Multiplicity, Jet, and Transverse Mass dependence of Bose-Einstein Correlations in e+e- Annihilation

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BEC Introduction

$$R_2 = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)} \Rightarrow \frac{\rho_2(Q)}{\rho_0(Q)}$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

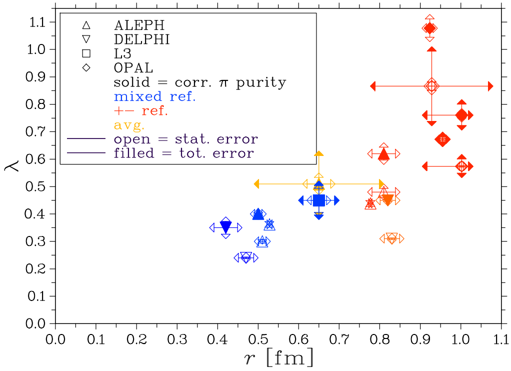
$$R_2(Q) = 1 + \lambda |\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$
$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent

Assuming $S(x)$ is a Gaussian with radius $r \Longrightarrow$

$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-Q^2 r^2}$$

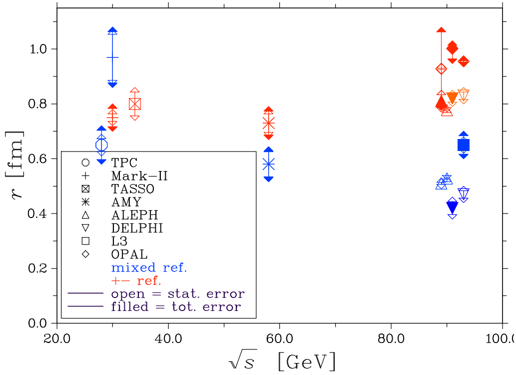
Results from R_2, $\sqrt{s} = M_Z$



– correction for π purity increases λ

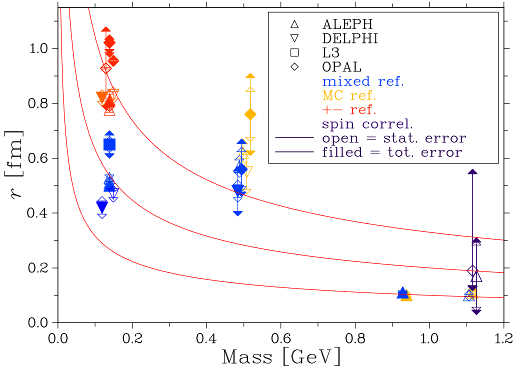
– mixed ref. gives smaller λ, r than $+-$ ref. – Average means little

$\sqrt{s}$ dependence of r



No evidence for $\sqrt{s}$ dependence

Mass dependence of *r* — BEC and FDC



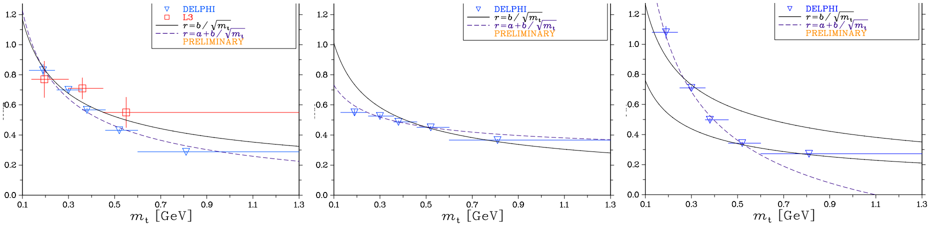
No evidence for $r \sim 1/\sqrt{m}$

$r_{\pi-\pi} \approx r_{\text{K-K}}$

$r(\text{mesons}) > r(\text{baryons})$

Transverse Mass dependence of r in LCMS

longitudinal | side | out



r decreases with m_t

but not equally fast in all components

The L3 Data

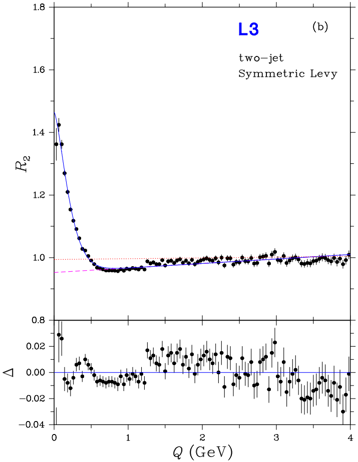
- $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} \approx M_Z$
- about $36 \cdot 10^6$ like-sign pairs of well measured charged tracks from about $0.8 \cdot 10^6$ events
- about $0.5 \cdot 10^6$ 2-jet events — Durham $y_{cut} = 0.006$
- about $0.3 \cdot 10^6 > 2$ jets, "3-jet events"
- use mixed events for reference sample, ρ_0
  corrected by MC (no BEC) for kinematics, resonances, etc.

$$\rho_0 \Longrightarrow \rho_0 \cdot \frac{\rho^{\text{MC}}}{\rho_0^{\text{MC}}}$$

Results – 'Classic' Parametrizations

$R_2 = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$ $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^{\alpha}\right)$
  $0 < \alpha \leq 2$
  $\alpha = 1.34 \pm 0.04$



	Gauss	Edgew	Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

Poor χ^2. Edgeworth and Lévy better than Gaussian, but poor.
Problem is the dip of R_2 in the region $0.6 < Q < 1.5\,\mathrm{GeV}$

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:

$$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$

  where for 2-jet events, $a = 1/m_\mathrm{t}$

$$\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2} \text{ is the “longitudinal” proper time}$$

$$\text{and } m_\mathrm{t} = \sqrt{E^2 - p_z^2} \text{ is the “transverse” mass}$$

- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of prod. points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is

$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau) \delta_\Delta(x - a\tau p)\rho_1(p)$$

- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.

$$\rho_2(p_1,p_2) = \int \mathrm{d}^4 x_1 \mathrm{d}^4 x_2 S(x_1,p_1) S(x_2,p_2) \left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$

- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then

$$R_2(p_1,p_2) = 1 + \lambda \mathrm{Re} \widetilde{H}\left(\frac{a_1 Q^2}{2}\right) \widetilde{H}\left(\frac{a_2 Q^2}{2}\right), \quad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau) \exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction,
  $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\frac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\frac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \;, \quad \alpha \neq 1$$
  where
  - α is the index of stability;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$\begin{aligned} R_2(Q, a_1, a_2) \;=\; & \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right. \\ & \left.\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\}\cdot(1 + \epsilon Q) \end{aligned}$$

BEC in the τ-model

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^{\alpha} + a_2^{\alpha}}{2}$
- Particle production begins immediately, $\tau_0 = 0$
- Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

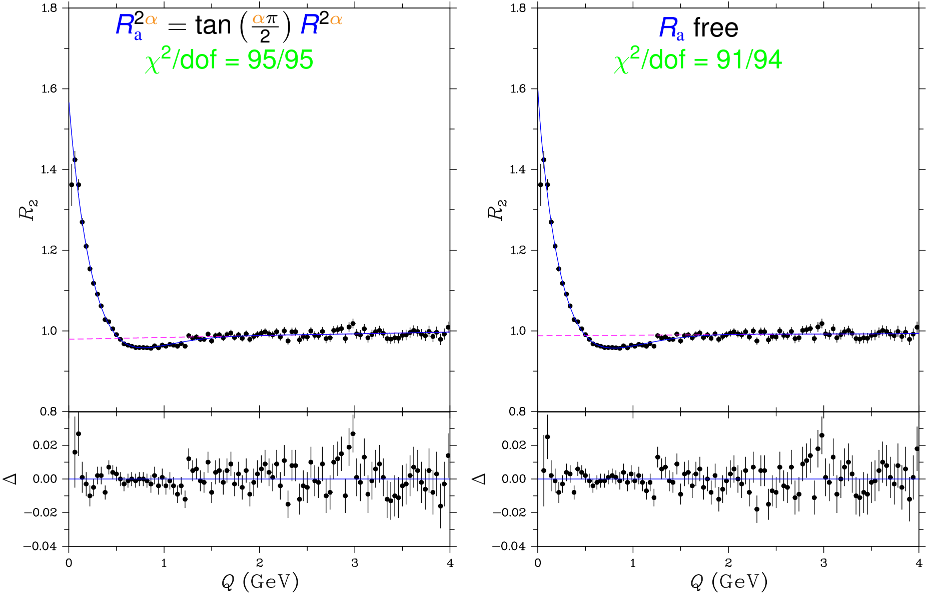
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

Compare to sym. Lévy parametrization:

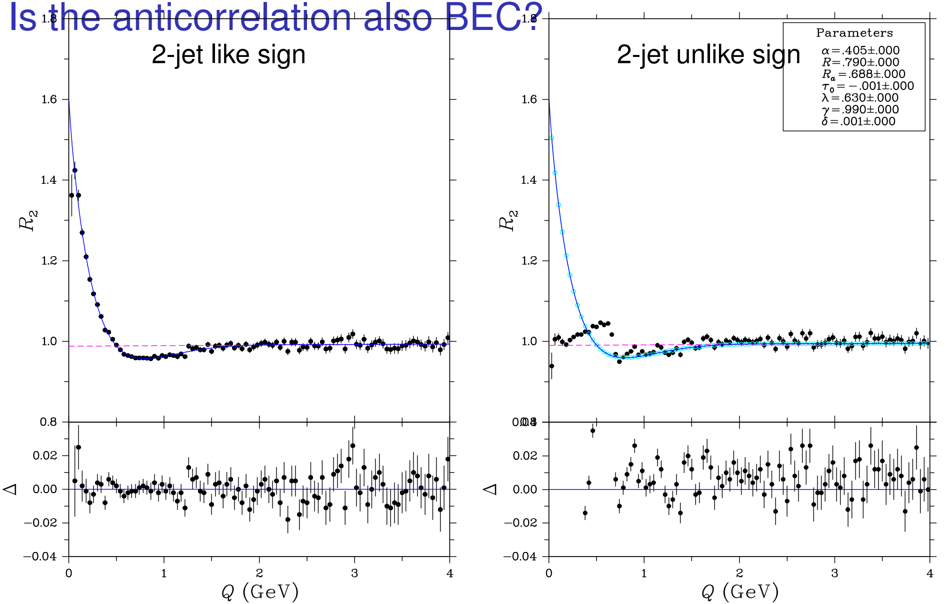
$$R_2(Q) = \gamma \left[1 + \lambda \exp\left[-|rQ|^{\alpha} \right] \right] (1 + \epsilon Q)$$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay



Is the anticorrelation also BEC?



Resonances in anticorrelation region confuse things

But anticorrelation may be present in unlike sign

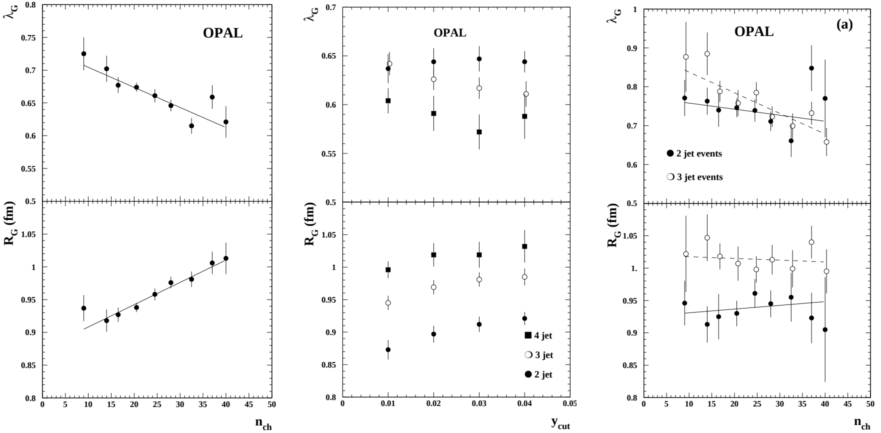
Multiplicity/Jet dependence – OPAL

$$R_2(Q) = \gamma(1 + \lambda\, e^{-Q^2 r^2})(1 + \delta Q + \epsilon Q^2)$$

OPAL,Z.Phys.C72(1996)389

λ

r



$\lambda \searrow$ with n_{ch}
$r \nearrow$ with n_{ch}

$\lambda \searrow$ with n_{jet}
$r \nearrow$ with n_{jet}

$\lambda_{\text{n-jet}} \approx$ indep. of n_{ch}
$r_{\text{n-jet}}$ indep. of n_{ch}

Multiplicity dependence appears to be largely due to number of jets.

Multiplicity/Jet dependence in τ-model

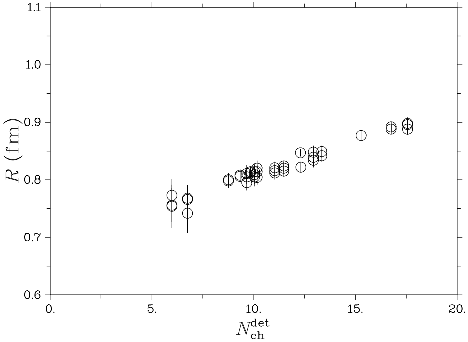
Use simplified τ-model, $\tau_0 = 0$
to investigate multiplicity and jet dependence

To stabilize fits against large correlation of parameters α and *R* fix $\alpha = 0.44$

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

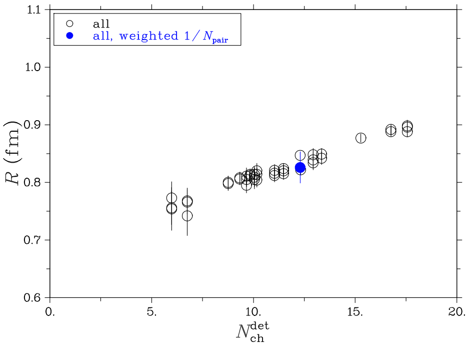
PRELIMINARY



R increases with multiplicity

Multiplicity dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ PRELIMINARY



R not constant

$\Longrightarrow$ R from fit is an average

But maybe not the average we want

To get R at avg. multiplicity of sample, should weight pairs by $1/N_{\text{pairs in event}}$

or calculate average multiplicity as

$$\frac{\sum_{\text{events}} N_{\text{event}} N_{\text{pairs in event}}}{N_{\text{pairs}}}$$

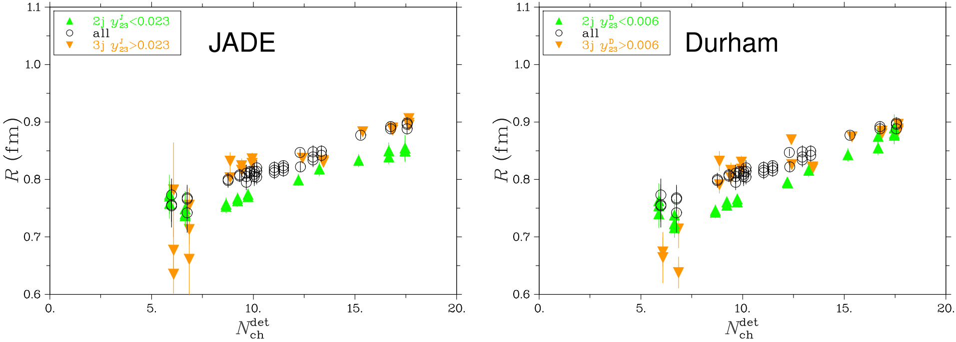
But the difference is small

So I ignore it.

R increases with multiplicity

Multiplicity/Jet dependence in τ-model

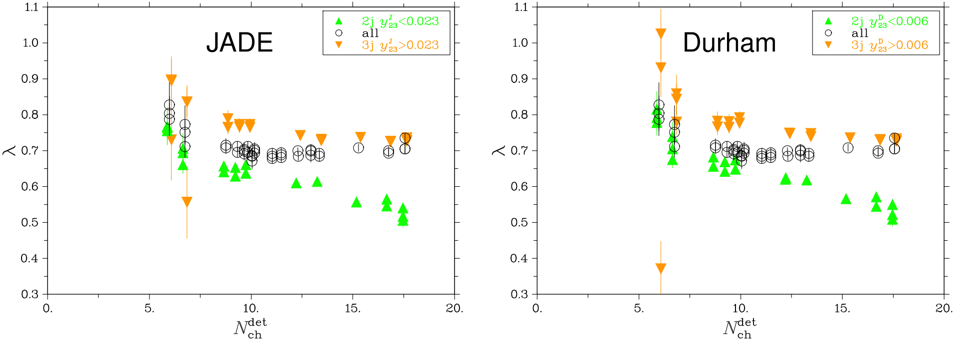
Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ PRELIMINARY



- R increases with N_{ch} and with number of jets whereas OPAL found $r_{n\text{-jet}}$ approx. indep. of N_{ch}
- Increase of R with N_{ch} similar for 2- and 3-jet events
- However, $R_{3\text{-jet}} \approx R_{all}$

Multiplicity/Jet dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ PRELIMINARY

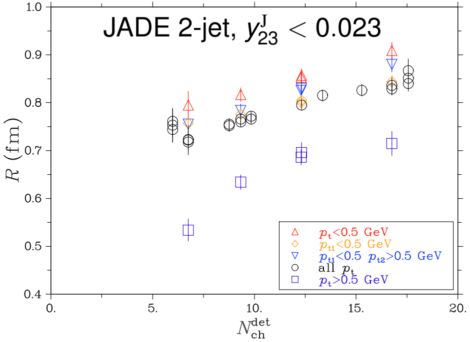


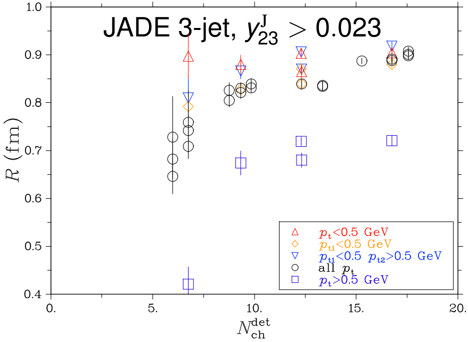
- $\lambda_{\text{3-jet}} > \lambda_{\text{2-jet}}$ opposite of OPAL
- λ initially decreases with N_{ch}
- then λ_{all} and $\lambda_{\text{3-jet}}$ approx. constant
  while $\lambda_{\text{2-jet}}$ continues to decrease, but more slowly
- whereas OPAL found λ_{all} decreasing approx. linearly with N_{ch}

m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$ PRELIMINARY

and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)





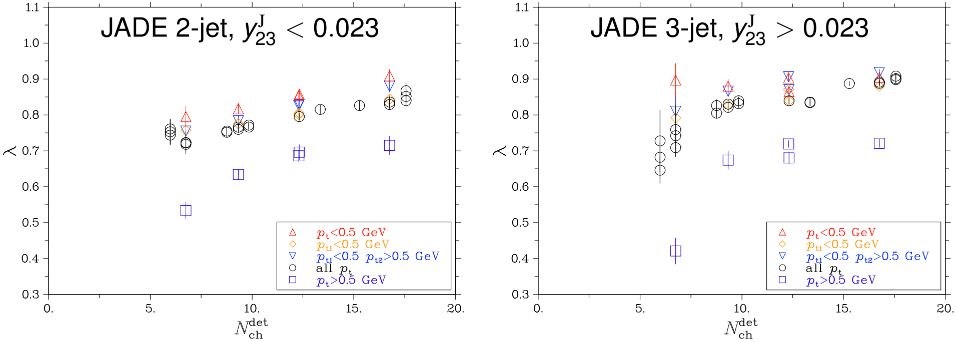
- R decreases with m_t for all N_{ch}
  smallest when both particles at high p_t

m_t dependence in τ-model

Using simplified τ-model, $\alpha = 0.44$, $\tau_0 = 0$

PRELIMINARY

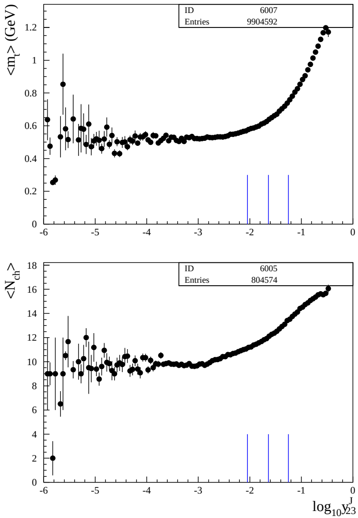
and cutting on $p_t = 0.5\,\mathrm{GeV}$ ($m_t = 0.52\,\mathrm{GeV}$)



- λ decreases with m_t
  smallest when both particles at high p_t

On what do r, R, λ depend?

- r, R increase with N_{ch}
- r, R increase with N_{jets}
- for fixed number of jets, R increases with N_{ch}
  but r constant with N_{ch} (OPAL)
- r, R decrease with m_t
- Although m_t, N_{ch}, N_{jets} are related, each contributes to the increase/decrease of R
  but only m_t, N_{jets} contribute to the increase/decrease of r
- λ decreases with N_{ch}, N_{jets}
  though somewhat differently for τ-model, Gaussian (OPAL)
- λ decreases with m_t



ADDITIONAL MATERIAL

Introduction — Correlations

q-particle density $\quad \rho_q(p_1, \ldots, p_q) = \frac{1}{\sigma_{\text{tot}}} \frac{d^q \sigma_q(p_1,\ldots,p_q)}{dp_1 \ldots dp_q}$

where σ_q is inclusive cross section

Normalization:

$$\int \rho_1(p)\, dp = \langle n \rangle$$
$$\int \rho_2(p_1, p_2)\, dp_1\, dp_2 = \langle n(n-1) \rangle$$

In terms of 'factorial cumulants', C

$$\rho_1(p_1) = C_1(p_1)$$
$$\rho_2(p_1, p_2) = C_1(p_1)C_1(p_2) + C_2(p_1, p_2)$$
$$\rho_3(p_1, p_2, p_3)) = C_1(p_1)C_1(p_2)C_1(p_3)$$

"trivial" 3-particle correlations $\quad + \sum_{3 \text{ perms}} C_1(p_1)C_2(p_2, p_3)$

"genuine" 3-particle correlations $\quad + C_3(p_1, p_2, p_3)$

2-particle correlations $\quad C_2 = \rho_2(p_1, p_2) - C_1(p_1)C_1(p_2)$

Convenient to normalize $\quad R_q = \frac{\rho_q}{\prod_{i=1}^{q} \rho_1(p_i)} \qquad K_q = \frac{C_q}{\prod_{i=1}^{q} \rho_1(p_i)}$

e.g., $\quad R_2 = 1 + \frac{C_2}{\rho_1(p_1)\rho_1(p_2)} = 1 + K_2$

Introduction — BEC

To study BEC, not other correlations, replace $\prod_{i=1}^{q} \rho_1(p_i)$ by $\rho_0(p_1, ..., p_q)$, the q-particle density if no BEC (reference sample)

e.g., 2-particle BEC are studied in terms of

$$R_2(p_1, p_2) = \frac{\rho(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since 2-π BEC only at small $Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M_{12}^2 - 4m_\pi^2}$, integrate over other variables

$$R_2(Q) = \frac{\rho(Q)}{\rho_0(Q)}$$

Assuming incoherent particle production and spatial source density $S(x)$,

$$R_2(Q) = 1 + |G(Q)|^2$$

where $G(Q) = \int \mathrm{d}x \, e^{iQx} S(x)$ is the Fourier transform of $S(x)$

Assuming $S(x)$ is a Gaussian with radius r $\Longrightarrow$

$$R_2(Q) = 1 + e^{-Q^2 r^2}$$

$R_2(Q) \propto 1 + \lambda e^{-Q^2 r^2}$

Assumes

- incoherent average over source
  λ tries to account for
  - partial coherence
  - multiple (distinguishable) sources, long-lived resonances
  - pion purity
- spherical (radius r) Gaussian density of particle emitters
  seems unlikely in $\mathrm{e}^+\mathrm{e}^-$ annihilation — jets
- static source, *i.e.*, no *t*-dependence certainly wrong

Nevertheless, this Gaussian formula is the most often used parametrization

And it works fairly well

But what do the values of λ and r actually mean?

When Gaussian parametrization does not fit well,

- can expand about the Gaussian (Edgeworth expansion).
  Keeping only the lowest-order non-Gaussian term,
  $\exp\left(-Q^2 r^2\right)$ becomes
  $$\exp\left(-Q^2 r^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(Qr)\right]$$
  (H_3 is third-order Hermite polynomial)
- Assume source radius is a symmetric Lévy distribution rather than Gaussian $\exp\left(-Q^2 r^2\right)$ becomes
  $$\exp\left(-Q^2 r^\alpha\right) \quad , 0 < \alpha \leq 2$$
  α is the Lévy index of stability

Experimental Problems I

I. Pion purity

1. mis-identified pions – K, p
   – correct by MC. – But is it correct?
2. resonances
   - long-lived affect λ
   BEC peak narrower than resolution
   - short-lived, *e*.g., ρ, - affect *r*
   – correct by MC. – But is it correct?
3. weak decays
   $\sim$ 20% of Z decays are $\mathrm{b\bar{b}}$

   like long-lived resonances, decrease λ

- per Z: 17.0 $\pi^{\pm}$, 2.3 K$^{\pm}$, 1.0 p (15% non-π)

Origin of π^+ in Z decay	(%) (JETSET 7.4)
direct (string fragmentation)	16
decay (short-lived resonances) $\Gamma > 6.7\,\mathrm{MeV}, \tau < 30$ fm (ρ, ω, K*, Δ, ...)	62
decay (long-lived resonances) $\Gamma < 6.7\,\mathrm{MeV}, \tau > 30$ fm	22

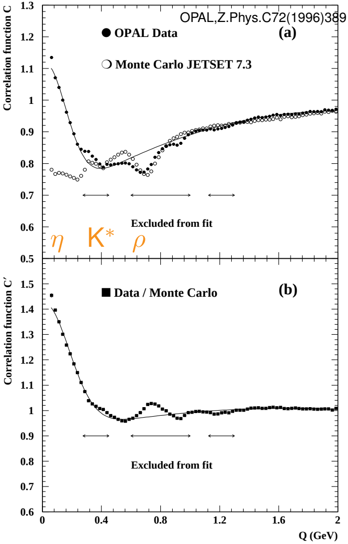
Experimental Problems II

II. Reference Sample, ρ_0
— it does NOT exist

Common choices:

1. $+,-$ pairs
   But different resonances than $+,+$
   – correct by MC. – But is it correct?
2. Monte Carlo — But is it correct?
3. Mixed events – pair particles from different events
   But destroys all correlations, not just BEC
   – correct by MC. – But is it correct?
4. Mixed hemispheres (for 2-jet events)
   – pair particle with particle reflected from opposite hemisphere
   But destroys all correlations
   – correct by MC. – But is it correct?

R_2



Experimental Problems III, IV

III. Final-State Interactions

1. Coulomb
   - form not certain
     (usually use Gamow factor)
     overcorrects!
   - for R_2, a few % in lowest Q bin
   - double if $+, -$ ref. sample
   - often neglected for R_2
   - but not negligible for R_3
2. Strong interaction - $S = 0$ $\pi\pi$ phase shifts can be incorporated together with Coulomb into the formula for R_2

Osada, Sano, Biyajima, Z.Phys. C72(1996)285)

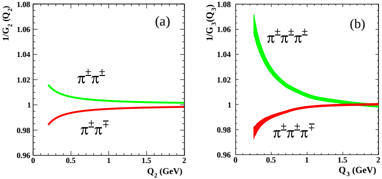
tends to increase λ, decrease r

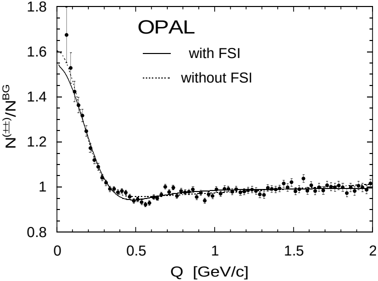
e.g., using OPAL data:

$\lambda_{\mathrm{noFSI}} = 0.71$, $\lambda_{\mathrm{FSI}} = 1.0$

$r_{\mathrm{noFSI}} = 1.34$, $r_{\mathrm{FSI}} = 1.09\,\mathrm{fm}$

- Not used by experimental groups



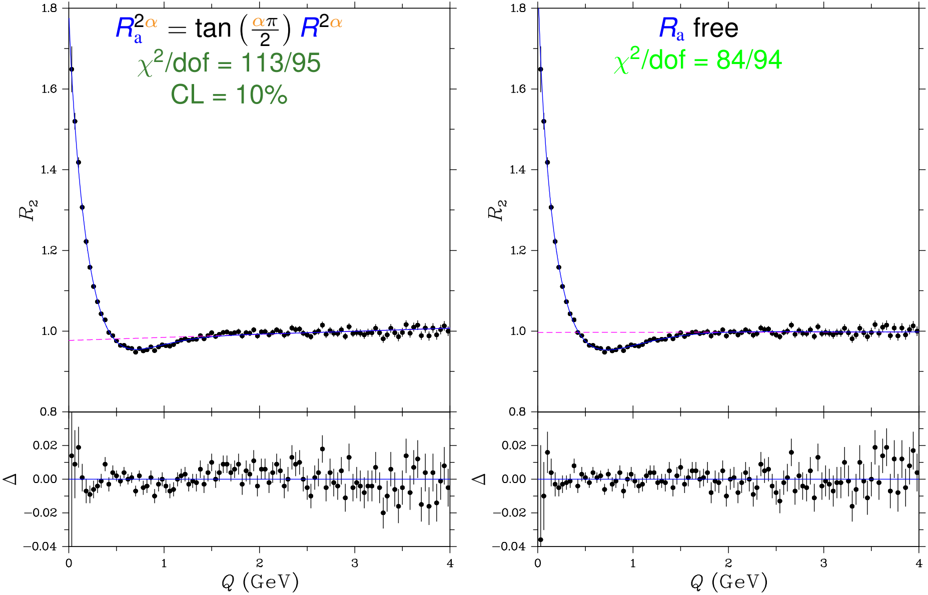


IV. Long-range correlations

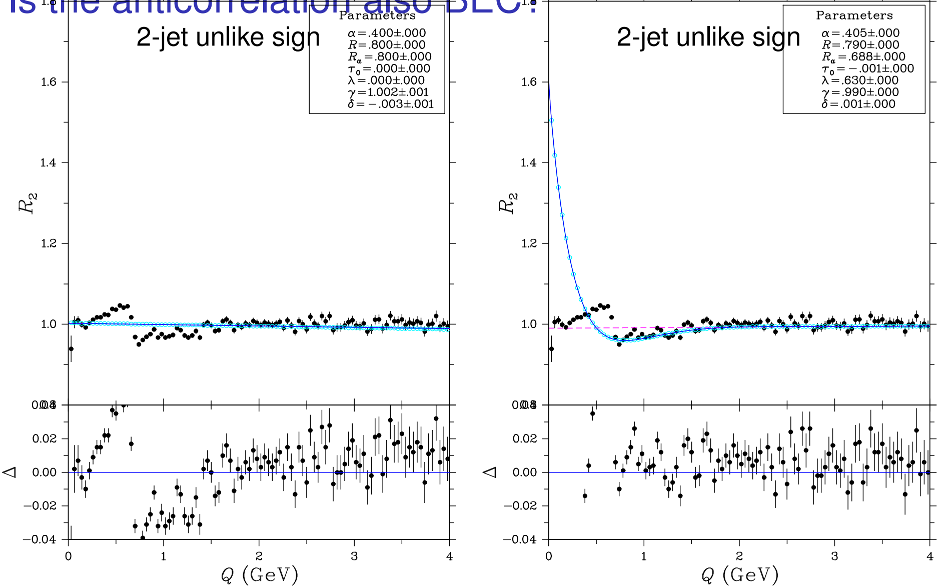
inadequately treated in ref. sample:

$$R_2(Q) \propto (1 + \lambda e^{-Q^2 r^2})(1 + \delta Q)$$

3-jet Results on Simplified τ-model from L3 Z decay



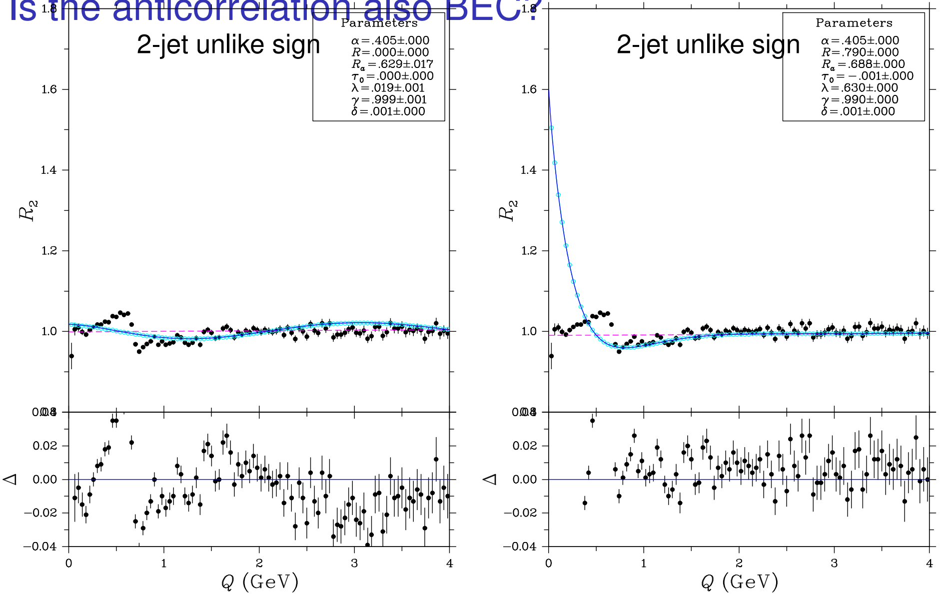
Is the anticorrelation also BEC?



Resonances in anticorrelation region confuse things

But anticorrelation may be present in unlike sign

Is the anticorrelation also BEC?

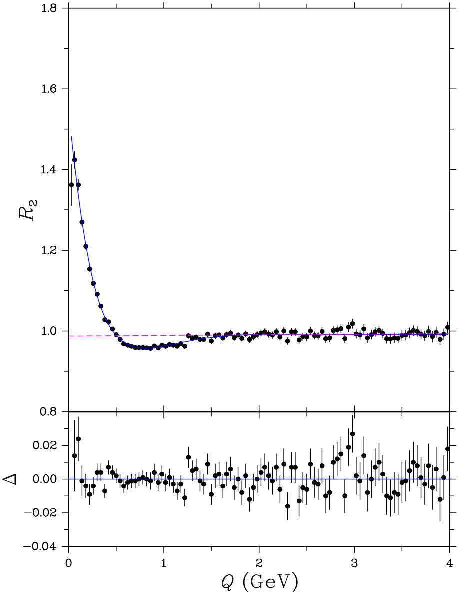


If anticorrelation is present in unlike sign,

it requires the damping of the exp of the BEC peak

Full τ-model for 2-jet events — $a = 1/m_t$

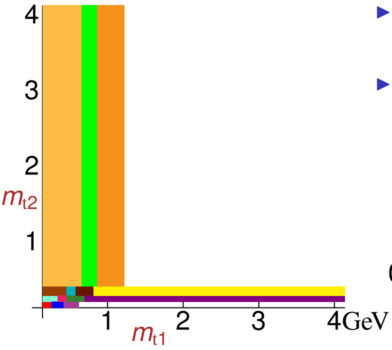
$$R_2(Q, m_{t1}, m_{t2}) = \gamma \left\{1 + \lambda \cos\left[\frac{\tau_0 Q^2(m_{t1}+m_{t2})}{2(m_{t1} m_{t2})} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}}\right] \cdot \exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{m_{t1}^{\alpha}+m_{t2}^{\alpha}}{2(m_{t1} m_{t2})^{\alpha}}\right]\right\} \cdot (1 + \epsilon Q)$$



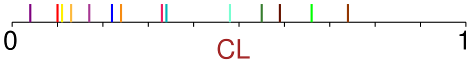
- Fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin, $m_{t1} > m_{t2}$
- $\tau_0 = 0.00 \pm 0.02$ so fix to 0
- χ^2/dof = 90/95

Full τ-model for 2-jet events

- τ-model predicts dependence on m_t, $R_2(Q, m_{t1}, m_{t2})$
- Parameters α, $\Delta\tau$, τ_0 are independent of m_t
- λ (strength of BEC) can depend on m_t



- divide m_{t1}-m_{t2} plane in regions (equal statistics)
- in each region fit $R_2(Q)$ using avg m_{t1}, m_{t2} in each Q bin with α, $\Delta\tau$, fixed to values found for entire plane and $\tau_0 = 0$



Elongation?

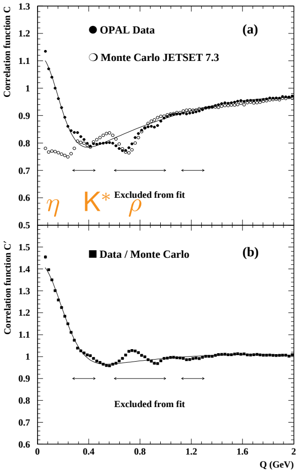
- Previous results using fits of Gaussian or Edgeworth found (in LCMS) $R_{\mathrm{side}}/R_{\mathrm{L}} \approx 0.64$
- But we find that Gaussian and Edgeworth fit $R_2(Q)$ poorly
- τ-model predicts no elongation and fits the data well
- Could the elongation results be an artifact of an incorrect fit function? or is the τ-model in need of modification?
- So, we modify *ad hoc* the τ-model description to allow elongation
- and find $R_{\mathrm{side}}/R_{\mathrm{L}} = 0.61 \pm 0.02$ – elongation is real
- Perhaps, $\overline{x}^{\mu}(p^{\mu}) = a\tau p^{\mu}$ should only apply to $\mu =$ longitudinal

OPAL

OPAL,Z.Phys.C72(1996)389

- Ref. sample is $+,-$ pairs different resonances than $+,+$
- Correction by MC insufficient
- Exclude 'resonance regions'

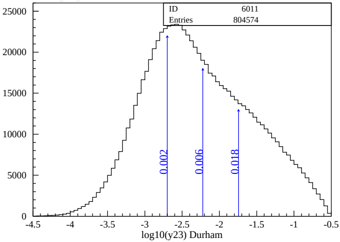
$$R_2(Q) = \gamma(1 + \lambda\, e^{-Q^2 r^2})(1 + \delta Q + \epsilon Q^2)$$



Jets

Jets — JADE and Durham algorithms

- force event to have 3 jets:
  - normally stop combining when all 'distances' between jets are $> y_{\rm cut}$
  - instead, stop combining when there are only 3 jets left
  - y_{23} is the smallest 'distance' between any 2 of the 3 jets
- y_{23} is value of $y_{\rm cut}$ where number of jets changes from 2 to 3



define regions of $y_{23}^{\rm D}$ (Durham):

$y_{23}^{\rm D} < 0.002$	narrow two-jet	or		
$0.002 < y_{23}^{\rm D} < 0.006$	less narrow two-jet		$y_{23}^{\rm D} < 0.006$	two-jet
$0.006 < y_{23}^{\rm D} < 0.018$	narrow three-jet		$0.006 < y_{23}^{\rm D}$	three-jet
$0.018 < y_{23}^{\rm D}$	wide three-jet			

and similarly for $y_{23}^{\rm J}$ (JADE): 0.009, 0.023, 0.056