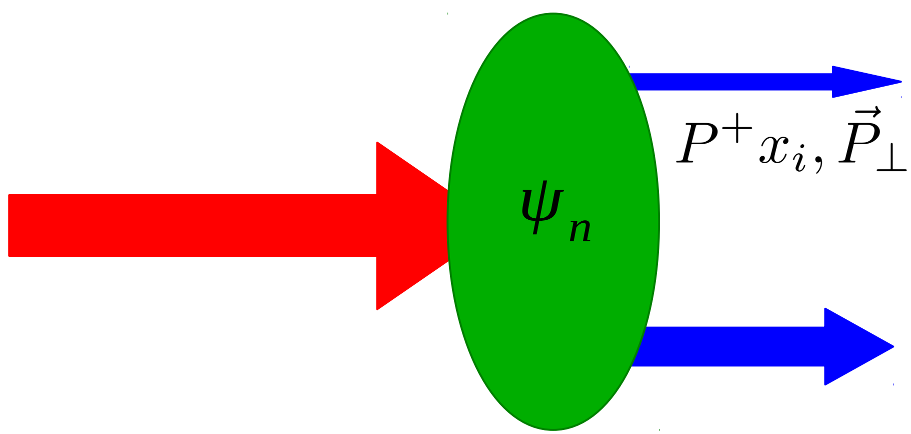
The proton wave function can be represented in the following way:

see *e.g.:* **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+ \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle + \psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots .$$



$p : P^+, \vec{P}_\perp$



$$\begin{cases} x_i = \dfrac{k_i^+}{P^+} \\ \sum_i^n x_i = 1 \\ \sum_i^n \vec{k}_{\perp i} = 0 \end{cases}$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i) \longleftrightarrow$ Invariant under LF boosts!

A Light Front wave function representation

It is possible to connect the front-form description of states and the canonical, instant-form one:

See *e.g.:* **B. D. Keister, W. N. Polyzou Adv. Nucl .Phys. 20, 225 (1991)**

$$|\vec{k}_\perp, \lambda, \tau\rangle_{[l]} \propto \sqrt{2k_0} \sum_{\lambda'} D^{1/2}_{\lambda\lambda'}(R_{il}(\vec{k})) |\vec{k}_\perp, \lambda', \tau\rangle_{[i]}$$

Melosh rotations

A relation between hadron wave functions $\psi^{[l]}_\lambda$, $\psi^{[i]}_\lambda$ can be obtained, *e.g.* for n=3:

$$\psi^{[l]}_\lambda(\beta_1, \beta_2, \beta_3) \propto \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right] \sum_{\mu_1\mu_2\mu_3} D^{1/2*}_{\mu_1\lambda_1}(R_{il}(\vec{k}_1)) D^{1/2*}_{\mu_2\lambda_2}(R_{il}(\vec{k}_2)) D^{1/2*}_{\mu_3\lambda_3}(R_{il}(\vec{k}_3)) \psi^{[i]}_\lambda(\alpha_1, \alpha_2, \alpha_3)$$

$$\begin{cases} \beta_i = x_i, \vec{k}_i, \lambda_i, \tau_i \\ \alpha_i = \vec{k}_i, \mu_i, \tau_i \\ \omega_i = k_{0i} \\ M_0 = \sum_i \omega_i \end{cases}$$

We need a CQM in order to describe:

$$\psi^{[i]}_\lambda$$

dPDFs in a Light-Front approach

Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)**, for GPDs, we obtained the following expression of the dPDF, from the Light-Front description of quantum states in the intrinsic system: $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$

$$F_{12}(x_1, x_2, k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, k_\perp)\Phi(\{\vec{k}_i\}, -k_\perp)$$

$$\times \quad \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

$$M_0 = \sum_i \sqrt{\vec{k}_i^2 + m^2}$$

The "support problem" is now solved!

$$x_1 + x_2 > 1 \Rightarrow F_{12}(x_1, x_2, k_\perp) = 0$$

$$\Phi(\{\vec{k}_i\}, \pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

Now we need a model to properly describe the hadron wave function in order to estimate the Light-Front dPDF

$$\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3) = D^{\dagger 1/2}(R_{il}(\vec{k}_1))D^{\dagger 1/2}(R_{il}(\vec{k}_2))D^{\dagger 1/2}(R_{il}(\vec{k}_3))\psi^{[i]}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

A Hyper-central CQM

We have chosen the following CQM for its capability to basically describe the hadron spectrum, despite of its simplicity (**P. Faccioli, M. Traini, V. Vento, Nucl. Phys. A 656, 400-420 (1999)**)

$$\psi^{[i]} = \frac{1}{\pi\sqrt{\pi}} \Psi(k_\xi) \times SU(6)_{spin-isospin}$$

$$k_\xi = \sqrt{2(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)}$$

Where the function $\Psi(k_\xi)$ is solution of the Mass equation:

$$(M_0 + V)\Psi(k_\xi) \equiv \left(\sum_i^3 \sqrt{\vec{k}_i^2 + m^2} - \frac{\tau}{\xi} + \kappa_l \xi \right) \Psi(k_\xi) = M\Psi(k_\xi)$$

$$\tau = 3.30, \qquad \kappa_l = 1.80 \text{ fm}^{-2}$$

$$\Psi(k_\xi) = \sum_{\nu=0}^{16} c_\nu \frac{(-1)^\nu}{\alpha^3} \left[\frac{2\nu!}{(\nu+2)!} \right]^{1/2} e^{-k_\xi^2/(2\alpha^2)} \sum_{m=0}^{\nu} \frac{(-1)^m}{m!} \frac{(\nu+2)!}{(\nu-m)!(m+2)!} \left(\frac{k_\xi^2}{\alpha^2} \right)^m$$

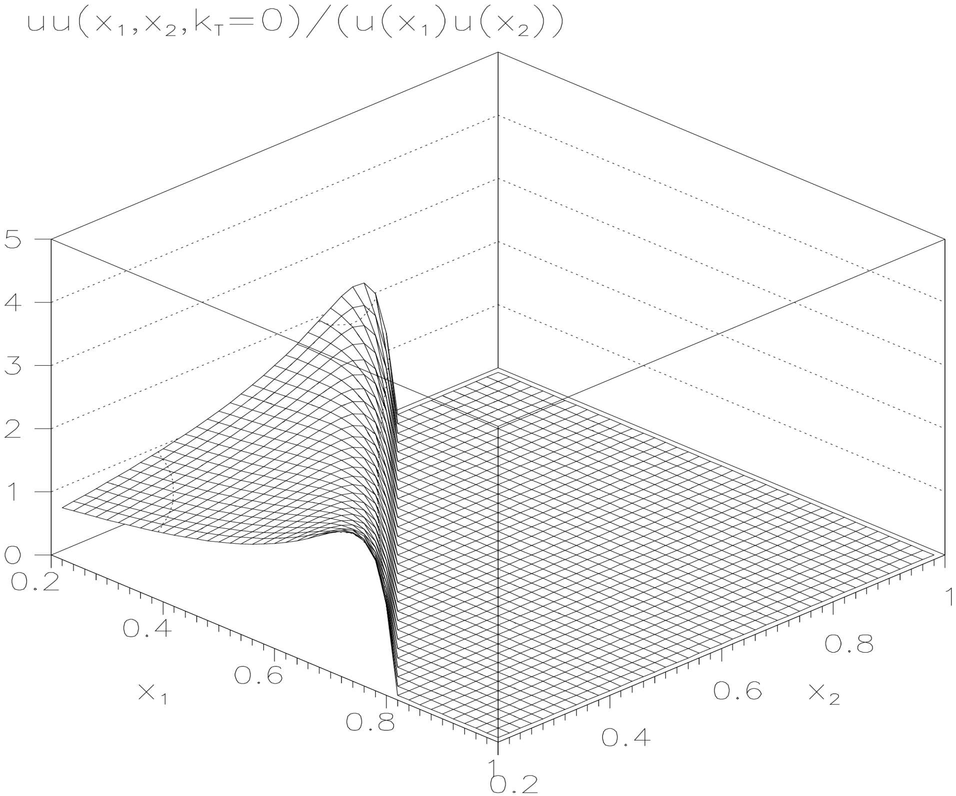
$$\alpha = 7.9 \text{ fm}^{-1}$$

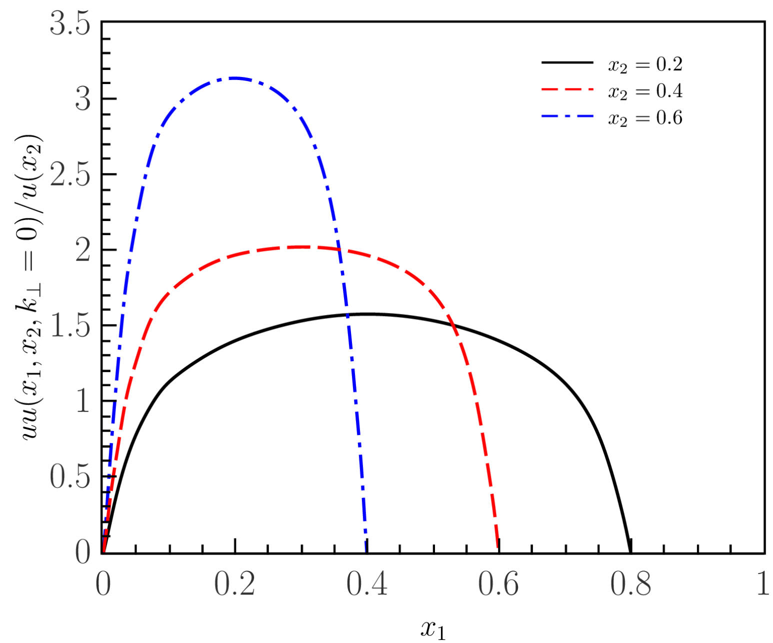
c_ν: expansion coefficients

This model has been used in:

P. Faccioli, M. Traini, V. Vento, NPA 656, 400-420 (1999)
S. Boffi, B. Pasquini and M. Traini, NPB 649, 243 (2003)
S. Boffi, B. Pasquini and M. Traini, NPB 680, 147-163 (2004)...
M. Traini, PRD89, 034021 (2014)

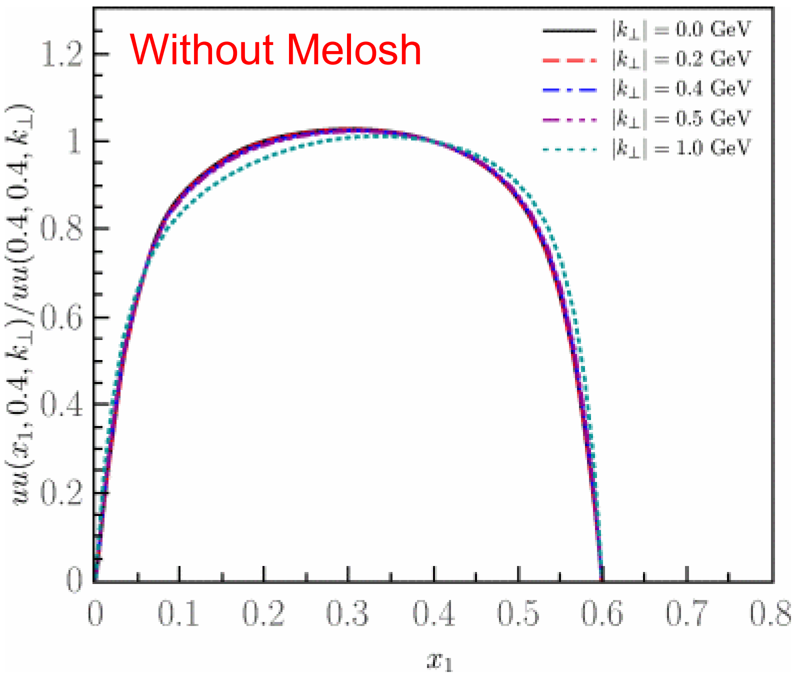
RESULTS: $x_1 - x_2$ factorization

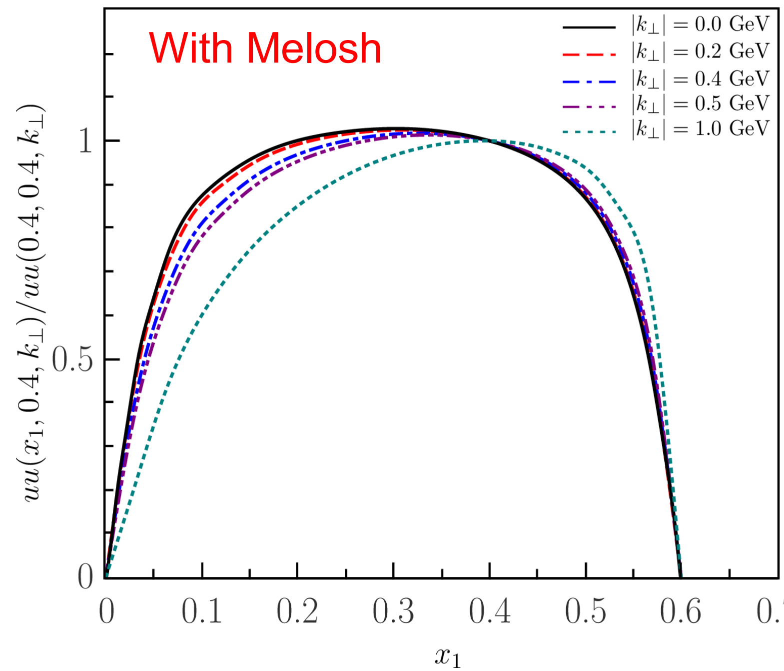




- The “support problem” is clearly solved!
- Thanks to the solution of the “support problem”, the symmetry $uu(x_1, x_2, 0) = uu(x_2, x_1, 0)$ due to the particle indistinguishability, is restored!
- Also in this relativistic case, the $x_1 - x_2$ factorization is strongly violated!

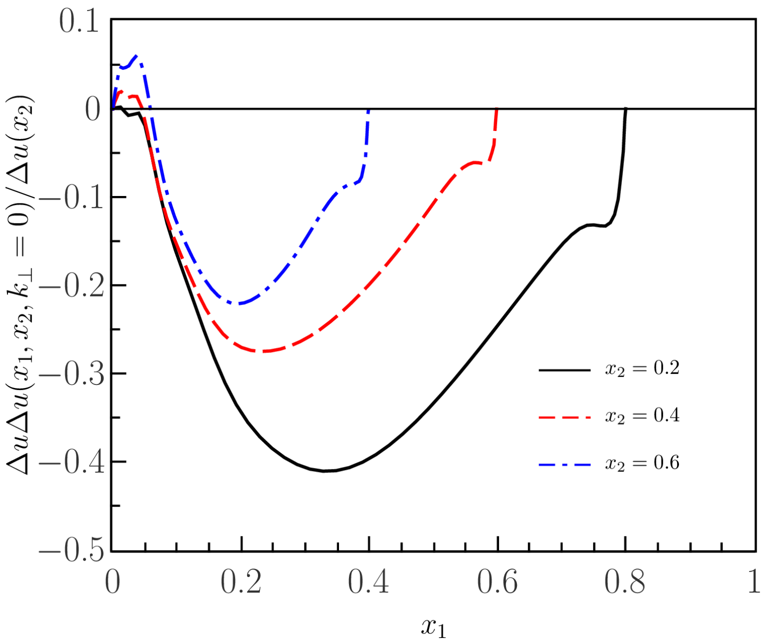
RESULTS: $(x_1, x_2) - k_\perp$-factorization

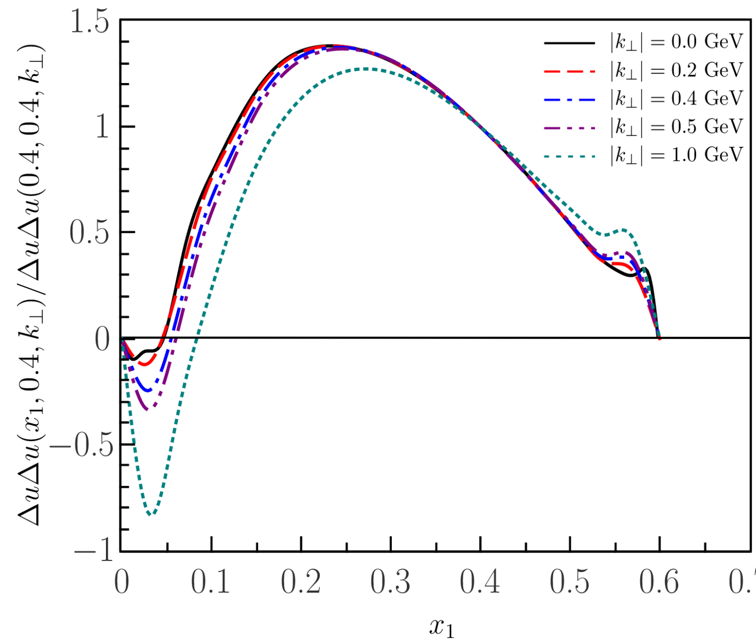




- In this relativistic case the factorization is clearly violated;
- It is remarkable how the Melosh rotations, properly taken into account in the calculation, increase the violation of the $(x_1, x_2) - k_\perp$ factorization!

Results for spin correlations





$$
\begin{aligned}
u_{\uparrow(\downarrow)} u_{\uparrow(\downarrow)}(x_1, x_2, k_\perp) \;&=\; 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \, \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right) \\
&\times \; \Phi^*(\{\vec{k}_i\}, k_\perp) \frac{1 \pm \sigma_3(1)}{2} \frac{1 \pm \sigma_3(2)}{2} \Phi(\{\vec{k}_i\}, -k_\perp)
\end{aligned}
$$

Here we have calculated: $\Delta u \Delta u(x_1, x_2, k_\perp) = \sum_{i=\uparrow,\downarrow} u_i u_i - \sum_{i \neq j = \uparrow,\downarrow} u_i u_j;$

(defined in **M. Diehl et Al, JHEP 1203, 089 (2012),**
M. Diehl and T. Kasemets, JHEP 1305, 150 (2013))

$$|\Delta u \Delta u| \leq uu$$

Positivity bound

This particular distribution, different from zero also in an unpolarized proton, contains more information on spin correlations, which could important at small x and large *t* (LHC) !

Also in this case, both factorizations, $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ are strongly violated!

pQCD evolution of dPDFs calculations

The evolution equations for the dPDFs are based on a generalization of the DGLAP equations used, *e.g.*, for the single PDFs (**Kirschner 1979, Shelest, Snigirev, Zinovev 1982).**
Introducing the Mellin moments:

$$\langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} = \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n_1-2}\, x_2^{n_2-2}\, x_1 x_2 F_{q_1,q_2}(x_1,x_2,Q^2)\ ,$$

defining the moments of the quark-quark NS splitting functions at LO as follows:

$$P_{NS}^{(0)}(n_1) = \int dx\ x^{n_1} P_{NS}^{(0)}(x)\ ,$$

using the modified DGLAP evolution equations, without the inhomogeneus term, since we are evaluating the valence dPDFs, one gets

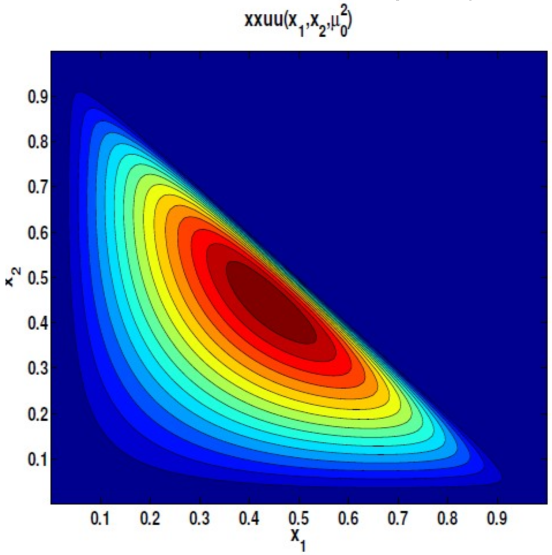
$$\langle x_1 x_2 F_{i_q 1, i_q 2}(Q^2)\rangle_{n_1,n_2} = \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)}\right)^{\frac{-P_{NS}^{(0)}(n_1) - P_{NS}^{(0)}(n_2)}{\beta_0}} \langle x_1 x_2 F_{i_q 1, i_q 2}(\mu_0^2)\rangle_{n_1,n_2}$$

The dPDF at any high energy scale is obtain by inverting the Mellin transformation:

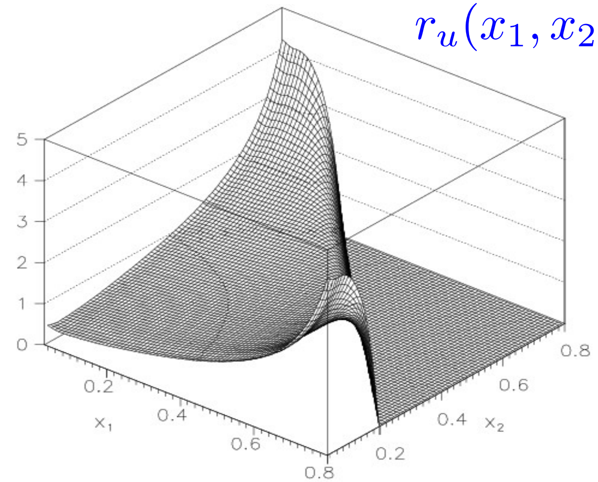
$$\begin{aligned} x_1 x_2 F_{q_1,q_2}(x_1,x_2,Q^2) &= \frac{1}{2\pi i}\oint_{\mathcal{C}} dn_1\, \frac{1}{2\pi i}\oint_{\mathcal{C}} dn_2 \\ &\times\ x_1^{(1-n_1)}\, x_2^{(1-n_2)}\, \langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} \end{aligned}$$

pQCD evolution of the LF dPDFs

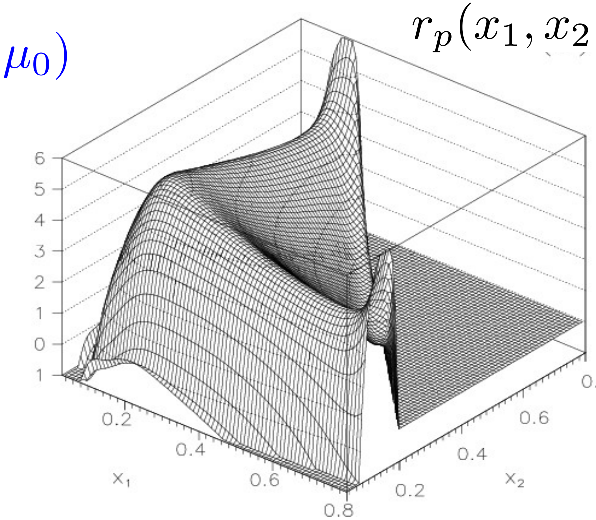
Hadronic scale (only valence quarks): $Q^2 = \mu_0^2 \simeq 0.1 \text{ GeV}^2$



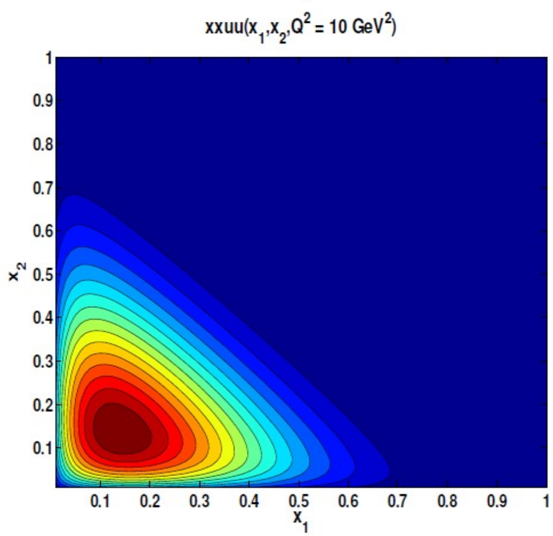
$r_u(x_1, x_2; \mu_0)$



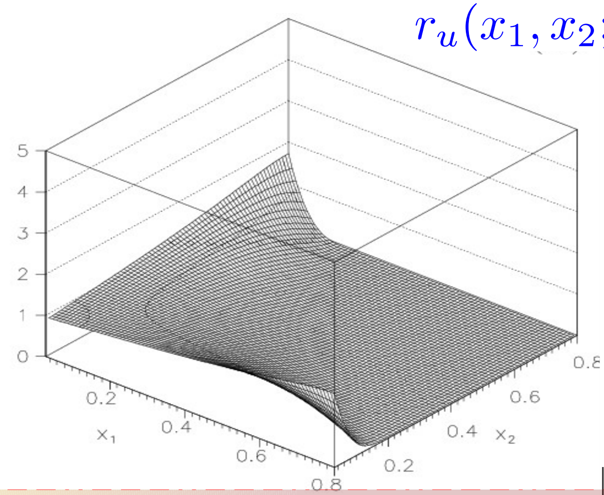
$r_p(x_1, x_2; \mu_0)$



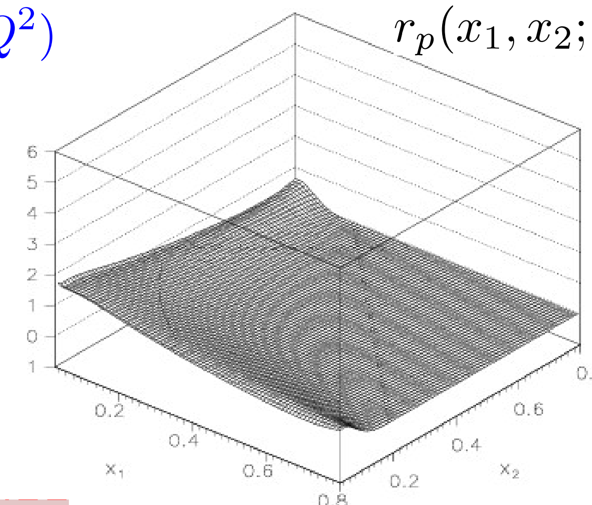
$Q^2 = 10 \text{ GeV}^2$



$r_u(x_1, x_2; Q^2)$



$r_p(x_1, x_2; Q^2)$



$$r_u(x_1, x_2; Q^2) = 2\frac{uu(x_1, x_2; Q^2)}{u(x_1; Q^2)u(x_2; Q^2)} \quad r_p(x_1, x_2; Q^2) = -6.17\frac{\Delta u \Delta u(x_1, x_2; Q^2)}{\Delta u(x_1; Q^2)\Delta u(x_2; Q^2)}$$

All these ratios would 1 if there were no correlations!

Conclusions

A CQM calculation of the dPDFs in NR cases:

M. R., S.Scopetta and V.Vento, PRD 87, 114021 (2013)

- violation of both the x_1, x_2 and $(x_1, x_2) - k_\perp$ factorizations;
- Problems with support and pQCD evolution .

A CQM calculation of the dPDFs with a fully covariant approach:

M. R., S.Scopetta, M. Traini and V.Vento, arXiv:1409.1500v1 [hep-ph]

- solution to the “bad support” problems in the calculation of the dPDFs;
- symmetry in the exchange of two partons in the dPDFs correctly restored;
- violations of both the $(x_1, x_2) - k_\perp$ and x_1, x_2 factorizations for the polarized and unpolarized dPDFs;

pQCD evolution of the LF dPDFs:

- the results show that the factorization ansatz is not justified in the valence quark region, also at high energy scales;
- at very small x , the role of the correlations is less important after the evolution to
- the experimental scales;
- The role of spin correlations is still relevant after pQCD evolution, even at small x .

What are we working on

- pQCD evolution of the calculated dPDF with the contribution of the singlet sector;
- Non perturbative Gluons and sea quarks (higher fock states) to be included into the scheme.

Direct link to LHC Physics

A link between dPDFs and GPDs

M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089

The dPDF is formally defined through the Light-cone correlator:

$$F_{12}(x_1,x_2,\vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^{2} dl_i^-\, e^{ix_i l_i^- p^+}\right] \langle p|O(z,l_1)|X\rangle\langle X|O(0,l_2)|p\rangle\Big|_{l_1^+=l_2^+=z^+=0}^{\vec{l}_{1\perp}=\vec{l}_{2\perp}=0}$$

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle\langle p'|$$

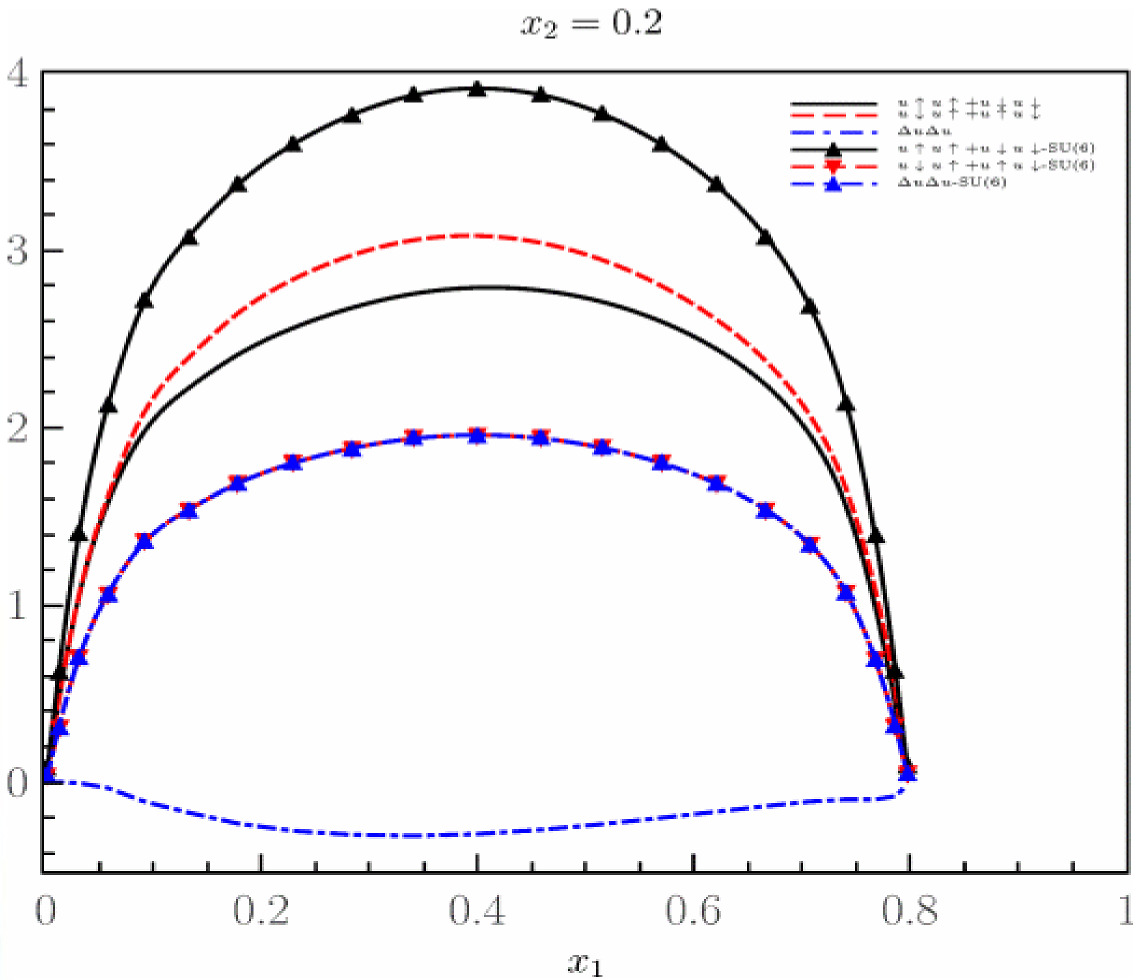
GPDs depending on the impact parameter

$$F_{12}(x_1,x_2,\vec{z}_\perp) \sim \int d\vec{b}\, f(x_1,0,\vec{b}+\vec{z}_\perp)\, f(x_2,0,\vec{b})$$

In GPDs, the variables $\vec{b}$ and x are correlated

Correlations between $\vec{z}_\perp$ and $x_1,\ x_2$ could be present in dPDFs !

Results for spin correlations



$$\frac{uu(x_1, x_2, 0) + \Delta u \Delta u(x_1, x_2, 0)}{2}$$

$$\frac{uu(x_1, x_2, 0) - \Delta u \Delta u(x_1, x_2, 0)}{2}$$

$$\Delta u \Delta u(x_1, x_2, 0)$$

$\triangle$ = SU(6) symmetry