# TMD parton densities and final states at the LHC

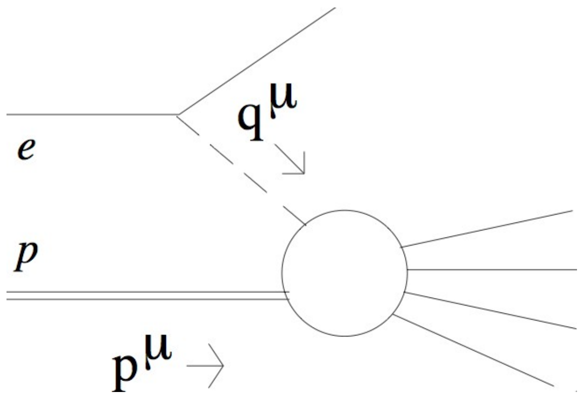
F Hautmann

- *Proton structure with transverse momentum dependence*
- *What can we learn from high-precision DIS data*
- *Application to LHC: Drell Yan + jets production*

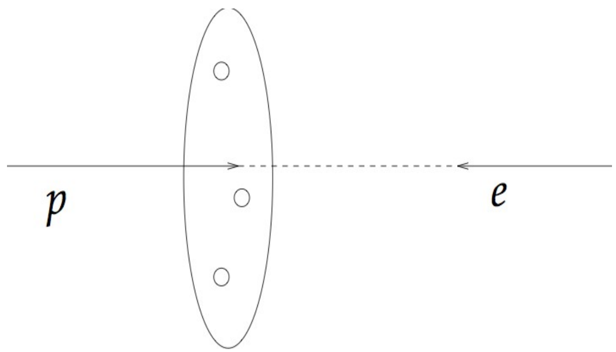
Based on collaborations with S Dooling, H Jung, S Taheri Monfared

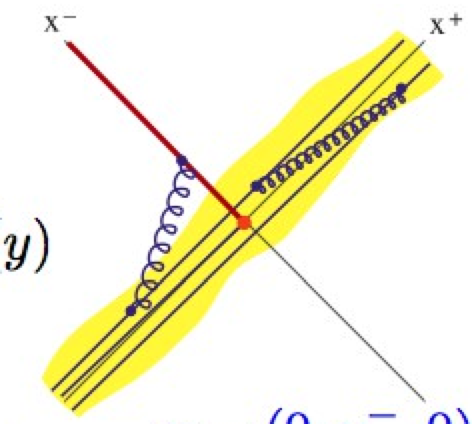
ISMD 2014, Bologna, September 2014

# The collinear parton density functions




DIS in
infinite momentum frame




$$\mathrm{Pdf's}: \quad f(x,\mu) = \int \frac{dy^-}{2\pi} \, e^{-ixp^+y^-} \, \widetilde{f}(y)$$

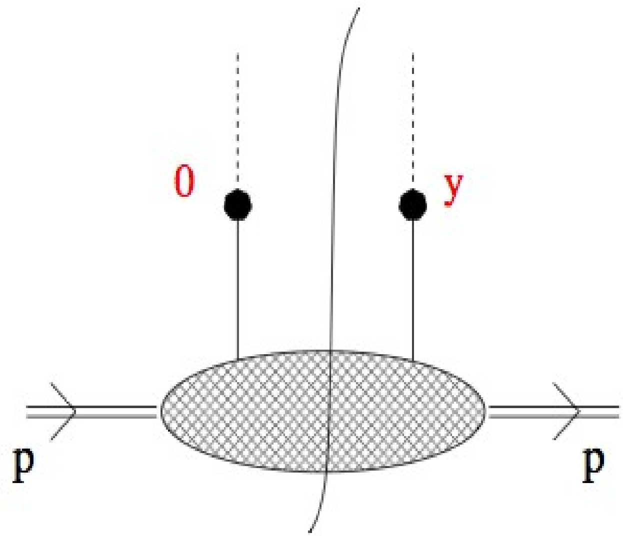
$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, 0)$$

correlation of parton fields at lightcone distances

$$V_y(n) = \mathcal{P} \exp\left( i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \, n) \right)$$

# Transverse momentum dependent (TMD) parton density functions

Generalize matrix element to non-lightlike distances:




$$p = (p^{+}, m^{2}/2p^{+}, 0_{\perp})$$

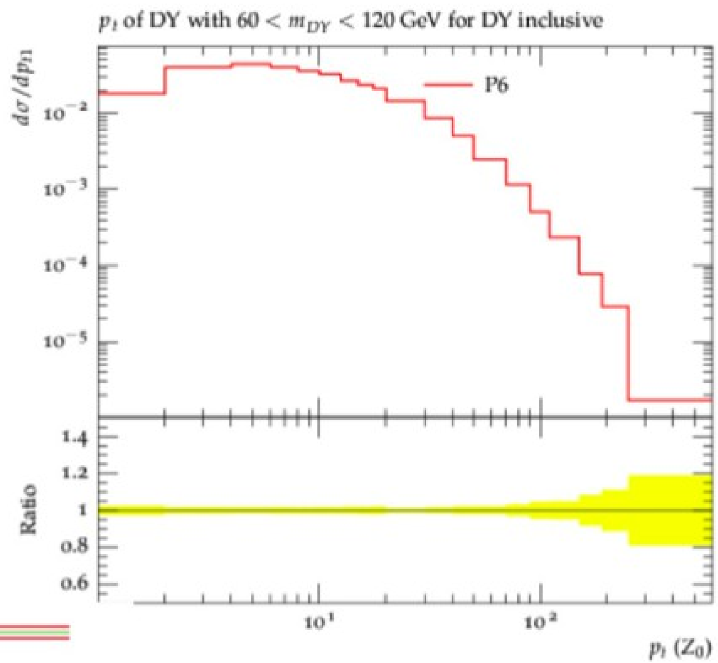
$$\tilde{f}(y) = \langle\, P \mid \overline{\psi}(y)\, V_y^{\dagger}(n)\, \gamma^{+}\, V_0(n)\, \psi(0) \mid P \,\rangle \quad , \qquad y = (0, y^{-}, y_{\perp})$$

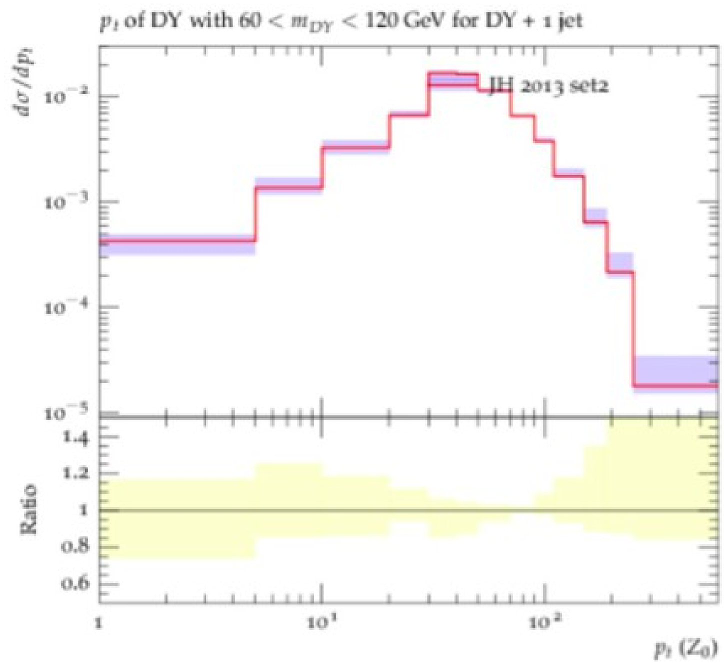
TMD pdf's:

$$f(x, k_{\perp}) = \int \frac{dy^{-}}{2\pi} \frac{d^{d-2} y_{\perp}}{(2\pi)^{d-2}} e^{-ixp^{+}y^{-} + ik_{\perp}\cdot y_{\perp}} \tilde{f}(y)$$

# Classic motivations for TMDs (I)
# Drell Yan hadroproduction pT spectra

H Jung, talk at DIS2014, Warsaw, April 2014





- inclusive DY
  - small $p_t$ spectrum, peaked at ~ 5 GeV
  - soft gluon resummation, CSS
  - standard TMD approach

- DY + jet, $p_t^{jet} > 30$ GeV
  - peak shifted to larger $p_t$
  - “mini-jet” resummation
  - need TMDs in truly perturbative region

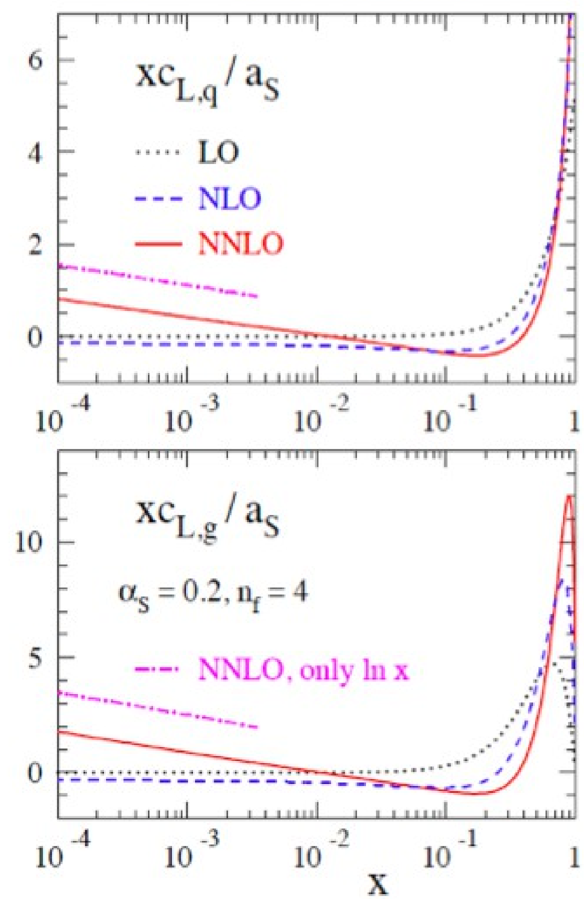
Talk by S. Dooling, WG2, Wednesday

# Classic motivations for TMDs (II)

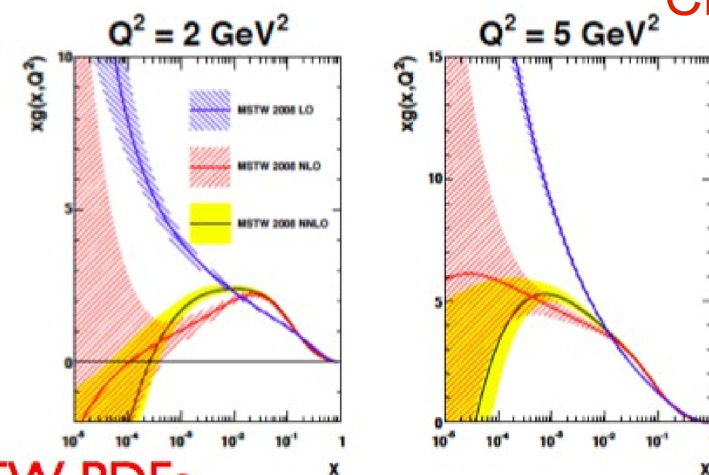
## DIS at x → 0 : large corrections to fixed order

A Stasto, talk at VHECR2014, CERN, Aug 2014

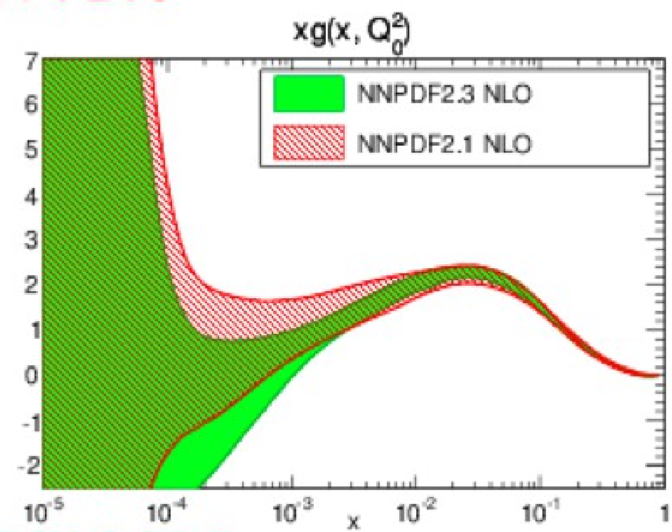
$$x^{-1}F_L = C_{L,\mathrm{ns}} \otimes q_{\mathrm{ns}} + \langle e^2 \rangle \left( C_{L,\mathrm{q}} \otimes q_{\mathrm{s}} + C_{L,\mathrm{g}} \otimes g \right)$$




Moch, Vermaseren, Vogt




MSTW PDFs




NNPDF2.3 PDFs

# “The TMDlib project” http://tmdlib.hepforge.org/

- a platform for theory and phenomenology of TMD pdfs
- library of fits and parameterizations LHApdf style

arXiv:1408.3015v1 [hep-ph] 13 Aug 2014

**TMDlib and TMDplotter:**
**library and plotting tools for**
**transverse-momentum-dependent parton distributions**
**Version 1.0.0**

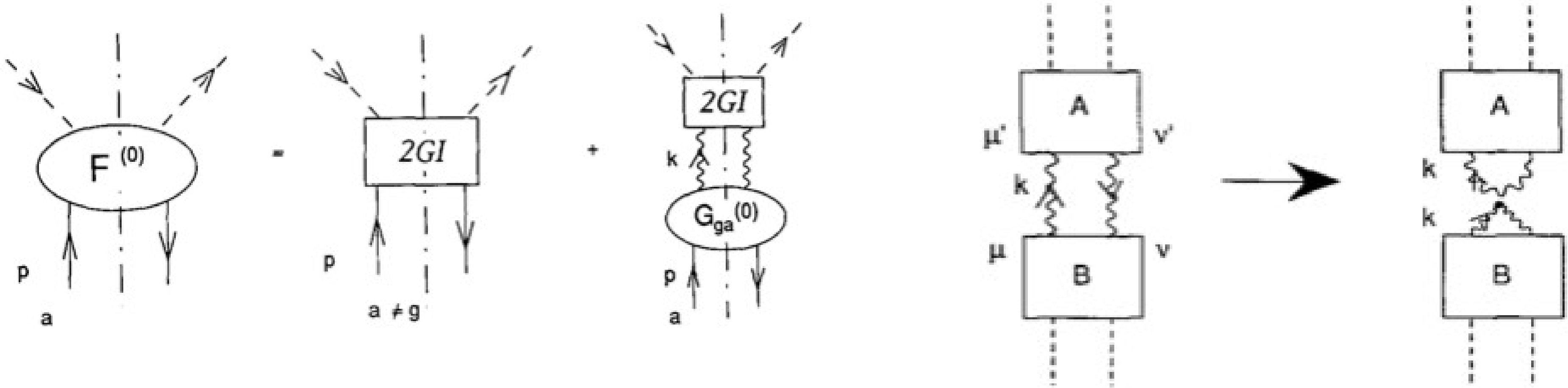
F. Hautmann[1,2], H. Jung[3,4], M. Krämer[3],
P. J. Mulders[5,6], E. R. Nocera[7], T. C. Rogers[8], A. Signori[5,6]

[1] Rutherford Appleton Laboratory, UK
[2] Dept. of Theoretical Physics, University of Oxford, UK
[3] DESY, Hamburg, FRG
[4] University of Antwerp, Belgium
[5] Department of Physics and Astronomy, VU University Amsterdam, the Netherlands
[6] Nikhef, the Netherlands
[7] Università degli Studi di Milano and INFN Milano, Italy
[8] C.N. Yang Institute for Theoretical Physics, Stony Brook University, USA

**Abstract**

Transverse-momentum-dependent distributions (TMDs) are central in high-energy physics from both theoretical and phenomenological points of view. In this manual we introduce the library, TMDlib, of fits and parameterisations for transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter. We provide a description of the program components and of the different physical frameworks the user can access via the available parameterisations.

# DIS at small x: transverse momentum dependent high-energy factorization



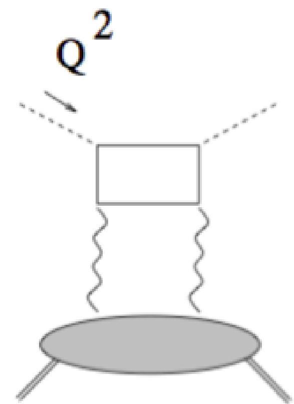

$$F_j(x,Q^2) = \int_x^1 \frac{dz}{z} \int d^{2+2\varepsilon}\mathbf{k}\ \underbrace{\hat{\sigma}_j\left(x/z, \mathbf{k}/Q, \alpha_s (Q/\mu)^{\varepsilon}, \varepsilon\right)}_{2GI\ kernel}\ \mathcal{A}(z, \mathbf{k}, \mu, \varepsilon) \qquad j = 2, L$$

where $$\mathcal{A}(z, \mathbf{k}, \mu, \varepsilon) = \int \frac{dk^2}{2(2\pi)^{4+2\varepsilon}}\ P_{\mu\nu}^{(H)}\ G^{\mu\nu}(k,p)$$

↖

*unintegrated (TMD) gluon density*

# Example: flavor singlet evolution at small x




$$\frac{d}{d\ln\mu^2}\begin{pmatrix}\Sigma\\ G\end{pmatrix} = \begin{pmatrix}P_{qq} & P_{qg}\\ P_{gq} & P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Sigma\\ G\end{pmatrix}$$

$$P_{gg} = \underbrace{\sum_{k=1}^{\infty} a_k \alpha_s^k x^{-1} \ln^{k-1} x}_{L(x)} + \underbrace{(b_0\alpha_s + \sum_{k=1}^{\infty} b_k \alpha_s \alpha_s^k x^{-1} \ln^{k-1} x)}_{NL(x)} + \dots$$

$$P_{qg} = c_0\alpha_s + \underbrace{\sum_{k=1}^{\infty} c_k \alpha_s \alpha_s^k x^{-1} \ln^{k-1} x}_{NL(x)} + \dots$$

- TMD factorization $\Rightarrow$ well-defined resummation of $\alpha_s^n \ln^{n-m} x$ corrections to splitting functions as well as DIS coefficient functions

# Example: DIS $F_2$ coefficient function from TMD factorization

$$C^g_{2,N}(\alpha_s, Q^2/\mu^2) = \int_0^1 dx\, x^{N-1}\, C^g_2(x, \alpha_s, Q^2/\mu^2)$$

$$C^g_{2,N}(\alpha_s, Q^2/\mu^2 = 1)$$
$$= \frac{\alpha_s}{2\pi} T_R N_f \frac{2}{3} \left\{ 1 + 1.49\, \frac{\overline{\alpha}_s}{N} + 9.71 \left(\frac{\overline{\alpha}_s}{N}\right)^2 + 16.43 \left(\frac{\overline{\alpha}_s}{N}\right)^3 + \mathcal{O}\left(\frac{\alpha_s}{N}\right)^4 \right\}$$

LO NLO NNLO

[Catani & H (1994)]

$$\alpha_s (\alpha_s/N)^k \leftrightarrow \alpha_s^2 (\alpha_s \ln x)^{k-1}$$

Need for single-log resummations
[cf. double-log (x → 1, x → 0 timelike): e.g., A. Vogt, talk at DIS2012, Bonn]

# Beyond x → 0: CCFM exclusive evolution

$$x\mathcal{A}(x,k_t,q) = x\mathcal{A}(x,k_t,q_0)\Delta_s(q) + \int dz \int \frac{dq'}{q'} \cdot \frac{\Delta_s(q)}{\Delta_s(q')} \tilde{P}(z,k_t,q') \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, q'\right)$$

- solve integral equation via iteration:

$$x\mathcal{A}_0(x,k_t,q) \quad = \quad x\mathcal{A}(x,k_t,q_0)\Delta(q)$$

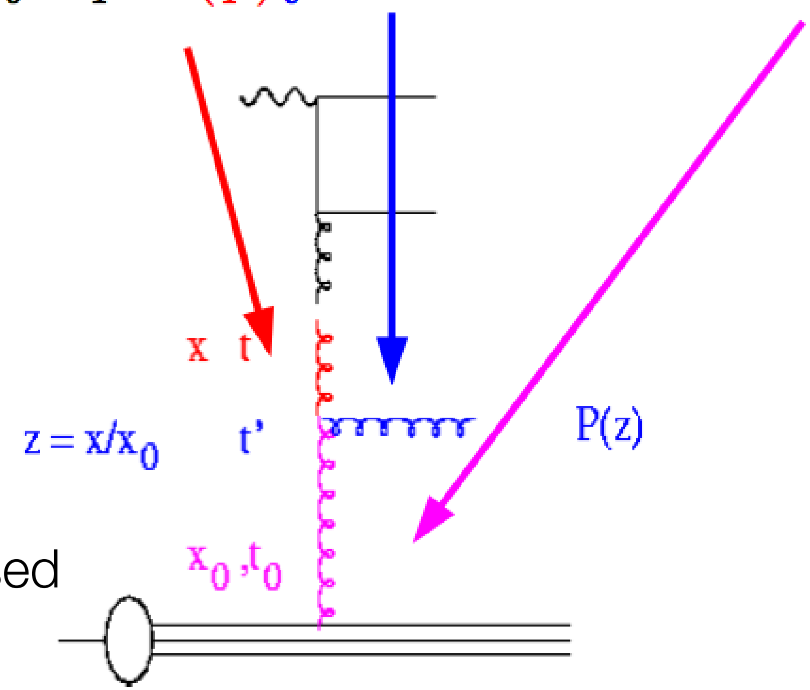
$$x\mathcal{A}_1(x,k_t,q) \quad = \quad x\mathcal{A}(x,k_t,q_0)\Delta(q) + \int \frac{dq'}{q'} \frac{\Delta(q)}{\Delta(q')} \int dz \tilde{P}(z) \frac{x}{z} \mathcal{A}(x/z, k_t', q_0)\Delta(q')$$

from q′ to q w/o branching

branching at q′

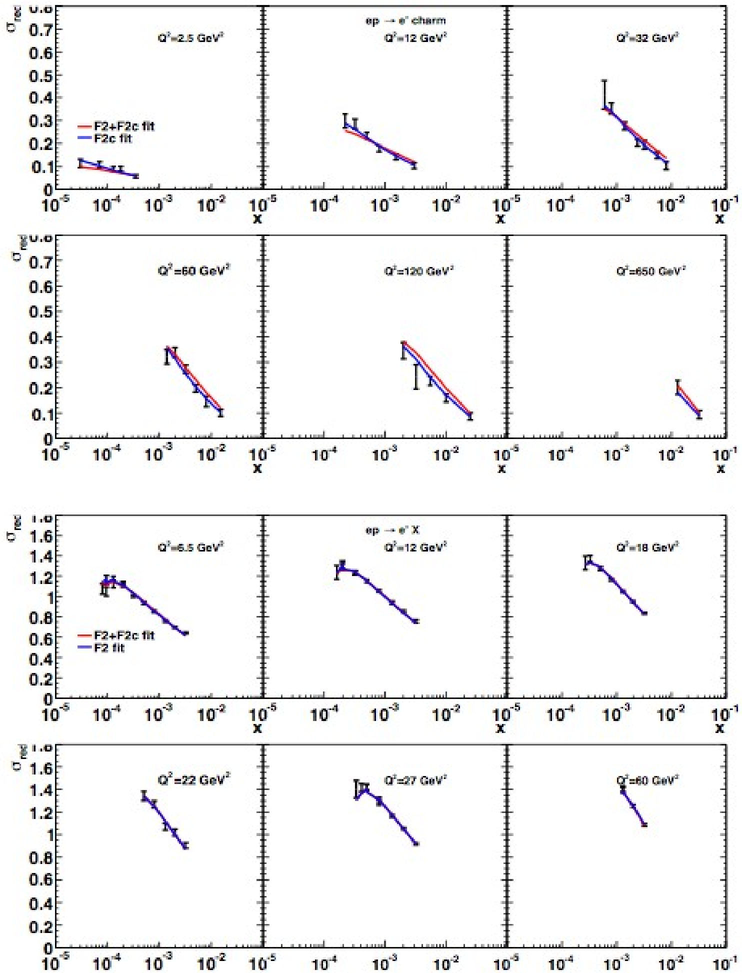
from $q_0$ to q′ w/o branching

- Note: evolution equation formulated with Sudakov form factor is equivalent to "plus" prescription, but better suited for numerical solution for treatment of kinematics




- evolution code uPDFevolv publicly released [Jung, Taheri Monfared & H, arXiv:1407.5935]

# kT-dependent gluon density from precision DIS data




[Jung & H, Nucl. Phys. B 883 (2014) 1]

- Good description of inclusive DIS data with TMD gluon
- Sea quark yet to be included at TMD level
- Fit performed with herafitter package https://www.herafitter.org/

| | $\chi^2/ndf\,(F_2^{(\mathrm{charm})})$ | $\chi^2/ndf\,(F_2)$ | $\chi^2/ndf\left(F_2 \text{ and } F_2^{(\mathrm{charm})}\right)$ |
|---|---|---|---|
| 3-parameter | 0.63 | 1.18 | 1.43 |
| 5-parameter | 0.65 | 1.16 | 1.41 |

# kT-dependent gluon density from precision DIS data

[Jung & H, Nucl. Phys. B 883 (2014) 1]

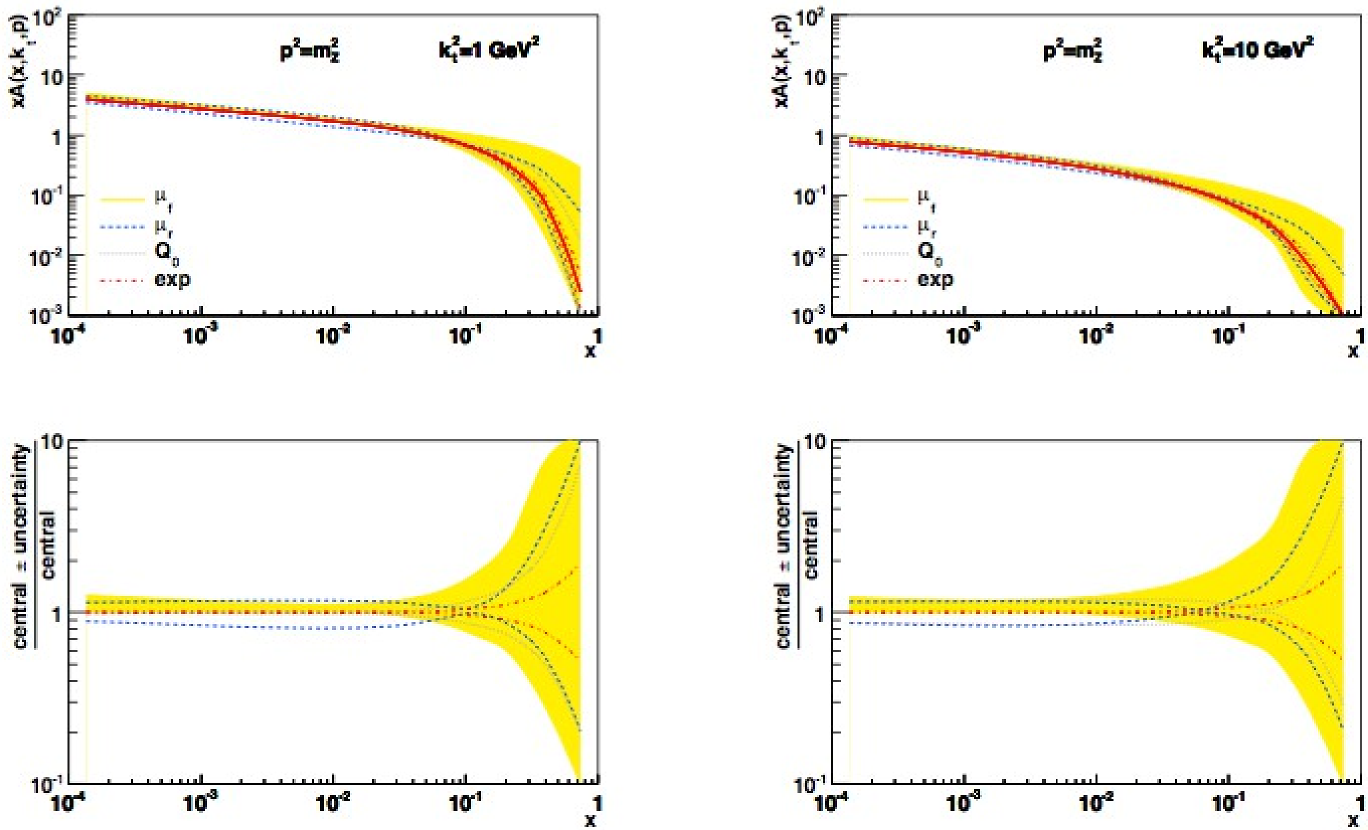



FIG. 6. *Experimental and theoretical uncertainties of the unintegrated TMD gluon density versus $x$ for different values of transverse momentum at $p^2 = m_Z^2$. The yellow band gives the uncertainty from the factorization scale variation; the curves indicate the uncertainties from the other sources.*

# Integrated pdfs

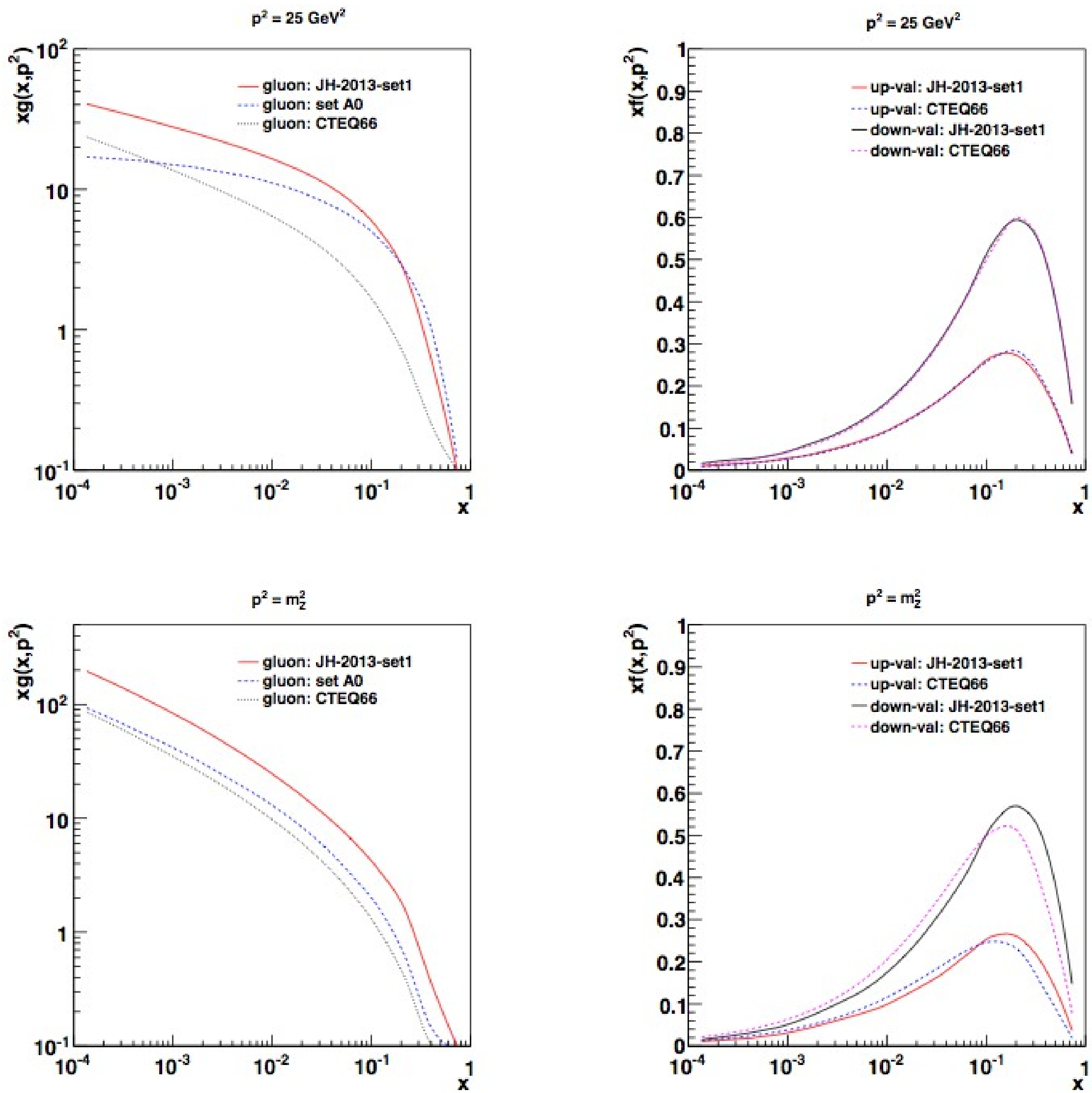
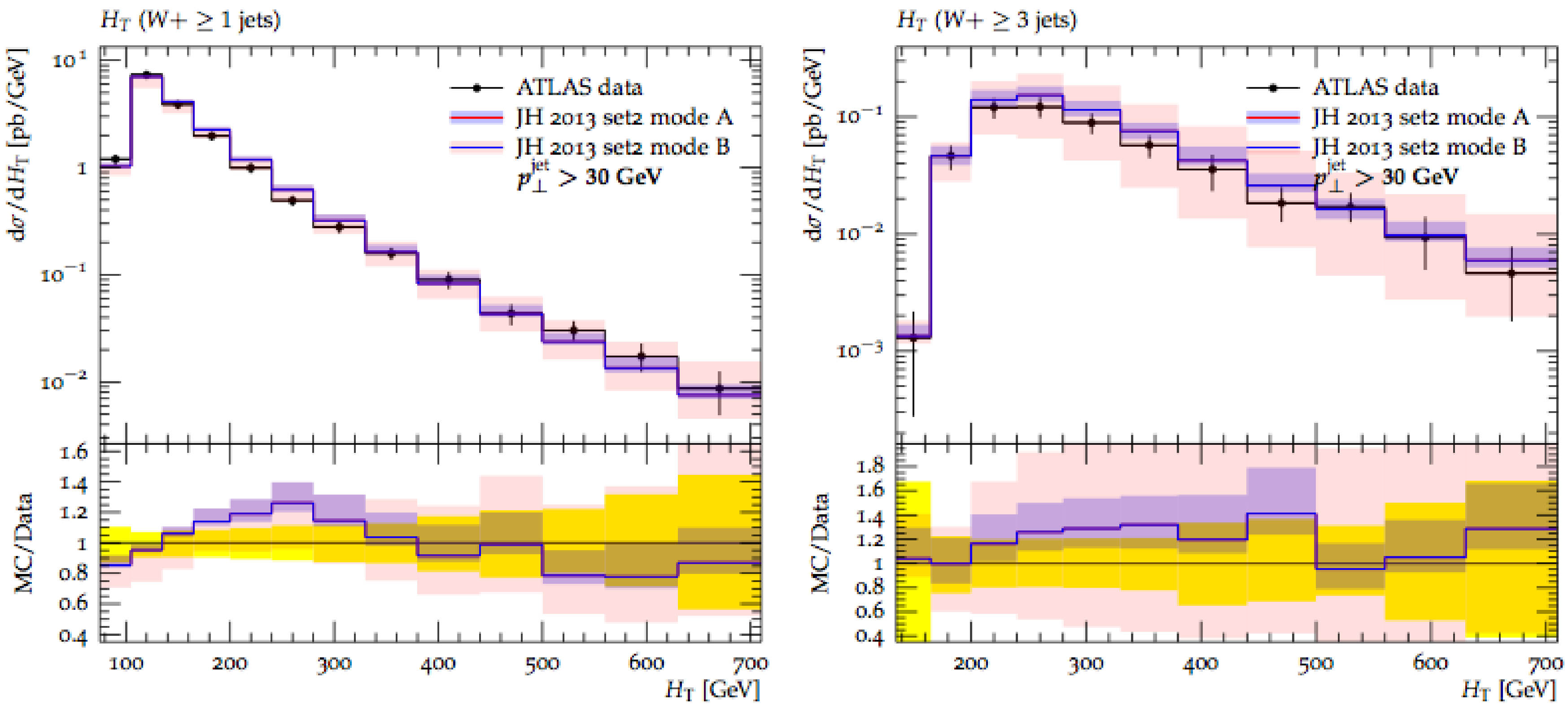



FIG. 8. *Integral over transverse momenta of the TMD distributions for (left) gluon and (right) valence quark at different evolution scales: (top)* $p^2 = 25$ $GeV^2$*; (bottom)* $p^2 = m_Z^2$.

# Can we go to large transverse momenta? Total H_T distribution in W + n jets final states at the LHC

Dooling, Jung & H, Phys. Lett. B736 (2014) 293




mode A: uncertainties from renorm. scale, starting evol. scale, expt. errors

mode B: include factorization scale uncertainties

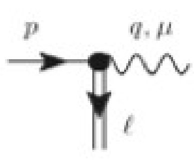
# Application to vector bosons + jets at high energy

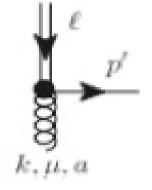
→ Use exclusive CCFM evolution

→ Determine TMD pdf from high-precision DIS data

→ Obtain predictions for final states associated with Drell-Yan using high-energy off-shell matrix elements

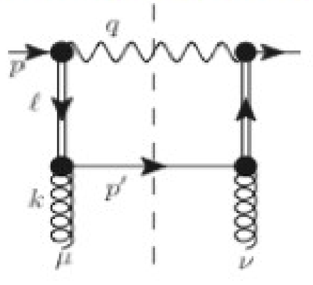
- High-energy effective theory → effective vertices




*[Bogdan & Fadin, NPB740 (2006) 36]*
*[Lipatov & Vyazovsky, NPB597 (2001) 399]*

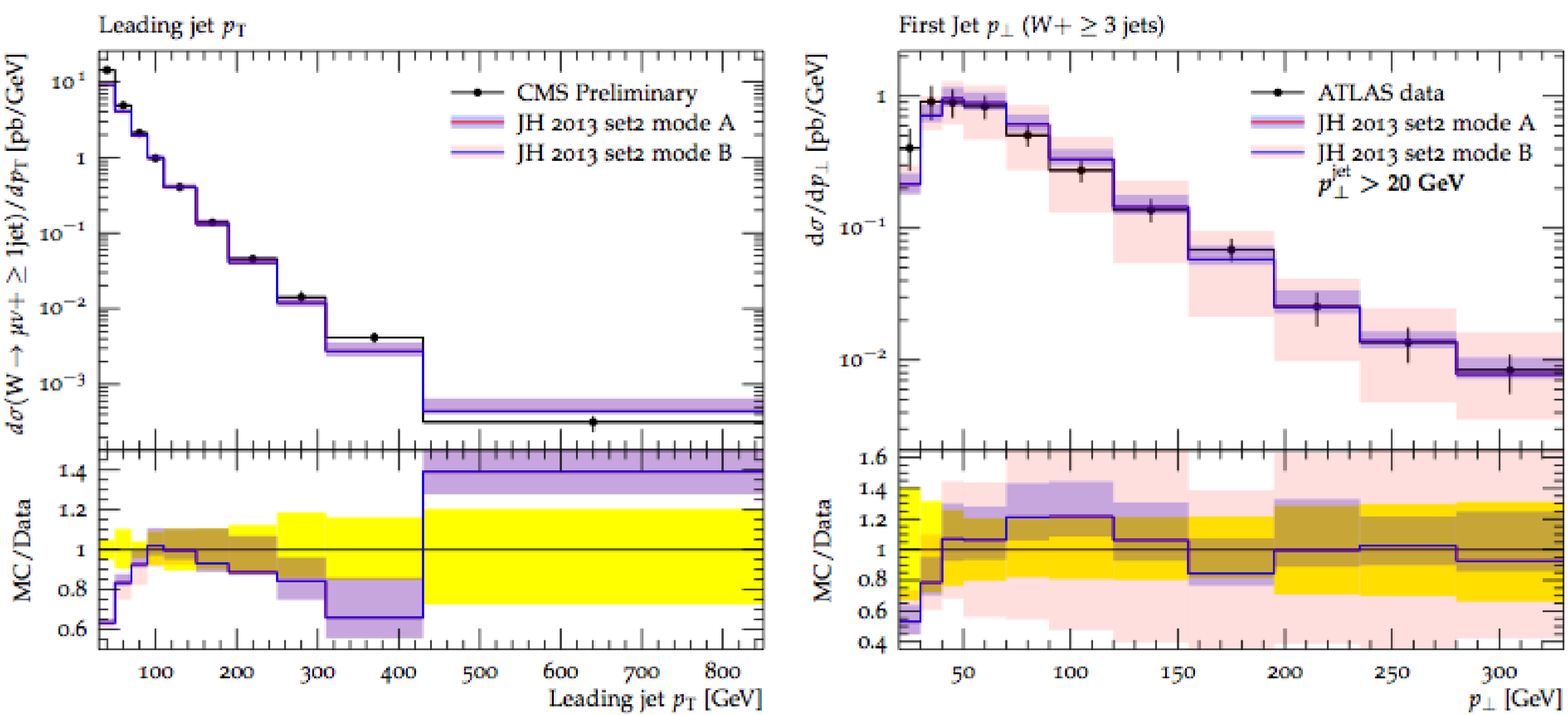
- Parton matrix elements (gauge-invariant, despite off-shell parton)




*[Ball & Marzani, NPB814 (2009) 246]*
*[Hentschinski, Jung & H, NPB865 (2012) 54]*

# W + n jets final states at the LHC

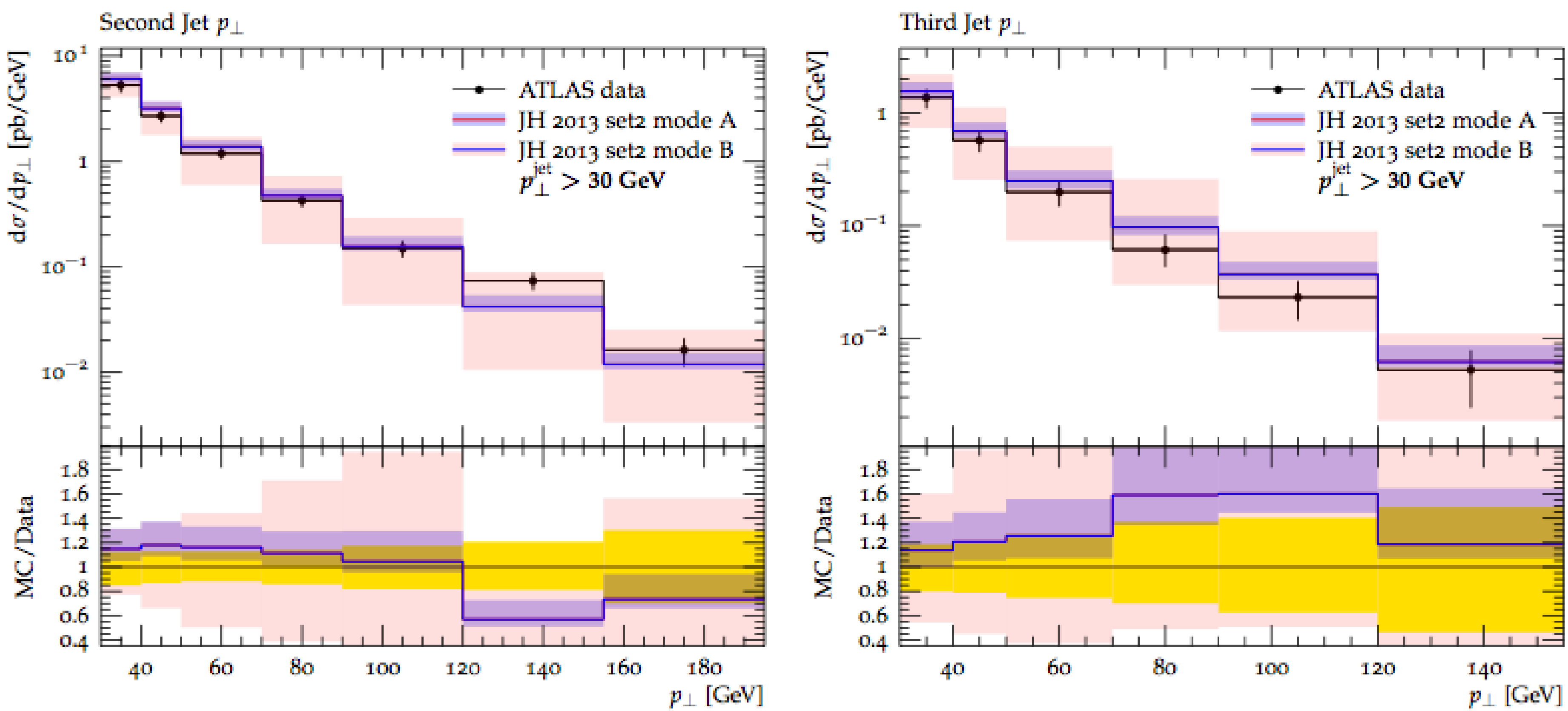
Dooling, Jung & H, Phys. Lett. B 736 (2014) 293




Leading jet pT: (left) inclusive; (right) n>=3

# W + n jets final states at the LHC

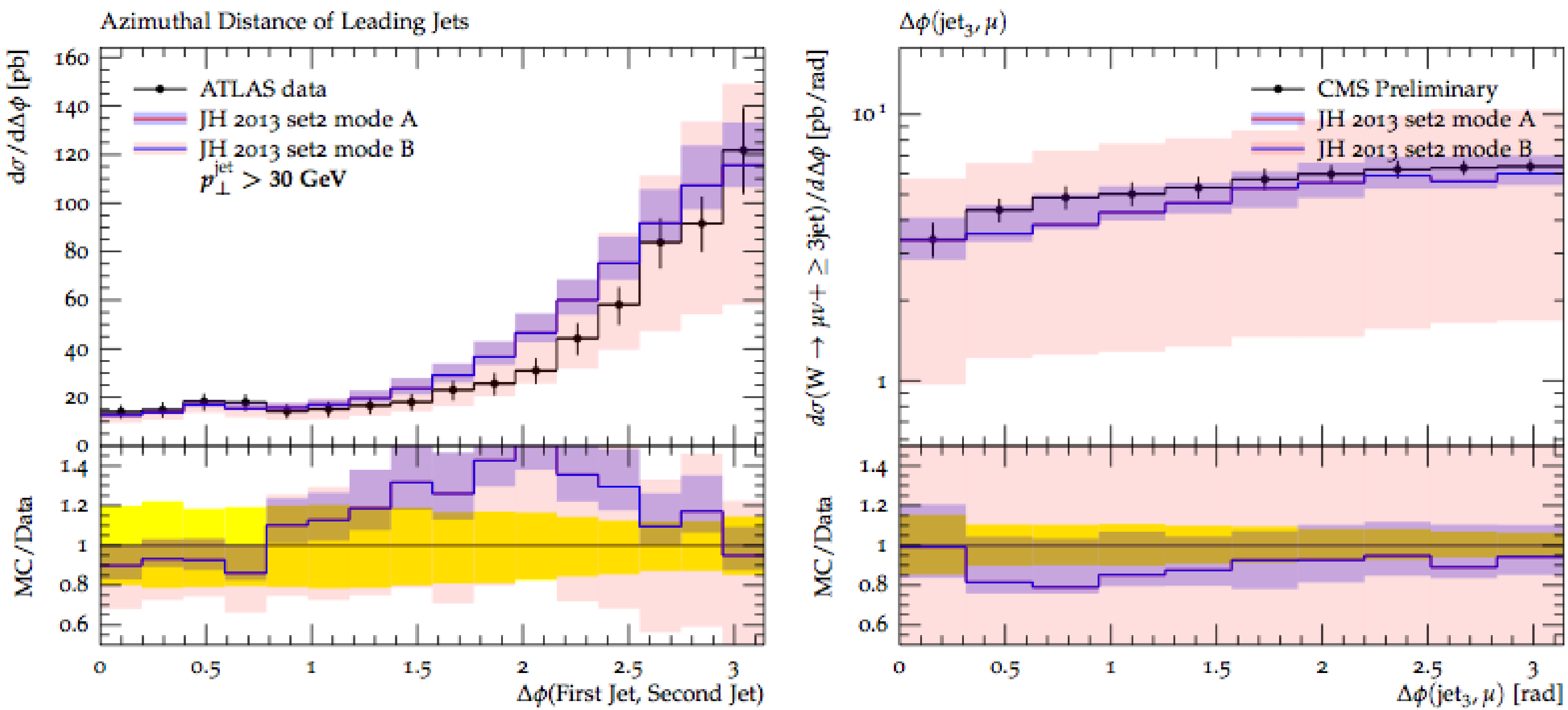
Dooling, Jung & H, Phys. Lett. B 736 (2014) 293




Subleading jets: (left) second jet pT; (right) third jet pT

# Angular correlations in W + n jets final states

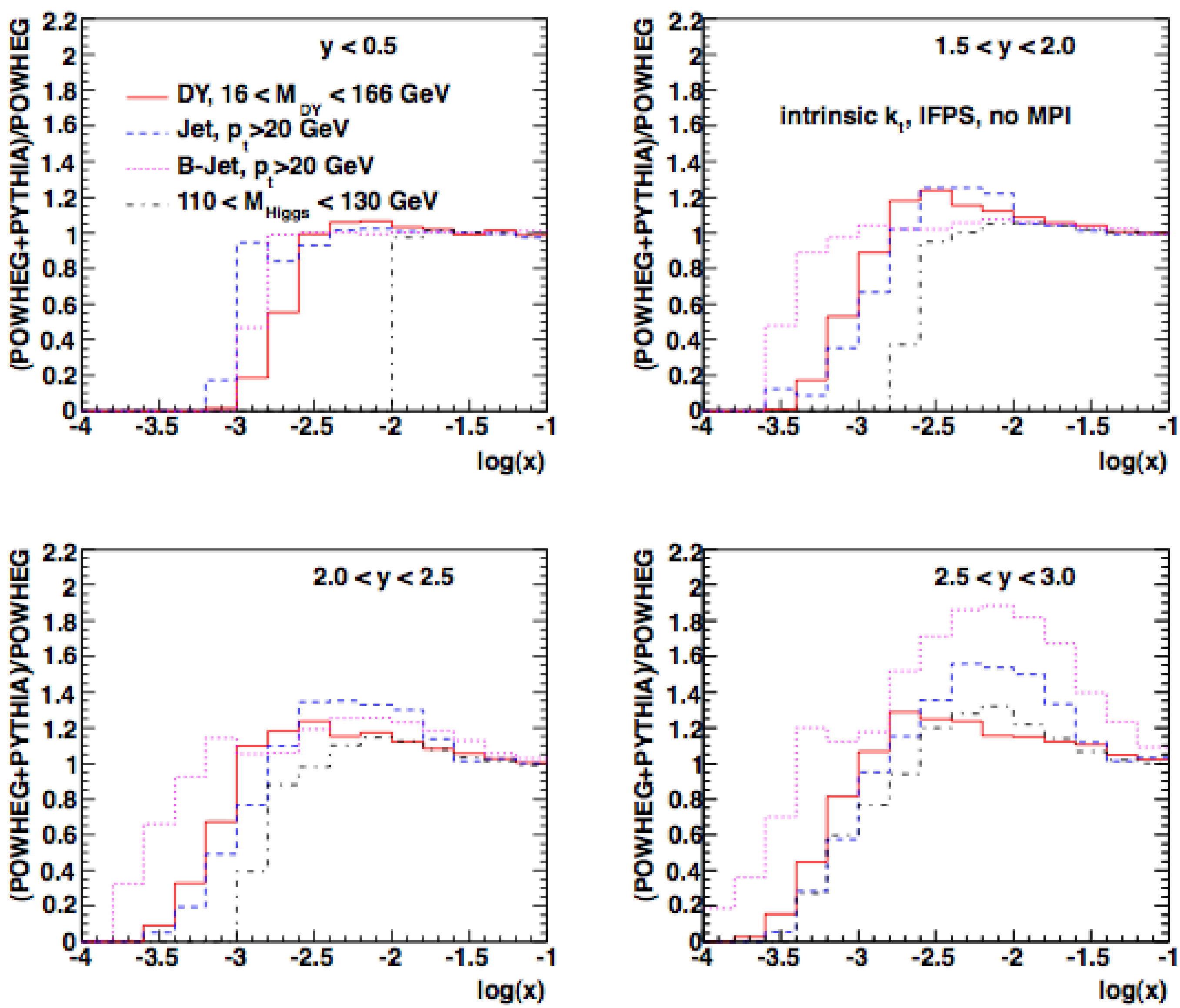
Dooling, Jung & H, Phys. Lett. B 736 (2014) 293




(left) Delta-phi between two hardest jets; (right) vector boson - third jet correlation

# Kinematic effects in parton shower evolution

- Longitudinal momentum shifts due to energy-momentum conservation combined with collinearity approximation [S. Dooling et al., arXiv:1212.6264]
- Non-negligible effect especially in data/theory comparisons for high rapidity jets.



# Conclusion

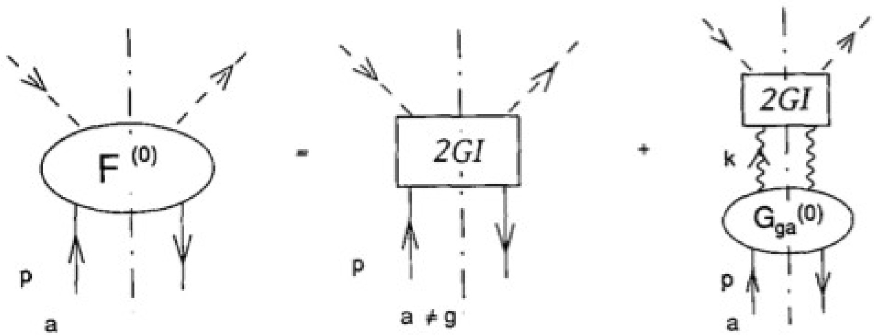
- TMD proton structure relevant to both large pT and small pT processes, high x and low x:
  TMDlib platform http://tmdlib.hepforge.org/

- First determination of TMD gluon from combined high-precision DIS data, including uncertainties [→ herafitter]

- The approach has far reaching implications for LHC physics:
  treatment of kinematic corrections to parton showers;
  studies of theor uncertainties in multi-particle final states;
  ex.: W + jets pT spectra and angular correlations

# Extra slides

# The renormalization group R factor




$$\int^{\mu} dk_{\perp} \, \mathcal{F}(k_{\perp}, \mu) \; = \; R \otimes f^{\overline{\rm MS}}(\mu)$$

- Perturbative expansion ($x \to 0$):

$$R(x) - \delta(1-x) \simeq 1.40 \; \alpha_s^3 \; \ln^2 x - 0.11 \; \alpha_s^4 \; \ln^3(1/x) + \ldots$$

- Relationship with RG evolution:

$$\mathcal{F}_N(k_{\perp}, \mu) = \underbrace{R_N \, [\gamma_N \, \frac{1}{k_{\perp}^2} \, \left( \frac{k_{\perp}^2}{\mu_F^2} \right)^{\gamma}_N ]}_{\varepsilon \to 0} \; \underbrace{\Gamma_N (\frac{\mu_F^2}{\mu^2})}_{\text{series of poles } 1/\varepsilon}$$

$\gamma_N$ = anomalous dimension
$\mu_F$ = factorization scale

# Motivation

Short-distance factorization:

$$m_H^2 \ \sigma = \sum_{a,b} \int dx_1 \ dx_2 \ f_a(x_1, \mu) \ C(\alpha_s(\mu_R), x/(x_1 x_2), m_H/\mu) \ f_b(x_2, \mu)$$

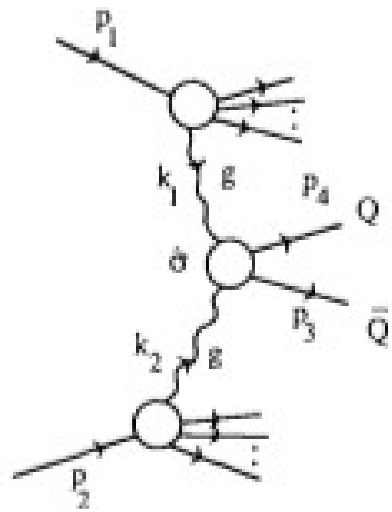
- High-energy logarithms in hard-scattering function...

$$\begin{aligned} C(\alpha_s, x, m_H/\mu) \ &= \ c^{(0)}(x) + \frac{\alpha_s}{\pi} \left[ c^{(1)}(x) + \overline{c}^{(1)}(x) \ L \right] \\ &+ \ \left( \frac{\alpha_s}{\pi} \right)^2 \left[ c^{(2)}(x) + \overline{c}^{(2)}(x) \ L + \overline{\overline{c}}^{(2)}(x) \ L^2 \right] + \cdots \end{aligned}$$

$$L = \ln(m_H^2/\mu^2), \ x = m_H^2/s$$

- ...as well as in the evolution of parton density functions

# Off-shell cross section



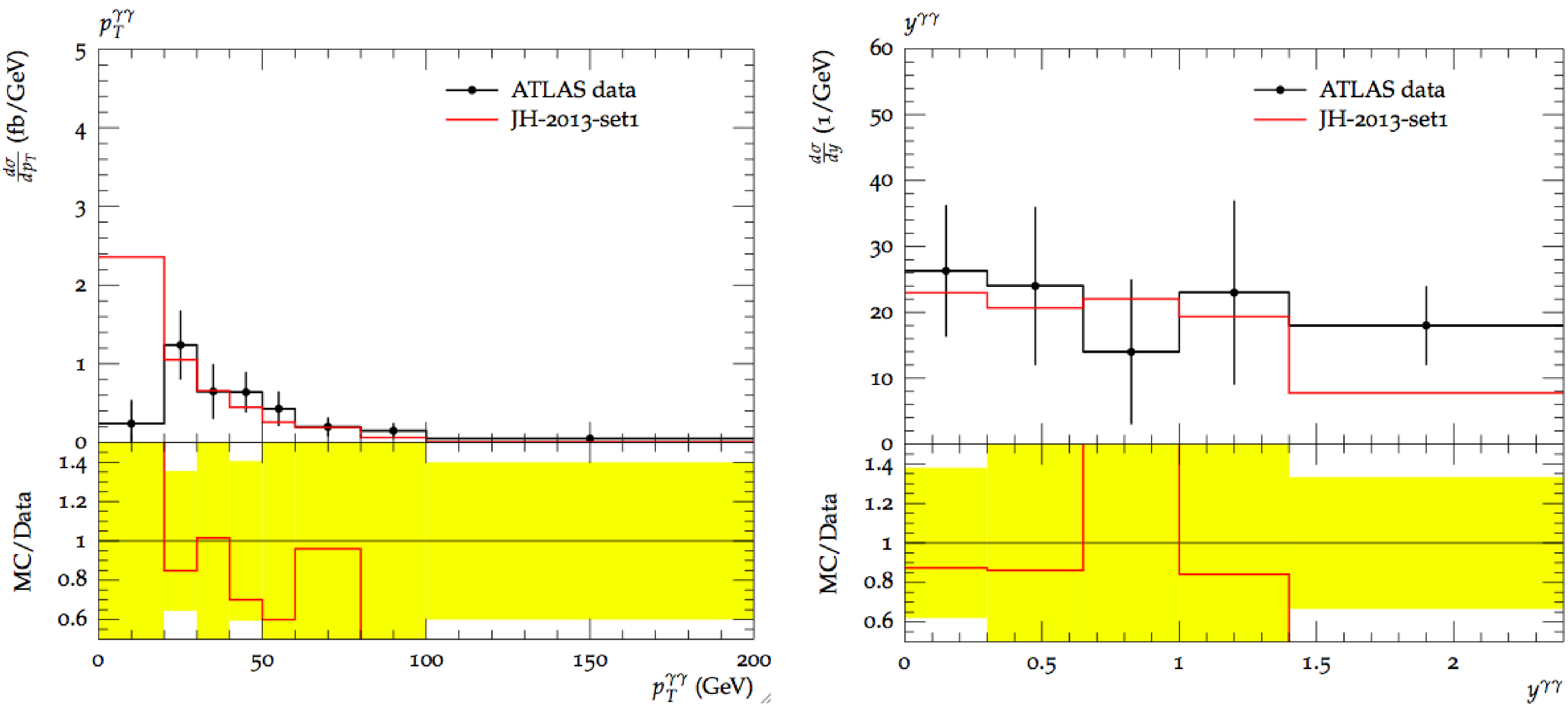

$$h_N(\gamma_1,\gamma_2) \;=\; \gamma_1\,\gamma_2 \int \frac{d^2\mathbf{k}_1}{\pi \mathbf{k}_1^2}\left(\frac{\mathbf{k}_1^2}{m_H^2}\right)^{\gamma_1} \int \frac{d^2\mathbf{k}_2}{\pi \mathbf{k}_2^2}\left(\frac{\mathbf{k}_2^2}{m_H^2}\right)^{\gamma_2}$$

$$\times \int_0^1 \frac{dx}{x}\, x^N\, \hat{\sigma}\left(x, \frac{\mathbf{k}_1}{m_H}, \frac{\mathbf{k}_2}{m_H}\right)$$

$\hat{\sigma}$ computed by coupling $gg \to H$ to eikonal gluon polarizations

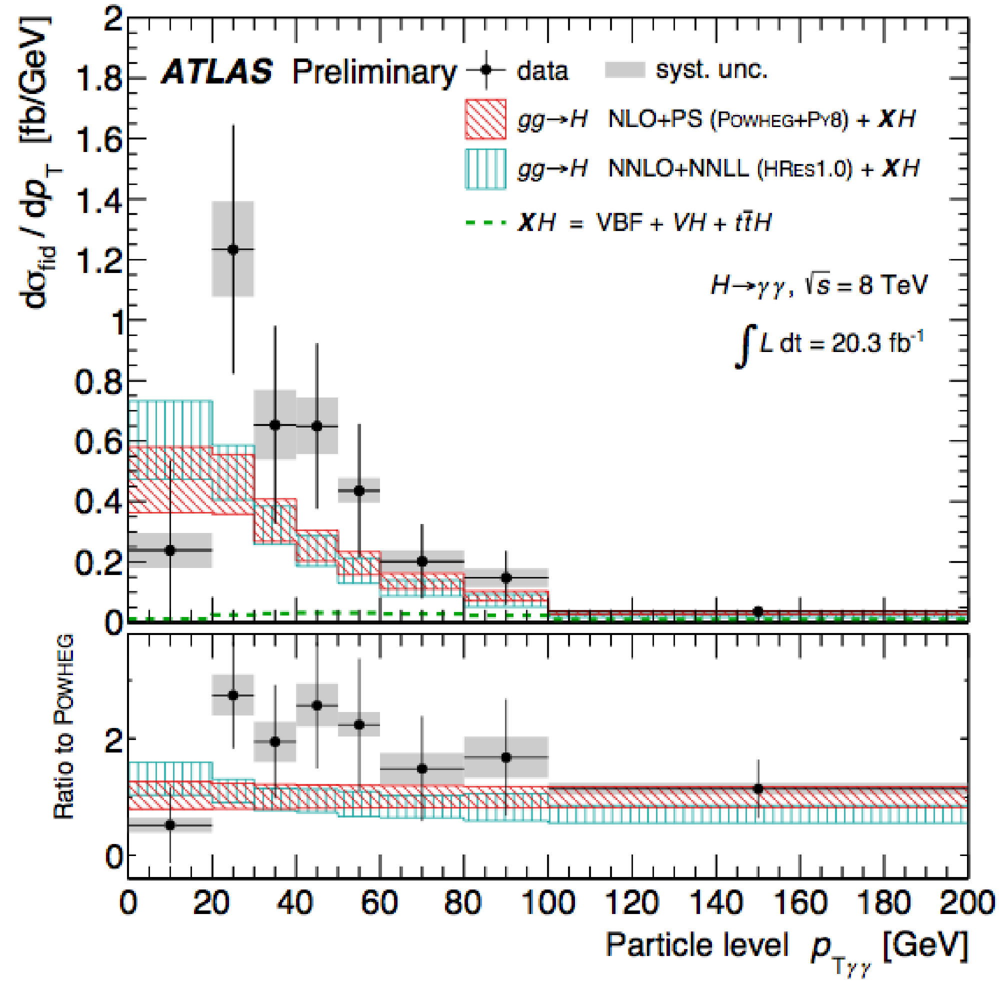
$$\mathcal{M}^{(eik)}(k_1,k_2,p) = \frac{2k_{1\perp}^{\mu_1}\,k_{2\perp}^{\mu_2}}{\sqrt{\mathbf{k}_1^2\mathbf{k}_2^2}}\mathcal{M}_{\mu_1\mu_2}(k_1,k_2,p)$$

# ATLAS pT and rapidity spectra
# Higgs → gamma gamma




JH-2013-set1: kT dependent gluon density from fits to precision DIS data

# ATLAS preliminary Conf-2013-072

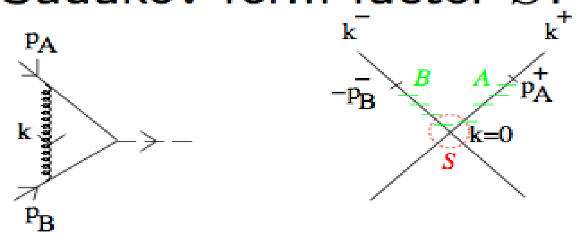



(a) $p_{\mathrm{T}}^{\gamma\gamma}$

# TMD evolution equations

Examples:

- Sudakov form factor $S$:



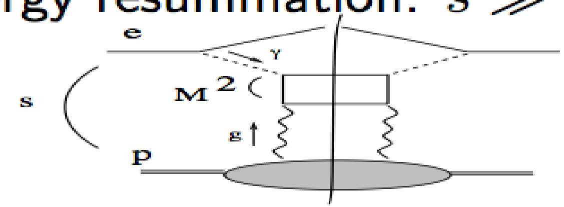

(a) (b)

▷ entering Drell-Yan production, W-boson $p_\perp$ distribution, . . .

$\Rightarrow \partial S/\partial \eta = K \otimes S$ CSS evolution equations [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

- High-energy resummation: $s \gg M^2 \gg \Lambda^2_{\rm QCD}$




◇ energy evolution: BFKL equation [Balitsky-Fadin-Kuraev-Lipatov]

↪ corrections down by $1/\ln s$ rather than $1/M$

CCFM equation is TMD branching equation which contains both Sudakov physics and BFKL physics