Multiparticle Dynamics in 2D Flux Tube

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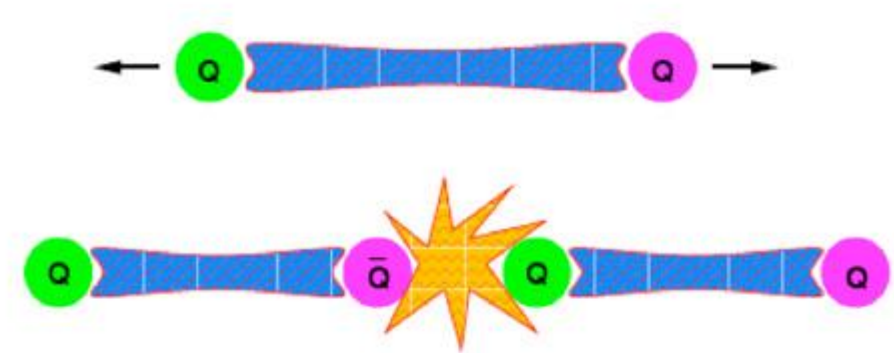
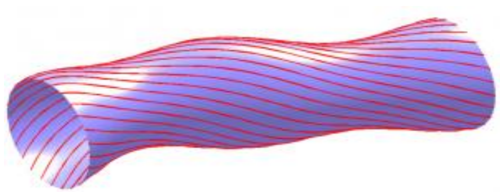
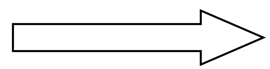
Contents

1.Introduction

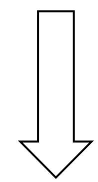


3D (3+1)

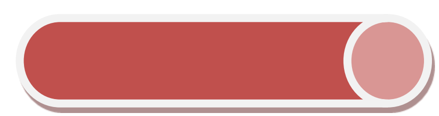




“3D” (3+1)



“3D” (2+ (1+1))



2. Compactification

(A.V.Koshelkin, C.Y.Wong, Phys.Rev. D86 (2012) 125026)

2.1. The main task

$$\mathcal{A}(4D)[\Psi(4D,x), F^a_{\mu\nu}(4D,x), g_{4D}] \Rrightarrow \mathcal{A}(2D)[\Psi(2D,x^0,x^3), F^a_{\mu\nu}(2D,x^0,x^3), g_{2D}] \qquad (1)$$

$$\mathcal{A}(4D) = \int d^4x \Bigg\{ Tr\Bigg[\frac{1}{2}\left[\bar{\Psi}(4D,x)\gamma^\mu(4D)\Pi_\mu(4D)\ \Psi(4D,x) - \bar{\psi}(4D,x)\ m(x)\ \Psi(4D,x)\right]$$
$$-\frac{1}{2}\left[\bar{\Psi}(4D,x)\gamma^\mu(4D)\overleftarrow{\Pi}_\mu(4D)\ \Psi(4D,x) + \bar{\Psi}(4D,x)\ m(x)\ \Psi(4D,x)\right]\Bigg] - \frac{1}{4}F^a_{\mu\nu}(4D,x)\ F^{\mu\nu}_a(4D,x)\Bigg\}. \qquad (2)$$

$$\Pi_\mu(4D) = i\partial_\mu + g_{4D}\ T_a A^a_\mu(4D,x) = p_\mu + g_{4D}\ T_a A^a_\mu(4D,x);$$

$$\overleftarrow{\Pi}_\mu(4D) = i\overleftarrow{\partial}_\mu - g_{4D}\ T_a A^a_\mu(4D,x) = \overleftarrow{p}_\mu - g_{4D}\ T_a A^a_\mu(4D,x);$$

$$F^a_{\mu\nu}(4D,x) = \partial_\mu A^a_\nu(4D,x) - \partial_\nu A^a_\mu(4D,x) + g_{4D}\ f^a_{bc}\ A^b_\mu(4D,x)\ A^c_\nu(4D,x) \qquad (3)$$

$$\equiv \partial_\mu A^a_\nu(4D,x) - \partial_\nu A^a_\mu(4D,x) - ig_{4D}[A^b_\mu(4D,x),\ A^c_\nu(4D,x)]^a.$$

2.2. The key approximation

The longitudinal dominance and transverse confinement

$$max\,\{|A_1^a|;|A_2^a|\} \ll min\,\{|A_0^a|;|A_3^a|\} \quad \Leftrightarrow \quad |A_1^a|,|A_2^a| \equiv 0$$

However!!!

$$A_0^a = A_0^a(x^0,\mathbf{x}),\;\; A_3^a = A_3^a(x^0,\mathbf{x})$$

The Lorentz Gauge

$$\partial_\mu A^\mu(x^0,\vec{x}) = 0$$

2.3.Compactification 4D -> 2D

We transform

(i) a fermion field

$$\Psi(4D,x) \equiv \begin{pmatrix} \varphi(x^0;\boldsymbol{x}) \\ \chi(x^0;\boldsymbol{x}) \end{pmatrix} \equiv \begin{pmatrix} \varphi_1(x^0;\boldsymbol{x}) \\ \varphi_2(x^0;\boldsymbol{x}) \\ \chi_1(x^0;\boldsymbol{x}) \\ \chi_2(x^0;\boldsymbol{x}) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} G_1(\boldsymbol{r}_\perp)\left(f_+(x^0;x^3)+f_-(x^0;x^3)\right) \\ -G_2(\boldsymbol{r}_\perp)\left(f_+(x^0;x^3)-f_-(x^0;x^3)\right) \\ G_1(\boldsymbol{r}_\perp)\left(f_+(x^0;x^3)-f_-(x^0;x^3)\right) \\ G_2(\boldsymbol{r}_\perp)\left(f_+(x^0;x^3)+f_-(x^0;x^3)\right) \end{pmatrix} , \quad (6)$$

$$\int dx^1 dx^2 \left(|G_1(\boldsymbol{r}_\perp)|^2 + |G_2(\boldsymbol{r}_\perp)|^2\right) = 1 \ . \quad (7)$$

(ii) a gauge field

$$A^a_\mu(4D,x^0,x^3,\boldsymbol{r}_\perp) = \sqrt{|G_1(\boldsymbol{r}_\perp)|^2 + |G_2(\boldsymbol{r}_\perp)|^2}\, A^a_\mu(2D,x^0,x^3) \ . \quad (8)$$

The coupling constant and gauge transformation become

$$g_{2D} = \int dx^1 dx^2 g_{4D}[|G_1(\boldsymbol{r}_\perp)|^2 + |G_2(\boldsymbol{r}_\perp)|^2]^{3/2} \quad (9)$$

$$\delta A^a_\mu(2D,x^0,x^3) = f^a_{\ bc}\varepsilon^b(x^0,x^3)A^c_\mu(2D,x^0,x^3) \quad (10)$$

2.4. 2D - action functional

$$\mathcal{A}(2D) = \int d^2X \left\{ Tr\left[\frac{1}{2}\left[\bar{\Psi}(2D,X)\gamma^k(2D)\Pi_k(2D)\Psi(2D,X) - \bar{\Psi}(2D,X)m_{qT}\Psi(2D,X)\right]\right.\right.$$
$$\left. -\frac{1}{2}\left[\bar{\Psi}(2D,X)\gamma^k(2D)\overleftarrow{\Pi}_k(2D)\ \Psi(2D,X) + \bar{\Psi}(2D,X)m_{qT}\Psi(2D,X)\right]\right]$$
$$\left. -\frac{1}{4}F^a_{\mu\nu}(2D)\ F^{\mu\nu}_a(2D) + \frac{1}{2}m^2_{gT}[A^\mu_a(2D)A^a_\mu(2D)]\right\}, \qquad (11)$$

where $\{\mu,\nu\}$=0,3, and

$$\Pi_\mu(2D) \ = \ i\partial_\mu + g_{2D}\ T_a A^a_\mu(2D,x) = p_\mu + g_{2D}\ T_a A^a_\mu(42,x),$$

$$\overleftarrow{\Pi}_\mu(2D) \ = \ i\overleftarrow{\partial}_\mu - g_{2D}\ T_a A^a_\mu(2D,x) = \overleftarrow{p}_\mu - g_{2D}\ T_a A^a_\mu(2D,x). \qquad (12)$$

$$\Psi(X) \equiv \Psi(2D,X) = \begin{pmatrix} f_+(X) \\ f_-(X) \end{pmatrix}, \qquad X = (x^0;x^3),$$

$$\gamma^0(2D) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^3(2D) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad g_{\mu\nu}(2D) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$m_{qT} = \int dx^1 dx^2 \left\{ m(\boldsymbol{r}_\perp)\left(|G_1(\boldsymbol{r}_\perp)|^2 - |G_2(\boldsymbol{r}_\perp)|^2\right) + \left(G_1^*(\boldsymbol{r}_\perp)(p_1 - ip_2)G_2(\boldsymbol{r}_\perp)\right) - \left(G_1(\boldsymbol{r}_\perp)(p_1 + ip_2)G_2^*(\boldsymbol{r}_\perp)\right)\right\} \qquad (13)$$

$$m^2_{gT} = \frac{1}{2}\int dx^1 dx^2 \left[\{\partial_1[|G_1(\boldsymbol{r}_\perp)|^2 + |G_2(\boldsymbol{r}_\perp)|^2]^{1/2}\}^2 + \{\partial_2[|G_1(\boldsymbol{r}_\perp)|^2 + |G_2(\boldsymbol{r}_\perp)|^2]^{1/2}\}^2\right] \qquad (14)$$

2.5. The Dirac fields in (1+1) space-time

$$\left\{i\gamma^{\mu}\left(\partial_{\mu}-ig_{2D}\cdot A_{\mu}^{a}(x)T_{a}\right)-m_{qT}\right\}\Psi(2D,X)=0,\quad \mu=0,3, \tag{15}$$

$$A_{\mu}^{a}(2D,X)=\left(A_{0}^{a}(X),-A_{3}^{a}(X)\right),\qquad X=(x^{0};x^{3}).$$

(i) Formal solution

$$\Psi(2D,X) = \int_{-\infty}^{+\infty}\frac{d\omega}{2\sqrt{L}}\sum_{p}\frac{m_{qT}}{\sqrt{\omega^{2}+p\omega}}\ \exp(-iP_{\mu}x^{\mu})\ a(p,\omega)\left[\delta(\omega-\varepsilon(p))+\delta(\omega+\varepsilon(p))\right]\ \begin{pmatrix}\frac{\omega+p}{m_{qT}}+1\\ \frac{\omega+p}{m_{qT}}-1\end{pmatrix} \tag{16}$$

$$\times\ \{T_{l(M_0;M)}\exp\}\left\{ig_{2D}T_{a}\int dx^{\mu}A_{\mu}^{a}\right\},$$

where $a(p,\omega)$ is an operator of creation of any particle (a particle or antiparticle).

(ii) Fermion current

$$J_a^\mu = g_{2D}\, Tr\left\{\bar{\Psi}(x)\gamma^\mu T_a \Psi(x')\right\}; \qquad x' \to x. \tag{17}$$

$$J_a^\mu(x) \; = \; \frac{g_{2D}^2}{2\pi}\; N_f \; \left[A_a^\mu - \partial^\mu \frac{1}{\partial^\lambda \partial_\lambda}\partial_\nu A_a^\nu\right] \tag{18}$$

Gauge transformation

$$\delta J_a^\mu = \varepsilon_b f_a^{bc} J_c^\mu \tag{19}$$

$$\delta A_a^\mu = \varepsilon_b f_a{}^{bc} A_c^\mu$$

$$\delta \mathcal{L}_{m_{gT}} \; = \; 0.$$

2.6. Equation of motion for the 2D Gauge Fields

(In the Lorentz gauge)

$$\mathcal{A}(2D) = \int d^2X \left\{ \frac{i}{2}\left[\bar{\Psi}(2D,X)\gamma^k(2D)\partial_k(2D)\Psi(2D,X) - \bar{\Psi}(2D,X)m_{qT}\Psi(X)\right] \right.$$
$$-\frac{i}{2}\left[\bar{\Psi}(2D,X)\gamma^k(2D)\overleftarrow{\partial}_k(2D)\,\Psi(2D,X) + \bar{\Psi}(2D,X)m_{qT}\Psi(X)\right] \qquad (20)$$
$$\left. -\frac{1}{4}F^a_{\mu\nu}(2D)\,F^{\mu\nu}_a(2D) + \frac{1}{2}M^2_{gT}A^a_\nu(2D)A^\nu_a(2D)\right\},$$

$$M^2_{gT} = \frac{1}{2}\int dx^1dx^2\left[\{\partial_1[|G_1(r_\perp)|^2 + |G_2(r_\perp)|^2]^{1/2}\}^2 + \{\partial_2[|G_1(r_\perp)|^2 + |G_2(r_\perp)|^2]^{1/2}\}^2\right] + \frac{g^2_{2D}N_f}{2\pi}$$
$$\equiv m^2_{gT} + m^2_{gfT} \geq 0.$$

(21)

(i) The motion equation

$$\Box\, A_a^\nu(2D, X) \;=\; M_{gT}^2\, A_a^\nu(2D, X) \qquad (22)$$

(ii) Its solution

$$A_a^\nu(2D, X) \;=\; \sum_k \frac{e_a^\nu\, M_{qT}}{\sqrt{\left(k^2 + M_{qT}^2\right)^3}} \left\{ \exp(-ikX)\, b_a(k, \nu) \; + \exp(+ikX)\, \bar{b}_a^\dagger(k, \nu) \right\} \qquad (23)$$

$$k^\mu = (k^0; \boldsymbol{k}), \qquad E(k) \equiv k^0 = +\sqrt{\boldsymbol{k}^2 + M_{qT}^2}.$$

(iii) Colorless particles

$$A_{\text{color-singlet}}^\nu = \frac{1}{\sqrt{8}} \sum_a A_a^\mu |8, a\rangle \qquad (24)$$

$$\Box\, A_{\text{color-singlet}}^\nu(2D, X) \;=\; M_{gT}^2\, A_{\text{color-singlet}}^\nu(2D, X)$$

2.7. Transverse motion in a tube

$$\mathcal{F} = \mathcal{A}(4D) + \frac{\lambda}{2} \int dx^1 dx^2 \left(|G_1(\vec{r}_\perp)|^2 + |G_2(\vec{r}_\perp)|^2\right) \int dx^0 dx^3 \left(\bar{\psi}(2D,X)\, \psi(2D,X)\right) \tag{25}$$

$$\begin{aligned}
(p_1 + ip_2)G_1(\vec{r}_\perp) &= (m(\vec{r}_\perp) + \lambda)G_2(\vec{r}_\perp),\\
(p_1 - ip_2)G_2(\vec{r}_\perp) &= (\lambda - m(\vec{r}_\perp))G_1(\vec{r}),\\
(p_1 + ip_2)G_2^*(\vec{r}_\perp) &= (m(\vec{r}_\perp) - \lambda)G_1^*(\vec{r}_\perp),\\
(p_1 - ip_2)G_1^*(\vec{r}_\perp) &= -(m(\vec{r}_\perp) + \lambda)G_2^*(\vec{r}).
\end{aligned} \tag{26}$$

$$\lambda = \lambda^* = m_q T \quad (!) \tag{27}$$

3.The confinement potential

$$m(\vec{r}_\perp) = \kappa r_\perp\,, \quad \kappa > 0$$

$$\begin{pmatrix} H_0 & -i\kappa e^{-i\varphi} \\ +i\kappa e^{+i\varphi} & H_0 \end{pmatrix} \begin{pmatrix} G_+(r_\perp) \\ G_-(r_\perp) \end{pmatrix} = \lambda^2 \begin{pmatrix} G_+(r_\perp) \\ G_-(r_\perp) \end{pmatrix}$$

$$HG = \lambda^2 G,$$

$$G_1 \equiv G_+ \Rightarrow \uparrow,\ G_2 \equiv G_- \Rightarrow \downarrow$$

$$J,\quad \sigma = \pm 1/2$$

3.1. J=0, the ground state

$$\xi = \kappa r_\perp^2$$

$$\lambda_{n+1/2}^2 = 4\kappa\ (n+1/2)\,; \qquad n = 0,1,2,\dot{}$$

$$G_- = \frac{1}{2}\sqrt{\frac{2\kappa n!}{\Gamma(n+3/2)}}\ \frac{e^{(-i\varphi/2)}}{\sqrt{2\pi}}\ \xi^{1/4}e^{-\xi/2}\left(L_n^{1/2}(\xi) + \frac{\sqrt{n+1/2}}{\xi^{1/2}}L_n^{-1/2}(\xi)\right)$$

$$G_+ = \frac{1}{2i}\sqrt{\frac{2\kappa n!}{\Gamma(n+3/2)}}\ \frac{e^{(+i\varphi/2)}}{\sqrt{2\pi}}\ \xi^{1/4}e^{-\xi/2}\left(L_n^{1/2}(\xi) - \frac{\sqrt{n+1/2}}{\xi^{1/2}}L_n^{-1/2}(\xi)\right)$$

$$\lambda_n^2 = 4\kappa\ (n+1);\quad n = 0,1,2,\ldots$$

$$G_- = \frac{1}{2}\ \sqrt{\frac{2\kappa n_r!}{\Gamma(n_r+3/2)}}\ \frac{e^{(-i\varphi/2)}}{\sqrt{2\pi}}\ \xi^{1/4}e^{-\xi/2}\left(L_n^{1/2}(\xi) - \frac{\sqrt{n+1}}{\xi^{1/2}}L_{n+1}^{-1/2}(\xi)\right)$$

$$G_+ = -\frac{1}{2i}\ \sqrt{\frac{2\kappa n_r!}{\Gamma(n_r+3/2)}}\ \frac{e^{(+i\varphi/2)}}{\sqrt{2\pi}}\ \xi^{1/4}e^{-\xi/2}\left(L_n^{1/2}(\xi) + \frac{\sqrt{n+1}}{\xi^{1/2}}L_{n+1}^{-1/2}(\xi)\right)$$

3.2.Spectra ($\lambda^2_{J,n} / \kappa$)

$$(J=0)$$

$n_r=7/2$

$n_r=5/2$

$n_r=3/2$

$n_r=1/2$

$n_r = 3$

$n_r = 2$

$n_r = 1$

3.1.Masses

$$m_{gT}^2 \approx 0.3 \frac{4\kappa}{\sqrt{\pi}} \approx 0.7\kappa.$$

$$g_{2D} \approx \frac{1.92 g \kappa^{1/2}}{\pi^{5/4}} \approx 0.46 g \kappa^{1/2}, \qquad m_{gf,T}^2 \approx 3.4 \cdot 10^{-2} g^2 \kappa N_f.$$

$$M_{gT} = \sqrt{m_{gT}^2 + m_{gf\,T}^2} \approx 0.8\kappa \sqrt{1 + 5 \cdot 10^{-2} g^2 N_f}.$$

$$(|J| \neq 0 - numerical\ \ calculations)$$

$N = 3 + |\Lambda|/2$

$N = 2 + |\Lambda|/2$

$N = 1 + |\Lambda|/2$

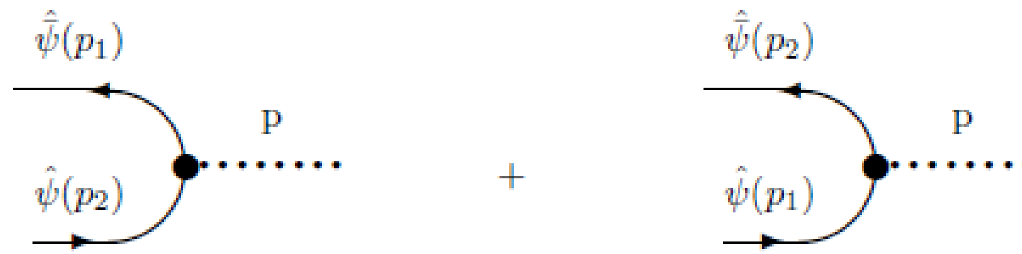
$$g_{2D} 0.45 \approx 0.286 g\kappa^{1/2}$$
$$m^2_{gf,T} \approx 1.3 \cdot 10^{-2} g^2 \kappa N_f$$
$$M^2_T \approx 0.8\kappa^{1/2}\sqrt{1 + 1.7 \cdot 10^{-2} g^2 N_f}$$

4. p_T - distribution

$$dP(\vec{p}) = (2\pi)^4 < |M(p_1, p_2; p)|^2 > \delta^{(4)}(p - p_1 - p_2) \frac{d^3 p}{2E(p)\ (2\pi)^3} \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3},$$

where

$$-M(p_1, p_2; p) \quad =$$



$$\frac{2E(p) dP(\vec{p}_\perp, p_z)}{d^3 p_\perp} \cong f_{qq\pi} \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \delta^{(3)}(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

$$\sum_{\nu,\mu} \left(|\psi_\nu(\vec{p}_1)|^2\ |\bar{\psi}_\mu(\vec{p}_2)|^2 n_\nu(\vec{p}_1) \bar{n}_\mu(\vec{p}_2) + |\psi_\mu(\vec{p}_1)|^2\ |\bar{\psi}_\nu(\vec{p}_2)|^2 n_\mu(\vec{p}_1) \bar{n}_\nu(\vec{p}_2) \right)$$

$$\frac{E(p)dP(\vec{p}_\perp, p_z)}{d^3p} \cong \frac{f_{qq\pi}}{2\pi^{1/2}\kappa^{3/2}(\pi)^6}\left(\int\limits_0^{p_\perp} p_{2\perp}dp_{2\perp}\int\limits_{p_\perp - p_{2\perp}}^{p_\perp + p_{2\perp}} p_{1\perp}dp_{1\perp} + \int\limits_{p_\perp}^{+\infty} p_{2\perp}dp_{2\perp}\int\limits_{p_{2\perp}-p_\perp}^{p_{2\perp}+p_\perp} p_{1\perp}dp_{1\perp}\right)\cdot$$

$$\frac{1}{\sqrt{4p_\perp^2 p_{1\perp}^2 - (p_\perp^2 + p_{1\perp}^2 - p_{2\perp}^2)^2}}\sum_{s_1=0}^{\infty}(-1)^{s_1}\int\limits_0^\infty dt_1\, e^{-\frac{(1+s_1)^2\kappa}{t_1^2T^2}}\ \frac{e^{-(p_{1\perp}^2/\kappa)\tanh(t_1^2/4)}}{\sinh(t_1^2/2)}\left(\sqrt{\frac{2}{\pi z_1}}e^{-2z_1} + erf(\sqrt{2z_1})\right)$$

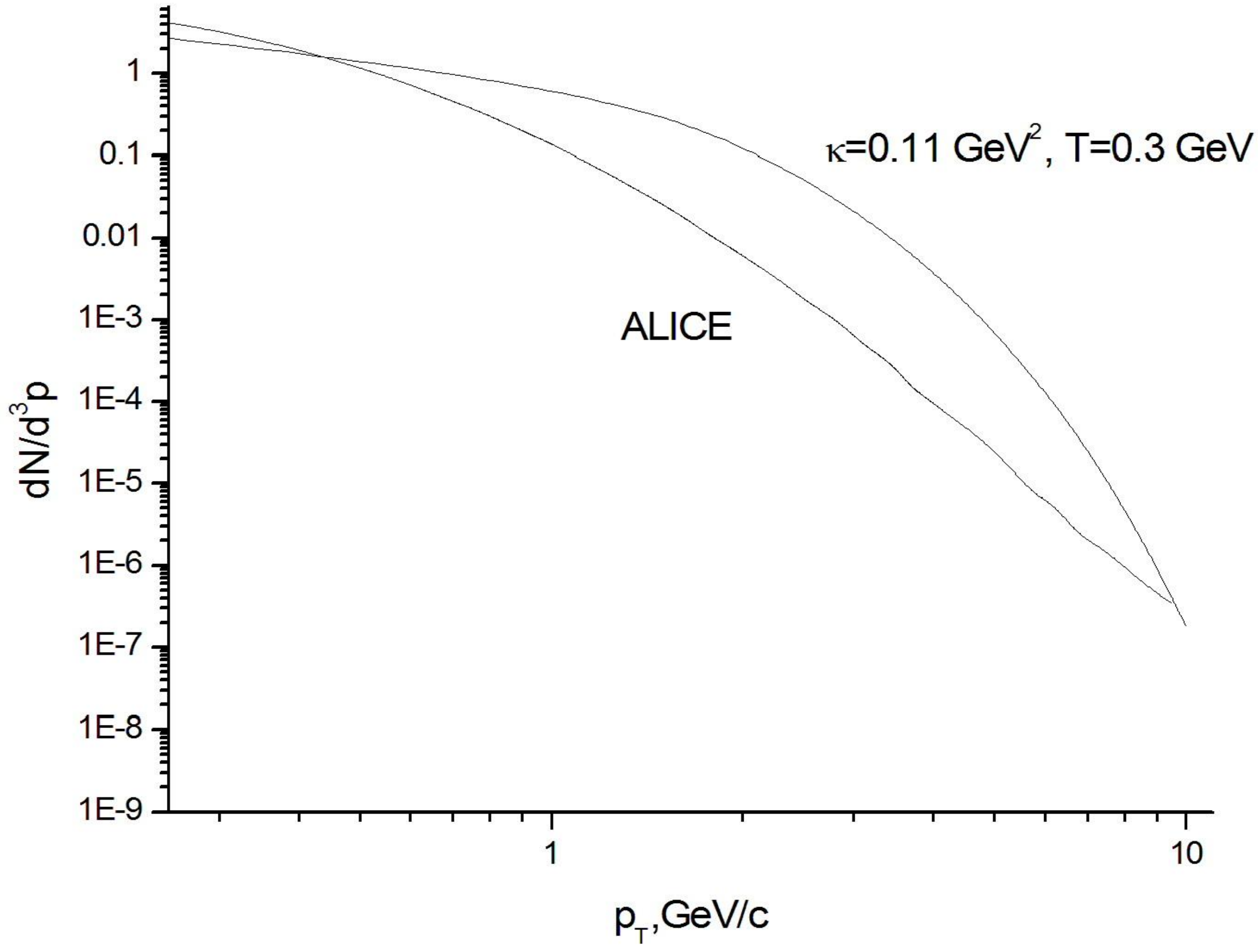
$$\sum_{s_2=0}^{\infty}(-1)^{s_2}\int\limits_0^\infty dt_2\, e^{-\frac{(1+s_2)^2\kappa}{t_2^2T^2}}\ \frac{e^{-(p_{2\perp}^2/\kappa)\tanh(t_2^2/4)}}{\sinh(t_2^2/2)}\left(\sqrt{\frac{2}{\pi z_2}}e^{-2z_2} + erf(\sqrt{2z_2})\right)\frac{e^{-\frac{p_z t_1^2 t_1^2}{4\kappa(t_1^2+t_2^2)^2}}}{(t_1^2+t_2^2)^{1/2}};$$

$$z_i = \frac{p_{i\perp}^2}{\kappa\sinh(t_i^2/2)}$$

5.Comparison with experiments

The ALICE experiment on p-p collisions – PLB.**693**, p53 (2010).

$$\sqrt{s} = 0.9\ TeV$$


κ=0.11 GeV², T=0.3 GeV
ALICE
dN/d³p
1
0.1
0.01
1E-3
1E-4
1E-5
1E-6
1E-7
1E-8
1E-9
1
10
p_T,GeV/c

6.Conclusion

1. The 4D -> 2D$ compactification is realized in terms of the action principle in the SU(N) gauge field theory.
2. The exact gauge invariant 2D-Lagrangian is derived. On a basis of the obtained Lagrangian, the gauge invariant fermion current and the transverse mass of a fermion are calculated.
3. The derived current is found to be proportional to the amplitude of the gauge field,similar to the case of 2D QED (J.Schwinger,1962).
4. The obtained mass for SU(N) field theory depends strongly on the 2D coupling constant, number of flavors, as well as on the transverse mass of the fermion.
5. The results allow us to calculate p_T - distribution of hadrons produced in collisions of heavy ions of high energies.

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