## Double Parton Interactions in pA collisions and Partonic Correlations

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## A simplest model for Double Parton Interactions

Multiple Parton Interactions have been introduced to solve the unitarity problem generated by the fast raise of the inclusive hard cross sections at small $x$. At small $x$ the hard cross section can in fact become larger than the total inelastic cross section.

For a given the final state, multiple parton interactions are the processes which maximize the incoming parton flux


Unitarity is restored because the inclusive cross section counts the multiplicity of interactions. In this way, when the average multiplicity of interactions is large, the inclusive cross section is no more bounded by the value of the total inelastic cross section.

The simplest case is Double Parton Scattering. The incoming parton flux is maximal when the hard component of the interaction is disconnected and, in the case of the DPS, one thus obtains the geometrical picture here below, where the non-perturbative components are factorized into functions which depend on two fractional momenta and on the relative transverse distance $b$ between the two interaction points


When neglecting spin and color, the inclusive double parton-scattering cross-section, for two parton processes A and B in a pp collision, is thus given by

$$
\sigma_{(A, B)}^{D}=\frac{m}{2} \sum_{i, j, k, l} \int D_{i j}\left(x_{1}, x_{2} ; b\right) \hat{\sigma}_{i k}^{A}\left(x_{1}, x_{1}^{\prime}\right) \hat{\sigma}_{j l}^{B}\left(x_{2}, x_{2}^{\prime}\right) D_{k l}\left(x_{1}^{\prime}, x_{2}^{\prime} ; b\right) d x_{1} d x_{1}^{\prime} d x_{2} d x_{2}^{\prime} d^{2} b
$$

Which, with leads to the "pocket formula" of the cross section utilized in the experimental analysis:

$$
\sigma_{d o u b l e}^{p p(A, B)}=\frac{m}{2} \frac{\sigma_{A} \sigma_{B}}{\sigma_{e f f}}
$$

## Comparison with Experiment

In the "pocket formula" all unknowns are summarized in the value of a single quantity $\boldsymbol{\sigma}_{\text {eff }}$

CMS talk
Antwerp 2013


D0 Collaboration, Phys.Rev. D81 (2010) 052012


FIG. 11: Effective cross section $\sigma_{\text {eff }}(\mathrm{mb})$ measured in the three $p_{T}^{\text {jet2 }}$ intervals.
F. Abe et al. [CDF Collaboration], Phys. Rev. D 56, 3811(1997).

Dependence of $\boldsymbol{\sigma}_{\text {eff }}$ on x



Distributions of x are plotted in Figs. 20(a) and 20(b), along with a prediction obtained by applying the DS $<1.2$ selection to the admixture $90 \%$ MIXDP+ $10 \%$ PYTHIA. No systematic deviation of the DP rate vs x , and thus no $\mathbf{x}$ dependence to $\boldsymbol{\sigma}_{\text {eff }}$, is apparent over the x range accessible to this analysis ( $0.01-0.40$ for the photon+jet scattering, $0.002-0.20$ for the dijet scattering).



## $\sigma_{\text {eff }}=15 \mathbf{m b}$

R.Maciula and A.Szczurek, Phys. Rev. D87, 074039


FIG. 12 (color online). Inclusive transverse momentum distributions of different charmed mesons measured by different groups at the LHC. The long-dashed line corresponds to the standard SPS $c \bar{c}$ production, and the dotted line represents the DPS $c \bar{c} c \bar{c}$ contribution.

The the "pocket formula" of the inclusive cross-section has thus shown to be able to describe the experimental results of the direct search of double parton collisions in rather different kinematical regimes with a value of $\sigma_{\text {eff }}$ compatible with a universal constant, while the study of CDF, of the dependence of $\sigma_{\text {eff }}$ on the fractional momenta of the incoming partons, is again compatible with a value of $\sigma_{\text {eff }}$ independent on $x$.

In the simples model, not inconsistent with present experimental evidence, DPS are therefore given by the disconnected contribution, which maximizes the incoming parton flux at small $x$, and leads to the "pocket formula" utilized in the experimental analyses, with a universal value of $\sigma_{\text {eff }}$


## $\sigma_{\text {eff }}$ and partonic correlations

One may write the double parton distribution functions as

$$
\Gamma\left(x_{1}, x_{2} ; b\right)=G\left(x_{1}, x_{2}\right) f_{x_{1} x_{2}}(b), \quad G\left(x_{1}, x_{2}\right)=K_{x_{1} x_{2}} G\left(x_{1}\right) G\left(x_{2}\right)
$$

where $f$ is normalized to one and the transverse scales, characterizing $f$, may still depend on fractional momenta.

$$
\begin{aligned}
& \int f_{x_{1} x_{2}}(b) d^{2} b=1 \quad G(x)=\langle n\rangle_{x}, \quad G\left(x_{1}, x_{2}\right)=\langle n(n-1)\rangle_{x_{1}, x_{2}} \\
& K_{x_{1} x_{2}}=\frac{\langle n(n-1)\rangle_{x_{1}, x_{2}}}{\langle n\rangle_{x_{1}}\langle n\rangle_{x_{2}}}
\end{aligned}
$$

In the simplest case one would have $K_{x x},=1$ which, after integrating on $b$, would be the case of a Poissonian multi-parton distribution in multiplicity.

In $p p$ one thus has

## correlations in multiplicity

$$
\begin{aligned}
& \sigma_{\text {double }}^{p p(A, B)}\left(x_{1}, x_{1}^{\prime}, x_{2}, x_{2}^{\prime}\right)= \frac{m}{2} K_{x_{1} x_{2}} K_{x_{1}^{\prime} x_{2}^{\prime}} G\left(x_{1}\right) \hat{\sigma}_{A}\left(x_{1}, x_{1}^{\prime}\right) G\left(x_{1}^{\prime}\right) \\
& \times G\left(x_{2}\right) \hat{\sigma}_{B}\left(x_{2}, x_{2}^{\prime}\right) G\left(x_{2}^{\prime}\right) \int f_{x_{1} x_{2}}(b) f_{x_{1}^{\prime} x_{2}^{\prime}}(b) d b \\
& \text { where } \quad \int f_{x_{1} x_{2}}(b) f_{x_{1}^{\prime} x_{2}^{\prime}}(b) d b=\frac{K_{x_{1} x_{2}} K_{x_{1}^{\prime} x_{2}^{\prime}}}{\pi \Lambda^{2}\left(x_{1}, x_{1}^{\prime}, x_{2}, x_{2}^{\prime}\right)} \sigma_{A}\left(x_{1}, x_{1}^{\prime}\right) \sigma_{B}\left(x_{2}, x_{2}^{\prime}\right) \\
& \pi \Lambda^{2}\left(x_{1}, x_{1}^{\prime}, x_{2}, x_{2}^{\prime}\right)
\end{aligned} \underbrace{}_{\text {typical transverse interaction area }} .
$$

$$
\sigma_{d o u b l e}^{p p(A, B)}=\frac{m}{2} \frac{\sigma_{A} \sigma_{B}}{\sigma_{e f f}}
$$

All new information on the hadron structure is thus summarized in the effective cross section

## Limiting case

$$
\Gamma\left(x_{1}, x_{2} ; b\right)=G\left(x_{1}, x_{2}\right) f_{x_{1} x_{2}}(b), \quad G\left(x_{1}, x_{2}\right)=K_{x_{1} x_{2}} G\left(x_{1}\right) G\left(x_{2}\right) \quad K_{x_{1} x_{2}}=\frac{\langle n(n-1)\rangle_{x_{1}, x_{2}}}{\langle n\rangle_{x_{1}}\langle n\rangle_{x_{2}}}
$$

a) If partons are not correlated in multiplicity one has

$$
K_{x_{1} x_{2}}=1
$$

b) If partons are not correlated in transverse coordinates one may write:

$$
\Gamma(x ; b)=G(x) f_{x}(b), \quad \int f_{x}(b) d^{2} b=1 \quad f_{x_{1}, x_{2}}(b)=\int f_{x_{1}}\left(b^{\prime}\right) f_{x_{2}}\left(b-b^{\prime}\right) d^{2} b^{\prime}
$$

Two gluon form factor

In this way one however obtains $\sigma_{\text {eff }}=\pi \Lambda^{2}=\mathbf{3 2} \mathbf{~ m b}$, which is about a factor 2 too large as compared with available experimental evidence

Either $K$ is NOT equal to $\mathbf{1}$ or $\boldsymbol{\pi} \Lambda^{\mathbf{2}}$ is NOT equal to $\mathbf{3 2} \mathbf{~ m b}$ or both

$$
\sigma_{e f f}\left(x_{1}, x_{1}^{\prime}, x_{2}, x_{2}^{\prime}\right)=\frac{\pi \Lambda^{2}\left(x_{1}, x_{1}^{\prime}, x_{2}, x_{2}^{\prime}\right)}{K_{x_{1} x_{2}} K_{x_{1}^{\prime} x_{2}^{\prime}}}
$$

The experimental indication is that the effective cross section depends only weakly on fractional momenta.

## weak dependence of $\Lambda$ and $K$ on fractional momenta

Since all new information on the hadron structure is summarized by a single quantity,

## the effective cross section does not provide enough information to discriminate between $\Lambda$ and $K$.

To obtain additional information on multi-parton correlations one may study MPI in $p A$ collisions. In the case of a double parton interaction, in a collision of a proton with a nucleus, the effects of longitudinal and transverse correlations are in fact different when a single nucleon or both target nucleons participate in the hard process.

## DPS in $p-\boldsymbol{A}$ collisions

Additional information can be obtained from DPS in $p-A$ collisions:


## WJJ production in pA collisions

We have evaluated the cross section below with $\mathbf{K}, \boldsymbol{\Lambda}$ and $\boldsymbol{\sigma}_{\text {eff }}$ independent on x

$$
\begin{aligned}
& \sigma^{p A}(W J J)=\sigma_{S}^{p A}(W J J)+\sigma_{D}^{p A}(W J J), \text { where } \quad \sigma_{D}^{p A}(W J J)=\left.\sigma_{D}^{p A}(W J J)\right|_{1}+\left.\sigma_{D}^{p A}(W J J)\right|_{2} \\
& \left.\sigma_{D}^{p A}(W J J)\right|_{1}=\frac{1}{\sigma_{e f f}}\left[Z \sigma_{S}^{p[p]}(W) \sigma_{S}^{p[p]}(J J)+(A-Z) \sigma_{S}^{p[n]}(W) \sigma_{S}^{p[n]}(J J)\right]
\end{aligned}
$$

$$
\left.\sigma_{D}^{p A}(W J J)\right|_{2}=K\left[\frac{Z}{A} \sigma_{S}^{p p}(W)+\frac{A-Z}{A} \sigma_{S}^{p n}(W)\right] \sigma_{S}^{p p}(J J)
$$



We have evaluated the cross sections assuming that $\mathbf{K}, \boldsymbol{\Lambda}$ and $\boldsymbol{\sigma}_{\text {eff }}$ are independent on $x \quad\left(\sigma_{\text {eff }}=\pi \Lambda^{2} / \mathbf{K}^{2}\right)$, in two extreme cases
a) $K^{2}=\mathbf{1}$ and $\pi \Lambda^{2}=\sigma_{\text {eff }}:$ No correlation in multiplicity, $\sigma_{\text {eff }}$ gives the typical value of the transverse area where the DPS takes place
b) $\mathbf{K}^{\mathbf{2}}=\mathbf{2}$ and $\boldsymbol{\pi} \Lambda^{\mathbf{2}}=\mathbf{K}^{\mathbf{2}} \boldsymbol{\sigma}_{\text {eff }}$ : The observed value of $\boldsymbol{\sigma}_{\text {eff }}$ is completely due to the correlation in multiplicity

In both cases we have evaluated the cross sections with two different values for $\boldsymbol{\sigma}_{\text {eff }}$ :

$$
\sigma_{\text {eff }}=15 \mathrm{mb}(\text { ATLAS }) \text { and }
$$

$\sigma_{\text {eff }}=20.7 \mathbf{~ m b}$ (CMS)

## Expectations according with the simplest model

a) $K^{2}=\mathbf{1}$ and $\pi \Lambda^{2}=\sigma_{\text {eff }}$ (No correlation in multiplicity)

$$
\frac{\left.\sigma_{D}^{p A}\right|_{2}}{\left.\sigma_{D}^{p A}\right|_{1}} \approx 2 \quad \quad[200 \% \text { anti-shadowing corrections }]
$$

b) $\mathbf{K}^{\mathbf{2}}=\mathbf{2}$ and $\boldsymbol{\pi} \Lambda^{\mathbf{2}}=\mathbf{K}^{\mathbf{2}} \boldsymbol{\sigma}_{\text {eff }}$ (No correlations in the transverse coordinates)

$$
\frac{\left.\sigma_{D}^{p A}\right|_{2}}{\left.\sigma_{D}^{p A}\right|_{1}} \approx 3 \quad \quad[300 \% \text { anti-shadowing corrections }]
$$

while the amount of anti-shadowing changes only by about $6 \%$ when $\sigma_{\text {eff }}$ changes from 15 (ATLAS) to 20.7 mb (CMS)

A more detailed information from the transverse spectra:
$\mathrm{p}_{\mathrm{t}}$ spectrum of the leading jet in $p-p$ and in $p-P b$


The shape in $\mathrm{p}_{\mathrm{t}}$ of the leading jet is very different in $p-p$ and in $p-\mathrm{Pb}$ for $p_{\mathrm{t}}<40 \mathrm{GeV}$
$\mathrm{p}_{\mathrm{t}}$ spectrum of the W decay-lepton in $p-p$ and in $p-P b$


While the spectrum does not change much in $p-p$ collisions, the effect in $p-P b$ collisions is dramatic and one expects an increase of about $90 \%$ at $\mathrm{p}_{\mathrm{t}} \sim 40 \mathrm{GeV}$
$\mathrm{p}_{\mathrm{t}}$ spectrum of the W decay-lepton in $p-\mathrm{Pb}$ : dependence on K and on the measured value of $\sigma_{\text {eff }}$ in $p-p$ collisions


The spectra in $p-P b$ collisions do not change much, when $\sigma_{\text {eff }}$ increases from 15 to 20 mb in $p-p$ collisions. The effect of increasing $\mathrm{K}^{2}$ from 1 to 2 is on the contrary sizable and, in $p-P b$ collisions, one expects an increase from 60 to $90 \%$ of the observed cross section at $p_{t} \sim 40 \mathrm{GeV}$

## Concluding Summary

In the simpest model for DPS, not inconsistent with present experimental evidence, one considers only disconnected hard interactions and $\boldsymbol{\sigma}_{\text {eff }}$ does not dependent on fractional momenta.

In the model $\boldsymbol{\sigma}_{\text {eff }}$ is given by the ratio of the typical transverse interaction area $\left(\boldsymbol{\pi} \boldsymbol{\Lambda}^{\mathbf{2}}\right)$ and the multiplicity of parton pairs ( $\mathbf{K}^{\mathbf{2}}$ ). DPS in $p-p$ collisions can thus provide information only on the ratio between $\Lambda$ and $\mathbf{K}$.

In $p-A$ collisions the DPS interaction is simpler for processes which do not involve identical partons. We have thus studied in some detail the production of WJJ in $p-P b$ collisions.

In $p-A$ collisions the DPS cross section is characterized by a very strong antishadowing ( $\sim 2-300 \%$ positive correction term). The anti-shadowing correction term is a) proportional to the factor $\mathbf{K}$, which gives the multiplicity of parton pairs in the projectile proton and $\mathbf{b}$ ) it depends weakly on the partonic correlations in the transverse coordinates.

In particular the $p_{t}$ spectra of the leading jet and of the large $p_{t}$ lepton are characterized by the following peculiar features:

- the spectrum of the leading jet is expected to show an evident change of shape at $\mathrm{p}_{\mathrm{t}}<40 \mathrm{GeV}$;
- while the spectrum of the lepton should show a substantial increase at $p_{t} \sim 40$ GeV (namely at transverse momenta of about $1 / 2$ the mass of the W boson).

Once the expectations of the model were verified, both qualitatively and, at least to some extent, also quantitatively, one would have a reasonable argument to consider seriously the possibility of gaining reliable indications on the average number of pairs of partons in the proton, by measuring the amount of antishadowing in DPS in $p-P b$ collisions.

As soon as an estimate of $\mathbf{K}$ is available, one obtains $\Lambda$ from the measured value of $\sigma_{\text {eff }}$, acquiring in this way non trivial information on the 3D structure of the proton.

## Thank you



One has two different contributions to the forward scattering amplitude, in the case of two active target nucleons


In the case of two active target nucleons, when the two target partons are identical, in addition to the usually considered diagonal term one needs in fact to keep into account also the contribution of an interfrence term



A*

In the interference term the nucleon's fractional momenta are different in the right and in the left hand side of the cut: $Z-Z^{\prime}=x_{1}^{\prime}-x_{2}^{\prime}$ $\square$ the interference term is proprotional to the nuclear form factor, as a function of $Z-Z^{\prime}$
$x_{1}^{\prime}$ and $x^{\prime}{ }_{2}$ are measured in the final state. When $x_{1}{ }_{1}-x^{\prime}$, is large, the contribution of the interference term therefore is small. In many cases of interest the contribution of the interference term is however sizable.

The two interactions are localized in two points in transverse space. Given two interaction points the parton with fractional momentum $x_{1}$ may be provided by nucleon- 1 and the parton with fractional momentum $x^{\prime}{ }_{2}$ by nucleon-2 (configuration in $\mathbf{A}$ ) or vice-versa (configuration in $\mathbf{A}^{*}$ )

